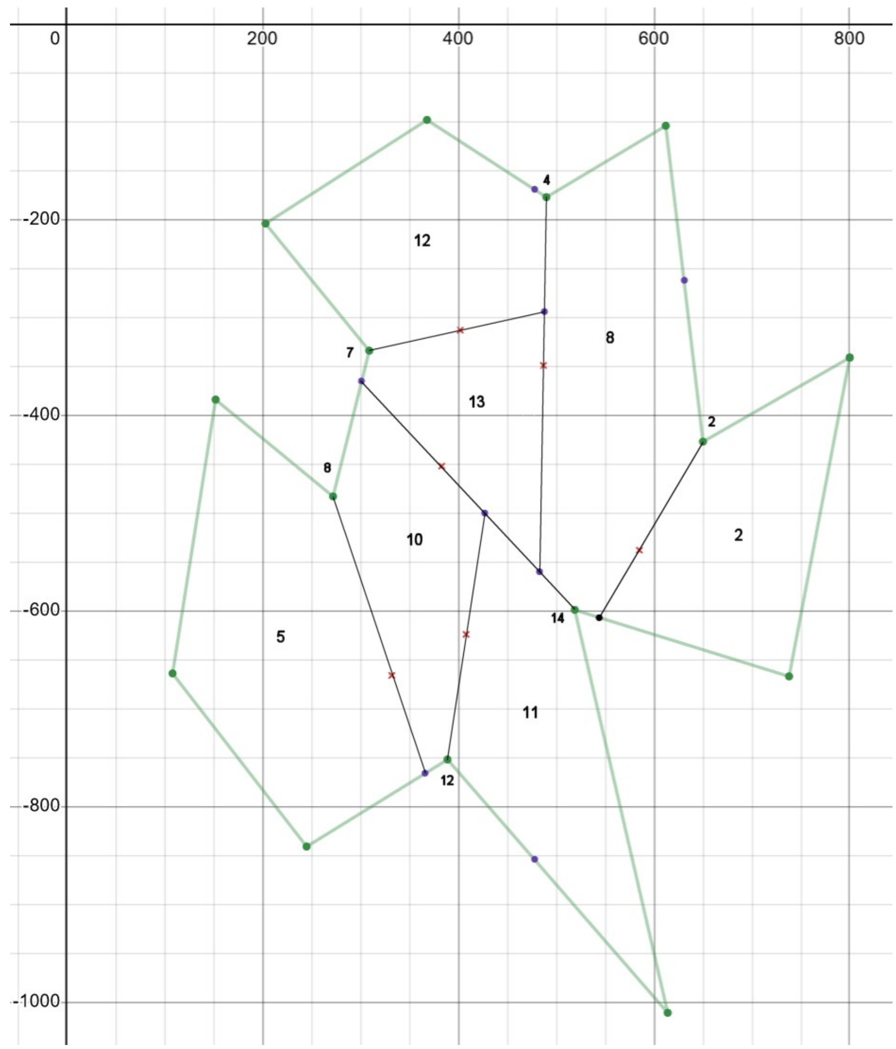
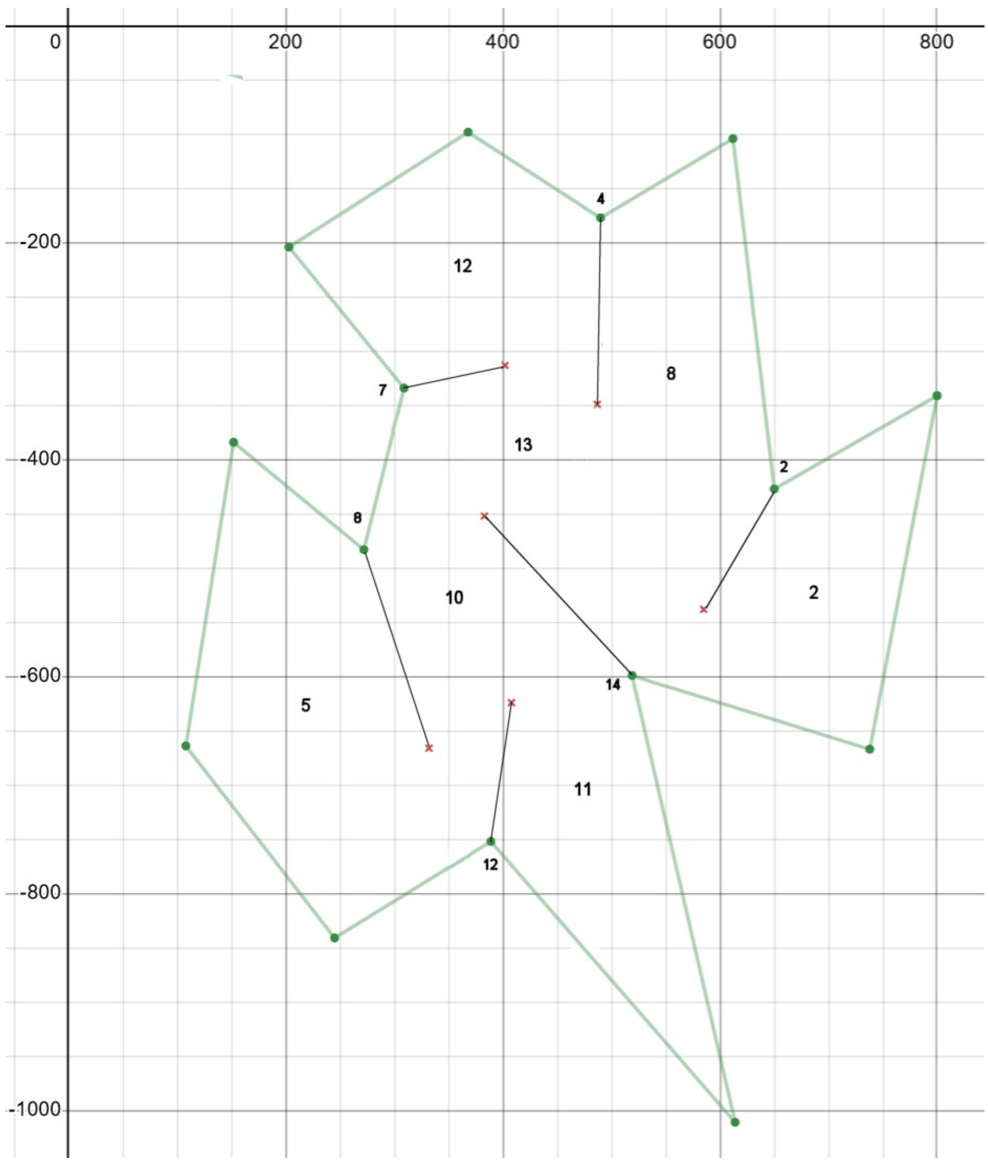
Determination of a Straight Skeleton from a Motorcycle Graph
April 11, 2024

