1.

使用講義的這三個公式實作題目要求

a. Composite Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a)+2\sum_{i=1}^{2n-1} f(x_i)+f(b)]-\frac{b-a}{12}h^2 f''(\xi)$$

b. Composite Midpoint rule

$$\int_a^b f(x)dx = 2h[f(x_1)+f(x_3)+\ldots+f(x_{2n-1})]+\frac{b-a}{6}h^2 f''(\xi)$$

c. Composite Simpson's rule

$$\int_a^b f(x)dx = \int_{x_0}^{x_{2n}} f(x)dx = \sum_{i=1}^{n}\int_{x_{2(i-1)}}^{x_{2i}} f(x)dx = \sum_{i=1}^{n}\frac{h}{3}[f(x_{2i-2})+4f(x_{2i-1})+f(x_{2i})]$$

$$= \frac{h}{3}[f(x_0)+2\sum_{i=2}^{n} f(x_{2i-2})+4\sum_{i=1}^{n} f(x_{2i-1})+f(x_{2n})]-\frac{b-a}{180}h^4 f^{(iv)}(\xi)$$

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PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-1.cpp -o 4-1
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-1
The integral of f(x) = exp(x)*sin(4*x) using Trapezopidal_rule: 0.396148
The integral of f(x) = exp(x)*sin(4*x) using Simpson_rule: 0.385664
The integral of f(x) = exp(x)*sin(4*x) using Midpoint rule: 0.364696
```

2.

使用講義公式實作 n=3 和 n=4 的 Gaussian Qurature

Remarks:

1. $n=2$, $x_1=-x_2=-0.577$, $c_1=c_2=1.$
2. $n=3$, $x_1=-x_3=-0.775$, $x_2=0$, $c_1=c_3=0.556$, $c_2=0.889$
3. $n=4$, $x_1=-x_4=-0.861$, $x_2=-x_3=0.340$, $c_1=c_4=0.348$, $c_2=c_3=0.652$

Consider $\int_a^b f(x)dx$

Let $\frac{x-a}{b-a}=\frac{\eta-(-1)}{1-(-1)}$ or $x=\eta\frac{b-a}{2}+\frac{1}{2}(b+a)$

Then $\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f[\frac{a+b}{2}+\frac{b-a}{2}\eta]d\eta$

$$\int_a^b f(x)dx = \frac{b-a}{2}\sum_{i=1}^{n} c_i f[\frac{a+b}{2}+\frac{b-a}{2}\eta_i]$$

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PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-2.cpp -o 4-2
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-2
the integral of f(x) = x^2*log(x) using Gaussian Quadrature with n = 3: 0.1922593773
Absolute error of n = 3: 0.0000000196
the integral of f(x) = x^2*log(x) using Gaussian Quadrature with n = 4: 0.1922593578
Absolute error of n = 4: 0.0000000001
```

3.

a+c

使用講義提供 Simpson's rule n=4 的公式，並改成 Multiple integral

c. $n=4$

$$\int_{x_0}^{x_4} f(x)dx = \frac{2h}{45}[7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)]-\frac{8h^7}{945}f^{(iv)}(\xi)$$

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-3-a.cpp -o 4-3a
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-3a
The integral of f(x) = 2y*sin(x) + cos(x)*cos(x) using Simpson's rule: 0.511823
Absolute error: 2.19929e-005
```

b+c

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-3-b.cpp -o 4-3b
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-3b
The integral of f(x) = 2y*sin(x) + cos(x)*cos(x) using Gaussian Quadrature: 0.785398
Absolute error: 0.273553
```

4.

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PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-4.cpp -o 4-4
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-4
The integral of f(x) = sin(x)*x^(-0.25) using Simpson's rule: 0.525931
The integral of g(t) = sin(1/t)*t^4 using Simpson's rule: 0.181118
```

a

下界使用10^{-6}來逼近 0，避免發生除以 0 的情況

b

經過代換後方程式為$\int_0^1 t^4 \sin\frac{1}{t}\, dt$，同樣下界使用$10^{-6}$來逼近 0，避免發生除以 0 的情況