Introduction to Artificial Intelligence

VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

COMP307/AIML420

Evolutionary Computation 3: GP for Regression and Classification

Dr Andrew Lensen

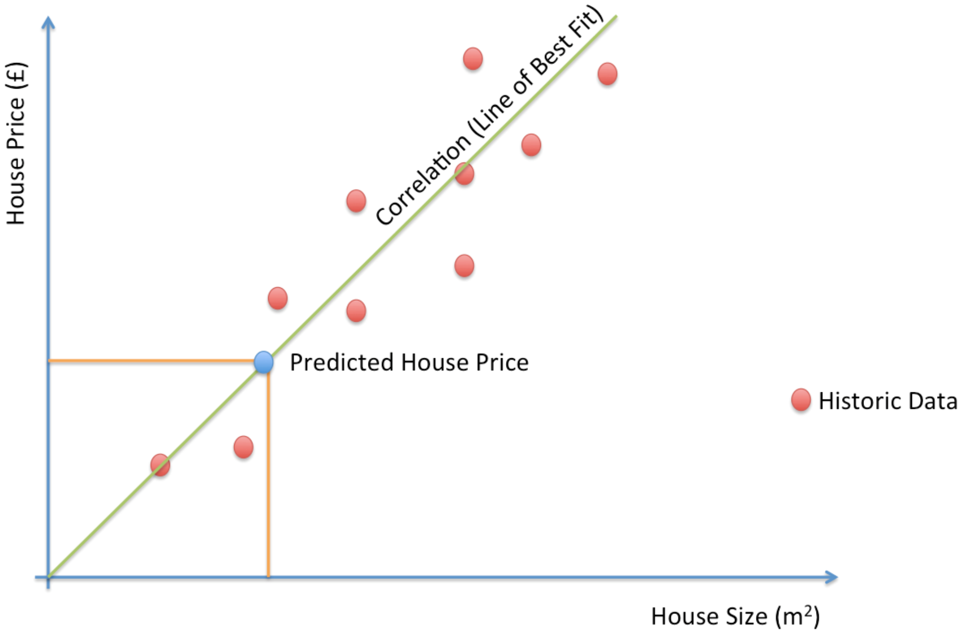
Andrew.Lensen@vuw.ac.nz

Outline

- Statistical parameter regression
- Symbolic regression
- GP for symbolic regression
- GP for binary classification

House Price Prediction

- How much should your tender be?



Price	Floor space	Rooms	Lot size	Appartment	Row house	Corner house	Detached
250000	71	4	92	0	1	0	0
209500	98	5	123	0	1	0	0
349500	128	6	114	0	1	0	0
250000	86	4	98	0	1	0	0
419000	173	6	99	0	1	0	0
225000	83	4	67	0	1	0	0
549500	165	6	110	0	1	0	0
240000	71	4	78	0	1	0	0
340000	116	6	115	0	1	0	0

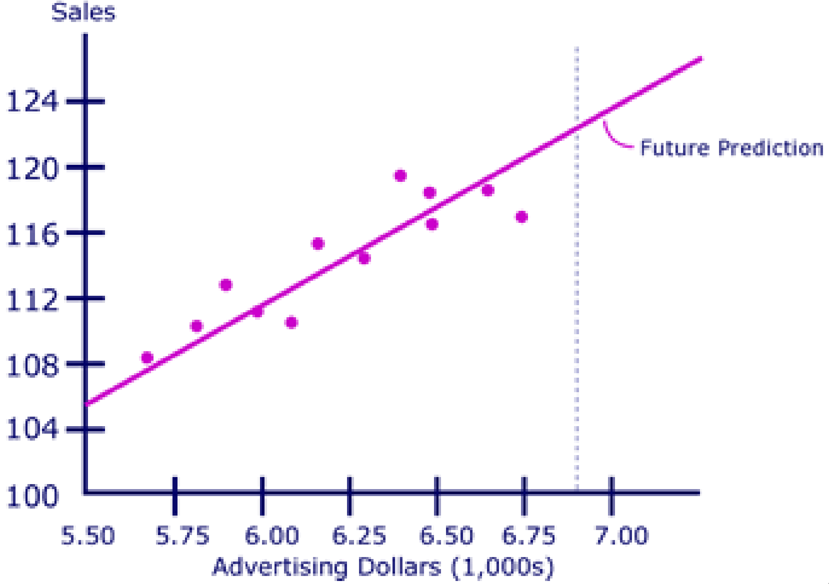
(Statistical) Regression Analysis

- In statistics, regression analysis examines the relation of a dependent variable (*response variable*) to specified independent variables (*explanatory variables*)

