Determination of a Straight Skeleton from a Motorcycle Graph
April 21, 2024

Below is the starting point, a motorcycle graph of a polygon. As is noted elsewhere, a motorcycle graph can be generated from a triangulation of an n-sided polygon [order n·log(n)], ray tracing of v convex vertices in the triangulation [order v·sqrt(n)] and brute force intersection of motorcycle paths [order v·v]. Note that each of the motorcycle regions is a convex polygon.

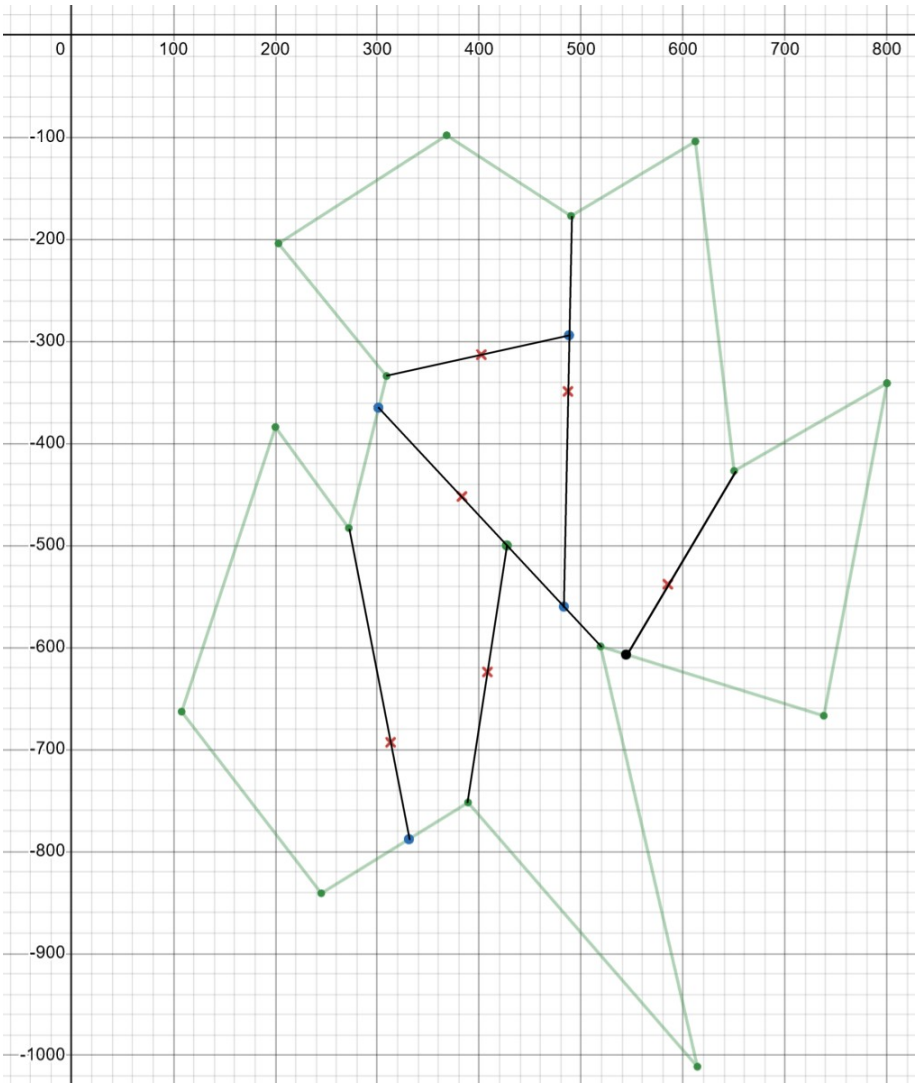

Fig. 1

The first step in the process is to calculate the "B" points for each of the motorcycle traces (red x in the graph above). These are the intersections of the bisector of a convex vertex and the bisector of a side forming the vertex and the target of the vertex bisector. These points are the ends of certain of the valleys defined by the convex vertices. Note that each of the regions is convex.

Eliminating the portion of the motorcycle track from a B point to the target side yields the following result.

