Technical University Khaenri'ah
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Quantum Aspects of the Art of Khemia*

Doctoral Thesis
by

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Abstract

> Khaenri'ah was a nation without a god – not because it had a god that died or abandoned them, but because it never had a god to begin with. It was a powerful nation, built purely by humans, an unprecedented flourishing and glorious civilization – it was the pride of humankind.
>
> *(Dainsleif)*

The purpose of this thesis is the (pre-)calculation of the damage a character in specific team compositions deals against an enemy and to find balanced stat ratios. Furthermore it aims to give players a better underlying understanding of damage reduction methods to increase the total damage, both screenshot and average damage.

Methods used to find acceptable models and the theoretical background will be added in a later version. Some principles may already be listed in chapter 3 and if not clearly stated otherwise the mention of errors is neglected as long as they are lower than the error caused by rounding the values displayed in the game, e.g. character stats and damage numbers. It is also worth noting that for all regression models the sum of squared residuals was minimized while trying to keep the models as "simple" as possible. Since it's easier to present the results, instead of showing exactly the deviations to reality, complex examples for teams and their damage can be found in section 1.6.

Finally this is an alpha version that hasn't yet been proofread by all contributors, therefore we would appreciate any kind of feedback. For this please use the hoyolab post(s) from Holou.

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Chapter 1.

Damage

> Khaenri'ah was an underground realm, and its natural fauna were few indeed. As such, its alchemy focused more heavily on the creation of life. This art of creation was known as "The Art of Khemia."
>
> *(Albedo)*

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1.1. Base Damage

The base damage DMG_{Base} is defined as the non-critical damage dealt to an object without consideration of the object's defense, resistance, any form of damage reduction or elemental reactions. The formula for the base damage is given by

$$DMG_{\text{Base}} = ATK \cdot Mult_{\text{Talent}} \cdot (1 + DMG_{\text{Bonus}}) \cdot (1 + DMG_{\text{Increase}}) \,, \tag{1.1}$$

whereby the definitions of attack ATK, talent multiplier $Mult_{\text{Talent}}$, damage bonus DMG_{Bonus} and damage increase DMG_{Increase} can be found in the following sections. Although some characters have skills that scale of attributes other than attack (e.g. defense DEF, health HP), this doesn't change the general outline of the damage formula (1.1). If there are multiple different stats influencing the damage they are added additively. E.g. Zhongli's 4th ascension passive giving him, e.g. for normal attacks, a damage increase by 1.39 % of his max HP. The formula (1.1) would change to (no damage increase)

$$DMG_{\text{Base}} = (ATK \cdot Mult_{\text{Talent 1}} + HP \cdot Mult_{\text{4th Ascension}}) \cdot (1 + DMG_{\text{Bonus}}) .$$

Another example would be the sword Cinnabar Spindle, equipped on Albedo it would increase his elemental skill (Transient Blossom) damage as follows (no damage increase):

$$\begin{aligned} DMG_{\text{Base}} &= (DEF \cdot Mult_{\text{Talent 2}} + DEF \cdot Mult_{\text{Weapon,passive}}) \cdot (1 + DMG_{\text{Bonus}}) \\ &= DEF \cdot (Mult_{\text{Talent 2}} + Mult_{\text{Weapon,passive}}) \cdot (1 + DMG_{\text{Bonus}}) . \end{aligned}$$

1.1.1. Attack

With the base attack

$$ATK_{\text{Base}} = ATK_{\text{Character}} + ATK_{\text{Weapon}} \tag{1.2}$$

of a character given by the sum of the innate level-dependent value of the character $ATK_{\text{Character}}$ and the weapon's main stat ATK_{Weapon} the total attack value ATK of a character can be calculated with

$$ATK = ATK_{\text{Base}} \cdot (1 + ATK_{\%}) + ATK_{\text{Flat}}. \tag{1.3}$$

$ATK_{\%}$ is the sum of all percentage attack bonuses in regards to the character and ATK_{Flat} is the sum of all flat attack bonuses applied to the character. For enemies the total attack (and defense) value just depends on the level and it's not relevant to go into any details in this thesis about that topic.

1.1.2. Talent

In the talent section of each character the talent multiplier $Mult_{\text{Talent}}$, depending only on the corresponding talent level, can be found. Although in some cases this multiplier can increase depending on the amount of stacks (this is just an example)

$$Mult_{\text{Talent}} = Mult_{\text{Talent,Base}} + \sum_{i=1}^{n} Mult_{\text{Stack,i}} = Mult_{\text{Talent,Base}} + n \cdot Mult_{\text{Stack}} \tag{1.4}$$

with n the number of stacks gained, whereby when $Mult_{\text{Stack,i}}$ is a constant $Mult_{\text{Stack}}$ this simplifies the equation and $Mult_{\text{Talent,Base}}$ is the base value.

Examples for stacks are Eula's Lightfall Swords (burst) gaining energy stacks for the explosion at the end or Raiden's Secret Art: Musou Shinsetsu (burst) gaining resolve stacks to increase the initial hit of her burst and normal attacks during the burst.

1.1.3. Damage Bonus

The variable DMG_{Bonus} in equ. (1.1) is the sum of all damage bonuses. These are always mentioned in-game with the keyword "DMG" and are neither considered as damage increase

(see 1.1.4), damage increase based on a characters stats nor are they described as damage stacks of character skills (see 1.1.2).

Examples are elemental-, physical-, elemental burst-, skill-, i-hit- or normal attack-, charged attack-, plunge-, all DMG, weapon passives with increased DMG against enemies affected by electro, hydro, pyro, ...

1.1.4. Damage Increase

The last type of damage DMG_{Increase} occurs very rarely and is multiplicative. In-game descriptions mention them as an increase of a specific skill of characters (not based on character stats or the whole character) and examples are:

1. Yoimiya's Niwabi Fire-Dance: "During this time, arrows fired by Yoimiya's Normal Attack will be Blazing Arrows, and their DMG will be increased and converted to Pyro DMG." The multiplier $1 + DMG_{\text{Increase}}$ is the Blazing Arrow DMG mentioned in the talent description depending on the talent level.
2. Xingqiu's 4th constellation Evilsoother: "Throughout the duration of Guhua Sword: Raincutter, the DMG dealt by Guhua Sword: Fatal Rainscreen is increased by 50 %."

1.1.5. Critical Hit

A critical hit gets increased by the crit-damage value $CD \in [0, \infty[$ of the character, resulting in

$$DMG_{\text{Base,Crit}} = DMG_{\text{Base}} \cdot (1 + CD)\,. \tag{1.5}$$

The crit-rate[1] $CR \in [0, 100]$ of the character is the probability that a critical hit occurs. The average damage[2] is then given by

$$\begin{aligned} \langle DMG_{\text{Base}} \rangle &= DMG_{\text{Base}} \cdot (1 - CR) + DMG_{\text{Base}} \cdot CR \cdot (1 + CD) \\ &= DMG_{\text{Base}} \cdot (1 + CR \cdot CD)\,. \end{aligned} \tag{1.6}$$

1.2. Total Damage

The total damage a character or enemy takes without consideration of elemental reactions can be expressed with

$$DMG_{\text{Total}} = DMG_{\text{Base}} \cdot Mult_{\text{DEF}} \cdot Mult_{\text{RES}} \cdot Mult_{\text{DMG Reduction}}, \tag{1.7}$$

whereby the multipliers for defense, resistance and damage reduction are described in the following sections.