  – The mathematical model of their relationship is the regression equation (e.g. $f(x, y) = 0$)

  – estimates of one or more hypothesized regression parameters ("constants")

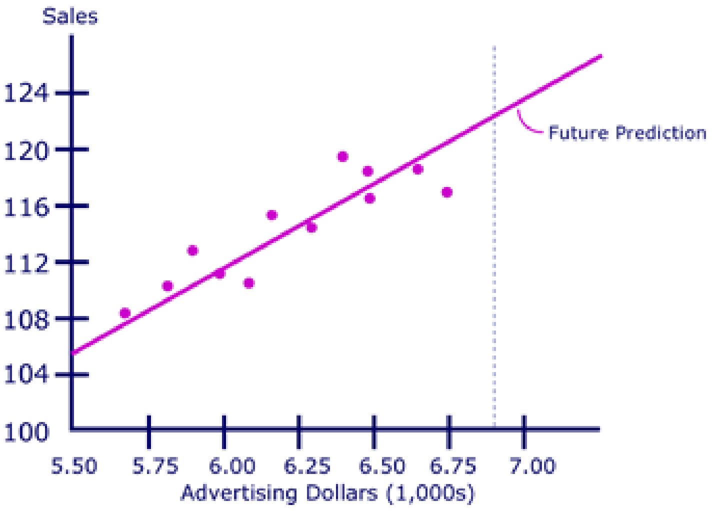
- Examples
  – Financial prediction
  – Saving prediction
  – Ad cost vs sales
  – Natural law discovery



Regression

- Two main tasks:
  - **Regression equation**: relationship between a dependent variable (response variable) and independent variables (explanatory variables)
  - **Parameters/Coefficients**

- Example: linear regression
  - Regression equation is a linear model: $y = \alpha \cdot x + \beta + \epsilon$
  - Coefficients: α and β
  - α is the slope, β is the intercept
  - ϵ is the *error* term (assume normally distributed)
  - Estimate α and β to minimise ϵ

