

Optimization Problems

Exercise 1: Univariate Optimization

Let $f : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto -\frac{1}{2} \cos(x) \exp\left(\frac{1}{4}(x+2)^2\right) + 3$, $x \in [-2, 2]$

- (a) Create a function-plot in Python
- (b) Calculate the optimal x after two iterations of Golden Ratio Nesting procedure
- (c) Calculate the optimal x after two iterations of Quadratic Interpolation
- (d) Calculate the optimal x after two iterations of Gradient Decent with starting-point $x = 2$ and fixed step-length 1.0

Exercise 2: Multivariate Optimization

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}, (x_1, x_2) \mapsto \left(\frac{3}{2}x_2 - 3\right)^2 + (2x_1 - 2)^2 + x_1x_2$, $x_1, x_2 \in [-4, 4]$

- (a) Create a contour plot and a 3D surface plot
- (b) Calculate the optimal \mathbf{x} after two iterations of Gradient Decent with starting-point $\mathbf{x} = (-4, -4)^T$ and fixed step-length 1.0
- (c) Calculate the optimal \mathbf{x} after two iterations of Gradient Decent with starting-point $\mathbf{x} = (-4, -4)^T$ and an adaptive step-length with backtracking and Armijo-rule with $\gamma_1 = 0.8$ and $\alpha_{init} = 4.0, \tau = 0.5$