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Part 1, Q1

Q1: 1. In order to compute the table of the joint distribution $P(X, Y, Z)$, we need to expand and simplify it first, then substitute corresponding Value into it, so Following is the step:

$P(X, Y, Z)$

Step 1: Use Product Rule: treat X, Y as the whole:

$\cancel{= P(X, Y) * P(Z|X, Y)}$

$= P(Z) \times P(X, Y|Z)$

Step 2: Use Bayes Rule:

$= \cancel{P(Z)} \times \frac{P(X,Y) \cdot P(Z|X,Y)}{\cancel{P(Z)}}$

Step 3. Since Z is $\boxed{\text{independent}}$ from X given Y, which is $\boxed{Z \perp X|Y}$, so, we can get: $\boxed{P(Z|X, Y) = P(Z|Y)}$ and $P(Z, X|Y) = P(Z|Y) \cdot P(X|Y)$, thus, by using ~~Indep~~ these, we can get:

$= P(X, Y) \cdot P(Z|X, Y)$ simplified from Step 2

$\cancel{= P(X, Y) = P(}$

$= P(X, Y) \cdot P(Z|Y)$: Use this formula

Step 4: Use Product Rule again:

$= P(X) \cdot P(Y|X) \cdot P(Z|Y)$

Finally, we got that $\boxed{P(X, Y, Z) = P(X) \cdot P(Y|X) \cdot P(Z|Y)}$

Next, subsitute corresponding Values and ~~draw~~ get result and draw table.

By using the provided two conditional distribution, we get:

e.g. ~~P(X,Y,Z) = P(~~ $P(X=0, Y=0, Z=0)$

$= P(X=0) \times P(Y=0 \mid X=0) \times P(Z=0 \mid Y=0)$

$= 0.35 \times 0.10 \times 0.70$

$= 0.0245$

The above example shows how it's computed, in order for saving the space, I just draw the table below.

X	Y	Z	P(X,Y,Z) which is P(X)·P(Y|X)·P(Z|Y)
0	0	0	$0.0245 = 0.35 \times 0.1 \times 0.7$
0	0	1	$0.0105 = 0.35 \times 0.1 \times 0.3$
0	1	0	$0.063 = 0.35 \times 0.9 \times 0.2$
0	1	1	$0.252 = 0.35 \times 0.9 \times 0.8$
1	0	0	$0.273 = 0.65 \times 0.6 \times 0.7$
1	0	1	$0.117 = 0.65 \times 0.6 \times 0.3$
1	1	0	$0.052 = 0.65 \times 0.4 \times 0.2$
1	1	1	$0.208 = 0.65 \times 0.4 \times 0.8$

Q1, 2. $\boxed{P(X,Y) = P(X) \cdot P(Y \mid X) \text{ [Product Rule]}}$

$\therefore P(X=0, Y=0) = P(X=0) \cdot P(Y=0 \mid X=0)$

$= 0.35 \times 0.1$

$= 0.035$

$P(X=0, Y=1) = P(X=0) \cdot P(Y=1 \mid X=0)$

$= 0.35 \times 0.9$

$= 0.315$

$$P(X=1, Y=0) = P(X=1) \cdot P(Y=0 \mid X=1)$$
$$= 0.65 \times 0.6$$
$$= 0.39$$

$$P(X=1, Y=1) = P(X=1) \cdot P(Y=1 \mid X=1)$$
$$= 0.65 \times 0.40$$
$$= 0.26$$

The full joint probability table of X and Y $P(X,Y)$ is shown below:

X	Y	P(X,Y)
0	0	0.035
0	1	0.315
1	0	0.39
1	1	0.26

Q1 3a) $P(Z=0) = P(X=0, Y=0, Z=0) + P(X=0, Y=1, Z=0)$

Sum Rule $\cancel{[\text{Sum Rule}]} + P(X=1, Y=0, Z=0) + P(X=1, Y=1, Z=0)$

$= P(X=0) \times P(Y=0 | X=0) \times P(Z=0 | Y=0) +$

Formular from previous Question

$P(X=0) \times P(Y=1 | X=0) \times P(Z=0 | Y=1) +$

$P(X=1) \times P(Y=0 | X=1) \times P(Z=0 | Y=0) +$

$P(X=1) \times P(Y=1 | X=1) \times P(Z=0 | Y=1)$

$= 0.0245 + 0.063 + 0.273 + 0.052$

$P(Z=0) = 0.4125$

Q1,3b) $P(X=0, Z=0)$

$= P(X=0, Y=0, Z=0) + P(X=0, Y=1, Z=0)$ Sum Rule

$= 0.0245 + 0.063$

$= 0.0875$

Q1 3c) $P(X=1, Y=0 | Z=1)$

$= \dfrac{P(X=1, Y=0) \cdot P(Z=1 | X=1, Y=0)}{P(Z=1)}$ Bayes Rule

$= \dfrac{P(X=1, Y=0) \cdot P(Z=1 | Y=0)}{1 - P(Z=0)}$ $\dfrac{\text{InDepedence}}{\text{Normalisation Rule}}$

