Module 3: Mathematical Induction and Recurrence Relations

This Assignment is worth 5% of your final grade.

Total number of marks to be earned in this assignment: 25

Assignment 3, Version 1[1]:

After completing Module 3, including the learning activities, you are asked to complete the following written assignment in the space provided.

1)

a. Find an explicit formula for the sequence $1, -4, 9, -16, \ldots$ *(2 marks)*

b. Determine the first four terms given by $a_n = 1 + \left\lfloor (-3/2)^n \right\rfloor$. *(2 marks)*

[1] Unless otherwise cited, all questions are created by the author. Any resemblance to existing texts is purely coincidental.

2)

a. Use mathematical induction to prove that for all integers $n \geq 1$, $1+7+13+19+\cdots+(6n-5)=n(3n-2)$. *(4 marks)*

b. Use mathematical induction to prove that for all integers $n \geq 1$, $6^n - 1$, is divisible by 5. *(3 marks)*

3) Let $a_0 = a_1 = a_2 = 2$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. Prove, using strong mathematical induction, that $a_n < 2^{n+2}$. *(2 marks)*

4)

a. Determine the first five terms in the sequence defined by $a_n = 4a_{n-1} - 3a_{n-2}$ for all integers $n \geq 2$, $a_0 = 1$, $a_1 = 2$. *(3 marks)*

b. Find a recurrence relation and initial conditions for the sequence 1, 4, 13, 40, 121, 364, … *(3 marks)*

5) Use iteration to guess an explicit formula for the sequence defined by $a_n = a_{n-1} + n$ for all integers $n \geq 1$, $a_0 = 1$. *(3 marks)*

6) Solve the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2}$ for all integers $n \geq 2$, $a_0 = 1$ and $a_1 = 6$. *(3 marks)*

Bibliography

Epp, S. S. (2020). *Discrete mathematics with applications* (5th Edition). Boston: Cengage.