Determination of a Straight Skeleton from a Motorcycle Graph
May 4, 2024

Below is the starting point, a motorcycle graph of a polygon. As is noted elsewhere, a motorcycle graph can be generated from a triangulation of an n-sided polygon [order n·log(n)], ray tracing of v convex vertices in the triangulation [order v·sqrt(n)] and brute force intersection of motorcycle paths [order v·v]. Note that each of the motorcycle regions is a convex polygon.

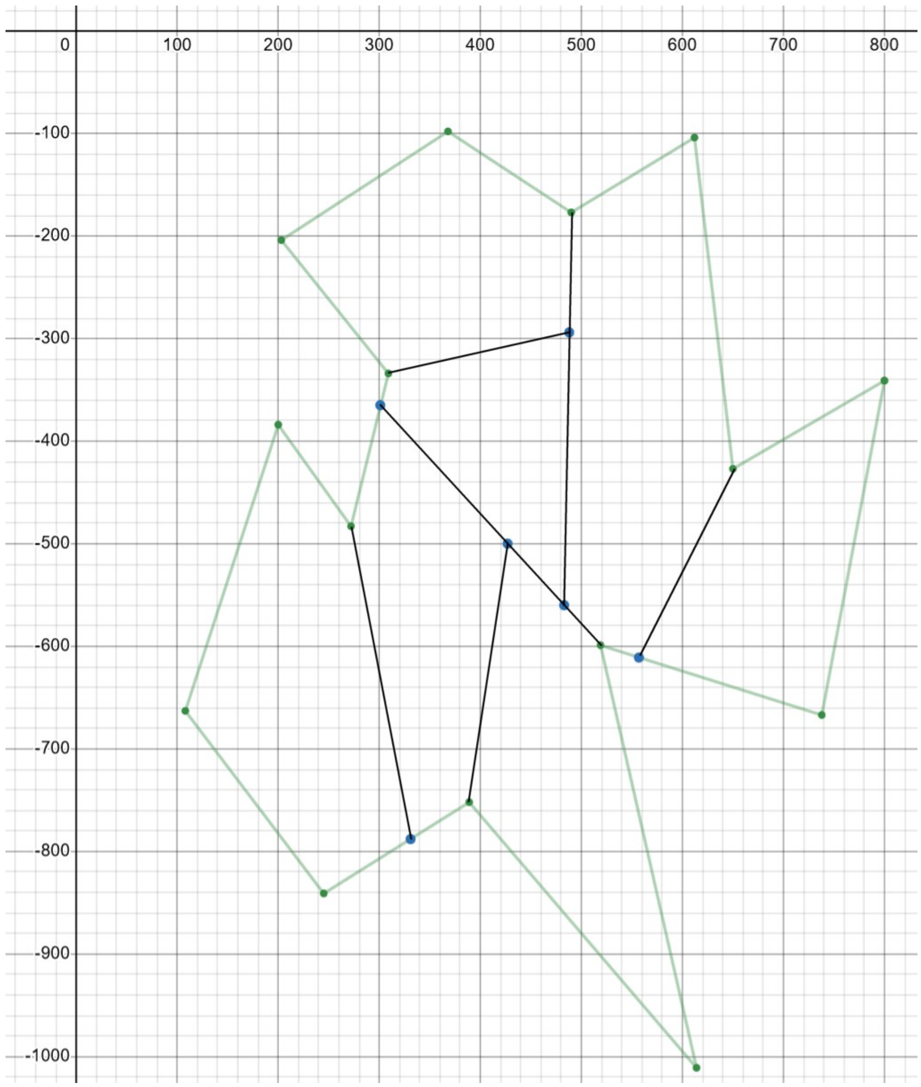

Fig. 1

The next step is to identify those regions having two or more adjacent sides on the boundary. Skeletons for these regions can be immediately determined. The skeleton ends on encountering a side with vertex/target points, as shown below. Where a node has less than four attached segments, the node is complete; if not, the node is incomplete. In the figure below there are six incomplete nodes, they having two attached segments. The vertex from 14 has one attached segment. We will next show how to complete the incomplete nodes.

