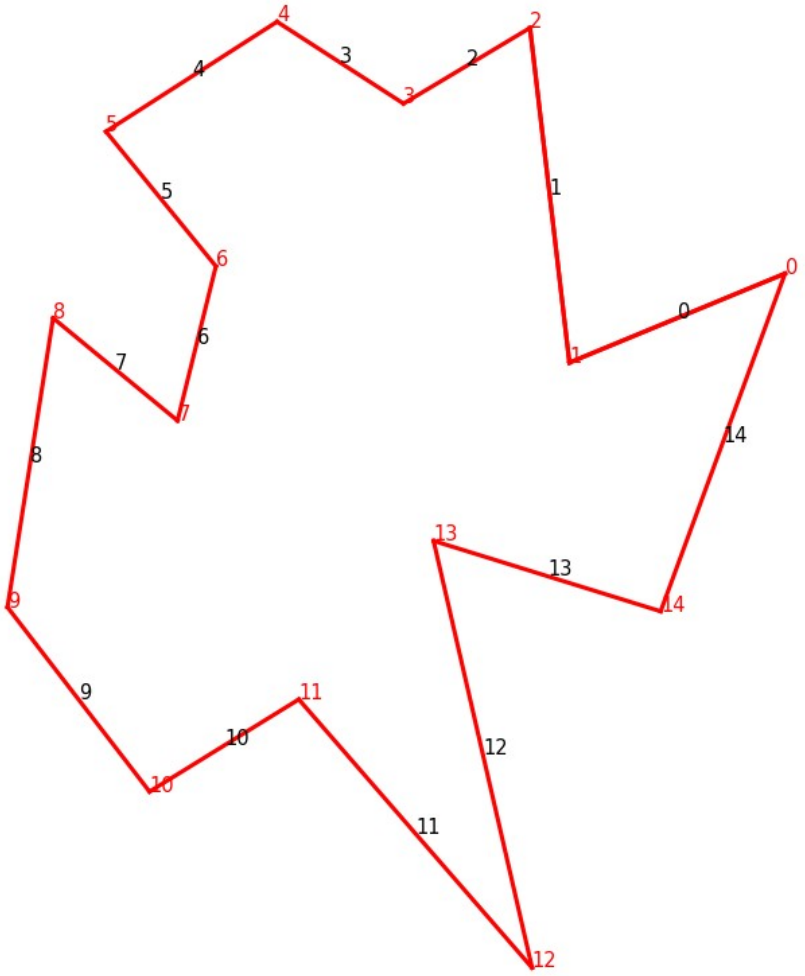
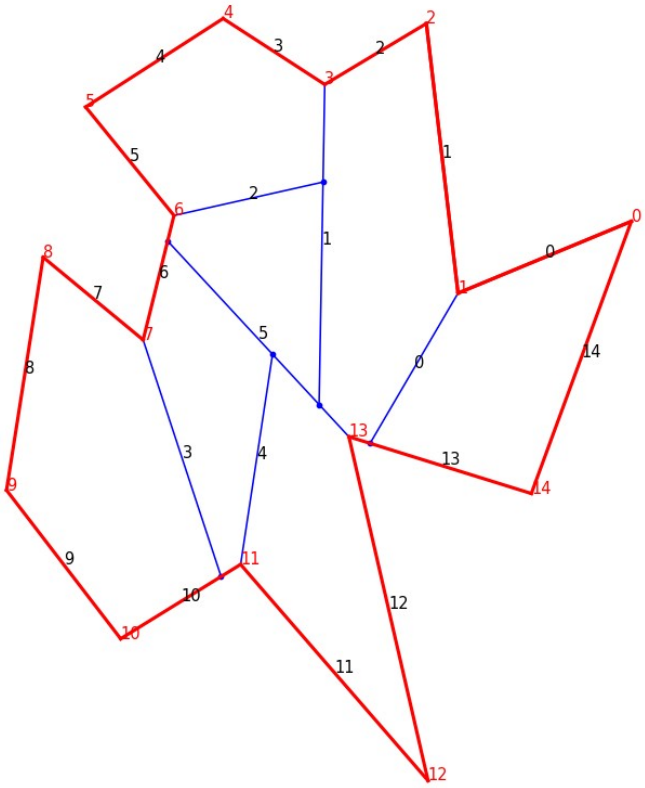
A novel method for determining the straight skeleton of a simple polygon

A given planar simple (no holes) polygon is shown below. Note that the 15 edges are numbered 0 - 14; the 14 corners are similarly numbered 0-14. There are 6 reflex (non convex) corners at corners 1, 3, 6, 7, 11 and 13.

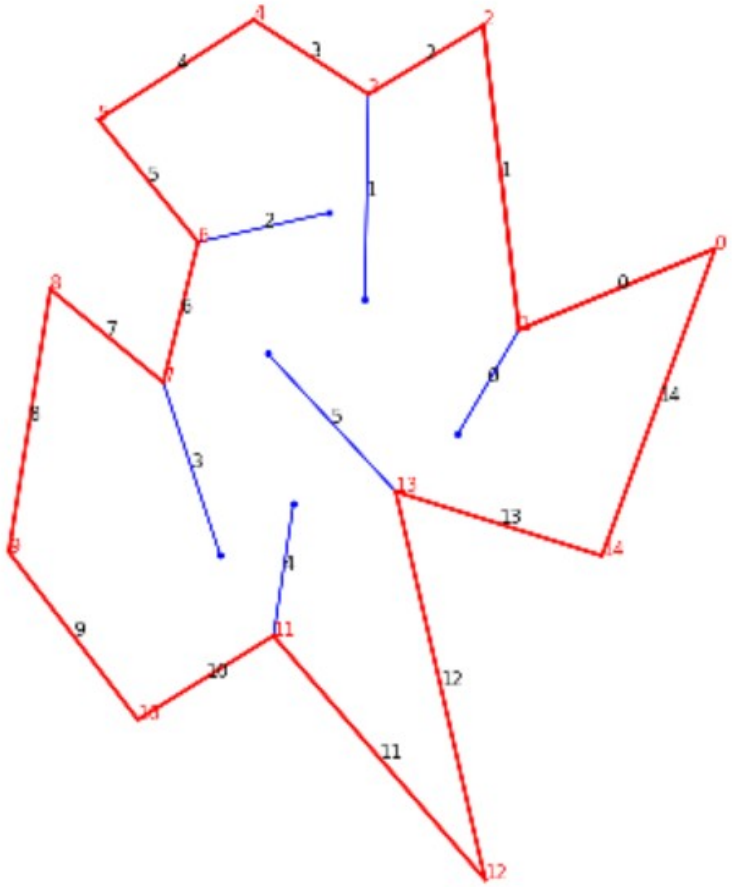


The first step is to generate the motorcycle tracks of the figure. Below is shown the result of that determination.

