

Determination of a Straight Skeleton from a Motorcycle Graph

April 23, 2024

Below is the starting point, a motorcycle graph of a polygon. As is noted elsewhere, a motorcycle graph can be generated from a triangulation of an n -sided polygon [order $n \cdot \log(n)$], ray tracing of v convex vertices in the triangulation [order $v \cdot \sqrt{n}$] and brute force intersection of motorcycle paths [order $v \cdot v$]. Note that each of the motorcycle regions is a convex polygon.

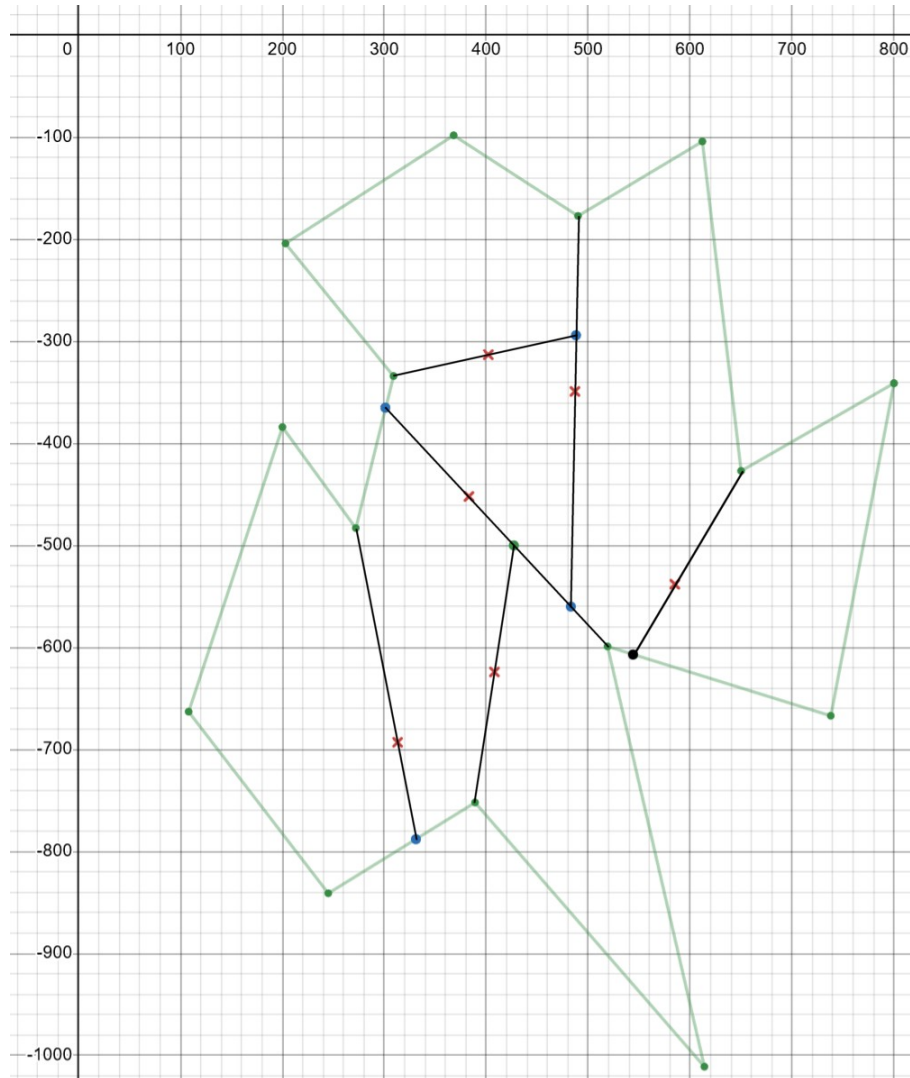


Fig. 1

The first step in the process is to calculate the “B” points for each of the motorcycle traces (red x in the graph above). These are the intersections of the bisector of a convex vertex and the bisector of a side forming the vertex and the target of the vertex bisector. These points are the ends of certain of the valleys defined by the convex vertices. Note that each of the regions is convex.

Eliminating the portion of the motorcycle track from a B point to the target side yields the following result.

