# Quantum Teleportation

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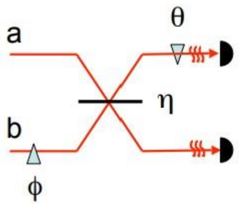
# Part 2: Experimental Setup(Optical) - Yunjie

1. Motivation
2. Initial State Preparation
3. Building Blocks
4. Hadamard Gate and Control-NOT Gate

# Motivation

### Why Photonics Qubit?

- Photons exhibit quantum phenomena
- They do not interact very strongly with each others, even with most matter
- They can be guided along long distances with low loss in optical fibers, delayed by phase shifters, combined by beamer splitters




Beam splitter and phase-shifter circuit for producing an arbitrary single qubit evolution on a spatial dual-rail qubit
Source: `https://arxiv.org/pdf/1103.6071.pdf`

### State Representation

- Usually, a state contains a superposition of one or zero photon, $|\Psi\rangle = a_0 |0\rangle + a_1 |1\rangle$
- **New!** Dual-Rail Representation: $|\Psi\rangle = a_0 |01\rangle + a_1 |10\rangle \equiv a_0 |\mathbf{0}\rangle + a_1 |\mathbf{1}\rangle$

  $|10\rangle$ One photon in mode 1, No photon in the mode 2

  $|01\rangle$ One photon in mode 2, No photon in the mode 1

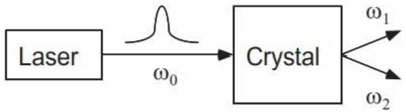
# Initial State Preparation

**Generate Single Photons**

- Attenuating the output of a laser, $|\alpha\rangle = e^{-\frac{|a|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$
- A coherent state could be attenuated to be a weaker coherent state
- For $\alpha = 0.1$, $e^{-\frac{|a|^2}{2}} \approx \sqrt{0.9}$,

  $\sqrt{0.9}\,|0\rangle + \sqrt{0.09}\,|1\rangle + \sqrt{0.002}\,|2\rangle + ...$

  The single photon with the probability over 95%




**Sychronicity between several single photon sources**

- Sending photon with frequency $\omega_0$ into a nonlinear optical medium to generate photon pair at frequncies at $\omega_1 + \omega_2 = \omega_0$
- When a photon $\omega_2$ is detected, the photon $\omega_1$ is known to exist
- Delaying the output appropriately, the synchronous multiple photons sources could be obtained

# Building Blocks – Phase Shifters

**Classical**

- A slab of transparent medium with index of refraction n different from that of free space, $n_0$
- A photon propagating through a phase shifter will experience a phase shift of $e^{i(n-n_0)L\omega/c_0}$ compared to a photon going the same distance through free space.

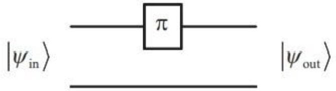
**Quantum**

- A phase shifter P acts just like normal time evolution, but at a different rate, and localized to only the modes going through it

$$P|0\rangle = |0\rangle, P|1\rangle = e^{i\Delta}|1\rangle$$

- The unitary transformation introduced by the phase shifters is $P = \exp(-iHL/c_0)$ and the Hamiltonian is $H = (n_0 - n)Z$
- For dual-rail states this transforms $c_0|01\rangle + c_1|10\rangle$ to $c_0 e^{-i\Delta/2}|01\rangle + c_1 e^{i\Delta/2}|10\rangle$

$$|\psi_{\text{out}}\rangle = \begin{bmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{bmatrix} |\psi_{\text{in}}\rangle$$




$$\begin{aligned} R_z(\theta) &\equiv e^{-i\theta Z/2} \\ &= \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2} Z \\ &= \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \end{aligned}$$

# Building Blocks – Beam Splitters

**Classical**

- A partially silvered piece of glass, which reflects a fraction $R$ of the incident light, and transmits $1-R$
- Define $R = \cos\theta$

$$\begin{aligned} a_{\text{out}} &= a_{\text{in}}\ \cos\theta + b_{\text{in}}\ \sin\theta \\ b_{\text{out}} &= b_{\text{in}}\ \cos\theta - a_{\text{in}}\ \sin\theta \end{aligned}$$

**Quantum**

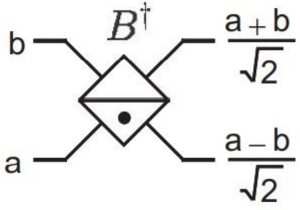
- The unitary transformation introduced by the beam splitters is $B = \exp[\theta(a^\dagger b - ab^\dagger)]$ and the Hamiltonian is $H = i\theta(-a^\dagger b + ab^\dagger)$
- For dual-rail states this transforms