Below is the starting point, a motorcycle graph of a polygon. As is noted elsewhere, a motorcycle graph can be generated from a triangulation of an n-sided polygon [order n·log(n)], ray tracing of v convex vertices in the triangulation [order v·sqrt(n)] and brute force intersection of motorcycle paths [order v·v]. Note that each of the motorcycle regions is a convex polygon.

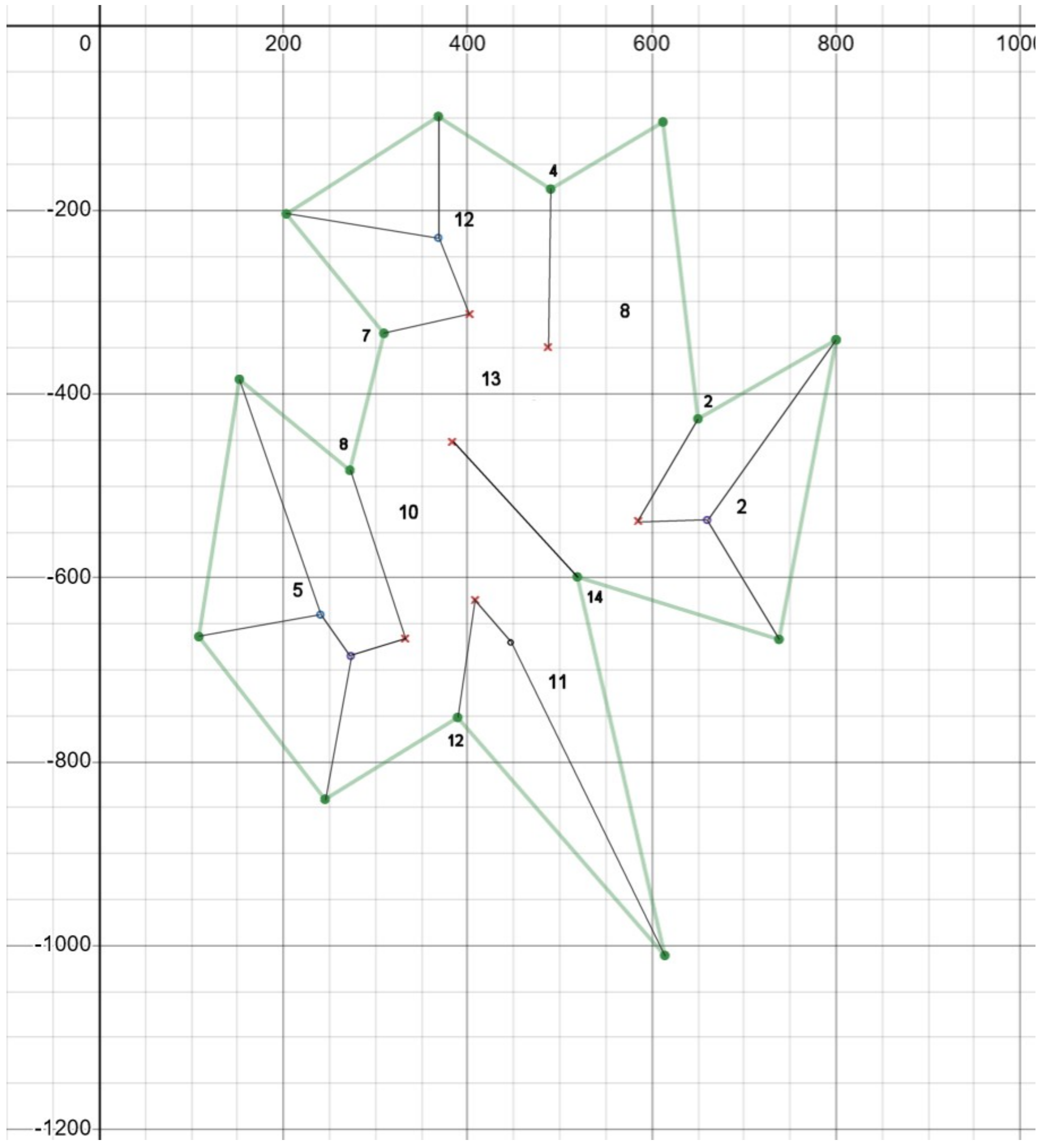


The first step in the process is to calculate the "B" points for each of the motorcycle traces (red x in the graph above). These are the intersections of the bisector of