CS 2383: Data Structures and Algorithms

Assignment 1: Assignment Title (if any)

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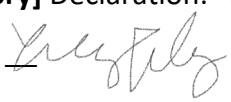
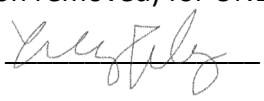
[Mandatory] Declaration: "I warrant that this is my own work."

Signed by 

[Optional] "I hereby give my permission for this work to be used (with my name and identifying information removed) for UNB Faculty of Computer Science program accreditation purposes."

Signed by 

Yulong Wang Assignment 3

Q1: $(1+2+\cdots+n) \leq (n+n+\cdots+n)$

$\leq n^2$ $\quad c=1 \quad n_0=1 \quad g(n)=n^2$

$f(n) \leq c \cdot g(n)$ for $n \geq n_0$

So this algo is $O(n^2)$

Q2: A: n columns, k rows; B: k columns, m rows

the run time will be $n \cdot m \cdot k$

assume n is the largest number such that $n \geq m \geq k$

so $n \cdot m \cdot k \leq n \cdot n \cdot n = n^3$

this algo is $O(n^3)$

Q3: a) $T(n) = T(n-1) + T(n-1) = 2T(n-1)$ $T(n-1) = 2T(n-2)$

$T(n) = 2T(n-2) + 2T(n-2)$

$T(n) = 4T(n-2)$

$T(n) = 2(Tn-1)$ $\quad n-1 \quad 2^1$

$T(n) = 4T(n-2)$ $\quad n-2 \quad 2^2$

$T(n) = 8T(n-3)$ $\quad n-3 \quad 2^3$

$\vdots$

$T(n) = 2^n T(0)$ $\quad 0$ $\quad T(n) = 2^n \cdot 1$

$\Rightarrow$ this algo is $O(2^n)$

$T(0) = 2^{-1} + 2^{-1}$

$2^0 = \frac{1}{2} + \frac{1}{2}$

$2^0 = 1 \Rightarrow T(0) = 1$

Q3 b) $0 \to n-1$, This algo is $O(n)$

Q4 $T(n) = 2T(n/2) + n \quad , n > 1 \quad T(1) = 1$

$T(\frac{n}{2}) = 2T(\frac{n}{4}) + \frac{n}{2}$

$\Rightarrow T(n) = 2 \cdot (2T(\frac{n}{4}) + \frac{n}{2}) + n$

$= 4T(\frac{n}{4}) + n + n$

$= 4T(\frac{n}{4}) + 2n$

$\Rightarrow T(n) = 4 \cdot (2T(\frac{n}{8}) + \frac{n}{4}) + 2n$

$= 8 \cdot T(\frac{n}{8}) + 3n$

$\Rightarrow \quad T(n) = 2T(\frac{n}{2}) + n \quad$ ①

$T(n) = 4T(\frac{n}{4}) + 2n$ ⋮

$T(n) = 8T(\frac{n}{8}) + 3n$ ⋮

⋮

$T(n) = 2^k T(\frac{n}{2^k}) + kn \quad k$

let $\frac{n}{2^k} = 1$

$n = 2^k$

$k = \log_2 n$

$\Rightarrow T(n) = n \cdot T(1) + \log n \cdot n \quad , T(1) = 1$

$= n + \log n \cdot n \leq \log n \cdot n + \log n \cdot n = 2n \log n \in O(n \cdot \log n)$