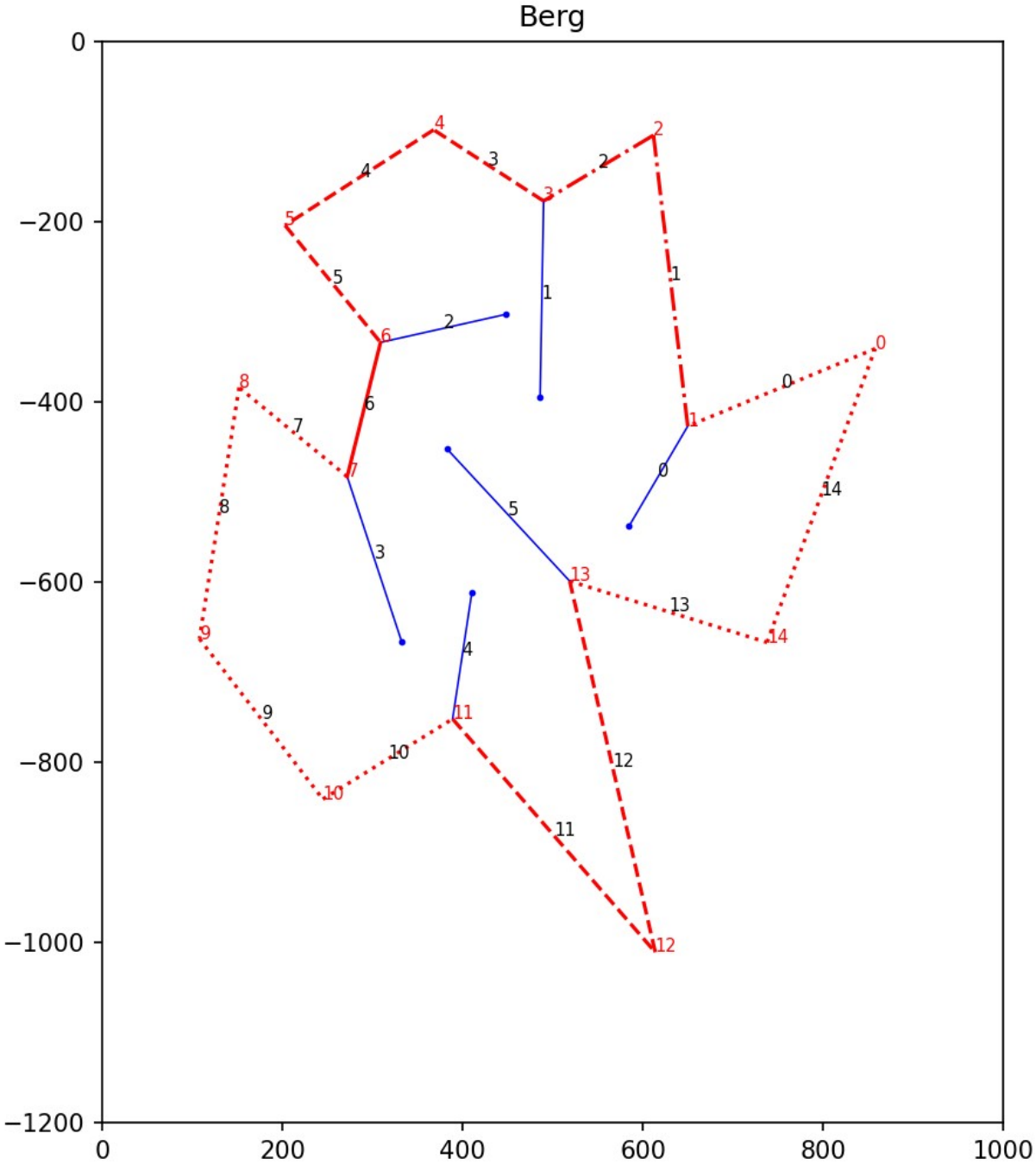


There is a track for each reflex corner. There are two types of tracks: one type extends from a vertex to and edge, the other stops at another track. Tracks 0, 3 and 5 are the former; 1, 2 and 4 the latter.

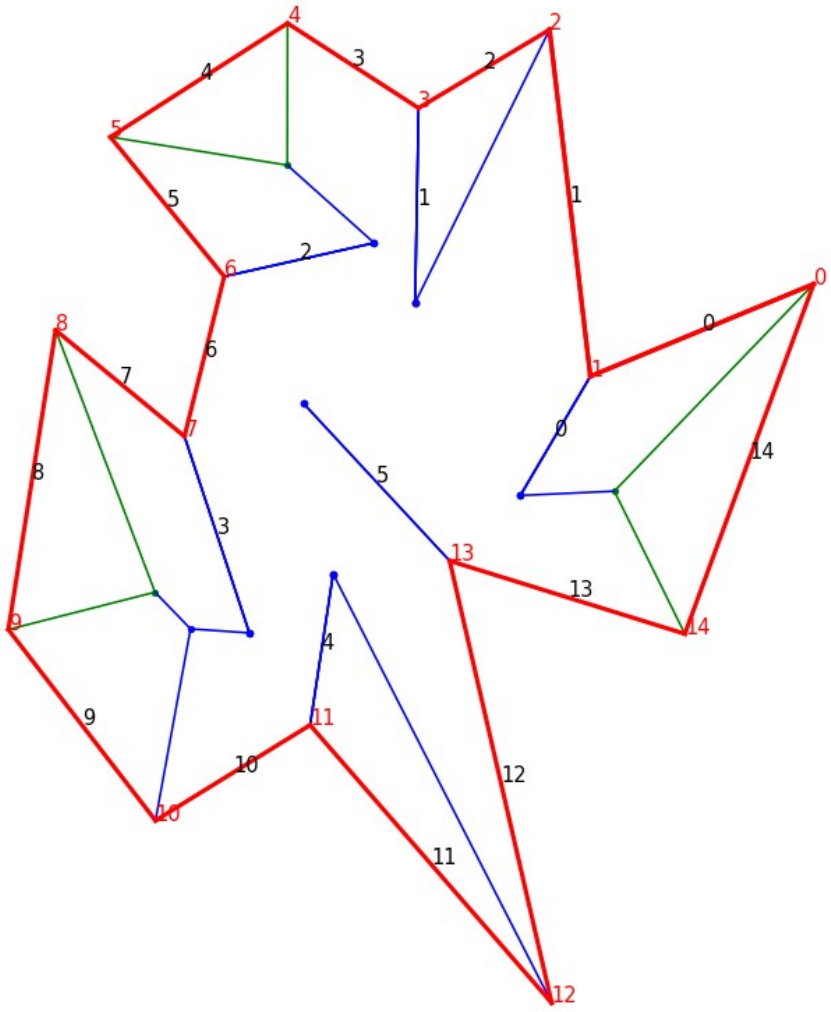
For those tracks terminating at another track, the motor cycle track target is replaced by the skeleton target. For example, for track 1, the motorcycle target is track 5. Its skeleton target is 13. This is determined by the edges at the vertex of the motorcycle target. Those edges are 12 and 13, with the closest being edge 13. Similarly for track 2, the edges at vertex 3 are 2 and 3, so that the skeleton edge for track 2 is edge 3. The endpoints of the tracks with changed targets are recalculated. These new endpoints are commonly called "B" points. They are the intersection of the bisector at the start of the track and the bisector of an edge at the base and the skeleton target. For example, for track 2, the track bisector is formed by sides 5 and 6. The skeleton target is side 3. The endpoint of track 2 is determined by the intersection of the bisector of 5/6 and the bisetor of 5/3. Note that this is the same result of using 5/6 and 6/3.



