

Retirement Income Analysis

with scenario matrices

William F. Sharpe

20. Lockbox Spending

Lockboxes

Chapter 15 introduced lockboxes. The idea is to establish, at the present time, a series of such boxes – one for each future year in which spending is desired. In the most general version, each such box can contain TIPS maturing in the designated year, shares in the market portfolio of traded world bonds and stocks, and/or m-shares providing income in that year that will be a non-decreasing function of the cumulative return on the market portfolio from the present to the maturity year.

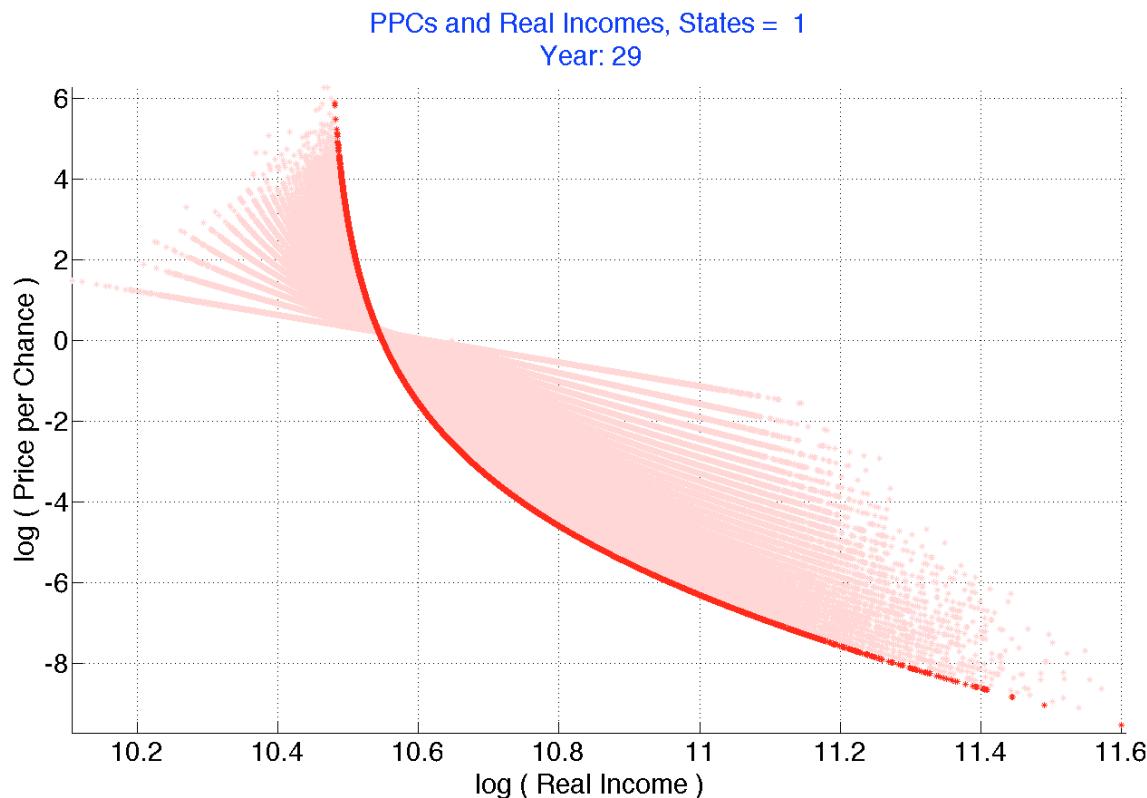
Since low-cost m-shares are not a reality at the time this is being written, we continue to focus only on lockboxes containing TIPS and/or shares in the market portfolio. More generally, we consider here only strategies that can be followed with currently available financial products.

Chapter 15 provided functions to create sets of such lockboxes. The first functions, *AMDnLockboxes_create* and *AMDnLockboxes_process*, are designed to produce distributions of income that are approximately the same as that provided by the market portfolio held for a specified number of years (beginning after the chosen base years). The second functions, *CMULockboxes_create* and *CMULockboxes_process*, provide distributions approximately consistent with the a constant implied marginal utility of incomes in each future year. The third functions, *combinedLockboxes_create* and *combinedLockboxes_process*, can be used to create lockboxes that are combinations of the first two types. The lockbox annuities analyzed in Chapter 16 utilized outputs from such functions, as will the approaches described in this chapter.

Before proceeding, it is useful to review the relationships between lockbox contents and the implied marginal utility functions introduced in chapter 9.

However created, each lockbox will initially contain TIPS and/or shares in the market portfolio. Accordingly, the distribution of income in the year the lockbox matures will be a combination of the ending value of the TIPS and/or the market portfolio. The value of the TIPS portfolio will be the same no matter what has happened to the market in the years since the lockbox was created. In a diagram with $\log(PPC)$ on the vertical axis and $\log(income)$ on the horizontal axis, this would plot as a vertical line. Holding only TIPS in a lockbox would thus be consistent with infinitely large risk-aversion. On the other hand, the value of the shares of the market portfolio in the box will be equal to the initial value invested times the cumulative return on that portfolio. In a diagram with $\log(PPC)$ on the vertical axis and $\log(income)$ on the horizontal axis, this would plot as a straight line with a slope equal to the market coefficient of relative risk aversion (our data element *market.b*). This part of income would thus be consistent with maximization of a utility function exhibiting constant relative risk aversion (CRRA). For any lockbox with both TIPS and shares in the market portfolio, the overall income produced when the box matures will be consistent with a utility function that exhibits *hyperbolic absolute risk aversion* (HARA), but not the special case of constant relative risk aversion.

Here is a graph from Chapter 15, showing the relationships between the logarithm of PPC and real income for a strategy designed to produce approximately the same real income distribution each year:



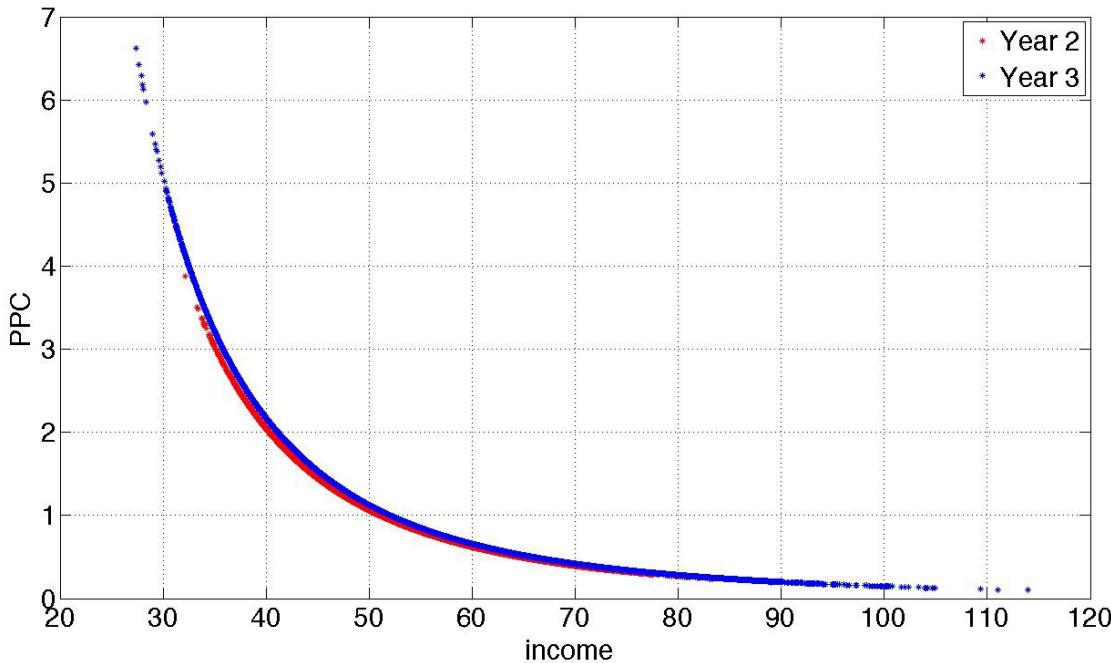
Consider, for example, the darkest curve, for year 29. As one moves to the left, the curve becomes steeper. If extended to even higher PPC values, it would move closer and closer to a vertical line showing the real income produced by the TIPS in the lockbox.. The overall result can thus be considered a combination of (a) TIPS, with infinite relative risk aversion, and (b) the market portfolio, with constant relative risk aversion. As we will see next, this interpretation proves helpful for considering mortality risk.

Mortality Adjustments

At the risk of overestimating the relevance of inferring utility functions from investment and spending choices, it is useful to consider possible adjustments to take mortality into account. We start with a very simple setting.

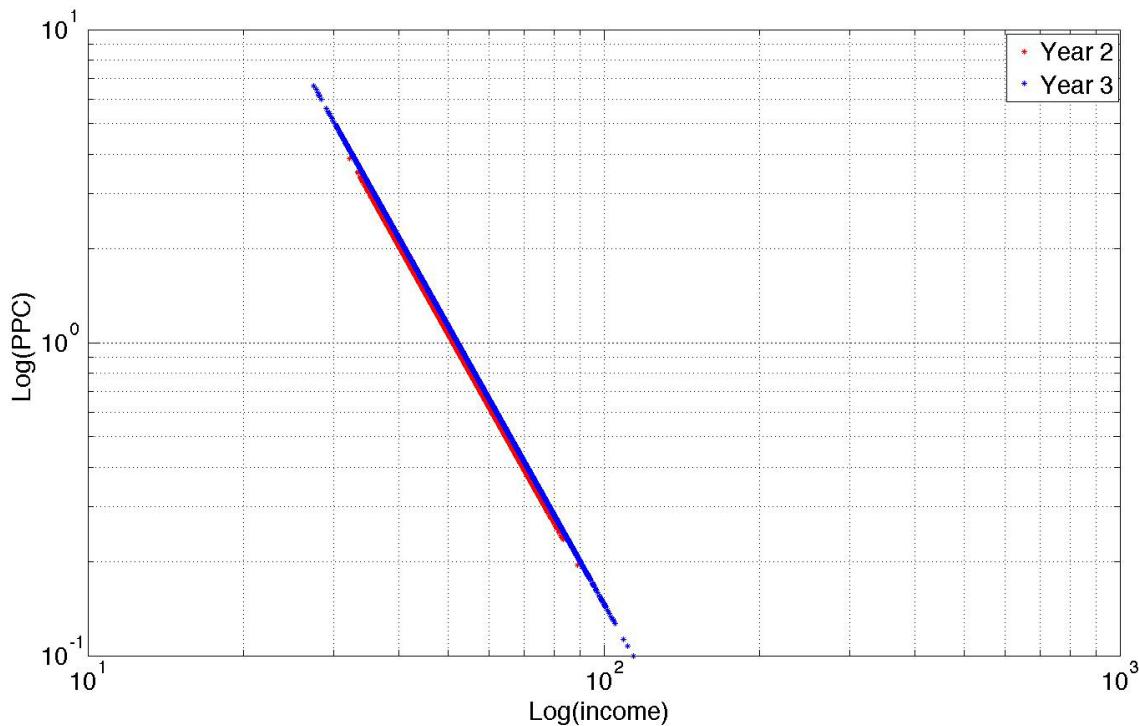
Assume that Jane Smith, a single retiree, has \$100,000 to invest. She is asked to consider a setting in which she will live long enough to enjoy income received at the beginning of year 2 and at the beginning of year 3 (and will then die at the end of the latter). After considerable deliberation, she has decided to put \$50,000 in a lockbox maturing in one year, the other \$50,000 in a lockbox maturing in two years, and to invest the money in each lockbox entirely in the market portfolio.

