

Intro to Programming for Public Policy Week 4 Simulation

Eric Potash

April 19, 2018

Randomness

Random numbers

In statistics we sample a random variable X with, say, a Bernoulli distribution with $p = .5$ representing a fair coin flip:

$$P(X = 0) = .5$$

$$P(X = 1) = .5$$

Random number generator

A random number generator is a computer program that that samples from a probability distribution.

NumPy library

- ▶ NumPy (short for Numerical Python) is a numerical module for python. It provides many useful features, including random number generation.

NumPy library

- ▶ NumPy (short for Numerical Python) is a numerical module for python. It provides many useful features, including random number generation.
- ▶ To use NumPy we import it. While we could simply do

```
import numpy
```

NumPy library

- ▶ NumPy (short for Numerical Python) is a numerical module for python. It provides many useful features, including random number generation.
- ▶ To use NumPy we import it. While we could simply do

```
import numpy
```

- ▶ It is common practice to use

```
import numpy as np
```

Random [0,1]

To generate a random float between 0 and 1 use
`np.random.random()`:

```
>>> import numpy as np
>>> np.random.random()
0.45986965699341753
```


Multiple draws

```
>>> import numpy as np
>>> np.random.random()
0.45986965699341753
>>> np.random.random()
0.5894272144763195
>>> np.random.random()
0.32955445470728895
```

Random Bernoulli

- We can transform a random number from 0 to 1 into a Bernoulli distribution with $p = .5$:

```
x = np.random.random()
if x > .5:
    y = 1
else:
    y = 0
```

Random Bernoulli

- ▶ We can transform a random number from 0 to 1 into a Bernoulli distribution with $p = .5$:

```
x = np.random.random()
if x > .5:
    y = 1
else:
    y = 0
```

- ▶ Technically you could apply other transformations to `np.random.random()` to sample from *any* distribution.

Random Bernoulli

- ▶ We can transform a random number from 0 to 1 into a Bernoulli distribution with $p = .5$:

```
x = np.random.random()
if x > .5:
    y = 1
else:
    y = 0
```

- ▶ Technically you could apply other transformations to `np.random.random()` to sample from *any* distribution.
- ▶ But in practice we use specialized NumPy functions for each distribution:

```
np.random.binomial(1, .5)
```

Random integer

```
>>> np.random.randint(0, 10)  
3
```

The parameters to `randint` are low (inclusive) and high (exclusive).

Random normal

- ▶ By default `np.random.normal` is the standard (mean 0, variance 1) normal:

```
>>> np.random.normal()  
-0.27054108268825433
```

Random normal

- ▶ By default `np.random.normal` is the standard (mean 0, variance 1) normal:

```
>>> np.random.normal()  
-0.27054108268825433
```

- ▶ You can pass a location (mean) and shape (standard deviation) parameter:

```
>>> np.random.normal(10,2)  
11.88791395360905
```

Random permutation

```
>>> np.random.permutation(10)  
array([4, 8, 5, 1, 7, 9, 2, 3, 0, 6])
```


Random seed

- Sometimes it can be useful for the “random” numbers to be deterministic. You can achieve that by setting the random seed:

```
>>> np.random.seed(42)
>>> np.random.random()
0.3745401188473625
>>> np.random.random()
0.9507143064099162
>>> np.random.random()
0.7319939418114051
```

Random seed

- Sometimes it can be useful for the “random” numbers to be deterministic. You can achieve that by setting the random seed:

```
>>> np.random.seed(42)
>>> np.random.random()
0.3745401188473625
>>> np.random.random()
0.9507143064099162
>>> np.random.random()
0.7319939418114051
```

- Every time you run this code it will produce the same “random” numbers

Plotting

Matplotlib

- ▶ Matplotlib is a python module for plotting
- ▶ It has a *submodule* called `pyp1ot` that provides an (relatively) easy interface to the most common plotting functions

Plot a sequence

- ▶ You can plot the sequence [1,2,3,4]:

```
>>> import matplotlib.pyplot as plt  
>>> plt.plot([1,2,3,4])
```

show

To actually see what you've plotted use the `show` function:

```
>>> import matplotlib.pyplot as plt  
>>> plt.plot([1,2,4,8])  
>>> plt.show()
```

