Q1 a) $f_1(n) = n^2 + 10n\log n - 2n + 4$ is $O(n^2)$

$$n^2 + 10n\log n - 2n + 4 \leq n^2 + 10n\log n + 2n + 4n \leq n^2 + 10n \cdot n + 2n^2 + 4n^2$$

So $n^2 + 10n\log n - 2n + 4 \leq 17n^2$ for $n \geq 1$

Let $n_0 = 1$ $c = 17$ $g(n) = n^2$, we show that

$f_1(n) \leq c \cdot g(n)$ for $n \geq n_0$, then $f_1(n)$ is $O(n^2)$

b) $f_2(n) = (n+1)^5$ is $\Theta(n^5)$

Step I i) Since $(n+1)^5 \leq (n+n)^5$ for $n \geq 1$

$(n+1)^5 \leq (2n)^5$

$(n+1)^5 \leq 32n^5$

Let $c_1 = 32$, $n_0 = 1$, $g(n) = n^5$

we shows that $f_2(n) \leq c \cdot g(n)$ for all $n \geq n_0$

So $f_2(n)$ is $O(n^5)$

Step II ii) since $(n+1)^5 \geq n^5$ for $n \geq 1$

Let $c = 1$ $n_0 = 1$ $g(n) = n^5$

we show that $f_2(n) \geq c \cdot g(n)$ for $n \geq n_0$

So $f_2(n)$ is $\Omega(n^5)$

Since $f_2(n)$ is $O(n^5)$ and $f_2(n)$ is $\Omega(n^5)$,

we know that $f_2(n)$ is $\Theta(n^5)$

Q2 a) $\sum_{i=1}^{n} i^2$ is $O(n^3)$

Since $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+4^2+\cdots+n^2$

So $\sum_{i=1}^{n} i^2 \le n^2+n^2+n^2+\cdots+n^2$ for $n \ge 1$

$\sum_{i=1}^{n} i^2 \le n \cdot n^2$

$\sum_{i=1}^{n} i^2 \le n^3$

let $c=1$ $g(n)=n^3$ $n_0=1$

we shown that $\sum_{i=1}^{n} i^2 \le c \cdot g(n)$ for $n \ge n_0$

So $\sum_{i=1}^{n} i^2$ is $O(n^3)$

b) $\sum_{i=1}^{n} \log(i)$ is $O(n \log n)$

Since $\sum_{i=1}^{n} \log(i) = \log 1 + \log 2 + \cdots \log n$

So $\sum_{i=1}^{n} \log(i) \le \log n + \log n + \cdots \log n$ for $n \ge 1$

So $\sum_{i=1}^{n} \log(i) \le n \cdot \log n$

let $c=1$ $g(n)=n \cdot \log n$ $n_0=1$

we shown that $\sum_{i=1}^{n} \log i \le c \cdot g(n)$ for $n \ge n_0$

So $\sum_{i=1}^{n} \log(i)$ is $O(n \cdot \log n)$

3: a) $10n \cdot \log n + 2n^2$ is $O(n^2)$

b) $100 \log n + 2n$ is $O(n)$

c) $400 n^{\frac{1}{2}}$ is $O(n^{\frac{1}{2}})$

d) $20n^5 + 2^n$ is $O(2^n)$

e) $n^3 + 10n^2 + 100n$ is $O(n^3)$

f) $\log(n^5)$ is $O(\log n)$

$O(\log n) \leq O(n^{\frac{1}{2}}) \leq O(n) \leq O(n^2) \leq O(n^3) \leq O(2^n)$