Here are the relationships between incomes and PPC values for the two years in a standard diagram with amounts (not logarithms) on each axis:



As can be seen, the plot for year 3 income lies slightly to the right and above that for year 2, but we presume that Jane knew this and found it the best choice. As with all market portfolios, for any year income is greater, the lower the cost (PPC). Looked at the other way, Jane has chosen to accept lower incomes for states in which income costs more, but the rate of decrease in income per unit increase in PPC is greater, the lower is income (that is, the curve becomes steeper as one moves from right to left).

Here is the same information, but with the logarithms of the two variables plotted:



As we know from earlier chapters, each curve plots as a straight line in this diagram, reflecting the constant elasticity (percent change in income per unit percent change in PPC) of any constant relative risk aversion function, which in our model applies to the market portfolio as a whole. And we know that the slope of each of these curves is equal to the element *market.b*, computed by the *market_process* function. Using our standard assumptions, this equals 2.9428, as reflected here.

Given the way in which Jane chose to invest, we make the heroic assumption that she has implicitly or explicitly maximized a utility function for future income. We thus interpret the graphs as showing properties of her marginal utility functions . In economists' jargon, these are her *revealed preferences*. As indicated in Chapter 9, we do not have numeric values for all the parameters of her utility functions (any positive linear transform would do), but we do know how she has chosen income in states of the world with different prices (costs) per chance.

We now change the story. Imagine that Jane has just returned from a medical examination which found that she has a 40% chance of dying just before the beginning of year 3, so that there is only a 60% probability that she will be alive to receive the contents of the second lockbox. Moreover, she has said that knowing she will be leaving wealth to heirs and/or charities provides her no satisfaction at all. In our terms, there is no utility associated with income for her estate.

Recall (from chapter 9) the first-order condition for an optimal strategy for providing income in a given year t :

$$\frac{\pi_{st} m(y_{st})}{p_{st}} = \lambda_t$$

The expression on the left is the expected marginal utility of income in a state divided by the price of a dollar of income in that state, which is in turn the probability of the state times the marginal utility of income if the state takes place. The goal is to select incomes for the states so that the expected marginal utility per dollar is the same in every state. Otherwise, it would be possible to rearrange incomes across states in a way that would cost the same but provide more expected utility.

As we know, rearranging this equation gives

$$m(y_{st}) = \lambda_t \frac{p_{st}}{\pi_{st}} = \lambda_t PPC_t$$

Which allows us to infer the characteristics of the marginal utility curve for a year from the relationship between price per chance (PPC) and income (y) for that year.

Now assume the probability that Jane will be alive in year t is π_{at} . The probability that Jane will obtain utility in a given state will now equal the probability that she is alive times the probability that the state will occur. And, since she derives no utility from income generated when she is dead, our first-order condition becomes:

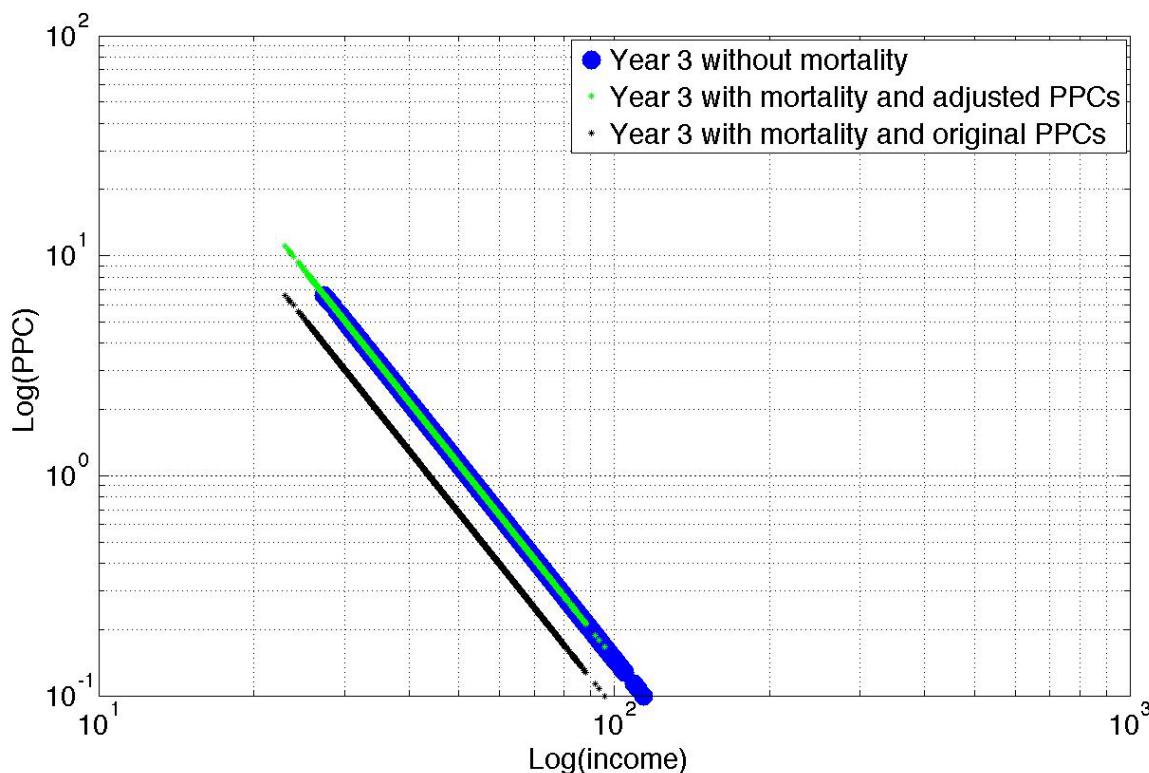
$$\frac{\pi_{at} \pi_{st} m(y_{st})}{p_{st}} = \lambda_t$$

Re-arranging and simplifying gives:

$$m(y_{st}) = \lambda_t \frac{PPC_t}{\pi_{at}}$$

When deciding on desired income for year 3, Jane thus should consider not the cost per chance for a state, but rather the cost per chance divided by the probability that she will be alive in that state. In our example, π_{at} for year 3 is 0.60, so income in any state is $1/0.60$ (roughly 1.67) times as expensive as it would be otherwise, and Jane should choose her incomes accordingly.

The figure below provides an illustration. The blue points show incomes that would be optimal in year 3 if it were guaranteed that Jane would be alive. These values are the same as in the previous figure but the points are plotted with larger dots for contrast. The green points show the incomes that would be optimal taking mortality into account. In this case the values for the vertical axis are the adjusted PPC's (1.67 times the actual PPC's), reflecting the higher cost per chance when both market probability and the probability of being alive are considered.

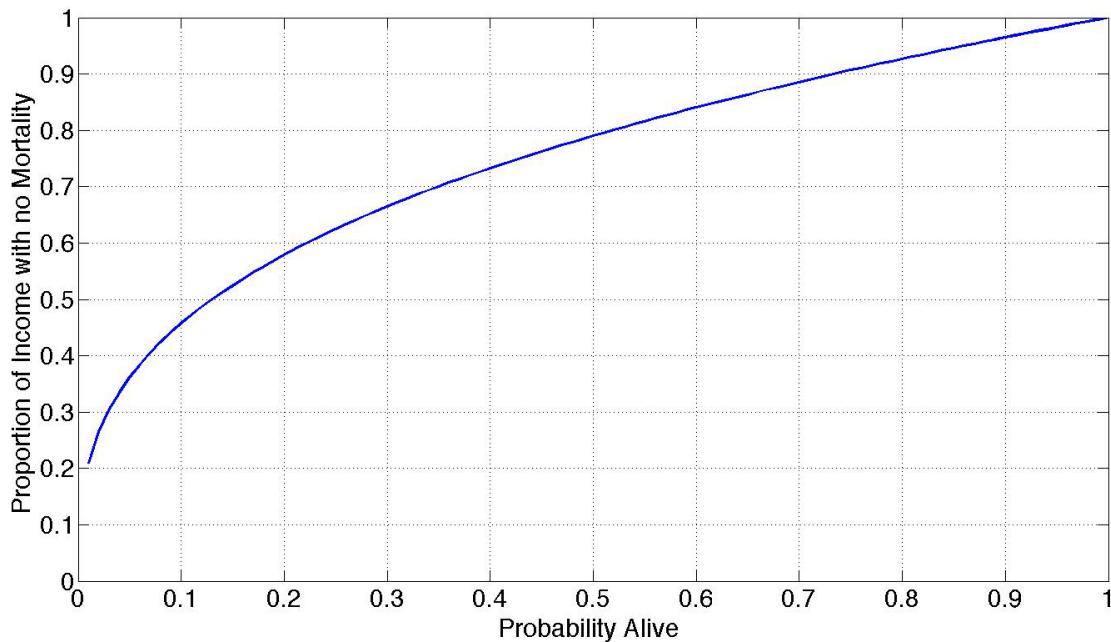


The black points in the graph plot the relationship between Jane's incomes and the original PPC values for the states. As can be seen, the black points fall on a line that lies to the left of that for the blue points parallel to it. And, since income is plotted on a logarithmic scale, this shows that in each state she chooses to reduce income by a given percentage to take mortality into account. Letting the mortality-adjusted income in a state be y'_s and the original income y_s , the formula for the adjustment is:

$$\frac{y'_s}{y_s} = e^{\ln(\frac{\pi_{al}}{b})}$$

Since Jane has a 0.60 probability of being alive in year 3 and the value of b for the market is 2.9428, Jane should choose to obtain roughly 84% as much income in each state as she would if she were certain to live to enjoy income in that year.

The figure below shows the relationship between the probability of being alive (on the horizontal axis) and the proportion of income to be received relative to the amount that would be chosen were there no mortality (on the vertical axis). Note that the income ratio is higher than the mortality ratio in every case. The reduction in income in each state is less than the chance of not being able to enjoy it.



We are almost done helping Jane, but one step remains. Recall that she has \$100,000 to invest. She indicated that, absent mortality, she would put \$50,000 in the market to be spent in year 2 and \$50,000 to be spent in year 3. But we have seen that when mortality is taken into account she should invest \$50,000 for year 2 and \$42,015 ($0.8406 * \$50,000$) for year 3. The total cost for this plan would be only \$92,015. The proportions invested for the two years would thus be $50,000/92,015$ and $42,015/92,105$, that is 0.5434 and 0.4566. Thus Jane should invest 54.34% of her money in the lockbox for year 2 and 45.66% in the lockbox for year 3.