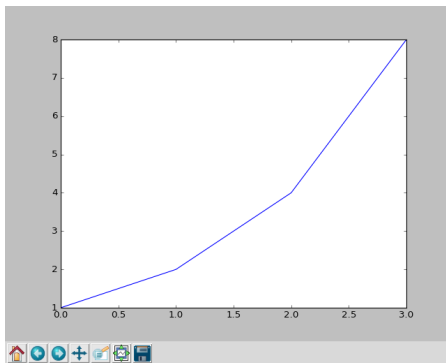


Figure 1: Show plot

savefig

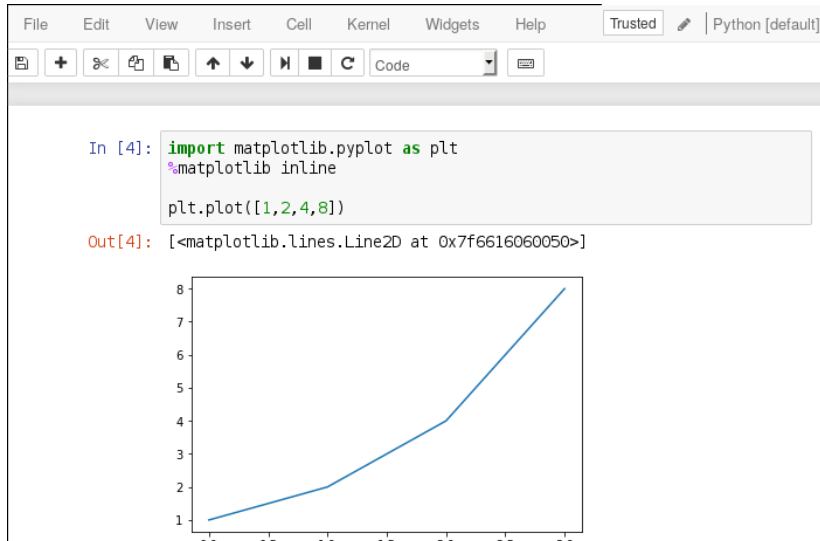
Alternatively, save the figure to a file:

```
>>> import matplotlib.pyplot as plt  
>>> plt.plot([1,2,4,8])  
>>> plt.savefig('plot.png')
```

Matplotlib will determine which format to use from the file extension (png, pdf, etc.)

Jupyter

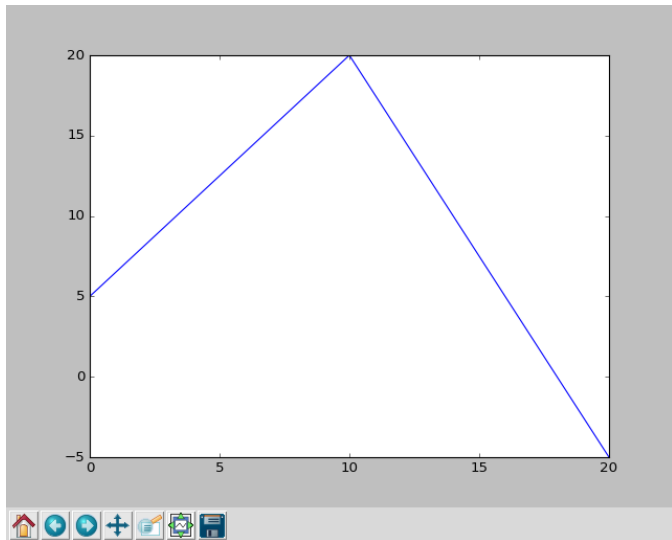
Use the “magic” command `%matplotlib inline` to simply have your plots appear inline in a Jupyter notebook:



Line plot

With two arguments, you can specify the x and y coordinates:

```
plt.plot([0,10,20], [5, 20, -5])
```



Formatting

- ▶ You can remove the lines and only show points using a format parameter which is a string.
- ▶ The default format string is 'b-', in which the 'b' is the color blue and the '-' represents a line.
- ▶ We can instead plot red circles using the format string 'ro'
- ▶ See `plot` documentation for more on formatting.

