Introduction to Artificial Intelligence

COMP307

Uncertainty and Probability 3: Introduction to Bayesian Network

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Outline

- Rules from previous lectures
- What is Bayesian Networks
- Why Bayesian Networks
- Cause — Effect
- Summary

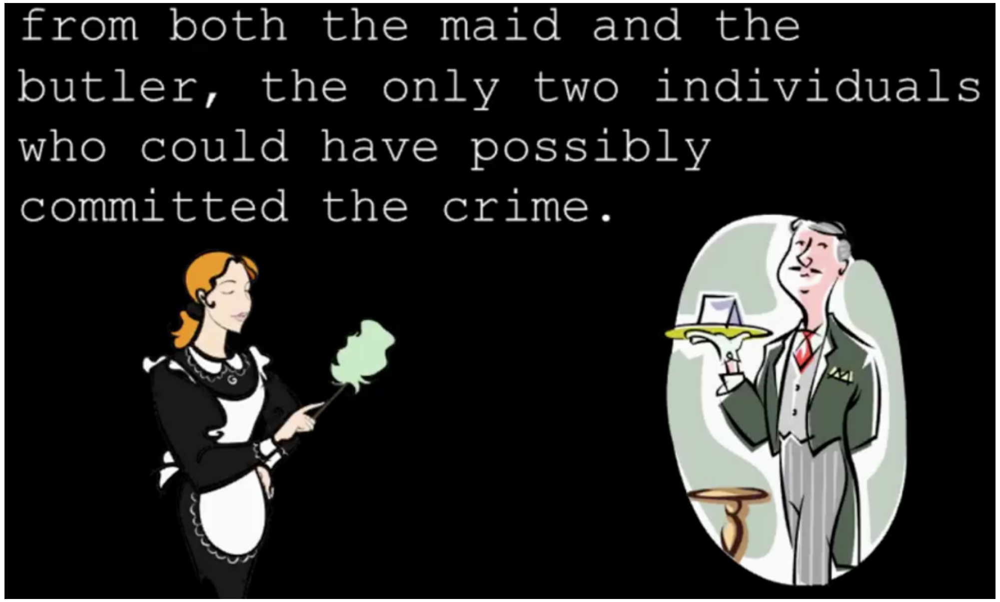
Thomas Bayes (/ˈbeɪz/; c. 1701 – 7 April 1761)

Rules from Previous Lectures

- Product Rule
  - P(X,Y) = P(X)*P(Y I X) = P(Y)*P(X I Y)
- Sum Rule:
  - $P(X) = \sum_y P(X,Y)$
- Normalisation Rule
  - $\sum_x P(X) = 1$, $\sum_x P(X|Y) = 1$
- Independence
  - $X \perp Y$, $P(X|Y) = P(X)$, $P(X,Y) = P(X) * P(Y)$
  - $X \perp Y|\, Z$, $P(X|\, Y,Z) = P(X|Z)$, $P(X,Y|Z) = P(X|Z) * P(Y|Z)$
- Bayes Rule
  - $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
  - $P(Y|X_1, \dots, X_n) = \frac{P(X_1|Y)\dots P(X_n|Y)P(Y)}{P(X_1,\dots,X_n)}$ [assume conditional independence]

A Lazy Detective

- Early one morning, Mr. Boddy was found dead



- Possible weapons:
  - Vacuum cleaner pipe / Candle stick
- Possible times:
  - Evening / Midnight
- Possible murderer:
  - Maid / Butler

Variable	**Domain**
Weapon	{VCP, CS}
Time	{Evening, Midnight}
Murderer	{Maid, Butler}

