

Mathematical Concepts 2

Exercise 1: Matrix Calculus

Let $\mathbf{x}, \mathbf{c} \in \mathbb{R}^d$, $\mathbf{x} = [x_1, x_2, x_3]^T$, $\mathbf{c} = [1, 2, 3]^T$, $\mathbf{u} : \mathbb{R}^d \rightarrow \mathbb{R}^d$, $\mathbf{u} \in \mathcal{C}^2$, $\mathbf{u} = \mathbf{x} - \mathbf{c}$

- (a) Compute $\frac{\partial \mathbf{x}^T \mathbf{x}}{\partial \mathbf{x}}$
- (b) Compute $\frac{\partial \|\mathbf{x} - \mathbf{c}\|_2^2}{\partial \mathbf{x}}$
- (c) Compute $\frac{\partial \|\mathbf{x} - \mathbf{c}\|_2}{\partial \mathbf{x}}$

Exercise 2: Optimality in 1d

Let $f : [-1, 2] \rightarrow \mathbb{R}$, $x \mapsto \exp(x^3 - 2x^2)$

- (a) Compute f'
- (b) Create a plot of f and f' in the range $x \in [-1, 2]$ with Python
- (c) Find all possible candidates x^* for maxima and minima.
Hint: \exp is a strictly monotone function.
- (d) Compute f''
- (e) Determine if the candidates are local maxima, minima or neither.
- (f) Find the global maximum and global minimum of f