Scatter plot

```
plt.plot([0,2,4,6,8], [1,2,3,4,3], 'ro')
```

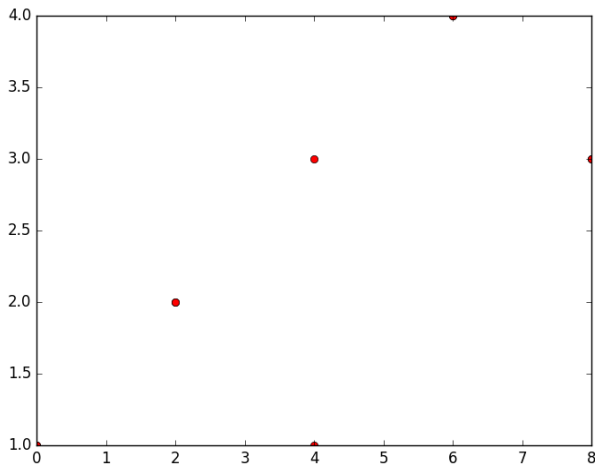


Figure 4: Scatter plot

Multiple plots

Multiple plots (before calling `show()` or in the same cell in Jupyter) will appear together. To clear the figure use `plt.clf()`.

```
plt.plot([0,10,20], [5, 20, -5])  
plt.plot([0,2,4,6,8], [1,2,3,4,3], 'ro')
```

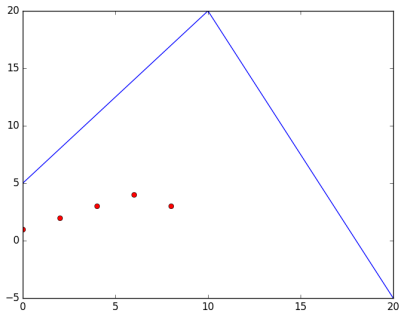
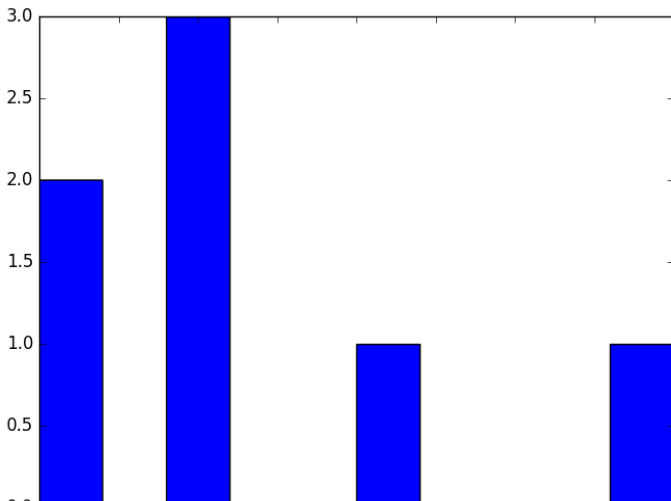


Figure 5: Multiple plots

Histogram

You can plot a histogram of a list of values:

```
plt.hist([0,1,1,0,1,2,4])
```



Probability distribution

```
xs = []  
for i in range(100):  
    xs.append(np.random.normal())  
  
plt.hist(xs)
```

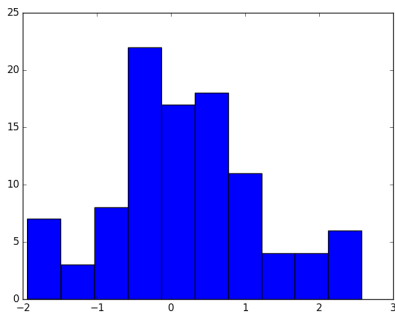


Figure 7: Normal histogram

More matplotlib

Next week we'll see more options including axis labels, titles, etc.

