

# Introduction to Video Processing

## HW4

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# HW4: Fourier Transform for Translating Sinusoidal Video

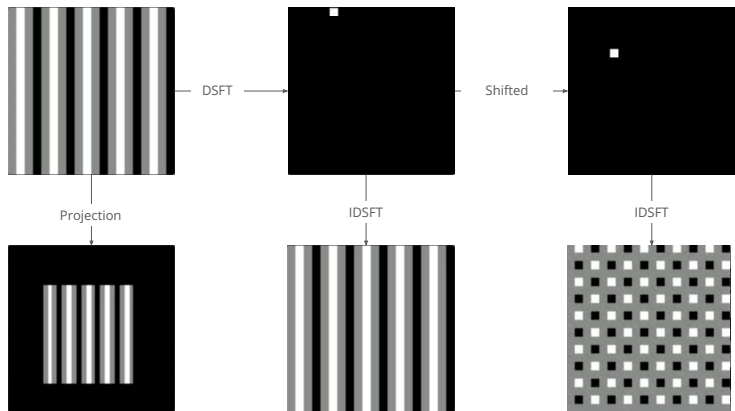


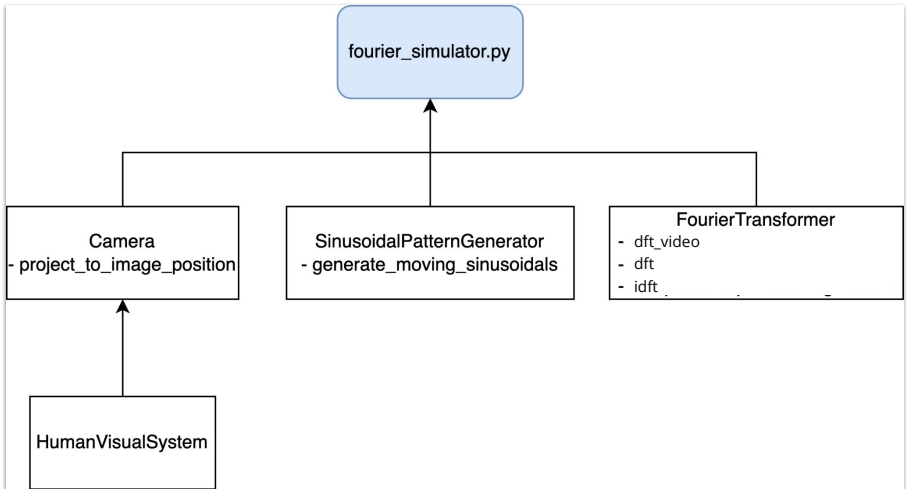
Figure: Overview of the workflow.

You should complete the following tasks:

- Complete the provided codes.
- Provide a HW report (see report guidelines).

# Code Completion

(Code Architecture)



(Tracing on **fourier\_simulator.py**)

```
from sinusoidal_pattern_generator import *
from fourier_transformer import *
from visualizer import *
import os.path

fx_vid = 0.5 # Cycles per mm along horizontal direction
fy_vid = 0.0 # Cycles per mm along vertical direction
vx_vid = 0.2 # Speed along horizontal direction mm/sec
vy_vid = 0.2 # Speed along vertical direction in mm/sec
Tb = 0 # Beginning of the time
dt = 1 # Frame time in seconds
Te = 30 # End of the time in seconds
fps = (Te - Tb) / dt
```

- The spatial temporal signal is a moving sinusoidal pattern.
- $fx\_vid$ ,  $fy\_vid$ ,  $vx\_vid$ ,  $vy\_vid$ ,  $Tb$ ,  $dt$ ,  $Te$  characterize frequencies of the 3D sinusoidal signal.

(Continue)

```
# Physical Length of the image plane
h_mm = 10
w_mm = 10
d_mm = 15

# Height, width, and interval in pixels
height = 100
width = 100
intv = 5 # Interval for downsampling the frequency and spatial temporal
         variables
```

- *h\_mm, w\_mm, d\_mm, height, width, intv* characterizes the resolution of the signal.

