

A few instructions

- Codes should be compatible with *Python3* and should run on Ubuntu.
- Code for each question should be placed in a separate stand-alone files.
- Codes should be well-commented.
- **Appropriate exceptions should be raised and handled.**

1. Write a program to visualize actual derivative ($f'(x)$) and forward finite difference approximation ($\delta_{0.01}^+(x)$) of the function $\sin(x^2)$ in the interval $[0, 1]$. [10]
2. Write a program to visualize the absolute errors of approximation $\delta_{0.01}^+(x)$, $\delta_{0.01}^-(x)$ and $\delta_{0.01}^c(x)$ of function $\sin(x^2)$ in the interval $[0, 1]$. [10]
3. Write a program to visualize, as a function of h , the maximum absolute error of approximations $\delta_h^+(x)$ and $\delta_h^c(x)$ of function $\sin(x^2)$ in the interval $[0, 1]$. In the same figure, also plot the theoretical maximum absolute error of approximations $\delta_h^+(x)$ and $\delta_h^c(x)$. [20]
4. Write a program to visualize, as a function of M (number of intervals), area under the curve $y(x) = 2x \cdot e^{x^2}$ in the interval $[1, 3]$ computed using the trapezoidal formula. In the figure, also indicate the exact area. [20]
5. Write a program to visualize, as a function of u , area under the curve $y(x) = 2x \cdot e^{x^2}$ in the interval $[0, u]$ computed using various integration functions available in Python's `scipy.integrate` module. In the figure, also indicate the actual area under the curve. [20]
6. Enhance the class `Polynomial`, developed in the last coding assignment, as follows [10]
 - Add a method `derivative` that will return the polynomial's derivative.

```
p = Polynomial([1, 2, 3])
pd = d.derivative()
print(pd)
```

Expected output:

```
Coefficients of the polynomial are:
2 6
```

- Add a method `area` that takes two arguments a and b , and returns the exact area under the polynomial in the interval $[a, b]$

```
p = Polynomial([1, 2, 3])  
print(p.area(1,2))
```

Expected output:

```
Area in the interval [1, 2] is: 11
```

7. Write a program that uses the enhanced `Polynomial` class to approximate area under the curve $y(x) = e^x \cdot \sin x$ in the interval $[0, 1/2]$ within a guaranteed error of 10^{-6} .

[10]

NOTE: Your code should not use any numerical integration techniques, and should not compute the actual area under the curve.