COMP0037 ASSIGNMENT 3

Group: Group L

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April 24, 2020

1 Decision Re-Plan Policy

Suppose B1 is the cell where the robot first detects the obstacle O_B.

Suppose C1 is a cell on the aisle C at the same horizontal level as B1, as illustrated in the Figure 1-c on the assignment sheet.

Let T_W be the time the robot has to wait after the obstacle has been discovered.

Let c be the cost function associated with the path, where $c(L(\pi)) = \mathbb{E}(L(\pi))$.

Note that:

L_{XY} is the shortest path length between two cells X and Y,

T is the number of timesteps the obstacle remains in front of the robot where $T = 0.5/\lambda_B + \widetilde{T}$ with $\mathbb{E}(t) = 1/\lambda_B$ and $\mathbb{E}(\tilde{t}) = 0.5/\lambda_B$,

L_W is the cost of waiting a timestep.

1.1

Let π_1 be the policy that the robot waits for the obstacle O_B to clear.

Let π_2 be the policy that the robot plans a new path down aisle C.

Then,

$$c(L(\pi_1)) = c(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG})$$
$$c(L(\pi_2)) = c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$$

When, on average, the waiting policy is better than the other one:

$$c(L(\pi_1)) \leq c(L(\pi_2))$$
$$\mathbb{E}(L(\pi_1)) \leq \mathbb{E}(L(\pi_2))$$
$$\mathbb{E}(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) \leq \mathbb{E}(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$$
$$\mathbb{E}(T_W \cdot L_W + L_{B_1B} + L_{BC}) \leq \mathbb{E}(L_{B_1C_1} + L_{C_1C})$$

Because L_{XY} is constant, L_W is constant, and $L_{B_1B} = L_{C_1C}$:

$$\begin{aligned} \mathbb{E}(T_W) \cdot L_W + L_{BC} &\leq L_{B_1C_1} \\ \mathbb{E}(T_W) \cdot L_W &\leq L_{B_1C_1} - L_{BC} \\ \mathbb{E}(T_W) &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W} \end{aligned}$$

In this case, $T_W = 1 \cdot T = T$, so:

$$\begin{aligned} \mathbb{E}(T) &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \frac{1}{\lambda_B} &\leq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \lambda_B &\geq \frac{L_W}{L_{B_1C_1} - L_{BC}} \end{aligned}$$

Therefore, the smallest value of λ_B which guarantees on average that waiting is the better strategy is $\frac{L_W}{L_{B_1C_1} - L_{BC}}$ and $\lambda_B \neq 0$.

1.2

Let π_0 be the policy that the robot drives directly down aisle B.

Let π_1 be the policy that the robot drives down aisle B, encounters an obstacle and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle, drives down aisle C.

Let π_3 be the policy that the robot drives directly down aisle C.

As $L(\pi_0) = L(\pi_3)$ and thus $c(L(\pi_0)) = c(L(\pi_3))$, we will not discuss π_0 here.

Then,

$$\begin{aligned} c(L(\pi_1)) &= c(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) \\ c(L(\pi_2)) &= c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG}) \\ c(L(\pi_3)) &= c(L_{IC_1} + L_{C_1C} + L_{CG}) \end{aligned}$$

If the robot decides to drive directly down aisle C, then:

$$c(L(\pi_3)) \leq c(L(\pi_1)) \tag{1}$$
$$c(L(\pi_3)) \leq c(L(\pi_2)) \tag{2}$$

by (1) - (2):

$$\begin{aligned} 0 &\leq c(L(\pi_1)) - c(L(\pi_2)) \\ 0 &\leq \mathbb{E}(L_{IB_1} + T_W \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) - \mathbb{E}(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG}) \\ 0 &\leq \mathbb{E}(T_W) \cdot L_W + L_{BC} - L_{B_1C_1} \\ \mathbb{E}(T_W) &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \mathbb{E}(T) &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \frac{1}{\lambda_B} &\geq \frac{L_{B_1C_1} - L_{BC}}{L_W} \\ \lambda_B &\leq \frac{L_W}{L_{B_1C_1} - L_{BC}} \end{aligned}$$

Therefore, the maximum value of λ_B at which the robot will decide to drive directly down C and not attempt to drive down aisle B is $\frac{L_W}{L_{B_1C_1}-L_{BC}}$ and $\lambda_B \neq 0$.

