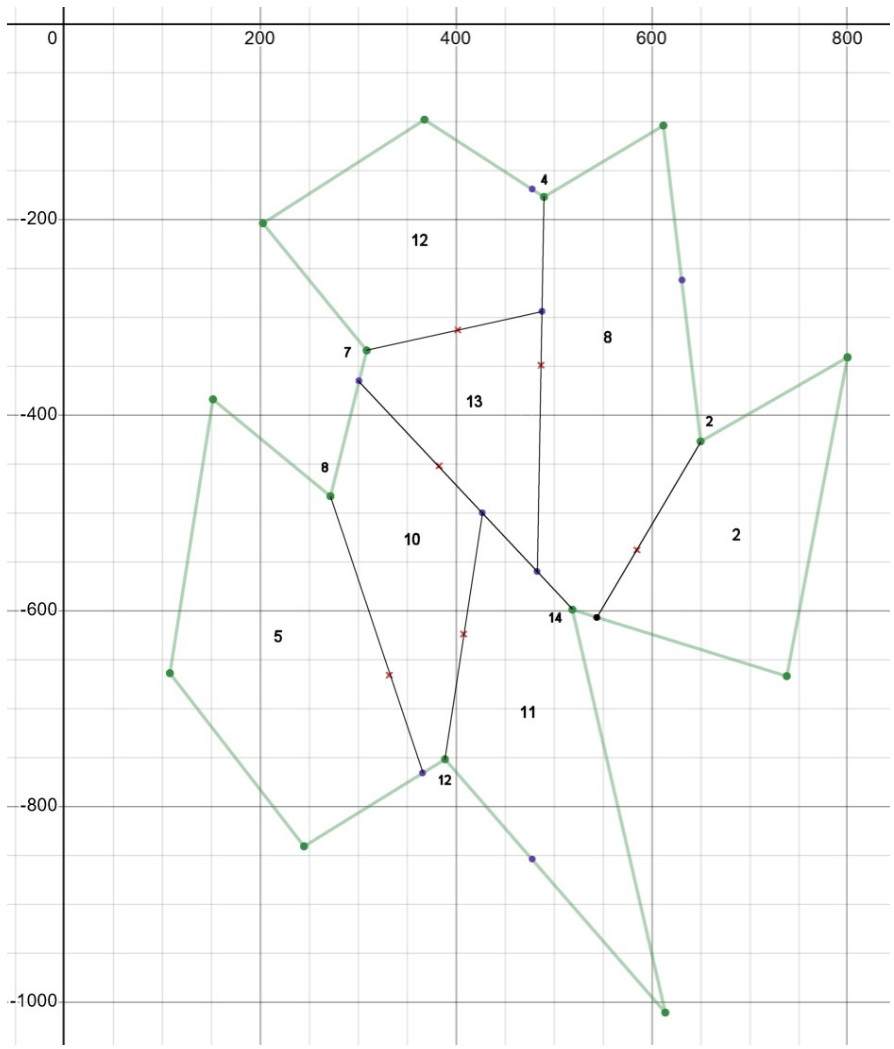
Determination of a Straight Skeleton from a Motorcycle Graph
April 12, 2024

Below is the starting point, a motorcycle graph of a polygon. As is noted elsewhere, a motorcycle graph can be generated from a triangulation of an n-sided polygon [order n·log(n)], ray tracing of v convex vertices in the triangulation [order v·sqrt(n)] and brute force intersection of motorcycle paths [order v·v]. Note that each of the motorcycle regions is a convex polygon.



The first step in the process is to calculate the "B" points for each of the motorcycle traces (red x in the graph above). These are the intersections of the bisector of a convex vertex and the bisector of a side forming the vertex and the target of the vertex bisector. These points are the ends of the valleys defined by the convex vertices.

Eliminating the portion of the motorcycle track from the B point to the target yields in the following result. This is a graph of the valleys associated with each convex vertex.

