INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD

COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology, Palakkad
CS5016: Computational Methods and Applications

23 Jan, 2023

Coding Assignment 2
Networks, Random Graphs and Percolation

Max points: 150

A few instructions

- Codes should be compatible with *Python3* and should run on Ubuntu.
- Code for each question should be placed in a separate stand-alone files.
- Codes should be well-commented.

1. Create a class `UndirectedGraph` that represents an *undirected graph* such that [40]

- The graph should be stored in its adjacency list representation.
- It should be possible to create an undirected graph with a fixed number of vertices/nodes indexed as $\{1, 2, \ldots, n\}$ or a free graph (without any pre-defined number of vertices).

```
# The following code should create a free graph
g = UndirectedGraph()
```

```
# The following code should create a graph with 10 vertices
g = UndirectedGraph(10)
```

- It should be possible to add a node to the graph. If the graph is not free, an exception should be raised if index (only a positive integer should be allowed) of the node is larger than number of nodes in the graph.

```
g = UndirectedGraph()
g.addNode(1)
g.addNode(100)
```

```
g = UndirectedGraph(10)
g.addNode(4)
```

```
g = UndirectedGraph(10)
g.addNode(11)
```

Expected output:

Question 1 continues on the next page. . .

```
<class 'Exception'>
Node index cannot exceed number of nodes
```

- It should be possible to add undirected edges to the graph.

  NOTE: Adding an edge (a, b) to the graph implies that nodes a and b are also added to the graph.

```python
# The following code adds an undirected edge (10,25) to the graph
g = UndirectedGraph()
g.addEdge(10, 25)
```

```python
# The following code adds an undirected edge (1,2) to the graph
g = UndirectedGraph(10)
g.addEdge(1,2)
```

- Overload the + operator so that it will be possible to add nodes and edges to the graph using this operator.

```python
# The following code adds node 10 to the graph
g = UndirectedGraph()
g = g + 10
```

```python
# The following code adds an undirected edge (12,15) to the graph
g = UndirectedGraph()
g = g + (12, 15)
```

```python
# The following code adds undirected edges (1,2) and (3,4) to the graph
g = UndirectedGraph(10)
g = g + (1, 2)
g = g + (3, 4)
```

- It should be possible to `print` an object of type `UndirectedGraph`.

```python
g = UndirectedGraph()
print(g)
```

Expected output:

```
Graph with 0 nodes and 0 edges. Neighbours of the nodes are belows:
```

```python
g = UndirectedGraph(5)
print(g)
```

Question 1 continues on the next page...

Expected output:

```
Graph with 5 nodes and 0 edges. Neighbours of the nodes are belows:
Node 1: {}
Node 2: {}
Node 3: {}
Node 4: {}
Node 5: {}
```

```
g = UndirectedGraph()
g = g + 10
g = g + (11, 12)
print(g)
```

Expected output:

```
Graph with 3 nodes and 1 edges. Neighbours of the nodes are belows:
Node 10: {}
Node 11: {12}
Node 12: {11}
```

```
g = UndirectedGraph(5)
g = g + (1, 2)
g = g + (3, 4)
g = g + (1, 4)
print(g)
```

Expected output:

```
Graph with 5 nodes and 3 edges. Neighbours of the nodes are belows:
Node 1: {2, 4}
Node 2: {1}
Node 3: {4}
Node 4: {1, 3}
Node 5: {}
```

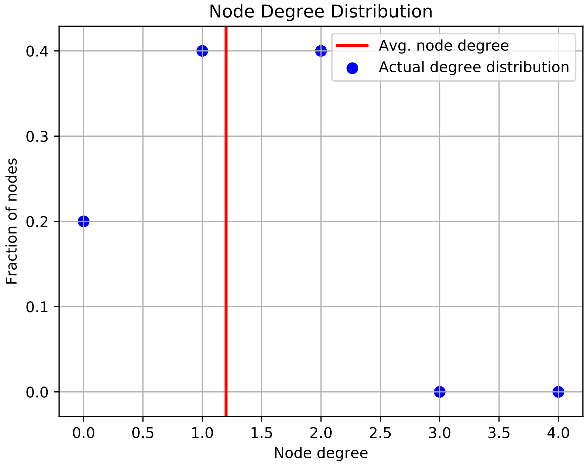
- It should be possible to plot the degree distribution[1] of a graph.

```
g = UndirectedGraph(5)
g = g + (1, 2)
g = g + (3, 4)
g = g + (1, 4)
g.plotDegDist()
```

