Intro to Programming for Public Policy Week 4 Simulation and Plotting

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Randomness

Random numbers

In statistics we sample a random variable X with, say, a Bernoulli distribution with $p = .5$ representing a fair coin flip:

$$P(X = 0) = .5$$
$$P(X = 1) = .5$$

Random number generator

A random number generator is a computer program that samples from a probability distribution.

NumPy library

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```
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```

- It is common practice to use

```
import numpy as np
```

Random [0,1]

To generate a random `float` between 0 and 1 use `np.random.random()`:

```
>>> import numpy as np
>>> np.random.random()
0.45986965699341753
```

Multiple draws

```
>>> import numpy as np
>>> np.random.random()
0.45986965699341753
>>> np.random.random()
0.5894272144763195
>>> np.random.random()
0.32955445470728895
```

Random Bernoulli

- We can transform a random number from 0 to 1 into a Bernoulli distribution with $p = .5$:

```
x = np.random.random()
if x > .5:
    y = 1
else:
    y = 0
```

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- Technically you could apply other transformations to `np.random.random()` to sample from *any* distribution.
- But in practice we use specialized NumPy functions for each distribution:

```
np.random.binomial(1, .5)
```

Random integer

```
>>> np.random.randint(0, 10)
3
```

The parameters to `randint` are `low` (inclusive) and `high` (exclusive).

Random normal

- By default `np.random.normal` is the standard (mean 0, variance 1) normal:

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- You can pass a `location` (mean) and `shape` (standard deviation) parameter:

```
>>> np.random.normal(10,2)
11.88791395360905
```

Random permutation

```
>>> np.random.permutation(10)
array([4, 8, 5, 1, 7, 9, 2, 3, 0, 6])
```

Random seed

- Sometimes it can be useful for the "random" numbers to be deterministic. You can achieve that by setting the random seed:

```
>>> np.random.seed(42)
>>> np.random.random()
0.3745401188473625
>>> np.random.random()
0.9507143064099162
>>> np.random.random()
0.7319939418114051
```

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```

- Every time you run this code it will produce the same “random” numbers

Plotting

Matplotlib

- Matplotlib is a python module for plotting
- It has a *submodule* called `pyplot` that provides an (relatively) easy interface to the most common plotting functions

Plot a sequence

- You can plot the sequence `[1,2,3,4]`:

```
>>> import matplotlib.pyplot as plt
>>> plt.plot([1,2,3,4])
```

show

To actually see what you've plotted use the show function:

```
>>> import matplotlib.pyplot as plt
>>> plt.plot([1,2,4,8])
>>> plt.show()
```

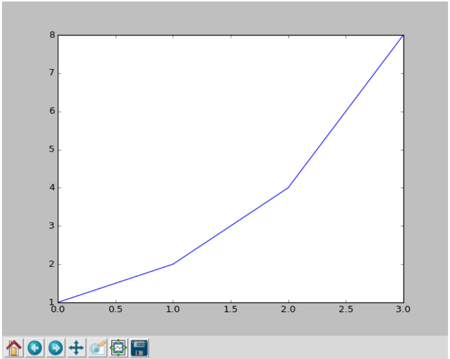


Figure 1: Show plot

savefig

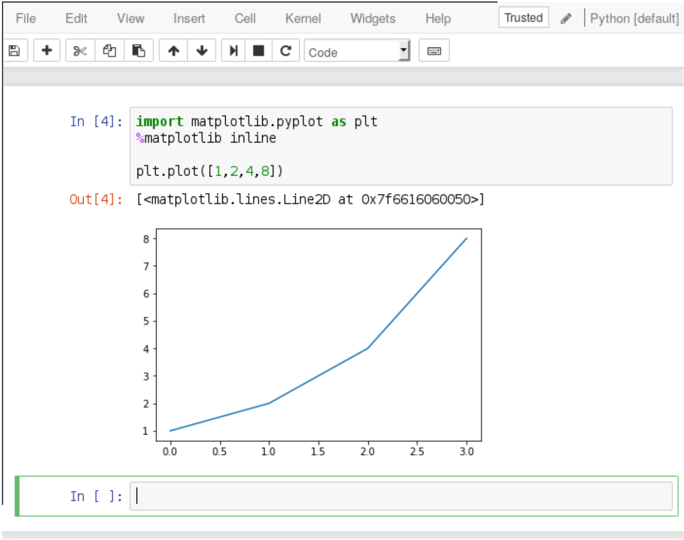
Alternatively, save the figure to a file:

```
>>> import matplotlib.pyplot as plt
>>> plt.plot([1,2,4,8])
>>> plt.savefig('plot.png')
```

Matplotlib will determine which format to use from the file extension (`png`, `pdf`, etc.)