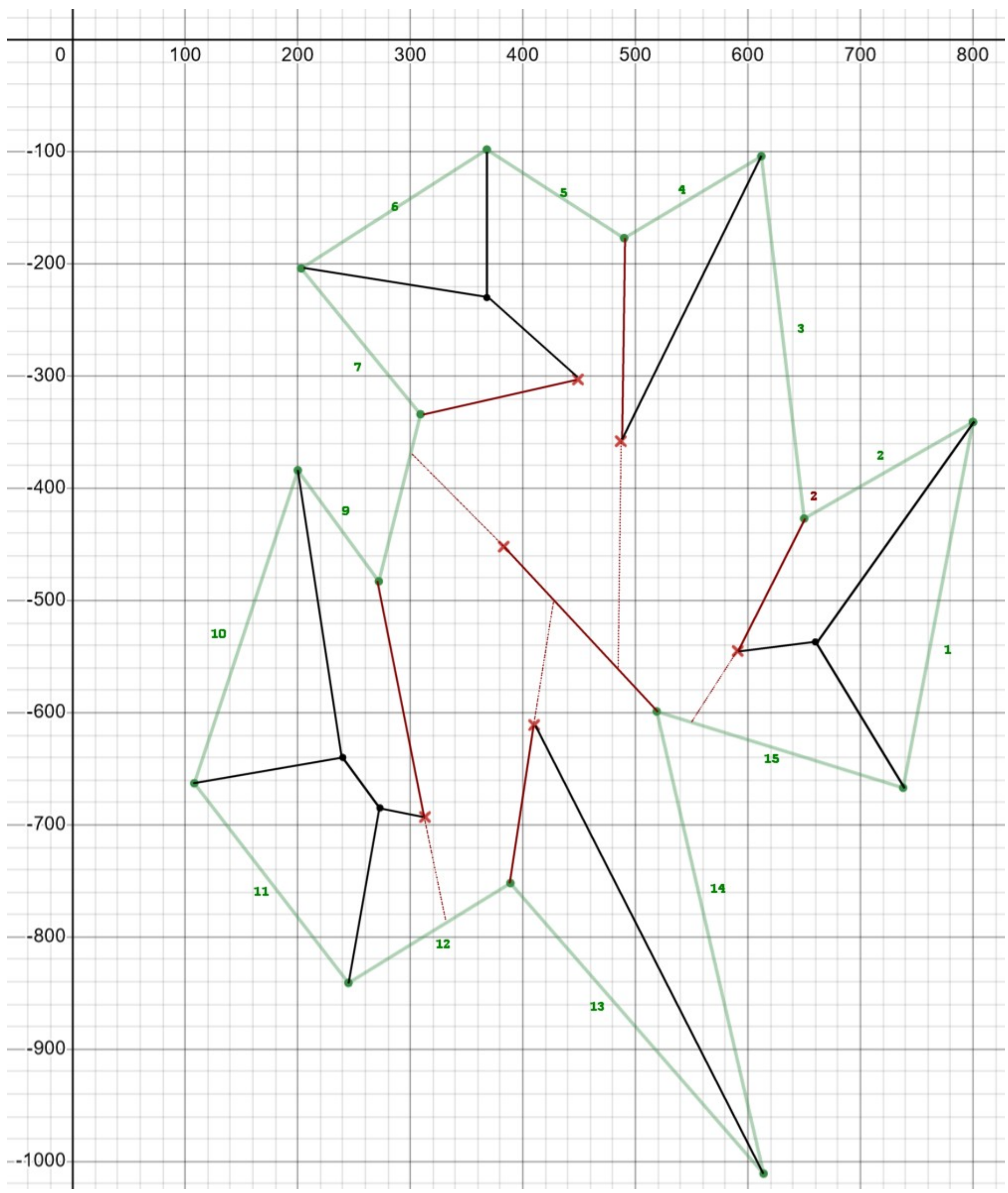

Fig. 3

At this point we will relink the motorcycle regions by removing links at target points except for those for a through path not associated with a peripheral region. Note that the through path from vertex point 14 divides the polygon into parts: a clockwise traversal in the lower part, counter-clockwise in the upper. This results in the paths around the interior of the polygon, as shown in the figure below (not all of the arrows are shown).

