

1.

a. Composite Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)] - \frac{b-a}{12} h^3 f''(\xi)$$

b. Composite Midpoint rule

$$\int_a^b f(x)dx = 2h[f(x_1) + f(x_2) + \dots + f(x_{2n-1})] + \frac{b-a}{6} h^3 f''(\xi)$$

c. Composite Simpson's rule

$$\begin{aligned} \int_a^b f(x)dx &= \int_{x_0}^{x_n} f(x)dx = \sum_{i=1}^n \int_{x_{2i-2}}^{x_{2i}} f(x)dx = \sum_{i=1}^n \frac{h}{3} [f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})] \\ &= \frac{h}{3} [f(x_0) + 2\sum_{i=2}^n f(x_{2i-2}) + 4\sum_{i=1}^n f(x_{2i-1}) + f(x_{2n})] - \frac{b-a}{180} h^5 f^{(4)}(\xi) \end{aligned}$$

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PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-1.cpp -o 4-1
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-1
The integral of f(x) = exp(x)*sin(4*x) using Trapezoidal_rule: 0.396148
The integral of f(x) = exp(x)*sin(4*x) using Simpson_rule: 0.385664
The integral of f(x) = exp(x)*sin(4*x) using Midpoint rule: 0.364696
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使用講義的這三個公式實作題目要求

2.

Remarks:

1.  $n = 2$ ,  $x_1 = -x_2 = -0.577$ ,  $c_1 = c_2 = 1$ .
2.  $n = 3$ ,  $x_1 = -x_3 = -0.775$ ,  $x_2 = 0$ ,  $c_1 = c_3 = 0.556$ ,  $c_2 = 0.889$
3.  $n = 4$ ,  $x_1 = -x_4 = -0.861$ ,  $x_2 = -x_3 = 0.340$ ,  $c_1 = c_4 = 0.348$ ,  $c_2 = c_3 = 0.652$

Consider  $\int_a^b f(x)dx$

$$\text{Let } \frac{x-a}{b-a} = \frac{\eta-(-1)}{1-(-1)} \text{ or } x = \eta \frac{b-a}{2} + \frac{1}{2}(b+a)$$

$$\text{Then } \int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^1 f\left[\frac{a+b}{2} + \frac{b-a}{2}\eta\right]d\eta$$

$$\int_a^b f(x)dx = \frac{b-a}{2} \sum_{i=1}^n c_i f\left[\frac{a+b}{2} + \frac{b-a}{2}\eta_i\right]$$

使用講義公式實作  $n=3$  和  $n=4$  的

Gaussian Quadrature

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-2.cpp -o 4-2
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-2
the integral of f(x) = x^2*log(x) using Gaussian Quadrature with n = 3: 0.1922593773
Absolute error of n = 3: 0.0000000196
the integral of f(x) = x^2*log(x) using Gaussian Quadrature with n = 4: 0.1922593578
Absolute error of n = 4: 0.0000000001
```

3.

a+c

c.  $n = 4$

$$\int_{x_0}^{x_4} f(x)dx = \frac{2h}{45}[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)] - \frac{8h^7}{945} f^{(6)}(\xi)$$

使用講義提供 Simpson's rule  $n=4$  的公式，並改成 Multiple integral

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-3-a.cpp -o 4-3a
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-3a
The integral of f(x) = 2y*sin(x) + cos(x)*cos(x) using Simpson's rule: 0.511823
Absolute error: 2.19929e-005
```

b+c

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> g++ .\4-3-b.cpp -o 4-3b
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\HW4> ./4-3b
The integral of f(x) = 2y*sin(x) + cos(x)*cos(x) using Gaussian Quadrature: 0.785398
Absolute error: 0.273553
```

4.

```
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\Hw4> g++ ./4-4.cpp -o 4-4
PS C:\Users\yunyu\Documents\大學\三下\數值方法\Numerical_class\Hw4> ./4-4
The integral of f(x) = sin(x)*x^(-0.25) using Simpson's rule: 0.526304
The integral of g(t) = sin(1/t)*t^4 using Simpson's rule: 0.181317
```

同樣使用講義提供 Simpson's rule  $n=4$  的公式

a

下界使用  $10^{-6}$  來逼近 0，避免發生除以 0 的情況

b

經過代換後方程式為  $\int_0^1 t^4 \sin \frac{1}{t} dt$ ，同樣下界使用  $10^{-6}$  來逼近 0，避免發生除以 0 的情況