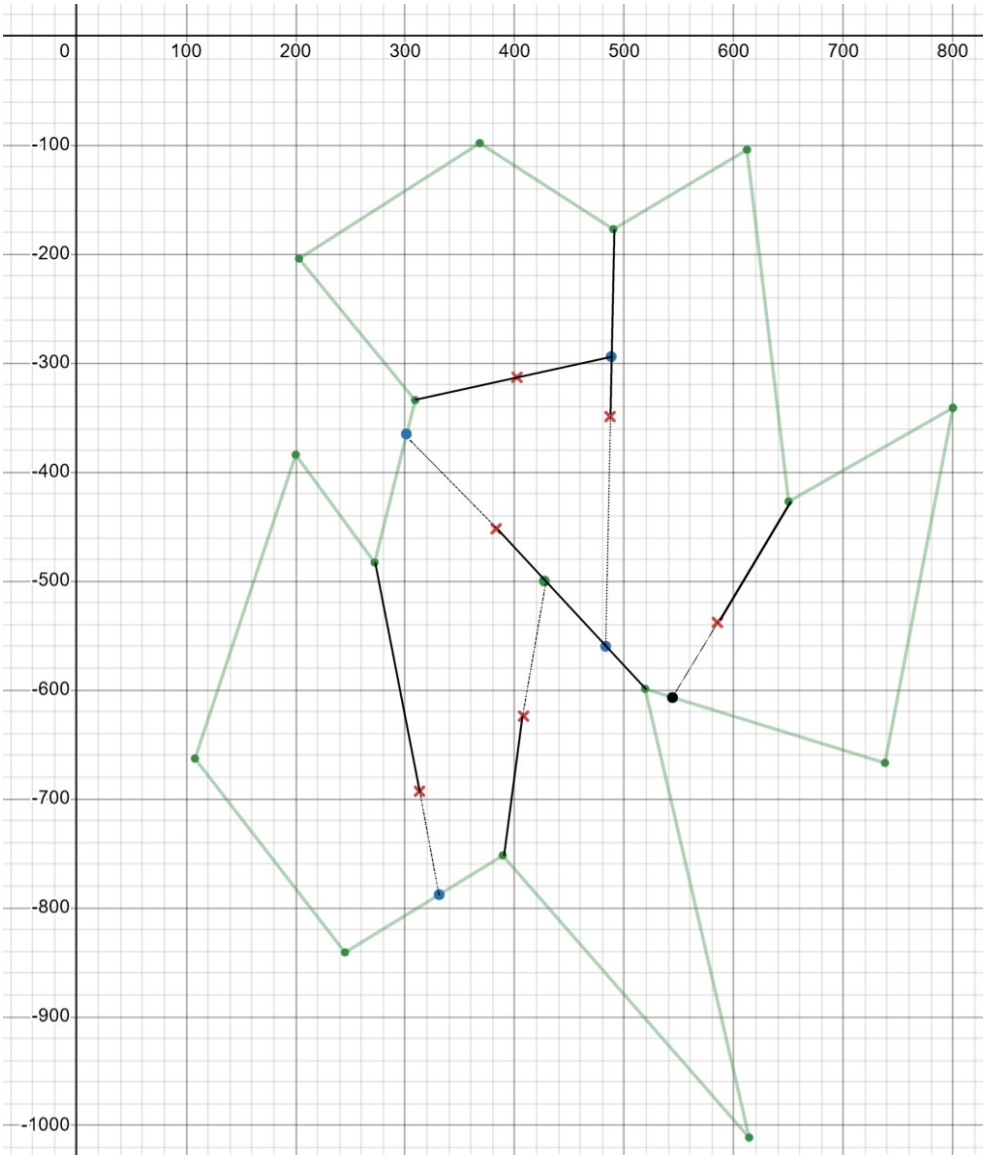

Fig. 2

The next step is to identify those regions having two adjacent sides on the boundary. Skeletons for these regions can be immediately determined. Generation of the skeleton is stopped when a "B" point is encountered. Where a node has three segments, the node is complete; where just one or two, the node is incomplete. In the figure there are six incomplete nodes. We will next show how to complete the incomplete nodes.