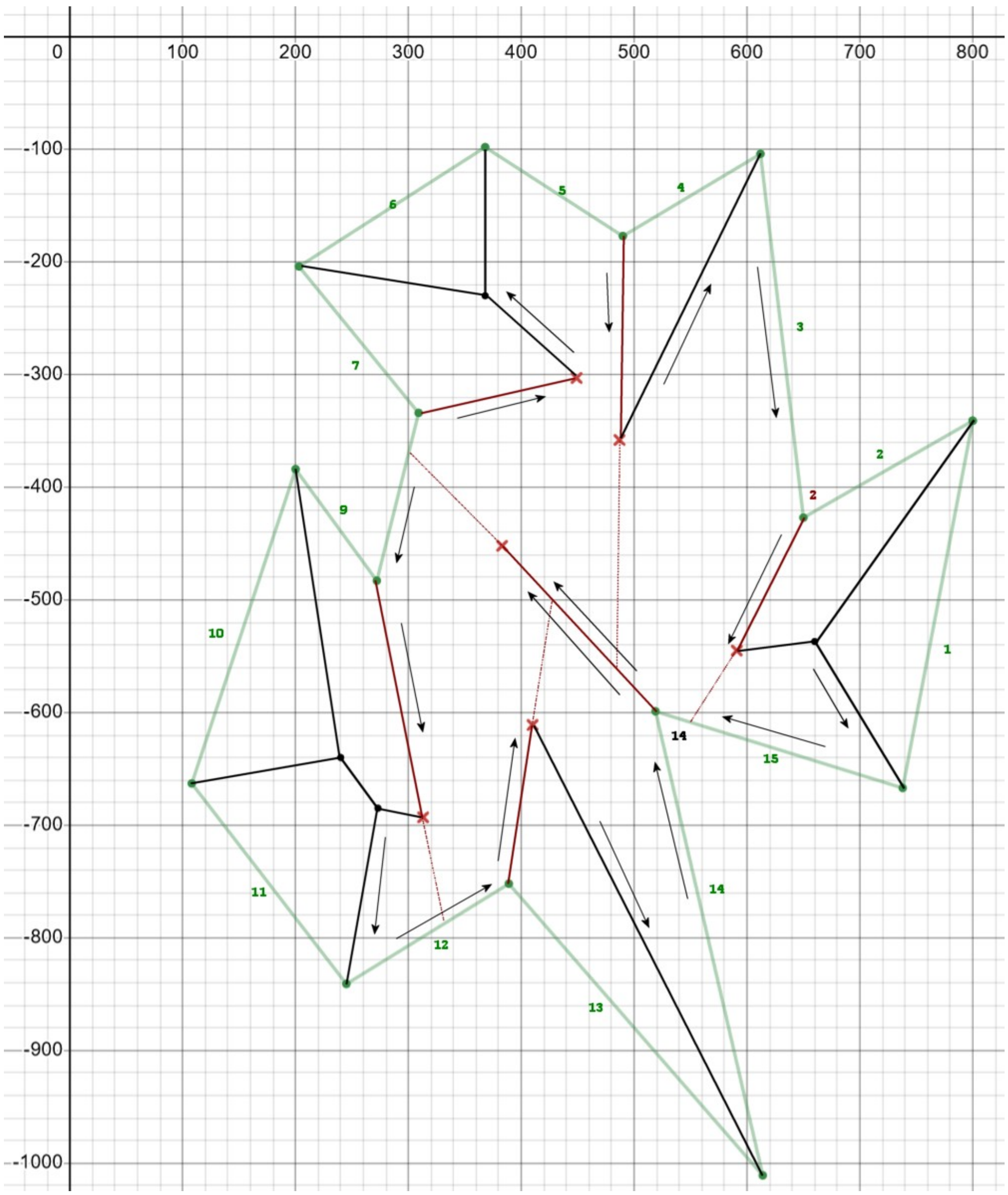

Fig. 4

The internal connections of the incomplete nodes can now be shown. From the target side of the bisector of the vertex at point 14 (point A), we develop two vectors from point B, one to the left and one on the right by traversing the paths determined by the relinking just described. The traversal ends when an incomplete point is encountered. For the traversal to the left, that end point is marked with an X. The one the right with Y. Vectors connect B with X and B with Y.