Next, the clipping regions of the polygon are determined. These regions are defined by successive reflex corners. In the case of our figure, there are five regions. Region 0 is defined by corners 1 and 3, region 1 by corners 3 and 6, region 2 by corners 7 and 11, region 3 by 11 and 13, and region 4 by corners 13 and 1. Side 6 does not belong to any region.



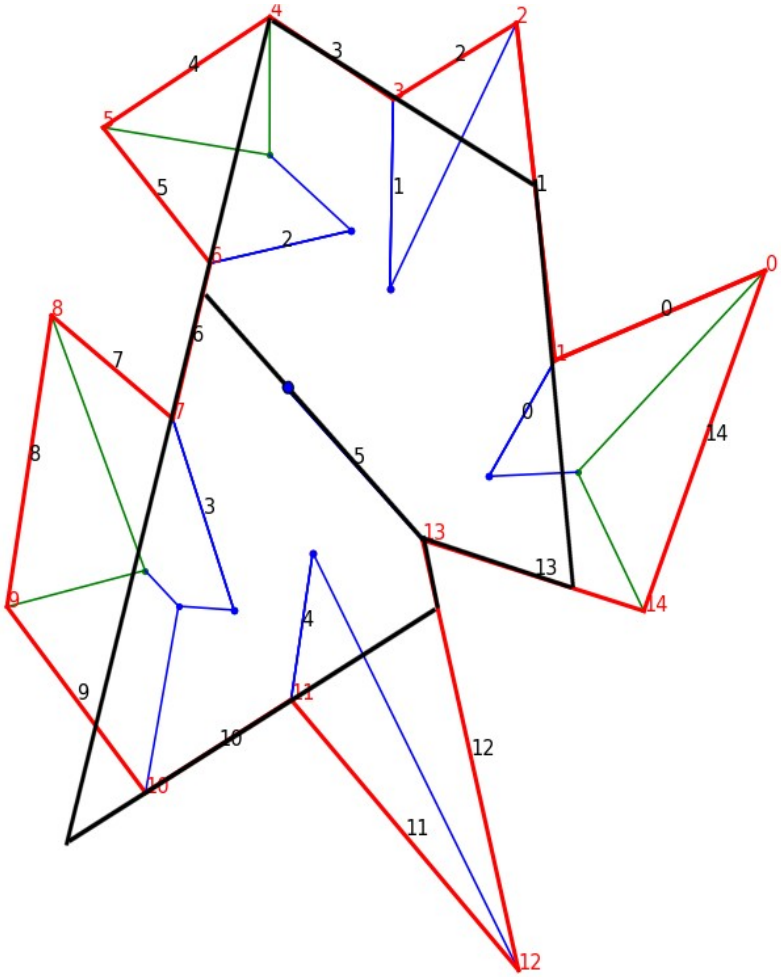
The next step is to determine the skeletons for each of the five regions. This is done by considering the defining tracks as constraints, as shown in the figure below. For example, the region from corner 7 to corner 10 can be considered a convex figure if sides 7 and 10 are extended until they meet. A skeleton for that convex figure is straightforward. The constraining figure results in a truncation of the convex figure by track 3. In the case of the region of corners 3 and 5, a skeleton is developed until the skeleton trail is between sides 5 and 3, the two sides of the region.



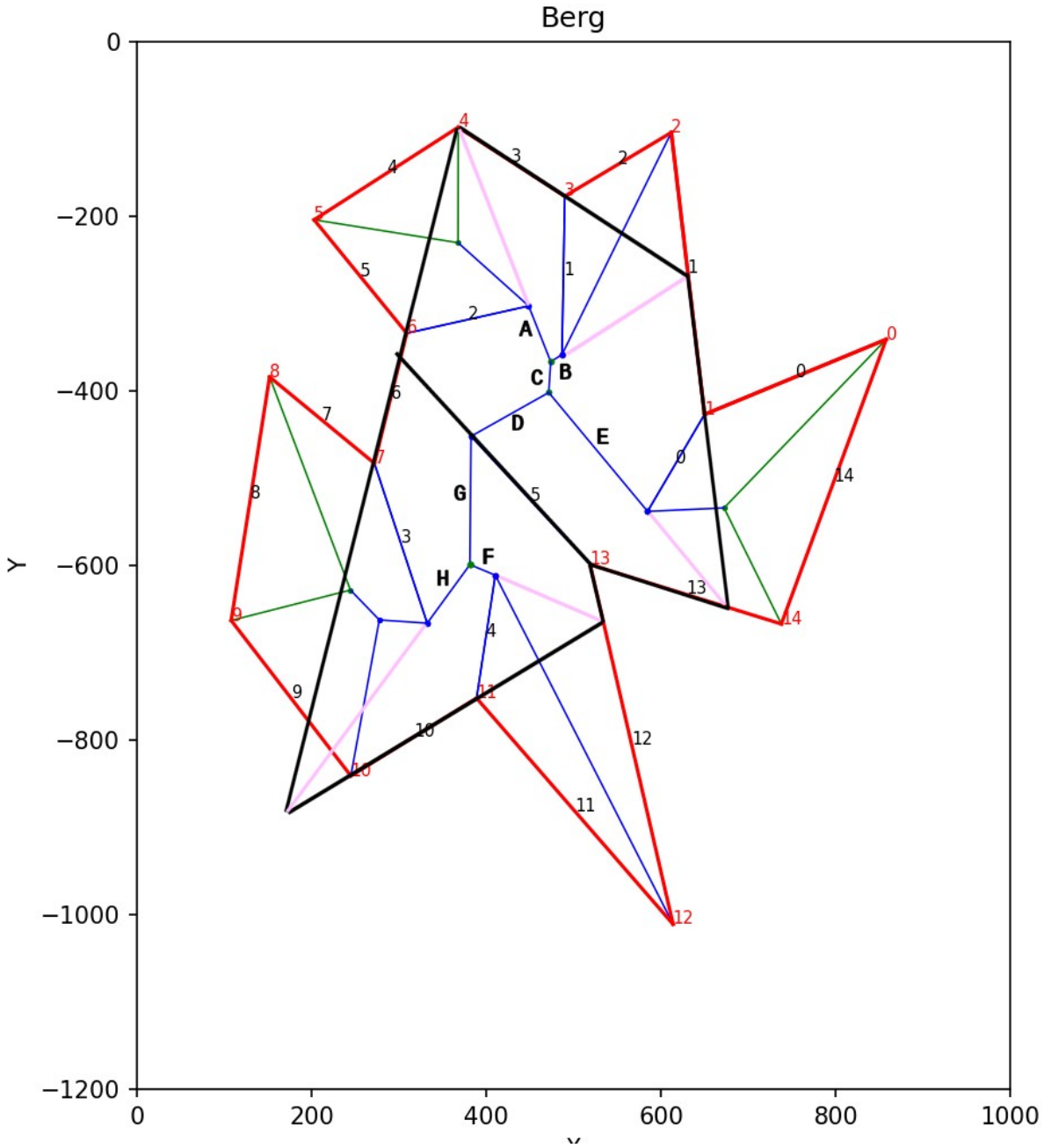
The edges are now re-linked to omit the edges of the partial skeletons to produce a reduced figure: Starting at edge 6 and proceeding counterclockwise, we encounter 10, 12, 13, 1, 3 and back to 6. Note that there is one reflex angle at corner 13 corresponding to track 5.