[1] https://en.wikipedia.org/wiki/Degree_distribution

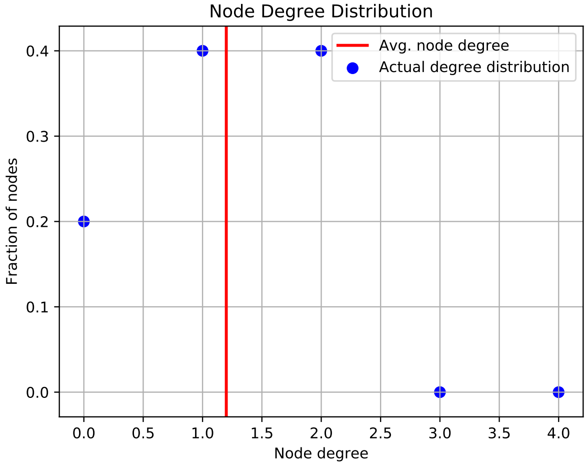
Question 1 continues on the next page...

Expected output:




```
g = UndirectedGraph()
g = g + 100
g = g + (1, 2)
g = g + (1, 100)
g = g + (100, 3)
g = g + 20
```

Expected output:



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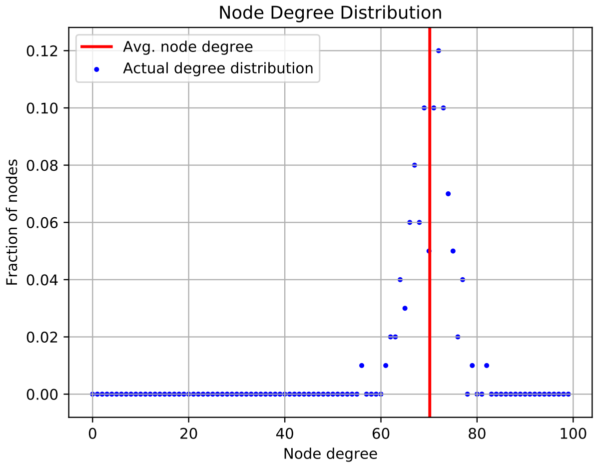
2. Create a derived class `ERRandomGraph` from the base class `UndirectedGraph` such that it should be possible to create a Erdős-Rényi random graph $G(n, p)$ [20]

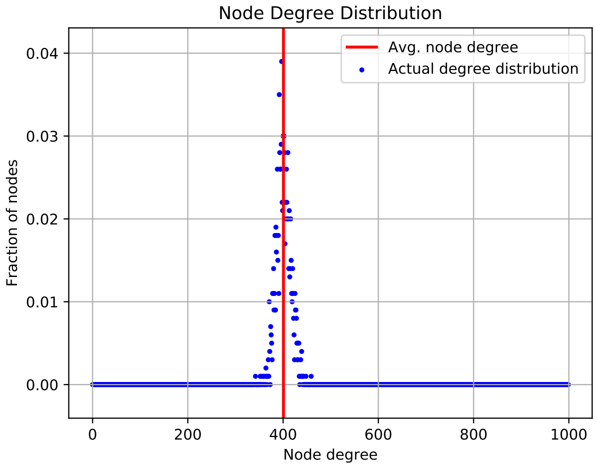
```
# The following code creates a G(100, 0.7) random graph and
# plots its degree distribution
g = ERRandomGraph(100)
g.sample(0.7)
g.plotDegDist()
```

Expected output:




```
# The following code creates a G(1000, 0.4) random graph and
# plots its degree distribution
g = ERRandomGraph(1000)
g.sample(0.4)
g.plotDegDist()
```

Expected output:



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3. *Connectivity* is a basic concept in Graph Theory. It defines whether a graph is *connected* or *disconnected*. A graph with fixed number of vertices is *connected* if there is a path between every pair of vertices. [20]

   - Add a method `isConnected` to the class `UndirectedGraph` that returns `True` if the graph is connected, and `False` otherwise.

     NOTE: This method should use BFS to determine connectedness and should not use any Python libraries/modules.

```
g = UndirectedGraph(5)
g = g + (1, 2)
g = g + (2, 3)
g = g + (3, 4)
g = g + (3, 5)
print(g.isConnected())
```

     Expected output:

```
True
```

```
g = UndirectedGraph(5)
g = g + (1, 2)
g = g + (2, 3)
g = g + (3, 5)
print(g.isConnected())
print(g)
```

     Expected output:

```
False
```

   - Write a function that uses the `isConnected` method of the `ERRandomGraph` class to verify the following statement

"Erdős-Rényi random graph $G(100, p)$ is almost surely connected only if $p > \frac{\ln 100}{100}$."