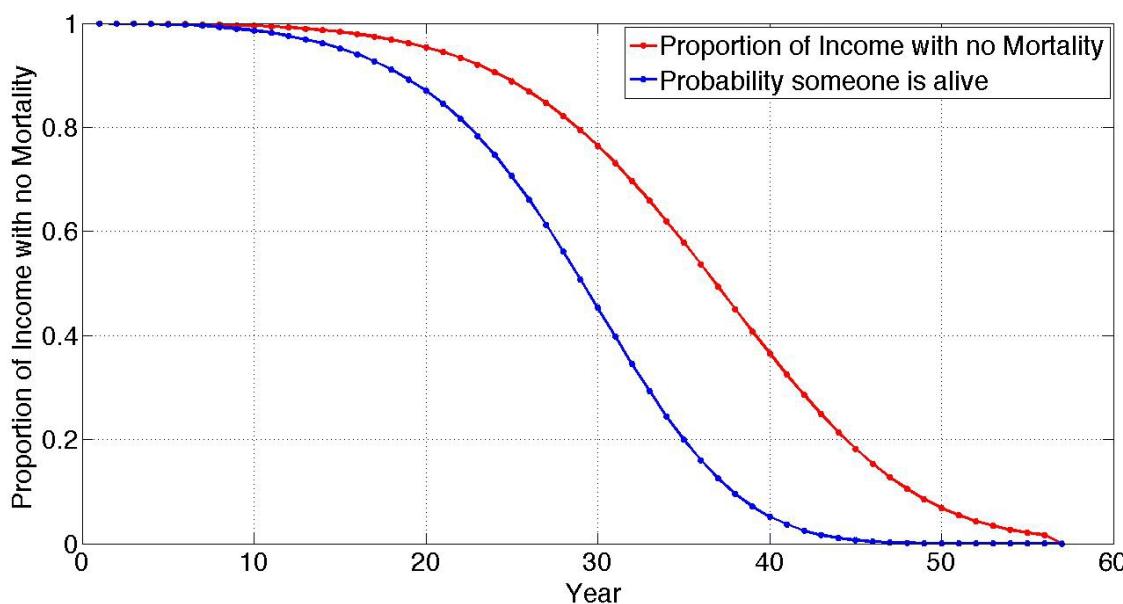
Before generalizing to more complex cases, it is useful to review the approach taken here. We initially asked Jane to assume she would be certain to live for a given number of years and to construct lockboxes that would be best for her in that situation. Next we asked how much satisfaction she would get by knowing that if she did indeed die within that period, some money would be provided for her estate. Her answer to the latter was “none”. We then estimated the probability that she would die before the end of the horizon. Based on these three sets of information, we advised her to choose a different allocation of funds among the lockboxes.

Our next task is to generalize this approach to cover multiple years and personal states.

Bequest Motives

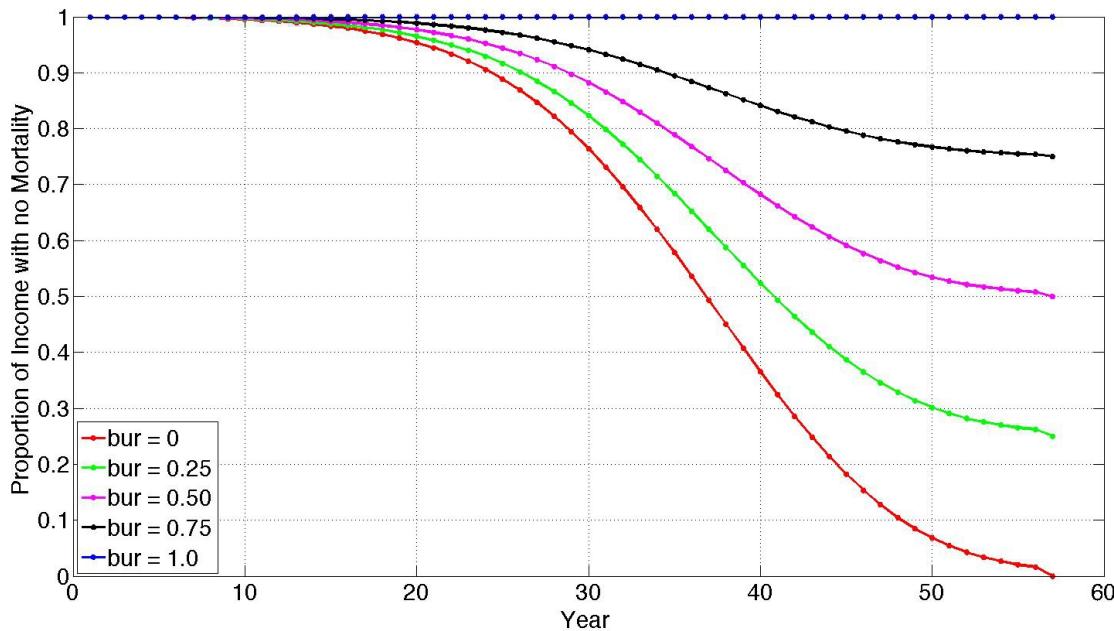
Mirriam-Webster defines *bequest* as “the act of giving or leaving something by will.” In our world, this is represented a positive value for income in personal state 4 (the first year in a scenario in which neither of a pair of retirees is alive). Jane had no such motive, but many people do. We now expand the approach taken with Jane to (1) cases with multiple future years and (2) retirees that may take some pleasure from knowing that there could be money left for their heirs (people, charities, etc.).

In our previous example, Jane could have died before year 3 began, thus leaving a bequest. But she said that knowing that a bequest might be left would give her no satisfaction (utility), hence we assigned no utility to such payments. However, others might feel differently. For many retirees, thinking about the possibility of leaving an estate to individuals and/or organizations at some future date provides satisfaction today. Consider Bob and sue Smith. In the diagram below, the blue curve plots the probability that one or both will be alive in each future year. Applying the formula derived in the previous section, which assumes that no utility is associated with money left to an estate, gives the points on the red curve. If this truly reflects Bob and Sue's feelings, the amounts in the lockboxes invested in the market portfolio should be adjusted using these factors and the resulting proportions scaled to sum to the initial amount invested in the market.



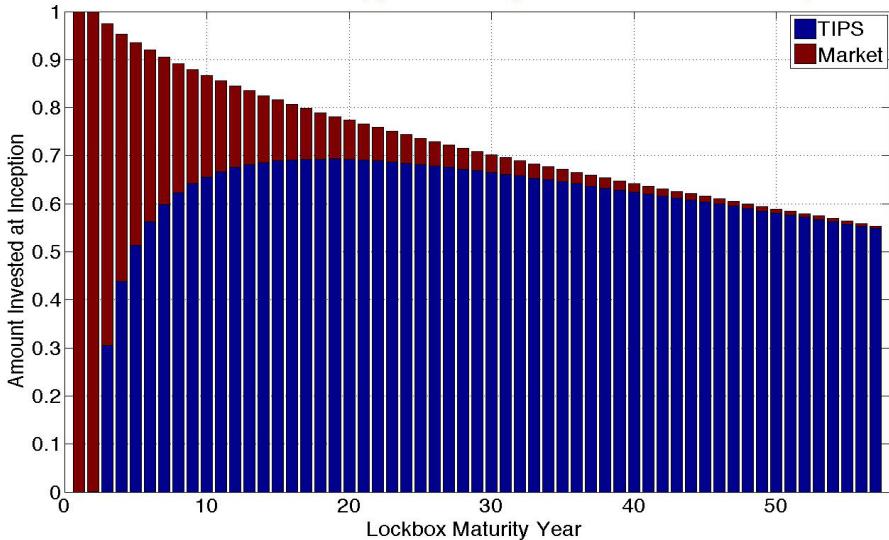
Consider now the other extreme case, in which Bob and Sue consider money left to an estate as just as desirable as an equal amount spent at the time. In this case, the original lockbox proportions in the market portfolio should be used. In effect, the adjustment proportions lie on a horizontal curve at the top of the diagram.

For most retirees, preferences fall between these two extremes. A simple (perhaps simplistic) way to measure such attitudes is to make adjustments using a weighted average of the horizontal line at 1.0 and the red line. We will call the weight the *bequest utility ratio (bur)*. The diagram below shows the proportions of the original lockboxes to the used for five values of this ratio, based on the Smith's mortality projections.

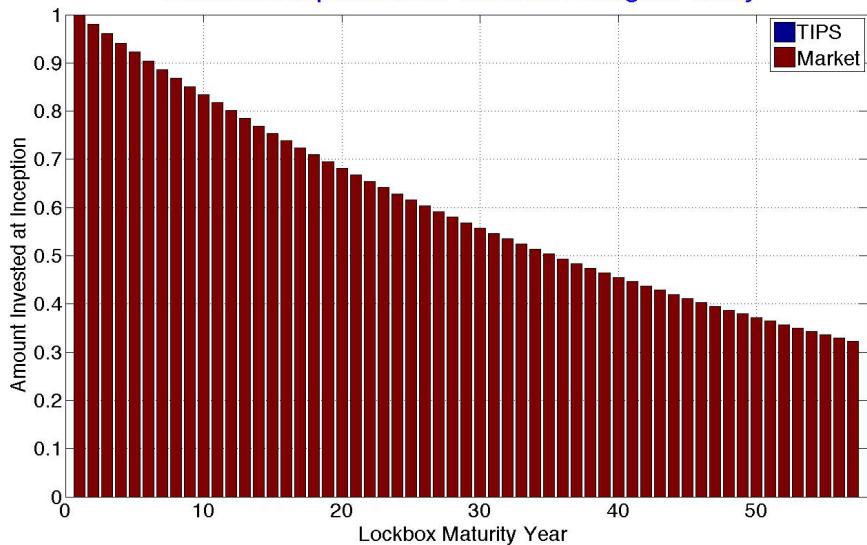


To see the possible effects of such adjustments, we consider three cases, using the lockbox functions described in Chapter 15. As discussed there, the first function is designed to provide lockbox incomes in every year after year 2 that approximate the market distribution in year 2. The second provides incomes consistent with a constant marginal utility in each year based on investment entirely in the market portfolio. The third is a composite derived by taking equal weights of the first two proportions. The contents of the three sets of lockboxes are shown in the diagrams on the next page.