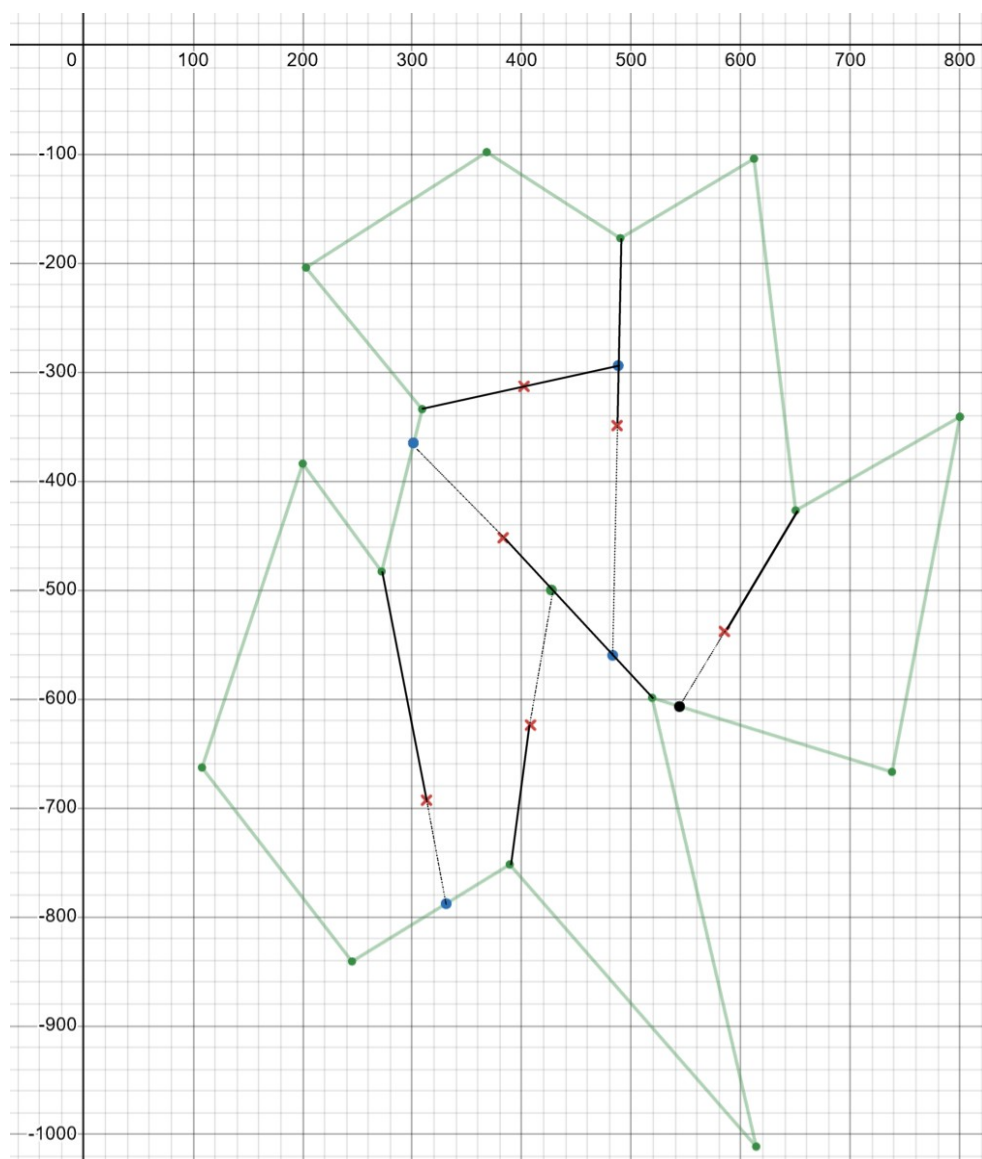


Fig. 2

The next step is to identify those regions having two adjacent sides on the boundary. Skeletons for these regions can be immediately determined. The skeleton on these regions does not include vertex or target points. Generation of the skeleton is stopped when a target point is encountered, as shown below. Where a node has three attached segments, the node is complete; where just one or two, the node is incomplete. In the figure below there are six incomplete nodes. We will next show how to complete the incomplete nodes.