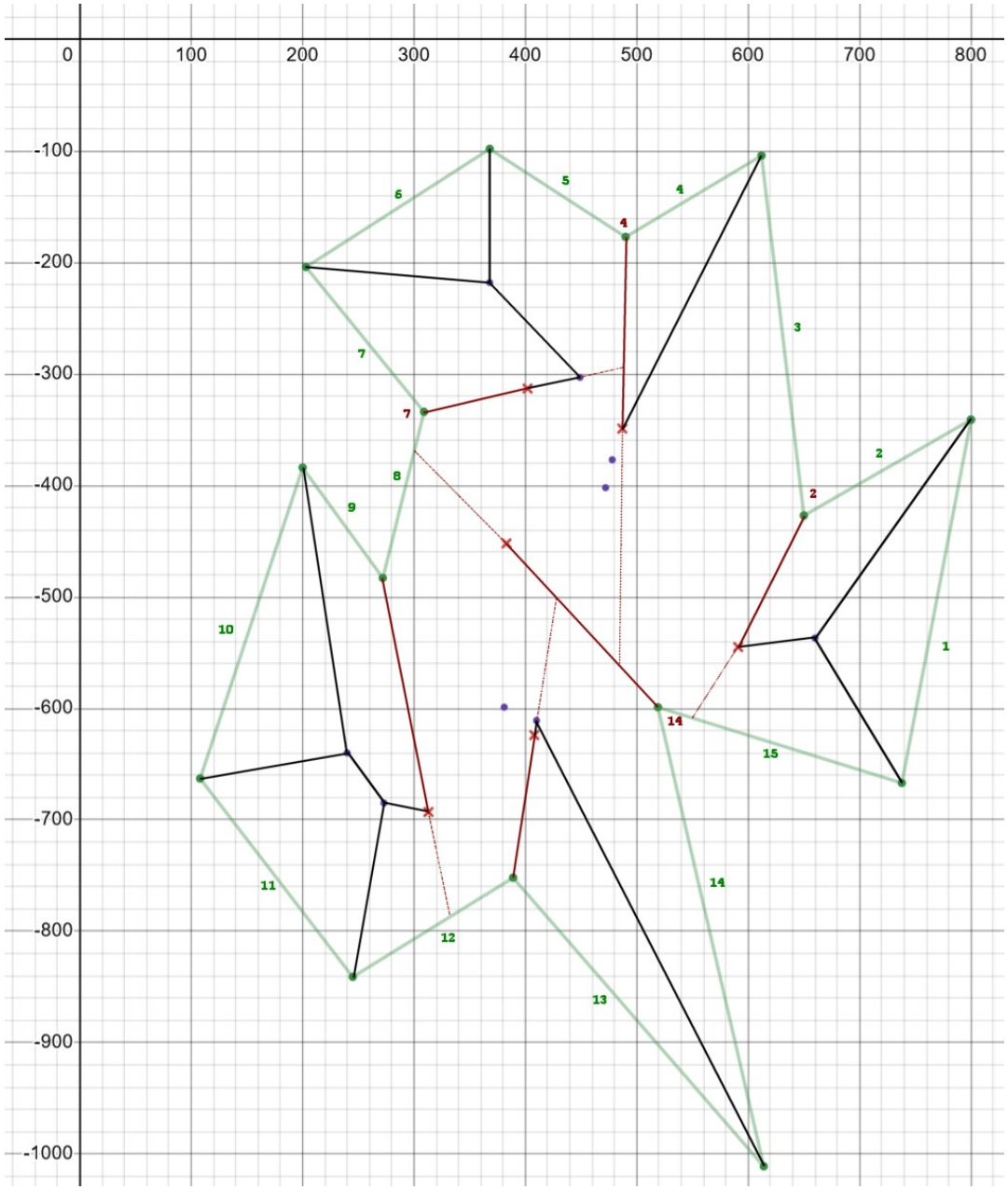

Fig. 3

Starting at each incomplete point construct a pair of skeleton segments that make the smallest possible angle away from the direction of the point. In the figure below skeleton segments for two of the incomplete points have been drawn as arrows, two starting at point A (green), and two at point B (blue).