A Lazy Detective

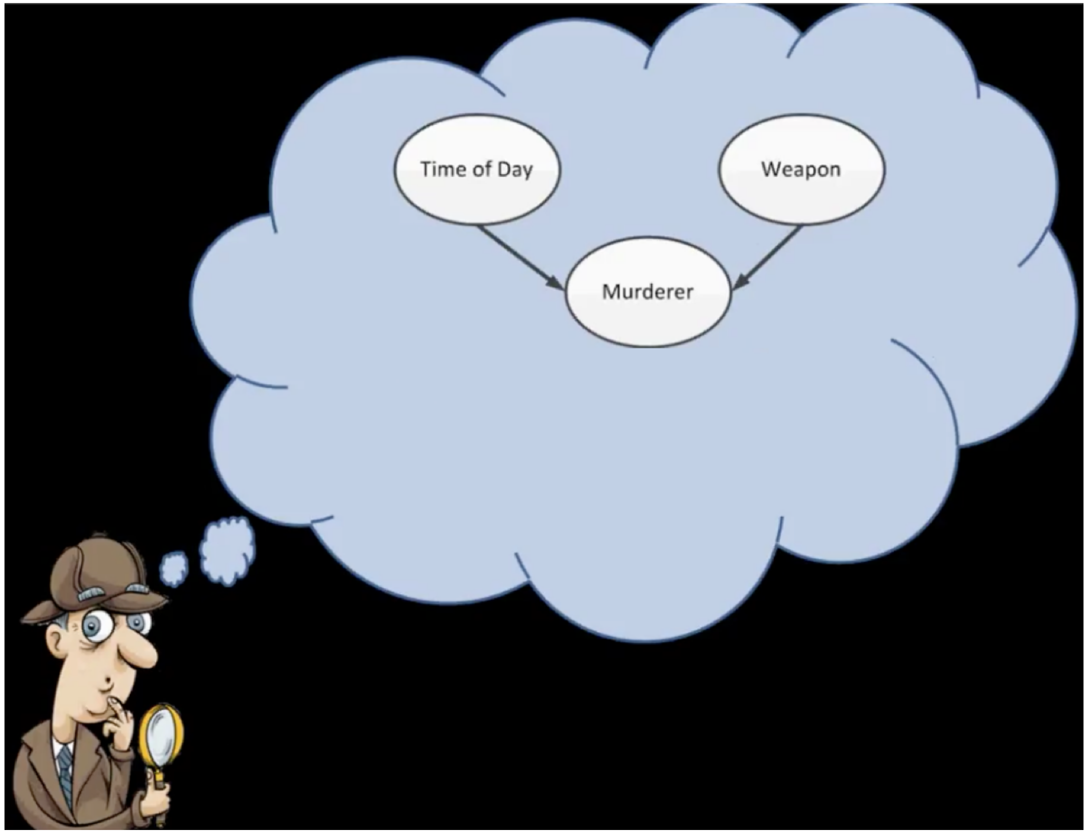
- Not wanting to stay in the scene any longer
- Not wanting to gather more information
- Reasoning based on only the current information

- If the weapon is vacuum cleaner pipe, then the murderer is very likely to be the maid
- If the weapon is candle stick, then the murderer is very likely to be the butler
- If the murder time is evening, then the murderer is likely to be the maid
- If the murder time is midnight, then the murderer is likely to be the butler
- …

Conditional Dependencies

- Model the conditional dependencies between the variables as a directed acyclic graph (DAG)
  – Time of Day -> Murder
  – Weapon -> Murder
  – Time of Day and Weapon are independent

- What is missing?



Bayesian Networks

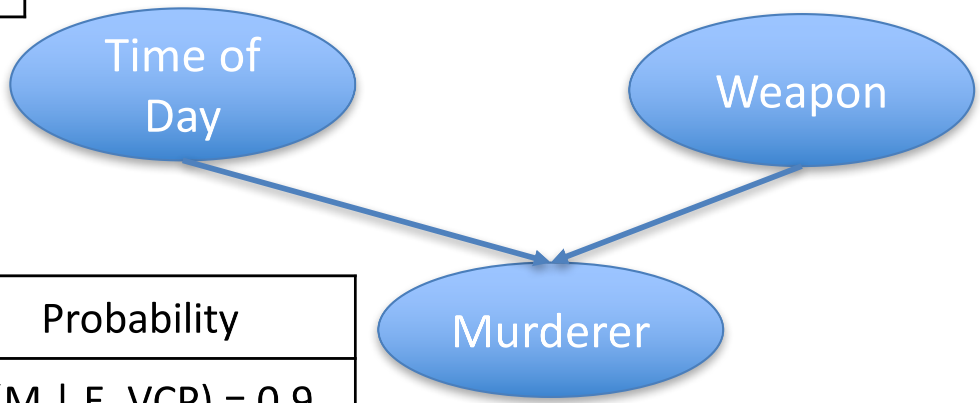
- Beliefs as conditional probabilities
  - Report from the lab:
    - P(Time = evening) = 0.05, P(Time = midnight) = 0.95
    - P(Weapon = VCP) = 0.8, P(Weapon = CS) = 0.2
  - From detective
    - P(Murderer = maid | Time = evening, Weapon = VCP) = 0.9
    - P(Murderer = maid | Time = evening, Weapon = CS) = 0.55
    - P(Murderer = maid | Time = midnight, Weapon = VCP) = 0.35
    - P(Murderer = maid | Time = midnight, Weapon = CS) = 0.05