We will now divide the figure at the remaining reflex angles, which is just one at track 5. The dividing track 5 and its target are duplicated. One copy of the duplicates is linked to edges 10 and 12; the other linked to edges 3 and 13. Also, any track associated with a remaining reflex corner if drawn from it corner to "B" point.

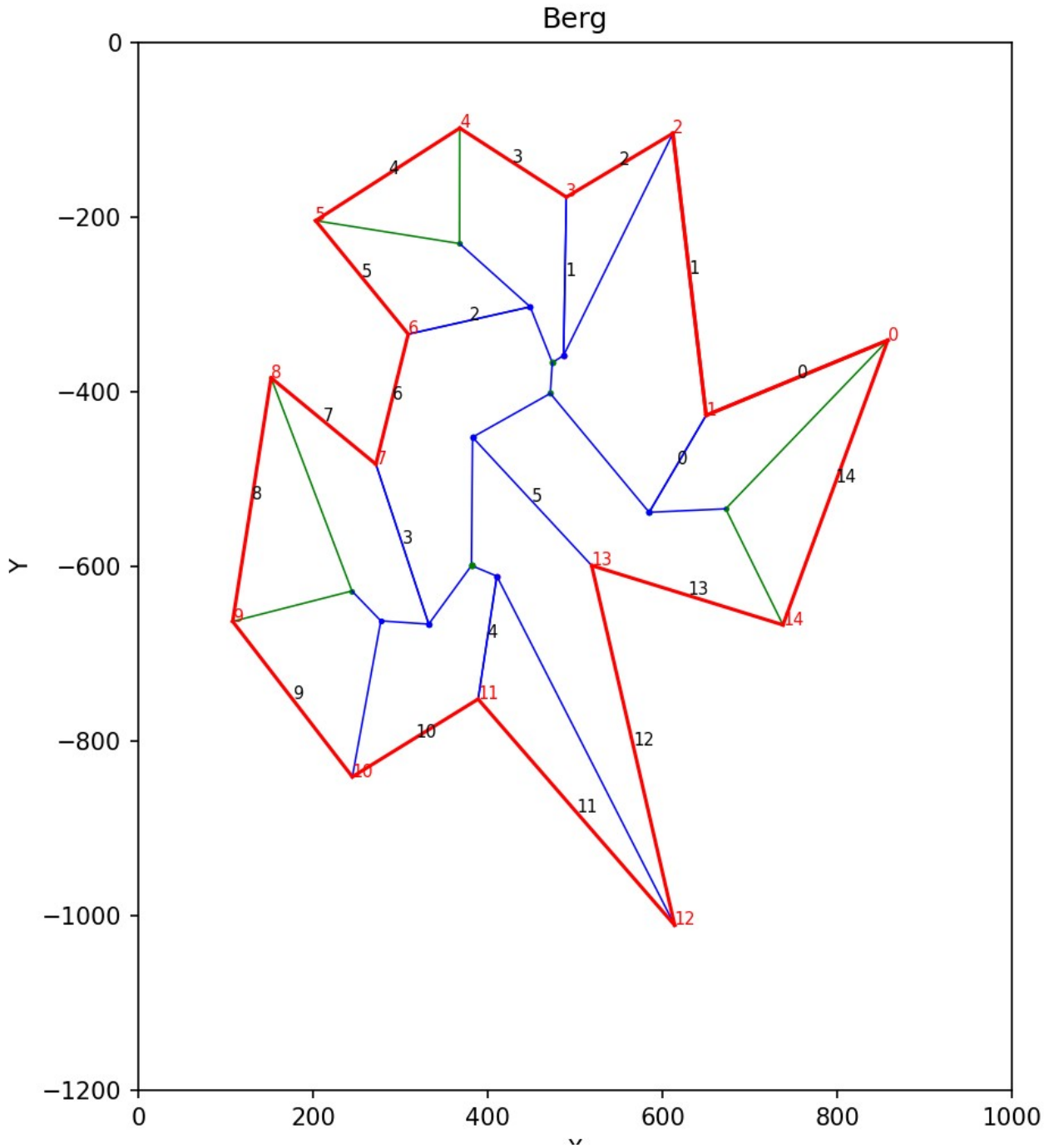
Note that the figure has now been reduced to two convex figures. Skeletons for these two figures can be determined using the same process as was used with the original regions except that the skeleton origins are the unfinished ends of the region process rather than track bases or adjacent sides as was formerly the case. Any remaining track not used in defining one of the partial skeletons is a through track that divides the reduced figure into convex polygons. The two resulting polygons of our reduced figure consist of sides 6, 10, 12 and track 5; and sides 13, 1, 3, 6 , and track 5 for the other.