a convex vertex and the bisector of a side forming the vertex and the target of the vertex bisector. These points are the ends of the valleys defined by the convex vertices.

Eliminating the portion of the motorcycle track from the B point to the target yields in the following result. This is a graph of the valleys associated with each convex vertex.

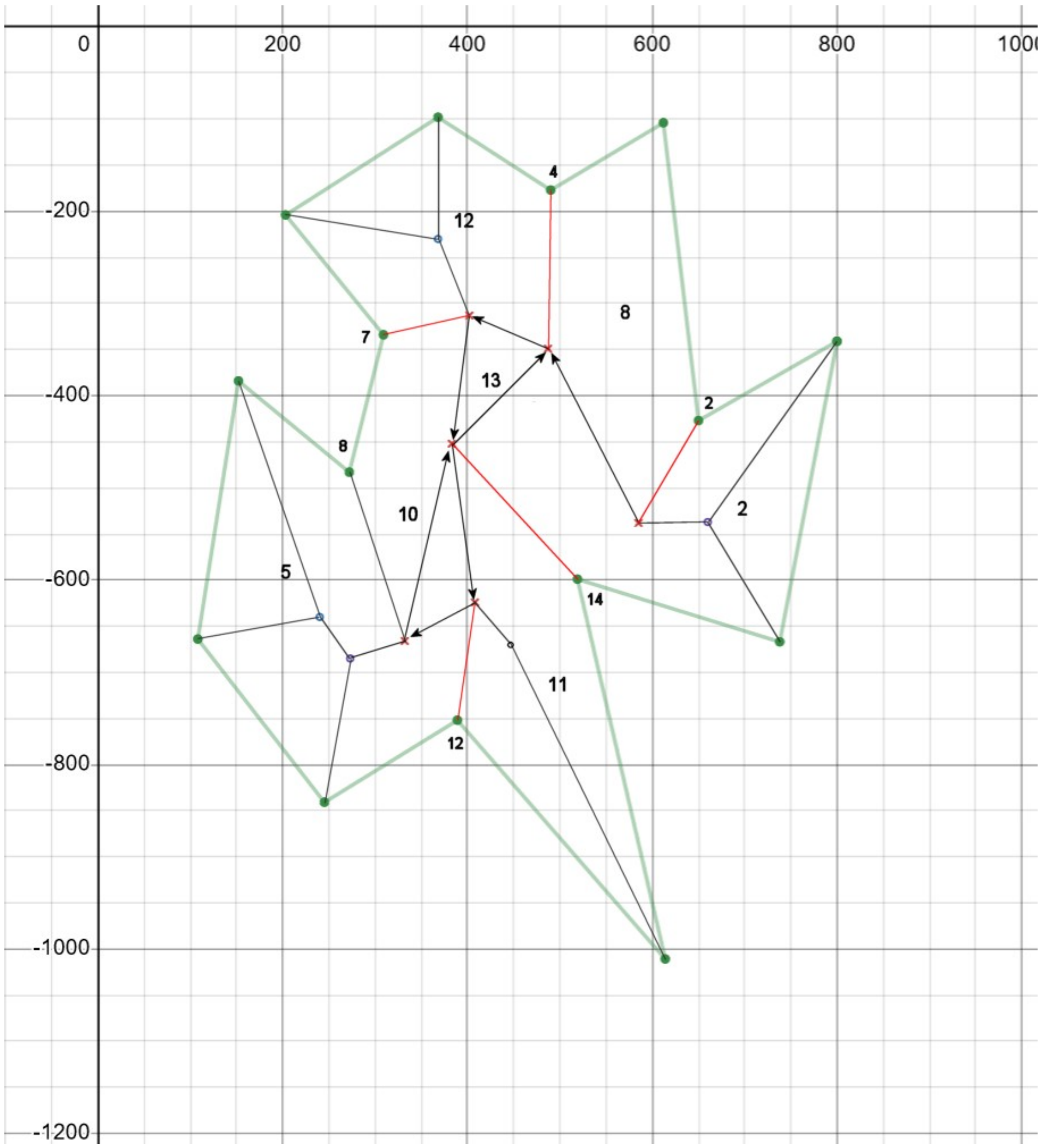


The next step is to identify those regions containing either just one boundary point, or having two adjacent points on the boundary. Skeletons for these regions can be immediately determined.