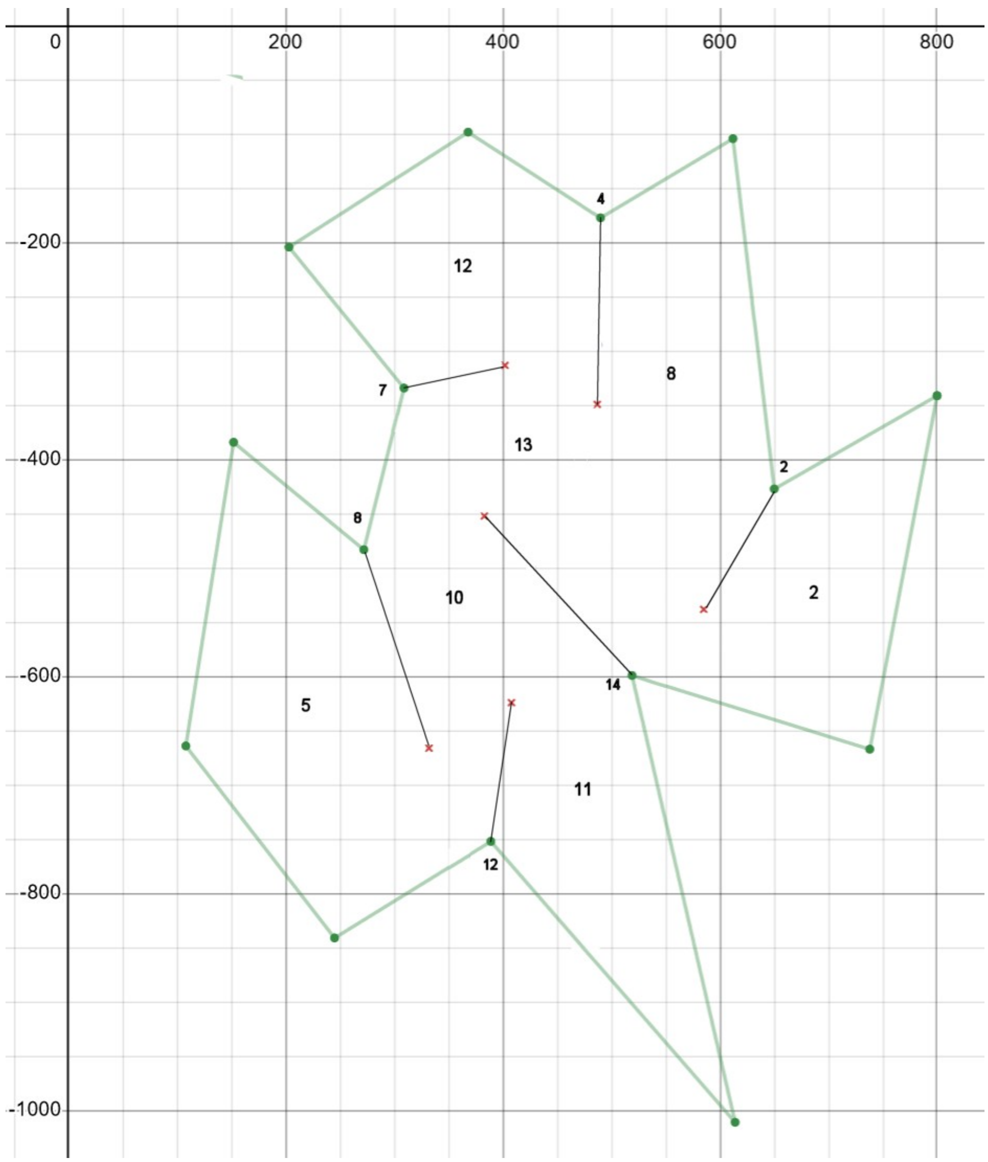


Fig. 1

The next step is to identify those regions containing either just one boundary point, or having two adjacent points on the boundary. Skeletons for these regions can be immediately determined.

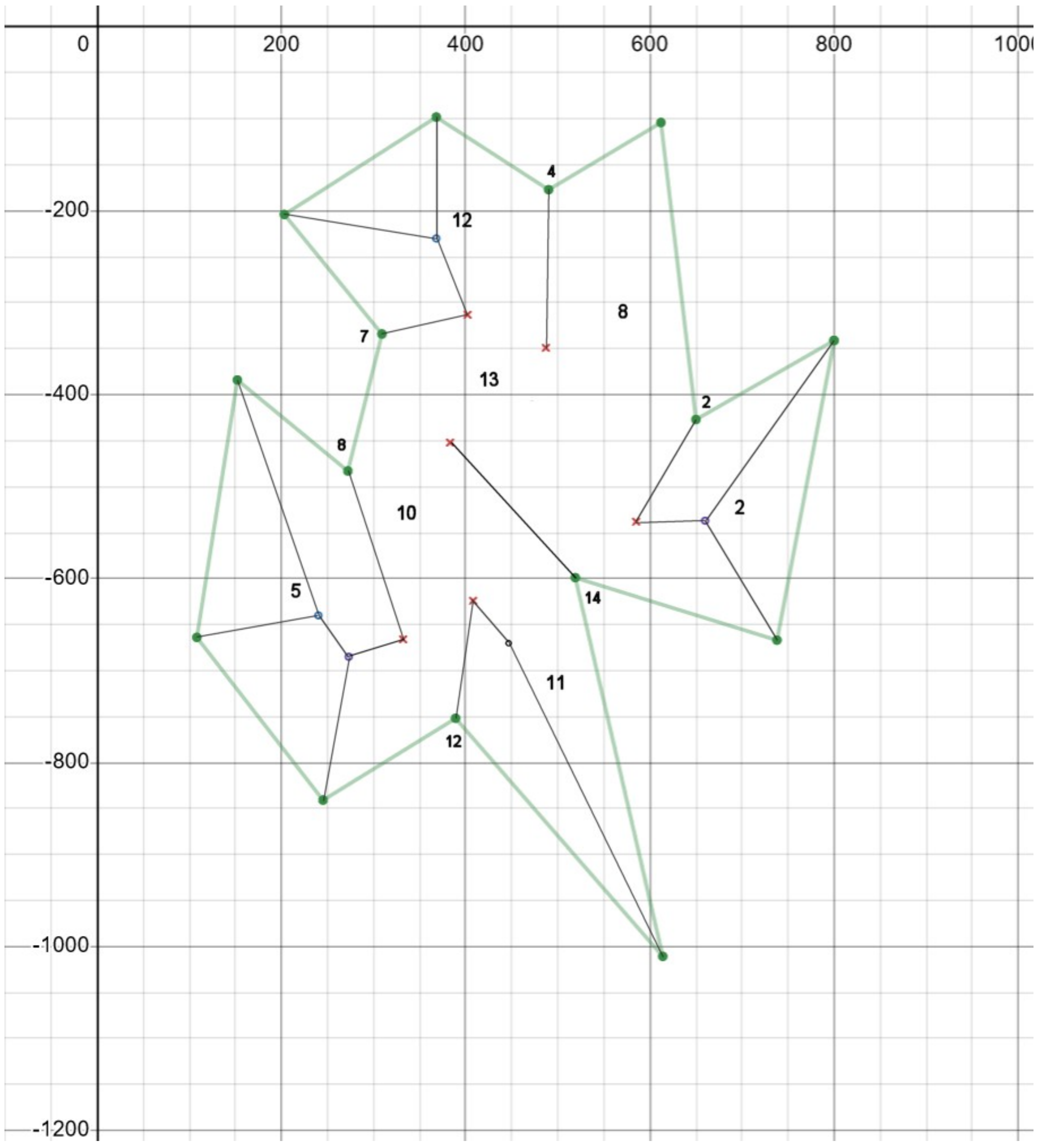


Fig. 2

For each B point there is a pair of skeleton segments that make the sharpest possible angle at the B point. These segments are shown (when possible) as pointed lines below when the segment is entirely within a region. (Previously determined segments lying outside the convex hull of the B points are not considered candidate segments.)

