Uncertainty Quantification and Quantification of Margins and Uncertainties

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Outline

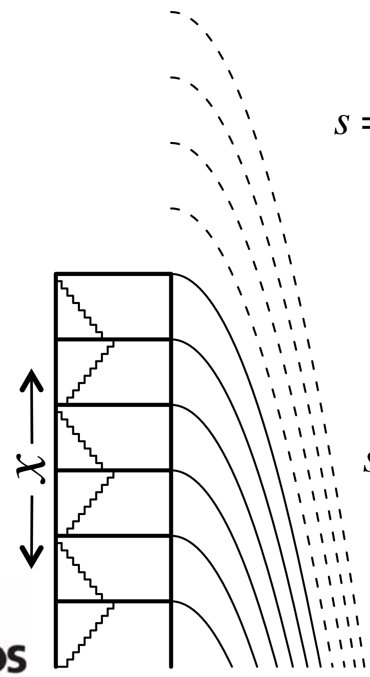
- Introduction
- Sensitivity Analysis
  - Exploration of input-output relationships
- Calibration
  - Probabilistic constraints from field data on input parameters
  - Discrepancy
- Quantification of Margins and Uncertainties (QMU)
  - Confidence ratio
  - Reliability

Introduction

- Sensitivity analysis techniques are useful for exploring input-output relationships
  - Provide information on individual and joint parameter effects on output variation
- Calibration techniques probabilistically tune computer models to experimental data
  - Many sources of uncertainty accounted for, including uncertainty in physics parameters, experimental error, and model inadequacy
- QMU methodology used to make certification or assessment decisions
  - Margin (M): Difference between expected nominal performance and threshold
  - Uncertainty (U): Combined performance and threshold uncertainties
  - Confidence Ratio (CR = M/U): CR > 1 defined as certification success

Example: Dropping Objects from a Tower

- Experiment: Drop a solid ball from a specified height
  - Output: Measured flight time (*y*)
- Computer Model: Implements Newton's Law with drag coefficient
  - Two parameters: *x* = height (controlled); θ = drag coefficient (uncertain physics)
  - Output: Calculated flight time ($\eta(x, \theta)$)



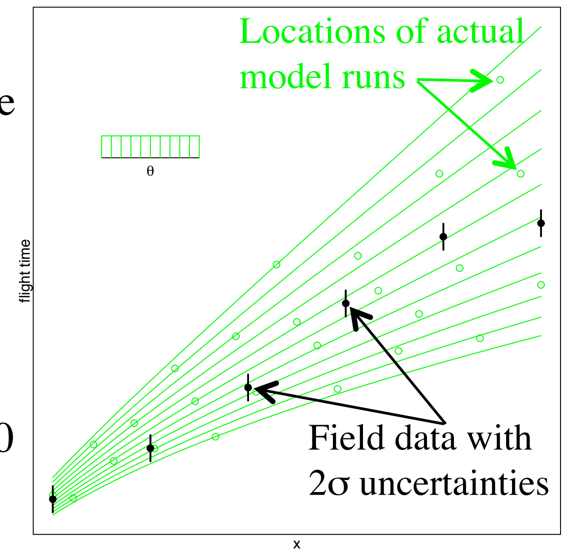
computer model

s = position; τ = time

$$\frac{d^2 s}{d\tau^2} = -1 - \theta \frac{ds}{d\tau}$$

initial conditions

$$s(0) = x, \quad \left.\frac{ds}{d\tau}\right|_{\tau=0} = 0$$



flight time

$\eta(x, \theta)$ is the root of the equation $s(\tau) = 0$.

