

COMPUTER SCIENCE AND ENGINEERING

Indian Institute of Technology, Palakkad

CS5016: Computational Methods and Applications

Coding Assignment 5

Least-Square Function Approximations

20 Feb, 2023

Max points: 100

A few instructions

- Codes should be compatible with *Python3* and should run on Ubuntu.
- Code for each question should be placed in a separate stand-alone files.
- Codes should be well-commented.
- Appropriate exceptions should be raised and handled.
- 1. Write a function to compute the polynomial of degree *n* that is the best fit for a given set of points. It should take as its arguments an array of tuples and an integer *n*, and return an object of type Polynomial (developed in the last coding assignment). The function is also expected to produce a plot of the input points along with the best-fit polynomial together.

[20]

2. Write a function to compute the polynomial of degree n that best approximates the function $\sin(x) + \cos(x)$ in the interval $[0, \pi]$. It should take as its argument an integer n, and return an object of type Polynomial (developed in the last coding assignment). The function is also expected to plot the actual and approximate function together.

[15]

3. Write a function that uses the enhanced Polynomial class (developed in the last coding assignment) to compute the n^{th} Legendre polynomial. It should take as its argument an integer n, and return an object of type Polynomial that represents the n^{th} Legendre polynomial.

[15]

4. Write a function to compute the least-square approximation of e^x in the interval [-1,1] using the first n Legendre polynomials. It should take as its argument an integer n, and return an object of type Polynomial. Also, plot the actual and approximate function together.

[10]

5. Write a function that uses the enhanced Polynomial class (developed in the last coding assignment) to compute the $n^{\rm th}$ Chebyshev polynomial. It should take as its argument an integer n, and return an object of type Polynomial that represents the $n^{\rm th}$ Chebyshev polynomial.

[10]

6. Write a function to compute the first 5 Chebyshev polynomials and numerically demonstrate that they are orthogonal with respect the weight function $w(x) = 1/\sqrt{1-x^2}$ in the interval [-1,1].

[10]

7. Write a function to compute coefficients of the best-fit Fourier approximation $S_n(x)$ of function e^x in the interval $[-\pi, \pi]$. It should take as its arguments an integer n, and print the coefficients of $S_n(x)$. The function is also expected to generate a plot with the actual and approximate function together.

[10]

8. Using Python's scipy.fft package, write a function, with time complexity of $O(n \log n)$, to multiply two large n-digit integers.

[10]