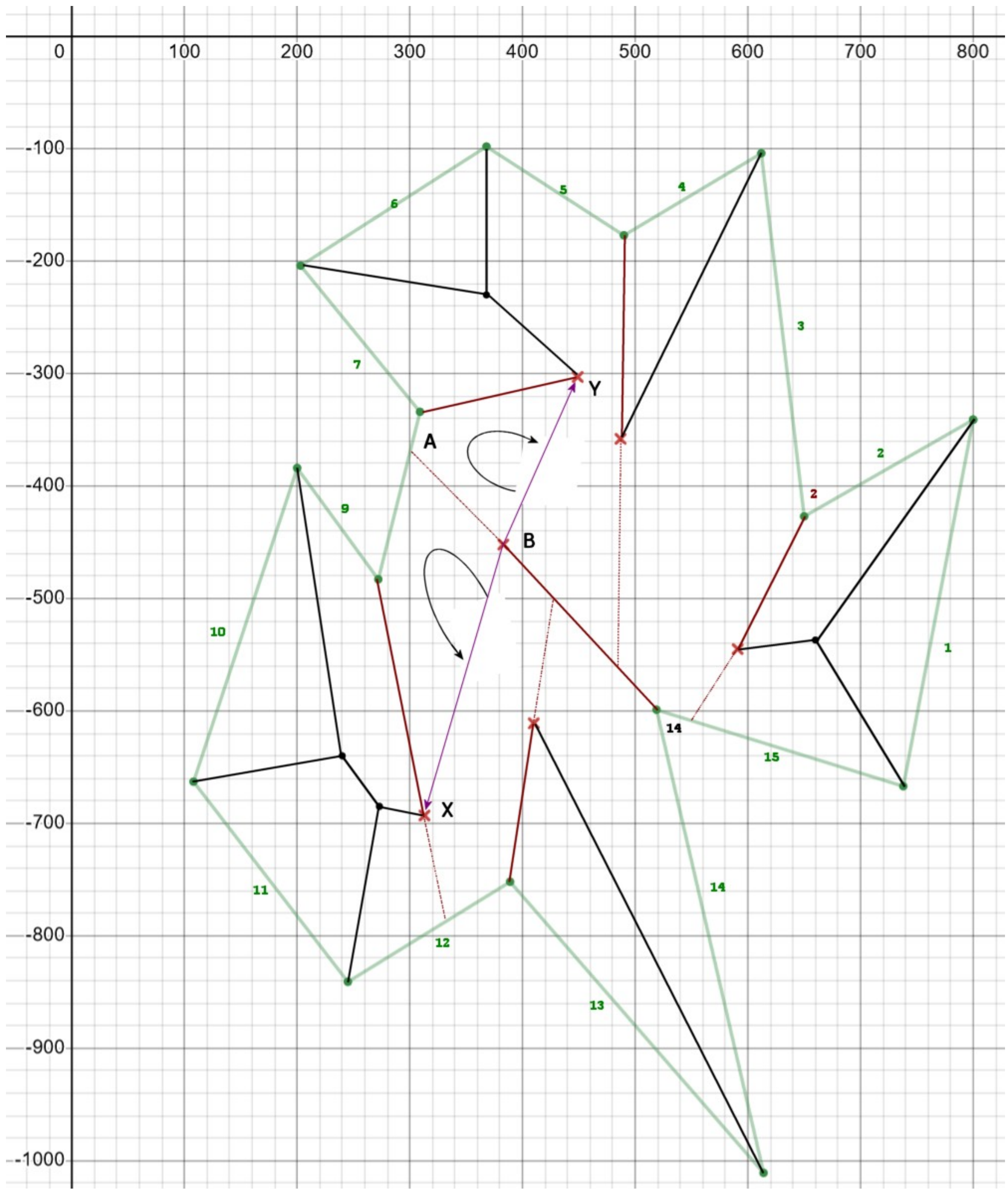

Fig. 5

Next we will consider the three regions having more than two adjacent boundary segments, shown below. We traverse the paths for the side of the incomplete node without a connecting segment, stopping when an incomplete node is encountered. A vector is placed between the two incomplete nodes, as shown below.

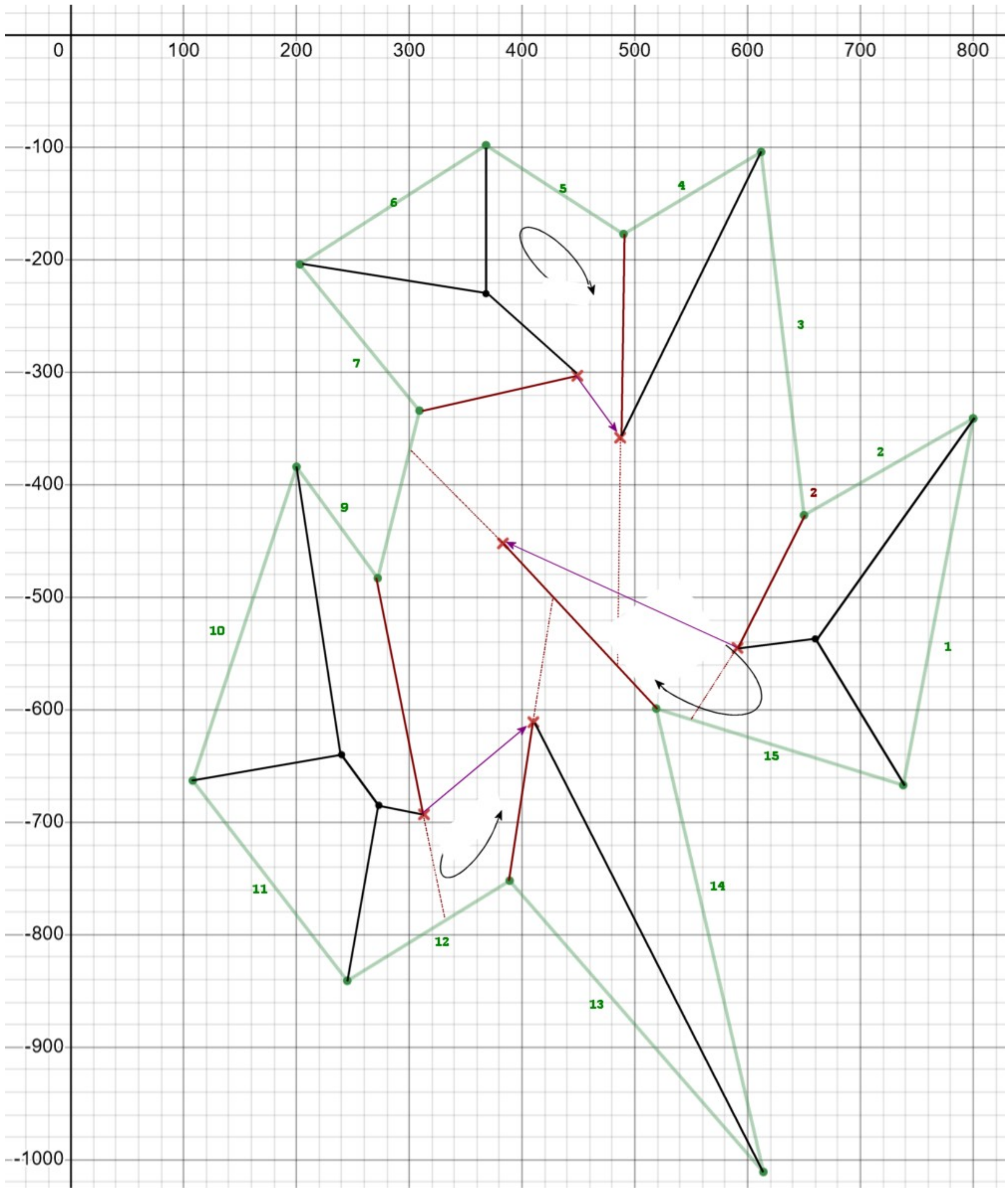

Fig. 6

For the regions having just two adjacent boundary segments, we undertake the same process as is shown below. For the lower region the traversal starts at s and continues along sides A, B and C to point f. A vector connects points s and f. And similarly for the upper region.

