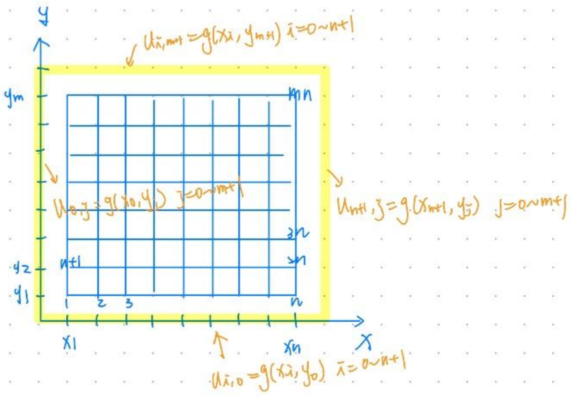
1.

先建立邊界的值




```
for (int j = 0; j <= ny; ++j) {
    double y = j * k;
    u[0][j] = cos(y);          // u(0, y) = cos(y)
    u[nx][j] = -cos(y);        // u(π, y) = -cos(y)
}

for (int i = 0; i <= nx; ++i) {
    double x = i * h;
    u[i][0] = cos(x);          // u(x, 0) = cos(x)
    u[i][ny] = 0.0;            // u(x, π/2) = 0
}
```

之後根據以下公式可知當前某點的相鄰係數關係

$$\alpha u_{i,j-1} + u_{i-1,j} - 2(1+\alpha)u_{i,j} + u_{i+1,j} + \alpha u_{i,j+1} = h^2 f_{i,j}$$

$$\begin{array}{ccc} & u_{i,j+1} & \\ & \alpha & \\ u_{i-1,j} & u_{i,j} & u_{i+1,j} \\ 1 & -2(1+\alpha) & 1 \\ & u_{i,j-1} & \\ & \alpha & \end{array}$$

且 F = $h^2 f_{i,j}$

並依據公式做 F 邊界的修正

$$\text{Case 1} \quad j=1: \quad u_{i-1,1} - 2(1+\alpha)u_{i,1} + u_{i+1,1} + \alpha u_{i,2} = \boxed{h^2 f_{i,1} - \alpha u_{i,0}}$$

$$\text{a.} \quad i=1: \quad -2(1+\alpha)u_{1,1} + u_{2,1} + \alpha u_{1,2} = \boxed{h^2 f_{1,1} - \alpha u_{i,0} - u_{0,1}} \triangleq F_{1,1}$$

$$\text{Case 3} \quad j=m: \quad \alpha u_{i,m-1} + u_{i-1,m} - 2(1+\alpha)u_{i,m} + u_{i+1,m} = \boxed{h^2 f_{i,m} - \alpha u_{i,m+1}}$$

$$\text{a.} \quad i=1: \quad \alpha u_{1,m-1} - 2(1+\alpha)u_{1,m} + u_{2,m} = \boxed{h^2 f_{1,m} - \alpha u_{1,m+1} - u_{0,m}} = F_{1,m}$$

輸出結果:

PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw12> ./1

y/x	0	0.314159	0.628319	0.942478	1.25664	1.5708	1.88496	2.19911	2.51327	2.82743	3.14159
1.5708	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.2566	0.3090	0.1728	0.0531	-0.0589	-0.1667	-0.2699	-0.3642	-0.4413	-0.4863	-0.4679	-0.3090
0.9425	0.5878	0.3681	0.1763	-0.0050	-0.1823	-0.3539	-0.5116	-0.6420	-0.7244	-0.7255	-0.5878
0.6283	0.8090	0.5646	0.3476	0.1326	-0.0869	-0.3056	-0.5112	-0.6862	-0.8099	-0.8589	-0.8090
0.3142	0.9511	0.7532	0.5559	0.3332	0.0858	-0.1732	-0.4243	-0.6452	-0.8145	-0.9158	-0.9511
0.0000	1.0000	0.9511	0.8090	0.5878	0.3090	0.0000	-0.3090	-0.5878	-0.8090	-0.9511	-1.0000

2.

3.

和第一題概念類似，只是公式不太一樣

$$\alpha u_{i,j-1} + (r_i^2 - \frac{h}{2} r_i) u_{i-1,j} - 2(\alpha + r_i^2) u_{i,j} + (r_i^2 + \frac{h}{2} r_i) u_{i+1,j} + \alpha u_{i,j+1} = r_i^2 h^2 f_{i,j}$$

Case 1. $j=1$: $(r_i^2 - \frac{h}{2} r_i) u_{i-1,1} - 2(\alpha + r_i^2) u_{i,1} + (r_i^2 + \frac{h}{2} r_i) u_{i+1,1} + \alpha u_{i,2} = \boxed{r_i^2 h^2 f_{i,1} - \alpha u_{i,0}}$

a. $i=1$: $-2(\alpha + r_1^2) u_{1,1} + (r_1^2 + \frac{h}{2} r_1) u_{2,1} + \alpha u_{1,2} = \boxed{r_1^2 h^2 f_{1,1} - \alpha u_{1,0} - (r_1^2 - \frac{h}{2} r_1) u_{0,1}} \triangleq F_{1,1}$

Case 3 $j=m$:

$$\alpha u_{i,m-1} + (r_i^2 - \frac{h}{2} r_i) u_{i-1,m} - 2(\alpha + r_i^2) u_{i,m} + (r_i^2 + \frac{h}{2} r_i) u_{i+1,m} = \boxed{r_i^2 h^2 f_{i,m} - \alpha u_{i,m+1}}$$

a. $i=1$:

$$\alpha u_{1,m-1} - 2(\alpha + r_1^2) u_{1,m} + (r_1^2 + \frac{h}{2} r_1) u_{2,m} = \boxed{r_1^2 h^2 f_{1,m} - \alpha u_{1,m+1} - (r_1^2 - \frac{h}{2} r_1) u_{0,m}} \triangleq F_{1,m}$$

