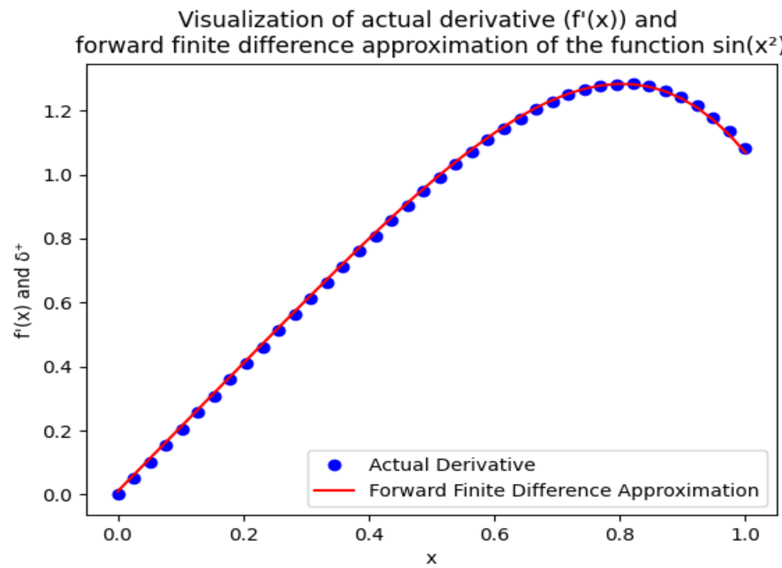
Coding Assignment 4
Numerical Differentiation and Integration

-Yukta Salunkhe
-112001052

Q1.Approach mentioned in comments.

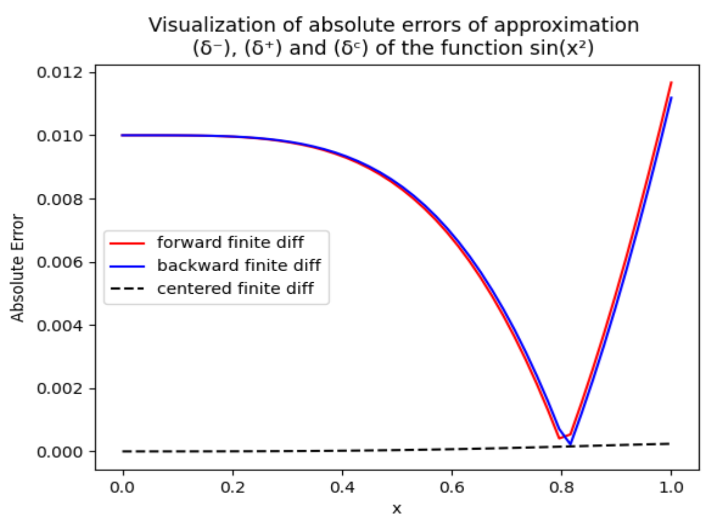
We choose a few random points within the interval [0,1]. Plot the derivatives and the forward finite Difference approximation at those points for the function $\sin(x^2)$

As is visible from the plot, both the curves overlap with each other.



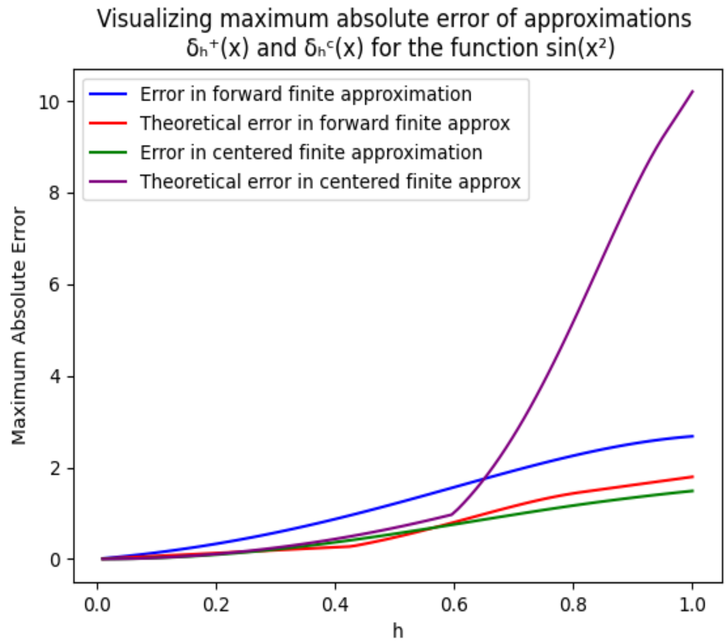
Q2. Approach mentioned in comments.

Here over the interval [0,1] absolute errors of approximation for forward finite, backward finite and centered finite difference are plotted for the function $\sin(x^2)$. Uniform random points from the interval are taken. The graph I got is as following.We can see that the centered finite difference gives almost 0 error.



