COMP0037 ASSIGNMENT 3

Group: Group L

Members: Yun Fang, Yusi Zhou

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1 Decision Re-Plan Policy

Suppose B1 is the cell where the robot first detects the obstacle O_B.

Suppose C1 is a cell on the aisle C at the same horizontal level as B1, as illustrated in the Figure 1-c on the assignment sheet.

Let T_W be the time the robot has to wait after the obstacle has been discovered.

Let c be the cost function associated with the path, where $c(L(\pi)) = \mathbb{E}(L(\pi))$.

Note that:

L_{XY} is the shortest path length between two cells X and Y,

T is the number of timesteps the obstacle remains in front of the robot where $T = 0.5/\lambda_B + \widetilde{T}$ with $\mathbb{E}(t) = 1/\lambda_B$ and $\mathbb{E}(\tilde{t}) = 0.5/\lambda_B$,

L_W is the cost of waiting a timestep.

1.1

Let π_1 be the policy that the robot waits for the obstacle O_B to clear.

Let π_2 be the policy that the robot plans a new path down aisle C.

Then,

$$c(L(\pi_1)) = c(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG})$$
$$c(L(\pi_2)) = c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$$

When, on average, the waiting policy is better than the other one:

$$c(L(\pi_1)) \leq c(L(\pi_2))$$
$$\mathbb{E}(L(\pi_1)) \leq \mathbb{E}(L(\pi_2))$$
$$\mathbb{E}(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) \leq \mathbb{E}(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$$
$$\mathbb{E}(T_W \cdot L_W + L_{B_1B} + L_{BC}) \leq \mathbb{E}(L_{B_1C_1} + L_{C_1C})$$

Because L_{XY} is constant, L_W is constant, and $L_{B_1B} = L_{C_1C}$:

$$\begin{aligned}\mathbb{E}(T_W) \cdot L_W + L_{BC} &\leq L_{B_1C_1} \\ \mathbb{E}(T_W) \cdot L_W &\leq L_{B_1C_1} - L_{BC} \\ \mathbb{E}(T_W) &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W}\end{aligned}$$

In this case, $T_W = 1 \cdot T = T$, so:

$$\begin{aligned}\mathbb{E}(T) &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \frac{1}{\lambda_B} &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \lambda_B &\geq \frac{L_W}{L_{B_1C_1} - L_{BC}}\end{aligned}$$

Therefore, the smallest value of λ_B which guarantees on average that waiting is the better strategy is $\frac{L_W}{L_{B_1C_1} - L_{BC}}$ and $\lambda_B \neq 0$.

1.2

Let π_0 be the policy that the robot drives directly down aisle B.

Let π_1 be the policy that the robot drives down aisle B, encounters an obstacle and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle, drives down aisle C.

Let π_3 be the policy that the robot drives directly down aisle C.

As $L(\pi_0) = L(\pi_3)$ and thus $c(L(\pi_0)) = c(L(\pi_3))$, we will not discuss π_0 here.

Then,

$$\begin{aligned}c(L(\pi_1)) &= c(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) \\ c(L(\pi_2)) &= c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG}) \\ c(L(\pi_3)) &= c(L_{IC_1} + L_{C_1C} + L_{CG})\end{aligned}$$

If the robot decides to drive directly down aisle C, then:

$$c(L(\pi_3)) \leq c(L(\pi_1)) \tag{1}$$

$$c(L(\pi_3)) \leq c(L(\pi_2)) \tag{2}$$

by (1) - (2):

$$\begin{aligned}0 &\leq c(L(\pi_1)) - c(L(\pi_2)) \\ 0 &\leq \mathbb{E}(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) - \mathbb{E}(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG}) \\ 0 &\leq \mathbb{E}(T_W) \cdot L_W + L_{BC} - L_{B_1C_1} \\ \mathbb{E}(T_W) &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \mathbb{E}(T) &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \frac{1}{\lambda_B} &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \lambda_B &\leq \frac{L_W}{L_{B_1C_1} - L_{BC}}\end{aligned}$$

Therefore, the maximum value of λ_B at which the robot will decide to drive directly down C and not attempt to drive down aisle B is $\frac{L_W}{L_{B_1C_1}-L_{BC}}$ and $\lambda_B \neq 0$.

1.3

In this case, $T_W = p_B \cdot T + (1-p_B)\cdot 0 = p_B \cdot T$.