Notice that we have one cycle "hole" near the Region 10 label, and an unfinished "hole" to the left, right and above the Region 13 label. We know that this hole is unfinished, as the B point for vertex 14 has only three connecting segments, whereas B for vertex 2 has only one. (We have to have an even number of segments at each B point.)

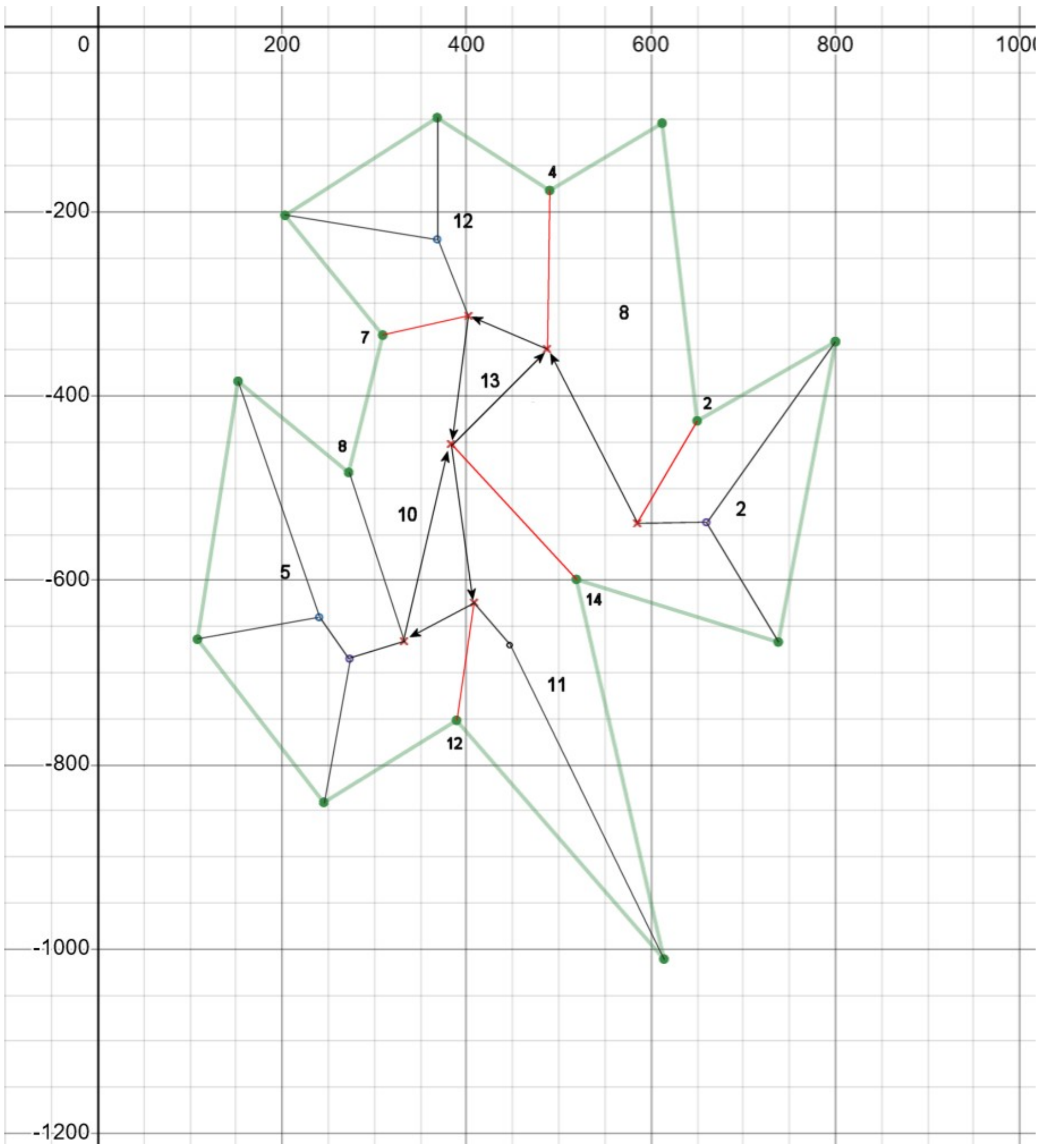


Fig. 3

To close the unclosed chain we step backwards in the unfinished chain starting at the tail of the segment with an odd number of segments at its head. Proceeding to the tail of the segments that shares a head, this second time we find (see next page) that the tail of that segment satisfies the "must be an even number of connectors" condition.

