Investigating Conditions for Maximising Magnetic Field Confinement Time in Tokamaks

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1 AIM

To investigate requirements for maximising magnetic confinement time of charged particles in a tokamak; in particular, optimal magnetic field, electric field and particle speed conditions.

2 INTRODUCTION AND BACKGROUND THEORY

Tokamaks are devices designed to confine plasma with toroidal magnetic fields induced by electromagnets to facilitate nuclear fusion. The key behind such confinement is that charges tend to gyrate (spiral) along magnetic field lines.

The general equation of motion for charges in a magnetic field $\vec{B}$ acted on by non-magnetic force $\vec{F}$ is:

$$m\frac{d\vec{v}}{dt} = \vec{F}_{net} = \vec{F} + q\vec{v} \times \vec{B} \text{ (Chen, 2015)}$$

The additional force leads to a component of motion perpendicular to $\vec{B}$ called drift velocity:

$$\vec{v}_{drift\ due\ to\ \vec{F}} = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2} \text{ (Chen, 2015)}$$

Such drift velocity is a key factor behind charges escaping magnetic confinement in a tokamak. Overall, there are 4 main drift velocities.

The first two below are technically not due to “non-magnetic” forces, but nevertheless need to be treated so due to relevant mathematics.

Curvature drift: arises from centrifugal force due to $\vec{B}$ curvature; leads to opposite charges drifting in opposite directions parallel to the curvature’s rotational axis.

Grad – B drift: due to “grad – B force” arising from $\vec{B}$ non-uniformity; leads to charge separating drift that amplifies curvature drift.

$E \times B$ drift: arises from electrostatic force due to $\vec{E}$ resulting from charge separation caused by curvature/grad – B drift; leads to drift described by:

$$\vec{v}_{drift\ due\ to\ \vec{E}} = \frac{\vec{E} \times \vec{B}}{B^2} \text{ (Chen, 2015)}$$

$G \times B$ drift: arises from gravitational force due to gravitational field $\vec{g}$; leads to drift described by:

$$\vec{v}_{drift\ due\ to\ \vec{g}} = \frac{m}{q}\frac{\vec{g} \times \vec{B}}{B^2} \text{ (Chen, 2015)}$$

Consequently, charge separation and confinement escape are inevitable.[1] Nevertheless, depending on conditions, the finite confinement time could be substantially extended. Fusion requires sustained periods of plasmas confinement, thus it is essential to find settings where drift effects are limited, thereby maximising confinement time.

3 METHOD

3.1 GENERAL SETUP

Deuterium ions and electrons were used as positively and negatively charged particles as tokamaks in reality commonly use deuterium plasma fuel.

Charge motions were modelled via numerically solving the equation of motion expressed as coupled first order differentials using the Runge-Kutta fourth order method (RK4):

$$\vec{s}' = \vec{v}$$

$$\vec{v}' = \frac{\vec{F}_{net}}{m}$$

3.2 TOKAMAK SIMULATION

Tokamaks were simulated with:

- Toroidal $\vec{B}$ with strength B at radius r from toroid's rotational centre:
  $$B = \frac{\mu_0 N i}{2\pi r}\,(\text{Walker et al.}, 2014)$$
- Gravitational field of $-9.8ms^{-2}$ parallel to tokamak's rotational axis
- $\vec{E}$ parallel to tokamak's rotational axis
  - $\pm$ direction set to point from $+$ to – charges based on charge separation behaviour in tokamaks

Dependent variable was confinement time. Particles that did not escape within simulation time were treated as having confinement time of 3 × simulation time.

Two sets of scenarios with different (though effectively equivalent) independent variables each containing 2 tokamak base scenarios were investigated:

- 2 deuterium ion only scenarios (charges behave similarly overall in tokamaks, thus for probing general behaviour it suffices to examine ions only)
  - Independent variables
    - Coil number (equivalent to altering $\vec{B}$ as current was held constant)
    - $\vec{E}$ strength
    - Particle speed (by altering initial velocity vectors of particles)
- 2 tokamaks modelled after ITER (a major fusion powerplant currently under construction) according to Aymar et al. (2002)
  - Independent variables
    - Coil number

[1] Methods to combat confinement escape exist, such as adding a "poloidal pitch" magnetic field. However, they are not investigated in the simplified scenarios simulated here.