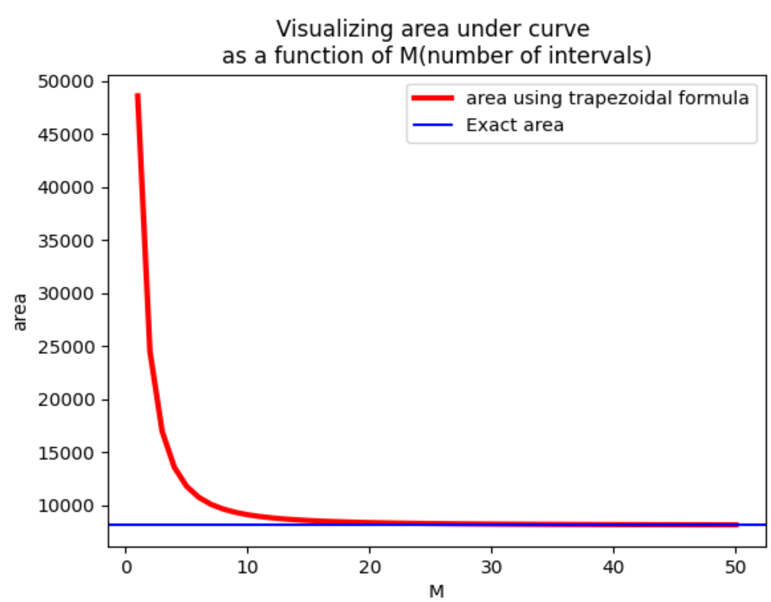
Q3. Approach mentioned in comments.

=>
For the function $\sin(x^2)$, the maximum absolute error of approximation by finite forward difference and centered finite difference along with the theoretical maximum errors for respective approximations are plotted.
The plot obtained is as follows:



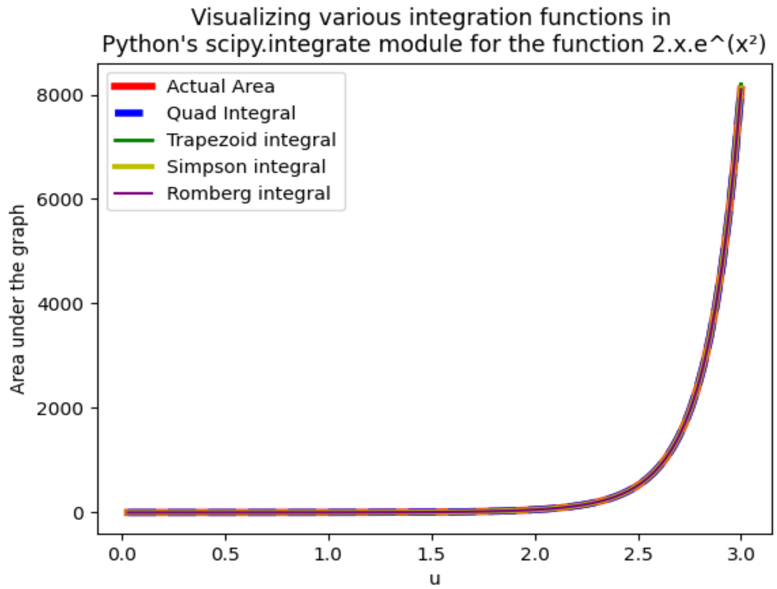
Q4. Approach mentioned in comments.

=> M is varied from 1 to 50. For each number of intervals within the x range of [1,3], the area under the curve for: $y(x) = 2x \cdot e^{\wedge}(x^2)$ as given by trapezoidal and actual area is plotted. As visible from the plot, the approximate area (using trapezoidal formula) becomes equal to the actual area, as the number of intervals(M) increases, thus decreasing error of approximation by trapezoidal formula.



Q5. Approach mentioned in comments.

=> Area under the curve for function : $y(x) = 2x \cdot e^{(x^2)}$ as computed by various integration functions available in Python's scipy.integrate module is plotted. Also the actual area computed from its integral function is plotted. As visible from the plot, all the functions compute the same area, thus we get overlapping lines for all, and is equal to the actual area under the curve within interval [0,x] for any given x.



Q6. Approach mentioned in comments.
=> Added derivative and integral functions in the polynomial class which returns the coefficients of the derivative and integral polynomials of a given function respectively. Then from the integral polynomial we can find the value of integration at any given x. And thus we can calculate the area under the curve for our function given

Q7. Approach mentioned in comments.
=> In this question we randomly take x points and find the corresponding function values $e^x.\sin(x)$ at those points. Then we try to find the polynomial expansion for the given function by trying to fit a polynomial on the points through which our given function curve passes. This is done by using the fitMatrixByPoints method of our polynomial class. This can also be done by considering the Maclaurin series of the given function. Then after getting the polynomial we find the area. The error in actual and approximate area is of 10^{-12} order. Thus it is guaranteed to be less than 10^{-6}.