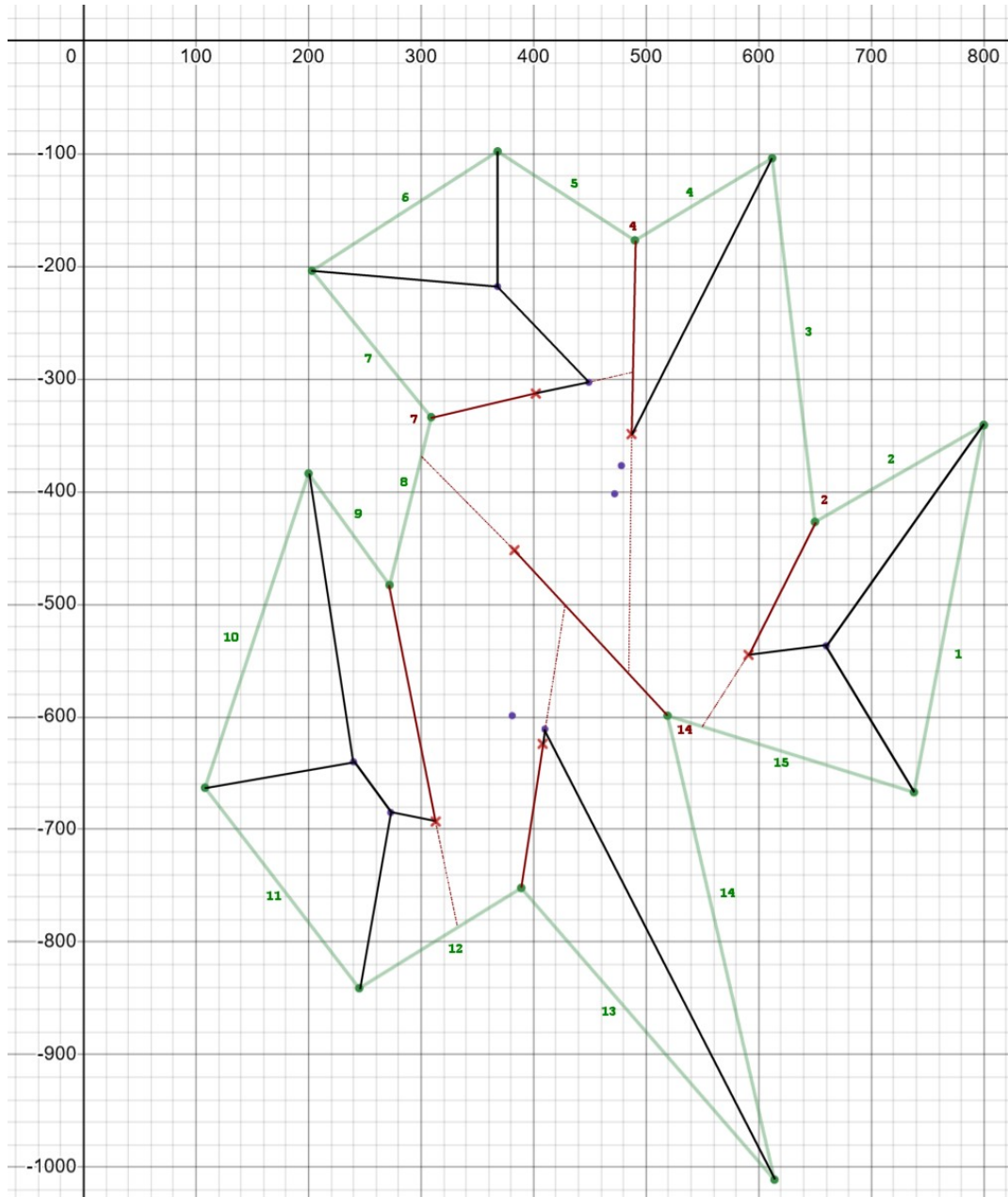


Fig. 3

At this point we will relink the motorcycle regions by removing the target points. This results in the paths show in the figure below (not all of the arrows are shown).

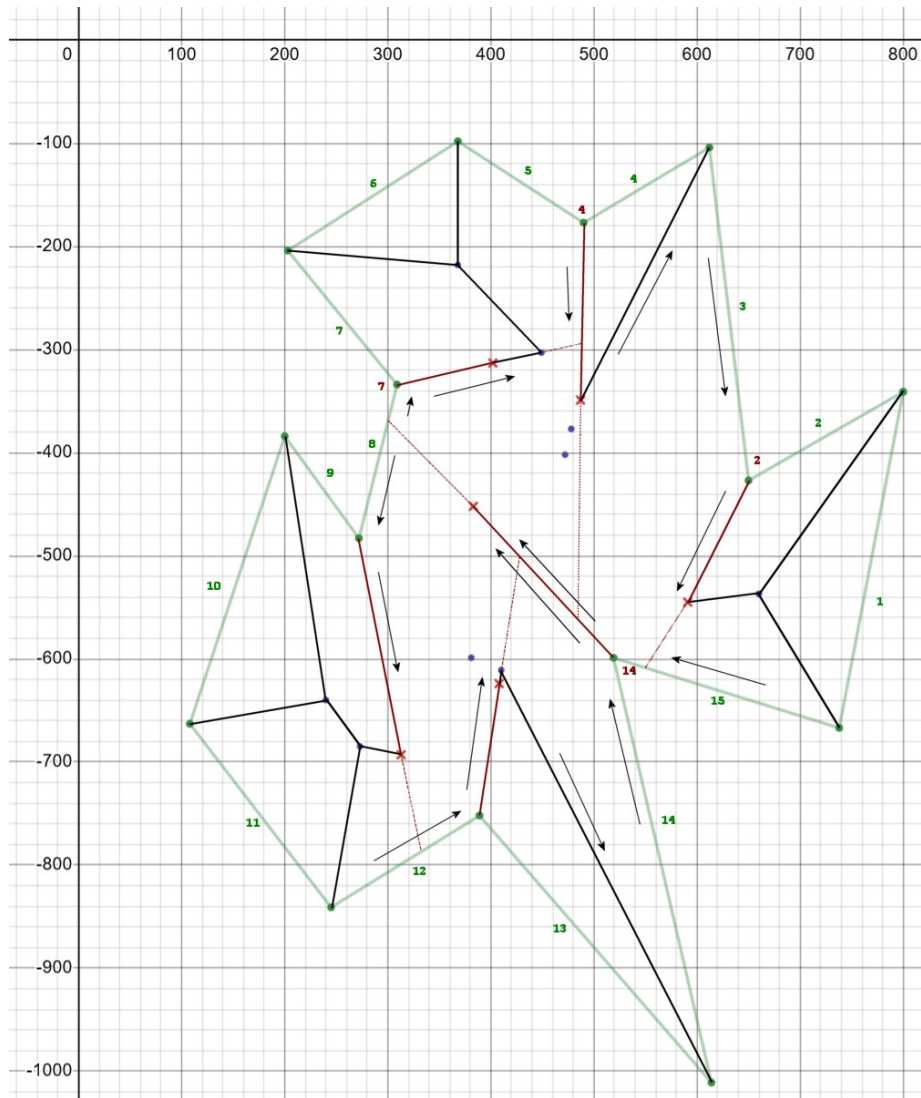


Fig. 4

The internal connections of the incomplete nodes can now be shown. From the B point for the bisector of vertex at point 14, we develop to vectors, one to the left and one to the right by traversing the paths determined by the relinking just described, ending the traversal when an incomplete point is found. For the traversal to the left, that point is marked with an X. The one on the right with Y. Vectors are placed between B and X and Y.