1.3

In this case, $T_W = p_B \cdot T + (1 - p_B) \cdot 0 = p_B \cdot T$.

As p_B is constant, $\mathbb{E}(T_W) = p_B \cdot \mathbb{E}(T) = p_B/\lambda_B$.

Let π_1 be the policy that the robot drives down aisle B, encounters an obstacle and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle, drives down aisle C.

Let π_3 be the policy that the robot drives directly down aisle C.

Since in this situation the robot attempts to drive down aisle B first:

$$c(L(\pi_3)) \geq c(L(\pi_1)) \tag{3}$$
$$c(L(\pi_3)) \geq c(L(\pi_2)) \tag{4}$$

Similarly as in the section 1.2, we can obtain that:

$$\mathbb{E}(T_W) \leq \frac{L_{B_1C_1} - L_{BC}}{L_W}$$

As $\mathbb{E}(T_W) = p_B/\lambda_B$, λ_B is a fixed value and $\lambda_B \neq 0$,

$$p_B/\lambda_B \leq \frac{L_{B_1C_1} - L_{BC}}{L_W}$$
$$p_B \leq \frac{\lambda_B(L_{B_1C_1} - L_{BC})}{L_W}$$

Therefore, when p_B is below $\frac{\lambda_B(L_{B_1C_1}-L_{BC})}{L_W}$, the robot will attempt to drive aisle B first.

1.4

Let π_1 be the policy that the robot drives down aisle B and waits.

Let π_2 be the policy that the robot drives down aisle B, encounters an obstacle O_B, and drives down aisle C.

Let π_3 be the policy that the robot drives down aisle B, encounters an obstacle O_B, drives down aisle C, and waits.

Let π_4 be the policy that the robot drives down aisle B, encounters an obstacle O_B, drives down aisle C, encounters an obstacle O_C, and drives down aisle D.

Let π_5 be the policy that the robot drives directly down aisle D.

Suppose D1 is a cell on the aisle D at the same horizontal level as B1 and C1.

Then,

$$
\begin{aligned}
c(L(\pi_1)) &= c(L_{IB_1} + T_{WB} \cdot L_W + L_{B_1B} + L_{BC} + L_{CG}) \\
c(L(\pi_2)) &= c(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG}) \\
c(L(\pi_3)) &= c(L_{IB_1} + L_{B_1C_1} + T_{WC} \cdot L_W + L_{C_1C} + L_{CG}) \\
c(L(\pi_4)) &= c(L_{IB_1} + L_{B_1C_1} + L_{C_1D_1} + L_{D_1D} + L_{DG}) \\
c(L(\pi_5)) &= c(L_{ID_1} + L_{D_1D} + L_{DG})
\end{aligned}
$$

And,

$$
\begin{aligned}
\mathbb{E}(T_{WB}) &= p_B/\lambda_B \\
\mathbb{E}(T_{WC}) &= p_C/\lambda_C
\end{aligned}
$$

We are looking to the path length, so if the robot drives directly down aisle D, it means:

$$
\begin{aligned}
L(\pi_5) &\leq L(\pi_1) && (5)\\
L(\pi_5) &\leq L(\pi_2) && (6)\\
L(\pi_5) &\leq L(\pi_3) && (7)\\
L(\pi_5) &\leq L(\pi_4) && (8)\\
& && (9)
\end{aligned}
$$

Sum up the four equation, we obtain that:

$$
\begin{aligned}
4L(\pi_5) &\leq 4L_{IB_1} + \mathbb{E}(T_{WB}) \cdot L_W + \mathbb{E}(T_{WC}) \cdot L_W + 3L_{B_1C_1} + L_{C_1D_1} + 4L_{B_1B} + L_{BC} + 3L_{CG} + L_{DG}\\
4L(\pi_5) &\leq 4L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot L_W + 3L_{B_1C_1} + L_{C_1D_1} + 4L_{B_1B} + L_{BC} + 3L_{CG} + L_{DG}\\
4L(\pi_5) &\leq 4L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot L_W + 4L_{B_1C_1} + 4L_{B_1B} + 4L_{CG}\\
L(\pi_5) &\leq L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot 0.25L_W + L_{B_1C_1} + L_{B_1B} + L_{CG}\\
L(\pi_5) &\leq L_{IB_1} + (p_B/\lambda_B + p_C/\lambda_C) \cdot 0.25L_W + L_{B_1C_1} + L_{C_1C} + L_{CG}
\end{aligned}
$$