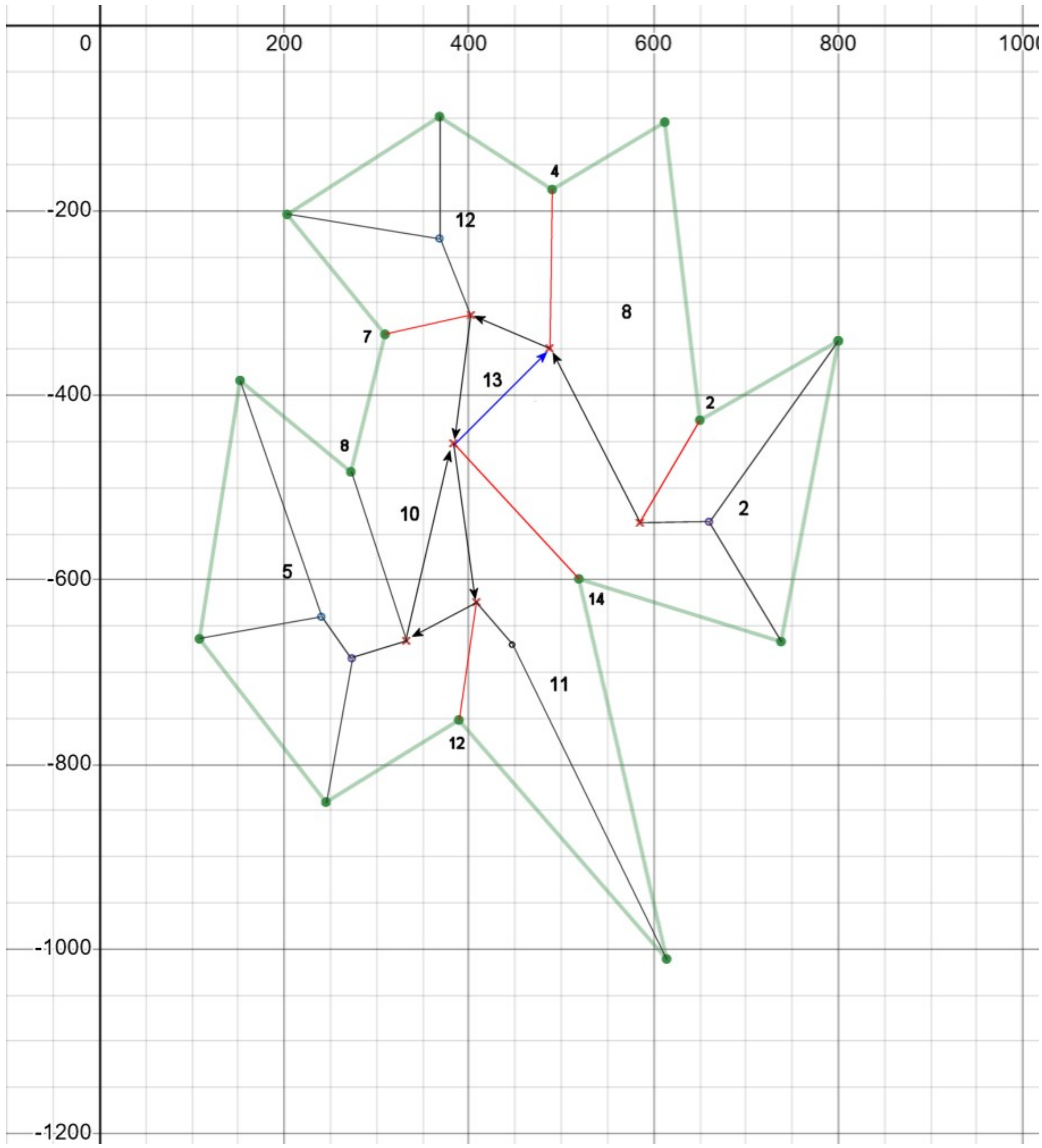
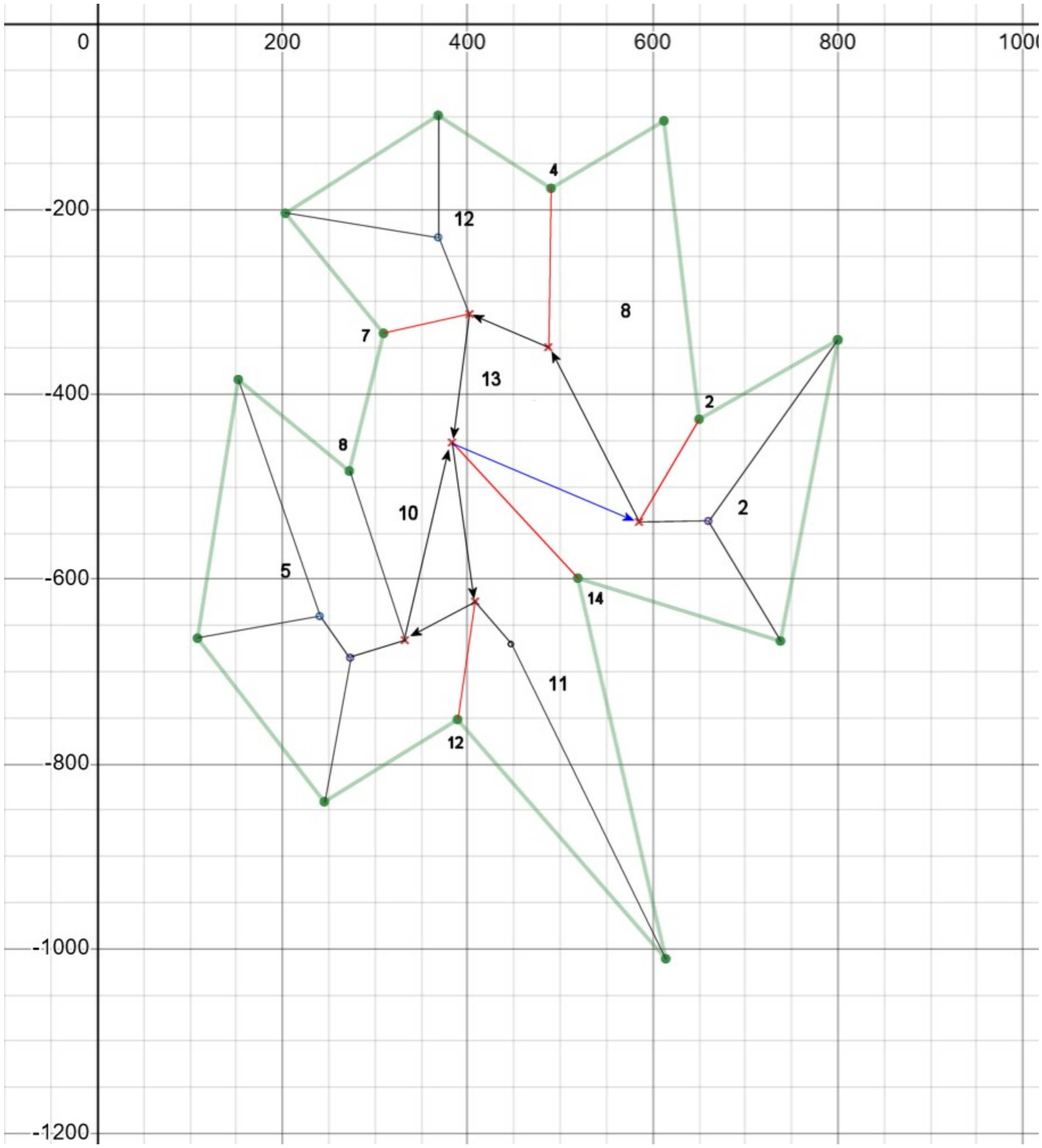


Fig. 4



0
200
400
600
800
-200
-400
-600
-800
-1000
-1200
4
12
8
7
13
2
8
10
2
5
14
11
12

Fig. 6

At this point we have two convex areas which contain a point higher than any edge of the area. In the case of the triangular area, the high point is simply the incenter of the edges ending at 8, 12 and 14.