1.2.1. Defense

Analogue to section 1.1.1 the base defense DEF_{Base} and the total defense value DEF of a character is defined by

$$DEF_{\text{Base}} = DEF_{\text{Character}} \tag{1.8}$$

$$DEF = DEF_{\text{Base}} \cdot (1 + DEF_{\%}) + DEF_{\text{Flat}}. \tag{1.9}$$

[1] The in-game stat screen can show negative or higher values than 100 for the crit-rate and the variable CR defined here is therefore 0 or 100 respectively.

[2] The average damage may also be influenced by attack speed buffs, although this will be omitted here, and than integration over time is necessary.

It was deduced that the defense multiplier $Mult_{\text{DMG,DEF}}$ depends on the defenders defense and the attackers level. Therefore for the enemy defense, without any defense reduction, a polynomial model with only the level has been used for the regression analysis and lead to

$$DEF_{\text{Enemy}} = 5 \cdot Level_{\text{Enemy}} + 500 \tag{1.10}$$

and under consideration of the D-symmetry (3.1) an intrinsic defense $\widetilde{DEF}_{\text{Object}}$ for every object has been defined

$$\widetilde{DEF}_{\text{Object}} = (1 - DEF_{\text{Reduction}}) \cdot (5 \cdot Level_{\text{Object}} + 500)\,, \tag{1.11}$$

whereby $DEF_{\text{Reduction}}$ considers a defense reduction applied to the object $\in \{\text{Enemy}, \text{Character}\}$ being attacked (defender).

Using a rational function model the damage reduction depending on the defenders defense and the attackers level (expressed through the intrinsic defense) has been deduced to

$$DMG_{\text{Reduction,DEF}} = \frac{DEF_{\text{Defender}}}{DEF_{\text{Defender}} + \widetilde{DEF}_{\text{Attacker}}}. \tag{1.12}$$

With this the defense multiplier

$$Mult_{\text{DEF}} = 1 - DMG_{\text{Reduction,DEF}}, \tag{1.13}$$

in the case the character attacks the enemy, can be simplified to

$$Mult_{\text{DEF}} = \frac{Level_{\text{Character}} + 100}{(1 - DEF_{\text{Reduction}}) \cdot (Level_{\text{Enemy}} + 100) + Level_{\text{Character}} + 100}, \tag{1.14}$$

whereby the last transformation of the equation is only valid when a character attacks an enemy and it shows that the defense multiplier only depends on the level of the character and enemy. One can see from the formula (1.14) or in the figure 1.1 that, without any defense reduction, the damage is halved when both the character and enemy are of equal level.

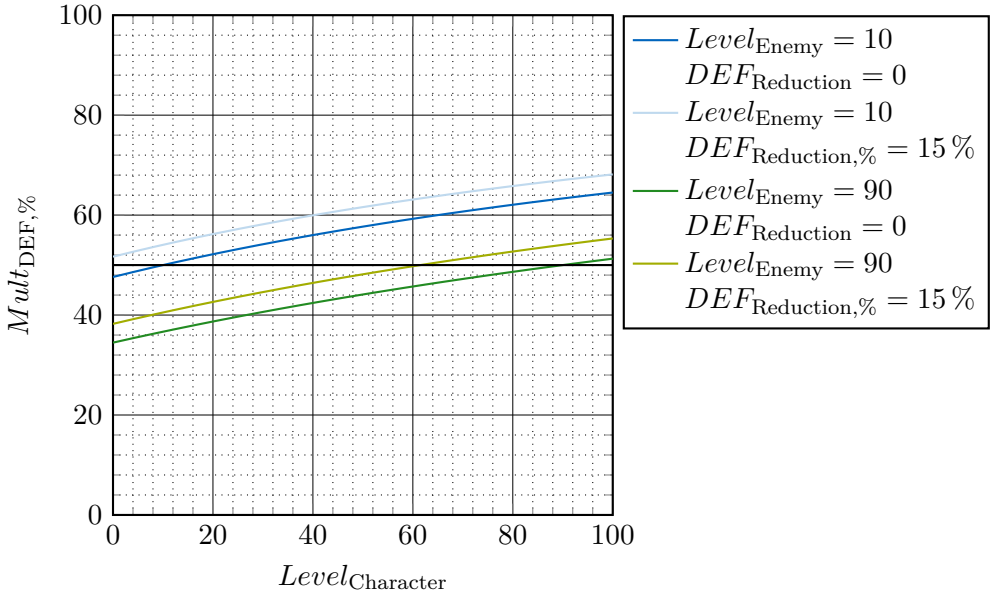


Figure 1.1.: Defense multiplier as a function of the attackers level for different enemy levels and defense reductions.

Example 1.1 (Defense Reduction with Lisa): *In this example Lisa's burst is used to reduce the defense of the enemy, Primo Geovishap level* 93*, by* $DEF_{\text{Reduction}} = 15\,\%$*. The character dealing damage is Keqing level* 90 *with her E ability. With the equation* (1.14) *the defense multiplier without defense reduction amounts to* 0.496 1 *and with changes to* 0.536 7*. Therefore the increase in total damage is* $\frac{0.5367}{0.4961} = 1.0818$*, in other words* 8.18 %*.*

a. Without Lisa's defense reduction: 14 147. **b.** With Lisa's defense reduction: 15 303.

Figure 1.2.: Damage comparison for defense reduction with Lisa's Q- and Keqing's E skill.

Multiplying the damage in figure 1.2.a with this number leads to 15 304*, which is, considering in-game rounding, equivalent to the damage in the right figure 1.2.b.*

1.2.2. Resistance

A character or enemy has different base resistances RES_{Base} to every elemental and physical damage. These can be increased or decreased, whereby RES_{Bonus} is defined as the sum of the first and RES_{Debuff} of the latter. The resistance multiplier varies non-linearly with the total resistance

$$RES = RES_{\text{Base}} + RES_{\text{Bonus}} - RES_{\text{Debuff}} \tag{1.15}$$

and has been determined by a regression analysis to have the following form

$$Mult_{\text{RES}} = \begin{cases} 1 - \left(\dfrac{RES}{2}\right) & \text{for } RES < 0, \\ 1 - RES & \text{for } 0 \leq RES \leq 0.75, \\ \dfrac{1}{4 \cdot RES + 1} & \text{for } RES > 0.75. \end{cases} \tag{1.16}$$

In most cases $RES_{\text{Base},\%}$ of enemies[3] is lower than 75 % and therefore one can casually say that in this case when the base resistance is positive, e.g. $RES_{\text{Base}} = 10\,\%$, and a Viridescent Venerer swirl reaction is caused ($RES_{\text{Debuff},\%} = 40\,\%$) the resistance gets reduced to an effective value of $\frac{RES_{\%}}{2} = -15\,\%$. Lastly keep in mind that an enemies resistance can also be affected by effects like "Ley Line Disorder", "Elemental Nodes", "elemental armor of Fatui Skirmishers", etc.

Example 1.2 (Resistance Reduction with Viridescent Venerer (VV) Swirl): *Sucrose, equipped with a 4pc VV artefact set, is used to reduce the electro resistance by* 40 % *of the enemy, Primo Geovishap* $RES_{\text{Base},\%} = 10\,\%$*. With equation* (1.16) *the resistance multiplier changes from* 0.9 *to* 1.15 *and the increase in total damage is* $\frac{1.15}{0.9} = 1.278$*, in other words* 27.8 %*.*

[3] The currently known resistances of enemies can be found either at https://genshin-impact.fandom.com/ or https://genshin.honeyhunterworld.com/, although take the latter with a grain of salt and think if it makes sense, since it may contain some mistakes.