As p_B is constant, $\mathbb{E}(T_W) = p_B \cdot \mathbb{E}(T) = p_B/\lambda_B$.

Let π_1 be the policy that the robot drives down aisle B, encounters an obstacle and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle, drives down aisle C.

Let π_3 be the policy that the robot drives directly down aisle C.

Since in this situation the robot attempts to drive down aisle B first:

$$c(L(\pi_3)) \geq c(L(\pi_1)) \tag{3}$$
$$c(L(\pi_3)) \geq c(L(\pi_2)) \tag{4}$$

Similarly as in the section 1.2, we can obtain that:

$$\mathbb{E}(T_W) \leq \frac{L_{B_1C_1} - L_{BC}}{L_W}$$

As $\mathbb{E}(T_W) = p_B/\lambda_B$, λ_B is a fixed value and $\lambda_B \neq 0$,

$$p_B/\lambda_B \leq \frac{L_{B_1C_1} - L_{BC}}{L_W}$$
$$p_B \leq \frac{\lambda_B(L_{B_1C_1} - L_{BC})}{L_W}$$

Therefore, when p_B is below $\frac{\lambda_B(L_{B_1C_1}-L_{BC})}{L_W}$, the robot will attempt to drive aisle B first.

1.4

Let π_1 be the policy that the robot drives down aisle B and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle O_B, and drives down aisle C.

Let π_3 be the policy that the robot drives down aisle B, encounters an obstacle O_B, drives down aisle C, and waits.

Let π_4 be the policy that the robot drives down aisle B, encounters an obstacle O_B, drives down aisle C, encounters an obstacle O_C, and drives down aisle D.

Let π_5 be the policy that the robot drives directly down aisle D.

Suppose D1 is a cell on the aisle D at the same horizontal level as B1 and C1.

Then,

$$c(L(\pi_1)) = c(L_{IB_1} + T_{WB}\cdot L_W + L_{B_1B} + L_{BC} + L_{CG})$$
$$c(L(\pi_2)) = c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$$
$$c(L(\pi_3)) = c(L_{IB_1} + L_{B_1C_1} + T_{WC}\cdot L_W + L_{C_1C} + L_{CG})$$
$$c(L(\pi_4)) = c(L_{IB_1} + L_{B_1C_1} + L_{C_1D_1} + L_{D_1D} + L_{DG})$$
$$c(L(\pi_5)) = c(L_{ID_1} + L_{D_1D} + L_{DG})$$

And,

$$\mathbb{E}(T_{WB}) = p_B/\lambda_B$$
$$\mathbb{E}(T_{WC}) = p_C/\lambda_C$$

We are looking to the path length, so if the robot drives directly down aisle D, it means:

$$
\begin{aligned}
L(\pi_5) &\leq L(\pi_1) && (5)\\
L(\pi_5) &\leq L(\pi_2) && (6)\\
L(\pi_5) &\leq L(\pi_3) && (7)\\
L(\pi_5) &\leq L(\pi_4) && (8)\\
& && (9)
\end{aligned}
$$

Sum up the four equation, we obtain that:

$$
\begin{aligned}
4L(\pi_5) &\leq 4L_{IB_1} + \mathbb{E}(T_{WB}) \cdot L_W + \mathbb{E}(T_{WC}) \cdot L_W + 3L_{B_1C_1} + L_{C_1D_1} + 4L_{B_1B} + L_{BC} + 3L_{CG} + L_{DG}\\
4L(\pi_5) &\leq 4L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot L_W + 3L_{B_1C_1} + L_{C_1D_1} + 4L_{B_1B} + L_{BC} + 3L_{CG} + L_{DG}\\
4L(\pi_5) &\leq 4L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot L_W + 4L_{B_1C_1} + 4L_{B_1B} + 4L_{CG}\\
L(\pi_5) &\leq L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot 0.25L_W + L_{B_1C_1} + L_{B_1B} + L_{CG}\\
L(\pi_5) &\leq L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot 0.25L_W + L_{B_1C_1} + L_{C_1C} + L_{CG}
\end{aligned}
$$

Let L be the path length of $(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$

Therefore, the upper bound on the value of the path length of the path going down aisle is $L + 0.25L_W \cdot (p_B/\lambda_B + p_C/\lambda_C)$.

2 Implement System in ROS

References

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