To find the triple connection points of the four-sided area, we will determine its skeleton and use that information to identify the edges contributing to these points. Note that there are two points in the figure where three lines meet. The upper triple is associated with the sides 7-8, 4-5 and 2-3. The lower is associated with 7-8, 2-3 and 14-15.

Note that the skeleton segment bisecting point three in Region 8 doesn't intersect beyond point B of vertex 4, therefore it must intersect the point 4 bisector at its B point.

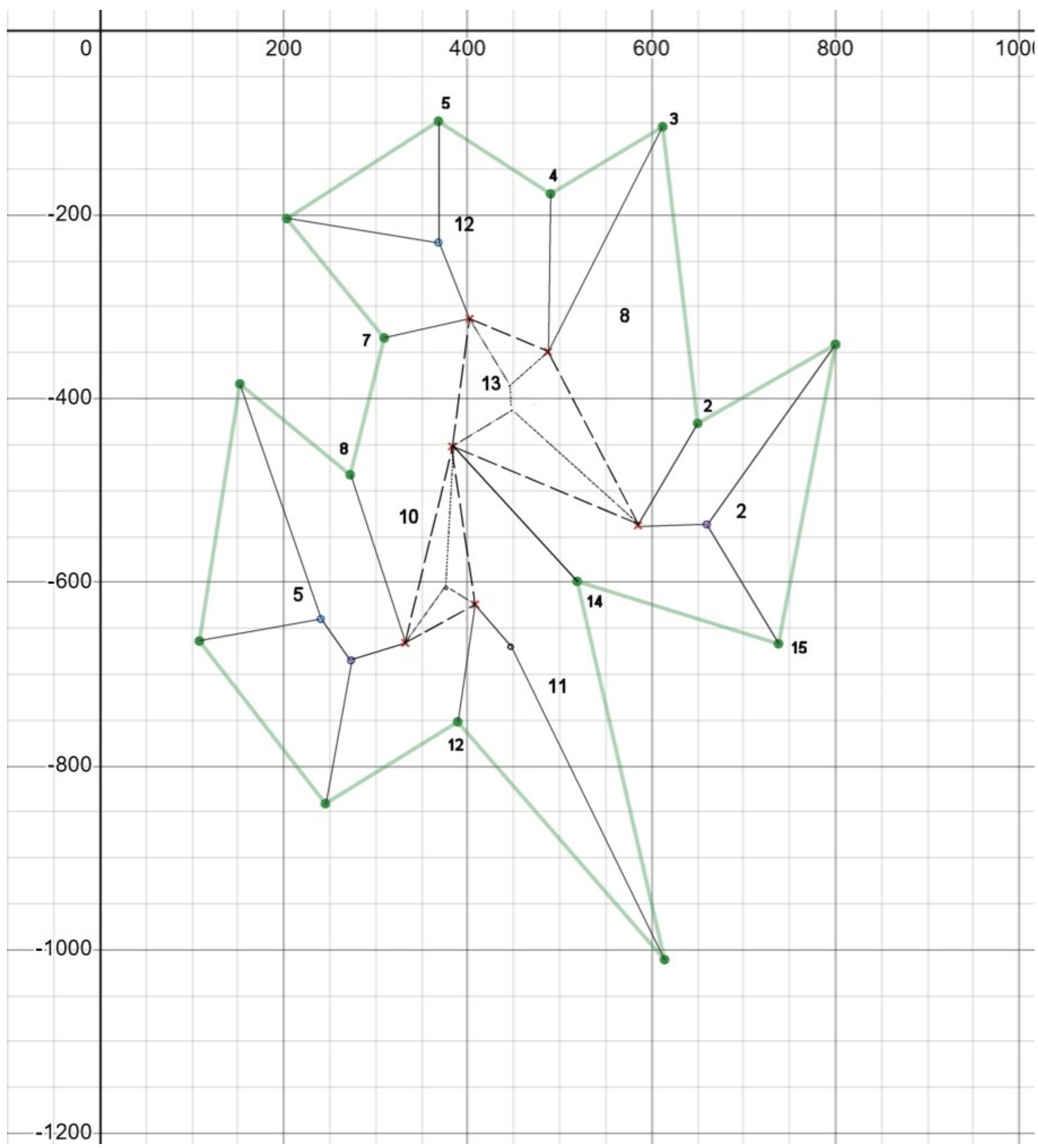


Fig. 7

Add segments between the incenter and the sides associated with sides 7-8, 14-15 and 2-3.