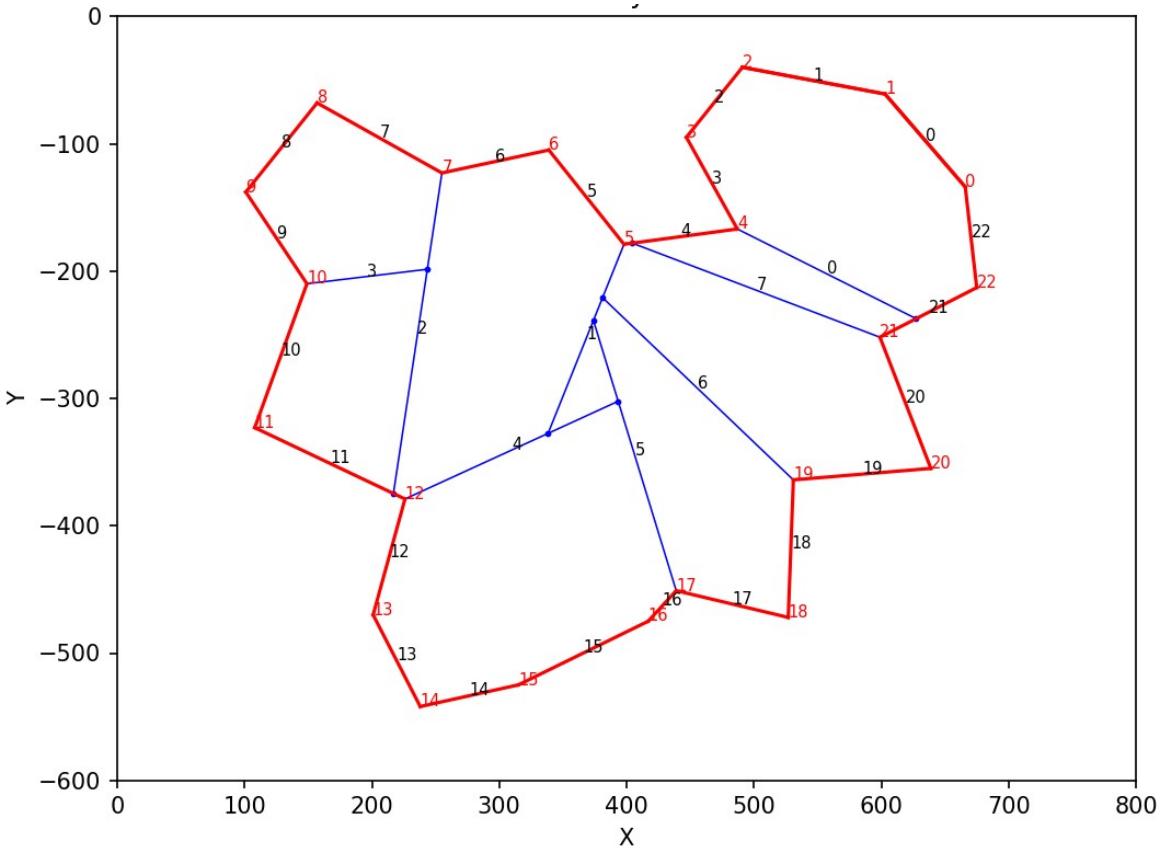
Determining the skeletons of the convex polygons of the reduced figure is straightforward. The partial skeletons at the ends of each region's defining track are extended in the usual manner. For example, the partial skeleton ending at the end of track 2 is extended by a line defined by the bisector of 3/6 that extends to the intersection of the line defined by the bisector of 1/3. the new skeleton segment is labeled A in the figure. All of the additional skeleton segments are labeled A through H. Several portions of bisectors not used are shown in pink.



Finally, eliminating all the lines except those for the skeleton, we have the final result.

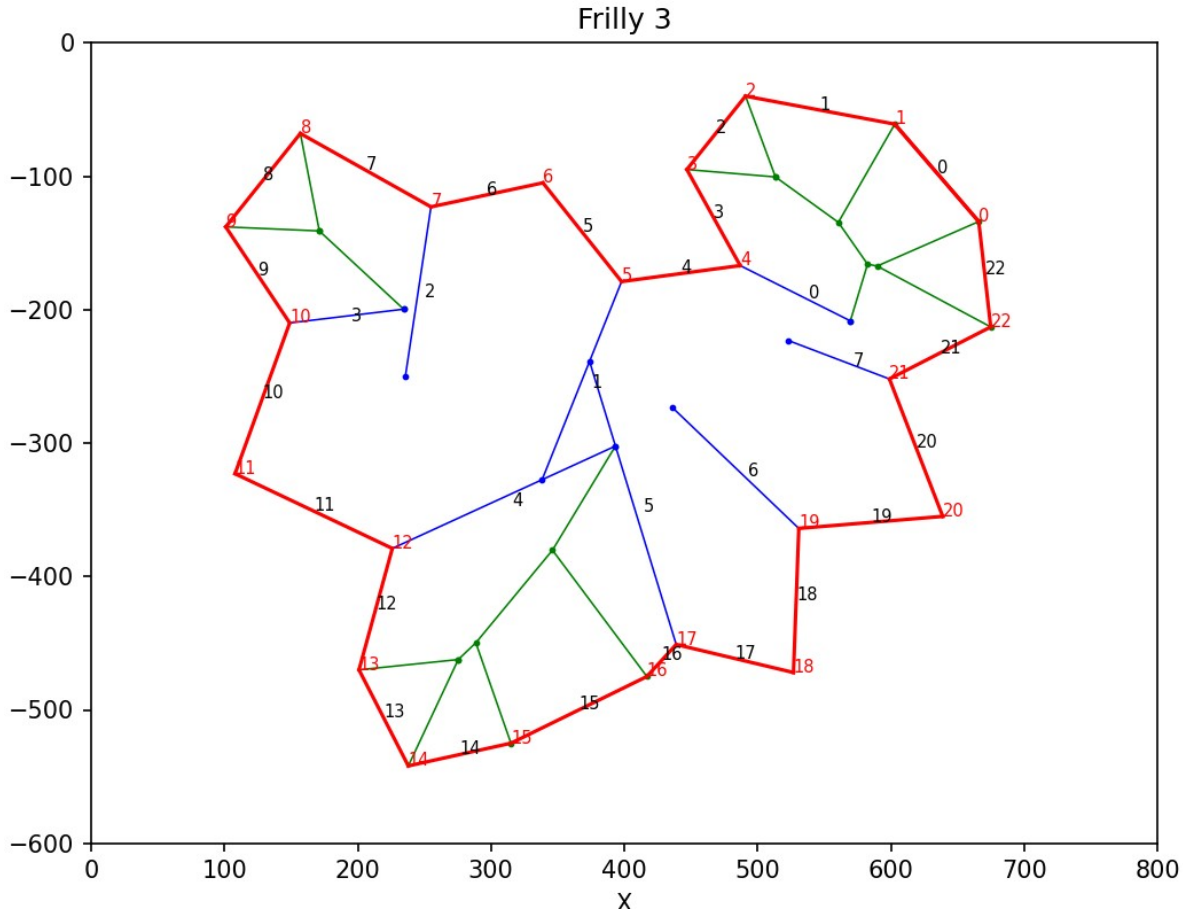


Final thoughts. The example given above shows a polygon with no "islands." An island is where tracks intersect one another as is shown in the figure below: track 4 intersects 5, 5 intersects 1 and 1 intersect 4.