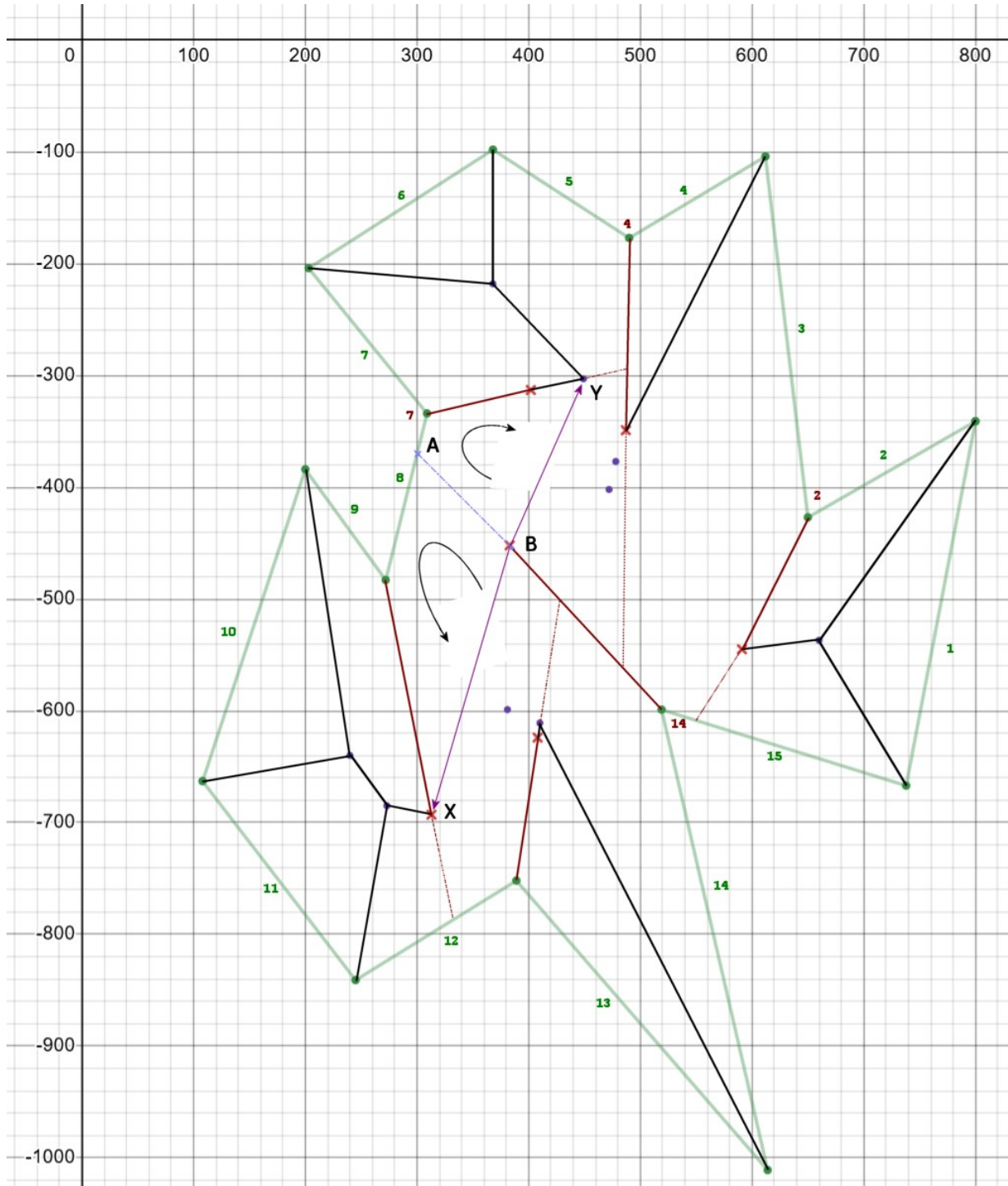


Fig. 5

Next we will consider the three regions having more than two adjacent boundary segments, shown below. We traverse the paths for the side of the incomplete node without a connecting segment, stopping when an incomplete node is encountered and a vector is placed between the two incomplete nodes, as shown below.