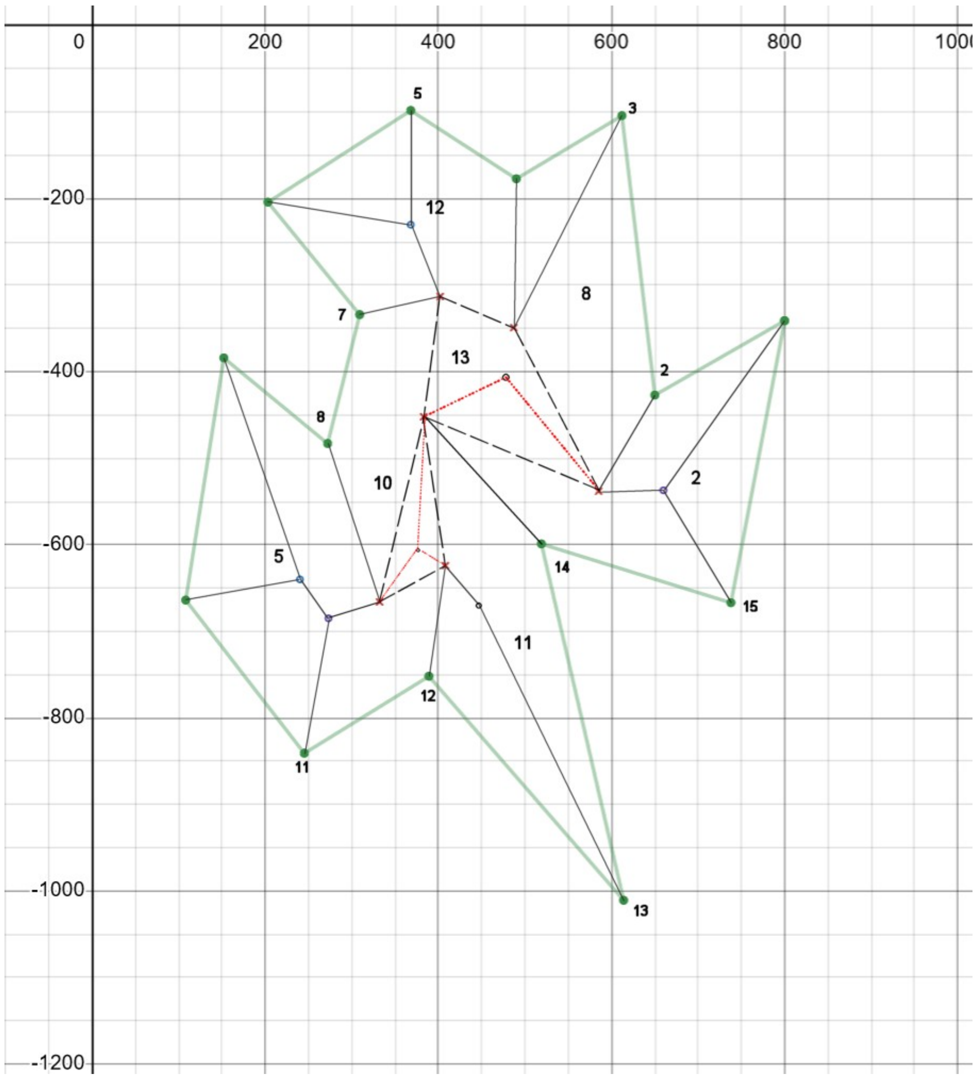


Fig. 8

Below is the situation when the skeleton is created for the incenter of sides 7-8, 2-3 and 4-5.

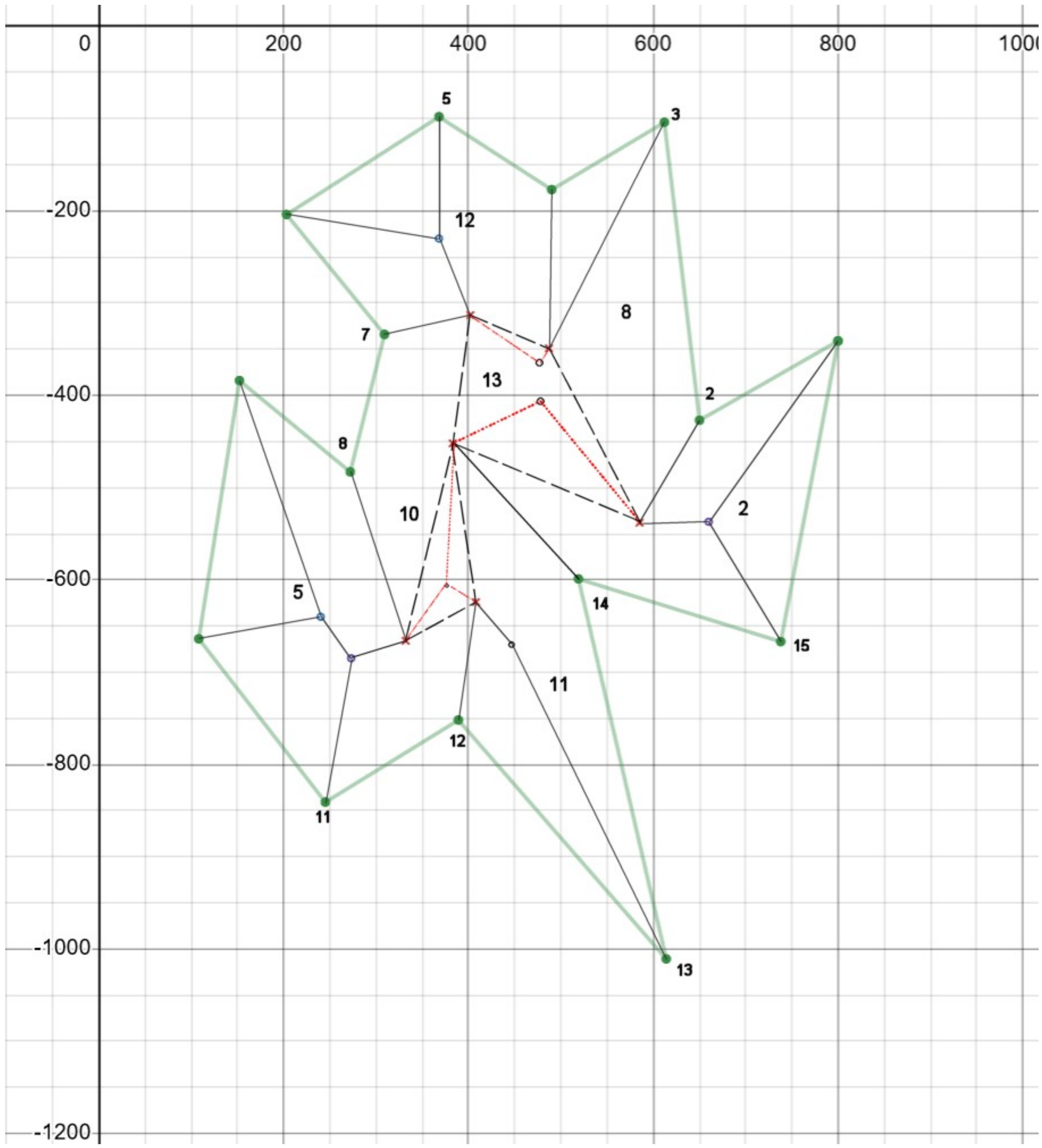


Fig. 9

Add the connector between the two incenters.

