8-1

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> g++ .\8_1.cpp -o 1
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> ./1
a. degree 2: a0 = 3.08639, a1 = -1.88375, a2 = 6.69118
a. error: 0.00524569
b. a = 0.398495, b = 21.4445
b. error: 94.983
c. n = 2.01963, b = 6.23895
c. error: 0.0117207
```

對於be^{ax}和bx^n這種非線性的函數，最常見的做法應該是把它轉成對數，但對數變換會改變誤差結構，所以可能會誤差很大(像這題的 b)

非線性最小二乘法是非線性形式的最小二乘法，用包含n個未知參數的非線性模型擬合m個觀測值（$m \geq n$），可用於某些形式的非線性回歸。該方法的基礎是使用線性模型近似並通過連續迭代來優化參數。它與線性最小二乘法既有相同之處、也有一些顯著差異。

感覺 b 應該要用非線性的 least square 來算會比較準確，但這章節沒有教到所以就沒有做了

8-2

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> g++ .\8_2.cpp -o 2
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> ./2
legendre polynomial approximation:
0.420735 * 1 + 0.326548 * x - 0.232631 * (x^2 - 1/3)
Sum of Squared Errors (SSE):0.0032424
```

使用講義的 legendre polynomials 公式+正交基底的 least square approximation

Ex. Legendre polynomials $w(x)=1$

$$p_0(x)=1,\ B_1=0,\ p_1(x)=x,\ B_2=0,\ C_2=\frac{1}{3},\ p_2(x)=x^2-\frac{1}{3},$$

$$p_3(x)=x^3-\frac{3}{5}x,\ \ldots,\ p_{n+1}(x)=xp_n(x)-\frac{n^2}{4n^2-1}p_{n-1}(x)$$

$$p(x)=c_0P_0(x)+c_1P_1(x)+c_2P_2(x)$$

$$c_k=\frac{\int_{-1}^{1} f(x)P_k(x)dx}{\int_{-1}^{1}[P_k(x)]^2dx}$$

8-3

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> g++ .\8_3.cpp -o 3
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\F74114095_numerical_hw8> ./3
Discrete least squares trigonometric polynomial
 S_4(x) = 0.229603 + -0.146756 * cos(1z) + 0.232287 * sin(1z) + 0.054608 * cos(2z) + -0.124941 * sin(2z) + -0.0389289 * cos(3z) + 0.082932 * sin(3z) + 0.0335423 * cos(4z)

(b) Integral of S_4(x) over [0,1]: 0.229603
(c) Integral of f(x) = x^2 sin x over [0,1]: 0.223244
(d) Error E(S_4): 0.0115505
```

使用講義 FFT 的計算公式

$a_0 = \frac{1}{m}\sum_{i=0}^{2m-1} y_i$, $a_l = \frac{1}{m}\sum_{i=0}^{2m-1} y_i \cos lx_i$, $b_l = \frac{1}{m}\sum_{i=0}^{2m-1} y_i \sin lx_i$

Ex. Use $S_n(x) = \frac{1}{2}a_0 + a_n \cos(nx) + \sum_{k=1}^{n-1}[a_k \cos kx + b_k \sin kx]$, $x \in [-\pi, \pi)$

to least square approximate to $\{x_i, y_i\}_{i=0}^{2m-1}$,

$$x_0 = c,\ x_{2m-1} = d,\ \Delta x = \frac{d-c}{2m-1},\ x_i = x_0 + i\Delta x$$

Transform the interval $[c,d]$ to $[-\pi,\pi]$

Let $z_i = \pi[2\frac{(x_i - c)}{d-c} - 1]$

Then the function $S_n(z) = \frac{1}{2}a_0 + a_n \cos(nz) + \sum_{k=1}^{n-1}[a_k \cos kz + b_k \sin kz]$, $z \in [-\pi, \pi)$

will least square approximate to $\{z_i, y_i\}_{i=0}^{2m-1}$, $z_i = -\pi + (\frac{i}{m})\pi$.