CS1303 - Fall 2021

Assignment # 3

Due: Monday, October 4, by 11:59 pm

Submission Instructions:

- Your answers should be submitted through Crowdmark. Contact Dr. Fleming if you have any questions.
- **Note:** Please submit the answer to only one question in each file that you submit on Crowdmark. Also, if possible, please leave try to leave some white space beside or below your solutions, to allow room for the marker to provide comments in Crowdmark.
- Assignments can be submitted up to 24 hours late with a 20% penalty. Assignments submitted more than 24 hours late will not be accepted, unless you have prior approval from Dr. Fleming.
- All answers you submit must be your own work. You may discuss general approaches to assignment problems with your classmates. However, these must be general and cannot include things such as detailed steps of an algorithm or a proof. Please see the course syllabus for more details.

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1. **(2 marks)** Rewrite the conditional statement below as a **disjunction** ('or' statement) that is logically equivalent to the original statement.

   You will buy apples if you do not buy bananas.

2. **(6 marks)**

   Note: For this problem, submit your answer to part (a) only. You should complete part (b) for your own benefit, but it will not be marked.

   For each of the following conditional statements, write (i) the contrapositive of the statement, (ii) the converse of the statement, and (iii) the negation of the statement.

   In part (a), your answers should be written in English (not in propositional logic).

   In part (b), all of your solutions should be rewritten so that negations are applied to single letters only.

   For both (a) and (b), you might have to use DeMorgan's Laws as part of the process.

   (a) (6 marks) If the schools are open and the roads are safe, then the game is not cancelled.

   (b) (0 marks) $(\neg Q \to P) \to (Q \wedge \neg R)$

3. **(8 marks)** Use the Laws of Equivalence (found in the "Resources" folder on Desire2Learn) to prove the following equivalences.

   State clearly which law(s) you are using in each step.

   Be sure to read the file in the "Resources" folder on Desire2Learn entitled "Important notes about equivalence proofs".

   Note: For this problem, submit your answer to part (a) only. You should complete parts (b) and (c) for your own benefit, but they will not be marked.

   (a) (8 marks) $R \to (\neg Q \to P) \equiv \neg Q \to (R \to P)$

   (b) (0 marks) $(P \wedge Q) \vee (P \wedge \neg(Q \vee \neg P)) \equiv P$

   (c) (0 marks) $\neg((P \to \neg Q) \wedge (P \to Q)) \equiv P$

4. **(3 marks)** For each of the sets specified below using set-builder notation, use the set-roster method to specify the elements.

   (a) $\{n \in \mathbb{Z} \mid 20 \leq n \leq 35$, and n is even$\}$

   Reminder: $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$ is the set of integers.

   (b) $\{x \in \mathbb{N} \mid 5x < 47$, and x is odd$\}$

   Reminder: $\mathbb{N} = \{1, 2, 3, 4, 5, \cdots\}$ is the set of natural numbers.

   (c) $\{a \in \mathbb{Z} \mid a^2 \leq 16\}$

5. **(5 marks)** Suppose that the universal set for variable x is $\mathbb{Z}$.
   Let $P(x)$ be the predicate $x^2 < 25$,
   let $Q(x)$ be the predicate $-4 \leq x \leq 4$,
   and let $R(x)$ be the predicate $|x| < 4$.

   (a) Let A be the truth set of $P(x)$. Use set-roster notation to specify the elements of A.
   (b) Let B be the truth set of $Q(x)$. Use set-roster notation to specify the elements of B.
   (c) Let C be the truth set of $R(x)$. Use set-roster notation to specify the elements of C.

   (d) Which of the following statements are true?
      i. $A \subseteq B$
      ii. $B \subseteq A$
      iii. $A = B$
      iv. $A \subseteq C$
      v. $C \subseteq A$
      vi. $A = C$
      vii. $B \subseteq C$
      viii. $C \subseteq B$
      ix. $B = C$

6. **(6 marks)** Suppose that the universal set for all variables is $\mathbb{Z}$, and let $P(a, b, c)$ be the predicate $a^2 = |b - c|$.

   Note: $|b - c|$ is referring to the absolute value of $b - c$.

   (a) Provide an example of values for a, b and c for which $P(a, b, c)$ is true and for which $b < a < c$.

   (b) Provide an example of values for r, s and t for which $P(r, s, t)$ is true and for which $r < 0$, $s < 0$, $t < 0$.

   (c) Provide an integer n such that $P(n, n, n)$ is true.

   (d) Provide an integer x such that $P(x, x + 9, x + 18)$ is true.