STAD37A2

Yulun Wu

24/10/2022

Q6

```
X = matrix(c(0.56, -0.01, 0.79, 1.07, -0.30, 2.35, 1.16, 0.12,
             -0.46, 0.41, 2.18, 1.73, -0.44, -0.14, 0.54, 5.36,
             1.00, 1.56, 0.58, 1.63),10,2,byrow=TRUE)
X
```

```
##        [,1]  [,2]
##   [1,]  0.56 -0.01
##   [2,]  0.79  1.07
##   [3,] -0.30  2.35
##   [4,]  1.16  0.12
##   [5,] -0.46  0.41
##   [6,]  2.18  1.73
##   [7,] -0.44 -0.14
##   [8,]  0.54  5.36
##   [9,]  1.00  1.56
##  [10,]  0.58  1.63
```

```
mu0 = c(1.4,1.2)
mu0
```

```
## [1] 1.4 1.2
```

(a)

```
n = nrow(X) # n
p = ncol(X) # p
X_bar = colMeans(X) # X_bar
X_bar
```

```
## [1] 0.561 1.408
```

```
S = cov(X) # S
S
```

```
##           [,1]      [,2]
## [1,] 0.6679656 0.1576911
## [2,] 0.1576911 2.6508844
```

```
T_sq = n*t(X_bar-mu0)%*%solve(S)%*%(X_bar-mu0) # Test statistics T^2
T_sq
```

```
##          [,1]
```

```
## [1,] 11.16917
```

```r
criticalValue = (((n-1)*p)/(n-p))*qf(1-0.05,p,n-p) # critical value
criticalValue
```

```
## [1] 10.03268
```

Since 11.16917 > 10.03268, we reject the null hypothesis that mu = (1.4,1.2)' at level a = 0.05.

(b)

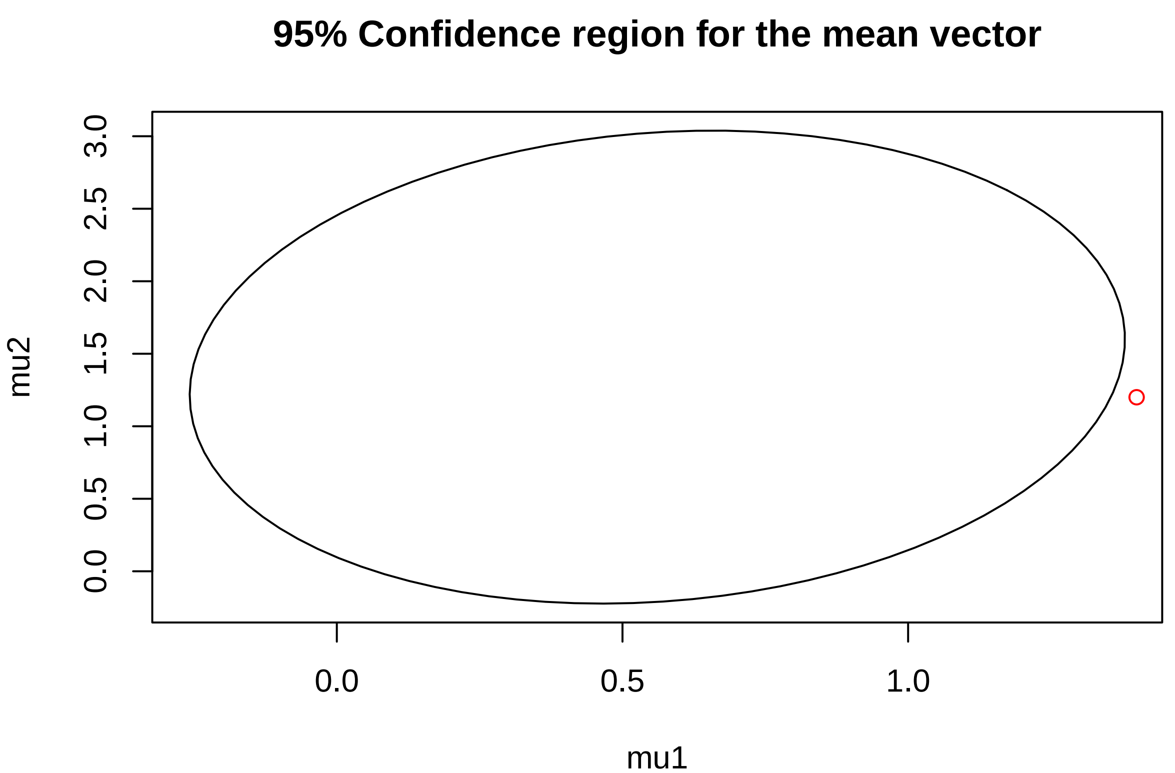
```r
library(ellipse)
```

```
## Warning: package 'ellipse' was built under R version 4.1.2
```

```
##
## Attaching package: 'ellipse'
```

```
## The following object is masked from 'package:graphics':
##
##     pairs
```

```r
# plot confidence ellipse
plot(ellipse(S,centre=X_bar,t=sqrt(criticalValue/n)),type="l",xlab="mu1",ylab="mu2",main="95% Confidenc
points(mu0[1],mu0[2],col="red") # mu0
```