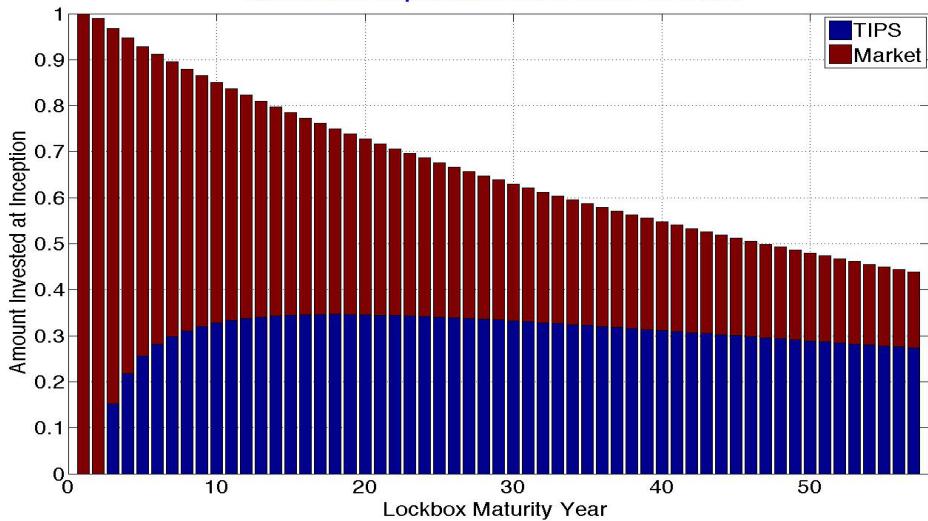
Lockbox Contents for Approximating Market Distribution in year 2



Lockbox Proportions for Constant Marginal Utility



Lockbox Proportions for AMD2 & CMU



The key observation is that the amounts originally invested in the market portfolio differ radically across the three cases. In the first, relatively little is invested in it after the initial years; thus any reduction in the portions invested in the market in the later years will have at best a minor effect on the distribution of incomes across scenarios with different personal states. In the second case, all assets are invested in the market portfolio; hence the effects of adjustments in market holdings to take mortality into account could be substantial. The third case falls between the extremes.

The table below shows the effects of adjustments for the Smiths based on different bequest utility ratios and the three alternative lockbox investments. For each type, we show the percentage of the present value going to the estate for two alternative values of the bequest utility ratio: 1.0 (a bequest is as desirable as income for living beneficiaries), and 0.0 (there is no utility associated with the possibility of a bequest). The final column shows ratio of the percentages in the previous columns.

Lockboxes	% Estate Bur = 1.0	% Estate Bur = 0.0	%Estate bur0/bur1
AMD2	43.9%	43.4%	0.989
CMU	37.8%	20.3%	0.537
.5*AMD2 + .5*CMU	41.2%	32.9%	0.799

The table shows that in every case, the present value of the possible payments to the estate is a substantial portion of the total (between 20% and 44%). The Smith's children and charities should be delighted if Bob and Sue adopt one of these lockbox strategies. If the Smiths really do consider a bequest just as desirable as income spent when they are alive (bur = 1.0), so be it. But if not, this type of adjustment ($0.0 \leq \text{bur} < 1.0$) can do only so much to allocate the possible payments from bequests to income, with the magnitude of the change dependent on the chosen lockbox strategy.

Of course these percentages refer only to incomes from the lockbox spending strategy. Had we included the Smith's social security incomes (which in this example have a present value similar to that invested in lockboxes), the percentages in columns 2 and 3 would have been roughly half as large.

Combining Lockbox Spending with a Fixed Annuity

Retirees who desire (or need) to use their savings primarily or entirely to provide income while they are alive, face a dilemma. Either run the risk of a long life with insufficient income in the latter years or spend less when they are likely to be alive in order to have money left for years in which they could well be gone. Adjusting planned incomes over time may help, but for those with modest savings, it maybe an insufficient solution.

In an ideal world, low-cost *lockbox annuities* of the type described in Chapter 16 would provide a solution to this problem. But at the time this is written no such instruments exist. There are two interesting alternatives. The first would combine lockbox spending for some number (n) of years in which one or both beneficiaries are highly likely to be alive with the immediate purchase of a deferred fixed income annuity that would provide income for the later years ($n+1$ and thereafter). The second alternative would also use a set of lockboxes to provide spending for n years but would add another lockbox, to be used in year $n+1$ to purchase a fixed annuity (or to pay the estate then or in an earlier year if both beneficiaries die beforehand). We will provide for each of these approaches later in the chapter. First, we provide functions for lockbox spending.

Creating an iLockboxSpending Data Structure

Relatively few elements are needed for lockbox spending. Here is the function to create a data structure for the approach.

```
function iLockboxSpending = iLockboxSpending_create( );
% create a lockbox spending data structure

% amount invested
iLockboxSpending.investedAmount = 100000;

% relative payments from lockboxes: size(2,client number of years)
% row 1: tips
% row 2: market portfolio
% may be provided by AMDnLockboxes.proportions, CMULockboxes.proportions,
% combinedLockboxes.proportions or otherwise
% note: lockboxes are to be spent for personal states 1,2,3 or 4
iLockboxSpending.lockboxProportions = [ ];

% bequest utility ratio
% ratio of utility per dollar for bequest versus spending
% note: this applies equally for personal states 1,2 and 3
iLockboxSpending.bequestUtilityRatio = 0.50;

% show adjusted lockbox amounts (y or n)
iLockboxSpending.showLockboxAmounts = 'y';

end
```

The amount to be invested is, as usual, a dollar value. The *lockbox proportions* element is to be filled with a matrix created by one of the three lockbox functions described earlier or with some other matrix with proportions invested in TIPS in the first row and proportions invested in the market in the second row. As before, each column indicates the proportions to be invested for income at the beginning of the associated year. For generality, the number of columns may be less than the number of years that the clients may live. If so, the proportions for the remaining years are assumed to be zero. The next element indicates the desired bequest utility ratio. The final element indicates whether or not it is desired to show the actual dollar amounts to be invested in each of the lockboxes.

As indicated in the comment lines, the final value of the amount in any given lockbox will be spent in the year for which it was intended as long as any beneficiary is alive (personal states 1, 2 or 3). In the event that both die before then, the remaining value at the time will be paid to the estate. We make no attempt to adjust the payments when only one of the beneficiaries is alive.

Processing an *iLockboxSpending* Data Structure

Processing an *iLockboxSpending* data structure is relatively straightforward. Since we need to compute the actual dollar amounts to be invested in each lockbox, the function returns revised versions of both the *client* data structure and the *iLockboxSpending* data structure. The beginning and end of the function are:

```
function [client,iLockboxSpending] = iLockboxSpending_process(iLockboxSpending, client, market);
% creates LB spending income matrix and fees matrix
% then adds values to client incomes matrix and fees matrices

% the lockbox proportions matrix can be computed by AMDnLockboxes_process
% or in some other manner. The first row is TIPS, the second is Market
% proportions, and there is a column for each year in the client matrix

.....
end
```

The first set of instructions gets the size of the matrices (number of scenarios and number of years), then, if needed, adjusts the lockbox proportions matrix to have as many columns as required:

```
% get number of scenarios and years
[ nsцен nyrs ] = size( client.pStatesM );

% fill lockbox proportions with zeros if needed
props = iLockboxSpending.lockboxProportions;
nlbyears = size( props, 2 );
props = [ props( :, 1:nlbyears ) zeros( 2, nyrs-nlbyears ) ];
if size( props, 2 ) > nyrs
    props = props( :, 1:nyrs );
end;
```

Next, vectors of survival rates are computed from the mortality tables in the client data structure:

```
% compute survival rates
surv1 = cumprod( 1 - client.mortP1 );
surv2 = cumprod( 1 - client.mortP2 );
survboth = surv1 .* surv2;
surv1only = surv1 .* (1-surv2);
surv2only = surv2 .* (1-surv1);
survanyone = survboth + surv1only + surv2only;
```

This information is utilized to compute new lockbox proportions, taking the bequest utility ratio into account. The adjusted proportions are then added to the *iLockboxSpending* data structure:

```
% adjust proportions to take bequest utility ratio into account
% adjust market lockbox values
ranyoneV = exp( log(survanyone) / market.b );
rmaxV = ones( 1, nyrs );
bur = iLockboxSpending.bequestUtilityRatio;
ratioV = bur*rmaxV + (1-bur)*ranyoneV;
% change market proportions to keep total the same
oldsum = sum( props( 2, : ) );
newmktprops = ratioV .* props( 2, : );
newsum = sum( newmktprops );
newmktprops = ( newmktprops / newsum ) * oldsum;
newprops =[ props(1,:) ; newmktprops ];
% save new proportions
iLockboxSpending.adjustedLockboxProportions = newprops;
```

It is then possible to compute the actual dollar amounts in the lockboxes, based on the adjusted proportions and the amount to be invested:

```
% compute lockbox dollar values
LBVals =( newprops / sum(sum(newprops)) ) * iLockboxSpending.investedAmount;
```

If requested, the dollar amounts invested in the lockboxes are then shown in a stacked bar chart:

```
% plot lockbox amounts if requested
if lower( iLockboxSpending.showLockboxAmounts ) == 'y'
    xs = LBVals;
    nyrs = size( xs, 2 );
    fig = figure;
    x = 1:1:size( xs, 2 );
    bar( x , xs', 'stacked' ); grid;
    set( gca, 'FontSize', 30 );
    ss = client.figurePosition;
    set( gcf, 'Position', ss );
    set( gcf, 'Color', [1 1 1] );
    xlabel( 'Lockbox Maturity Year ', 'fontsize', 30 );
    ylabel( 'Amount Invested at Inception ', 'fontsize', 30 );
    legend('TIPS ','Market ');
    ax = axis; ax(1) = 0; ax(2) = nyrs+1; ax(3) = 0; ax(4) = max(sum(xs)); axis(ax);
    t = [ 'Lockbox Amounts at Inception ' ];
    title( t, 'Fontsize', 40, 'Color', 'b' );
    beep; pause;
end; %if lower(combinedLockboxes.showContents) = 'y'
```

It remains to compute the incomes for each year and scenario, then post them to the client incomes matrix.

