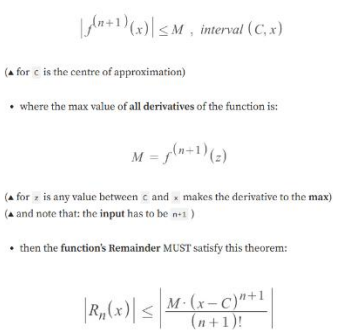
1.

因為題目只給了 4 個點，所以 Degree4 做不出來(他需要 5 個點)

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> g++ .\3-1.cpp -o 3-1
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> ./3-1
Degree: 1, Lagrange Approximation: 0.732077
Error Bound: 0.0003386291

Degree: 2, Lagrange Approximation: 0.731716
Error Bound: 0.0000018423

Degree: 3, Lagrange Approximation: 0.731704
Error Bound: 0.0000000269
```



$|f^{(n+1)}(x)| \le M$, $interval\ (C, x)$

(▲ for c is the centre of approximation)

- where the max value of **all derivatives** of the function is:

$$M = f^{(n+1)}(z)$$

(▲ for z is any value between c and x makes the derivative to the **max**)
(▲ and note that: the **input** has to be $n+1$)

- then the **function's Remainder** MUST satisfy this theorem:

$$|R_n(x)| \le \left| \frac{M \cdot (x-C)^{n+1}}{(n+1)!} \right|$$

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0).....(x-x_n)$$

網路上找到的拉格朗日 error bound 的公式如左圖所示，但是因為不知道這題的 center 是什麼，所以參考第四章看到感覺類似 error bound 的公式(x-x_0)(x-x_1)…(x-x_n)的方式來做

2.

用題目提供的 4 點來建立拉格朗日插值多項式，然後用 Secant Method 的方式求解，得到 x=0.567145 時，x-e^{-x}=0

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> g++ .\3-2.cpp -o 3-2
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> ./3-2
Inverse Interpolation using Secant
Secant Method (Initial guesses 0, 1):
Converged to root: 0.567145 in 5 iterations
```

3.

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> g++ .\3-3.cpp -o 3-3
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW3> ./3-3
The position at t = 10: 596.316 feet
The velocity at t = 10: -100.718 feet/sec
Speed exceeds 55 mi/h at t = 0.033 seconds
Speed at that moment: 80.7584 ft/s
The predicted maximum speed is: 398.204 ft/s at t = 12.411 seconds
```

a.

使用程式建立 divided differences 的表格，用來計算 H(10), H'(10)的值

position = 596.316 feet

speed = -100.718 feet/sec

b.

55mi/h = 80.67 feet/s

使用間隔為 0.001，從 t = 0-13 計算 H’(t)的值，找到第一次超速的時間點
第一次超速的時間點為 0.033，速度為 80.7584 feet/s

c.
使用間隔為 h = 0.001，從 t = 0-13 計算 H’(t)的值，找到最大的速度值
最大的速度為 398.204 feet/s，在 t=12.4111 時