    - P(Murderer = butler | Time = evening, Weapon = VCP) = 0.1
    - P(Murderer = butler | Time = evening, Weapon = CS) = 0.45
    - P(Murderer = butler | Time = midnight, Weapon = VCP) = 0.65
    - P(Murderer = butler | Time = midnight, Weapon = CS) = 0.95

Bayesian Networks

Time	Probability
Evening	P(T = E) = 0.05
Midnight	P(T = M) = 0.95

Weapon	Probability
VCP	P(W = VCP) = 0.8
CS	P(W = CS) = 0.2



T	W	Murderer	Probability
E	VCP	Maid	P(M | E, VCP) = 0.9
E	CS	Maid	P(M | E, CS) = 0.55
M	VCP	Maid	P(M | M, VCP) = 0.35
M	CS	Maid	P(M | M, CS) = 0.05
E	VCP	Butler	P(B | E, VCP) = 0.1
E	CS	Butler	P(B | E, CS) = 0.45
M	VCP	Butler	P(B | M, VCP) = 0.65
M	CS	Butler	P(B | M, CS) = 0.95

Bayesian Networks

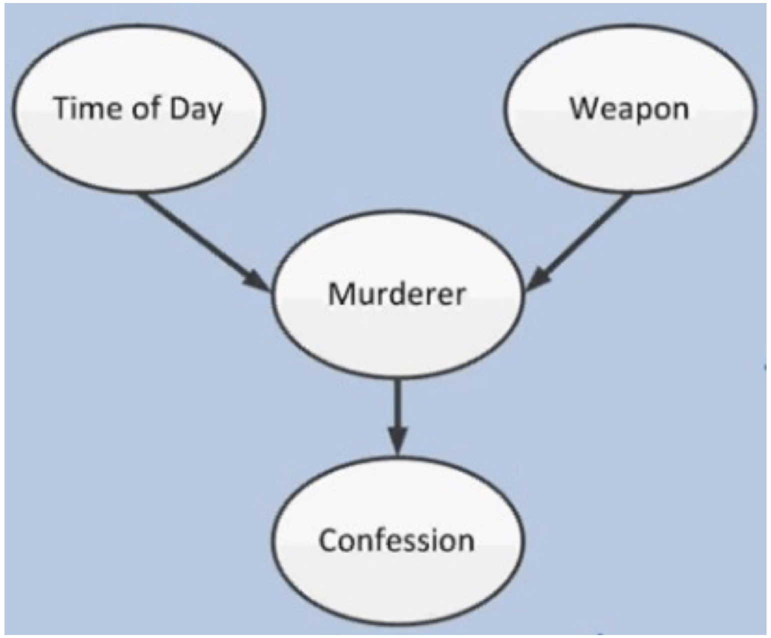
- Bayesian networks (BNs): a graphical representation of a probabilistic dependency model
  - also known as Belief networks (or Bayes nets for short)
  - Belong to the family of probabilistic graphical models (GMs).
    - Other GMs: Markov network …