Let L be the path length of $(L_{IB_1} + L_{B_1C_1} + L_{C_1C} + L_{CG})$

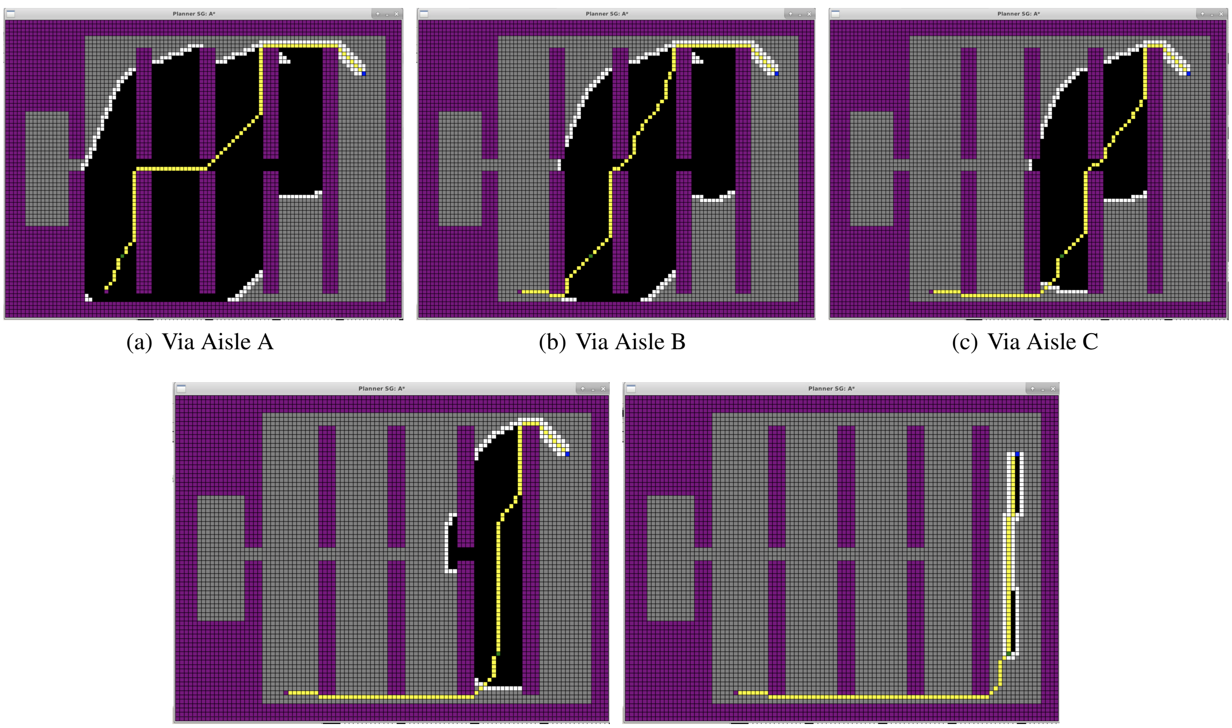
Therefore, the upper bound on the value of the path length of the path going down aisle is $L + 0.25L_W \cdot (p_B/\lambda_B + p_C/\lambda_C)$.

2 Implement System in ROS

2.1

We first add a function in the class which returns a cell coordinate as the intermediate destination, according to the aisle passed in. Then in the *planPathToGoalViaAisle()*, we call this function to get the intermediate cell coordinate and search a path from the given start cell to the intermediate cell. If there was a path then we extract and store this path and then search the second path which is from the intermediate cell to the given goal cell. If the second path existed, we extract it and link it to the first one using the provided function *addToEnd()*. At the end of this function we call the *searchGridDraw* to show the first path so that the two paths can be seen simultaneously.

The result is shown in Fig. 1. The start point is marked purple, the aisle cell is marked green, and the goal is marked blue.



(a) Via Aisle A (b) Via Aisle B (c) Via Aisle C

(d) Via Aisle D (e) Via Aisle E

Fig. 1. Planned Routes Down all the Different Initial Aisles

2.2

2.3

References

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