We begin by creating an income matrix with all zeros. Then we fill it, year by year. For each year we provide incomes separately for states in which anyone is alive and those in which the estate is to be paid. If someone is alive, we compute the current values of the Tips and market holdings in the lockbox for that year, taking cumulative Tips and market returns into account, then add the results to the appropriate cells in the income matrix. If the estate is to be paid, we cumulate the initial values of Tips and market holdings in the lockbox for that year plus the initial values of all lockboxes for subsequent years, determine the current values of the total amounts, then add the results to the cells in the income matrix:

```
% create incomes
incsM = zeros( nscen, nyrs );
for yr = 1:nyrs
    % scenarios with anyone alive
    ii = find( (client.pStatesM(:,yr)>0) & ( client.pStatesM(:,yr)<4 ) );
    % add cumulative value of tips
    incsM(ii,yr) = LBVals(1,yr) * market.cumRfsM(ii,yr);
    % add cumulative value of market
    incsM(ii,yr) = incsM(ii,yr) + LBVals(2,yr) * market.cumRmsM(ii,yr);
    % scenarios with estate
    ii = find( client.pStatesM(:,yr) == 4 );
    % values of current and remaining lockboxes
    m = sum( LBVals( :, yr:nyrs ), 2 );
    % add cumulative values of tips
    incsM(ii,yr) = m(1) * market.cumRfsM(ii,yr);
    % add cumulative value of market
    incsM(ii,yr) = incsM(ii,yr) + m(2)*market.cumRmsM(ii,yr);
end; % for yr = 1:nyrs
```

Finally, we add the incomes computed for this strategy to those previously in the client incomes matrix, providing a revised client data structure, which will be returned when the function has completed its work:

```
% add incomes to client incomes matrix
client.incomesM = client.incomesM + incsM;
```

Lockbox Spending plus Immediate Purchase of a Deferred Fixed Annuity

Some retirees will prefer to receive retirement income from multiple sources. Anyone with social security benefits plus some retirement savings will have at least two such sources of income. But many will also choose to use their discretionary savings to obtain income from two or more investment vehicles or other income sources. If so, decisions must be made concerning the amounts to invest in each vehicle.

A useful way to approach such a decision is to consider investing equal amounts in the sources under consideration. Each will provide a real income matrix. Next, divide each such matrix by the amount invested, giving the income per dollar invested in each scenario and personal state. If, for each source, incomes are proportional to initial investments, then one can compare the two income-per-dollar matrices and find desirable relative amounts to invest in the sources based on some metric of choice.

This is, of course, a highly abstract and conditional description. Here we provide a concrete example. Consider the following case. Bob and Sue have saved \$1,000,000 and wish to allocate it between a set of lockboxes to provide income for the next 20 years and a deferred fixed annuity to provide income thereafter. They realize that the lockbox for year 20 will provide a distribution of real income in that year, and that the deferred fixed annuity will provide a fixed amount of real income for year 21 and each year thereafter. Their goal is to have the annuity income in year 21 equal to some predetermined value from the probability distribution of incomes for year 20 from the lockbox spending strategy. How much should they invest in each approach?

The answer could be provided with a set of statements in the script for the Smiths (e.g. *SmithCase.m*). But this particular combination is likely to be of sufficient interest to warrant a separate set of functions that in turn use the functions developed for the two income sources.

Since the name *iLockboxSpendingPlusDeferredFixedAnnuity* is overly long, we will use the abbreviation *iLBSplusDFA*.

Here is the *iLBSplusDFA_create()* function:

```
function iLBSplusDFA = iLBSplusDFA_create()
% creates a data structure for a combination of lockbox spending
% and a deferred fixed annuity

% lockbox proportions (matrix with TIPS in top row, market in bottom row)
iLBSplusDFA.lockboxProportions = [ ];

% number of years of lockbox income
iLBSplusDFA.numberOfLockboxYears = 20;

% lockbox bequest utility ratio
iLBSplusDFA.bequestUtilityRatio = 0.50;

% percentile of last lockbox year income distribution for fixed annuity
% 100=lowest income; 50=median income, 0=highest income
iLBSplusDFA.percentileOfLastLockboxYear = 50;

% fixed annuity ratio of value to initial cost
iLBSplusDFA.annuityValueOverCost = 0.90;

% total amount invested
iLBSplusDFA.amountInvested = 100000;

end
```

The first data element is to be assigned a set of lockbox proportions in the usual format (as a matrix with TIPS proportions in the top row and market proportions in the bottom row). The next two elements specify the number of years income is to received from the lockboxes and the bequest utility ratio to be applied when revising the initial lockbox proportions.

The next element specifies the percentile of the distribution of real incomes from the final lockbox that is to be used to determine the real income from the deferred annuity. A value of 50 would indicate that the annuity income is to equal the median of the distribution of incomes from the final lockbox. A lower percentile will create a deferred annuity with more income, and a higher percentile will create an annuity with less income.

The next element indicates the ratio of the value of the annuity to its cost; and the final element specifies the total amount to be invested in the lockboxes plus the deferred annuity.

The *iLBSplusDFA_process()* function uses other income source functions. The goal is to first determine the allocation of funds between lockbox spending and the deferred annuity, then use the resulting amounts to produce income from the two sources.

The beginning and end of the function are:

```
function [client, iLBSplusDFA] = iLBSplusDFA_process(client, iLBSplusDFA, market );
% process lockbox spending plus deferred fixed annuity
.....
end
```

We provide the function with data structures for the client and the market plus one with parameters for the combined income source. The function returns a revised version of the client data structure with the new information added to the income and fees matrices. It also returns a revised version of the *iLBSplusDFA* data structure with the amounts invested in each of the two income sources.

The first section creates a deferred fixed real annuity with half the total amount for the combined strategy invested. A temporary version of the client with no prior incomes is used for the sole purpose of finding the real income in each year per dollar invested (in the last statement).

```
% create deferred fixed annuity with cost equal to 50% of total
iFixedAnnuity = iFixedAnnuity_create( );
% set deferral period
nLByrs = iLBplusDFA.numberOfLockboxYears;
iFixedAnnuity.guaranteedIncomes = zeros( 1, nLByrs );
% set relative incomes equal for personal states 1,2 and 3
iFixedAnnuity.pStateIncomes = [ 0 1 1 1 0 ];
% set incomes constant
iFixedAnnuity.graduationRatio = 1.00;
% set type of income to real;
iFixedAnnuity.realOrNominal = 'r';
% set ratio of value to initial cost
iFixedAnnuity.valueOverCost = iLBplusDFA.annuityValueOverCost;
% cost
iFixedAnnuity.cost = 0.50 * iLBplusDFA.amountInvested;
% create a temporary client with zero incomes
clientTemp = client;
[nscen nyrs] = size( clientTemp.incomesM );
clientTemp.incomesM = zeros( nscen, nyrs );
% process deferred fixed annuity with temporary client
clientTemp = iFixedAnnuity_process( iFixedAnnuity, clientTemp, market );
% find annuity real income per dollar invested
annuityIncomePerDollar = max( max(clientTemp.incomesM) ) / iFixedAnnuity.cost;
```

The next section does the same for the lockbox spending income, with the goal of determining the desired percentile of real income in the last year with lockbox incomes:

```
% create lockbox spending with cost equal to 50% of total
iLockboxSpending = iLockboxSpending_create( );
% set lockbox proportions for selected number of years
props = iLBSplusDFA.lockboxProportions( :, 1:nLByrs );
iLockboxSpending.lockboxProportions = props;
% set initial investment
iLockboxSpending.investedAmount = 0.50 * iLBSplusDFA.amountInvested;
% bequest utility ratio
iLockboxSpending.bequestUtilityRatio=iLBSplusDFA.bequestUtilityRatio;
% show lockbox amounts (y or n)
iLockboxSpending.showLockboxAmounts = 'n';
% create a new temporary client with zero incomes
clientTemp = client;
[ nsцен nyrs ] = size( clientTemp.incomesM );
clientTemp.incomesM = zeros( nsцен, nyrs );
% process lockbox spending with temporary client
[ clientTemp, iLockboxSpending ] = ...
    iLockboxSpending_process( iLockboxSpending, clientTemp, market );
% find incomes in final year per dollar invested
pstates = clientTemp.pStatesM( :, nLByrs );
ii = find( (pstates>0) & (pstates<4) );
incs = clientTemp.incomesM( ii, nLByrs );
incs = sort( incs , 'descend' );
incsPerDollar = incs / iLockboxSpending.investedAmount;
numIncsPerDollar = length( incsPerDollar );
% find percentile of income in final year per dollar invested
pctl = iLBSplusDFA.percentileOfLastLockboxYear;
incNum = round( .01 *pctl * numIncsPerDollar );
if incNum < 1; incNum = 1; end;
if incNum > numIncsPerDollar; incNum = numIncsPerDollar; end;
LBIncomePerDollar = incsPerDollar( incNum );
```

These preliminaries completed, it is straightforward to find the allocation between the two income sources that will provide the desired incomes, then to determine the dollar amounts to be invested in each one:

% find amounts to invest in lockbox and deferred annuity

```
r = annuityIncomePerDollar / ( LBIncomePerDollar + annuityIncomePerDollar );
LBInvestment = r * iLBplusDFA.amountInvested;
DFAInvestment = iLBplusDFA.amountInvested - LBInvestment;
```

Next we create a matrix of incomes and one of fees from the deferred fixed annuity:

% create incomes from deferred fixed annuity

```
clientTemp = client;
[ nscen nyrs ] = size( clientTemp.incomesM );
iFixedAnnuity.cost = DFAInvestment;
clientTemp = iFixedAnnuity_process( iFixedAnnuity, clientTemp, market );
DFAincsM = clientTemp.incomesM;
feesM = clientTemp.feesM;
```

And a matrix of incomes from the lockbox spending strategy (which, of course, has no fees):

% create incomes from lockbox spending

```
clientTemp = client;
[ nscen nyrs ] = size( clientTemp.incomesM );
clientTemp.incomesM = zeros( nscen, nyrs );
iLockboxSpending.investedAmount= LBInvestment;
[ clientTemp, iLockboxSpending ] = ...
    LockboxSpending_process( iLockboxSpending, clientTemp, market );
LBincsM = clientTemp.incomesM;
```

Next, the amounts invested in the two sources are added to the *iLBplusDFA* data structure so the required investments can be made:

```
% add amounts invested to iLBplusDFA data structure  
iLBplusDFA.DFAInvestment = DFAInvestment;  
iLBplusDFA.LBInvestment = LBInvestment;
```