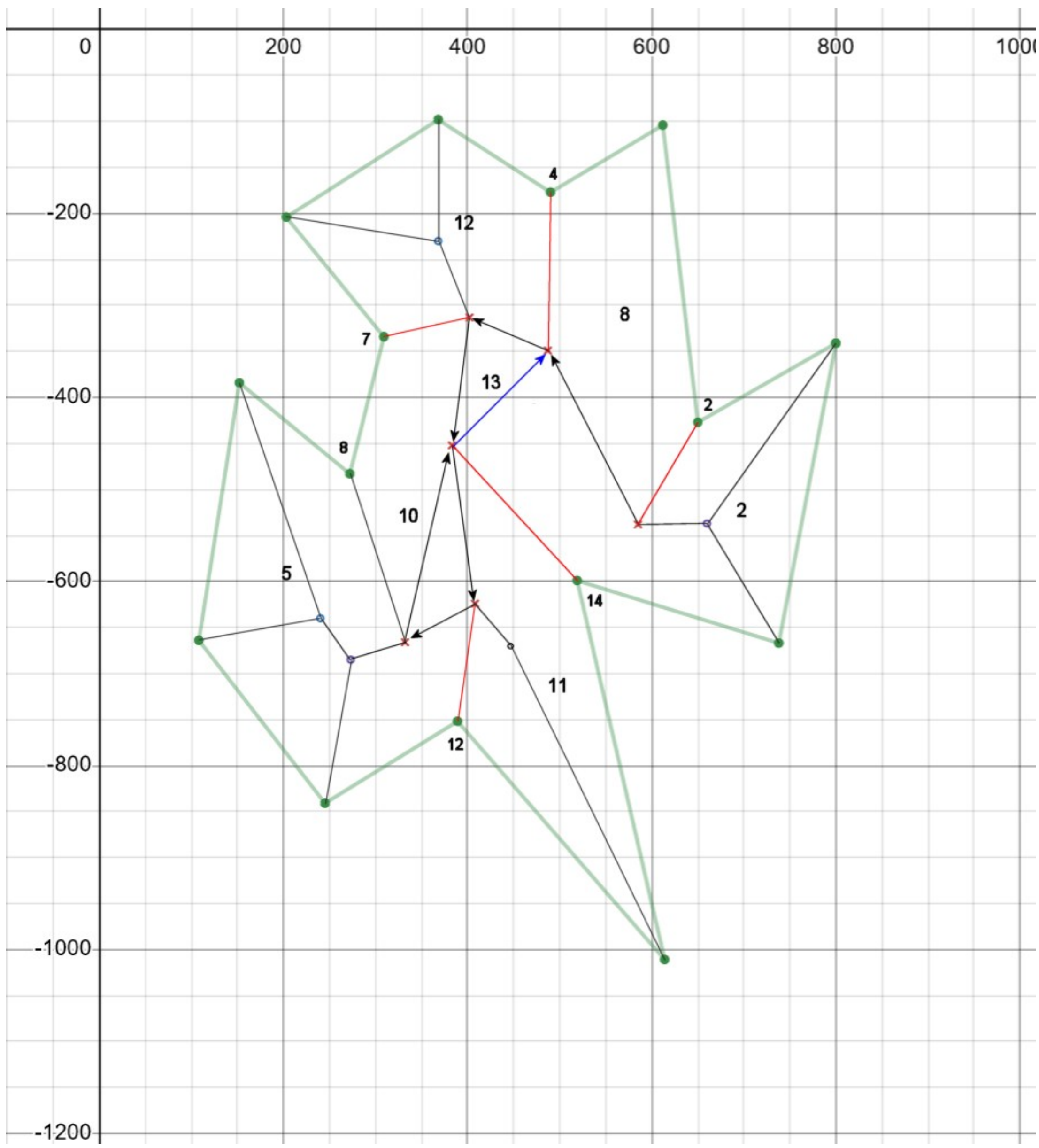


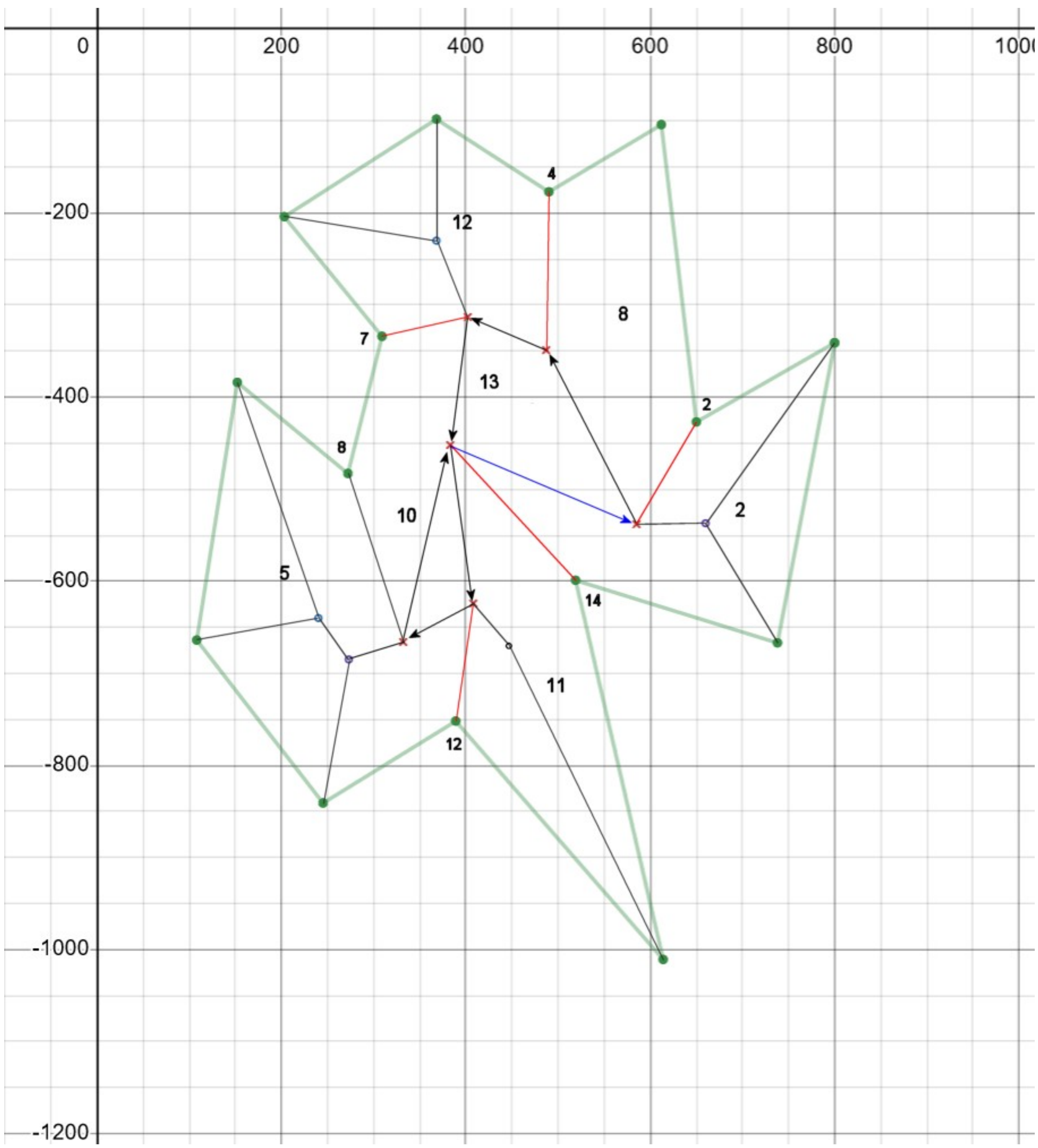
For each B point there is a pair of skeleton segments that make the sharpest possible angle at the B point. These segments are shown (when possible) as pointed lines below when the segment is entirely within a region. (Previously determined segments lying outside the convex hull of the B points are not considered candidate segments.)

Notice that we have one cycle "hole" near the Region 10 label, and an unfinished "hole" to the left, right and above the Region 13 label. We know that this hole is unfinished, as the B point for vertex 14 has only three connecting segments, whereas B for vertex 2 has only one. (We have to have an even number of segments at each B point.)



To close the unclosed chain we step backwards in the unfinished chain starting at the B node, so that the first examination is shown below, with the segment in blue the candidate closure. However, we find that there are already two connectors at end of the candidate, so we progress to the prior B point. This second time we find (on next page) that the added segment satisfies the "must be an even number of connectors" condition.