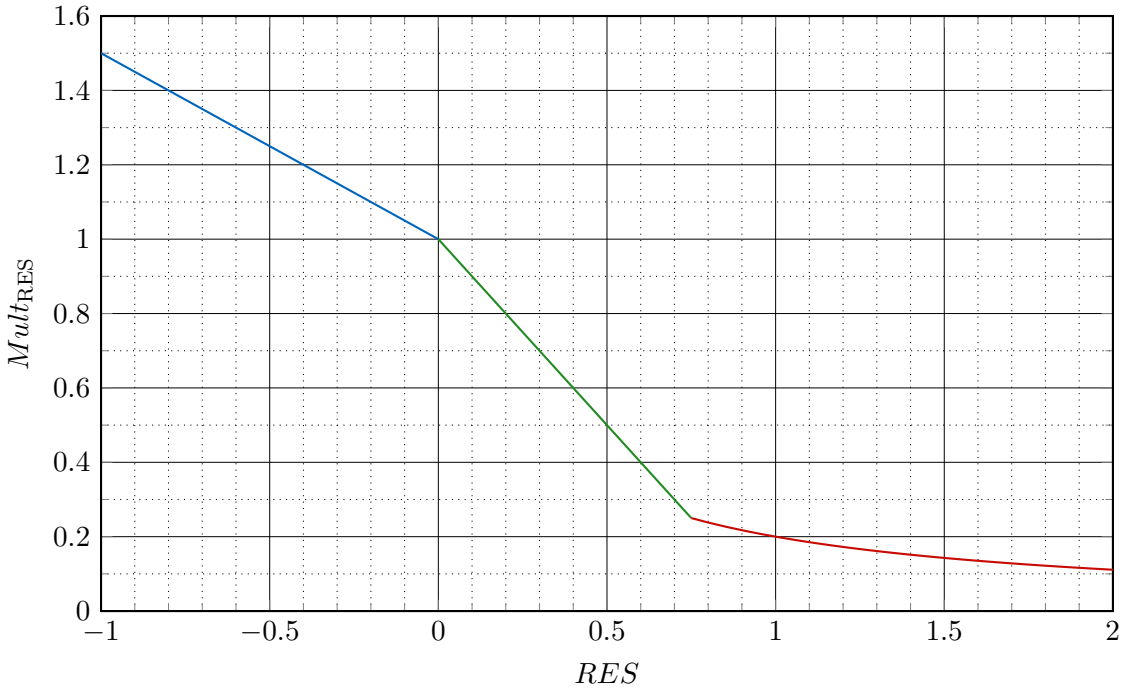


Figure 1.3.: Resistance multiplier as a function of the total resistance.

a. Without VV reduction: 14 147. **b.** With VV reduction: 18 076.

Figure 1.4.: Damage comparison for resistance reduction with 4pc VV artefact bonus.

Multiplying the damage in the left figure 1.4.a with this number leads to 18 077*, which is, considering in-game rounding, equivalent to the damage in the right figure 1.4.b.*

1.2.3. Reduction

The last multiplier for damage reduction occurs very rarely, in-game descriptions mention them directly as damage reductions of specific skills of characters and is given by

$$Mult_{\text{DMG Reduction}} = 1 - DMG_{\text{Reduction}}, \tag{1.17}$$

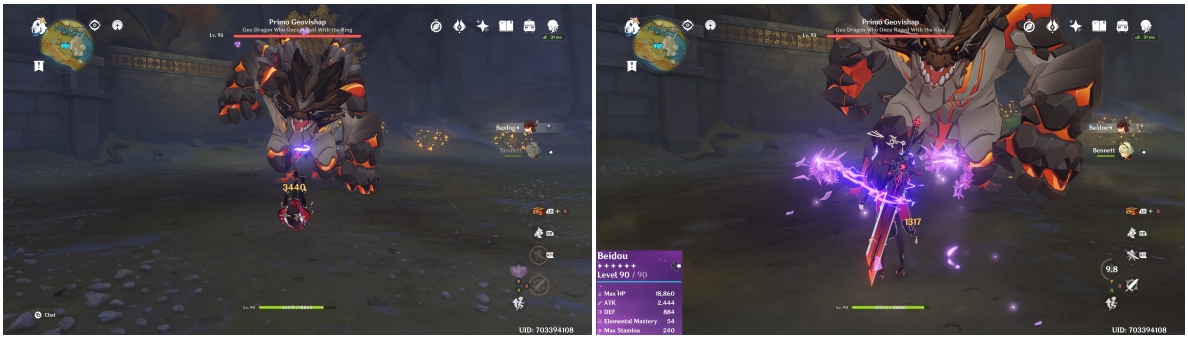
whereby multiple damage reduction abilities are added together additively. Examples are

1. Beidou's Thunderbeast's Targe: "Decreases DMG taken. (37 % at talent level 13)"
2. Xingqiu's Guhua Sword: "When a character takes DMG, the Rain Sword will shatter, reducing the amount of DMG taken. 20 % of Xingqiu's Hydro DMG Bonus will be converted to additional DMG Reduction for the Rain Swords."

3. Jean's 6th constellation Lion's Fang, Fair Protector of Mondstadt: "Incoming DMG is decreased by 35 % within the Field created by Dandelion Breeze."

4. Traveler's (anemo) 4th constellation Cherishing Breezes: "Reduces DMG taken while casting Palm Vortex by 10 %."

Example 1.3 (Damage Reduction with Beidou): *In this example the reduction of incoming damage is shown with a C6-Beidou, therefore the additional hp of the shield, based on* 16 % *of Beidou's max HP, has to be taken into consideration.*



a. Without damage reduction: 3 440.

b. With damage reduction at 2nd hit: 1 317 (+3 018 from shield).

Figure 1.5.: Incoming damage comparison for damage reduction with Beidou's Q.

One can see in figure 1.5 that Beidou's max HP is 18 860 *and therefore her shield HP amounts to* 3 018*. The talent level of Beidou's burst in this test is* 13*, providing* 37 % *damage reduction. The reduction multiplier* $Mult_{\text{DMG Reduction}}$ *changes from* 1 *to* 0.63*.*

Without any reduction the incoming damage is 3 440 *(see fig. 1.5.a), multiplying with the damage reduction multiplier leads to only* 2 167 *damage taken. Two hits with damage reduction, after destroying the shield, result in* 1 316 *incoming damage taken, which is, considering in-game rounding, equivalent to the damage taken in the right figure 1.5.b.*

1.3. Elemental Reactions

There are two types of elemental reactions: amplifying reactions that increase the damage of the hit triggering the reaction and transformative reactions dealing additional elemental damage when triggered.

The elemental bonus EM_{Bonus} is displayed in the stat screen in-game and given approx. by

$$EM_{Bonus,i} = k_i \cdot \frac{EM}{EM + c_i}, \tag{1.18}$$

with the elemental mastery EM of the character. The visualization in figure 1.6 serves for simple estimations. The two constants c_i and k_i depend on the type of reaction and are listed in table 1.1.

Table 1.1.: Regressed model parameters for the elemental bonus EM_{Bonus}.

