Math 106
Elementary Probability & Statistics

Homework Review

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CHARTERED 1693

Chapter 7

A **probability density function (pdf)** is an equation used to compute probabilities of continuous random variables. It must satisfy the following two properties:

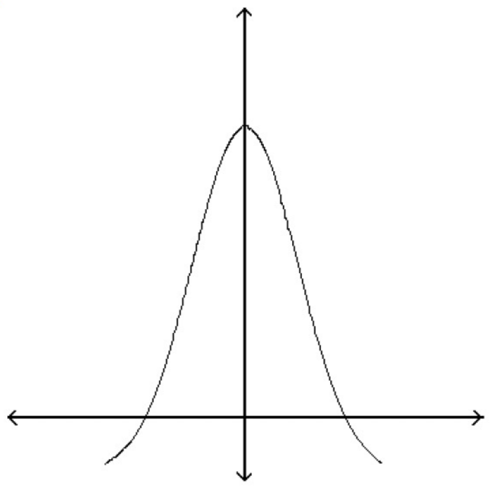
1. The total area under the graph of the equation over all possible values of the random variable must equal 1.

2. The height of the graph of the equation must be greater than or equal to 0 for all possible values of the random variable.

Chapter 7

Determine whether the graph can represent a normal curve. If it cannot, explain why.

8)



A) The graph cannot represent a normal density function because it is not symmetric.

B) The graph cannot represent a normal density function because the graph takes negative values for some values of x.

C) The graph can represent a normal density function.

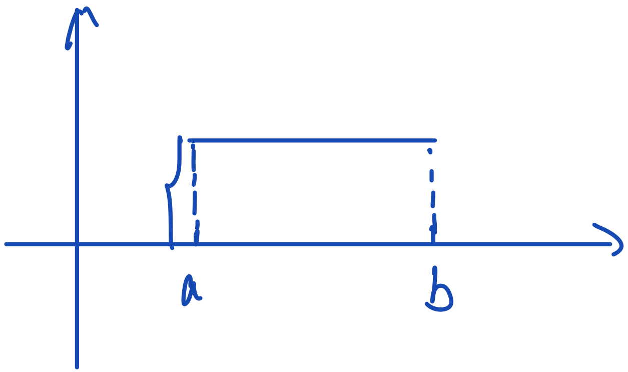
D) The graph cannot represent a normal density function because the area under the graph is less than 1.

8) B

15 wrongs

Chapter 7

Uniform Distribution: Equally likely probabilities for all outcomes



$\frac{1}{b-a}$

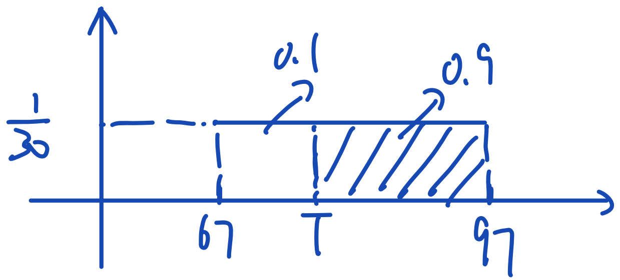
Chapter 7

1) High temperatures in a certain city for the month of August follow a uniform distribution over the interval 67°F to 97°F. Find the high temperature which 90% of the August days exceed.

