

## Different parameter settings

### 1. DATA\_N

- origin

```
By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 7/10
```

DATA\_N = 50

- DATA\_N increases

```
By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 9/10
By summing: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 9/10
```

DATA\_N = 100

由大數法則可知，隨著試驗次數的增加，事件發生的頻率會趨於一個穩定值。所以 DATA\_N 的增加能讓模型參數的估計更加穩定，使得正確區分模型的能力提高。

### 2. sampleRepeatNum

- origin

```
By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 7/10
```

sampleRepeatNum = 2000000

- sampleRepeatNum decreases

```
By sampling: Model1 data, correct selection 6/10
| | | | Model2 data, correct selection 4/10
By summing: Model1 data, correct selection 8/10
| | | | Model2 data, correct selection 5/10
```

sampleRepeatNum = 200

### 3. mu\_prior\_params - standard deviation

- origin

```
By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 7/10
```

SD = 4

- SD increases

```

By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 7/10
By summing: Model1 data, correct selection 8/10
| | | | Model2 data, correct selection 8/10

```

SD = 8

- SD decreases

```

By sampling: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 4/10
By summing: Model1 data, correct selection 5/10
| | | | Model2 data, correct selection 5/10

```

SD = 2

我認為過於嚴格(標準差小)有可能會限制模型，造成表現變差；反之如果標準差較大的話，有助於模型靈活調整參數，提升推論效果。

#### 4. sigma\_prior\_param\_a

- origin

```

By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 7/10

```

a = 0.5

- a increases

```

By sampling: Model1 data, correct selection 10/10
| | | | Model2 data, correct selection 8/10
By summing: Model1 data, correct selection 8/10
| | | | Model2 data, correct selection 8/10

```

a = 1

- a decreases

```

By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 5/10

```

a = 0.4

#### 5. sigma\_prior\_param\_b

- origin

```

By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 5/10
By summing: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 7/10

```

b = 2

- b increases

```

By sampling: Model1 data, correct selection 9/10
| | | | Model2 data, correct selection 7/10
By summing: Model1 data, correct selection 8/10
| | | | Model2 data, correct selection 8/10

```

$b = 8$

- $b$  decreases

```

By sampling: Model1 data, correct selection 7/10
| | | | Model2 data, correct selection 4/10
By summing: Model1 data, correct selection 5/10
| | | | Model2 data, correct selection 5/10

```

$b = 0.5$

Gamma 分布的概率密度函數是：

$$f(x; a, b) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b} \quad \text{for } x > 0$$

其中：

- $a$  是形狀參數 (shape)
- $b$  是尺度參數 (scale)
- $\Gamma(a)$  是 Gamma 函數

平均值： $\mathbb{E}[X] = a \cdot b$

變異數： $\text{Var}(X) = a \cdot b^2$

所以當  $a, b$  變大時，變異數也會變大，模型選擇的效果會變好；反之  $a, b$  變小，效果變差。

## Extra Credit

根據作業提供的相關公式來計算 2-component 的 Maximum likelihood，並且使用 Expectation-Maximization Algorithm 來實作

$$P[x_i | M_2, \mu_2, \sigma_2] = (1 - b) \mathcal{N}(x_i; \mu_{2a}, \sigma_{2a}) + (b) \mathcal{N}(x_i; \mu_{2b}, \sigma_{2b})$$

參考：<https://chih-sheng-huang821.medium.com/機器學習-em-演算法-expectation-maximization-algorithm-em-高斯混合模型-gaussian-mixture-model-gmm-和 gmm-em 詳細推導-c6f634410483>

```

Data generated with two components
generating data with: m; (μ1,σ1); (μ2,σ2) = 0.078; (-1.11,1.63); (-0.83,3.15)
1-component maximum likelihood: 3.29161e-70
2-component mixture maximum likelihood: 8.63218e-55
Integrals by sampling= (6.588e-58,9.03338e-58) by summing: (3.07579e-58,8.55344e-58)

```

Write in Gaussian\_pool0rNot1.c