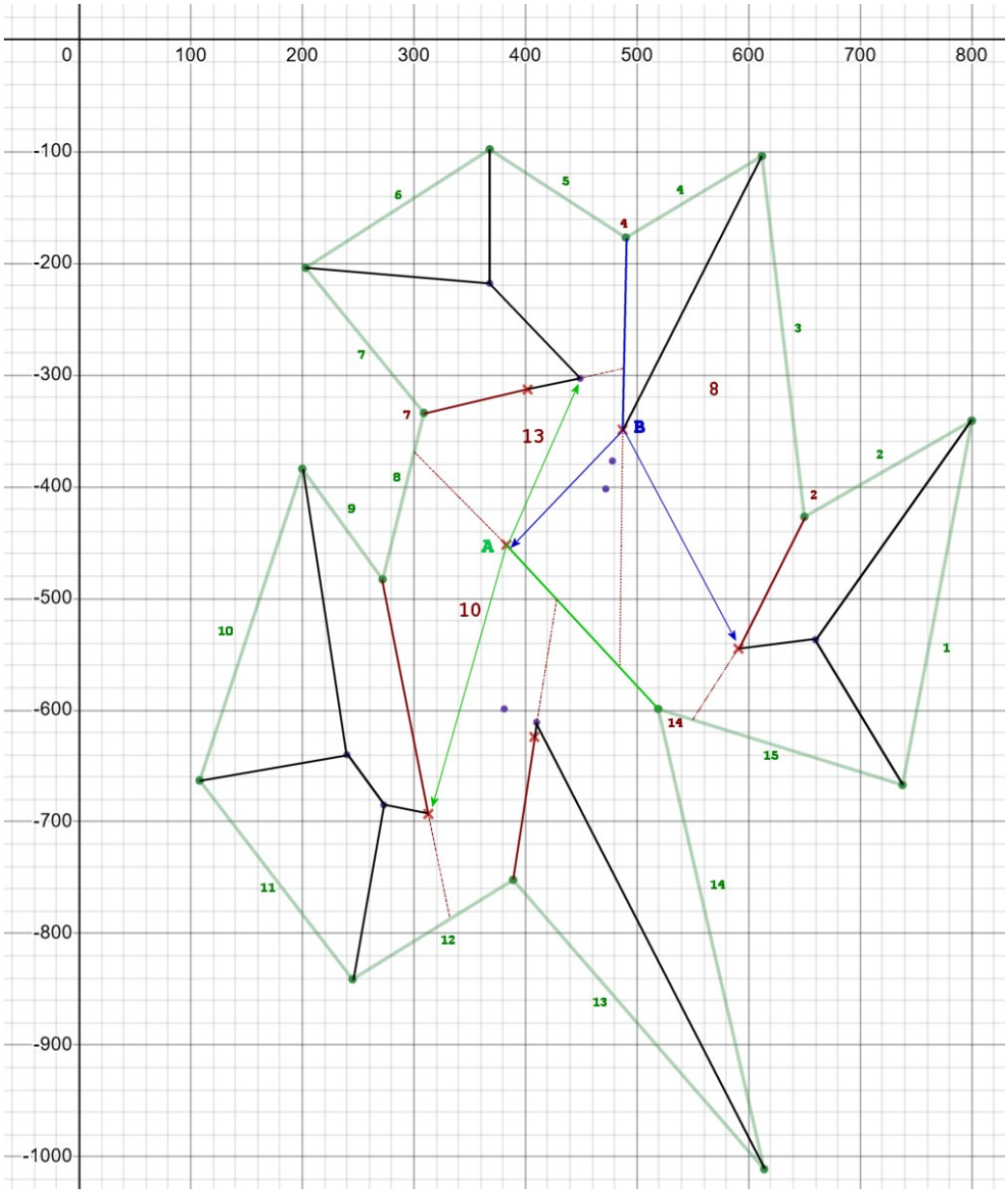

Fig. 4

The figure below shows all of the vectors for incomplete points using the rule for the process just described. Note that the vector immediately above point 14 is the only one where the head and tail of the arrow is not in the same region.