(Continue)

```
print('Generating the sinusoidal signal...')
pattern_generator = SinusoidalPatternGenerator(w_mm, h_mm, width,
        height, Tb, Te)
spatial_temporal_signal = pattern_generator.generate_moving_sinusoidals
        (fx_vid, fy_vid, fps, vx_vid, vy_vid, intv)

# Viz for Spatial Temporal Signals
visualize_spatial_temporal_signal(folder_out + '/spatial_temporal.mp4',
        spatial_temporal_signal, width, height, fps, intv)

# Viz the viewer perspective results
visualize_viewed_signal(folder_out + '/viewing.mp4', spatial_temporal_
        signal, width, height, fps, intv, w_mm, h_mm, d_mm)
```

- The spatial temporal signal is generated by *generate\_moving\_sinusoidals*.
- We utilize two visualization functions for visualizing original spatial temporal signal and its viewing version.

(Continue)

```
xformer = FourierTransformer()

# Compute frequency response
print('Computing Fourier transform for the sinusoidal signal...')
frequency_signal = xformer.dft_video(spatial_temporal_signal)

# Cache the results for testing or debugging
cache_signal(folder_out + '/frequency.sg', frequency_signal)
'''

# Load cached signal if you do not want to compute the same transform
  again, used for debugging.
frequency_signal = load_signal(folder_out + '/frequency.sg', spatial_
    temporal_signal.shape)
'''

# Viz for Frequency Response
visualize_frequency_signal(folder_out + '/frequency_response.mp4',
    frequency_signal, width, height, fps, intv)
```

- Create the fourier transformer.
- Use *dft\_video* for the video transformation.
- We can cache or load signal for debugging or testing.



(Continue)

```
# Reconstruct spatial temporal signal via inverse fourier transform
print('Recovering the spatial temporal signal by inverse Fourier
      transform...')
recovered_signal = xformer.idft(frequency_signal)

# Cache reconstructed signal
cache_signal(folder_out + '/reconstruction.sg', recovered_signal)

# Visualize reconstructed signal (SHOULD be visually the same as
    original spatial temporal signal)
visualize_recovered_signal(folder_out + '/reconstruction.mp4',
    recovered_signal, width, height, fps, intv)
```

- We use *idft* for performing the inverse Fourier transform.
- The reconstructed signal should be ideally the same as original spatial temporal signal.

(Continue)

```
# Shift the frequency signal
print('Shifting the frequency signal...')
dx = 0
dy = 5
dt = 0
resized_height, resized_width, frames = frequency_signal.shape[0],
    frequency_signal.shape[1], frequency_signal.shape[2]
shifted_frequency_signal = np.zeros([resized_height, resized_width,
    frames], dtype=complex)
# TODO #2: Implement a shifting operation to move the frequency
    responses. The shifted responses are stored in
# shifted_frequency_signal

# Viz for Shifted Frequency signal
visualize_frequency_signal(folder_out + '/shifted_frequency.mp4',
    shifted_frequency_signal, width, height, fps, intv)
```

- Implement the frequency response shifting here by using  $dx$ ,  $dy$ ,  $dt$ .

(Continue)

```
# Generate the corresponding spatial temporal signal, it SHOULD be
  changed.
print('Generating new spatial temporal signal according to the shifted
      frequency signal...')
edited_spatial_temporal_signal = xformer.idft(shifted_frequency_signal)

# Cache the new spatial temporal signal
cache_signal(folder_out + '/edited-spatial-temporal.sg', edited_spatial
             _temporal_signal)

# Visualize the new spatial temporal signal
visualize_recovered_signal(folder_out + '/edited-spatial-temporal.mp4',
                           edited_spatial_temporal_signal, width, height, fps, intv)
```

- Perform inverse Fourier transform on the shifted frequency signal, we will see the changed spatial temporal signal.