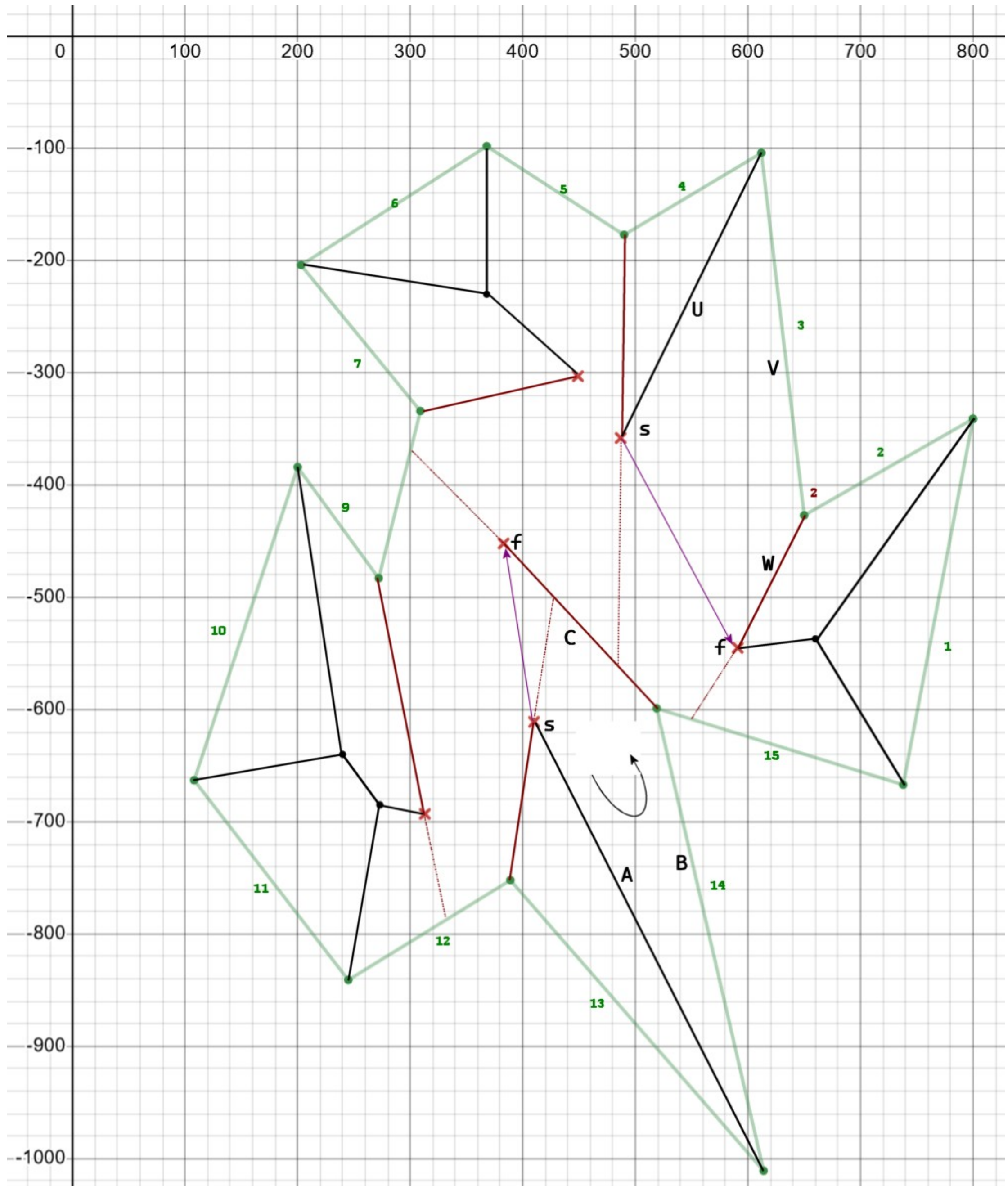

Fig. 7

In the figure below we bring together all of the vectors shown in Figures 5 through 7: Green is from Fig. 4; blue from Fig. 5; and orange from Fig. 6. Note that the previously incomplete nodes now all have at least four attached segments. We can see that two regions are formed, holes where skeletons need to be defined. The lower hole has three sides, the upper four.