The point of mu0 is outside of the 95% confidence ellipse, this means mu0 doesn't lies in the confidence ellipse, so we reject the null hypothesis that mu = (1.4,1.2)' at level a = 0.05.

(c)

```r
a = c(2,-1) # a
a
```

```
## [1]  2 -1
```

```
mu0=0.2 # 2mu1-mu2=0.2
SE = sqrt((t(a)%*%S%*%a)/n)
SE
```

```
##           [,1]
## [1,] 0.6849805
```

```
testStatistic = abs((t(a)%*%X_bar-mu0)/SE)
testStatistic
```

```
##           [,1]
## [1,] 0.7095093
```

```
criticalValue2 = qt(1-0.05/2,n-1)
criticalValue2
```

```
## [1] 2.262157
```

Since 0.7095093 < 2.262157, we fail to reject the null hypothesis that 2mu1-mu2=0.2 at level a = 0.05.

(d)

```
c_sq = ((n-1)*p/(n-p))*qf(1-0.05,p,n-p) # c_sq
c_sq
```

```
## [1] 10.03268
```

```
c = sqrt(c_sq)
c
```

```
## [1] 3.167441
```

```
f = c(1,0) # for mu1
g = c(0,1) # for mu2
SEmu1 = sqrt((t(f)%*%S%*%f)/n) # standard error for mu1
SEmu1
```

```
##           [,1]
## [1,] 0.2584503
```

```
SEmu2 = sqrt((t(g)%*%S%*%g)/n) # standard error for mu2
SEmu2
```

```
##           [,1]
## [1,] 0.5148674
```

```
atranspose_xbar_mu1 = t(f)%*%X_bar # f'X_bar for mu1
atranspose_xbar_mu2 = t(g)%*%X_bar # g'X_bar for mu2
# SCI for mu1
SCI_LL_mu1 = atranspose_xbar_mu1 - c*SEmu1
SCI_UL_mu1 = atranspose_xbar_mu1 + c*SEmu1
SCI_mu1 = c(SCI_LL_mu1,SCI_UL_mu1)
SCI_mu1
```

```
## [1] -0.2576261  1.3796261
```

```
# SCI for mu2
SCI_LL_mu2 = atranspose_xbar_mu2 - c*SEmu2
SCI_UL_mu2 = atranspose_xbar_mu2 + c*SEmu2
```

```r
SCI_mu2 = c(SCI_LL_mu2,SCI_UL_mu2)
SCI_mu2
```

```
## [1] -0.2228121  3.0388121
```

The simultaneous 95% confidence interval for mu1 is (-0.2576261,1.3796261). The simultaneous 95% confidence interval for mu2 is (-0.2228121,3.0388121).

```r
criticalValue3 = qt(1-(0.05/2)/2,n-1) # m=2 because we have mu1 and mu2
criticalValue3
```

```
## [1] 2.685011
```

```r
# BCI for mu1
BCI_LL_mu1 = atranspose_xbar_mu1 - criticalValue3*SEmu1
BCI_UL_mu1 = atranspose_xbar_mu1 + criticalValue3*SEmu1
BCI_mu1 = c(BCI_LL_mu1,BCI_UL_mu1)
BCI_mu1
```

```
## [1] -0.1329418  1.2549418
```

```r
# BCI for mu2
BCI_LL_mu2 = atranspose_xbar_mu2 - criticalValue3*SEmu2
BCI_UL_mu2 = atranspose_xbar_mu2 + criticalValue3*SEmu2
BCI_mu2 = c(BCI_LL_mu2,BCI_UL_mu2)
BCI_mu2
```

```
## [1] 0.02557543 2.79042457
```

The 95% Bonferroni simultaneous confidence interval for mu1 is (-0.1329418,1.2549418). The 95% Bonferroni simultaneous confidence interval for mu2 is (0.02557543,2.79042457). sad