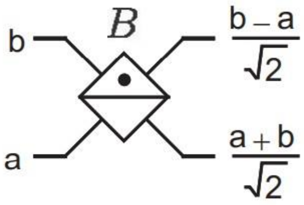
$$\begin{aligned} B|10\rangle &= \cos\theta|10\rangle - \sin\theta|01\rangle \\ B|01\rangle &= \cos\theta|01\rangle + \sin\theta|10\rangle \end{aligned}$$

$$BaB^\dagger = a\cos\theta + b\sin\theta \quad \text{and} \quad BbB^\dagger = -a\sin\theta + b\cos\theta$$

- In the matrix form, the operation B transform,

$$B = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = e^{i\theta Y}$$



# Building Blocks – Non-Linear Kerr Media

**Classical**

- The index of refraction n is proportional to the total intensity I of light going through i, $n(I) = n + n_2 I$
- two beams of light of equal intensity are nearly co-propagated through a Kerr medium, each beam will experience an extra phase shift of $e^{in_2 IL\omega/c_0}$ compared to what happens in the single beam case

**Quantum**

- The unitary transformation introduced by the beam splitters is $K = \exp(i\chi L a^\dagger a b^\dagger b)$ and the Hamiltonian is $H = \chi L a^\dagger a b^\dagger b$
- For single photon states this transforms

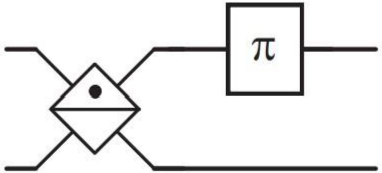
$$
\begin{aligned}
K|00\rangle &= |00\rangle \\
K|01\rangle &= |01\rangle \\
K|10\rangle &= |10\rangle \\
K|11\rangle &= e^{i\chi L}|11\rangle
\end{aligned}
$$

$$
\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{U_{CN}} = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{I\otimes H} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{K} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{I\otimes H}
$$

- Consider two dual-rail states, and take $\chi L = \pi$

$$|e_{00}\rangle = |1001\rangle, |e_{01}\rangle = |1010\rangle, |e_{10}\rangle = |0101\rangle, |e_{11}\rangle = |0110\rangle \rightarrow |e_{00}\rangle = |1001\rangle, |e_{01}\rangle = |1010\rangle, |e_{10}\rangle = |0101\rangle, |e_{11}\rangle = -|0110\rangle$$

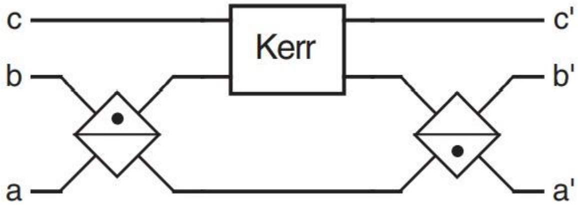
# (Optical) Hadamard Gate/ CNOT Gate




Hadamard gate on dual-rail single photon states,

$$|01\rangle \sim |\mathbf{0}\rangle \to (|01\rangle + |10\rangle)/\sqrt{2}$$
$$|10\rangle \sim |\mathbf{1}\rangle \to (|01\rangle - |10\rangle)/\sqrt{2}$$

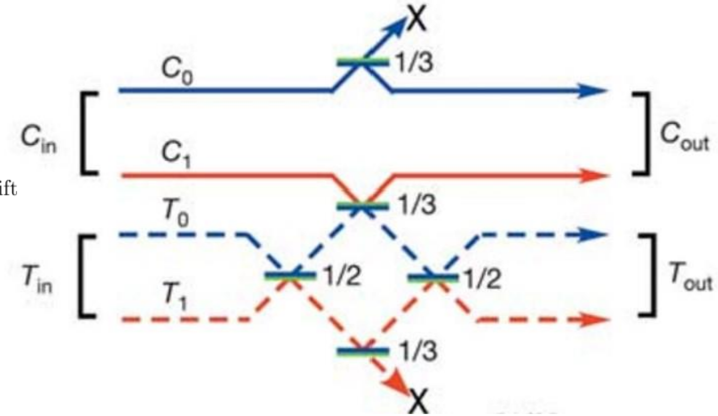
Create a superposition of the dual-rail states




For $\xi = \pi$,
When no photons are input at $c$, then $a' = a$ and $b' = b$,
When a single photon is input at $c$, then $a' = b$ and $b' = a$.

# Only Linear Optics?

Green side of the beam splitters reflects with a $\pi$ phase shift




Source: `https://arxiv.org/pdf/quant-ph/0403062.pdf`

- The gate works only when a single photon is detected on each qubit
  - $C_{\text{out}}$ and $T_{\text{out}}$ must be $|01\rangle$ or $|10\rangle$
- "When we do detect a single photon in each output which occurs with probability $\mathrm{P} = \frac{1}{9}$ we know that the CNOT operation has been correctly realised"