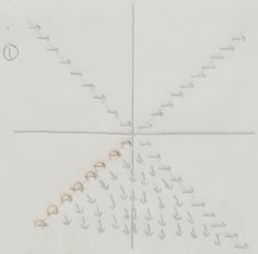
b10901154 金巧芸

2-1

18.

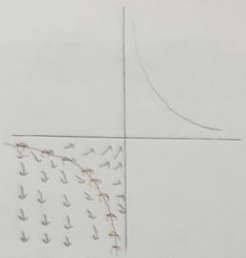
①



(a) 在 $x=y$ 及 $x=-y$ 上的所有點

(b) $x^2-y^2=0 \Rightarrow x=\pm y$

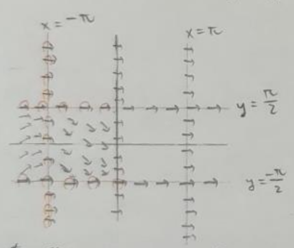
②



(a) 在 $y=\frac{1}{x}$ 上的所有點

(b) $1-xy=0 \Rightarrow y=\frac{1}{x}$

④



(a) 在 $x=n\pi$, $y=\frac{\pi}{2}+m\pi$ 上所有的點，$\forall n,m\in\mathbb{Z}$

(b) $\sin x\cdot\cos y=0$

$\Rightarrow \sin x=0$ or $\cos y=0$

$\Rightarrow x=n\pi$, $y=\frac{\pi}{2}+m\pi$ $\forall n,m\in\mathbb{Z}$

2-2

12.

$\sin 3x\,dx+2y\cos^3(3x)\,dy=0$

$\Rightarrow \frac{\sin(3x)}{\cos^3(3x)}dx=-2y\,dy$

$\Rightarrow \int \tan(3x)\sec^2(3x)\,dx=\int -2y\,dy$

$\Rightarrow \frac{1}{9}\sec^3(3x)=-y^2+C$

$\Rightarrow y=\pm\sqrt{-\frac{1}{9}\sec^3(3x)+C}$

check: $\cos^3(3x)=0 \Rightarrow \cos(3x)=0$

$\Rightarrow x=\frac{(2n+1)\pi}{6}$ $\forall n\in\mathbb{Z}$.

此時 $\frac{dx}{dy}=0$，$\sin 3x\frac{dx}{dy}+2y\cos^3(3x)=0$ 成立

Ans: $y=\pm\sqrt{-\frac{1}{9}\sec^3(3x)+C}$ or

$x=\frac{(2n+1)\pi}{6}$ $\forall n\in\mathbb{Z}$

25. $x^2\frac{dy}{dx}=y-xy$　　$y(-1)=-1$

$\Rightarrow \frac{1}{y}dy=\frac{1-x}{x^2}dx$

$\Rightarrow \ln|y|=-\frac{1}{x}-\ln|x|+C_1$

$\Rightarrow |y|=\frac{1}{|x|}e^{-\frac{1}{x}}\cdot e^{C_1}$

$\Rightarrow y=\frac{\pm C_2e^{-\frac{1}{x}}}{|x|}=\frac{C_3e^{-\frac{1}{x}}}{x}$

$\Rightarrow y(-1)=-C_3e=-1 \Rightarrow C_3=e^{-1}$

check: $y=0 \Rightarrow C_3$代入0時成立

$x=0$，$y(-1)$ 無值

Ans: $y=\frac{e^{-\frac{1}{x}-1}}{x}$

42.

$$\frac{dy}{dx} = (y-1)^2 - 0.01$$

$$\Rightarrow \frac{1}{(y-1-0.1)(y-1+0.1)}\,dy = dx$$

$$\Rightarrow x = \int \frac{5}{y-1.1} - \frac{5}{y-0.9}\,dy$$

$$\Rightarrow x = 5\ln\left|\frac{y-1.1}{y-0.9}\right| + C$$

$$\Rightarrow y(0) = 1$$

$$\Rightarrow 0 = 5\ln\left|\frac{-0.1}{0.1}\right| + C \Rightarrow C = 0$$

check $(y-1)^2 - 0.01 = 0$

$$\Rightarrow y = -0.9 \text{ or } 1.1 \Rightarrow y(0) \neq 1$$

Ans: $x = 5\ln\left|\frac{y-1.1}{y-0.9}\right|$

57.

$$\frac{dy}{dx} = \frac{W}{T_1} = \frac{x\rho}{T_1} \quad , \rho, T_1 \text{ are constant}$$

$$\Rightarrow y = \frac{x^2\rho}{2T_1} + C$$

initial-value: $y(0) = a$, $y(\frac{L}{2}) = h+a$

$$\Rightarrow y(0) = a \Rightarrow C = a$$

$$\Rightarrow y(\tfrac{L}{2}) = \frac{\frac{L^2}{4}\rho}{2T_1} + a = h + a$$

$$\Rightarrow \frac{\rho}{T_1} = \frac{8h}{L^2}$$

$$\Rightarrow y = \frac{4x^2h}{L^2} + a$$

Ans: $y = \frac{4x^2h}{L^2} + a$