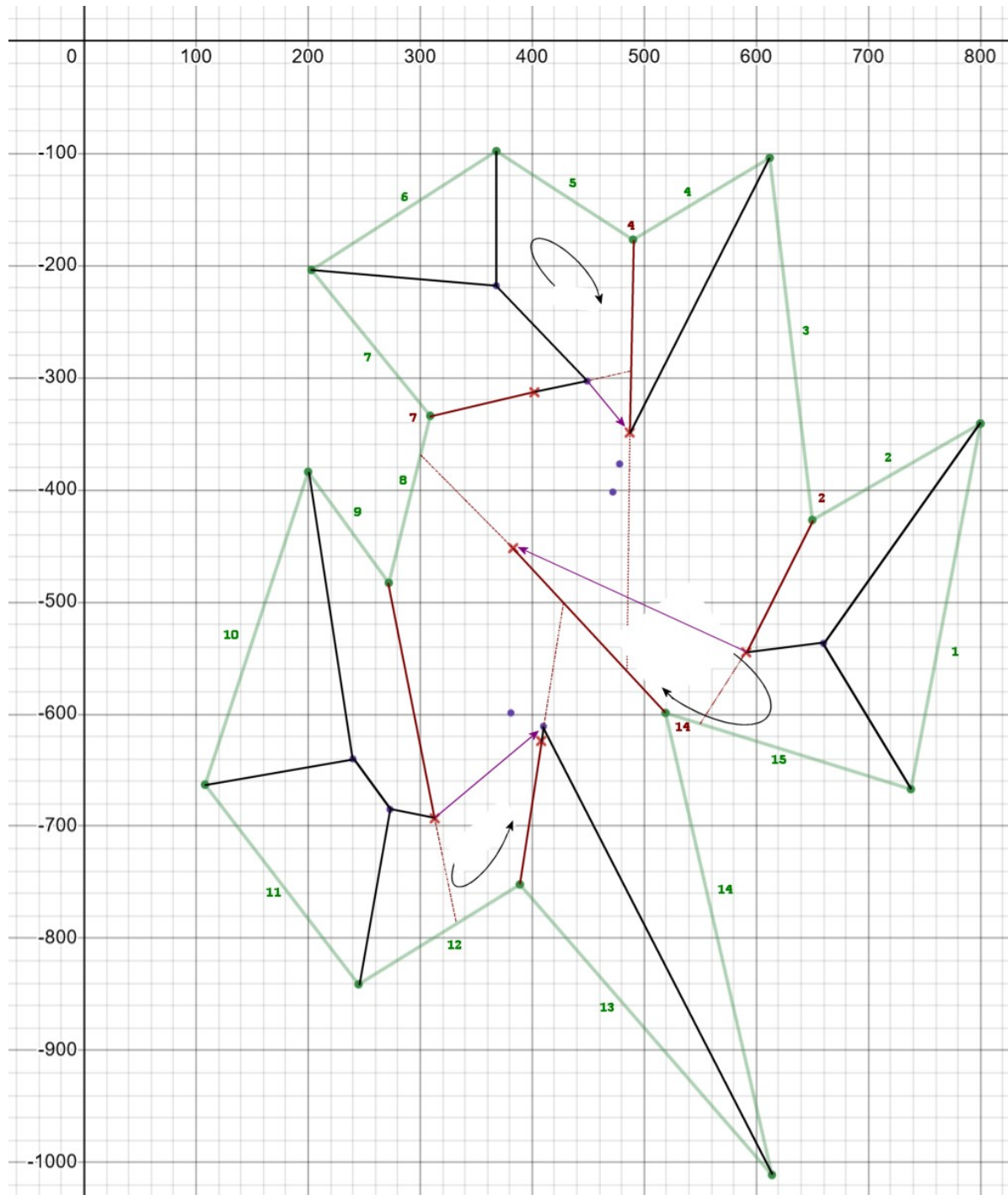


Fig. 6

For the regions having just two adjacent boundary segments, we undertake the same process as shown below. For the lower region the traversal starts at “s” and continues along sides A, B and C to point “f.” And similarly for the upper region.

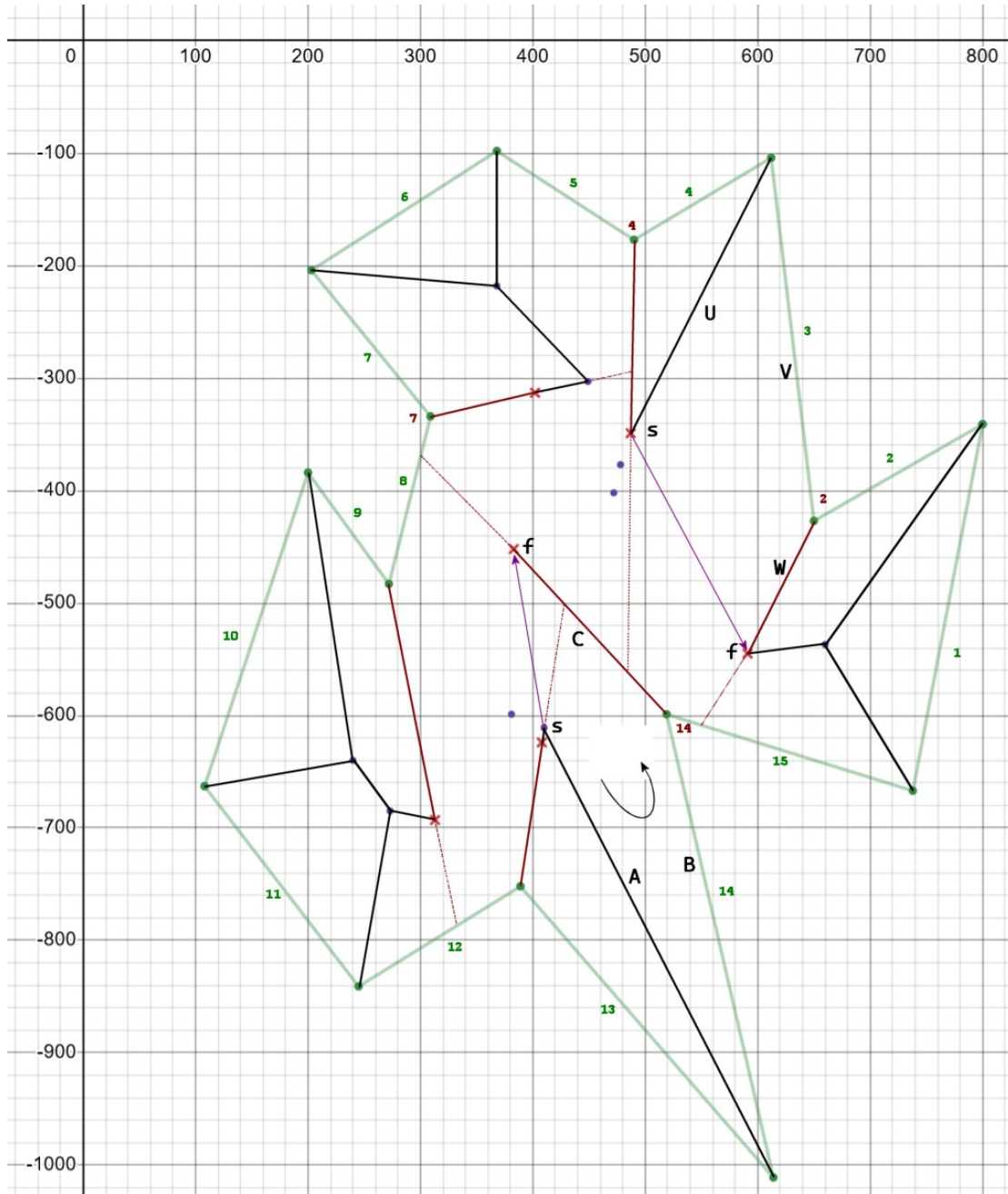


Fig. 7

In the figure below we bring together all of the vectors shown in Figures 5 through 7.: Green is from Fig. 4; blue is from Fig. 5; and orange is from Fig. 6. We can see that two regions are formed, holes where skeletons need to be defined.