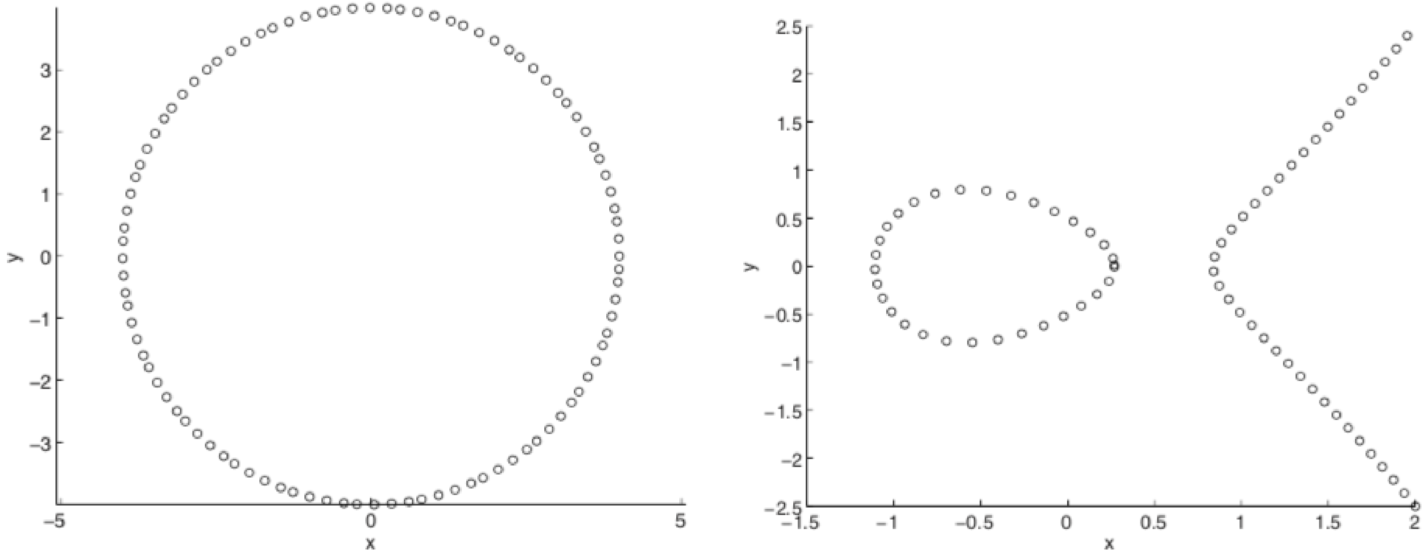


Assume model structure, estimate model parameters

Symbolic regression

- However, linear model is too simple for many real-world data

- Hard to find the proper regression equation



(a) **Circle:** $x^2 + y^2 - 4^2$ (b) **Elliptic Curve:** $x^3 + x - y^2 - 1.5$

Symbolic regression: A physics example

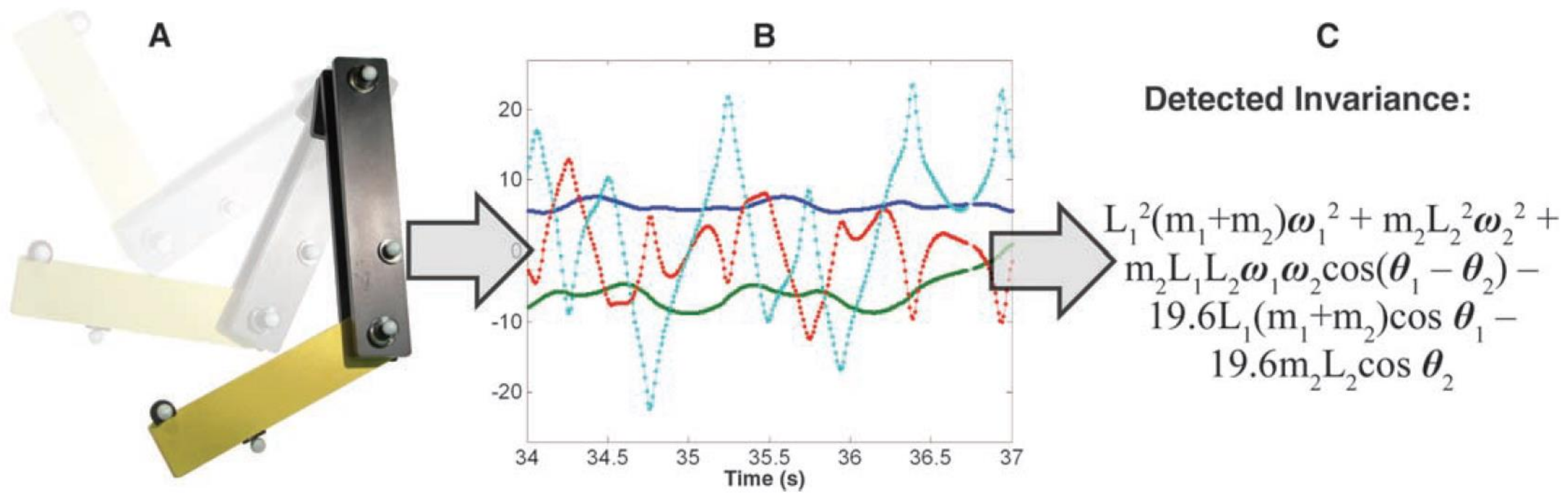


Fig. 1. … We captured the angles and angular velocities of a chaotic double-pendulum (A) over time using motion tracking (B), then we automatically searched for equations… without any prior knowledge about physics or geometry, the algorithm found the conservation law (C)