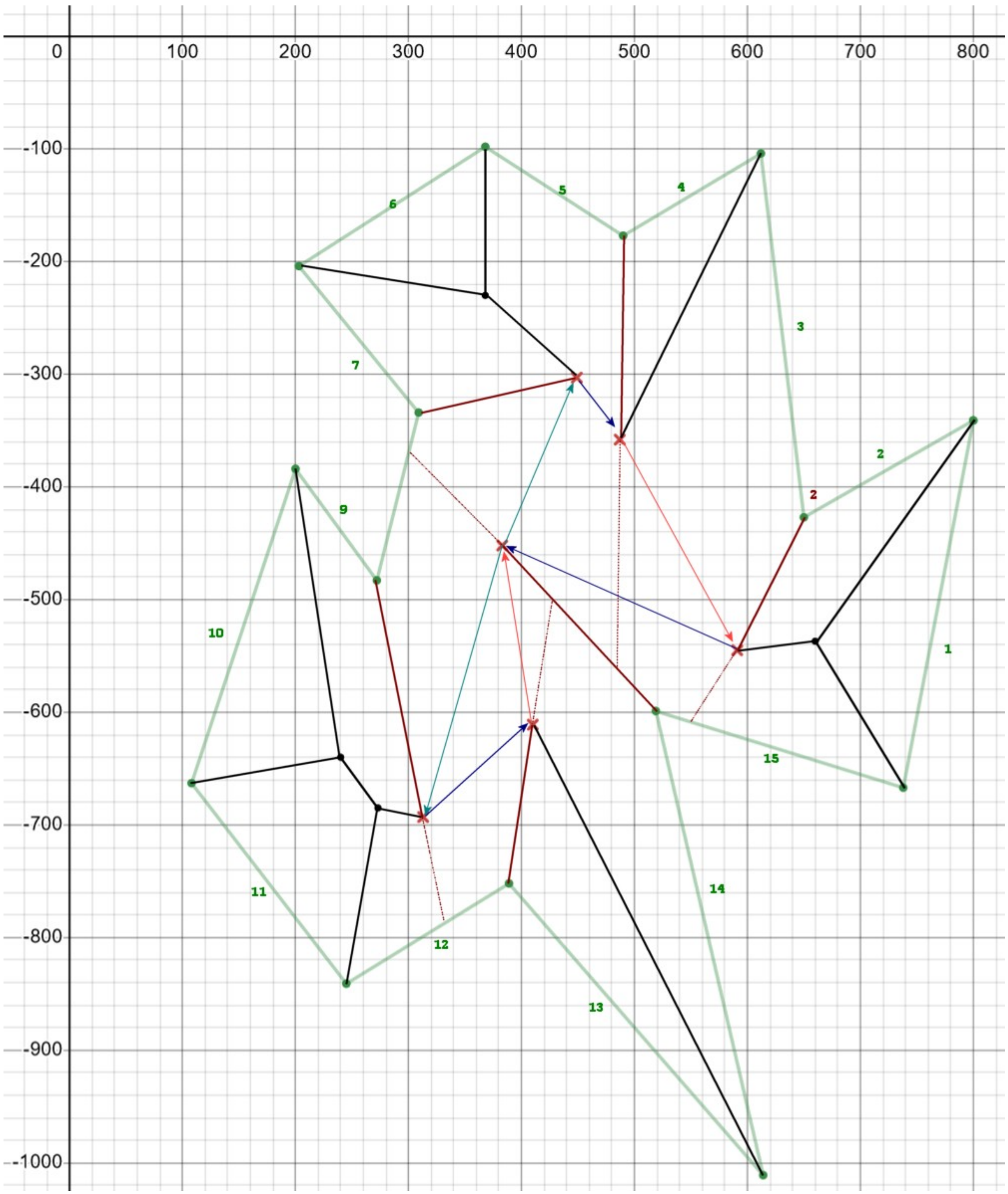

Fig. 6

The two convex hole areas contain points higher than along any edge of the area. In the case of the lower triangular area, the triangulation of the area indicates a high point is the incenter of the edges 8, 12 and 14. This incenter of these edges is shown as a dot in the triangular region. Note this incenter is not the same as the center of the skeleton of the region.

To find triple connection points of the four-sided upper area, the upper node of its skeleton corresponds to sides 8, 5 and 3, while the lower area associates sides 8, 15 and 3. The two sides that join these two incenters are 8 and 3. The two incenters are shown as dots in this region.