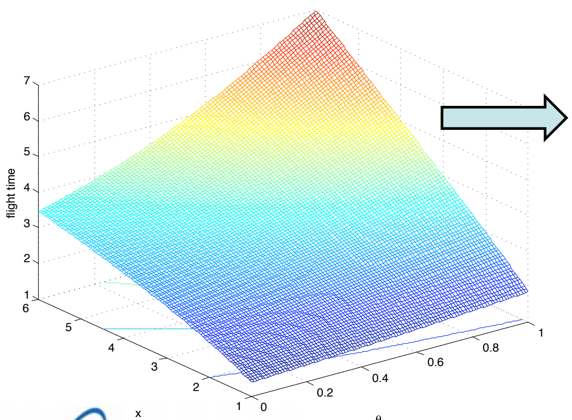
Sensitivity Analysis

- Single and multiple parameter effects (% total variation)

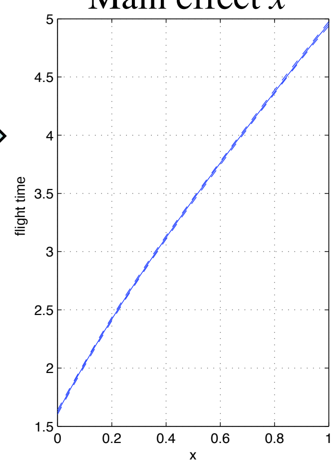
	x	θ
x	70.97%	5.46%
θ	•	23.57%

- Single parameter effects (averaged over all other parameters)

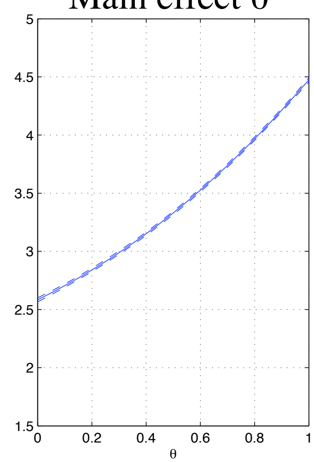
Response surface "emulator"



Main effect *x*



Main effect θ



Sensitivity analysis provides:

1. Decomposition of total output variance due to individual and joint input parameter variations
2. Plots of individual or joint input parameter effects

Calibration and Prediction

<u>Inputs</u>

x controllable

t uncertain physics

θ best, unknown value of **t**

field data, reality, observation error

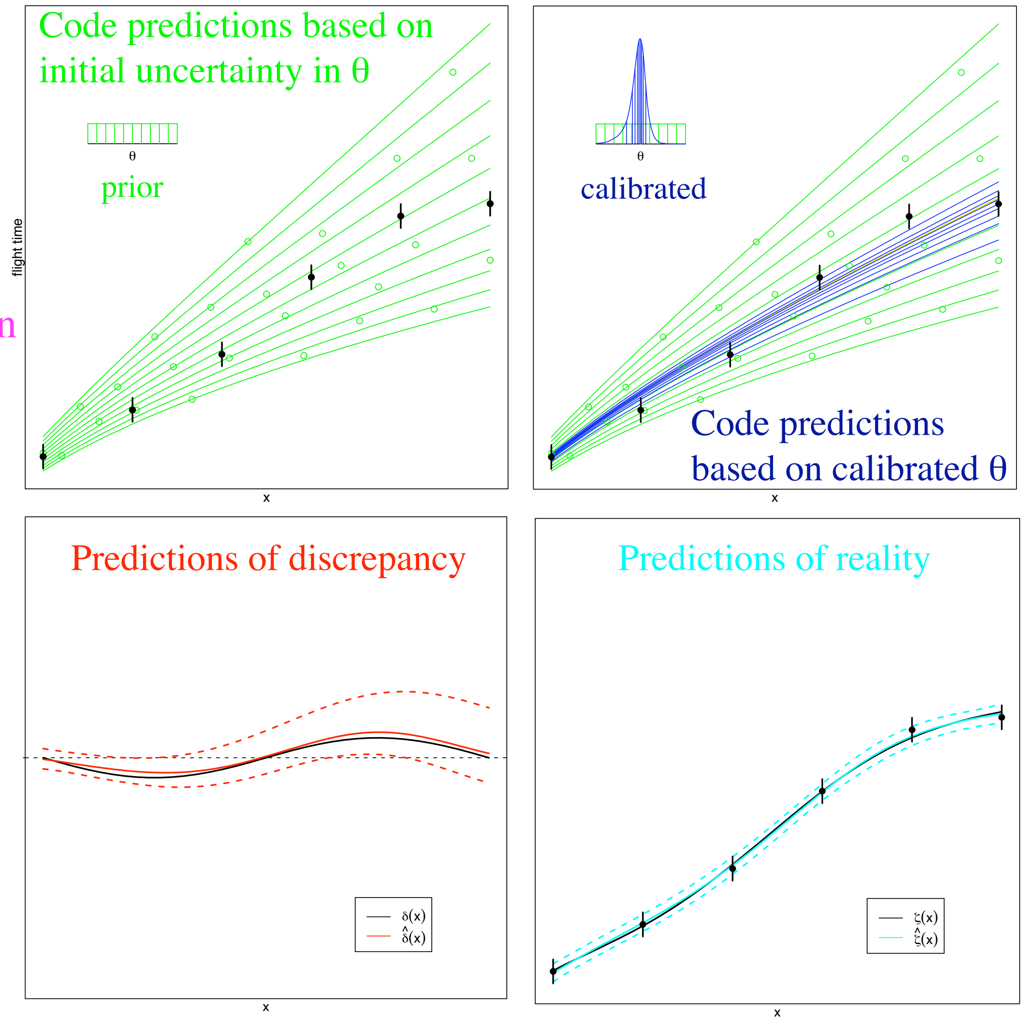
$$y(\mathbf{x}) = \zeta(\mathbf{x}) + \varepsilon(\mathbf{x})$$