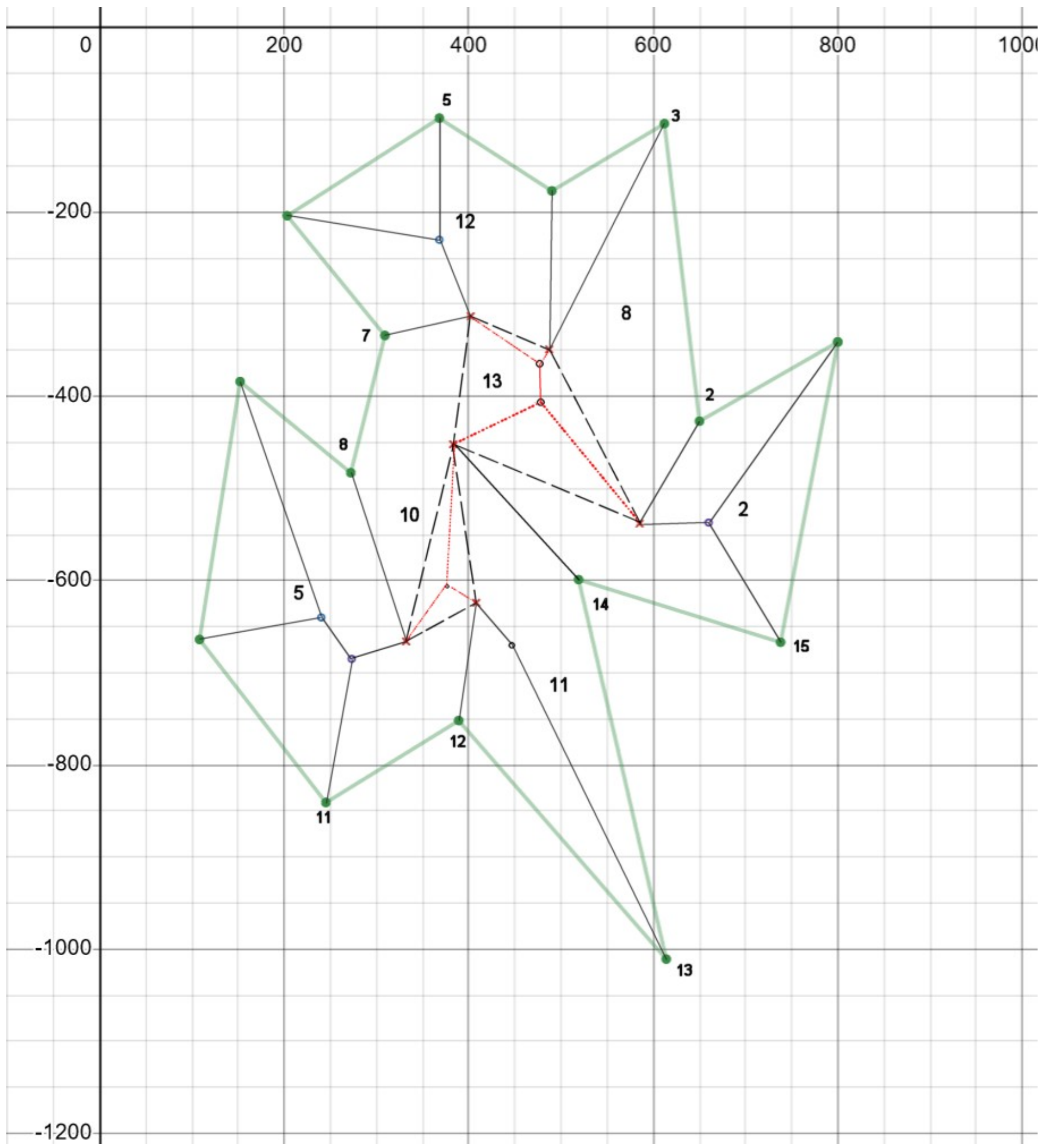


Fig. 10

Finally, we have the complete skeleton, shown below.