Jupyter

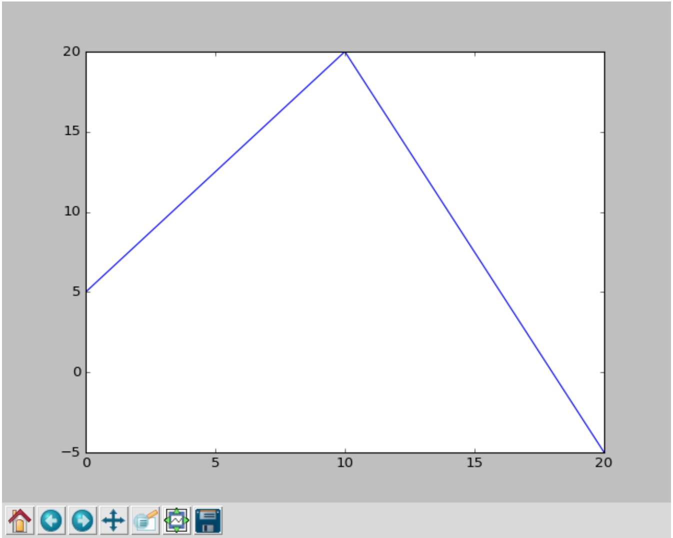
Use the “magic” command %matplotlib inline to simply have your plots appear inline in a Jupyter notebook:



Line plot

With two arguments, you can specify the x and y coordinates:

```
plt.plot([0,10,20], [5, 20, -5])
```



Formatting

- You can remove the lines and only show points using a format parameter which is a string.
- The default format string is `'b-'`, in which the `'b'` is the color blue and the `'-'` represents a line.
- We can instead plot red circles using the format string `'ro'`
- See `plot` documentation for more on formatting.

Scatter plot

```
plt.plot([0,2,4,6,8], [1,2,3,4,3], 'ro')
```

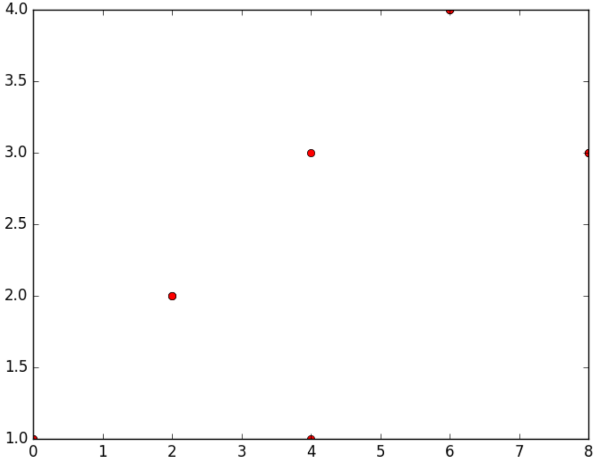


Figure 4: Scatter plot

Multiple plots

Multiple plots (before calling `show()` or in the same cell in Jupyter) will appear together. To clear the figure use `plt.clf()`.

```
plt.plot([0,10,20], [5, 20, -5])
plt.plot([0,2,4,6,8], [1,2,3,4,3], 'ro')
```

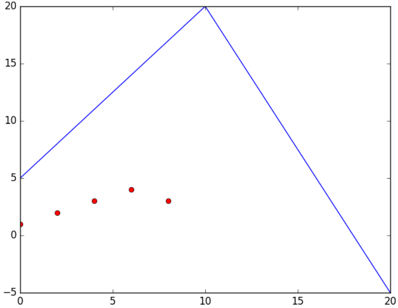
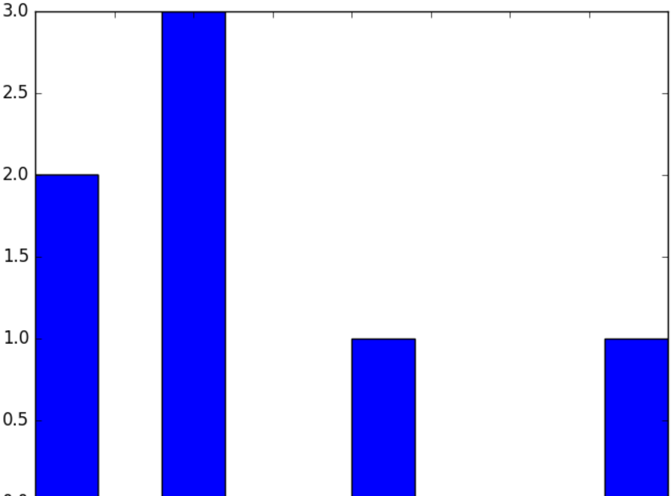


Figure 5: Multiple plots

Histogram

You can plot a histogram of a list of values:

```
plt.hist([0,1,1,0,1,2,4])
```



Probability distribution

```
xs = []
for i in range(100):
    xs.append(np.random.normal())

plt.hist(xs)
```

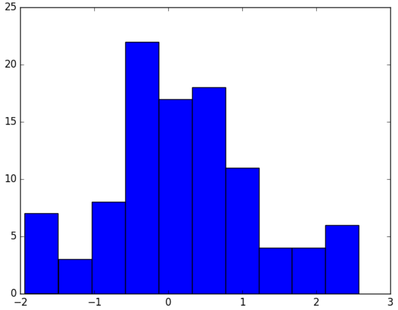


Figure 7: Normal histogram

More matplotlib

Next week we'll see more options including axis labels, titles, etc.