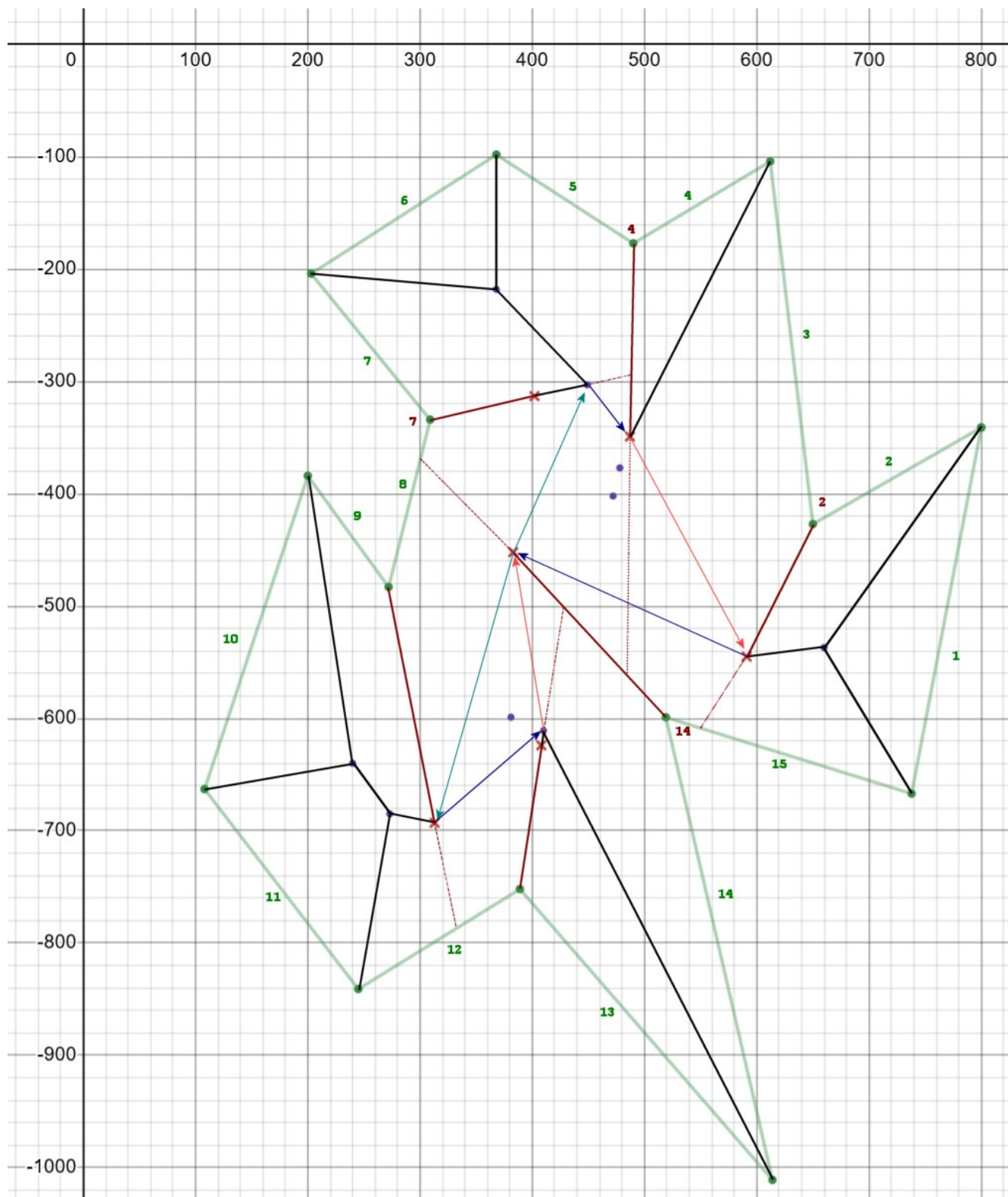


Fig. 6

The two convex hole areas contain points higher than along any edge of the area. In the case of the lower triangular area, the triangulation of the area indicates a high point as the incenter of the edges 8, 12 and 14. This incenter of these edges is shown as a dot in the triangular region.

To find triple connection points of the four-sided area, the upper node of the skeleton corresponds to sides 8, 5 and 3, while the lower area associates sides 8, 15 and 3. The two sides that join these two incenters are 8 and 3. The two incenters are shown as dots in this region.