	Amplifying	Transformative	Crystallize
k_i	2.78	16	4.44
c_i	1 400	2 000	1 400

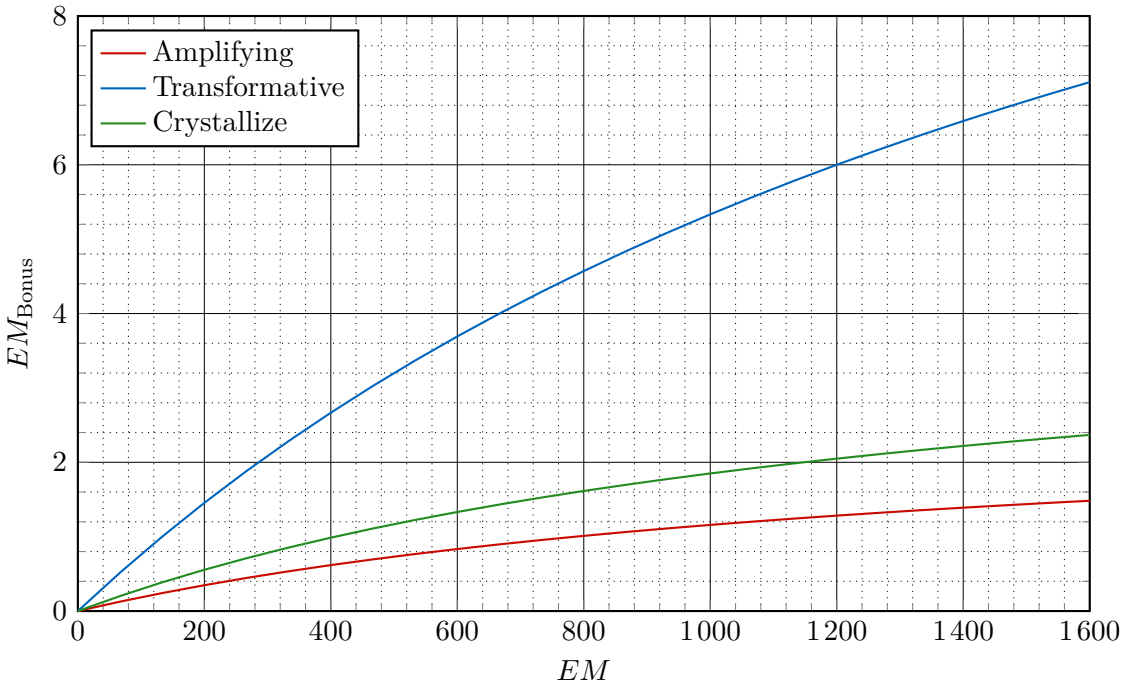


Figure 1.6.: Elemental bonus damage as a function of the elementary mastery.

In addition to the elemental mastery, reaction bonuses $Reaction_{\text{Bonus}}$, additive to EM_{Bonus}, can further increase elemental reactions. Examples are

- 4pc Crimson Witch of Flames (CWoF) artefact set: +15 % for melt and vaporize, +40 % for burning and overload.
- 4pc Thundering Fury (TF) artefact set: +40 % for overload, electro charge and superconduct.
- 4pc Viridescent Venerer (VV) artefact set: +60 % for swirl.
- 2pc Retracing Bolide (RB) artefact set: +35 % for crystallize.
- Mona's 1st constellation Prophecy of Submersion: +15 % for electro charge, vaporize and hydro swirl.

1.3.1. Amplifying Reactions

Amplifying reactions are melt and vaporize. Melt is the elemental reaction triggered by inflicting pyro on a target that is already affected by cryo or the other way around which often is more accurately termed reverse melt. Vaporize is triggered by inflicting hydro on an already pyro affected target and the opposite is called reverse vaporize.

These reactions add extra multipliers to the damage of the attack that triggered the reaction based on the triggering element, the elemental mastery and reaction bonuses of the triggering character. When an amplifying reaction occurs the total damage a character or enemy takes can be expressed with

$$DMG_{\text{Amplified}} = DMG_{\text{Total}} \cdot Mult_{\text{Amplified}} \tag{1.19}$$

$$Mult_{\text{Amplified}} = Mult_{\text{Reaction}} \cdot (1 + EM_{\text{Bonus,Amplified}} + Reaction_{\text{Bonus}}) , \tag{1.20}$$

$$Mult_{\text{Reaction}} = \begin{cases} 2 & \text{for melt/vaporize,} \\ 1.5 & \text{for reverse melt/vaporize.} \end{cases} \tag{1.21}$$

Example 1.4 (Amplifying Reaction (Reverse Melt) with Hu Tao): *Hu Tao, equipped with a 4pc CWoF artefact set, is used to cause a reverse melt reaction, whereby Kokomi is just used for the hydro application. Hu Tao's EM stat amounts to* 65 *(see fig. 1.7), giving her an elementary mastery bonus of* $EM_{\text{Bonus},\%} = 12.3\,\%$*, according to equation (1.18) and table 1.1. With the* 15 % *reaction bonus from the 4pc CWoF set, reverse melt reaction multiplier of* 1.5 *and equation (1.20) the amplifying multiplier is* $Mult_{\text{Amplified}} = 1.91$*.*

a. Without reaction: 5 502. **b.** With reverse vaporize: 10 513.

Figure 1.7.: Damage comparison for reverse vaporize with Hu Tao's normal attacks.

Multiplying the damage in the left figure 1.7.a with this number leads to 10 509*, which is, considering in-game rounding, especially for EM (*$[64.5, 65.5)$ *is all displayed as* 65*), equivalent to the damage in the right figure 1.7.b.*

1.3.2. Transformative Reactions

Transformative reactions are overload, shatter, electro-charge, superconduct and swirl[4]. These only scale with the triggering characters level[5], elemental mastery, reaction bonus and the resistance of the enemy (as always they aren't affected by any stats related to the character applying the element(s) first). In other words transformative reactions do not scale of the characters attack, can't crit, ignore the objects defense (therefore for enemies independent of there level), can't be buffed with damage bonuses, damage increase methods nor are they affected by damage reduction.

$$DMG_{\text{Transformative}} = Mult_{\text{RES}} \cdot Mult_{\text{Transformative}} \tag{1.22}$$

$$Mult_{\text{Transformative}} = Mult_{\text{Reaction}} \cdot Mult_{\text{Level}} \cdot (1 + EM_{\text{Bonus,Transformative}} + Reaction_{\text{Bonus}}) \tag{1.23}$$

$$Mult_{\text{Reaction}} = \begin{cases} 0.5 & \text{for superconduct} \\ 0.6 & \text{for swirl} \\ 1.2 & \text{for electro-charge} \\ 1.5 & \text{for shatter} \\ 2 & \text{for overload} \end{cases} \tag{1.24}$$

[4]The reaction "burning" will be omitted here.
[5]Ofc these reactions also can go the other way around and would than scale of the enemies level and resistance of the character.

The level multiplier for characters is listed in the appendix B.1, but since most players have their characters maxed out to a certain point the values $Mult_{\text{Level(70)}} = 765.6$, $Mult_{\text{Level(80)}} = 1\,077.4$ and $Mult_{\text{Level(90)}} = 1\,446.9$ or figure 1.8 should be enough for most. Side note: Simple regression models are currently not known or do not provide a good enough result.