A) 70°F B) 97°F C) 94°F D) 77°F

1) A

21 wrongs



$$(T-67) \times \frac{1}{30} = 0.1$$

$$T = 70°F$$

Chapter 7

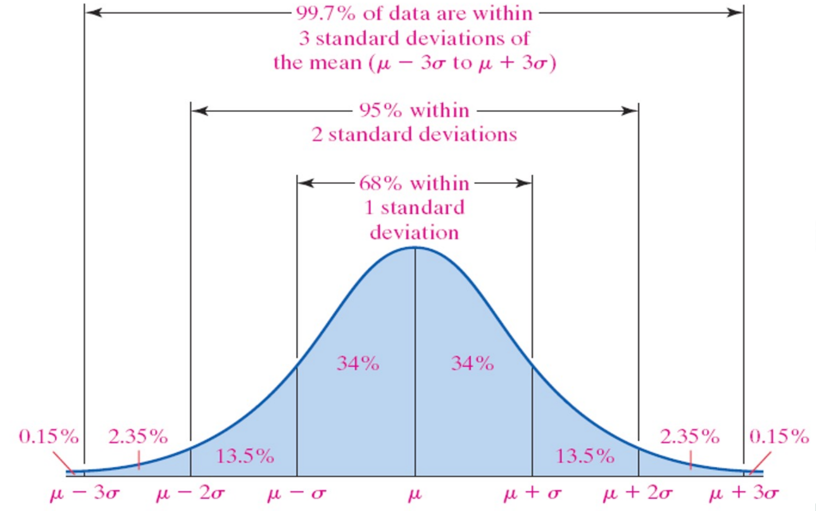
Normal Density Curve

1. It is symmetric about its mean, μ.
2. Because mean = median = mode, the curve has a single peak and the highest point occurs at $x = \mu$.
3. It has inflection points at $\mu - \sigma$ and $\mu + \sigma$.
4. The area under the curve is 1.
5. The area under the curve to the right of μ equals the area under the curve to the left of μ, which equals 1/2.
6. As x increases without bound (gets larger and larger), the graph approaches, but never reaches, the horizontal axis. As x decreases without bound (gets more and more negative), the graph approaches, but never reaches, the horizontal axis.

Chapter 7

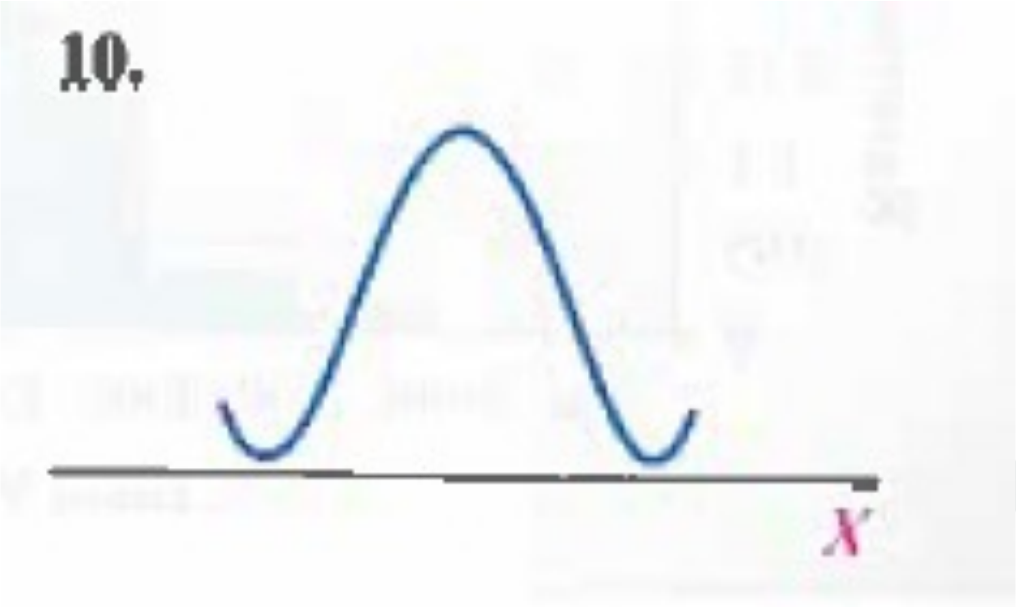
Normal Density Curve

7. The Empirical Rule:
   - Approximately 68% of the area under the normal curve is between x = μ – σ and x = μ + σ;
   - approximately 95% of the area is between x = μ – 2σ and x = μ + 2σ;
   - approximately 99.7% of the area is between x = μ – 3σ and x = μ + 3σ.



