Physics 514 Computational Physics, Fall 2020

Homework 4: 2D Ising Model

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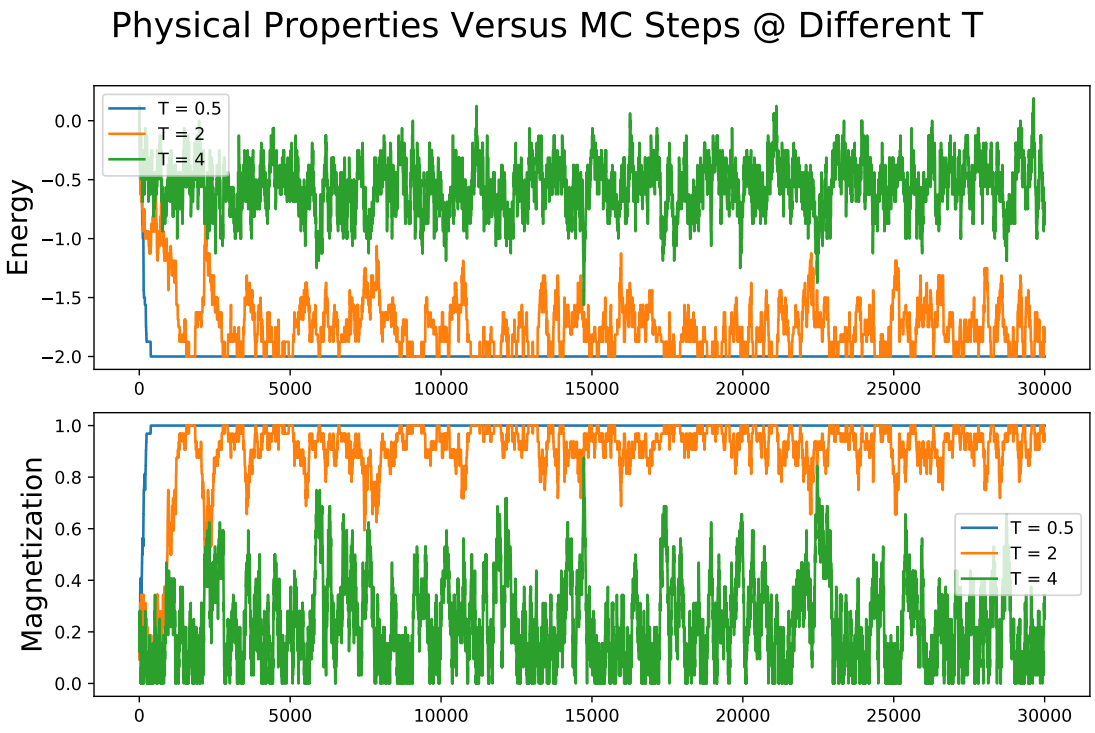
1 The Properties V.s MC steps