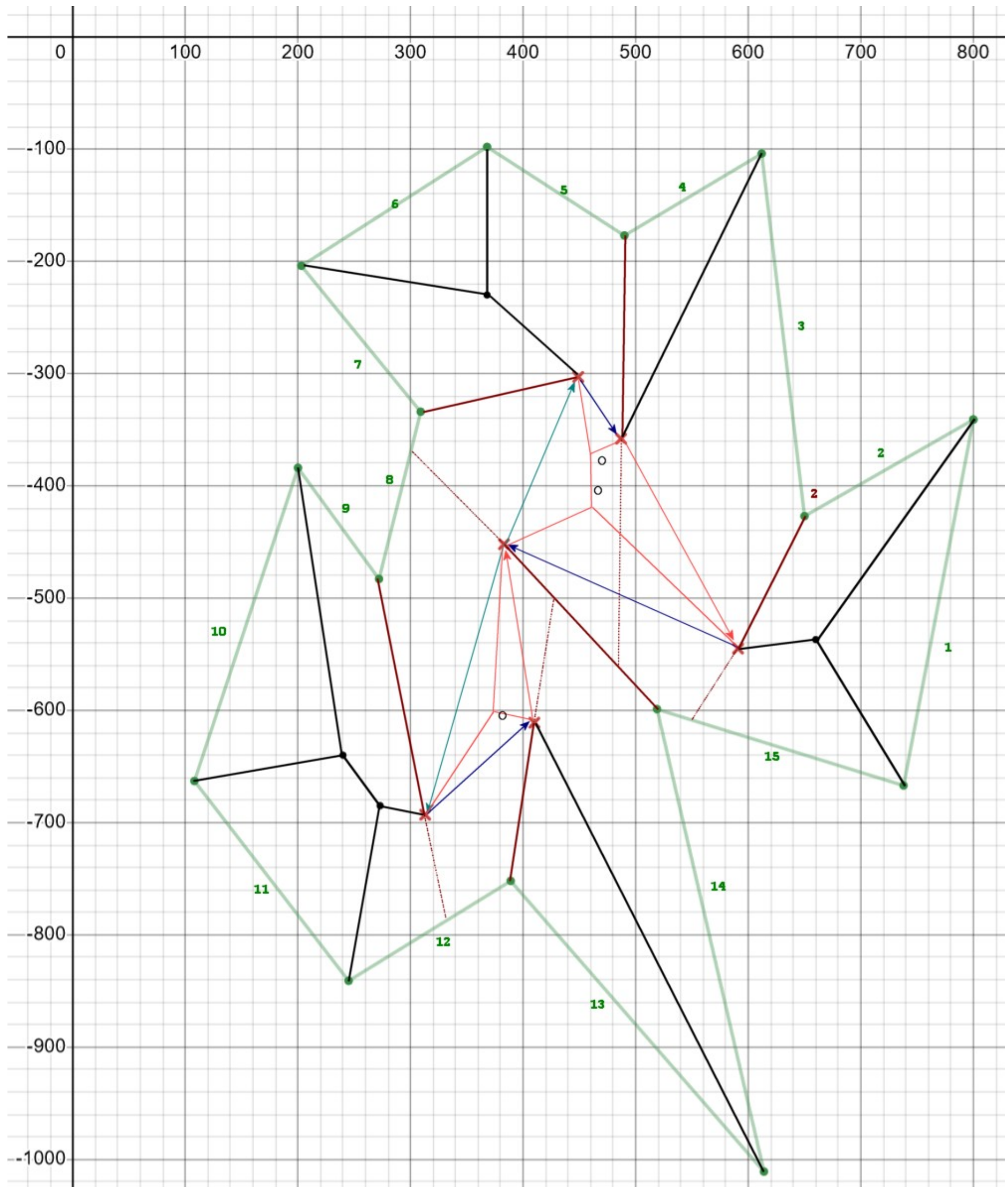

Fig. 7

Now, replace the region outlines and their internal skeletons with the incenters of the determining sides, and create lines from these points to the corners of the region. This replacement, shown below, is the result skeleton.