- Ion density (analogous to having $\vec{E}$ as a variable)
    - Corresponding $\vec{E}$ was computed by assuming separated charges form effectively charged parallel plates and using the following equation (see code for additional assumptions):

$$E = \frac{\sigma}{\varepsilon_0} \text{(Walker et al., 2014)}$$

- Temperature (analogous to having particle speed as a variable)

4 RESULTS AND DISCUSSION

4.1.1 General comments

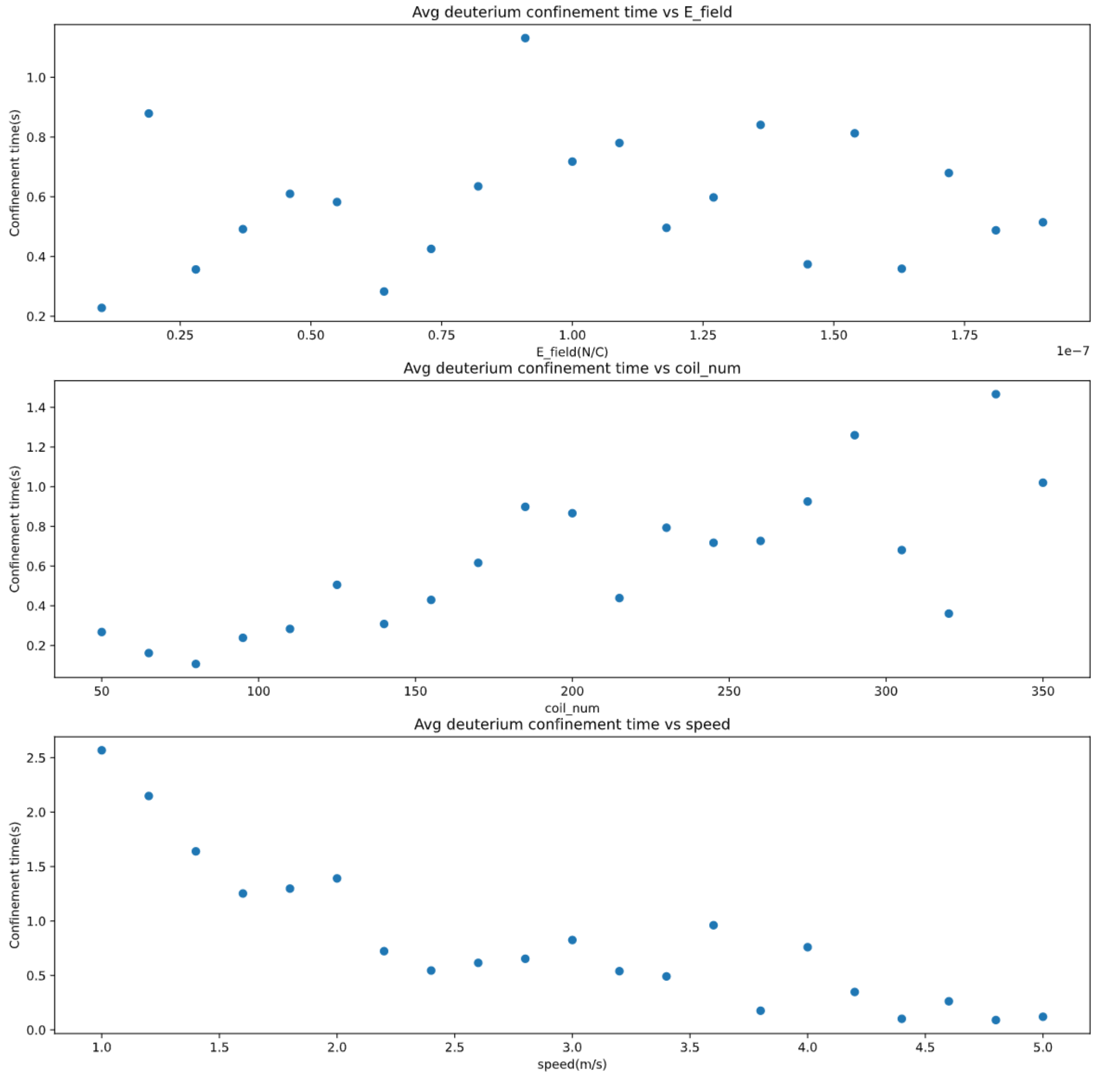
Electrons require smaller time step sizes (2-4 orders of magnitude less) than deuterium ions to maintain approximation precision. This is possibly explained by how electrons experience far greater centripetal acceleration than deuterium ions in gyration as $a_C = \frac{v_{\perp\vec{B}}^2}{r} = \frac{v_{\perp\vec{B}}^2}{mv_{\perp\vec{B}}/|q|B} = \frac{v_{\perp\vec{B}}|q|B}{m} \propto \frac{|q|}{m}$ and $\frac{|q|}{m}$ is larger for electrons than deuterium ions by an order of approximately 10^4.