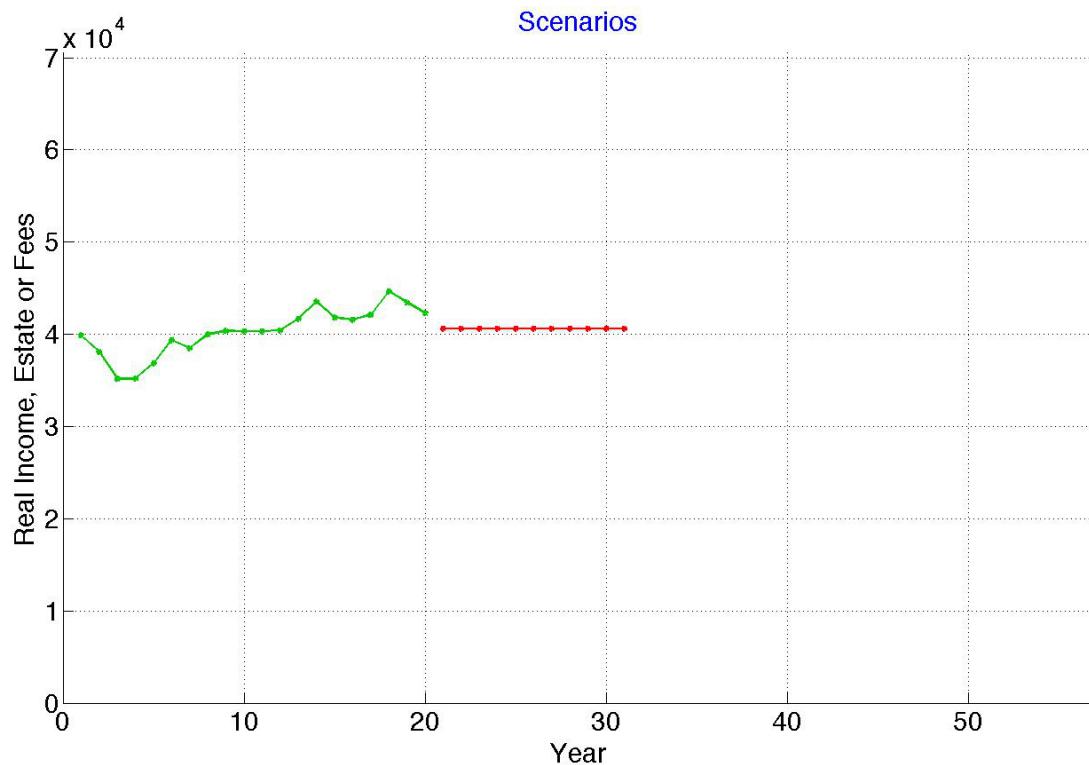
Finally, the fees and incomes are added to the prior fees and incomes in the matrices for the client data structure, to be returned after the overall function has done its work:

```
% add incomes to client income matrix  
client.incomesM = client.incomesM + DFAincsM + LBincsM;  
client.feesM = client.feesM + feesM;
```

A substantial amount of work, to be sure. But much of it is just housekeeping, and the entire operation took under 4 seconds on the author's venerable Macbook Pro. As usual, all the memory used for the variables and matrices created within the function is returned for other uses when the function has done its work (thank you Matlab).

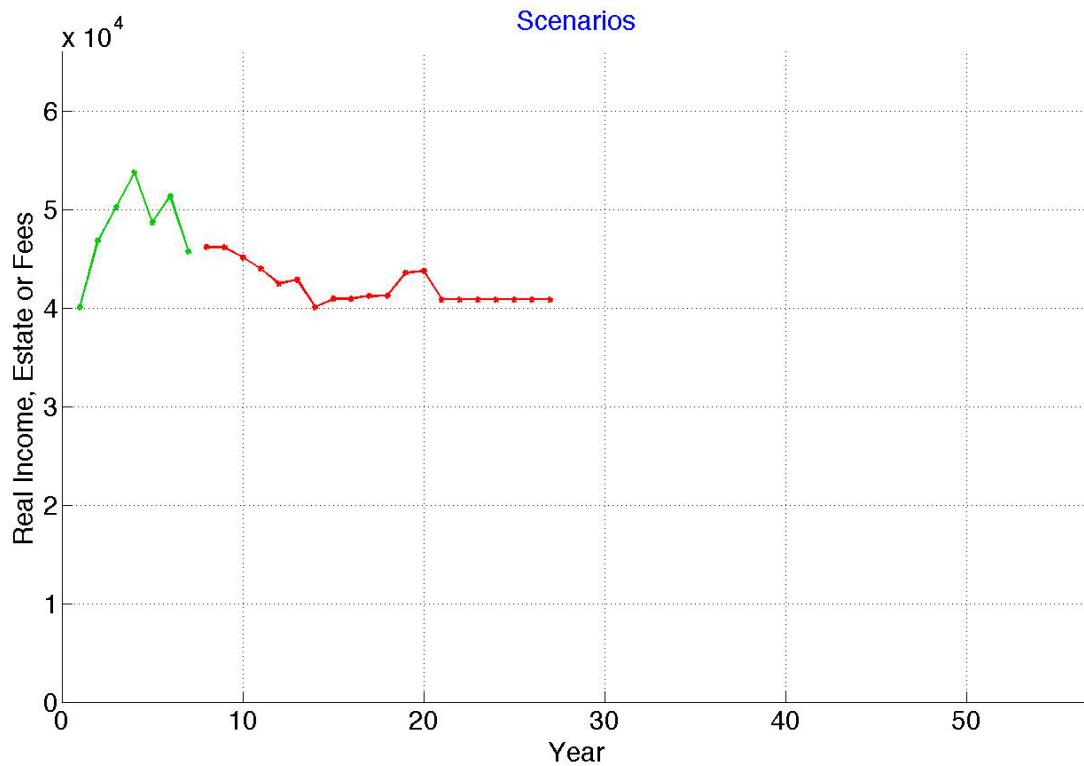
Now, let's see the results that this can produce for the Smiths. To focus on these income sources we exclude Bob and Sue's Social Security incomes and assume that they invest \$1,000,000 in this combination of lockboxes and a deferred fixed annuity.

First, some scenarios. Here is one possibility:



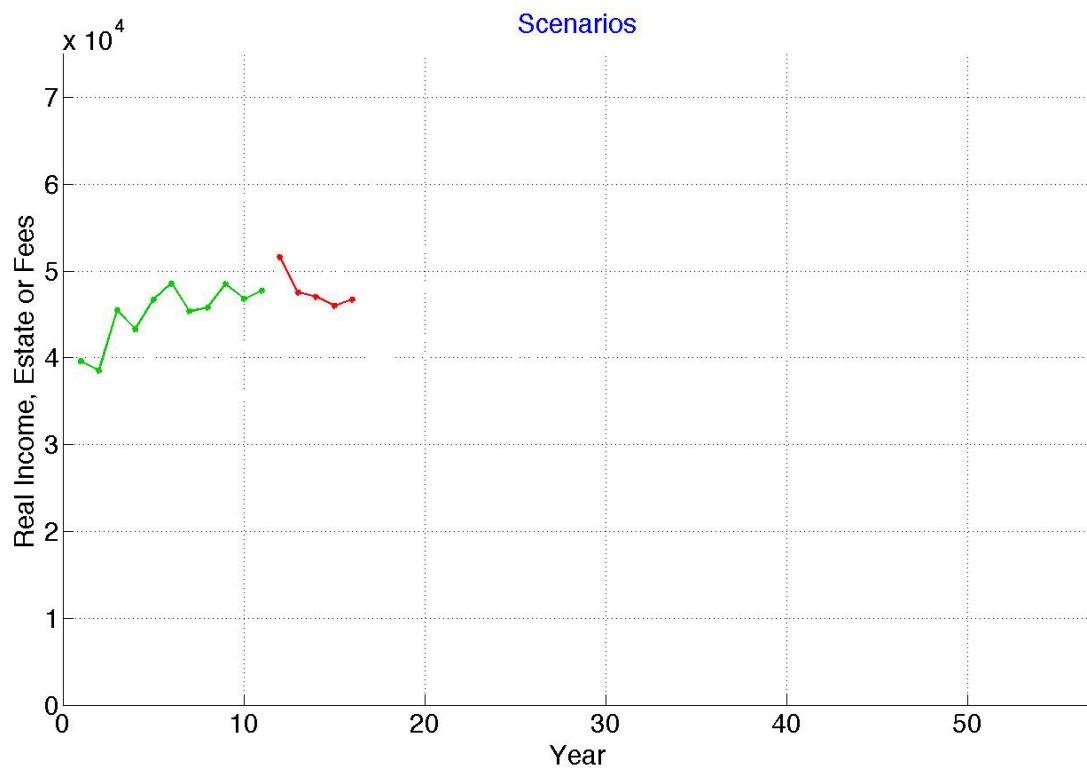
Real income starts at \$40,000 per year, then fluctuates, reaching close to \$42,000 in year 20. In year 21, the lockboxes run out, Bob dies and Sue then experiences eleven of income of slightly more than \$40,000.

Here is another possibility:



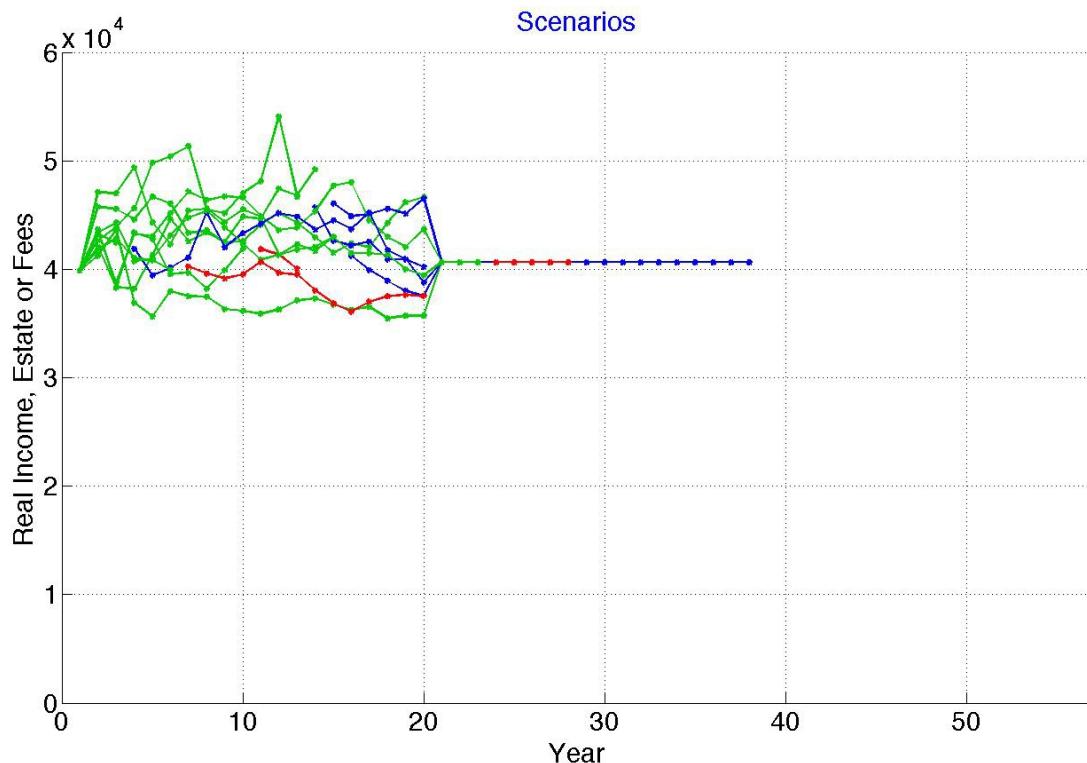
Once again, Sue outlives Bob (who dies after just seven years). Lockbox incomes fluctuate somewhat, rising to as high as \$58,000 but falling thereafter until reaching close to \$42,000 when the deferred annuity begins to make payments. Sue enjoys that income until she passes away in year 27 at the age of 92.