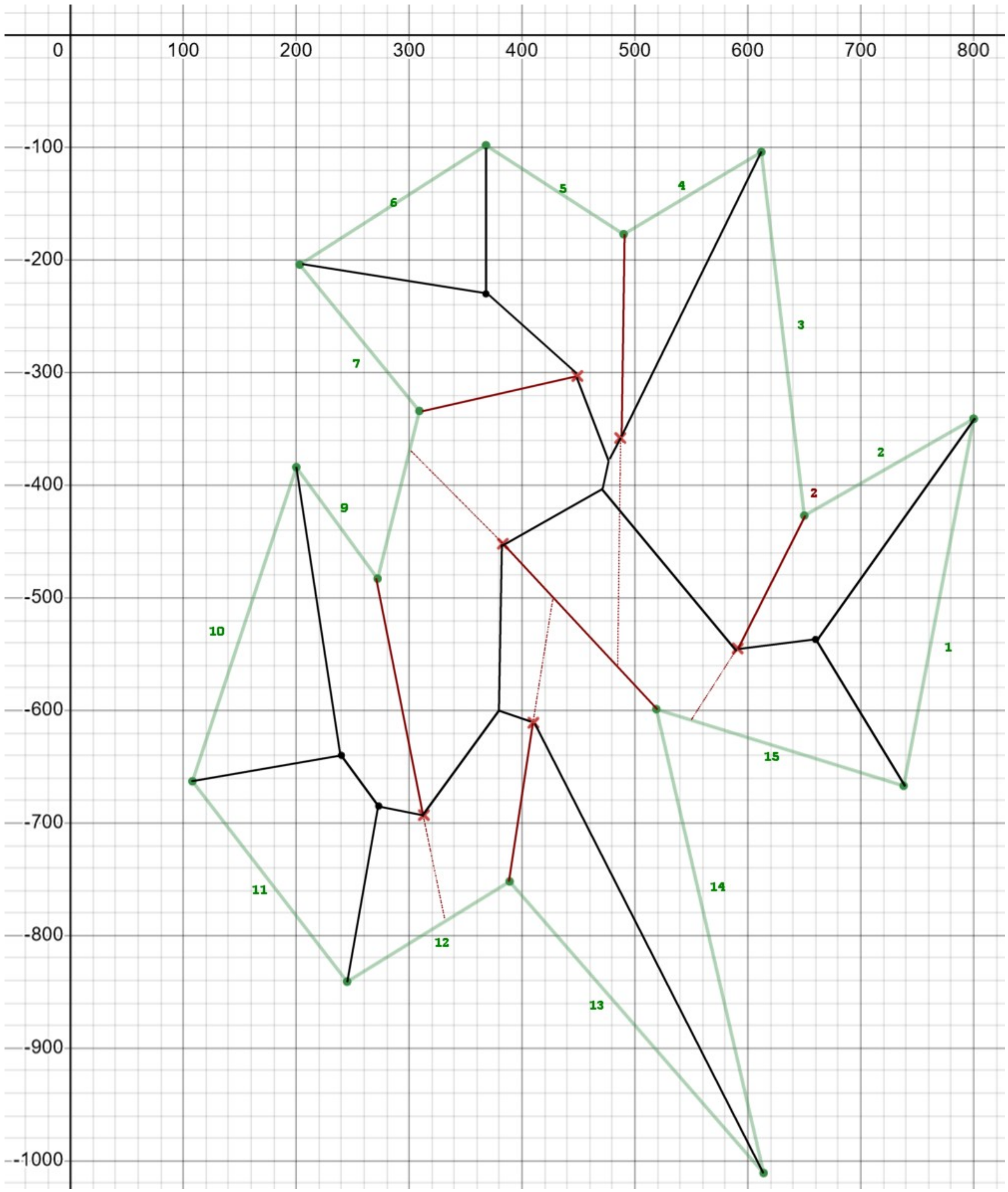

Fig. 8

For comparison, below is a medial axis skeleton of the same polygon.

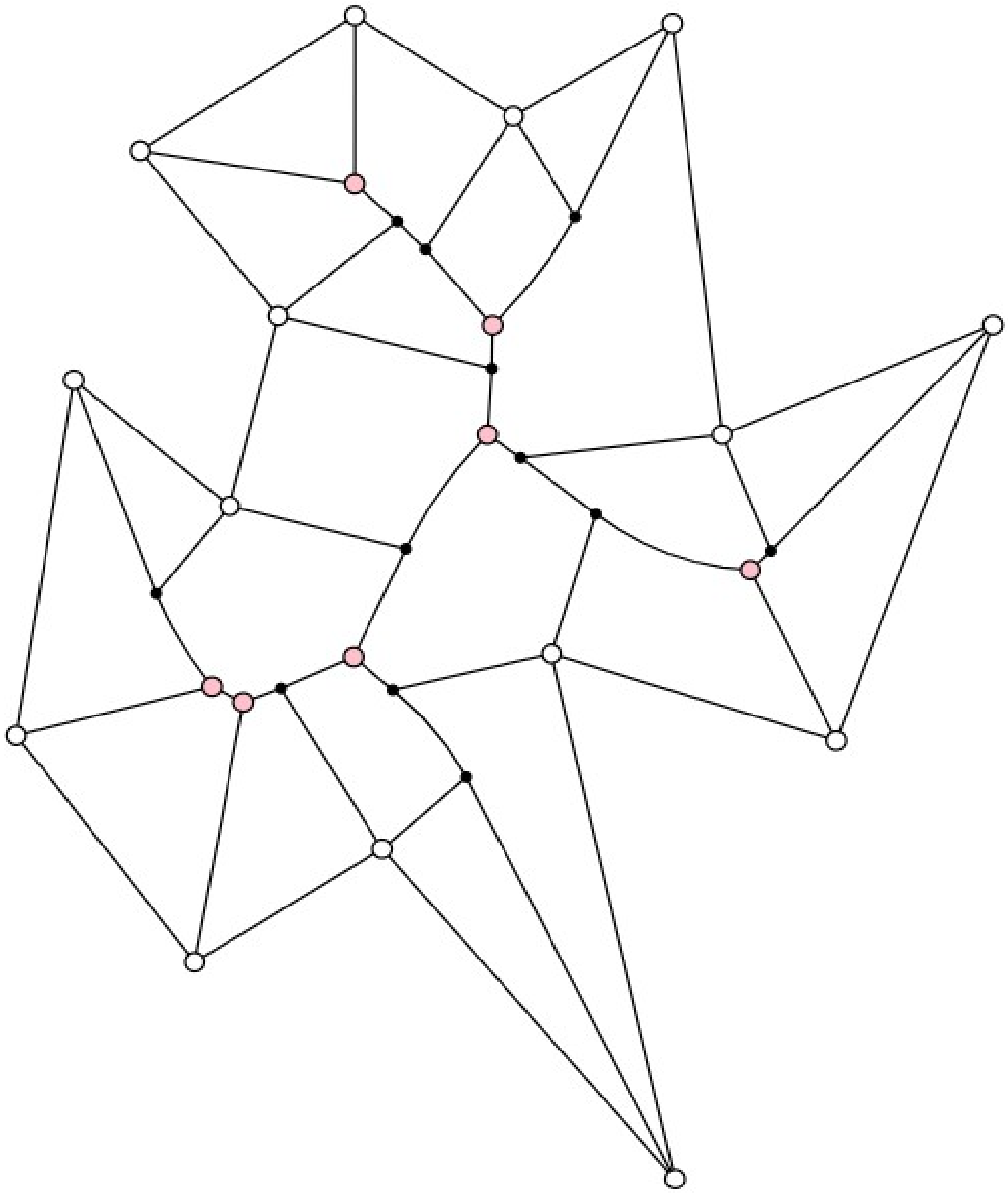

Fig. 9