4.2 TOKAMAK SIMULATIONS

4.2.1 Tokamak with small parameter value simulations, deuterium ion only

4.2.1.1 Small parameter value tokamak simulation

- Confinement time vs independent variable plots



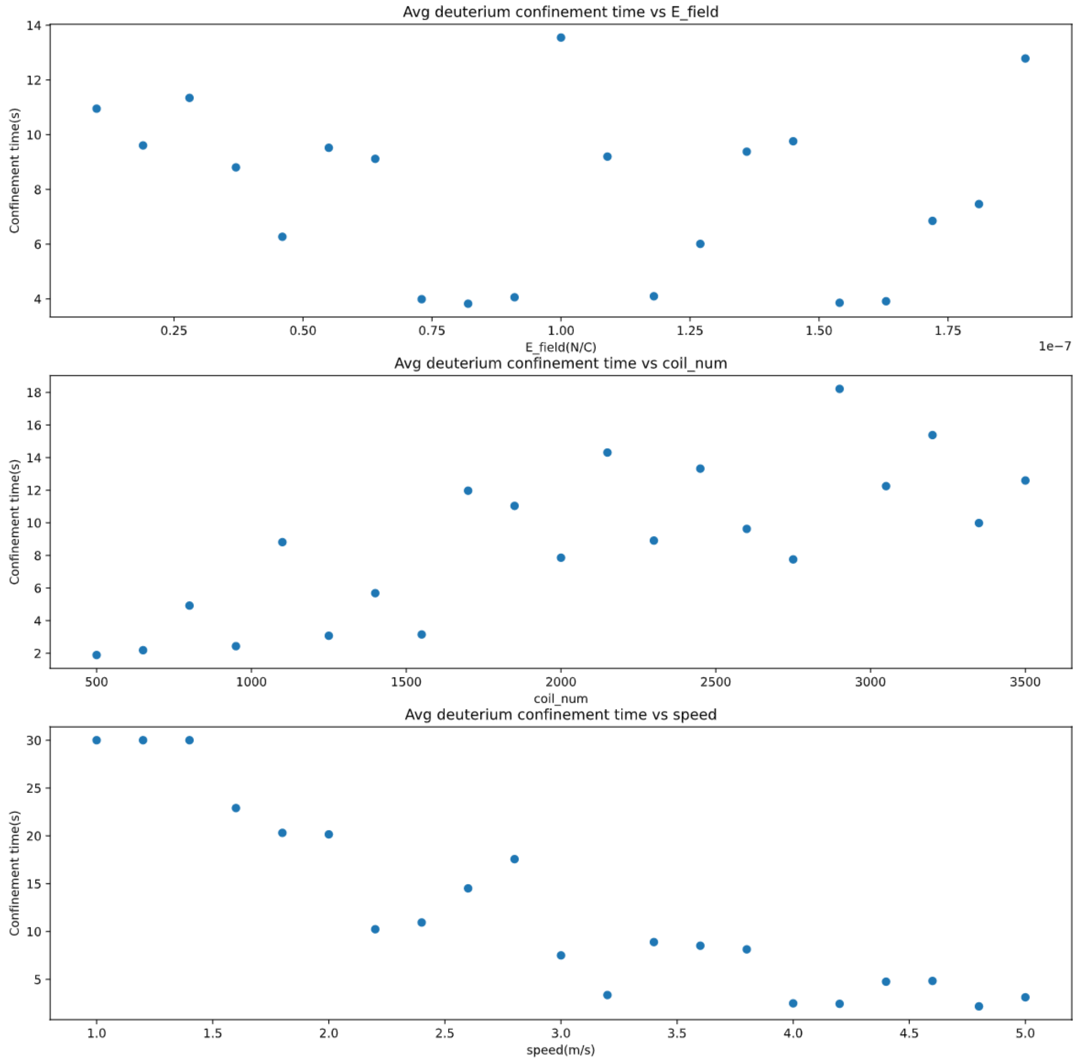
- Confinement time, independent variable correlation table

Variable	Deuterium Average Confinement Time (s)
E_field(N/C)	0.119114414
coil_num	0.766406794
speed(m/s)	-0.863807992

Here, increasing coil number ($\vec{B}$ strength) increases confinement time whilst increasing particle speeds decrease confinement time. $\vec{E}$ on the other hand has only a very weak positive correlation with confinement time that may well be a statistical fluctuation masking underlying independence, contrary to theory. Given the small variable values used for this simulation, a viable explanation is that for the set of conditions simulated, $E \times B$ drift is negligible compared to other drifts; consequently effects of $\vec{E}$ on confinement time is muted.