(Tracing on **fourier\_transformer.py**)

```
class FourierTransformer():
    def __init__(self):
        pass

    """
    Inputs:
        - video: 3D numpy array with real numbers
    Outputs:
        - output_signal: 3D numpy array with complex numbers
    """
    def dft_video(self, video):
        # Convert the input video type into complex type
        video_complex = video.astype(complex)
        return self.dft(video_complex)
```

- The *dft\_video* convert the type from real to complex.

(Continue)

```
'''
Inputs:
    - input_signal: 3D numpy array with complex numbers.
Outputs:
    - output_signal: 3D numpy array with complex numbers.
'''
def dft(self, input_signal):
    # Get width, height, frames of the input_signal.
    height, width, frames = input_signal.real.shape[0], input_
        signal.real.shape[1], input_signal.real.shape[2]

    # Get frequency response by computing fourier dft on the input
        signals
    output_signal = np.zeros([height, width, frames], dtype=complex
        )

    # TODO #3: Implement 3D fourier transform for output_signal
        according to the DSFT formulas provided in the slides.
    # You are NOT ALLOWED to use any third party API to execute the
        Fourier transform

    return output_signal
```

(Continue)

```
'''
Inputs:
    - input_signal: 3D numpy array with complex numbers
Outputs:
    - output_signal: 3D numpy array with complex numbers.
'''
def idft(self, input_signal):
    # Get width, height, frames of the input_signal.
    height, width, frames = input_signal.real.shape[0], input_
        signal.real.shape[1], input_signal.real.shape[2]

    # Get frequency response by computing fourier dft on the input
        signals
    output_signal = np.zeros([height, width, frames], dtype=complex
        )

    # TODO #4: Implement 3D inverse Fourier transform for output_
        signal according to the IDSFT formulas provided in the
        slides.
    # You are NOT ALLOWED to use any third party API to execute the
        inverse Fourier transform

    return output_signal
```

(Tracing on **visualizer.py**)

```
def visualize_spatial_temporal_signal(filename, signal, width, height,
    fps, intv):
    fourcc=cv2.VideoWriter_fourcc(*'mp4v')
    video = cv2.VideoWriter(filename, fourcc, fps, (width, height))
    image = np.zeros([height, width, 3], dtype = np.uint8)
    # Frame index of viz.
    f_viz = 0
    frames = signal.shape[2]
    for f in range(frames):
        for y in range(height):
            if y % intv == 0:
                # Get corresponding y in signal
                resized_y = y // intv
                for x in range(width):
                    if x % intv == 0:
                        # Get corresponding x in signal
                        resized_x = x // intv
                        (See next page)

    video.release()
```

- The visualization images have more pixels than original signal.
- *resized\_x, resized\_y*: corresponding spatial position in the signal.

(Continue)

```
# Get corresponding signal
b = signal[resized_y, resized_x, f]

# Dump to signal if current frame is filled.
if (f > f_viz):
    video.write(image)
    f_viz += 1

# Normalization
b_viz = (b + 1) / 2 # Mapping the value from
                    (-1, 1) to (0, 1)

# Draw response with thickness as the specified
  interval
image[y:y+intv, x:x+intv, :] = int(b_viz * 255)
# Mapping to integer
```

- Visualize the signal by filling square areas for every 3D variable.



(Continue)

(Tracing on **sinusoidal\_pattern\_generator.py**)

```
class SinusoidalPatternGenerator():  
    '''  
    Inputs:  
        w_mm: plane width in milli meters  
        h_mm: plane height in milli meters  
        width: number of pixels on the horizontal direction  
        height: number of pixels on the vertical direction  
        Tb: Beginning of the time  
        Te: End of the time  
    '''  
    def __init__(self, w_mm, h_mm, width, height, Tb, Te):  
        self.w_mm = w_mm  
        self.h_mm = h_mm  
        self.width = width  
        self.height = height  
        self.Te = Te  
        self.Tb = Tb
```

- A moving sinusoidal pattern in a plane.