但是這題比較有問題的是他的邊界條件在四個交點的地方會有衝突，我是依照他邊界條件給的順序依序做的，所以那四個點會是 50 或 100 而非 0

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw12> ./3
   t/r         0.5         0.6         0.7         0.8         0.9           1
1.0472     50.0000      0.0000      0.0000      0.0000      0.0000    100.0000
0.8727     50.0000     32.2567     32.9444     43.1044     63.5365    100.0000
0.6981     50.0000     45.8602     50.5938     62.4550     79.7182    100.0000
0.5236     50.0000     49.6786     55.8351     67.7253     83.3054    100.0000
0.3491     50.0000     45.8602     50.5938     62.4550     79.7182    100.0000
0.1745     50.0000     32.2567     32.9444     43.1044     63.5365    100.0000
0.0000     50.0000      0.0000      0.0000      0.0000      0.0000    100.0000
```

4.

我覺得這題比較符合講義這段的內容，但是邊界 p(0, t)≠ $p(1,t) \neq 0$

§ $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, $t > 0$, $u(0,t) = u(l,t) = 0$, $u(x,0) = f(x)$, $\frac{\partial u}{\partial t} = g(x)$

$$\frac{1}{k^2}(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) = \alpha^2 \frac{1}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}),$$

$$u_{i,j+1} = \lambda^2 u_{i-1,j} + 2(1-\lambda^2)u_{i,j} + \lambda^2 u_{i+1,j} - u_{i,j-1}, \quad \lambda^2 = \alpha^2 \frac{k^2}{h^2},$$

$$u_{1,j+1} = 2(1-\lambda^2)u_{1,j} + \lambda^2 u_{2,j} - u_{1,j-1},$$

$$u_{i,j+1} = \lambda^2 u_{i-1,j} + 2(1-\lambda^2)u_{i,j} + \lambda^2 u_{i+1,j} - u_{i,j-1},$$

$$u_{n,j+1} = \lambda^2 u_{n-1,j} + 2(1-\lambda^2)u_{n,j} - u_{n,j-1}.$$

$$\begin{Bmatrix} u_1 \\ \bullet \\ \bullet \\ u_n \end{Bmatrix}_{j+1} = \begin{bmatrix} 2(1-\lambda^2) & \lambda^2 & 0 & 0 \\ \lambda^2 & 2(1-\lambda^2) & \lambda^2 & 0 \\ 0 & \lambda^2 & 2(1-\lambda^2) & \lambda^2 \\ 0 & 0 & \lambda^2 & 2(1-\lambda^2) \end{bmatrix} \begin{Bmatrix} u_1 \\ \bullet \\ \bullet \\ u_n \end{Bmatrix}_j - \begin{Bmatrix} u_1 \\ \bullet \\ \bullet \\ u_n \end{Bmatrix}_{j-1}$$

教授上課沒有特別說這種問題怎麼處理，所以我只能仿造 forward-difference 的方式做邊界修正不知道這樣對不對 ouo

$$\begin{Bmatrix} u_1 \\ \bullet \\ \bullet \\ u_n \end{Bmatrix}_{j+1} = \begin{bmatrix} 1-2\lambda & \lambda & 0 & 0 \\ \lambda & \bullet & \bullet & 0 \\ 0 & \bullet & \bullet & \lambda \\ 0 & 0 & \lambda & 1-2\lambda \end{bmatrix} \begin{Bmatrix} u_1 \\ \bullet \\ \bullet \\ u_n \end{Bmatrix}_j + k \begin{Bmatrix} g_1 \\ \bullet \\ \bullet \\ g_n \end{Bmatrix}_j + \boxed{\lambda \begin{Bmatrix} q \\ 0 \\ 0 \\ r \end{Bmatrix}_j}$$

然後它的 t 只說≥0, 沒說到多少所以我先假設跟 x 一樣

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical class\F74114095 numerical hw12> ./4
```

t\x	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.00	1.00	0.81	0.31	-0.31	-0.81	-1.00	-0.81	-0.31	0.31	0.81	2.00
0.10	1.00	1.02	0.85	0.35	-0.29	-0.81	-1.02	-0.85	-0.35	0.29	2.00
0.20	1.00	1.04	1.06	0.87	0.35	-0.31	-0.85	-1.06	-0.87	0.84	2.00
0.30	1.00	1.04	1.06	1.06	0.85	0.31	-0.35	-0.87	0.13	0.84	2.00
0.40	1.00	1.02	1.04	1.04	1.02	0.81	0.29	0.84	0.84	1.29	2.00
0.50	1.00	1.00	1.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00
0.60	1.00	0.98	0.96	0.96	0.98	2.19	2.71	3.16	3.16	2.71	2.00
0.70	1.00	0.96	0.94	0.94	2.15	2.69	3.35	3.87	3.87	3.16	2.00
0.80	1.00	0.96	0.94	2.13	2.65	3.31	3.85	4.06	3.87	3.16	2.00
0.90	1.00	0.98	2.15	2.65	3.29	3.81	4.02	3.85	3.35	2.71	2.00
1.00	1.00	2.19	2.69	3.31	3.81	4.00	3.81	3.31	2.69	2.19	2.00