And one more story:



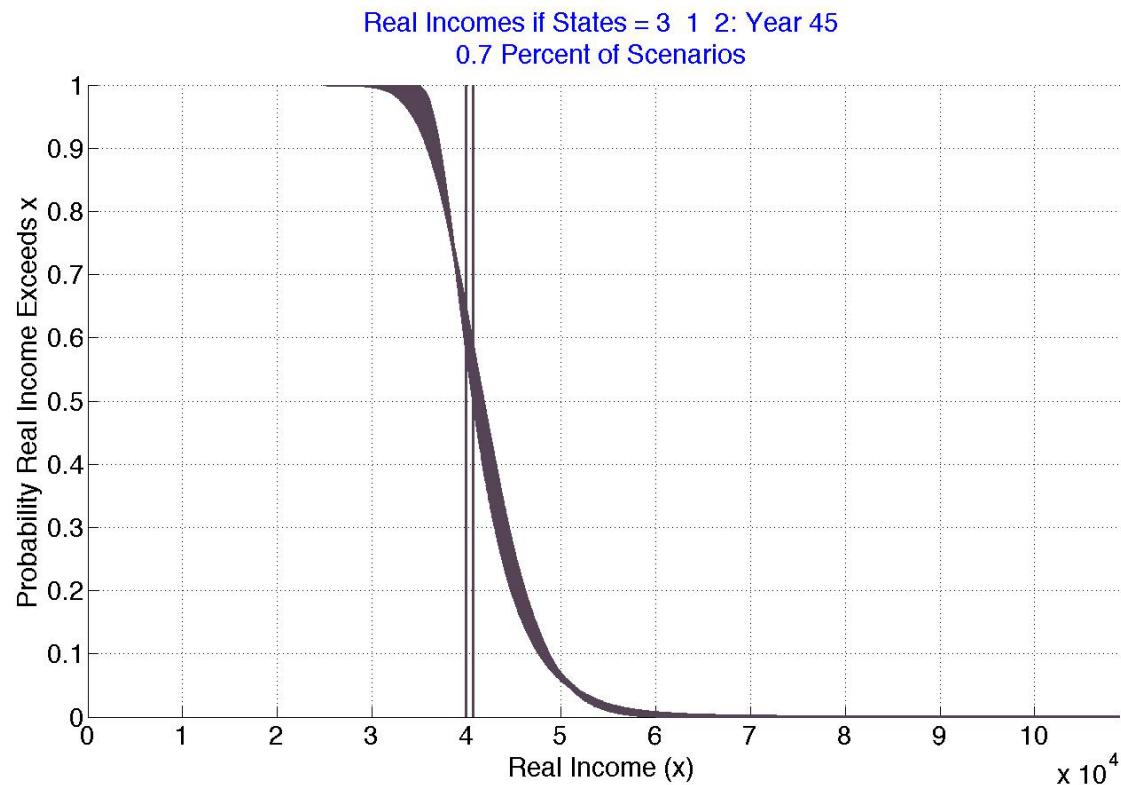
Here real incomes vary between slightly less than \$40,000 to close to \$50,000 over the 11 years that both Bob and Sue are alive. After Bob dies in year 16, Sue receives five more years of income between roughly \$52,000 and \$ 46,000, then passes away before receiving any money at all from the investment in the deferred annuity.

These are, of course, only a few of the huge number of potential future stories. But our choice to set the fixed annuity real income equal to the median value of the range of possible lockbox real incomes in year 20 means that incomes are as likely to rise after the last lockbox year as they are to fall. This can be seen in the following figure with ten scenarios (each in the same shade).



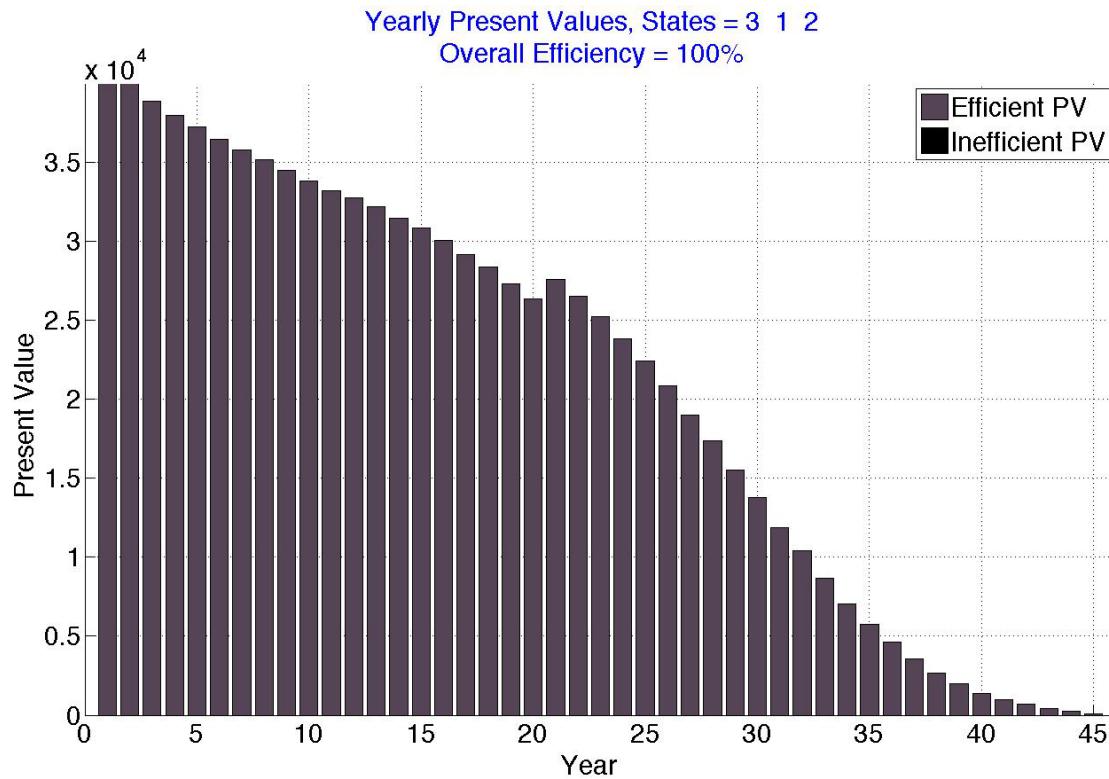
It is, of course, a bit of a jumble. And the last scenario shown can cover some outcomes for previous scenarios. In practice, one would watch it develop via animation, with the most recent scenario shown in a dark shade and the others in lighter shade. But the implication of the choice of the median income from the last lockbox for the annuity income is clear. Had a different percentile be used, the picture would be different, as intended.

The distributions of real income, shown below, are also as intended.



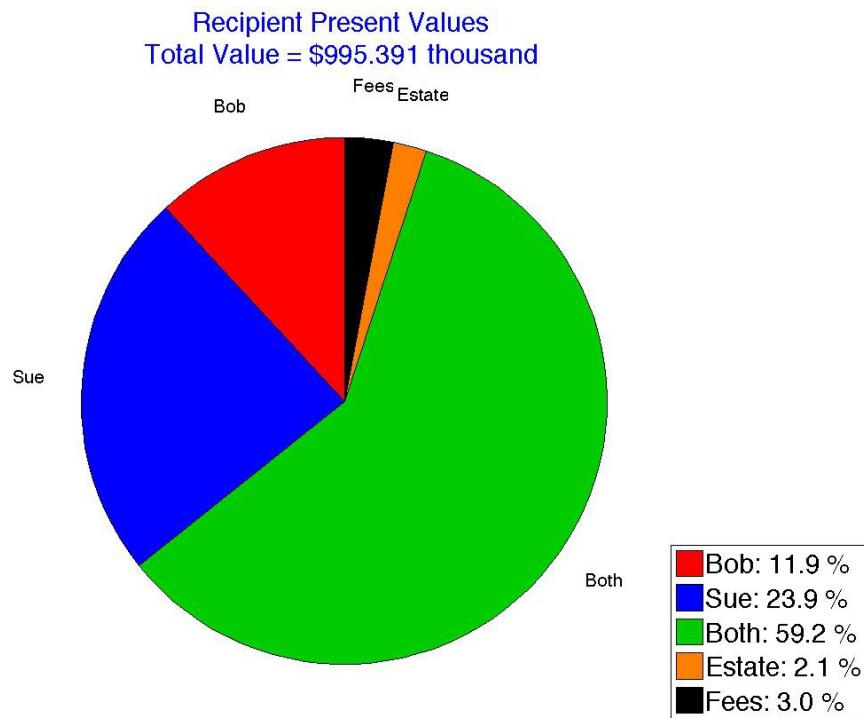
Since the lockbox spending is based on an AMD2 strategy, the ranges of incomes for each of the first 20 years plot on curves as close as possible (without m-shares) to that for year 2. The plot for year 21 and every subsequent years is the vertical line to the right of that for the first year. Moreover, it intersects the curve for year 20 at the 0.50 probability level (on the y-axis) by design (since we chose the 50'th percentile of the last lockbox income for the fixed annuity).

The graph of yearly present values is relatively unsurprising. There is a slight increase when the deferred fixed income annuity takes over as income source, since the median income from year 20 is repeated as the fixed income in year 21 and the present value of all possible incomes in year 20 is greater than that of the median income.



Importantly, the income distribution in each year is 100% cost efficient, as is the totality of all the incomes. This is not unexpected. Each lockbox contains TIPS and/or the market portfolio with the positions held without change until the year that the lockbox matures. Thus there will be a one-to-one relationship between terminal values and PPC values for each of the lockbox years. And each annual annuity payment is fixed in real terms so there is no cheaper way to produce the set of incomes for that year. While 100% cost efficiency may not be an absolute requirement for a retirement income strategy, it is a definite plus.

Finally, the present values of the claims of various participants on possible future income. Here is the graph:



As desired, a large part of the possible value accrues to Bob and Sue. The scenarios in which they both die within the first twenty years do provide some possible payments to their estate: collectively these have a present value equal to 2.1% of the total. The deferred annuity provider requires compensation for providing mortality pooling (and probably also overhead and/or profit); these costs have a present value equal to 3.0% of the total. But the present value of possible payments from the lockboxes and deferred annuity to the Smith's is close to 95% of the total amount invested in the two strategies. (As usual, due to sampling errors, the total is close to but not precisely equal to the amount invested).