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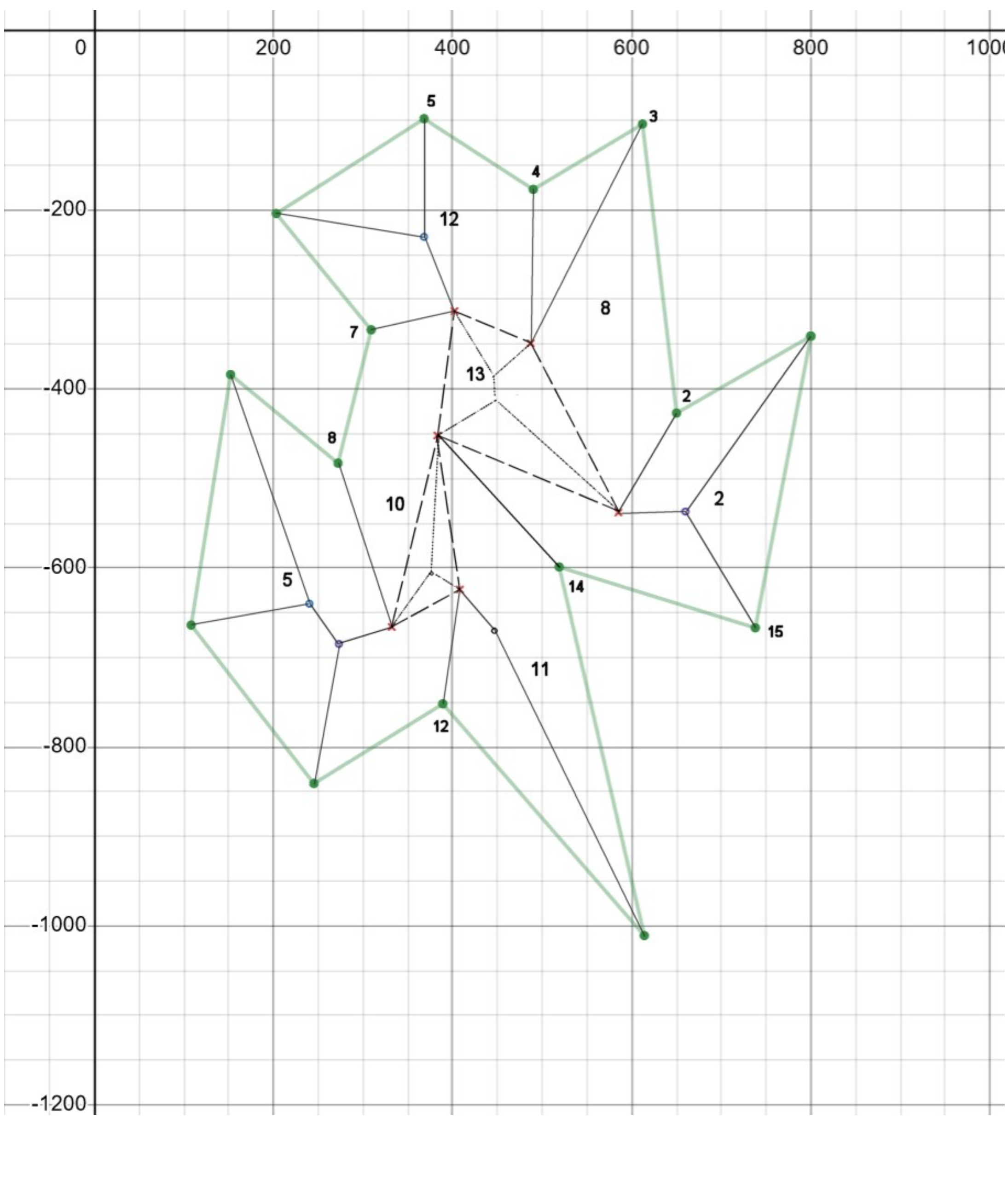
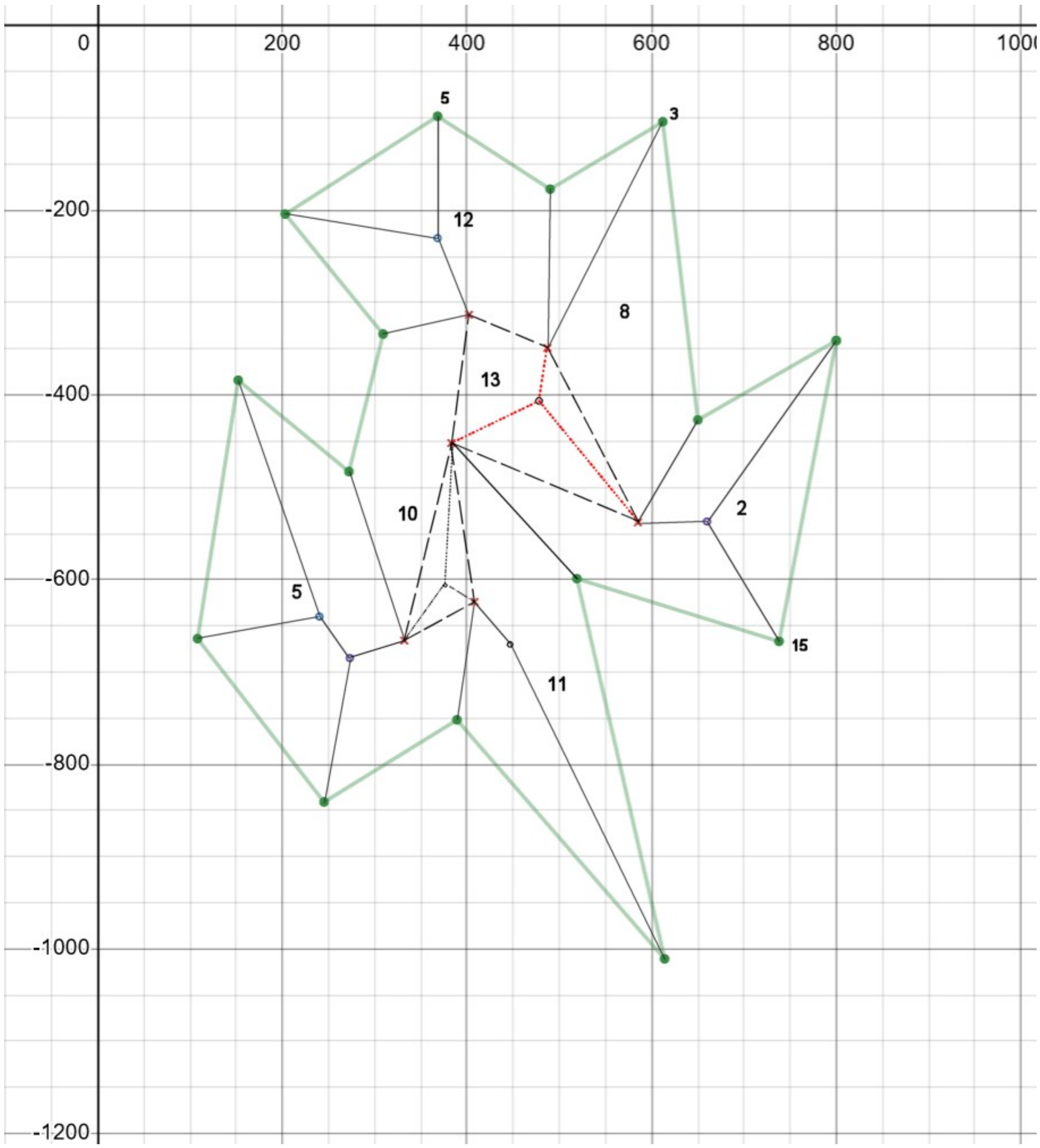
At this point we have two convex areas which contain a point higher than any edge of the area. In the case of the triangular area, the high point is simply the incenter of the edges ending at 8, 12 and 14.