$= \dfrac{P(X=1) \cdot P(Y=0 | X=1) \cdot 0.3}{1 - 0.4125}$ Product Rule

$= \dfrac{0.65 \times 0.6 \times 0.3}{0.5875}$ $\boxed{= \dfrac{234}{1175} \approx 0.1991}$

Q1,3 d) $P(X=0 \mid Y=0, Z=0)$

$$= \frac{P(X=0)\cdot P(Y=0, Z=0 \mid X=0)}{P(Y=0, Z=0)} \quad \text{Bayes Rule}$$

$$= \frac{P(X=0, Y=0, Z=0)}{P(Y=0)\cdot P(Z=0 \mid Y=0)} \quad \begin{array}{l}\text{Inverse use Product Rule} \\ \text{Product Rule}\end{array}$$

$$= \frac{P(X)\cdot P(Y \mid X)\cdot \cancel{P(Z \mid Y)}}{P(Y=0)\cdot \cancel{P(Z=0 \mid Y=0)}} \quad \text{expand it}$$

~~No~~

$$= \frac{P(X=0)\cdot P(Y=0 \mid X=0)}{P(X=0, Y=0, Z=0) + P(X=1, Y=0, Z=0) + P(X=1, Y=0, Z=1) + P(X=0, Y=0, Z=1)} \quad \text{Sum Rule}$$

$$= \frac{0.35 \times 0.1}{0.0245 + 0.273 + 0.117 + 0.0105}$$

$$\boxed{= \frac{0.035}{0.425} \approx 0.08235 = \frac{7}{85}}$$

Q2 i) $P(B,C) = P(B|C) \times P(C)$ Product Rule

$= 0.2 \times 0.4$

$= 0.08$

ii) $P(\neg A|B) = 1 - P(A|B)$ Normalisation Rule

$= 1 - 0.3$

$= 0.7$

~~iv) $P(A,B,C)$~~ =

iii) $P(A,B|C) = P(A|C) \cdot P(B|C)$

$= 0.5 \times 0.2$

$= 0.1$

Indepedence $A \perp B | C$

$P(A,B|C) = \cancel{P(A|C)}$

$= P(A|C) \cdot P(B|C)$ $\cancel{P(A|B)}$

iv) $P(A|B,C) = P(A|C)$

$= 0.5$

Independence $A \perp B | C$

$P(A|B,C)$

$= P(A|C)$

Q2 V) $P(A,B,C) = P(A) \cdot P(B,C|A)$ Product Rule

$= P(A) \cdot \frac{P(B,C) \times P(A|B,C)}{P(A)}$ Bayes Rule

$= P(B,C) \times P(A|C)$ Indepedence

$= P(C) \times P(B|C) \times P(A|C)$ Product

$= 0.4 \times 0.2 \times 0.5$

$= 0.04$

Q3 DragonflyDomain : { Common, Rare }, D=DragonFly

Step 1: ∵ 0.3% belongs to Rare Species, $\boxed{\therefore P(D=Rare)=0.003}$

And $\boxed{P(D=Common)=0.997}$, which is $1-P(D=Rare)=0.997$

Step 2: Then, 0.1% is mutation with extra wing, so, from this we can get : $\boxed{P(exWing \mid D=common)=0.001}$, which means the posterior/conditional probability that the dragonfly has extra set of wing given that the dragonfly belongs to the Common species is 0.001.

Step 3: Also, since All rare species DragonFly got extra Wing, which means the posterior probability that the dragonfly has extra set of wings given that it belongs to rare ~~is~~ specices is 1. it can be written as $\boxed{P(exWing \mid D=Rare)=1}$

Step 4: ~~This Question require us to calculate~~

This Question given us that the DragonFly:D has extra wings and require us to calculate the probability of this Dragonfly D belongs to Rare species, which is

$$\boxed{P(D=Rare \mid exWing) = ?}$$

Step 5: $P(D=Rare \mid exWing) = \dfrac{P(D=Rare) \times P(exWing \mid D=Rare)}{P(exWing)}$

Bayes Rule

~~Normalisation Rule~~ $\boxed{\text{Sum Rule}}$ $\Rightarrow$

$$= \frac{P(D=Rare) \times P(exWing \mid D=Rare)}{P(exWing, D=Rare) + P(exWing, \cancel{Comm}\ D=Common)}$$

Product Rule

$$= \frac{P(D=Rare) \times P(exWing \mid D=Rare)}{P(D=Rare) \times P(exWing \mid D=Rare) + P(D=Common) \times P(exWing \mid D=Common)}$$

$$= \frac{0.003 \times 1}{0.003 \times 1 + 0.997 \times 0.001}$$

Q3Ans $\boxed{= \dfrac{3000}{3997} \approx 0.7506}$