CS1303 - Fall 2021

Assignment # 2

Due: Monday, September 27, by 11:59 pm

Submission Instructions:

- Your answers should be submitted through Crowdmark. Contact Dr. Fleming if you have any questions.
- **Note:** Please submit the answer to only one question in each file that you submit on Crowdmark. Also, if possible, please leave try to leave some white space beside or below your solutions, to allow room for the marker to provide comments in Crowdmark.
- Assignments can be submitted up to 24 hours late with a 20% penalty. Assignments submitted more than 24 hours late will not be accepted, unless you have prior approval from Dr. Fleming.
- All answers you submit must be your own work. You may discuss general approaches to assignment problems with your classmates. However, these must be general and cannot include things such as detailed steps of an algorithm or a proof. Please see the course syllabus for more details.

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1. **(5 marks)** Consider the sentence "If x is a multiple of 12, then x is a multiple of 3." Rephrase this sentence in English in several ways:

   (a) using the word "implies"

   (b) using the word "whenever"

   (c) using the phrase "is sufficient for"

   (d) using the phrase "is necessary for"

   (e) using the phrase "only if"

2. **(2 marks)** Suppose that you are given the following logical statement:

   $$\neg\ P\ \wedge\ Q\ \rightarrow\ \neg\ R\ \vee\ S$$

   Based on our discussion of order of operations, which is the correct way to interpret this?

   (a) $\neg(P \wedge ((Q \rightarrow (\neg R)) \vee S))$
   (b) $(\neg(P \wedge (Q \rightarrow (\neg R)))) \vee S$
   (c) $((\neg P) \wedge (Q \rightarrow (\neg R))) \vee S$
   (d) $(\neg P) \wedge ((Q \rightarrow (\neg R)) \vee S)$
   (e) $(\neg(P \wedge Q)) \rightarrow (\neg(R \vee S))$
   (f) $((\neg P) \wedge Q) \rightarrow ((\neg R) \vee S)$
   (g) $((\neg P) \wedge (Q \rightarrow (\neg R))) \vee S$
   (h) $((\neg P) \wedge (Q \rightarrow (\neg R))) \vee S$
   (i) $(\neg P) \wedge ((Q \rightarrow (\neg R)) \vee S)$
   (j) $\neg((P \wedge Q) \rightarrow (\neg(R \vee S)))$

3. **(3 marks)** Suppose that P, Q and R are statements for which the only thing you are told is that $P \rightarrow (Q \wedge P)$ is false. This should give you some information about P and Q.

   Consider the statements below, given the information from above. For each statement below, answer one of the following:

   (i) we know it is true; (ii) we know it is false; (iii) we do not have enough information.

   Give very brief explanations for your answers.

   (a) $Q \wedge R$
   (b) $P \vee \neg R$
   (c) $Q \rightarrow P$

For questions 4 and 5, use the following order for the rows in your truth tables.

P	Q	
T	T	
T	F	
F	T	
F	F	

P	Q	R	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

4. **(8 marks)** Construct truth tables for the following statement forms.

   In your truth table, make sure that you include a column for each intermediate expression that you evaluate on your way to your final answer.

   (a) $(P \leftrightarrow \neg Q) \wedge (P \leftrightarrow Q)$

   (b) $(\neg P \vee (\neg Q \vee \neg R)) \vee (P \wedge Q)$

5. **(14 marks)** For each of the pairs of statement forms below, use the truth table method to determine if (i) is logically equivalent to (ii). Show the truth tables, state your answers, and explain briefly.

   (a) (i) $Q \rightarrow (P \vee R)$ (ii) $\neg P \rightarrow (Q \rightarrow R)$

   (b) (i) $R \rightarrow (P \wedge \neg Q)$ (ii) $(R \rightarrow P) \wedge \neg Q$

6. **(3 marks)** Based on your truth tables for each of the statement forms in questions 4 and 5 (4a, 4b, 5a(i), 5a(ii), 5b(i) and 5b(ii)), indicate whether each statement form is a tautology, a contradiction, or neither.

7. **(5 marks)** Use DeMorgan's Laws (and the double negation law) as many times as necessary to rewrite the following statement form:

$$\neg((R \wedge (P \ \vee \neg S)) \vee ((\neg P \vee Q) \ \vee \neg R))$$

In your final answer, the negation operator ($\neg$) should only be applied to atomic statements (single letters) rather than to compound statements.

For example, $\neg A \wedge \neg B$ would be an acceptable final answer, but $\neg(A \vee B)$ would not because the negation operator is applied to the compound statement $A \vee B$.