Simulation

Linear model

Given a slope β , consider the *data generating process*

$$y = \beta x + \epsilon$$

where ϵ is a standard normal error term:

$$\epsilon \sim \text{Normal}(0, 1)$$

Sampling

Given a point x , we can sample y by first sampling a random ϵ and then adding it to βx .

```
beta = 2
x = 1

epsilon = np.random.normal()
y = beta*x + epsilon
```

Sampling x

We can also sample x . For example we can make x uniform over some interval $[a, b]$:

```
beta = 2
a = 0
b = 10

x = np.random.uniform(a, b)

epsilon = np.random.normal()
y = beta*x + epsilon
```

Many samples

We can get many samples in a loop:

```
beta = 2
a = 0
b = 10

xs = []
ys = []

for i in range(1000):
    x = np.random.uniform(a, b)
    epsilon = np.random.normal(0, 3)
    y = beta*x + epsilon

    xs.append(x)
    ys.append(y)

plt.plot(xs, ys, 'ro')
```

Regression samples plot

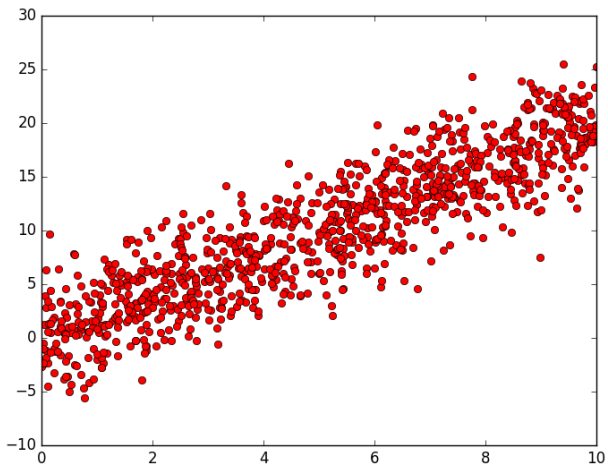


Figure 8: 1000 Samples from regression model with $\beta = 2$, $\sigma = 3$

Covariance

- ▶ We can use `np.cov` to calculate the covariance matrix for a sample of `xs` and `ys`:

```
>>> cov = np.cov(xs, ys)
>>> cov
array([[ 8.03709644, 15.72876449],
       [15.72876449, 31.79094298]])
```

- ▶ It's a 2d array (list):
 - ▶ `cov[0][0]` is the *variance* of `xs`
 - ▶ `cov[1][1]` is the variance of `ys`
 - ▶ `cov[0][1] == cov[1][0]` is their covariance

Regression

- ▶ Recall that the regression of y on x estimates

$$\hat{\beta} = \text{Cov}(x, y) / \text{Var}(x)$$

- ▶ So in code we can write

```
>>> beta_hat = cov[0][1] / cov[0][0]
>>> beta_hat
2.0235487499138292
```

Sample size

We can repeat this over subsets of the data:

```
beta_hats = []  
for i in range(1,1001):  
    cov = np.cov(xs[:i+1], ys[:i+1])  
    beta_hat = cov[0][1] / cov[0][0]  
    beta_hats.append(beta_hat)
```


Plot estimates

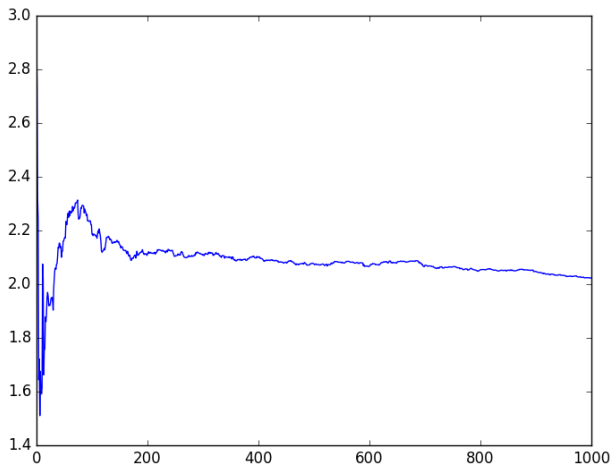


Figure 9: $\hat{\beta}$ as a function of sample size when $\beta = 2$, $\sigma = 3$