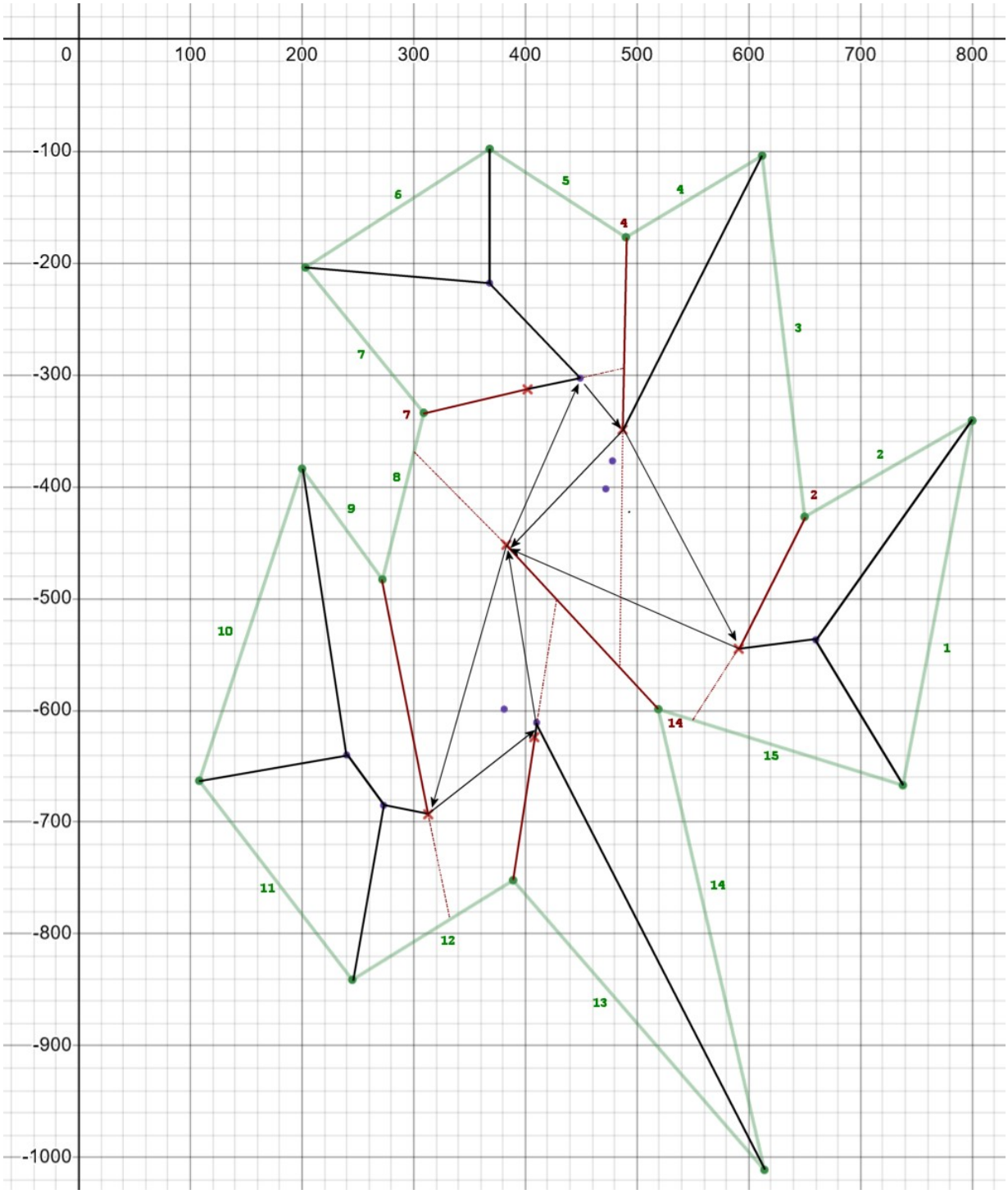

Fig. 4

Notice that there is always an even number of arrow heads and arrow tails where they meet except at two points, labeled A and B below. At A there are two arrow tails and one arrow head; at B there are three arrow heads and two tails. The anomalous situation can be resolved by eliminating the green arrow, resulting in an arrow head and tail at A and two arrow heads and two tails at B.