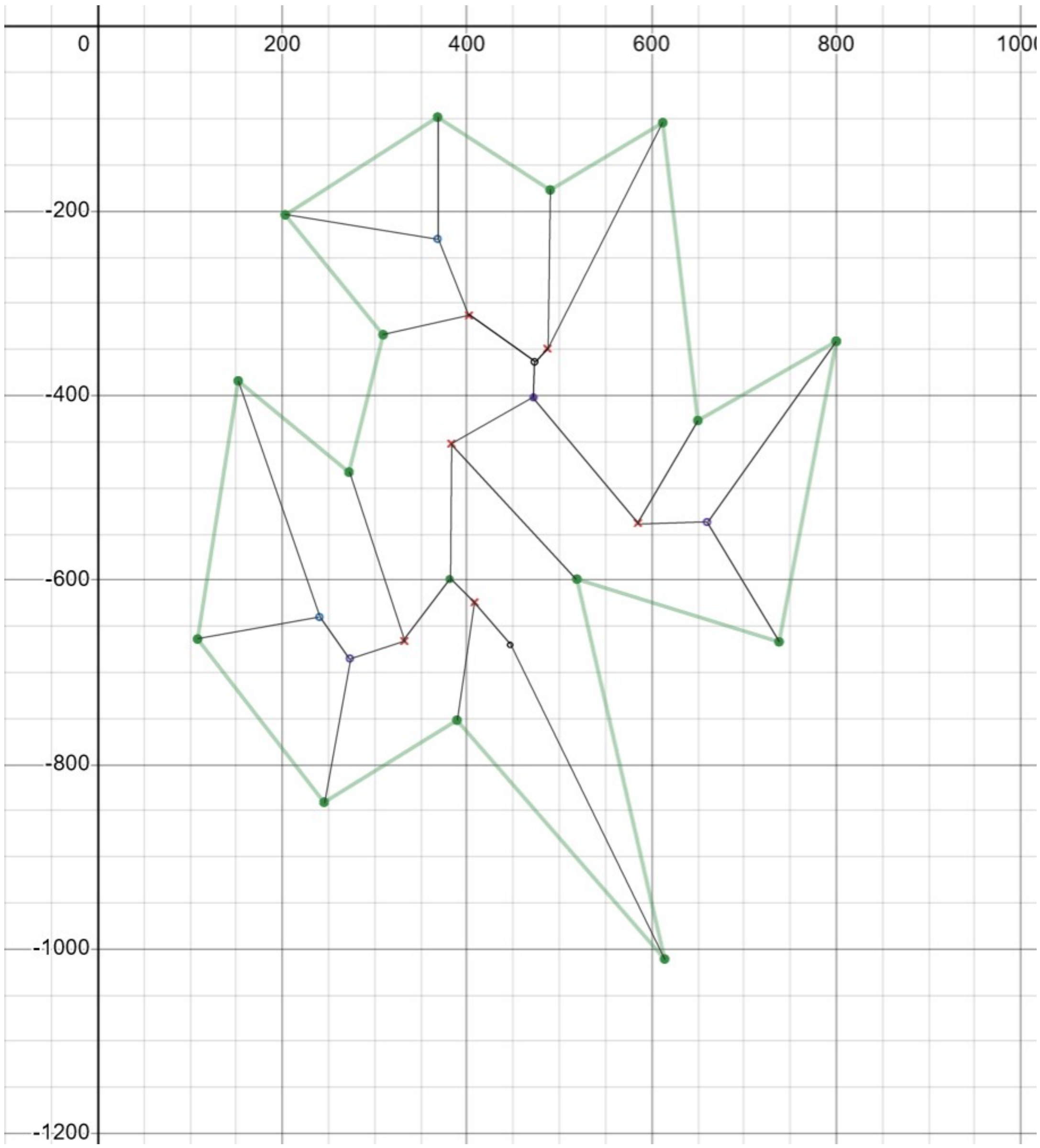


Fig. 11

For comparison, below is a medial axis skeleton of the same polygon.

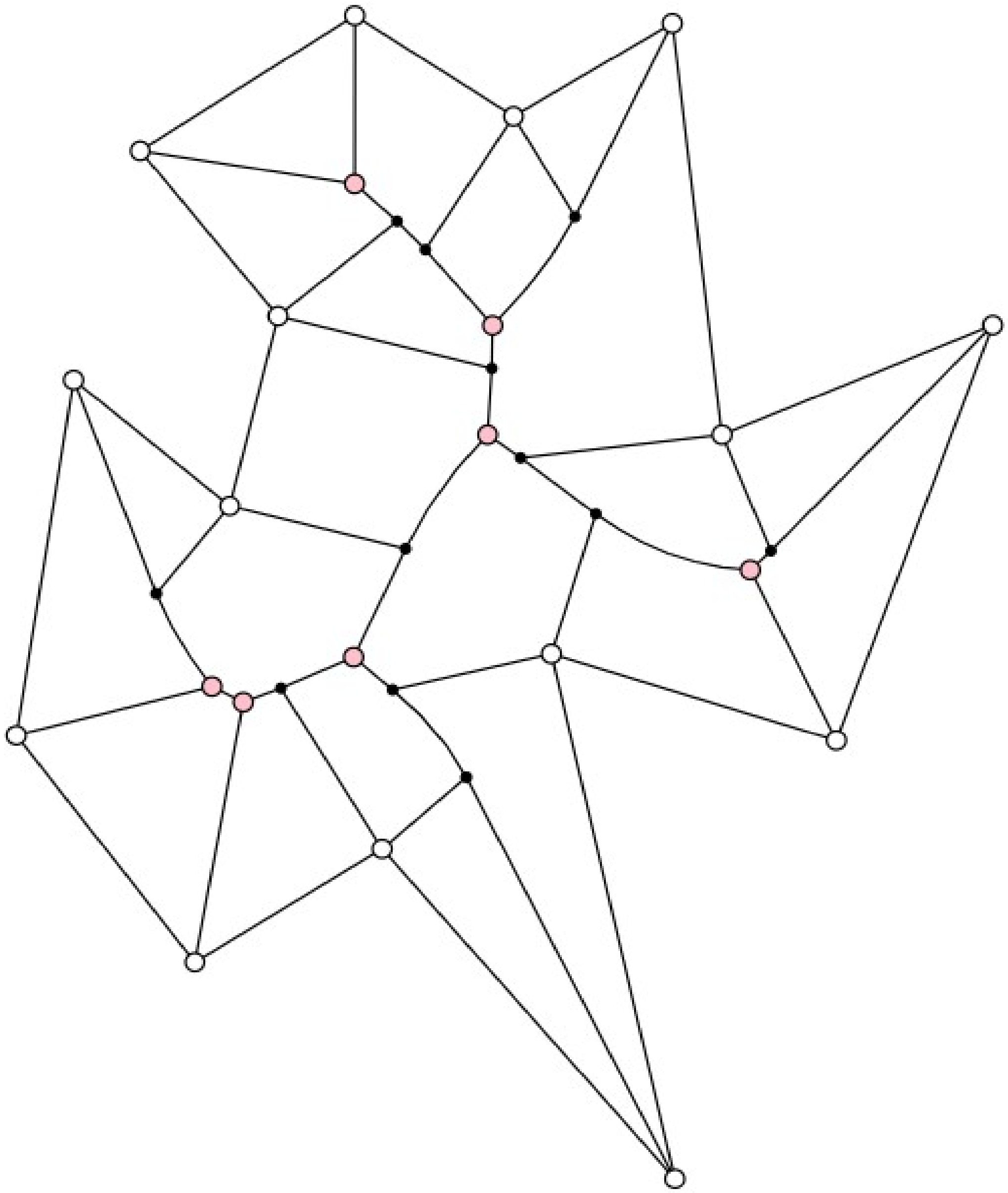


Fig. 12