4.2.1.2 Small parameter value tokamak simulation with larger coil number (stronger $\vec{\boldsymbol{B}}$)

- Confinement time vs independent variable plots



- Confinement time, independent variable correlation table

Variable	Deuterium Average Confinement Time (s)
E_field(N/C)	-0.185808444
coil_num	0.76968019
speed(m/s)	-0.907226644

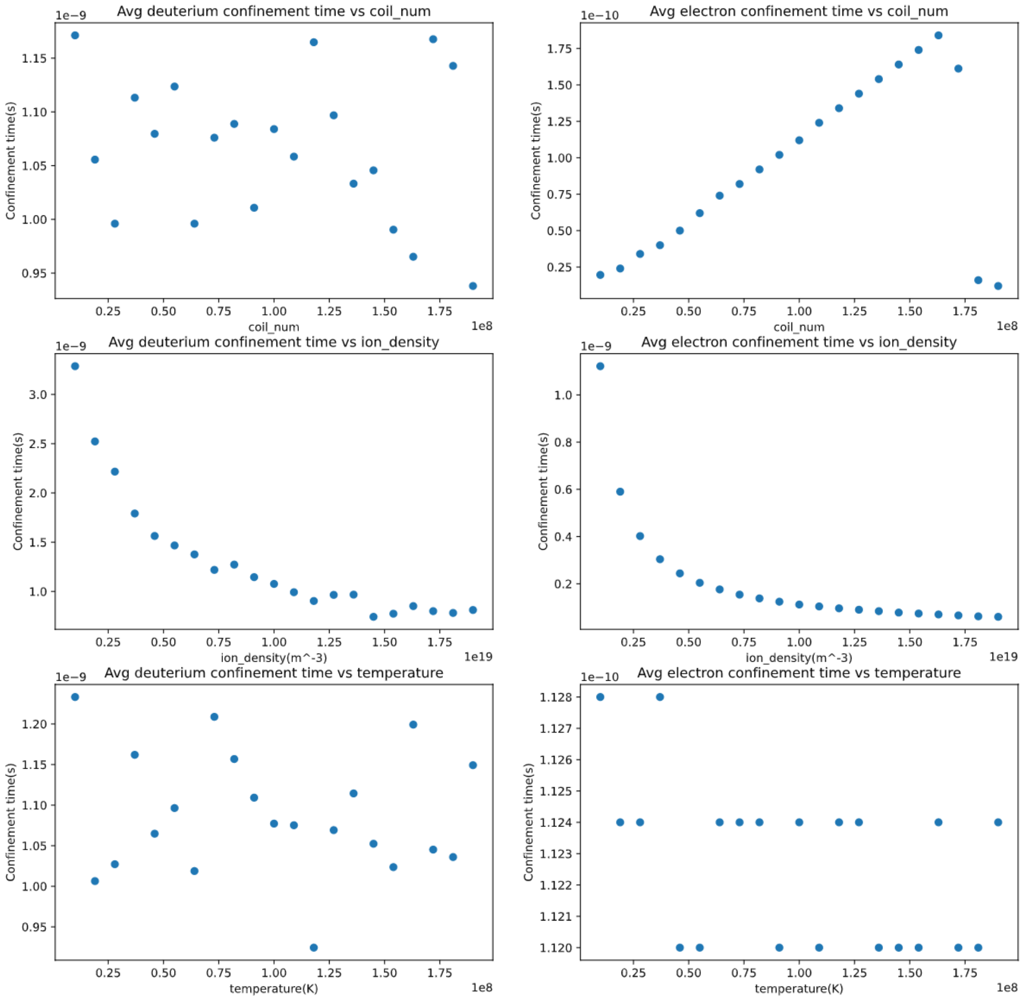
Significant positive correlations were found between coil number and confinement time and high negative correlations were found between particle speed and confinement time; both findings are consistent with those of the previous simulation.