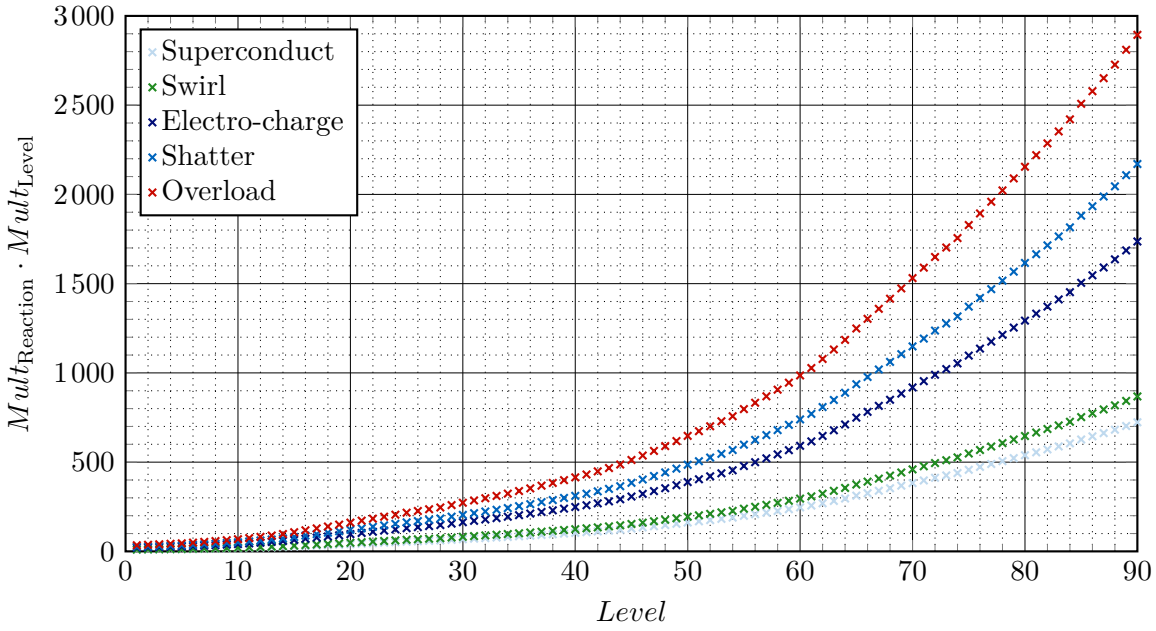


Figure 1.8.: Multipliers for transformative reactions as a function of the characters level.

1.3.2.1. Swirl-Induced Reaction Damage

Triggering swirl with multiple enemies affected by different elements will result in additional amplifying or transformative reactions being triggered due to the transfer of elements between the enemies. These so called swirl-induced reactions behave and scale, with the exception of the amplifyied/transformative multiplier being added, exactly in the same way as transformative reactions.

$$DMG_{\text{Swirl-Induced}} = Mult_{\text{RES,2nd Enemy}} \cdot Mult_{\text{Transformative,Swirl}} \cdot Mult_i \tag{1.25}$$

$$\begin{aligned} Mult_{\text{Transformative,Swirl}} &= Mult_{\text{Reaction,Swirl}} \cdot Mult_{\text{Level,Character}} \\ &\quad \cdot (1 + EM_{\text{Bonus,Swirl}} + Reaction_{\text{Bonus,Swirl}}) \end{aligned} \tag{1.26}$$

$$Mult_i = \begin{cases} Mult_{\text{Amplified}} & \text{see equ. (1.20) for } i = \text{Amplified} \\ Mult_{\text{Transformative}} & \text{see equ. (1.23) for } i = \text{Transformative} \end{cases} \tag{1.27}$$

In both cases of $Mult_i$ the level multiplier is based on the character triggering the swirl.

1.3.3. Crystallize

The elemental reaction triggered by inflicting geo on a target already affected by pyro, hydro, cryo or electro is called crystallize[6]. This reaction deals no damage, instead it generates corresponding elemental shards that can be picked up to gain an elemental shield (more see sec 2.1).

[6]The other way around also works but geo applications have a very low decay time and do not linger, which makes it nearly impossible to trigger this way.

1.4. Attack Speed

Attack speed[7] (ATK SPD) alters the speed of a character's attacks. This increases the average and overall dps in a multiplicative way in addition to other damage based bonuses and multiple atk speed buffs stack additively[8].

Since calculations strongly depend on the team one uses this thesis will not go any further in that regard, as example one may take Eula, where an atk speed increase leads to more possible stacks (burst) and therefore higher average and screenshot damage.

1.5. Snapshotting

To be continued...

1.5.1. Static and Dynamic Buffs

To be continued...

1.5.2. Elemental Reactions

To be continued...

1.6. Complex Examples

We would be glad to calculate any damage numbers, find optimizations, balance stat ratios, etc. for anybody, as long as the stats of each character, weapon, artefact bonuses, talents are provided.

1.6.1. Raiden

In this section first the initial burst damage (and charge attacks during her Q) of a C2-Raiden shall be calculated. The following characters (and enemy), artefacts and weapons are used (only the necessary information is displayed):

1. C2 Raiden: 4pc Emblem, R1 Engulfing Lightning, level 90, Q talent level: 10 (max. stacks), E talent level: 9.
2. C3 Bennet: 4pc Nobless, Aquillia Favonius, level 90, Q talent level: 10, Usage: $(191 + 674) \cdot \frac{100.8\,\% + 20\,\%}{100\,\%} = 1\,045$ atk buff.
3. C6 Sucrose: 4pc VV, Usage: 20 % electro damage buff and 40 % electro shred.
4. C6 Beidou: Usage: 15 % electro shred.
5. (pyro) Geovishap: Level 93, 10 % base electro resistance

Of course theoretically one could increase the damage further with other weapons characters and so on. Just for those interested we will at the end mention the damage with R5 Thrilling Tales of Dragon Slayers (TToDS) on Sucrose, R5 Wolf's Gravestone (WG) on Beidou with low hp enemy and C6 Sara with max. atk buff.

[7] Movement speed does not increase attack speed and vice versa.

[8] Analyzing the number of frames with different buffs leads to the conclusion that there may be either a general or for each character an individual (depending on the physics of the character) cap at around $(63 \pm 15)\,\%$... More high atk speed buffs are needed to make a better statement.

The first part consists of calculating Raiden's stats after applying every buff mentioned above:

$$\begin{aligned}
ER &= ER_{\text{Character}} + ER_{\text{Weapon}} + ER_{\text{Weapon,Q-passive}} + ER_{\text{Artefacts}} \\
&= 132\,\% + 55.1\,\% + 30\,\% + 90.6\,\% = 307.7\,\%, \\
ATK_{\text{Base}} &= ATK_{\text{Character}} + ATK_{\text{Weapon}} = 945, \\
ATK &= ATK_{\text{Base}} \cdot (1 + ATK_{\text{Artefacts,\%}} + ATK_{\text{Weapon,passive}} + ATK_{\text{Nobless,buff}}) \\
&\quad + ATK_{\text{Artefacts,Flat}} + ATK_{\text{Bennet,buff}} \\
&= 945 \cdot \left(1 + \frac{29.2\,\%}{100\,\%} + \frac{ER - 100\,\%}{100\,\%} \cdot \frac{28\,\%}{100\,\%} + \frac{20\,\%}{100\,\%}\right) + 375 + 1045 = 3379, \\
CD_{\%} &= CD_{\text{Base,\%}} + CD_{\text{Artefacts,\%}} = 50\,\% + 103.4\,\% = 153.4\,\%.
\end{aligned}$$

Next the talent and damage bonus multiplier[9]:

$$\begin{aligned}
Mult_{\text{Q-Talent,Initial}} &= Mult_{\text{Q-Talent,Initial}} + n_{\text{Resolve stacks}} \cdot Mult_{\text{Q-Talent,Initial bonus}} \\
&= 721\,\% + 60 \cdot 7\,\% = 1\,141\,\%, \\
Mult_{\text{Q-Talent,Charge 1.Hit}} &= Mult_{\text{Q-Talent,Charge 1.Hit}} + n_{\text{Resolve stacks}} \cdot Mult_{\text{Q-Talent,DMG bonus}} \\
&= 109.9\,\% + 60 \cdot 1.31\,\% = 188.6\,\%, \\
Mult_{\text{Q-Talent,Charge 2.Hit}} &= Mult_{\text{Q-Talent,Charge 2.Hit}} + n_{\text{Resolve stacks}} \cdot Mult_{\text{Q-Talent,DMG bonus}} \\
&= 132.7\,\% + 60 \cdot 1.31\,\% = 211.6\,\%, \\
DMG_{\text{Bonus,\%}} &= DMG_{\text{Artefacts,\%}} + DMG_{\text{Talent,passive,\%}} \\
&\quad + DMG_{\text{E-Talent,Bonus,\%}} + DMG_{\text{Sucrose,\%}} \\
&= 46.6\,\% + \frac{ER - 100\,\%}{100\,\%} \cdot 0.4\,\% + \frac{ER}{100\,\%} \cdot 25\,\% + 90 \cdot 0.3\,\% + 20\,\% \\
&= 251.68\,\%.
\end{aligned}$$

