EECS 541, Winter 2021

Homework 9: Quantum Teleportation: Building Blocks

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1 Phase Shifter, Beam Splitter, Hadamard Gate

For a phase shifter, we have relations,

$$P|0\rangle = |0\rangle, P|1\rangle = e^{i\Delta}|1\rangle$$

For a beam splitter. we have relations,

$$B = \exp[\theta(a^\dagger b - ab^\dagger)]$$

Also Pauli-Y and Pauli-Z matrices are listed here,

$$Y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(a) Show that the circuit in the slides transforms the dual-rail state by $|\psi_{\text{out}}\rangle = \begin{bmatrix} e^{-i\frac{\Delta}{2}} & 0 \\ 0 & e^{i\frac{\Delta}{2}} \end{bmatrix} |\psi_{\text{in}}\rangle$ Show also the equation $R_z(\theta) \equiv e^{-i\theta Z/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$ **Hints:**Write $|\psi_{\text{out}}\rangle$ and $|\psi_{\text{in}}\rangle$ in the single photon states for the calculation, then convert to the dual-rail states.

(b) Show that $BaB^\dagger = a\cos\theta + b\sin\theta$ and $BbB^\dagger = -a\sin\theta + b\cos\theta$ And the circuit for the beam splitter transform the dual state by $|\psi_{\text{out}}\rangle = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} |\psi_{\text{in}}\rangle$.show also the equation $R_y(\theta) \equiv e^{-i\theta Y/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Y = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$ **Hints:** Using the relation $e^{\lambda G} A e^{-\lambda G} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} C_n$ and $C_0 = A, C_1 = [G, C_0], C_2 = [G, C_1], C_3 = [G, C_2], \ldots,$ $C_n = [G, C_{n-1}]$ also $B|00\rangle = |00\rangle$

(c) Show the Hadamard gate in the slides transforms the dual-rail state $|01\rangle \to (|01\rangle + |10\rangle)/\sqrt{2}$ and $|10\rangle \to (|01\rangle - |10\rangle)/\sqrt{2}$ up to an overall phase