Notably, correlation between $\vec{E}$ and confinement time is very weakly negative. Although consistent with theory (weaker $E \times B$ drift means longer containment time), such result contradicts weakly positive correlations found in the previously. Again, effective independence of $\vec{E}$ to confinement time due to specific set of variable values could be behind such correlation values, in which case the simulations' discrepancies may be explained as statistical fluctuations.

4.2.2 Simulations modelled after ITER

4.2.2.1 ITER simulation

- Confinement time vs independent variable plots



- Confinement time, independent variable correlation table

Variable	Deuterium Average Confinement Time (s)	Electron Average Confinement Time (s)
coil_num	-0.205563948	0.524500223
ion_density(m^-3)	-0.853641908	-0.716935611
temperature(K)	-0.123836326	-0.465170524

Ion density ($\vec{E}$ strength) appear strongly negatively correlated to confinement time. Thus, $\vec{E}$ strength is an important factor in determining confinement time in this case consistent with theory. The fact that $\vec{E}$ is significant in this simulation (where $\vec{E}$ strength is high) but not in previous simulations (where $\vec{E}$ strengths were low) supports also theoretical predictions regarding $E \times B$ drift being proportional to $\vec{E}$ strength.

Temperature (particle speed) does not appear strongly related to confinement time unlike for small variable value simulations. Although correlation value between electron confinement time and temperature is of notable magnitude, credibility of any inferred relationship is impacted by the strange confinement time discreteness especially considering data points are averages of multiple values. Possible explanations include approximation error due to time step size considering how the discreteness is displayed only for electrons which as aforementioned demand smaller step size for maintaining accuracy.

As for coil number vs confinement time, there is no convincing trends for deuterium ions; however, for electrons, an almost linear relationship signalling strong positive correlation up until the final 3 data points is observable. The 3 data point anomaly may again be explained by approximation errors given the issue pertains to electrons only. An alternative cause would be Python floating point error which explain well why there is a well-defined threshold before linear pattern breakdown. Further simulation with smaller step sizes should be experimented to determine the cause.

Overall, the ITER simulation result appear to be the opposite of small value tokamak simulation results; variables with strong correlations to confinement time in the latter was found to have weak correlations in the former and vice versa. This suggests that whilst all independent variables examined are correlated to confinement time, in certain cases effects of some of the variable(s) on confinement time dominate over others.

4.2.2.2 ITER simulation with smaller ion density (weaker electric field)

No particle was found to escape confinement (despite same simulation time as previous ITER simulation). However, it should be noted that only $10^{-8}s$ were simulated; the results do not eliminate the chance that particles will escape over longer periods of time. Nevertheless, considering how the only difference this simulation has with the previous is a weaker $\vec{E}$, the results still supports the aforementioned negative correlation between ion density ($\vec{E}$ strength) and confinement time.

4.2.3 Comments on uncertainty and limitations

- The simplification of treating confinement times of particles that did not escape during the simulation timespan (as 3 times the simulation time) is rather arbitrary and thus impacts the validity of any quantitative trend deduced.
- Particles in multi-particle simulations were randomised according to a uniform distribution within appropriate initial bounds; consequently, the distribution of initial particle properties may not accurately reflect realistic distribution in plasmas, hence resulting in systematic average confinement time errors.
- Only 10 particles were simulated in multi-particle simulations, hence introducing considerable uncertainty in average confinement time that partly explain the lack of clear trend curves in many of the confinement time vs independent variable plots.

5 CONCLUSION

Tokamak confinement time was found to be maximised with weak electric fields (low ion density), low particle speeds (low temperature) and large coil number (strong magnetic field). Quantitatively however, each variable’s effects on confinement time depend on the specific set of variable values; in certain cases, some variables will dominate over others regarding influence over confinement time.

6 BIBLIOGRAPHY

Aymar, R., Barabaschi, P., & Shimomura, Y. (2002). The ITER design. *Plasma Physics And Controlled Fusion*, *44*(5), 519-565. doi: 10.1088/0741-3335/44/5/304

Chen, F. F. (2015). *Introduction to plasma physics and controlled fusion* (Third edition.). Springer.

Walker, J., Resnick, R., & Halliday, D. (2014). *Fundamentals of physics* (Tenth edition, extended.). John Wiley & Sons, Inc.