Schmidt, Michael, and Hod Lipson. "Distilling free-form natural laws from experimental data." ***Science*** 324, no. 5923 (2009): 81-85. **1600 citations!**

Symbolic Regression Applications

- Symbolic regression has many real-world applications:
  - Economics prediction: stock market prediction, GDP prediction
  - Industrial prediction: prediction of container handling capacity at a sea port; short/medium/long-term prediction of power load at a region
  - Experimental formula modelling in engineering: formulating the amount of gas emitted from a coal surface
  - Time series projection: CPI projection for a country or a region (or less capitalistic: Gross National Happiness (GNH) index?)
  - **Others?**

Symbolic regression

- Symbolic regression: finds a symbolic description of a model, not just a set of coefficients/parameters in a pre-specified model.

- Finds both:
  - the model structure, and
  - the corresponding coefficients/parameters

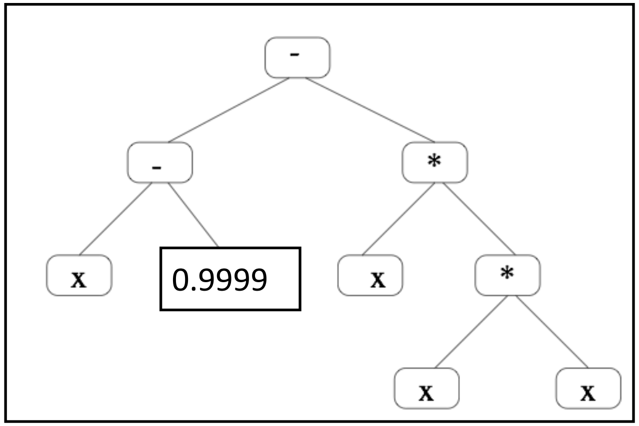
GP for Symbolic Regression (Example)

- Objective: Find a program/model that produces the correct value of the dependent variable y when given the value of an independent variable x
- Terminal Set: *x*, random constant *r*
- Function Set: $\{+, -, \times, \div\}$
- Fitness Cases: 50 cases of *x* and the corresponding *y* values (e.g. *50 instances/patterns/cases*)
- Fitness Measure: Sum of the *absolute* errors for the 50 cases
- Parameters: Population = 100, Generations = 50, MaxDepth = 17 reproduction rate: 5%, crossover rate: 90%, mutation rate: 5%
- Success: The fitness value is smaller than a pre-defined value, e.g. 0.01
- Termination criteria: satisfactory solutions found, or at generation 50

GP for Symbolic Regression (Example)

- One GP run gave: $y = (x - 0.9999) - x^3$
- Successful? If the true model is

$$y = (x - 1) - x^3$$



- Sometimes…:
  (% (% (* (* X 0.571) (* (- (* (+ (% 0.634094 0.68469) (+ (+ X X) -0.5992))(* (* (+ (% 0.634094 0.68469) (+ X -0.5992)) (* (% 0.354904 -0.7549) (* X 0.571))) (- X 0.395493))) - 0.4665) ....*another 15 lines*) ☹

- This example: one input variable (x), training set only

- Real-world applications: usually multiple variables, can have a separate test set, but use the same principles

GP for Symbolic Regression

- Compared with statistical parameter regression methods, GP method has the following properties:
  - Does **NOT** need to assume any distribution of data set,
  - Does **NOT** assume the independence of the input variables
  - Does **NOT** require any statistical background knowledge to assume any model

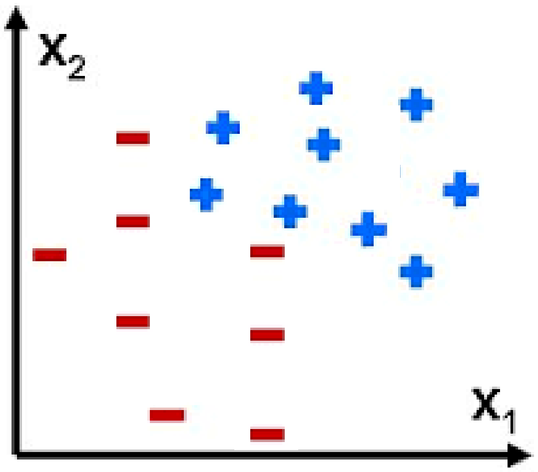
- Can automatically learn/evolve both the model structure and the model parameters at the same time