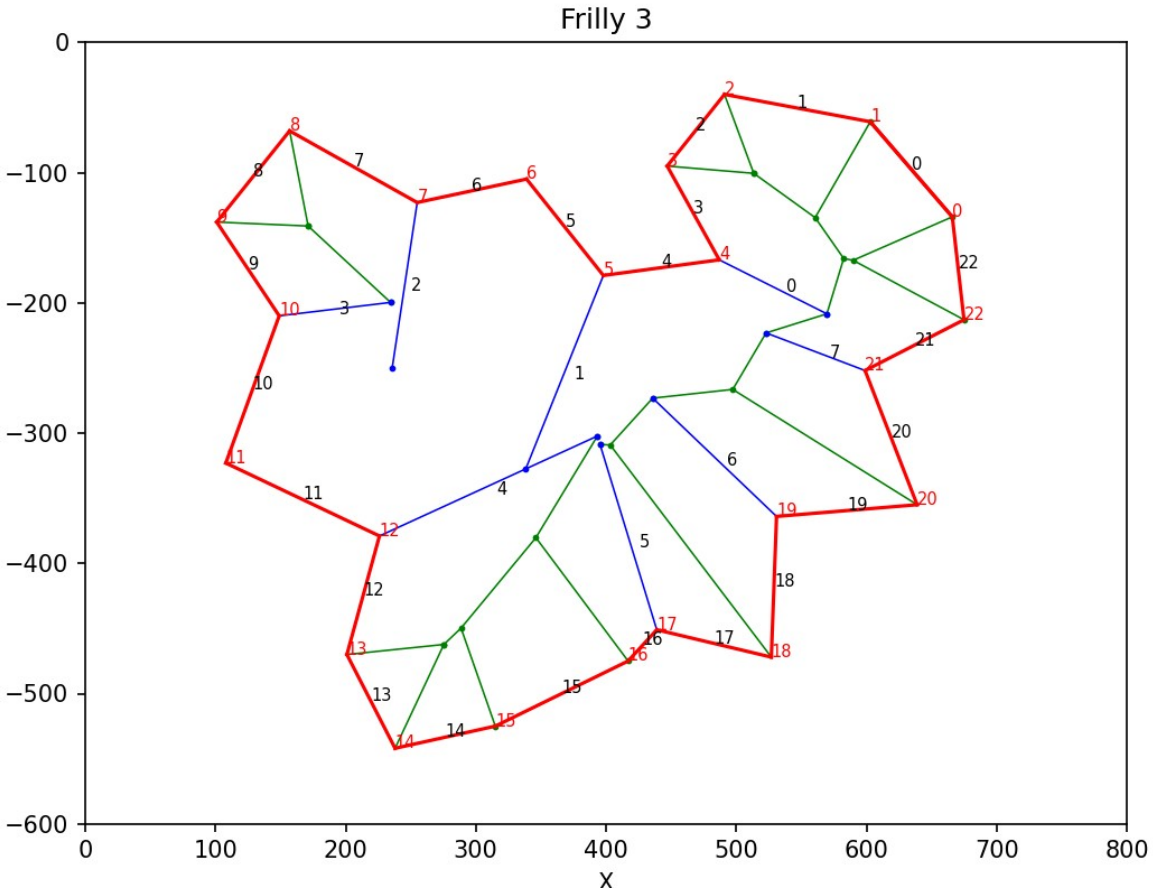


In the following we will show how developing the skeleton for this figure proceeds.

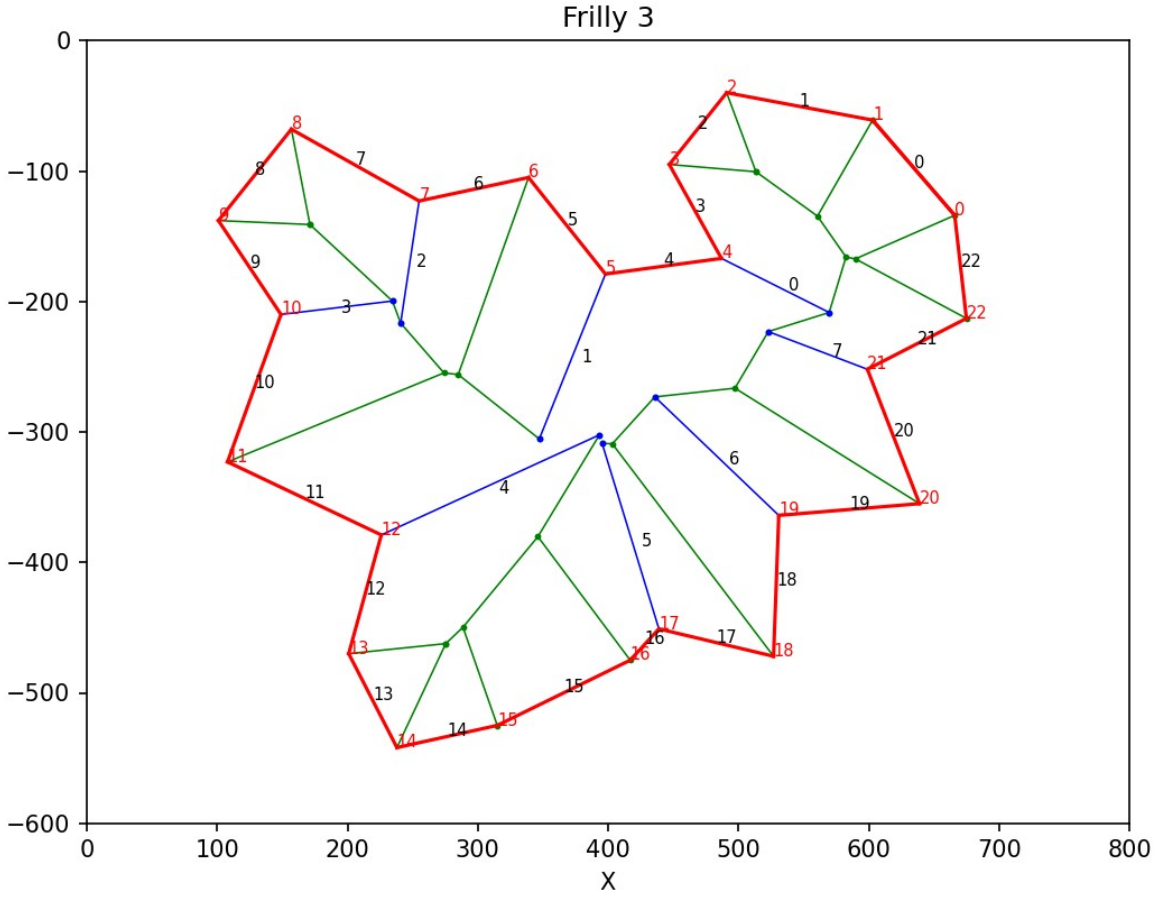
The first step in determining the skeleton is to start skeleton endpoints from regions having 3 or more sides. For this test polygon, the three regions are between corners 21 to 4, 7 to 10, and 12 to 17. The partial skeletons were paused on encountering a limiting reflex ray. The tracks forming the island (1,4 and 5) will play the same role as the dividing track 5 in the previous example.