With equation (1.1) this results in a base damage of

$$DMG_{\text{Base,Initial}} = 135.6\,\text{k}, \quad DMG_{\text{Base,Charge 1.Hit}} = 22.4\,\text{k}, \quad DMG_{\text{Base,Charge 2.Hit}} = 25.1\,\text{k}.$$

Lastly the calculation of the total damage according to chapter 1.2:

$$\begin{aligned}
RES_{\%} &= RES_{\text{Base,\%}} - RES_{\text{Debuff,VV,\%}} - RES_{\text{Debuff,Beidou,\%}} \\
&= 10\,\% - 40\,\% - 15\,\% = -45\,\%, \\
Mult_{\text{RES}} &= 1 - \left(\frac{RES_{\%}}{2 \cdot 100\,\%}\right) = 1 - \left(\frac{-45\,\%}{2 \cdot 100\,\%}\right) = 1.225, \\
Mult_{\text{DEF}} &= \frac{Level_{\text{Character}} + 100}{(1 - DEF_{\text{Reduction,C2-Raiden}}) \cdot (Level_{\text{Enemy}} + 100) + Level_{\text{Character}} + 100} = \\
&= \frac{190}{(1 - 0.6)\,(93 + 100) + 90 + 100} = 0.711, \\
DMG_{\text{Total,Crit,Initial}} &= DMG_{\text{Base}} \cdot Mult_{\text{RES}} \cdot Mult_{\text{DEF}} \cdot \left(1 + \frac{CD_{\%}}{100\,\%}\right) \\
&= 135.7\,\text{k} \cdot 1.225 \cdot 0.711 \cdot \left(1 + \frac{153.4\,\%}{100\,\%}\right) = 299\,\text{k}, \\
DMG_{\text{Total,Crit,Charge 1.Hit}} &= 49.4\,\text{k}, \\
DMG_{\text{Total,Crit,Charge 2.Hit}} &= 55.4\,\text{k}.
\end{aligned}$$

[9]For more accuracy one could take exacter values from e.g. https://genshin.honeyhunterworld.com/.

This numbers are equivalent to the damage in figure 1.10, considering in-game rounding. For the sake of reference Raiden at C0 would deal 209 k with the initial burst slash in this setup.

The next step would be to find the optimal stats for this setup and weapon. Therefore one has to hold the crit-damage constant, since else it would always lead to literally infinity there, and use attack and energy recharge as variables. Starting from 0 % $ATK_{\%}/ER$ artefact percentage bonuses[10] it can be seen in figure 1.9 that ER has a slightly higher value in terms of damage (and also in terms of getting energy for the team). In other words the usage of an ER main stat artefact for R1 Engulfing has now been confirmed (or in other words: Until 300 % ER (incl. weapon passive during burst) it's better to get ER than $ATK_{\%}$ rolls on artefacts). Furthermore after reaching 300 % ER one can ask oneself if an elemental damage bonus goblet or attack goblet would be better, the answer is elemental damage bonus but only by around 4 % here and therefore it's better to use the one with higher crit stats.

Lastly keep in mind that stat balancing depends strongly on the team and weapon used. E.g. if Bennet (or any similar attack buffer) is not used than one should preferable use ER sands and $ATK_{\%}$ goblet, or if Kazuha is used instead of C6 Sucrose the attack goblet would be of same value than the electro damage one after already hitting 300 % ER and so on.

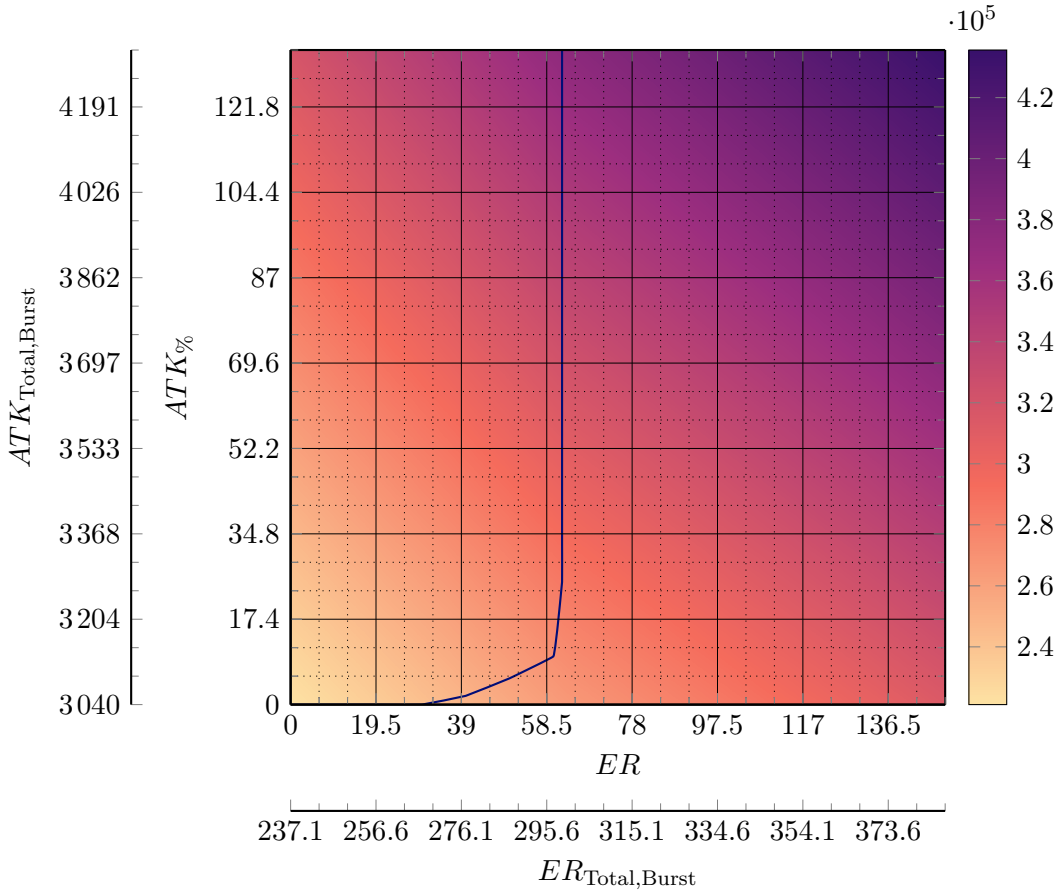
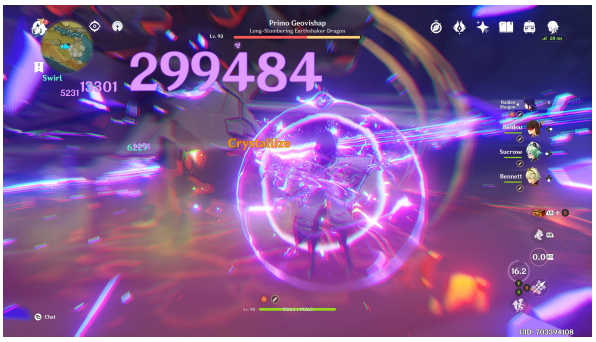


Figure 1.9.: Total damage as a function of ER and $ATK_{\%}$ artefact rolls (sand main stat and substats) for R1 Engulfing with this team (Bennet). The first thing one notices is that up to around 270 % ER (total) one gains more damage than with attack stats, afterwards nearly linearly up to 300 % ER and 12 % $ATK_{\%}$ and at that point $ATK_{\%}$ increases the damage more than ER (Reason: 4pc Emblem artefact set bonus).

