CS1303 - Fall 2021

Assignment # 4

Due: Monday, October 18, by 11:59 pm

Submission Instructions:

- Your answers should be submitted through Crowdmark. Contact Dr. Fleming if you have any questions.
- **Note:** Please submit the answer to only one question in each file that you submit on Crowdmark. Also, if possible, please leave try to leave some white space beside or below your solutions, to allow room for the marker to provide comments in Crowdmark.
- Assignments can be submitted up to 24 hours late with a 20% penalty. Assignments submitted more than 24 hours late will not be accepted, unless you have prior approval from Dr. Fleming.
- All answers you submit must be your own work. You may discuss general approaches to assignment problems with your classmates. However, these must be general and cannot include things such as detailed steps of an algorithm or a proof. Please see the course syllabus for more details.

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1. **(4 marks)** For each of the following universal statements, indicate if the statement is true or false. If it is false, provide a specific value that represents a counterexample and <u>show</u> why it makes the statement false.

   [**Reminder:** $\mathbb{Z}$ is the set of all integers, and $\mathbb{R}$ is the set of all real numbers.]

   [**Note:** For any number x, $|x|$ represents the absolute value of x.
   For example, $|7| = 7$, $|-11| = 11$, and $|0| = 0$.]

   (a) $(\forall n \in \mathbb{Z})\ (n > -3n)$

   (b) $(\forall n \in \mathbb{Z})\ (|n| \geq n)$

   (c) $(\forall n \in \mathbb{Z})\ (\frac{n^2}{n^2+1}$ is not an integer$)$

   (d) $(\forall x \in \mathbb{R})\ (x^2 \geq 7x - 10)$

2. **(3 marks)** For each of the following existential statements, indicate if the statement is true or false. If it is true, use a specific integer and <u>show</u> why it makes the statement true.

   (a) $(\exists n \in \mathbb{Z})\ (\frac{|n+5|}{|n|} = 2)$

   (b) $(\exists n \in \mathbb{Z})\ (n^2 + 3n < 0)$

   (c) $(\exists n \in \mathbb{Z})\ (4n \text{ is odd})$

3. **(5 marks)** For each of the statements below, indicate if it is true or false. You do not have to provide an explanation.

   (a) $(\forall n \in \mathbb{Z})\ (\exists m \in \mathbb{Z})\ (n = m^2 + 5)$
   (b) $(\forall n \in \mathbb{Z})\ (\exists m \in \mathbb{Z})\ (m = n^2 + 5)$
   (c) $(\exists a \in \mathbb{Z})\ (\forall b \in \mathbb{Z})\ (a \neq b + 10)$
   (d) $(\exists a \in \mathbb{Z})\ (\forall b \in \mathbb{Z})\ (a < b^2)$
   (e) $(\forall n \in \mathbb{Z})\ (\exists m \in \mathbb{Z})\ (\exists p \in \mathbb{Z})\ (mp + 3 = n)$

4. **(9 marks)** Negate each of the following quantified statements. Rewrite your answer so that negations are applied to basic predicates only (*i.e.*, they are not applied to any compound statements or outside any quantifiers). Show your steps.

   (a) $(\forall x \in D)\ ((P(x) \land \neg Q(x)) \rightarrow (R(x) \lor S(x)))$

   (b) $(\exists a \in X)\ (\forall b \in X)\ (\forall c \in X)\ (a > b + c)$

   (c) $(\forall x \in A)\ (\exists y \in B)\ (P(x, y) \land ((\forall z \in C)\ (Q(x, z) \rightarrow Q(y, z))))$

5. **(7 marks)** Use the definitions discussed in class to show that the following statements are true.

   For example, to show that $5 \mid 20$, we could state that $20 = 5(4)$, and since 4 is an integer, $5 \mid 20$ (by the definition of "divides").

   (a) $8 \mid (-56)$

   (b) $17 \mid 0$

   (c) 33 is a multiple of 11.

   (d) 12 is a divisor of 192.

   (e) $(\exists p \in \mathbb{Z})\ ((p \mid 7) \wedge (p \mid 4))$

   (f) $(\exists q \in \mathbb{Z})\ ((7 \mid q) \wedge (4 \mid q))$

   (g) $(\exists p, q \in \mathbb{Z})\ ((p \neq q) \wedge ((p \mid q) \wedge (q \mid p)))$

6. **(7 marks)**

   Note: For this problem, submit your answer to part (a) only. You should complete part (b) for your own benefit, but it will not be marked.

   For each of the statements shown below, write a formal mathematical proof.

   Be sure to start your proof with two lines that lay out what you **know** and what you are trying to **show**.

   (You can use a know-show table to help you to plan out your entire proof, but you do not have to submit this.)

   (a) (7 marks) For all integers n, if $6 \mid n$, then $9 \mid (n^2 + \frac{21n}{2})$

   (b) (0 marks) For all integers a, b and c, if $c \mid a$ and $c \mid b$, then $2c \mid (8a + 10b)$