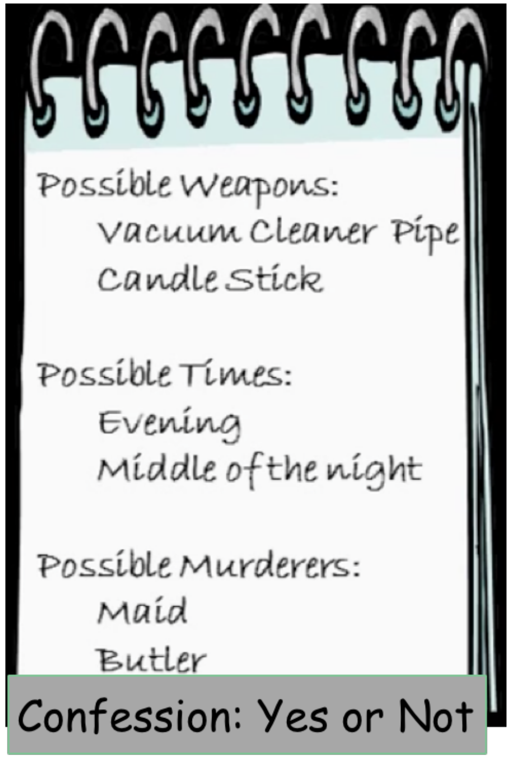
- These graphical structures are used to represent knowledge about an uncertain domain.
  - each node in the graph represents a random variable,
  - the edges between the nodes represent probabilistic dependencies among the corresponding random variables.
  - The conditional dependencies in the graph are often estimated by using known statistical and computational methods.

- BNs combine principles from graph theory, probability theory, computer science, and statistics.

Bayesian Networks

- Each node or variable may take one of a number of possible states or values.
- The belief in, or certainty of, each of these values is determined from the belief in each possible value of every node directly connected to it and its relationship with each of these nodes.
- The belief in each state of a node is updated whenever the belief in each state of any directly connected node changes.





Semantics of Bayesian Networks

- A set of nodes, one for a variable X
- A directed, acyclic graph
  - Each edge shows the direct influence between parent and child
  - A child depends on its parents
- A conditional probability table for each node
  - a collection of distributions over X, one for each combination of parents values

$$P(X \mid a_1, \dots, a_n)$$

  - (usually) description of a "causal" process

A Bayes Net = Topology (graph) + Local Conditional Probabilities

Why Bayesian Networks

- Several advantages for data analysis:
  - the model encodes dependencies among all variables, it readily handles situations where some data entries are missing.
  - a Bayesian network can be used to learn causal relationships, and hence can be used to gain understanding about a problem domain and to predict the consequences of intervention.
  - the model has both a causal and probabilistic semantics, it is an ideal representation for combining prior knowledge (which often comes in causal form) and data.
  - Bayesian statistical methods in conjunction with Bayesian networks offer an efficient and principled approach for avoiding the overfitting of data.

Cause and Effect

- A Cause is why something happens. An Effect is what actually happens (results of the Cause).

- A patient got a flu, and had a fever (high temperature).
  - Flu (disease) is the cause.
  - High temperature (symptom) is the effect.

- Causal Reasoning: solving a problem where only cause is known
  - P(Effect l Cause)
- Diagnostic Reasoning: reasoning about Cause when Effect is known
  - P(Cause l Effect)
- Inter-causal Reasoning: reasoning about the interactions between multiple causes influences