[10] Keep in mind that 5.8 % $ATK_{\%}$ and 6.5 % ER are of equal worth.

a. Raiden's stats without any form of buffs.



b. Initial burst hit: 299 484.

c. Charge attack 1.hit: 49 416, 2.hit: 55 389.

Figure 1.10.: Stats and damage of the C2 Raiden with R1 Engulfing with full team buffs as mentioned at the beginning of section 1.6.1.

1.6.2. Albedo

To be continued...

1.6.3. Hu Tao

To be continued...

1.6.4. Sucrose

To be continued...

Chapter 2.

Shields

Shields block damage to characters or enemies, whereby the ones protecting the character behave completely differently from enemy shields.

Characters can have multiple shields at once, but the damage dealt by an enemy affects every active shield simultaneously. The incoming damage DMG_{Incoming} is calculated as described in section 1.1 and 1.2, and therefore depends on the defense and resistance stat of the character, damage reduction multiplier and level (resistance multiplier) and atk of enemy. Any remaining damage afterward is subtracted from the character's health normally.

2.1. Crystallize Shields

As described in section 1.3.3 crystallize creates shields that can be picked up, last for 15 s and the shields base health is derived from the level of the triggering (geo) character and strengthened by a certain percent depending on the (geo) user's elementary mastery stat (see equ (1.18))

$$HP_{\text{Shield}} = HP_{\text{Base,Crystallize Shield}} \cdot (1 + EM_{\text{Bonus,Crystallize}}) \,. \tag{2.1}$$

For level scaling see figure 2.1 or take the in-game parameters listed in table B.3. Keep in mind that only one crystallize shield can be active at any time, picking up another one replaces the old shield.

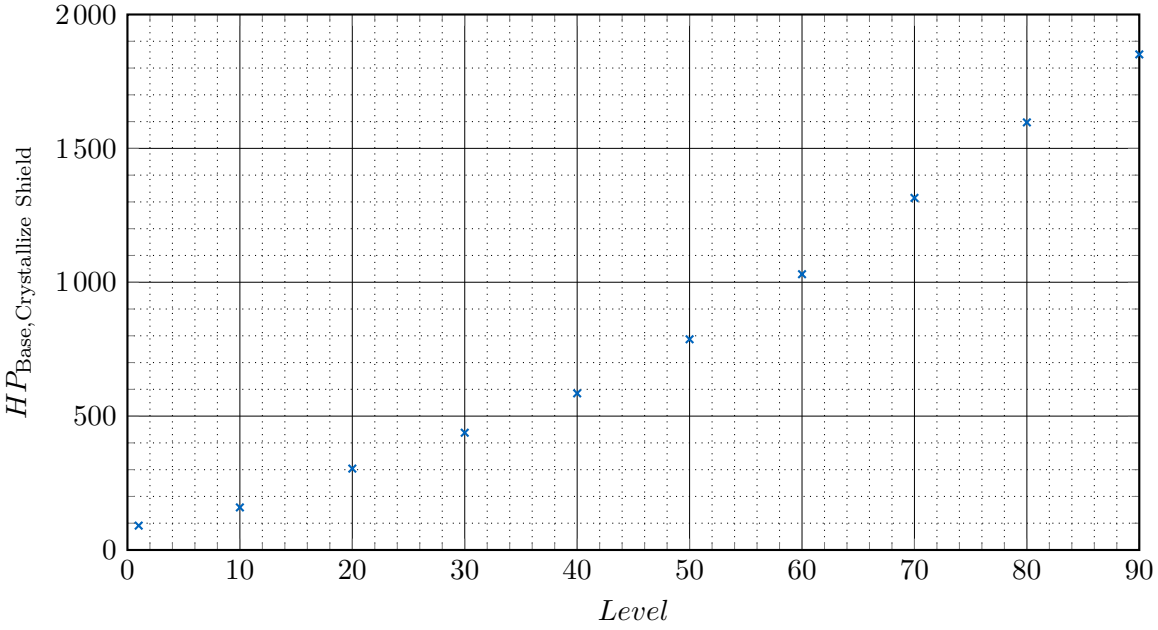


Figure 2.1.: Crystallize base shield health as a function of the characters level.

Remark 2.1 (Geo Elemental Shards): *Geovishap Hatchilings, Geo Hypostasis, ... can create geo elemental shards during combat that grant a geo shield when picked up by a character, but these are not considered crystallize shields since they aren't created via a crystallize reaction. These geo shields can stack with regular crystallize shields – it doesn't override them.*

2.2. Character Shields

These shields have a set amount of health $HP_{\text{Base,Shield}}$ defined during the creation of the shield (usually based on the character's DEF or HP stat) and are strengthened by intrinsic damage absorption bonuses/increases (e.g. Diona's shield)

$$HP_{\text{Shield}} = HP_{\text{Base,Character Shield}} \cdot (1 + \underset{\text{Bonus}}{DMG_{\text{Absorption}}}) \cdot (1 + \underset{\text{Increase}}{DMG_{\text{Absorption}}}). \tag{2.2}$$

Cooldowns and duration of character shields can be seen in the talent (or weapon, ...) descriptions.

2.3. Shield Strength and Elemental Effectiveness

When the active, via crystallize or character shield protected character takes damage, that doesn't ignore shields (e.g. corrosion), the health of the shield(s) reduces, whereby the incoming damage is scaled down based on the shields elemental effectiveness $Elemental_{\text{Effectiveness}}$ and the active character's shield strength $Shield\ Strength_{\text{Increase}}$. The absorbable damage is given with

$$DMG_{\text{Absorbable}} = HP_{\text{Shield}} \cdot Elemental_{\text{Effectiveness}} \cdot (1 + Shield\ Strength_{\text{Increase}}). \tag{2.3}$$

As said above shield strength increase depends on the active character and can be influenced with artefacts, e.g. Retracing Bolide, food and weapon passives. Keep in mind that $Elemental_{\text{Effectiveness}}$ may be called elemental absorption or -bonus sometimes in descriptions, but the meaning is not "bonus", rather always "effectiveness". In general geo characters create shields that have 150 % geo and physical effectiveness, character shields with other elements and crystallize shields have 250 % effectiveness against elemental damage of the same element, in other words they take only 40 % as much damage when hit by an attack of their own element.

After each hit $i \in \mathbb{N}$ the health and absorbable damage of the shield changes, as long as the incoming damage DMG_{Incoming} doesn't exceed the absorbable amount, according to

$$HP_{\text{Shield,i}} = HP_{\text{Shield,i}-1} \cdot \left(1 - \frac{DMG_{\text{Incoming}}}{DMG_{\text{Absorbable,i}-1}}\right), \tag{2.4}$$

$$DMG_{\text{Absorbable,i}} = (DMG_{\text{Absorbable,i}-1} - DMG_{\text{Incoming}}) \cdot \frac{1 + ShieldStrength_{\text{Increase,i}}}{1 + ShieldStrength_{\text{Increase,i}-1}}, \tag{2.5}$$

whereby we also included a possible change in shield strength (e.g. Zhongli's passive) after each hit.

2.4. Examples

In the following table 2.1 the absorbable damage from character shields was calculated for “typical” stats to give a feeling on how much the shields can absorb. Keep in mind that higher absorbable damage, does by far not mean the shield character is better than another. Cooldown, utility, energy recharge and so on play a huge part too.