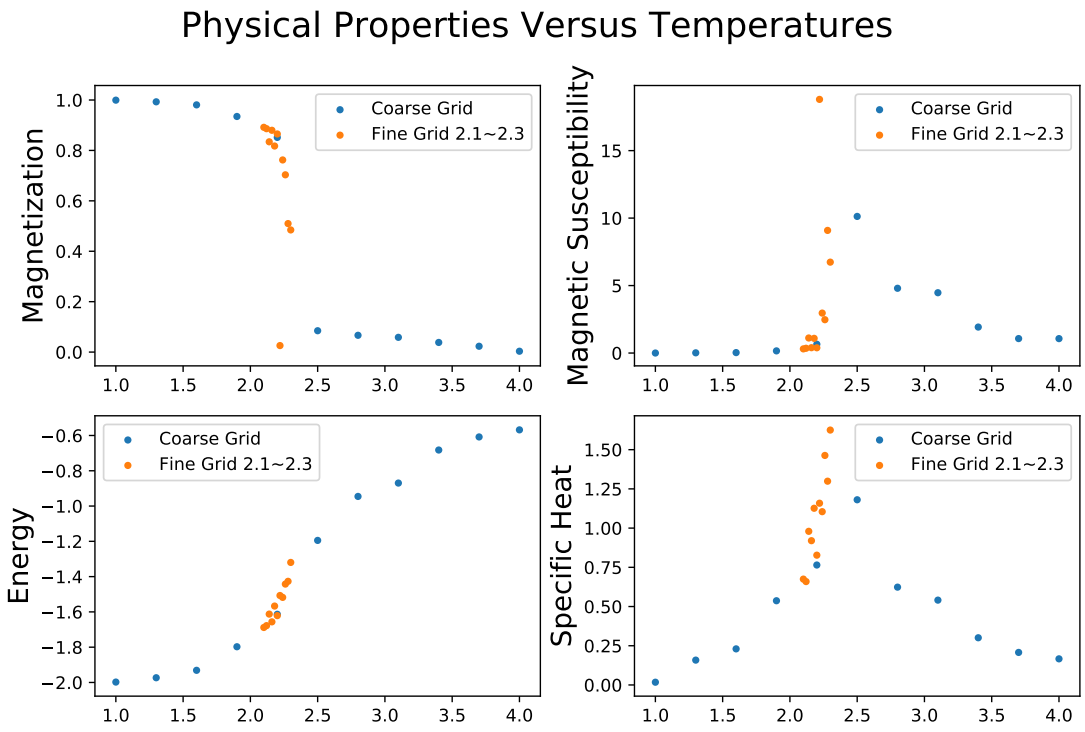


Here I plot the energy and magnetization versus the Monte Carlo steps at different T.

1. The $T = 0.5$ case converges quickly, while $T = 4$ case still oscillates at the end of the Monte Carlo steps. This is due to the fact that at the lower T, the configuration tends to reach the lowest energy configuration and stays there. In the single spin metropolis algorithm, we have probability $exp(-\Delta E/k_B T)$ switching to new configuration even it has higher energy, and the lower T makes this probability smaller. That is the reason we have quick convergence at lower T.

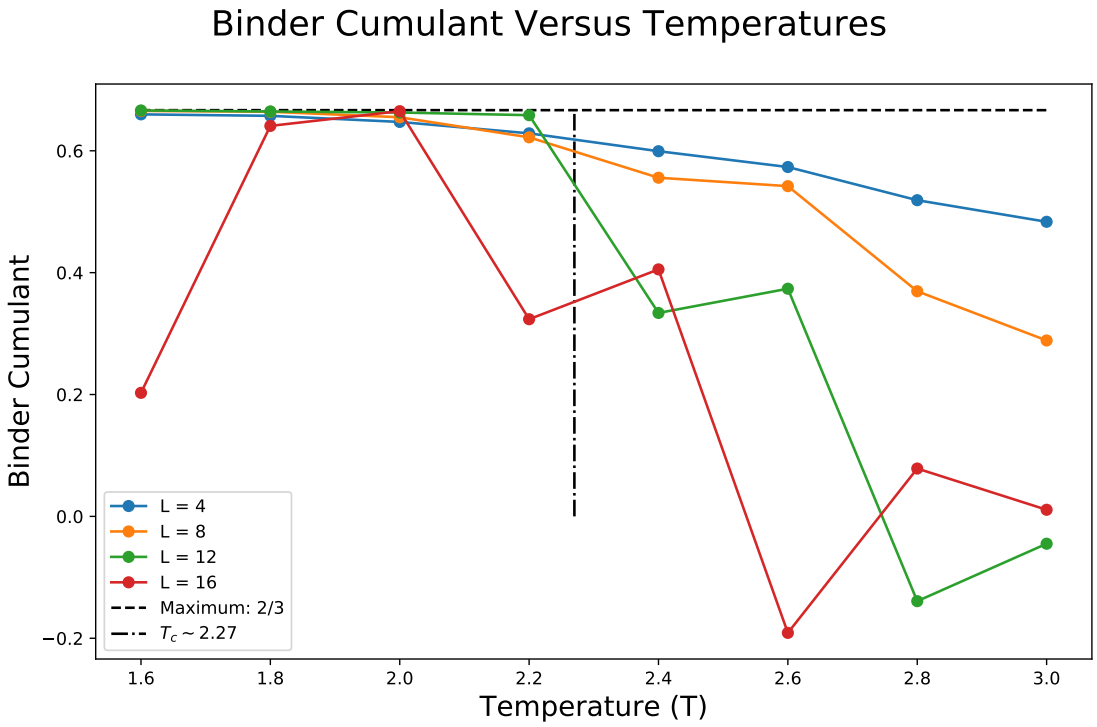
2. The $T = 0.5$ and 2 cases have lots of points at energy $= -2.0$ and magnetization $= 1.0$, while the $T = 4$ gets lots of points at energy $= -0.5$ and magnetization $= 0$. And in the next section, I will show this is related to the critical point.

