Summary of Multiple Integration

$\iiint_R f\, dV$

- Rect: $dV = dzdxdy$
- Cylender: $dV = dz \cdot rdrd\theta$
- Spherical: $dV = \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

Applications

- Mass
- Avg value of f
- Moment of inertia
- Gravitational attraction on mass at O

$\iint_S \vec{F} \cdot \hat{n}\, dS$ or $\iint_S \vec{F}\, d\vec{S}$

Calculate formulas for $\hat{n}dS$

<u>Ex</u>: $\hat{n}dS = \ldots dxdy$

Becomes $\iint \ldots dxdy$

+ general cases:

- $z = z(x, y)$

  $\hat{n}dS = \, < -z_x, -z_y, 1 > dxdy$

- $\vec{N}$ given normal vector

  $\hat{n}dS = \pm \dfrac{\vec{N}}{\vec{N} \cdot \hat{k}} dxdy$

Typical S

- Horizontal plane: yz-plane

  $dS = dydz$

  $\hat{n} = \pm \hat{i}$

- Sphere, centered at O

  $n = \pm \dfrac{< x, y, z >}{a}$, a is radius

  $dS = a^2 \sin\varphi\, d\varphi\, d\theta$

- Cylinder, centered at O

  $n = \pm \dfrac{< x, y, 0 >}{a}$, a is radius

  $dS = a\, dzd\theta$

$\int_C \vec{F} \cdot d\vec{r}$

$= \int_C Pdx + Qdy + Rdz$

With $\vec{F} = \, < P, Q, R >$

Parameterize C
-> express in terms of a single variable

$$\iiint_R f\,dV \xleftrightarrow{\text{div thm}} \iint_S \vec{F}\cdot\hat{n}\,dS \xleftrightarrow{\text{Stokes thm}} \int_C \vec{F}\cdot d\vec{r}$$

Divergence Theorem

$$\iint_S \vec{F}\cdot\hat{n}dS = \iiint_D \left(\nabla\cdot\vec{F}\right)dV$$

Stokes Theorem

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \left(\nabla\times\vec{F}\right)\cdot\hat{n}dS$$

Fundamental Theorem for Line Integrals

$f(P_1)-f(P_0)=\int_C(\nabla f)\cdot d\vec{r}$

Also: given $\vec{F}$ with $curl=0$, find potential