Table 2.1.: Shield health and absorbable damage of all character shields[a]

	Zhongli	Noelle[b]	Diona	Yanfei	Beidou[b]	Xinyan[b]	Thoma[d]
HP/DEF	35 000	2 400	32 000	32 000	20 000	1 000	28 000
$DMG_{\text{Absorption Bonus,\%}}$			75				
$DMG_{\text{Absorption Increase,\%}}$			15				
$Elemental_{\text{Effectiveness}}$	150	150	$250^{(\text{cryo})}$	$250^{(\text{pyro})}$	$250^{(\text{electro})}$	$250^{(\text{pyro})}$	$250^{(\text{pyro})}$
HP_{Shield}	10 122	9 650	12 842	14 400	$9\,307^{(\text{E})}$	$3\,362^{(1)}$	
					$3\,200^{(\text{Q})}$	$4\,653^{(3)}$	$5\,806^{(\text{E})}$
$DMG_{\text{Absorbable}}$	15 183[c]	14 475	12 842	14 400	$9\,307^{(\text{E})}$	$3\,362^{(1)}$	$5\,806^{(\text{E})}$
			$32\,104^{(\text{cryo})}$	$36\,000^{(\text{pyro})}$	$23\,268^{(\text{electro,E})}$	$8\,405^{(\text{pyro,1})}$	$14\,515^{(\text{pyro,E})}$
					$3\,200^{(\text{Q})}$	$4\,653^{(3)}$	$19\,756^{(\text{max})}$
					$8\,000^{(\text{electro,Q})}$	$11\,633^{(\text{pyro,3})}$	$49\,390^{(\text{pyro,max})}$
$DMG_{\text{Absorbable,Co-Op}}$	15 183[c]		6 421				
			$16\,052^{(\text{cryo})}$				

[a] Calculated for talent level 9, “typical (may vary around ±20 %), rounded” character stats, C0 for 5*-, C6 for 4* characters and shield strength increase from external sources is 0 %. (Kaeya’s shield is excluded here.)

[b] Full dps build, without consideration of the stats which would boost the shield.

[c] With Zhongli’s passive talent “Resonant Waves” the absorbable damage increases depending on the hits the Jade Shield takes, this can vary between 0 to 3 800 (normally ∼1 500).

[d] Thoma’s max absorbable damage was calculated under consideration of his passive talent, giving the active character shield strength increase, and all stacks (max. possible refreshes).

2.5. Enemy Shields

To be continued... (needs Theory to be finished first)

Chapter 3.

Theoretical Background

> The destruction of the nation of Khaenri'ah by the gods... is the reason why the Abyss Order now seeks to destroy the nations watched over by The Seven.
>
> *(Dainsleif)*

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3.1. Elements, Reactions and Resonances

To be continued... (give graphic of all elements, reactions and than resonances, keep text short because that's not the main part here.)

3.2. Models and Symmetries

To be continued...

3.2.1. D-Symmetry

One of the main concepts covers the calculations of damage numbers and therefore assumes that they are from the same form between all objects.

Definition 3.1 (Damage Reversal Symmetry (D-Symmetry)): *The D-symmetry is the theoretical symmetry of laws regarding damage under the transformation of objects*

$$D : object_{\mathrm{A}} \mapsto object_{\mathrm{B}}, \tag{3.1}$$

whereby in this thesis as objects only characters and enemies shall be considered.

3.3. Quantum Theory

This theory will explain how elements work in the game (mustn't violate D-Symmetry). It explains elemental reactions, and internal cooldowns (ICD) of elemental application. To be continued...

Chapter 4.

Computational Methods

To be continued...

Chapter 5.

Evaluation

To be continued...

Chapter 6.

Conclusion and Perspective

To be continued...

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Appendix A.

Experimental Setup Details

Appendix B.

Additional Parameters and Regression Results

B.1. Transformative Reaction Parameters

Table B.1.: Level multiplier for characters from level 1 to 90.

Level	Multiplier	Level	Multiplier	Level	Multiplier	Level	Multiplier
1	17.2	26	113.2	51	336.8	76	946.7
2	18.5	27	118.1	52	350.5	77	979.4
3	19.9	28	123.0	53	364.5	78	1 011.2
4	21.3	29	129.7	54	378.6	79	1 044.8
5	22.6	30	136.3	55	398.6	80	1 077.4
6	24.6	31	142.7	56	416.4	81	1 110.0
7	26.6	32	149.0	57	434.4	82	1 143.0
8	28.9	33	155.4	58	453.0	83	1 176.4
9	31.4	34	161.8	59	472.6	84	1 210.2
10	34.1	35	169.1	60	492.9	85	1 253.8
11	37.2	36	176.5	61	513.6	86	1 289.0
12	40.7	37	184.1	62	539.1	87	1 325.5
13	44.4	38	191.7	63	565.5	88	1 363.5
14	48.6	39	199.6	64	592.5	89	1 405.1
15	53.7	40	207.4	65	624.4	90	1 446.9
16	59.1	41	215.4	66	651.5		
17	64.4	42	224.2	67	679.5		
18	69.7	43	233.5	68	707.8		
19	75.1	44	243.4	69	736.7		
20	80.6	45	256.1	70	765.6		
21	86.1	46	268.5	71	794.8		
22	91.7	47	281.5	72	824.7		
23	97.2	48	295.0	73	851.2		
24	102.8	49	309.1	74	877.7		
25	108.4	50	323.6	75	914.2		

Table B.2.: Level multiplier for enemies from level 1 to 100.

Level	Multiplier	Level	Multiplier	Level	Multiplier	Level	Multiplier
1	17.2	26	113.2	51	336.8	76	854.4
2	18.5	27	118.1	52	350.5	77	877.8
3	19.9	28	123.0	53	364.5	78	900.1
4	21.3	29	129.7	54	378.6	79	923.8
5	22.6	30	136.3	55	398.6	80	946.4

Level	Multiplier	Level	Multiplier	Level	Multiplier	Level	Multiplier
6	24.6	31	142.7	56	416.4	81	968.6
7	26.6	32	149.0	57	434.4	82	991.0
8	28.9	33	155.4	58	452.6	83	1 013.5
9	31.4	34	161.8	59	471.4	84	1 036.1
10	34.1	35	169.1	60	490.5	85	1 066.6
11	37.2	36	176.5	61	509.5	86	1 090.0
12	40.7	37	184.1	62	532.8	87	1 115.0
13	44.4	38	191.7	63	556.4	88	1 141.7
14	48.6	39	199.6	64	580.1	89	1 171.9
15	53.7	40	207.4	65	607.9	90	1 202.8
16	59.1	41	215.4	66	630.2	91	1 202.8
17	64.4	42	224.2	67	652.9	92	1 233.9
18	69.7	43	233.5	68	675.2	93	1 264.7
19	75.1	44	243.4	69	697.8	94	1 305.7
20	80.6	45	256.1	70	720.2	95	1 346.1
21	86.1	46	268.5	71	742.5	96	1 468.9
22	91.7	47	281.5	72	765.2	97	1 524.0
23	97.2	48	295.0	73	784.4	98	1 577.0
24	102.8	49	309.1	74	803.4	99	1 627.6
25	108.4	50	323.6	75	830.9	100	1 674.8

B.2. Crystallize Parameters

Table B.3.: Base shield HP for crystallize shields created from characters with level 1 to 90.

Level	HP	Level	HP	Level	HP	Level	HP	Level	HP
1	91	20	304	40	585	60	1 030	80	1 597
10	159	30	438	50	787	70	1 315	90	1 851

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