- System input: just the data with unknown model/parameters
- System output: a “white-box” model structure with appropriate parameters and coefficients

Binary Classification

- Binary classification is the task of classifying the instances of a given set into two categories based on whether they have some property or not

- Two target classes, e.g.
  - Disease vs non-disease
  - normal vs abnormal
  - approve loan or reject
  - fault vs normal state
  - object vs non-object



GP for Binary Classification

- Compared with GP for symbolic regression problems, the terminal set and function set can be the same or very similar, but the fitness function is normally very different

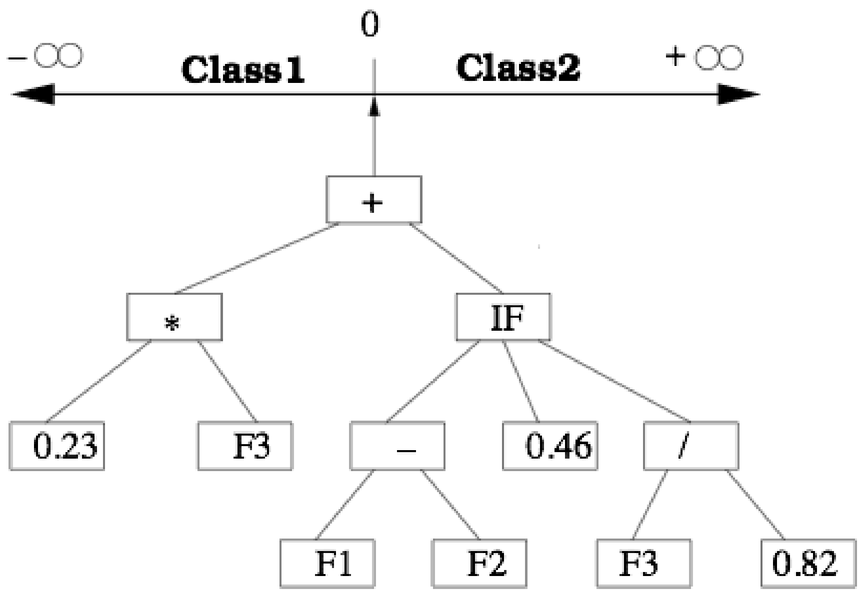
- For the fitness function, we can simply use classification accuracy or error rate.

- How do we determine which class an example belongs to?
  - Classification Strategy or Program Class Translation Rule

- For binary classification problems, this is quite easy: we can use the value "zero" in the real number space for the program output to separate the two classes

Program Class Translation Rule

- Is zero the best threshold?
- What if all features are positive?




```
Genetic Program: (+ (* 0.23 F3)
                    (IF (- F1 F2) 0.46 (/ F3 0.82))
                 )
```

```
if ProgOut < 0  then  Class1  else   Class2;
```

Where have we seen this before?

GP for Classification Example

- Task: (Image) Object classification: *objects vs non-objects*
- Objective: Find a program which can successfully *split* the instances into *two* classes
- Terminal Set: Object *attributes*: pixels, pixel statistics, or specific features, and *random numbers*.
- Function Set: $\{+,\ -,\ \times,\ \div,\ |x|,\ e^{x},\ \log x\,,\ \sin x\,,\ \cos x\}$
- Fitness Cases: Build a training set of *patterns* (*feature vectors*), some are objects, some not.
- Fitness Measure: classification accuracy or error rate
- Classification strategy: ProgOut > 0 for objects, else non-objects

Summary

- GP for symbolic regression
- Properties of GP for symbolic regression
- GP for binary classification

- How do you use GP for multi-class classification? Can we get better translation rules?

- That's it for this half!

- Tomorrow: Assignment 2, Q&A