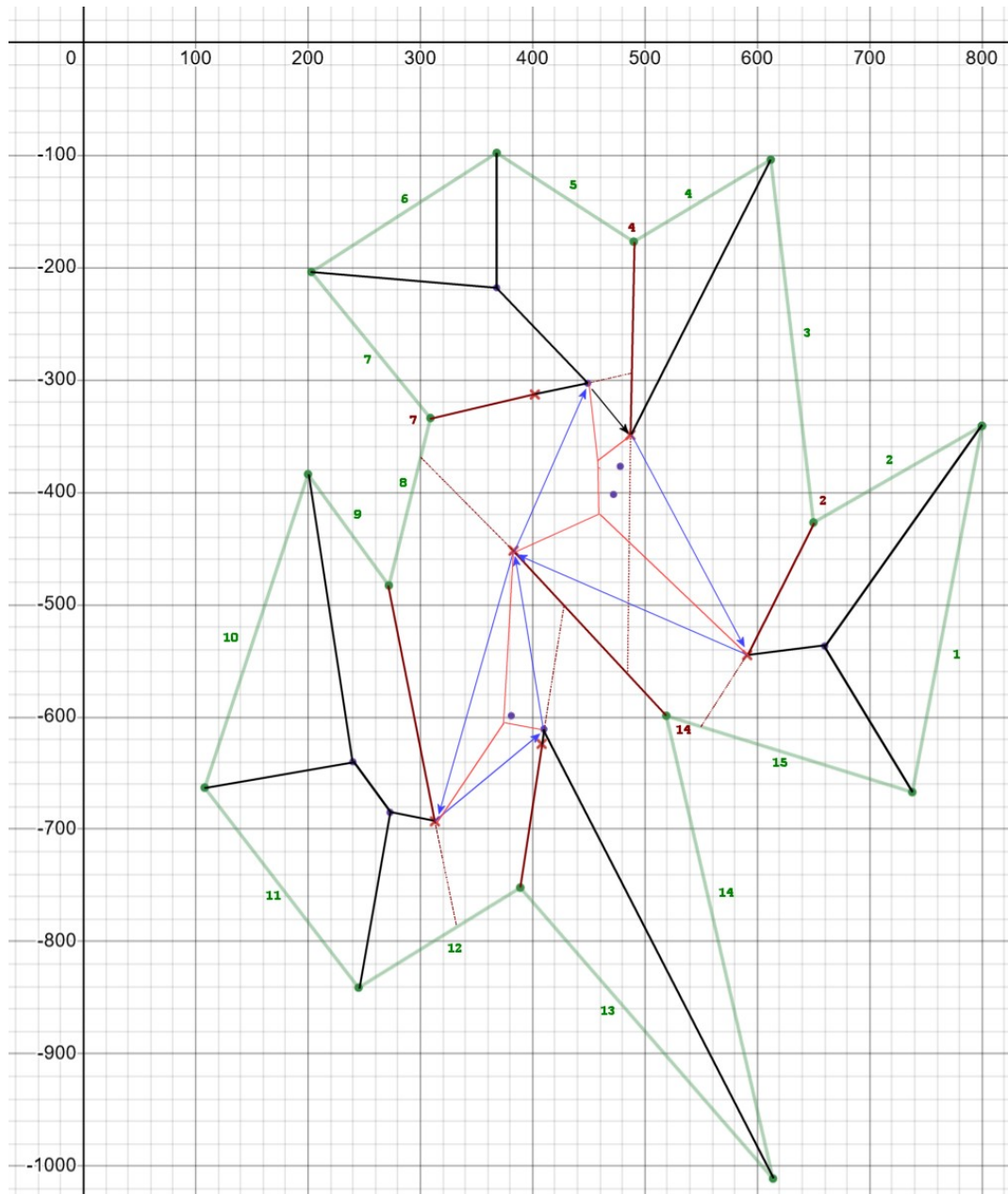


Fig. 7

Now, replace the region outlines and their internal skeletons with the incenter points extending to the appropriate corners of the regions. This replacement is shown below. This is the result skeleton.