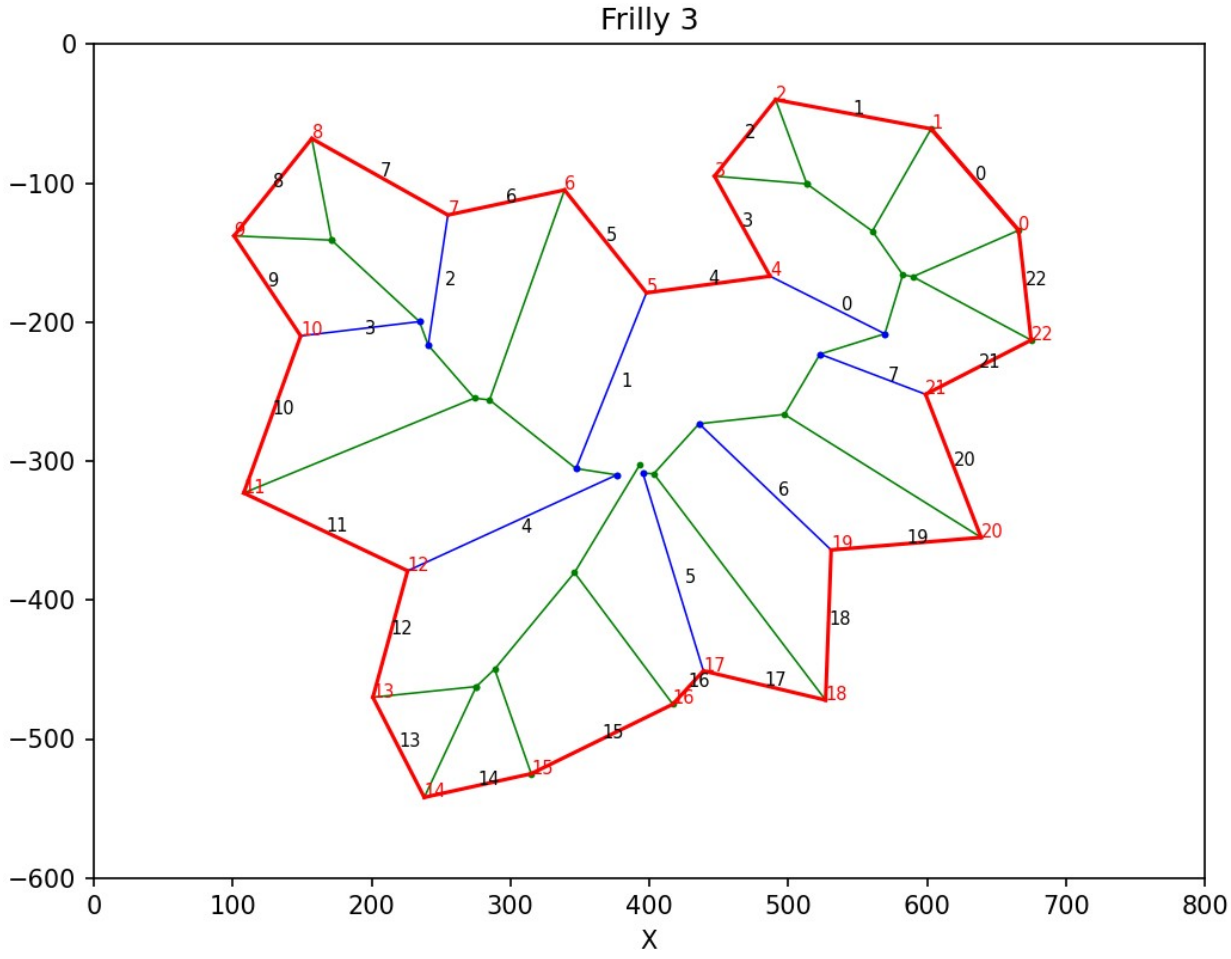
The next steps are to extend the partial skeletons to a terminating point, either a dividing line or as a condition indicating a transition to another region. Below is shown the extension of the region in the top right corner. The extension could proceed all the way to dividing line 5. Note that track 5 has been shortened slightly with the intersection of the skeleton coming from the right.