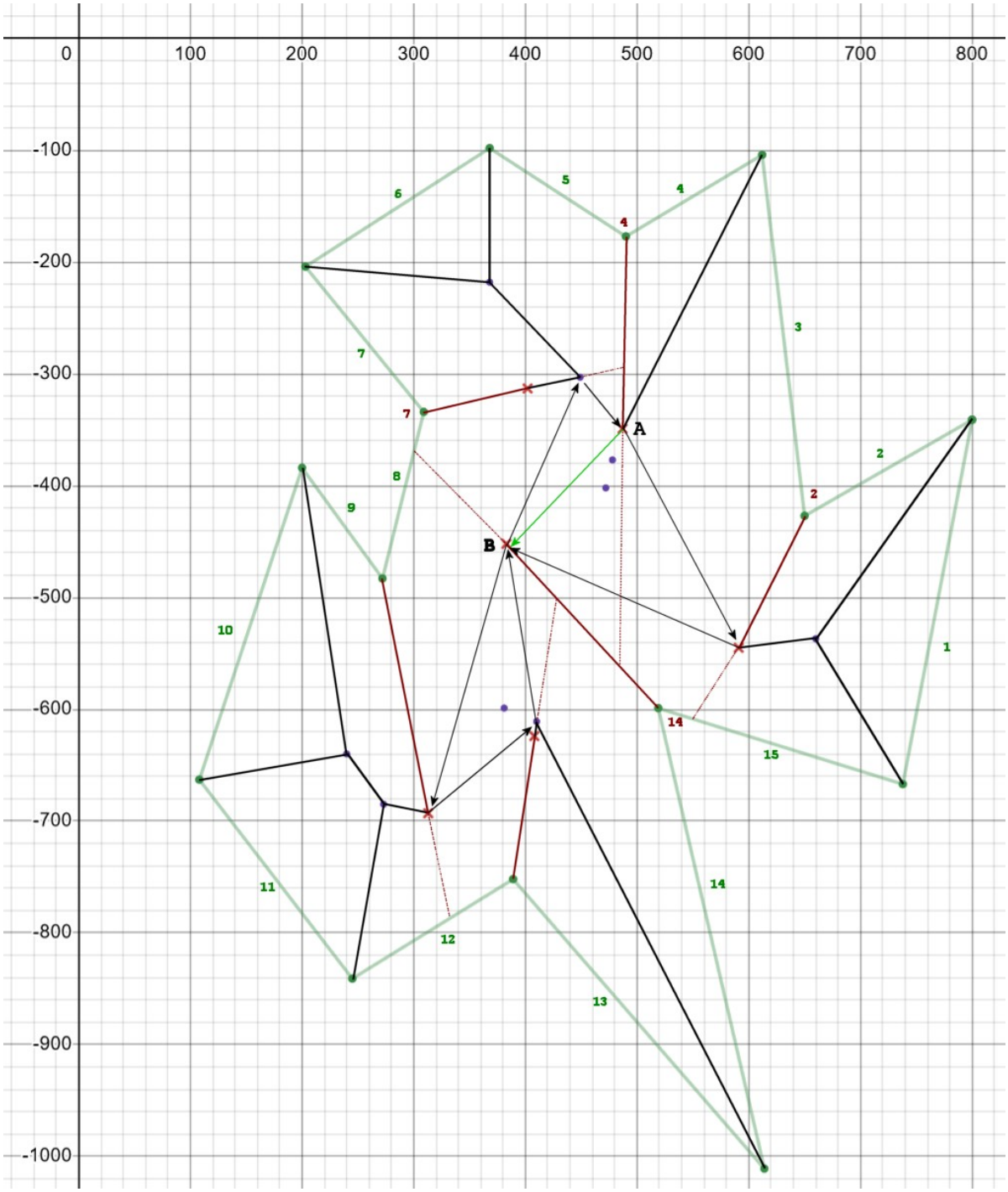

Fig. 6

At this point we have two convex areas which contain points higher than along any edge of the area. In the case of the lower triangular area, the region skeleton indicates a high point as the incenter of the edges 8, 12 and 14. This incenter of these edges is shown as a dot in the region

To find triple connection point of the four-sided area, the upper node of the skeleton corresponds to sides 8, 5 and 3, while the lower area associates sides 8, 15 and 3. The two sides that join these two incenters are 8 and 3. The two incenters are shown as dots in this region.