computer model, discrepancy

$$\zeta(\mathbf{x}) = \eta(\mathbf{x}, \theta) + \delta(\mathbf{x})$$

Basic steps in calibration analysis:

1. Assume initial probability dist’n for physics uncertainties **θ**.
2. Calibrate parameters **θ** to field data and simultaneously infer model discrepancies.



QMU for Certification

- *Margin (M)* is typically defined as the difference between expected performance, $E[\zeta(\mathbf{x})]$, and a threshold level, *a*, required for performance

Lower bound threshold

$$M(\mathbf{x}) = \max\left(E\left[\zeta(\mathbf{x})\right] - a, 0\right)$$

Upper bound threshold

$$M(\mathbf{x}) = \max\left(a - E\left[\zeta(\mathbf{x})\right], 0\right)$$

- *Uncertainty (U)* is defined by some combination of uncertainties in performance, $U_1(\mathbf{x})$, and in the threshold level, U_2. Performance uncertainty $U_1(\mathbf{x})$ is often taken to be the standard deviation of $\zeta(\mathbf{x})$. For example,

$$U(\mathbf{x}) = U_1(\mathbf{x}) + U_2 = SD[\zeta(\mathbf{x})] + U_2.$$

- *Confidence ratio (CR)* is defined as the ratio M/U. Margin *M* and, in particular, uncertainty *U* are chosen so that CR > 1 implies certification.

Reliability for Certification

- *Reliability (R)* is defined as the probability of successful performance. Assuming random threshold *A*, it is calculated as follows:

Lower bound threshold

$$R(\mathbf{x}) = \Pr[\zeta(\mathbf{x}) > A]$$

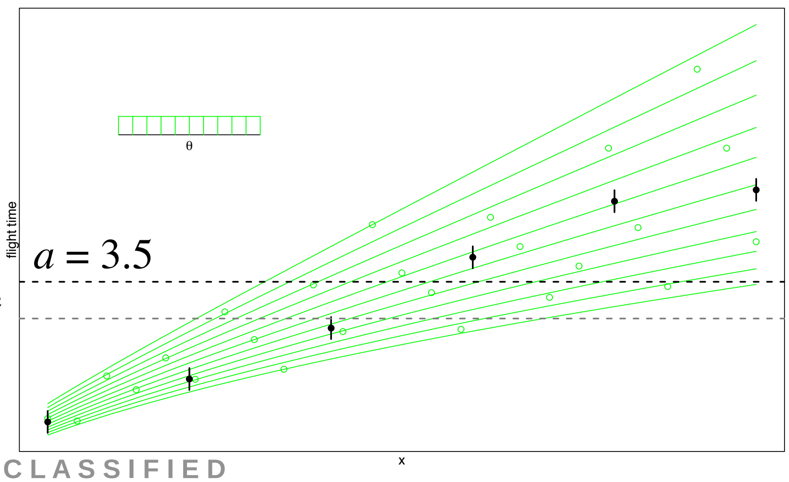
Upper bound threshold

$$R(\mathbf{x}) = \Pr[\zeta(\mathbf{x}) < A]$$

- Reliability exceeding a pre-specified level, e.g. 0.95, implies certification
- In general, no easily specified analytic relationship between *CR* and *R*
- For tower, assume upper bound threshold *A* has mean, *a* = 3.5, and standard deviation, $U_2 = 0.5$.

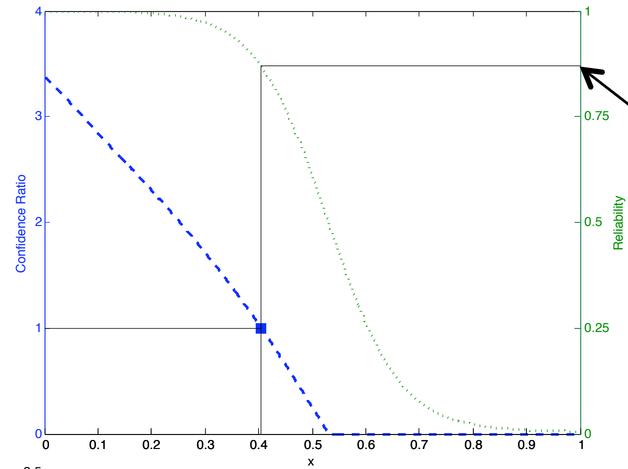
Confidence ratio and reliability will be calculated and compared.

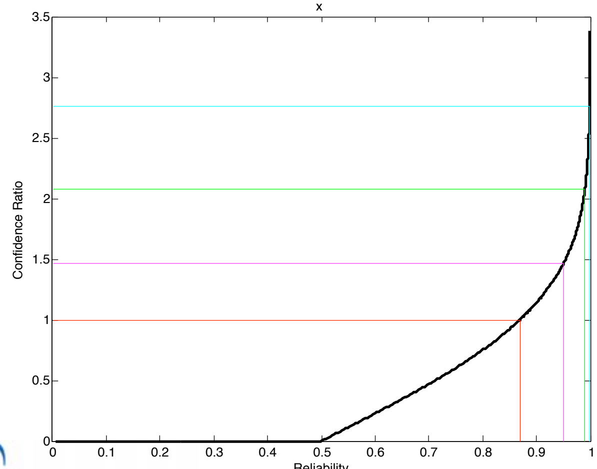
From statistical analysis, $\zeta(\mathbf{x})$ is a mixture of Gaussian dist'ns. Threshold *A* is assumed to be Gaussian and independent of $\zeta(\mathbf{x})$ for calculating *R*.



$U_2 = 0.5$

Results





If $x < 0.405$, then CR > 1: Performance is certified for heights up to 0.405.

Reliability at $x = 0.405$ (CR = 1) is only 87%!

Reliability is a more meaningful measure for making certification decisions; however, more assumptions are required (e.g. joint dist'n of $\zeta(\mathbf{x})$ and A).

Confidence Ratio	Reliability
1	0.87
1.46	0.95
2.07	0.99
2.76	0.999

Required confidence for certification increases at higher rate as reliability requirement approaches one.