To find the high point of the four-sided area, we will determine its skeleton and use that information to identify the edges contributing to the center of this area. Note that there are two points in the figure where three lines meet. The upper triple is associated with the sides 7-8, 4-5 and 2-3. The lower is associated with 7-8, 2-3 and 14-15. Calculating the distances from the candidate points to an edges shows that the lower triple has the greatest distance, so that will be the one to use. We can replace the four sided figure with the sides associated with the lower triple with the meeting at their incenter. We use the segment with the greatest distance from an edge to connect the Region 12 skeleton to the new skeleton and discard the other candidate connector. Notice that we now have a new "sharpest segments" for the B points of vertices 7, 2 and 14.

Note that the skeleton segment bisecting point three in Region 8 doesn't intersect beyond point B of vertex 4, therefore it must intersect the point 4 bisector at its B point.

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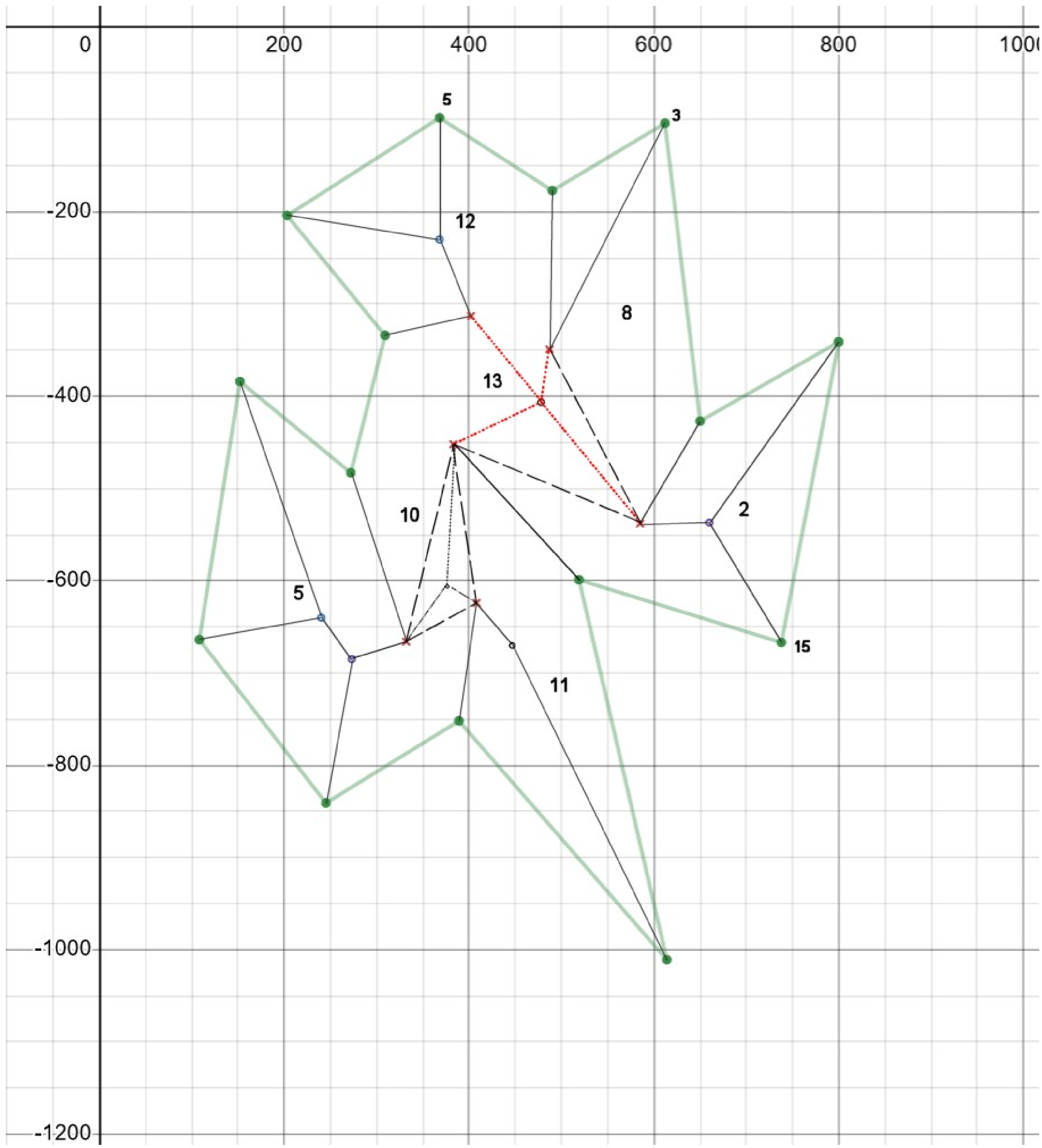