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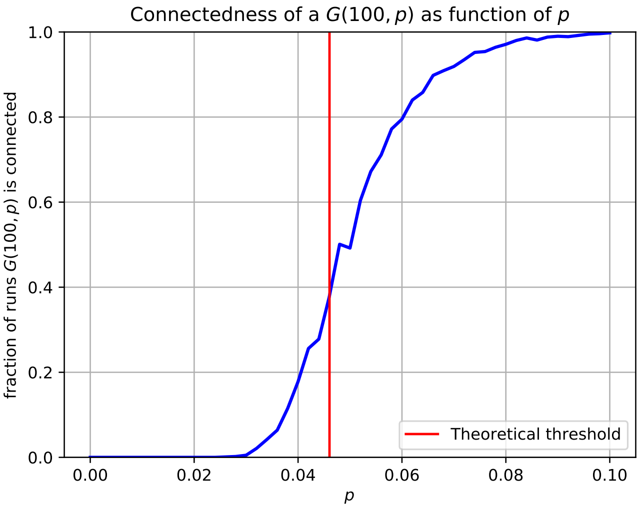
Expected output:



NOTE: The above plot shows connectedness averaged over 1000 runs.

4. A *connected component* of an undirected graph is a subgraph in which each pair of nodes is connected with each other via a path[2]. [20]

   - Add a method `oneTwoCompomentSizes` to the class `UndirectedGraph` that returns size of largest and second largest connected components in the graph.
   NOTE: This method should use BFS and should not use any Python libraries.

```
g = UndirectedGraph(6)
g = g + (1, 2)
g = g + (3, 4)
g = g + (6, 4)
print(g.oneTwoComponentSizes())
```

Expected output:

```
[3, 2]
```

```
g = RandomGraph(100)
g.sample(0.01)
print(g.oneTwoComponentSizes())
```

Expected output:

```
[23, 6]
```

[2] `https://www.baeldung.com/cs/graph-connected-components`

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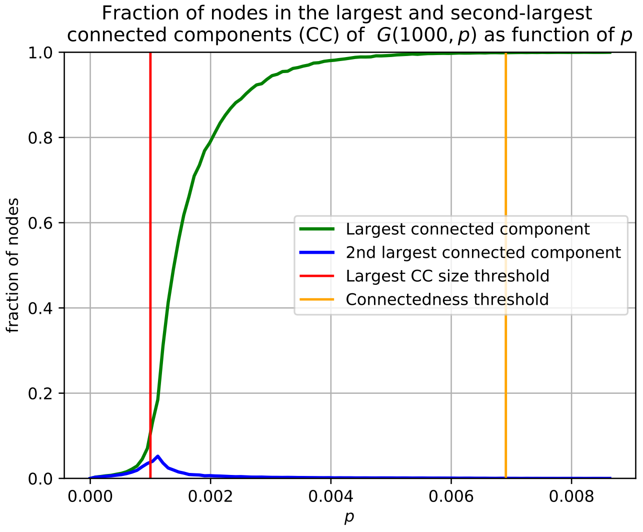
- Write a function that uses the `oneTwoComponentSizes` method of the `ERRandomGraph` class to verify the following statements

  "If $p < 0.001$, the Erdős-Rényi random graph $G(1000, p)$ will almost surely have only small connected components. On the other hand, if $p > 0.001$, almost surely, there will be a single giant component containing a positive fraction of the vertices."

  Expected output:



  NOTE: The above plot shows size of largest and second-largest components averaged over 50 runs.

5. You are expected to use Python's NetworkX package to solve this problem. Create a class `Lattice` such that [30]

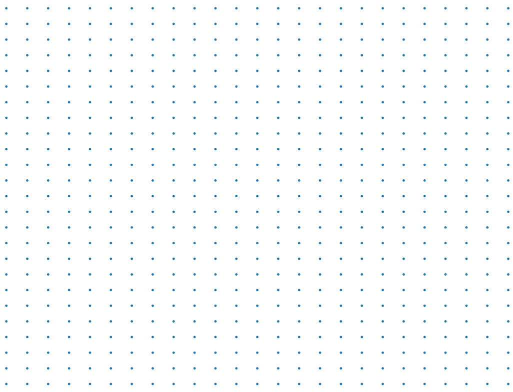
- It should be possible to create a $n \times n$ grid graph (a lattice).

```
# The following code will create a 10 x 10 grid graph
l = Lattice(10)
```

- It should be possible to display the grid graph.

```
l = Lattice(25)
l.show()
```