Cause and Effect

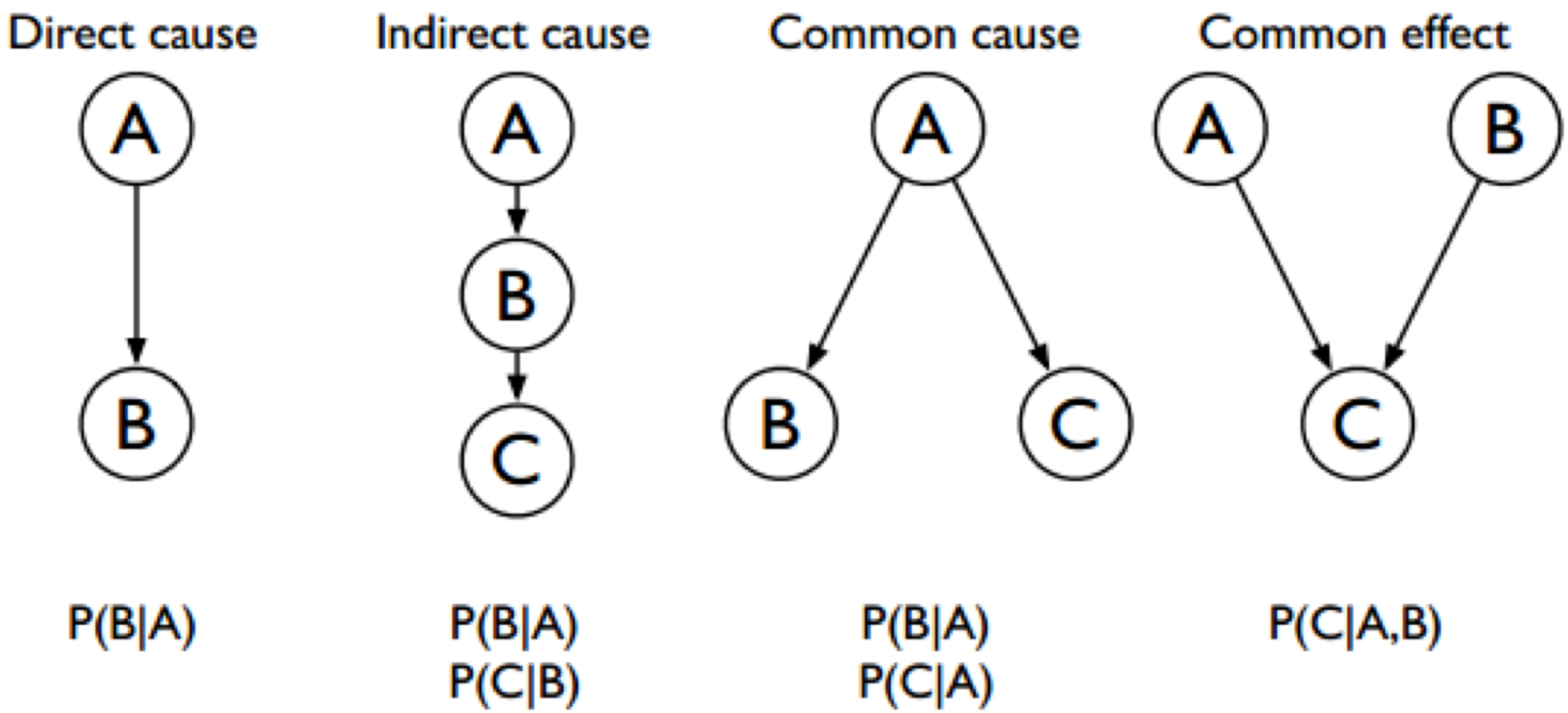
- Bayesian rules:
  - P(Cause | Effect) = P(Effect | Cause) * P(Cause) / P(Effect)

  - P(Cause) often called prior
  - P(Cause | Effect) is known as the posterior
  - P(Effect | Cause) is known as the likelihood

$$\underbrace{\boxed{P(Cause \mid Effect)}}_{\text{Posterior}} = \frac{\overbrace{\boxed{P(Effect \mid Cause)}}^{\text{Likelihood}} \times \overbrace{\boxed{P(Cause)}}^{\text{Prior}}}{\underbrace{\boxed{P(Effect)}}_{\text{Marginal likelihood or normalisation}}}$$

Cause and Effect

- Different conditional dependencies



- Common effect (multiple causes or “explaining away”. Suppose that there are exactly two possible causes of a particular effect, represented by a v-structure)
  - The causes are independent if the effect is unknown
  - Are the causes independent if the the effect is known?