Chapter 7

Determine whether the graph can represent normal curve

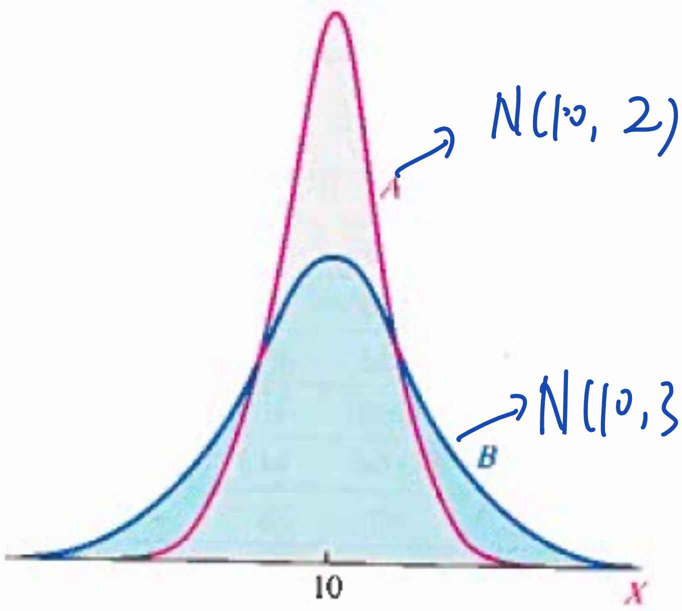


No

it does not approach the horizontal axis as x increases (and decreases) without bound

Chapter 7

One graph in the figure represents a normal distribution with mean $\mu = 10$ and standard deviation $\sigma = 3$. The other graph represents a normal distribution with mean $\mu = 10$ and standard deviation $\sigma = 2$. Determine which graph is which



N(10, 2)

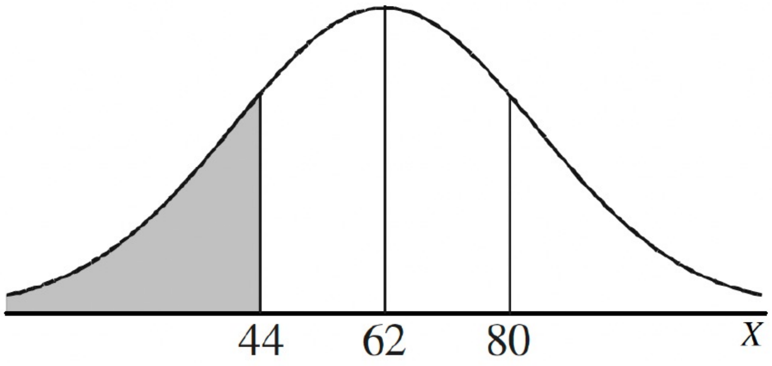
N(10,3)

higher standard deviation makes the graph lower and more spread out

Chapter 7

Monthly charges for cell phone plans in the United States are normally distributed with mean $\mu = \$62$ and standard deviation $\sigma = \$18$

Find the area under the normal curve to the left of $x = \$44$. Provide two interpretations of the results



$P(X \leq 44) = P(Z \leq \frac{44-62}{18}) = -1$

$= normalcdf(-1e99, 44, 62, 18) = 0.1587$

Interpretation 1: 15.87% of the cell phone plans in the United States are less than $44 per month

Interpretation 2: The probability is 0.1587 that a randomly selected cell phone plan in the United States is less than $44 per month

Chapter 7

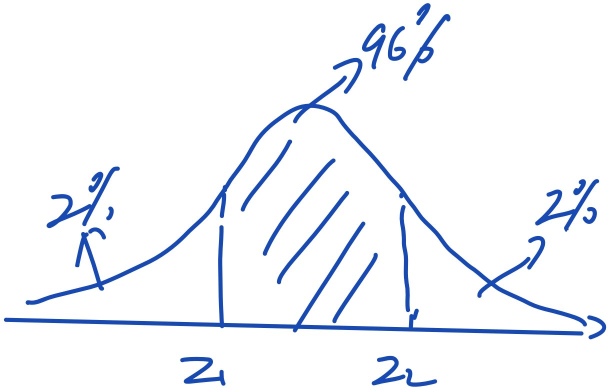
Find the indicated z-score.

6) Determine the two z-scores that separate the middle 96% of the distribution from the area in the tails of the standard normal distribution.

A) –1.75 and 1.75 B) –2.33 and 2.33 C) –2.05 and 2.05 D) 0 and 2.05

6) C

16 wrongs



invNorm(0.98, 0, 1)

Chapter 8

Summary: Shape, Center, and Spread of the Sampling Distribution of $\bar{x}$

Shape, Center, and Spread of the Population	Distribution of the Sample Mean: Shape	Center	Spread
Population is normal with mean μ and standard deviation σ	Regardless of the sample size n, the shape of the distribution of the sample mean is approximately normal	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Population is not normal with mean μ and standard deviation σ	As the sample size n increases, the distribution of the sample mean becomes approximately normal	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Chapter 8

15. A simple random sample of size $n = 49$ is obtained from a population with $\mu = 80$ and $\sigma = 14$.

(a) Describe the sampling distribution of $\bar{x}$.