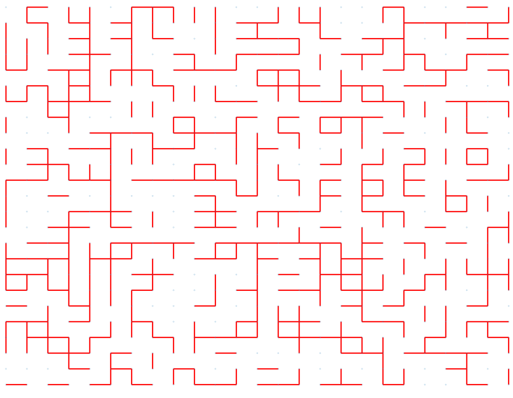
Expected output:



- It should be possible to simulate bond percolation[3] on the grid graph.

```
l = Lattice(25)
l.percolate(0.4)
l.show()
```

Expected output:



- The class should have a method `existsTopDownPath` that should return `True` if a path exists (along the open bonds) from the top-most layer to the bottom-most layer of the grid graph. The method should return `False` otherwise.

```
l = Lattice(25)
l.percolate(0.4)
l.existsTopDownPath()
```

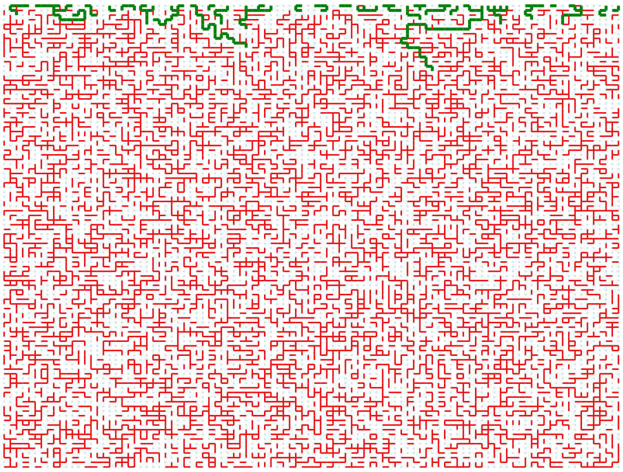
- The class should have a method `showPaths` that, for every node u in the top-most layer, does either of the following

[3] `https://en.wikipedia.org/wiki/Percolation_theory`

- If there is no path from u to nodes in the bottom-most layer, display the largest shortest path that originates at u.
- Otherwise, display the shortest path from u to the bottom-most layer.

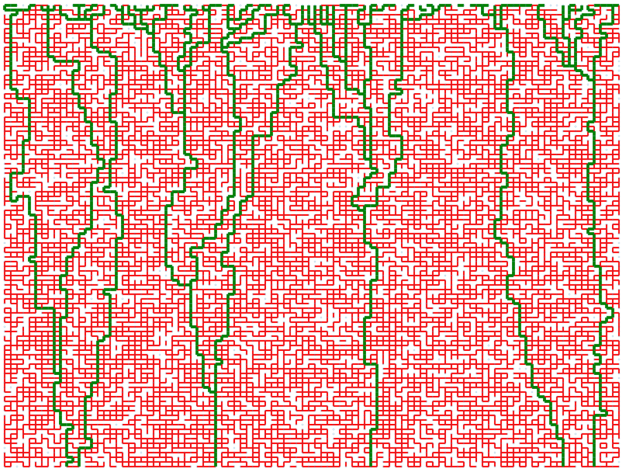
```
l = Lattice(100)
l.percolate(0.4)
l.showPaths()
```

Expected output:



```
l = Lattice(100)
l.percolate(0.7)
l.showPaths()
```

Expected output:



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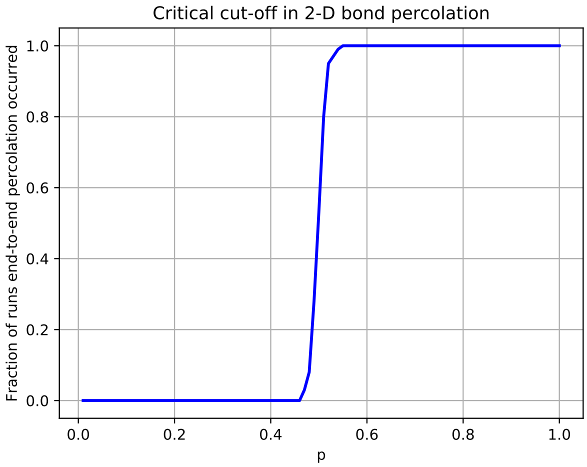
6. Write a function that uses the `Lattice` class to verify the following statement [20]

   "A path exists (almost surely) from the top-most layer to the bottom-most layer of a 100×100 *grid graph only if the bond percolation probability exceeds 0.5"*



NOTE: The above plot is obtained by averaging outcomes of 50 runs.