Bayesian Networks Example

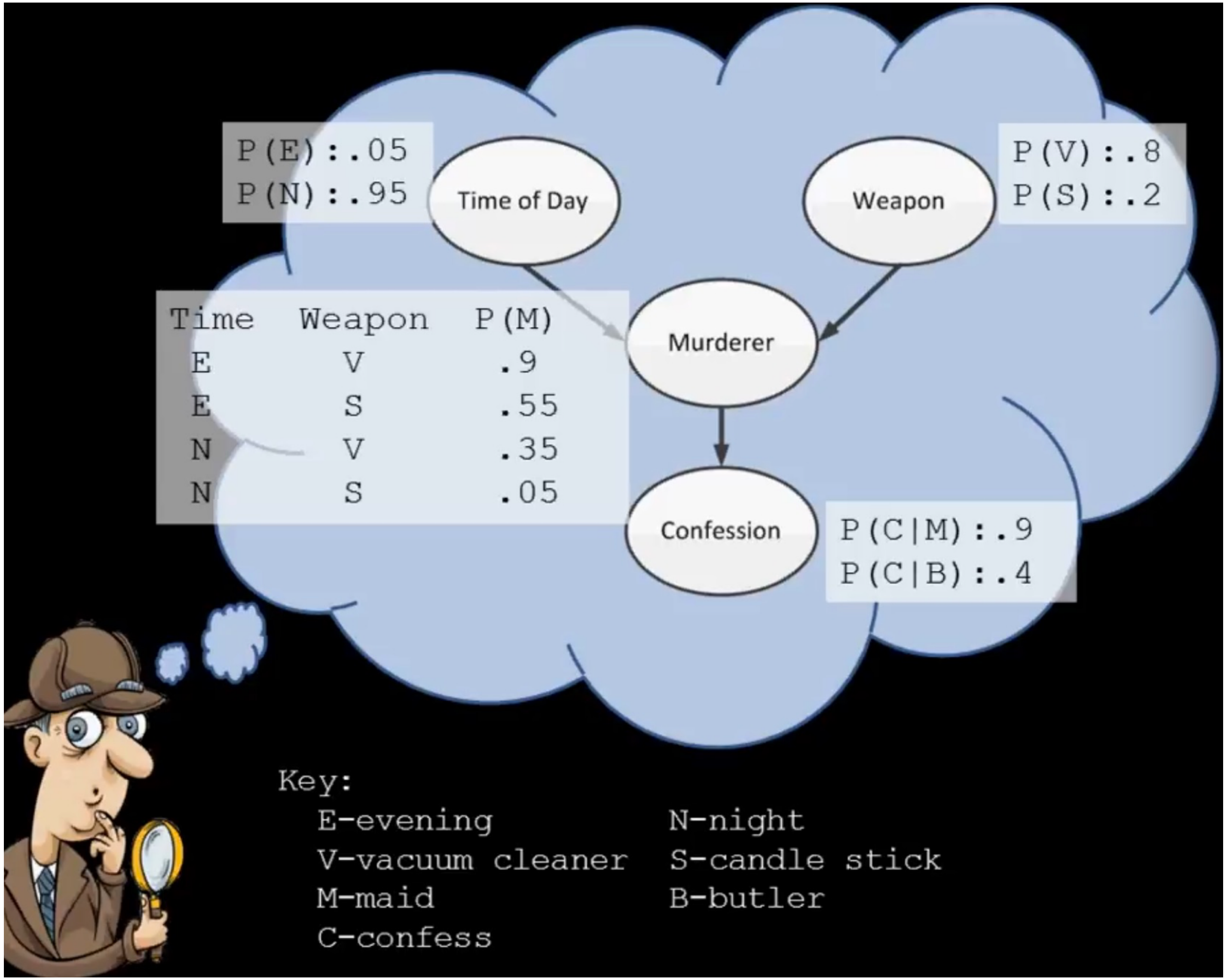
- Calculate the probability of the murderer being the maid

$$
\begin{aligned}
P(M) &= P(M,E,VCP) + P(M,E,CS) + P(M,M,VCP) + P(M,M,CS) \\
&= P(M|E,VCP)P(E,VCP) + P(M|E,CS)P(E,CS)+ \\
&\quad P(M|M,VCP)P(M,VCP) + P(M|M,CS)P(M,CS) \\
&= P(M|E,VCP)P(E)P(VCP) + P(M|E,CS)P(E)P(CS)+ \\
&\quad P(M|M,VCP)P(M)P(VCP) + P(M|M,CS)P(M)P(CS) \\
&= 0.9 \times 0.05 \times 0.8 + 0.55 \times 0.05 \times 0.2+ \\
&\quad 0.35 \times 0.95 \times 0.8 + 0.05 \times 0.95 \times 0.2 \\
&= 0.317
\end{aligned}
$$

- Which rules are used here?
- What is P(B)?
- Time and Weapon are independent if the murderer is unknown. Are they still independent if the murderer is known?

Bayesian Networks Example

- Calculate the probability that the murderer confesses



Summary

- Bayesian networks
  - A directed acyclic graph
  - Represent conditional dependencies between variables
  - Conditional distribution tables
- Cause and effect
  - Different relationships
- How to calculate probabilities in a Bayesian network

- Next lecture: how to build a BN