(b) What is $P(\bar{x} > 83)$?

(c) What is $P(\bar{x} \le 75.8)$?

(d) What is $P(78.3 < \bar{x} < 85.1)$?

(a) $\mu_{\bar{x}} = 80$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = \frac{14}{7} = 2$

(b) $P(\bar{x} > 83) =$ normalcdf(83, 1E99, 80, 2) $= 0.0668$

(c) $P(\bar{x} \le 75.8) = 0.0179$

(d) $P(78.3 < \bar{x} < 85.1) = 0.7969$

Chapter 8

21. **Reading Rates** The reading speed of second grade students is approximately normal, with a mean of 90 words per minute (wpm) and a standard deviation of 10 wpm.

(a) What is the probability a randomly selected student will read more than 95 words per minute?

(b) What is the probability that a random sample of 12 second grade students results in a mean reading rate of more than 95 words per minute?

(c) What is the probability that a random sample of 24 second grade students results in a mean reading rate of more than 95 words per minute?

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

(e) A teacher instituted a new reading program at school. After 10 weeks in the program, it was found that the mean reading speed of a random sample of 20 second grade students was 92.8 wpm. What might you conclude based on this result?

(f) There is a 5% chance that the mean reading speed of a random sample of 20 second grade students will exceed what value?

$X \sim N(90, 10)$

(f) $\bar{X} \sim N\left(90, \frac{10}{\sqrt{20}}\right)$

$P(\bar{X} > c) = P\left(Z > \frac{c-90}{10/\sqrt{20}}\right)$

$= 0.05$

$\frac{c-90}{10/\sqrt{20}} = 1.645$

$c = 93.7$

$c = \text{invNorm}(0.95, 90, 10/\sqrt{20})$

$= 93.7$

Chapter 8

Sampling Distribution of Proportions

For a simple random sample of size n with a population proportion p,

- The shape of the sampling distribution of $\hat{p}$ is approximately normal provided $np(1-p) \geq 10$
- The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$.
- The standard deviation of the sampling distribution $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

Chapter 8

11. A simple random sample of size $n = 75$ is obtained from a population whose size is $N = 10{,}000$ and whose population proportion with a specified characteristic is $p = 0.8$.

(a) Describe the sampling distribution of $\hat{p}$.

(b) What is the probability of obtaining $x = 63$ or more individuals with the characteristic? That is, what is $P(\hat{p} \geq 0.84)$?

(c) What is the probability of obtaining $x = 51$ or fewer individuals with the characteristic? That is, what is $P(\hat{p} \leq 0.68)$?

(a) $\mu_{\hat{p}} = 0.8$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8 \times 0.2}{75}} \approx 0.046$

$\hat{p} \sim N(0.8,\ 0.046)$

(b) $\hat{p} = \frac{63}{75} = 0.84$

$P(\hat{p} \geq 0.84) = P\left(z \geq \frac{0.84 - 0.8}{\sqrt{0.8 \times 0.2/75}}\right) = 0.1922$

(c) $P(\hat{p} \leq 0.68) = P\left(z \leq \frac{0.68 - 0.8}{\sqrt{0.8 \times 0.2/75}}\right) = 0.0047$

19. Afraid to Fly According to a study conducted by the Gallup organization, the proportion of Americans who are afraid to fly is 0.10. A random sample of 1100 Americans results in 121 indicating that they are afraid to fly. Explain why this is not necessarily evidence that the proportion of Americans who are afraid to fly has increased since the time of the Gallup study.

$\hat{p} = \frac{x}{n} = 0.11$

$P(X \geq 121) = P(\hat{p} > 0.11) = P\left(Z > \frac{0.11 - 0.10}{\sqrt{0.1 \times 0.9 / 1100}}\right) = 0.1335$

The result is not unusual, so the evidence is insufficient to conclude that the proportion of Americans who are afraid to fly have increased almost all