

5-1

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PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW5> g++ .\5-1.cpp -o 5-1
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW5> ./5-1
```

Euler method

t	y	value	absolute error
1.0	0.0000	0.0000	0.000000
1.1	0.1000	0.1052	0.005160
1.2	0.2099	0.2212	0.011325
1.3	0.3305	0.3491	0.018651
1.4	0.4624	0.4897	0.027328
1.5	0.6063	0.6439	0.037590
1.6	0.7630	0.8128	0.049711
1.7	0.9335	0.9975	0.064019
1.8	1.1185	1.1994	0.080902
1.9	1.3193	1.4201	0.100823
2.0	1.5369	1.6613	0.124338

Taylor method order 2

t	y	value	absolute error
1.0	0.0000	0.0000	0.000000
1.1	0.1050	0.1052	0.000160
1.2	0.2209	0.2212	0.000324
1.3	0.3486	0.3491	0.000509
1.4	0.4890	0.4897	0.000728
1.5	0.6429	0.6439	0.000993
1.6	0.8114	0.8128	0.001315
1.7	0.9958	0.9975	0.001707
1.8	1.1973	1.1994	0.002187
1.9	1.4173	1.4201	0.002772
2.0	1.6578	1.6613	0.003487

5-2

有去查感覺這題的題目像是剛性方程(因為 $h=0.1$ 數值真的超可怕 ouo)

在數學領域中，剛性方程 (stiffness equation) 是指一個微分方程，其數值分析的解只有在時間間隔很小時才會穩定，只要時間間隔略大，其解就會不穩定。目前很難去精確地去定義哪些微分方程是剛性方程，然而粗略而言，若此方程式中包含使其快速變動的項，則其為剛性方程。

題目沒說要從 $t=0$ 到多少的值，這邊假設到 $t=1$ 而已

step $h = 0.1$ result:

t	u1 (RK4)	u2 (RK4)	u1	u2	error u1	error u2
0.000000	1.333333	0.666667	1.333333	0.666667	0.000000	0.000000
0.100000	-3.052437	8.989305	1.793063	-1.032002	4.845500	-10.021308
0.200000	-23.847795	51.192704	1.423902	-0.874681	25.271697	-52.067385
0.300000	-130.165202	269.269193	1.131577	-0.724999	131.296778	-269.994192
0.400000	-680.231485	1399.368584	0.909409	-0.608214	681.140894	-1399.976798
0.500000	-3531.299585	7258.241839	0.738788	-0.515658	3532.038373	-7258.757497
0.600000	-18312.795052	37634.955483	0.605710	-0.440411	18313.400762	-37635.395894
0.700000	-94951.331907	195131.871735	0.499860	-0.377404	94951.831768	-195132.249139
0.800000	-492306.465639	1011721.872078	0.413671	-0.322954	492306.879311	-1011722.195031
0.900000	-2552513.623867	5245578.826590	0.341614	-0.274409	2552513.965482	-5245579.100999
1.000000	-13234278.789168	27197287.206587	0.279675	-0.229888	13234279.068843	-27197287.436475

$h = 0.1$ 可能已經超過穩定的最大步長了，所以會呈現爆炸式發散

step h = 0.050000 result:

t	u1 (RK4)	u2 (RK4)	u1	u2	error u1	error u2
0.000000	1.333333	0.666667	1.333333	0.666667	0.000000	0.000000
0.050000	1.721880	-0.499599	1.912059	-0.909077	0.190178	-0.409477
0.100000	1.726915	-0.832598	1.793063	-1.032002	0.066148	-0.199405
0.150000	1.617161	-0.890373	1.601967	-0.961459	-0.015194	-0.071086
0.200000	1.481687	-0.861042	1.423902	-0.874681	-0.057785	-0.013639
0.250000	1.348945	-0.807505	1.267646	-0.795221	-0.081299	0.012284
0.300000	1.227063	-0.750341	1.131577	-0.724999	-0.095487	0.025342
0.350000	1.117478	-0.695886	1.012999	-0.663060	-0.104480	0.032826
0.400000	1.019525	-0.645732	0.909409	-0.608214	-0.110117	0.037518
0.450000	0.931977	-0.599934	0.818630	-0.559389	-0.113347	0.040545
0.500000	0.853541	-0.558092	0.738788	-0.515658	-0.114753	0.042435
0.550000	0.783017	-0.519706	0.668275	-0.476225	-0.114743	0.043482
0.600000	0.719337	-0.484290	0.605710	-0.440411	-0.113627	0.043880
0.650000	0.661560	-0.451407	0.549909	-0.407635	-0.111651	0.043772
0.700000	0.608868	-0.420673	0.499860	-0.377404	-0.109007	0.043269
0.750000	0.560547	-0.391754	0.454695	-0.349296	-0.105852	0.042459
0.800000	0.515980	-0.364365	0.413671	-0.322954	-0.102309	0.041411
0.850000	0.474633	-0.338259	0.376158	-0.298076	-0.098475	0.040183
0.900000	0.436043	-0.313226	0.341614	-0.274409	-0.094428	0.038817
0.950000	0.399812	-0.289089	0.309583	-0.251739	-0.090229	0.037351
1.000000	0.365600	-0.265698	0.279675	-0.229888	-0.085925	0.035810

$h = 0.1$ 在穩定步長內所以呈現收斂