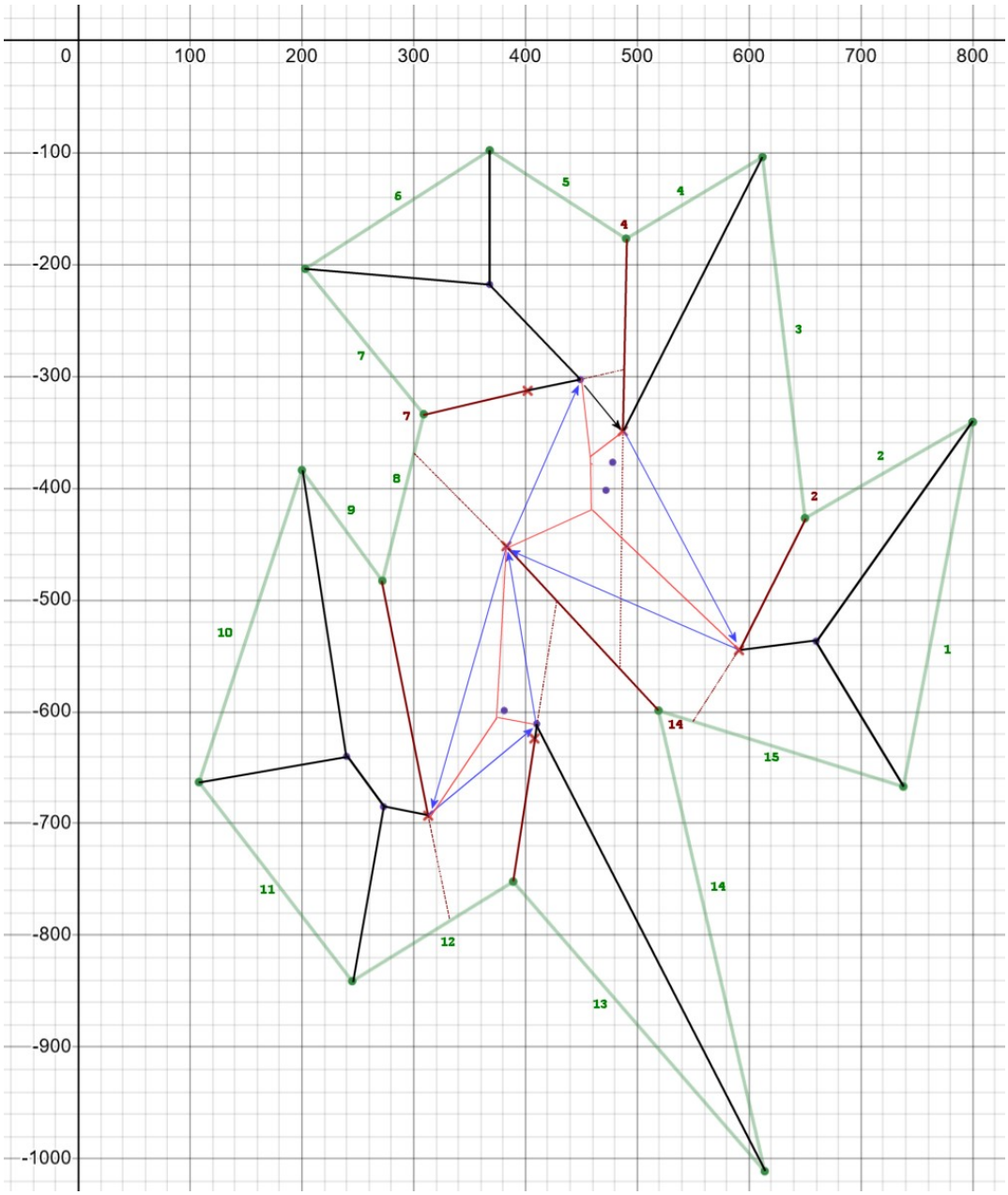

Fig. 7

Now, replace the region definitions and their internal skeletons with the incenter points and the corners of the regions. This replacement is shown below. This is the result skeleton.