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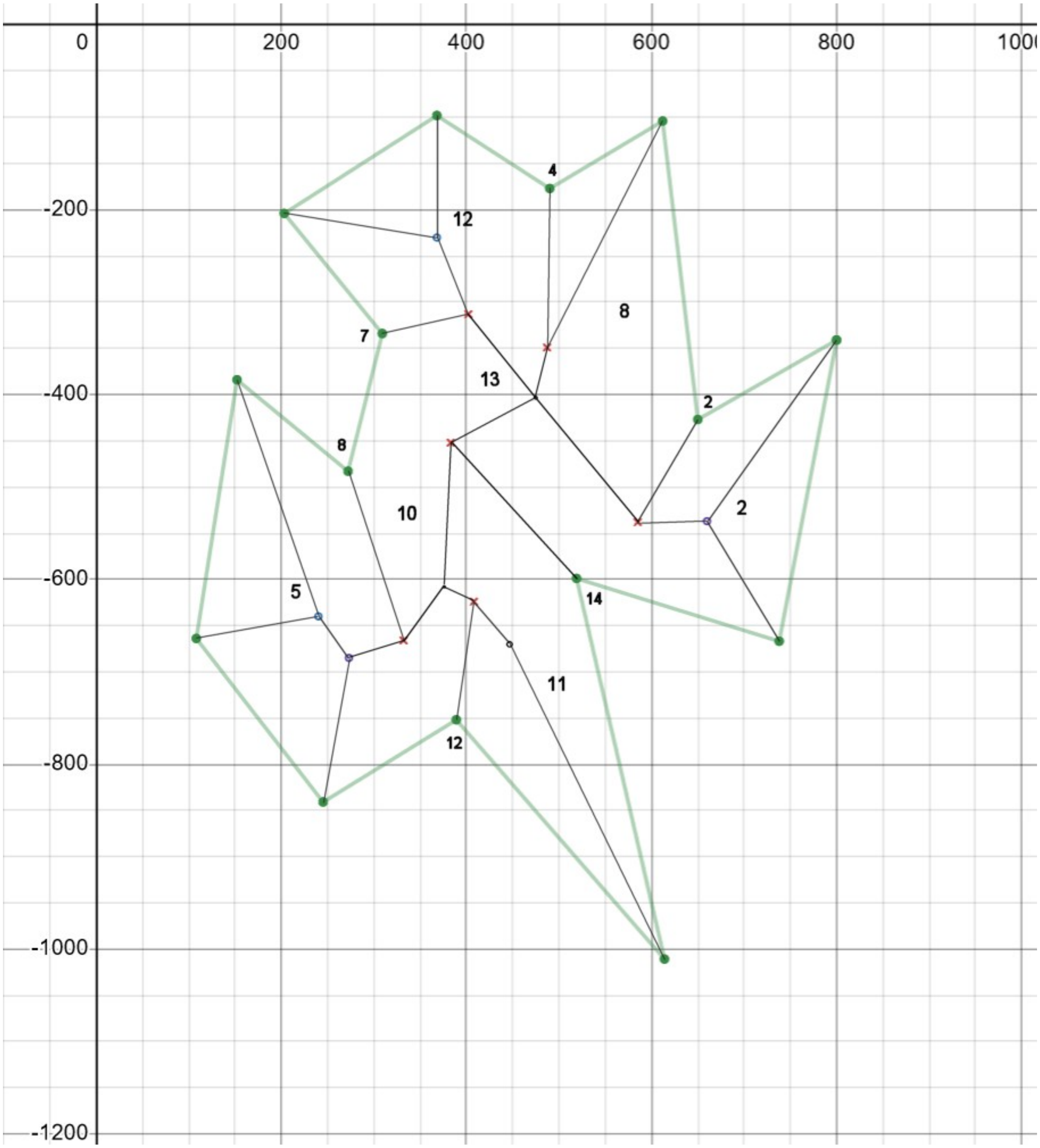
Finally, we have the complete skeleton, shown below.



For comparison, below is a medial axis skeleton of the same polygon.