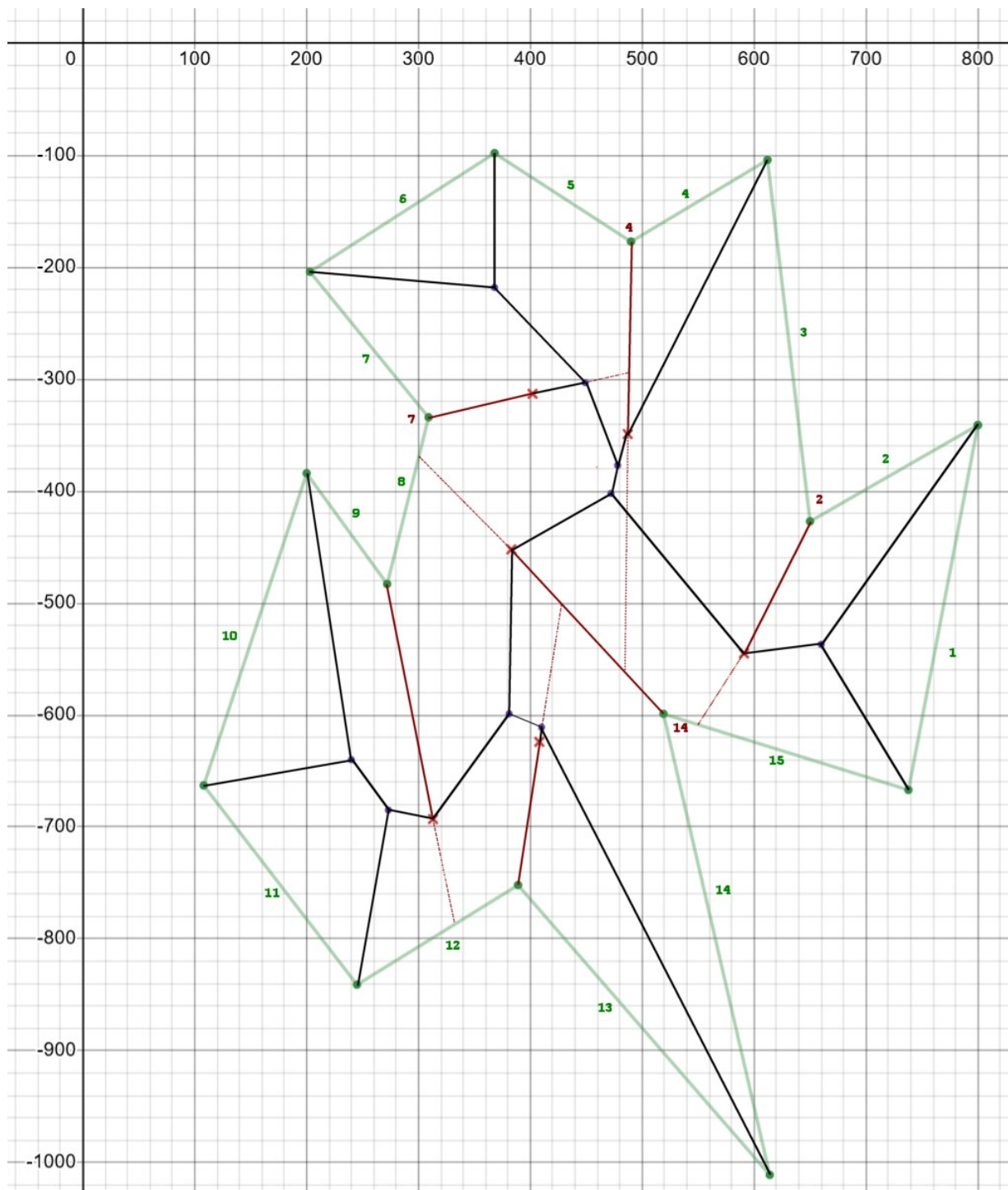


Fig. 8

For comparison, below is a medial axis skeleton of the same polygon.

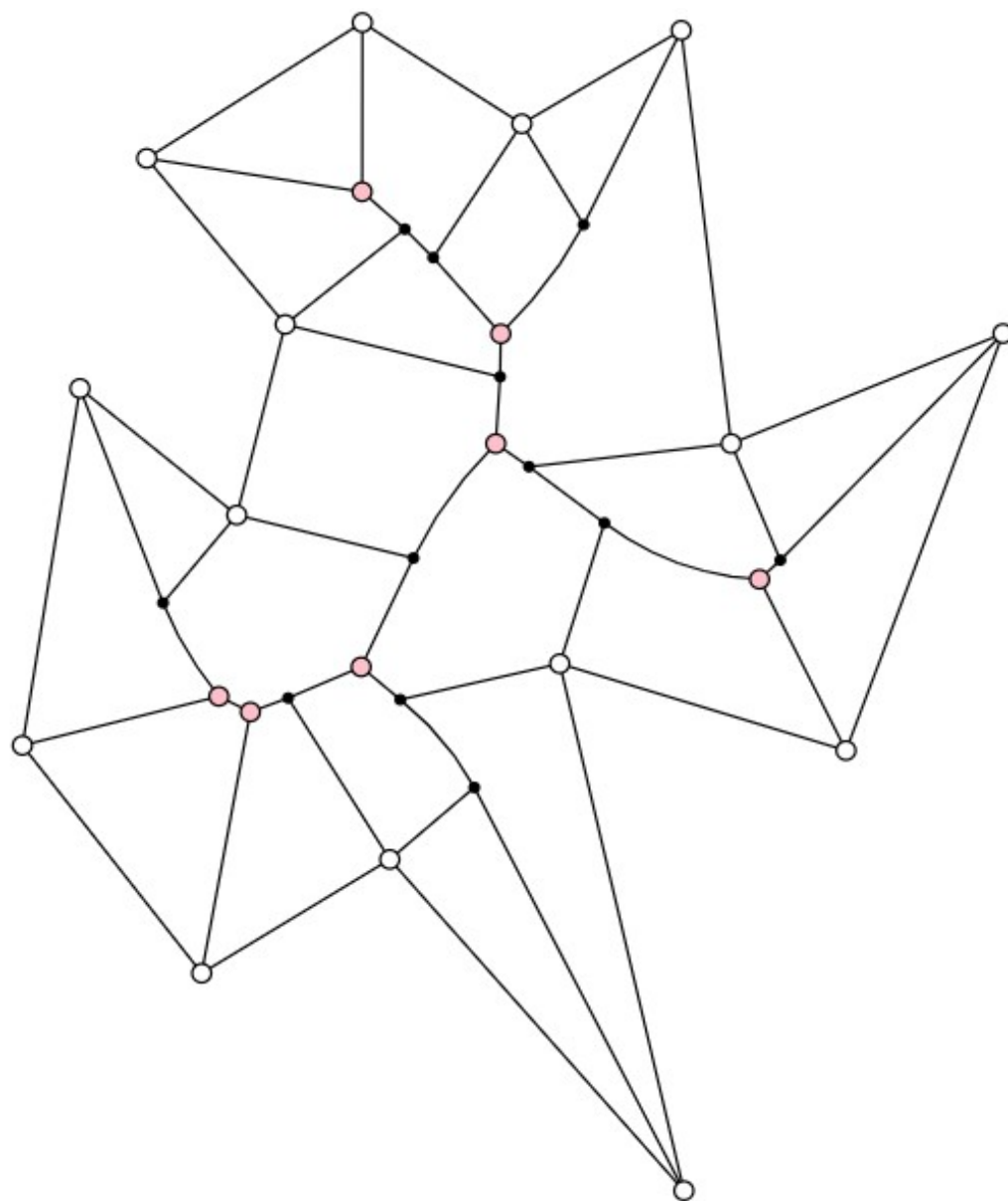


Fig. 9