

University of New Brunswick
Faculty of Computer Science
CS2333: Computability and Formal Languages
Homework Assignment 1, Due Time, Date 5:00 PM, January 21, 2022

Student Name: _____ Matriculation Number: _____

Instructor: Rongxing Lu

The marking scheme is shown in the left margin and [100] constitutes full marks.

1. Write formal descriptions of the following sets: (note that you should give the finite sets by listing their contents)
 - (a) The set of all integers that are less than 5
 - (b) The set of all positive integers that have a remainder of 3 if divided by 5
 - (c) The set of all substrings of *baab*
 - (d) The cross product of $\{0, 1, 3\}$ and $\{0, 1\}$
2. Give examples of relations that have the following combination of properties:
 - (a) symmetric and reflexive but not transitive
 - (b) symmetric and transitive but not reflexive
3. For each of the following equivalence relations, list their equivalence classes (using a representative element for each class):
 - (a) $\mathcal{R}_1 = \{(x, y) : x, y \in \mathbf{Z}^{nonneg} \text{ and } 3|(x - y)\}$
 - (b) $\mathcal{R}_2 = \{(x, y) : x, y \in \{0, 1\}^{2022} \text{ and the first symbol in } x \text{ is the same as the first symbol in } y\}$
4. Given languages L_1, L_2 , and L_3 such that
$$L_1(L_2 \cap L_3) = \emptyset$$
and
$$L_1 L_2 \cap L_1 L_3 \text{ is infinite}$$
and demonstrate these properties.
5. Show that $2 - \sqrt{2}$ is irrational. (Note that, we already know that $\sqrt{2}$ is not rational.)
6. Consider the relation between two sets defined by $S_1 \equiv S_2$ if and only if $|S_1| = |S_2|$. Show that this is an equivalence relation.