2 The property V.s Temperature and the Phase Transition



Here I plot different properties versus the temperature. And I add more points between 2.1 to 2.3 in order to see the transition within this area. From these plots, we could clearly see the there are sudden changes in magnetization, magnetic susceptibility and specific heat around the $T \sim 2.2$. Especially, the magnetization almost drops to zero after $T \sim 2.5$. This is also what we expect.

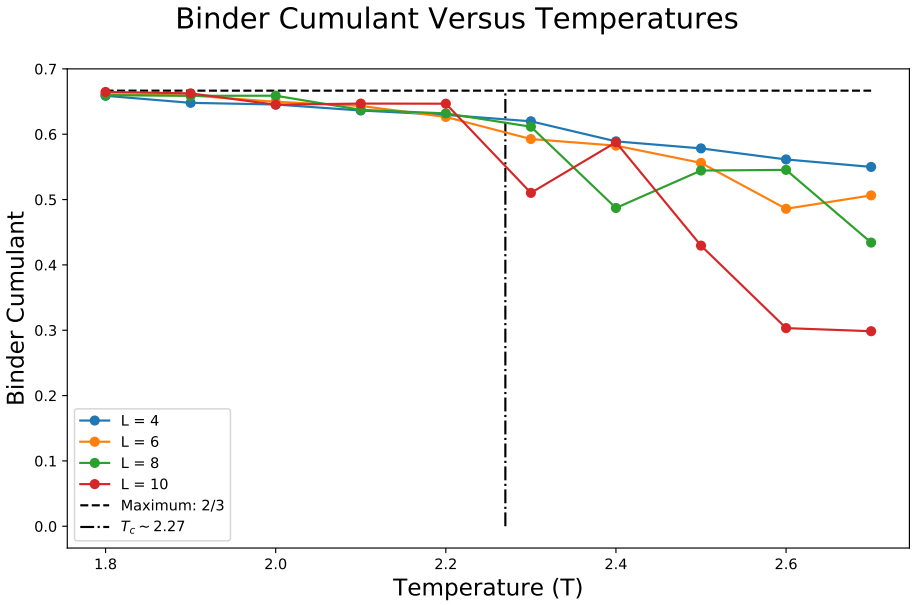
3 Binder Cumulant

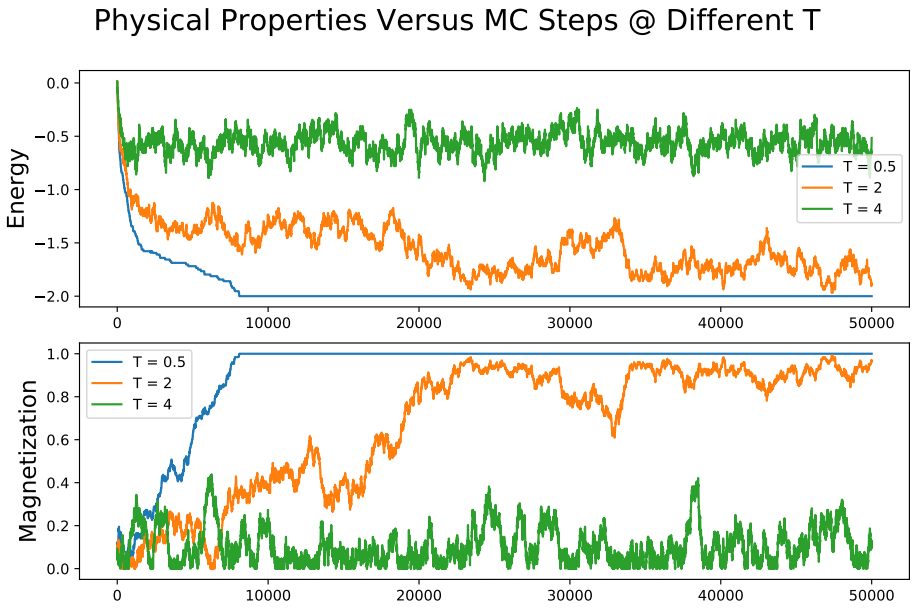


Here I plot the binder cumulant versus the temperature for different lattice size. Also, I indicate the maximum of the binder cumulant and the analytical answer to T_c of the 2D Ising model.

1. Although this plot does not show the exact intersection of all binder cumulant, especially for the $L = 16$ case, the initial value is not even $2/3$. This may due the fact that the thermalized steps are not enough.

2. The intersections are not perfect. This might caused by the relatively sparse intervals.





1. The surprising drop at the beginning of the $L = 16$ is caused be the overflow of the np.sum function($16 \times 16 = 2^8$) and the matrix I have for the spin is an integer matrix), and I solve them by specify the digits of the calculation.