Simulation

Linear model

Given a slope β, consider the *data generating process*

$$y = \beta x + \epsilon$$

where ϵ is a standard normal error term:

$$\epsilon \sim Normal(0, 1)$$

Sampling

Given a point x, we can sample y by first sampling a random ϵ and then adding it to βx.

```
beta = 2
x = 1

epsilon = np.random.normal()
y = beta*x + epsilon
```

Sampling x

We can also sample x. For example we can make x uniform over some interval $[a, b]$:

```
beta = 2
a = 0
b = 10

x = np.random.uniform(a, b)

epsilon = np.random.normal()
y = beta*x + epsilon
```

Many samples

We can get many samples in a loop:

```
beta = 2
a = 0
b = 10

xs = []
ys = []

for i in range(1000):
    x = np.random.uniform(a, b)
    epsilon = np.random.normal(0, 3)
    y = beta*x + epsilon

    xs.append(x)
    ys.append(y)

plt.plot(xs, ys, 'ro')
```

Regression samples plot

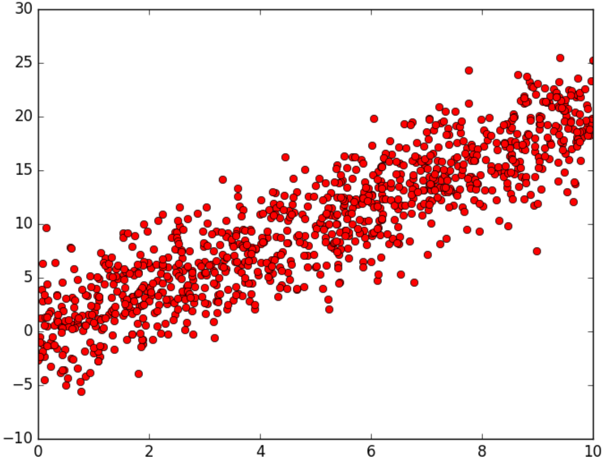


Figure 8: 1000 Samples from regression model with $\beta = 2$, $\sigma = 3$

Covariance

- We can use `np.cov` to calculate the covariance matrix for a sample of `xs` and `ys`:

```
>>> cov = np.cov(xs, ys)
>>> cov
array([[  8.03709644,  15.72876449],
       [ 15.72876449,  31.79094298]])
```

- It's a 2d array (list):
  - `cov[0][0]` is the *variance* of `xs`
  - `cov[1][1]` is the variance of `ys`
  - `cov[0][1] == cov[1][0]` is their covariance

Regression

- Recall that the regression of y on x estimates

$$\hat{\beta} = \frac{Cov(x, y)}{Var(x)}$$

- So in code we can write

```
>>> beta_hat = cov[0][1] / cov[0][0]
>>> beta_hat
2.0235487499138292
```

Sample size

We can repeat this over subsets of the data:

```
beta_hats = []
for i in range(1,1001):
    cov = np.cov(xs[:i+1], ys[:i+1])
    beta_hat = cov[0][1] / cov[0][0]
    beta_hats.append(beta_hat)
```

Plot estimates

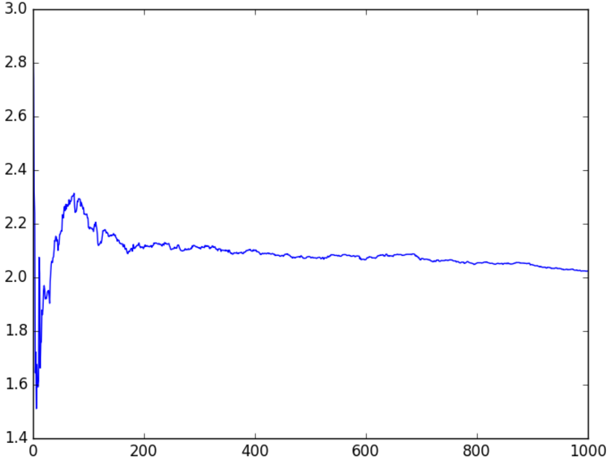


Figure 9: $\hat{\beta}$ as a function of sample size when $\beta = 2$, $\sigma = 3$