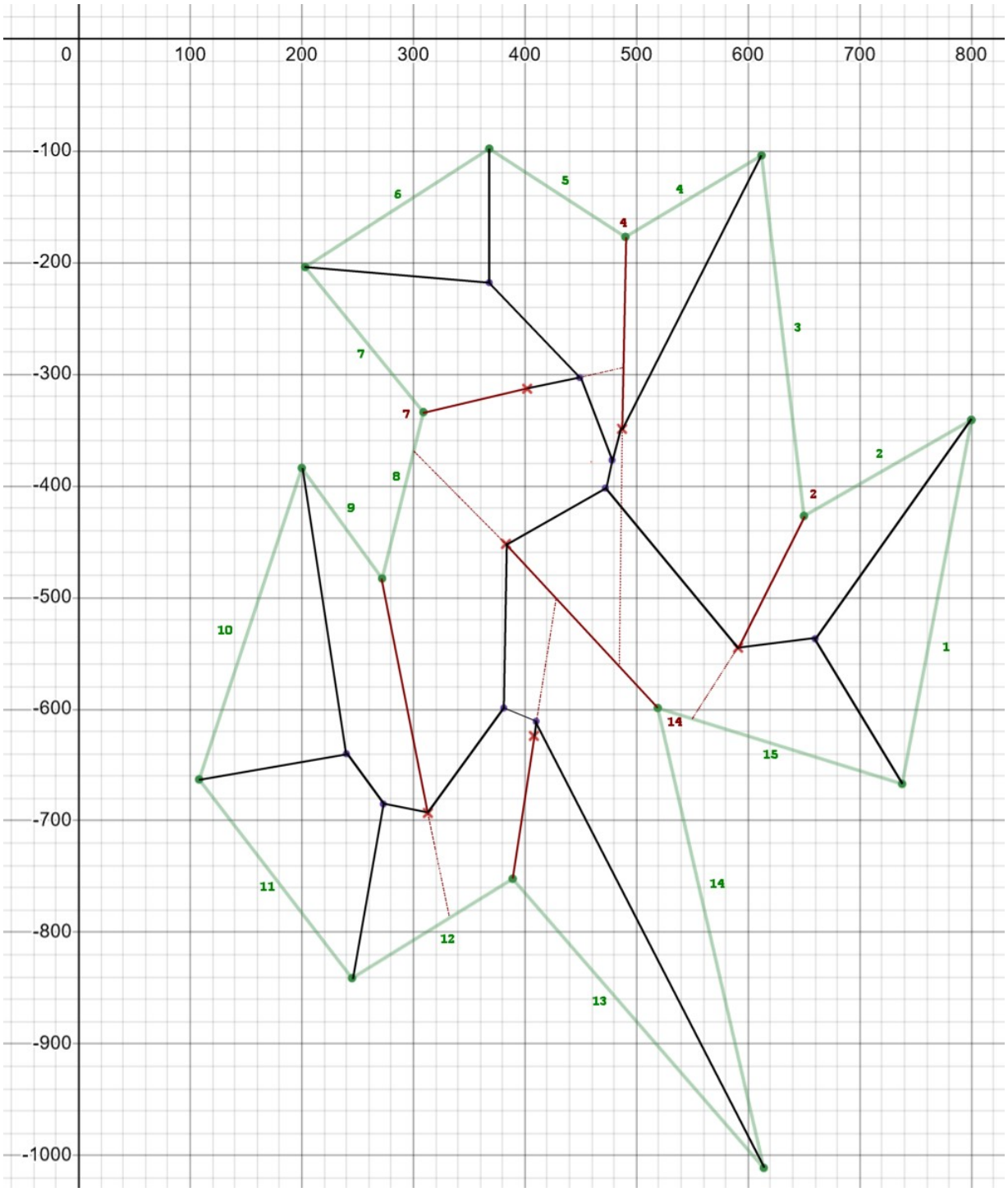

Fig. 7

For comparison, below is a medial axis skeleton of the same polygon.

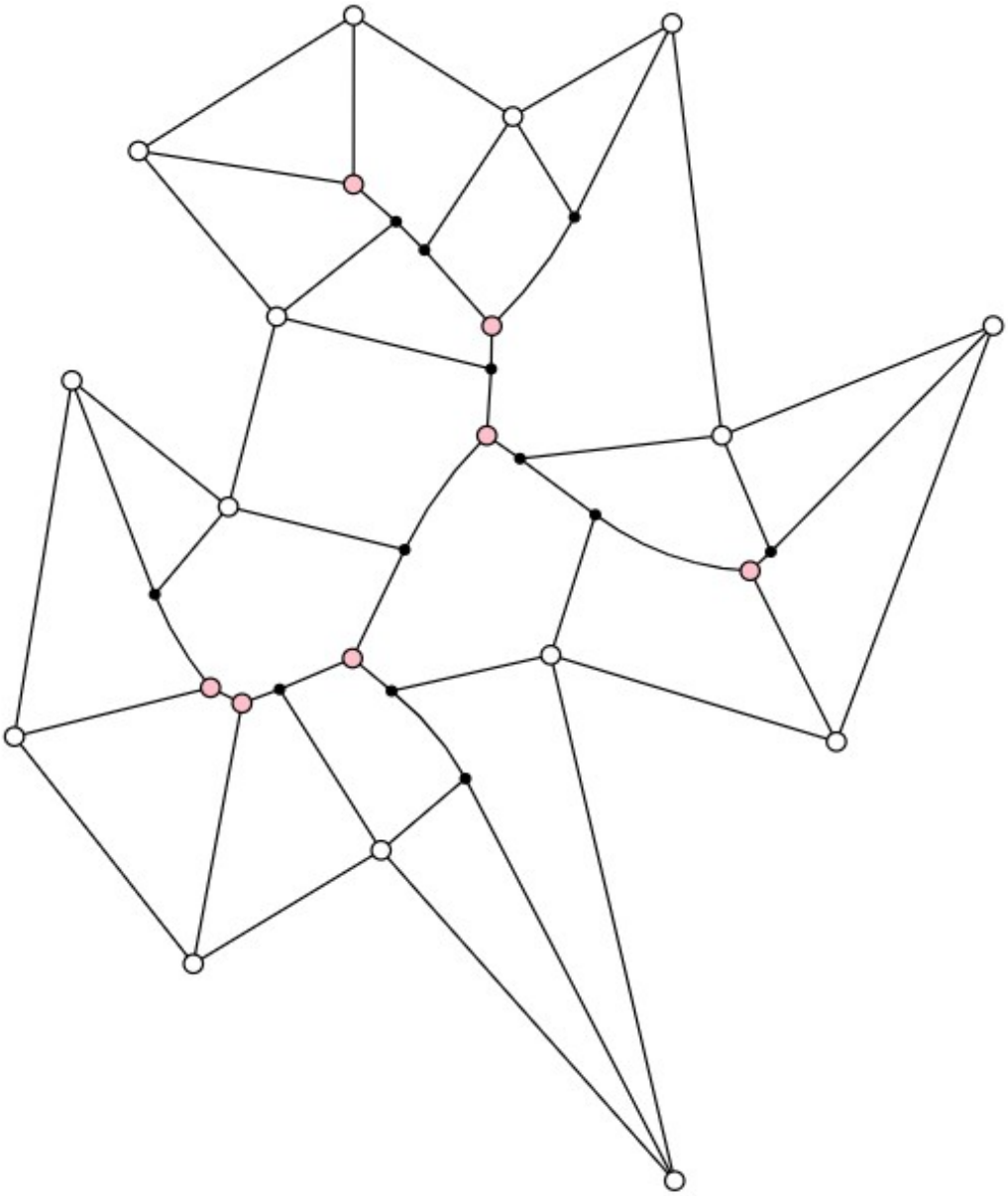

Fig. 8