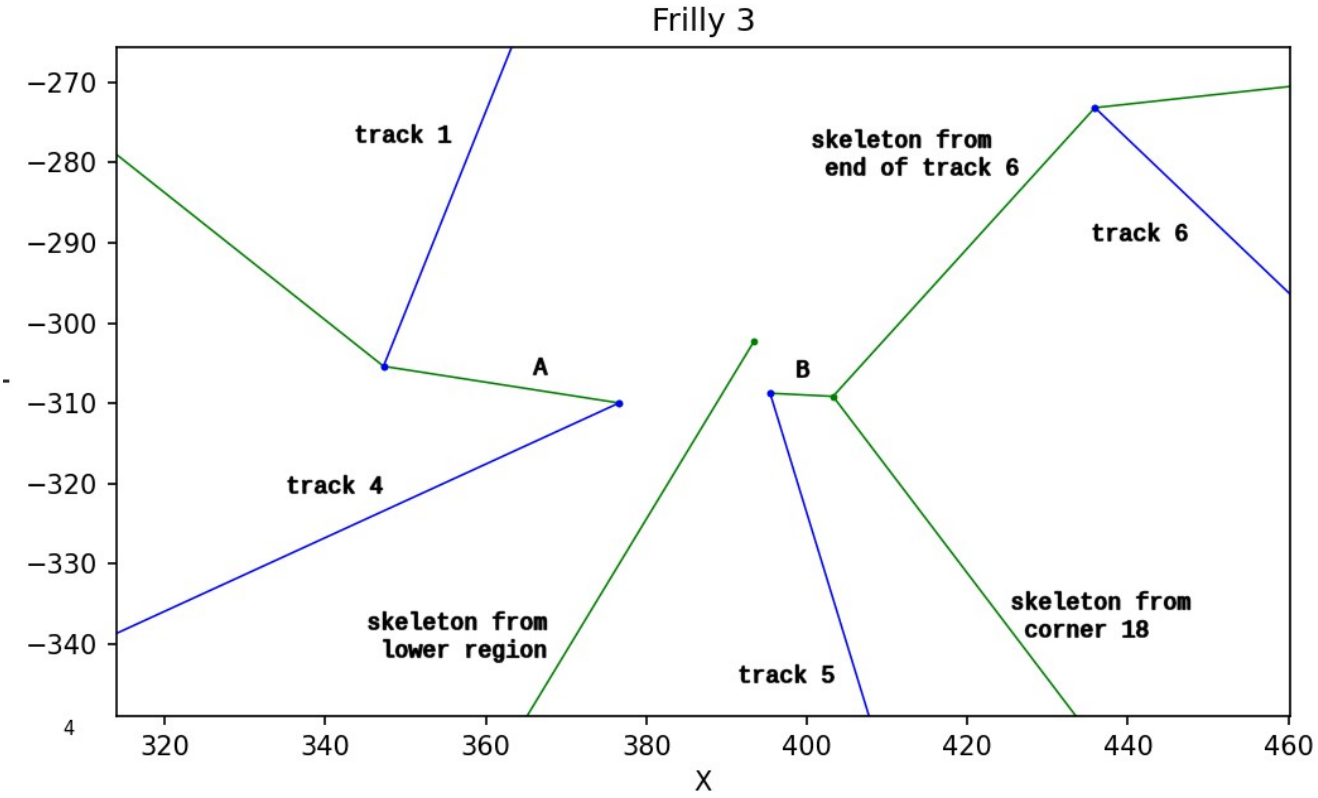
Proceeding to the region at upper left, we construct the partial skeleton for that region and pause at dividing track 1. We now recognize that the skeleton ending at the end of track 1 can be extended toward track 4.



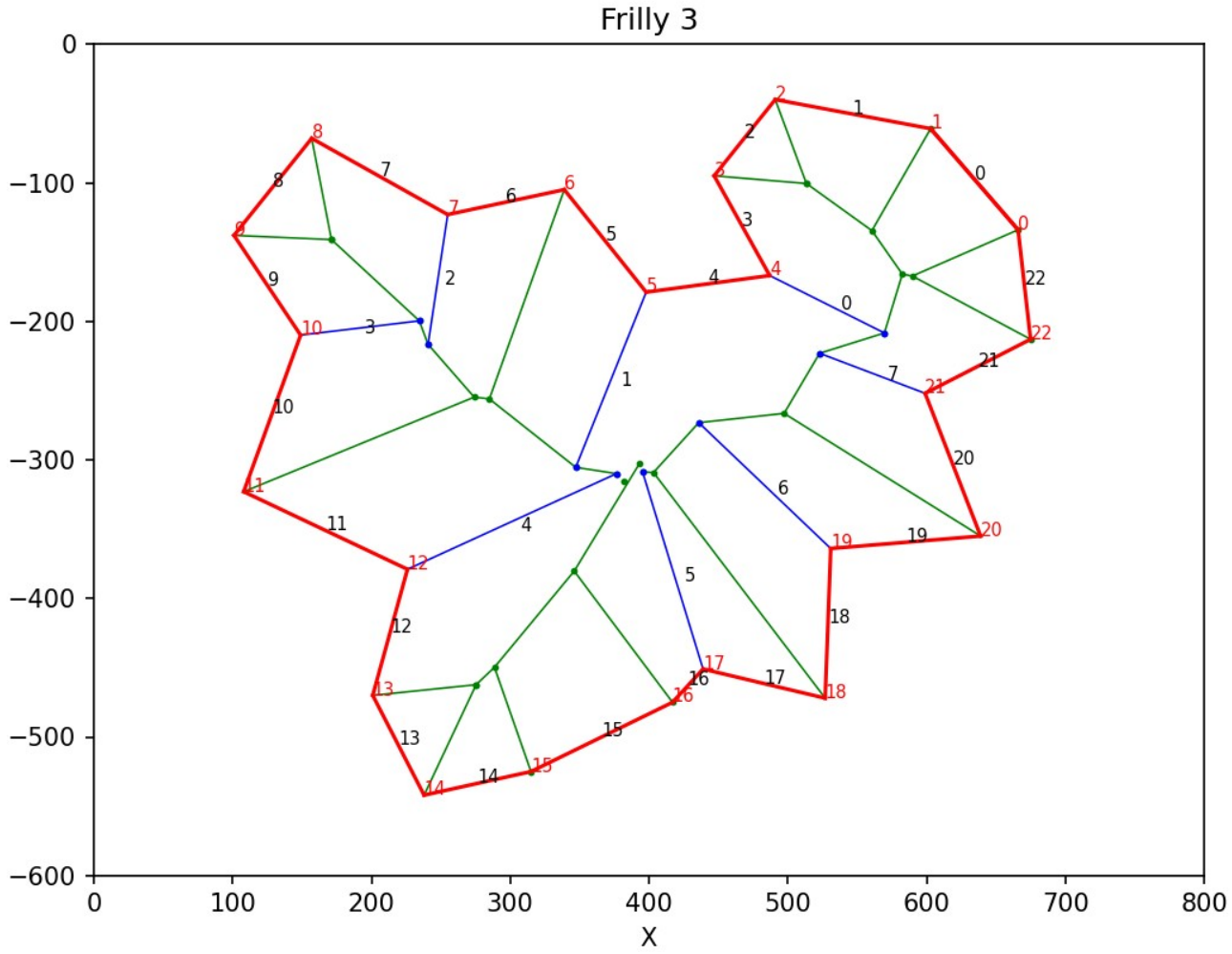
The connector between the end of track 1 and track 4 has been added. Note the shortening of track 4 from this addition.



Below is a blowup of the situation at the top of the lower region, the place where tracks 4 and 5 formerly met. The sides defining skeleton segment A are 4 and 12; those for B are 4 and 16. Since A and B both share side 4, the skeleton connector must also share side 4. Thus it must have an end point defined by sides 12 and 16 and 12 (or 16) and 4.



Below shows the placement of the determined endpoint for the top of the lower region. In the figure, the lower region is still connected to the track 4/5 intersection. The correct termination of the skeleton segment from the lower region is the isolated dot.



Thus, we have the complete skeleton.