(Continue)

```
'''
Inputs:
    fx: frequency on horizontal direction in cycles per mm
    fy: frequency on vertical direction in cycles per mm
    ft: frequency on temporal direction in frames per sec
    vx: velocity along horizontal direction in mm/sec
    vy: velocity along vertical direction in mm/sec
    intv: Interval for downsampling the frequency and spatial
        temporal variables
Outputs:
    video: temporal spatial signal. A list of 4D vectors: [b, x, y,
        t], where x, y, t represents the temporal
        sample, and b represents the brightness (singal) on (x, y, t)
'''
def generate_moving_sinusoidals(self, fx, fy, ft, vx, vy, intv):
    (See next page)
```

- The actual function to generate sinusoidal signal.

(Continue)

```
# Set video dimension
pix_per_mm = self.width / self.w_mm # Resolution

# Build time sequence with the interval (Tb, Te, dt)
times = []
dt = self.Te / ft
t = self.Tb
while t < self.Te:
    times.append(t)
    t += dt
```

- Build time sequence and set the resolution (*pix\_per\_mm*).

(Continue)

```
# Pre-compute video samples based on the interval
frames = len(times)
resized_height = (self.height - 1) // intv + 1
resized_width = (self.width - 1) // intv + 1
video = np.zeros([resized_height, resized_width, frames])
for f in range(frames): # Frame
    for y in range(self.height):
        if y % intv == 0: # Downsampling for reducing the
                           computing
            for x in range(self.width):
                if x % intv == 0: # Downsampling for reducing
                                   the computing
                    (See next page)
```

- Downsampling the signal by *intv*.

(Continue)

```
# Compute input signal value as the
    brightness according to the specified
    sin pattern
x_mm = x / pix_per_mm # In mm
y_mm = y / pix_per_mm # In mm
t = times[f]
dx = vx * t
dy = vy * t
b = np.sin(fx * 2 * np.pi * (x_mm + dx) +
    fy * 2 * np.pi * (y_mm + dy)) # Should
    located in (-1, 1)
resized_y = y // intv
resized_x = x // intv
video[resized_y, resized_x, f] = b
```

- The brightness is changed by the three factors  $fx$ ,  $fy$ ,  $dx$ ,  $dy$ .
- $dx$  and  $dy$  are varied with the speed and time.

# Report Guidelines

- Describe your implementation. (Do NOT paste code screen shot figures)
- Describe your understanding of theories.
- Describe your experiment settings and results.
  - Examine and analyze variations in results by adjusting the settings of the sinusoidal patten.
  - Examine and analyze variations in results by adjusting the response shift of the frequency signal.
  - Share any additional noteworthy insights.
  - Raise any additional problems or sharing any additional insights.
- Describe your proofs.
- Write your division of labor (same rules as those of previous homework).
- Only .pdf file format is allowed.

## Grading Criteria

- Richness of your experiments.
- Readability of the report.
- Clarity of your insights and the division of labor.

# Scoring

- Code completion (80%)
- Report (20%)



# Submission Guidelines

- Put all the codes into a folder *codes*.
- This time, the required videos/images are automatically put into the subfolder *results* in *codes*.
- Put all the experimented videos (with additional experiment setting) into a folder *supplementary-material*.
- Put **videos**, **supplementary-material** (optional) and report.pdf into a folder **hw4** (i.e., they are under same level of the file tree)
- Compress **hw4** and specify the file format as hw4.zip.
- Upload hw4.zip onto Digital Platform 3.
- Deadline: 12:00 a.m. on March 27th.

# Origin of The Latex Template

The latex template is downloaded from:  
<https://www.LaTeXTemplates.com>

The Author: Vel ([vel@latextemplates.com](mailto:vel@latextemplates.com))

## **Temporary page!**

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