A video of some of the graphs for a lockbox spending strategy using AMD2 lockboxes and a bequest utility ratio of 0.5 is available at:

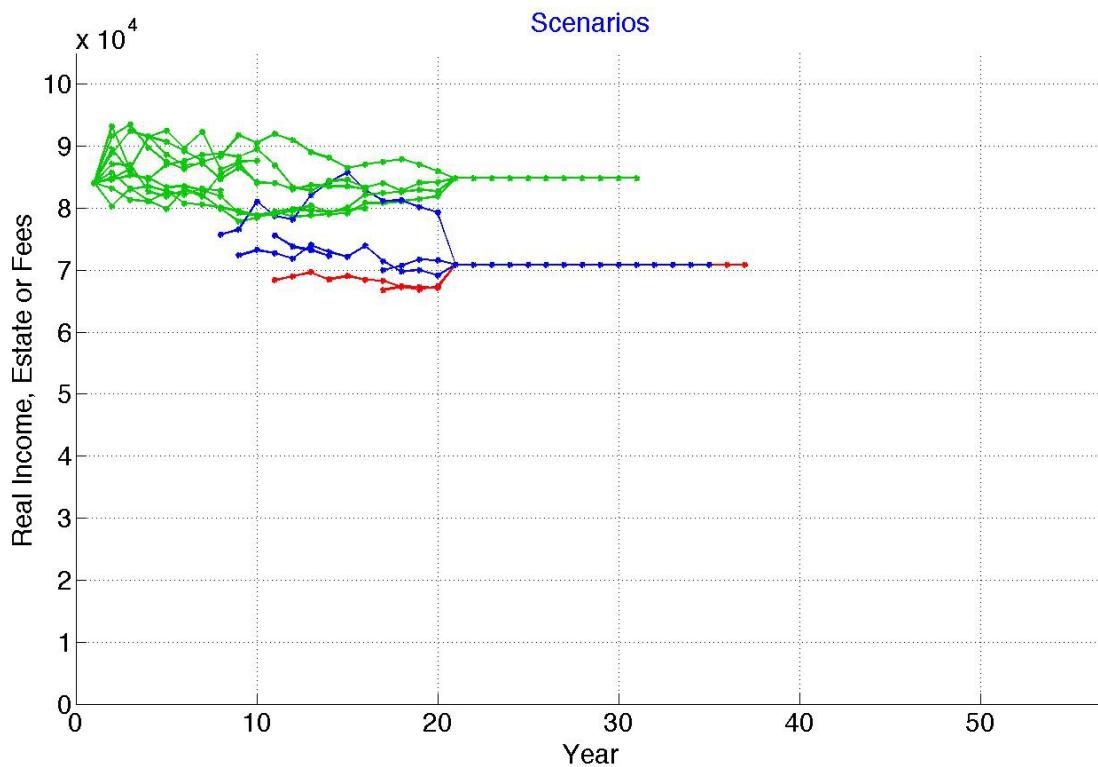
www.stanford.edu/~wfsharpe/RISMAT/SmithCase_Chapter20_LBS.mp4

Of course, this is only part of the income that the Smiths may receive. Recall from Chapter 14 that they have Social Security benefits worth almost as much as their discretionary savings. It is a simple matter to take both into sources into account. To the previous script we simply add:

```
% add Bob and Sue's Social Security (default values)
iSocialSecurity = iSocialSecurity_create();
client = iSocialSecurity_process(iSocialSecurity, client, market);
```

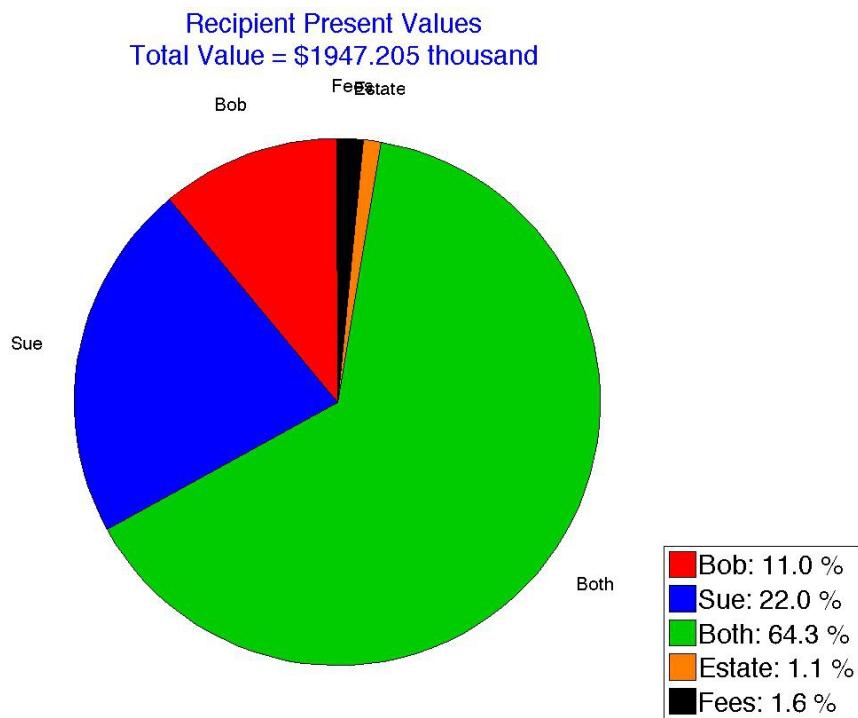
which especially simple since our default parameter values for *iSocialSecurity* are those for the Smiths.

Here are some scenarios for the combined sources of income:



Since Social Security provides more income when they are both alive, the green scenarios generally plot above those for the blue or red. Moreover, for years in which income is provided by Social Security plus the deferred fixed annuity, total income is constant, but at a higher level when both are alive.

Finally, the present values:



Overall, the values of Bob and Sue's claims on the income sources constitute almost 97.3% of the total. Their estate receives income only if they both die before the annuity takes effect. It gets nothing from Social Security after they die (besides the possibility of a small amount for burial fees, which we have ignored), and the deferred annuity has no benefits for survivors. Moreover, the cost of the deferred annuity is considerably less than half of the the total value of their income sources, so the associated fees take only a small percentage of their total retirement wealth.

Before leaving this case, it is useful to bring in a few aspects of the real world (or at least, the real world in the United States in early 2017). If a deferred annuity is to be purchased with after-tax dollars, at least some insurance companies allow for deferral periods as long as 30 years, but may require that the initial payment begin at some specified age. On the other hand, if before-tax money from an IRA, employer retirement plan or other tax-favored plan is to be used, payments may have to start at or before age 70½ (due to the required minimum distribution rules discussed in Chapter 18). However, there are exceptions to this rule. A *qualified annuity* is one funded entirely with pre-tax income. For such an annuity, payments may be deferred until age 85 (thus Bob Smith, who is 67, would only be able to defer for 18 years or less). Moreover, there are complex rules concerning the amount that may be invested in such a *QLAC* (*qualifying longevity annuity contract*) during the accumulation period. In 2017, total contributions to fund such an annuity for an employee were limited to \$125,000 across all sources, and contributions from a given funding source could not exceed 25% of that source's value.

Subject to such issues, deferred fixed annuities may still play a useful role in a retirement plan.

Lockbox Spending plus Deferred Purchase of a Fixed Annuity

We turn now to another possible combination of sources of retirement income. It involves the use of a set of lockboxes that will provide income for a number of years (n) plus an additional lockbox with assets to be used to purchase an immediate annuity that will provide income beginning at the start of year $n+1$, then continue to provide the same amount of real income thereafter as long as one or both of the beneficiaries is alive. As with the previous strategy, the goal would be to select n so that there is a relatively high probability that some beneficiary would be alive at the beginning of that year.

As we have indicated previously, one does not know today what the cost of an immediate annuity will be in some future year. Actuarial tables may well change in the interim, the interest rates for bonds used to provide funds to make the annuity payments could differ from present rates or even the forward rates implied by the current term structure of interest rates, and the status of the beneficiaries (alive or not, healthy or not) at the time is unknown at the present. That said, we will provide functions for the deferred purchase of an annuity, recognizing that some sources of uncertainty will be ignored.

For three reasons we again consider only annuities that promise constant real payments as long as one or both beneficiaries are alive. First, this conforms with our general position that for most retirees, real incomes are more relevant than nominal incomes. Second, there may be less uncertainty about future real interest rates than about future nominal interest rates. And finally, we choose to keep the analysis relatively simple (leaving more complex cases for others).

For tractability, in addition to our already strong assumption that the real interest rate today is the same for every horizon, we add a further assumption that future rates will be the same as present rates. While this will undoubtedly not be true, it may be that the uncertainty that we ignore is relatively minor. But only time will tell.

We also assume that future actuarial mortality tables are consistent with current ones. This may not be as strong an assumption as might first appear. Recall from Chapter 3 that our actuarial calculations are based on the RP-2014 mortality tables plus the MP-2014 mortality improvement tables, and that the latter tables project improvements in future mortality for each age. Thus the chance that Sue, who is now 65, will live for 20 years, then die in the next year is not the chance that a current 85-year old will die within a year. Instead, it is the chance that Sue will be alive at age 86 divided by the chance that she will be alive at age 85. The implicit assumption is that the mortality improvement tables correctly forecast future mortality tables. While this is not likely to be strictly true, the forecast mortality rates should be unbiased estimates of those used to price annuities at the time.

With these important caveats, we proceed.

To make analyses of combinations of lockbox spending and the future purchase of an annuity possible, we need functions that will provide the latter. To distinguish between the immediate purchase of a deferred annuity (utilized in the previous functions) and the future purchase of an immediate annuity, and also to keep the function name short, we utilize the abbreviation *iFAPlockbox*, to represent a *future annuity purchase* using proceeds from a lockbox invested at the present time.

Here is the *iFAPlockbox_create* function:

```
function iFAPlockbox = iFAPlockbox_create()
    % create a data structure for a lockbox to fund
    % future purchase of a fixed annuity

    % year in which annuity is to be purchased
    iFAPlockbox.yearOfAnnuityPurchase = 20;

    % initial proportion ($) in TIPS in lockbox (0 to 1.0)
    % with the remainder in the market portfolio
    iFAPlockbox.proportionInTIPS = 0.50;

    % initial amount ($) in the lockbox
    iFAPlockbox.investedAmount = 100000;

    % annuity ratio of value to initial cost
    iFAPlockbox.annuityValueOverCost = 0.90;

end
```

The elements are straightforward. If the default values are used, the annuity is to be purchased at the beginning of year 20, to provide income starting at that time and continuing until the estate is executed. The initial amount placed in the lockbox is \$100,000, of which half (0.50) is invested in TIPS, with the remainder in the market portfolio. When purchased, the annuity provides possible incomes worth 90% of the amount invested, with the other 10% going to the insurance provider as fees.

We do not attempt to cover graduated annuities, or those that might provide lower income to a surviving beneficiary, although one could certainly include such features.

The assumption is made that if both parties (e.g. Bob and Sue) die before the intended annuity purchase year, the value of this lockbox will be paid to the estate. Otherwise the securities in the lockbox will be sold and the proceeds used to purchase an immediate annuity that will make constant real payments until the last party dies (and no payments thereafter).

The *iFAPlockbox_process* function is somewhat lengthy. To some extent this is in order to make the computations as comprehensible as possible, but there a number of essential aspects. As usual, the reader is invited to skim or avoid the details, if desired.

The overall structure is:

```
function client = iFAPlockbox_process( client, iFAPlockbox, market );
    % processes an iFAPlockbox data structure
    % creating a future real annuity with constant payments as long as
    % anyone is alive

    .....

end
```

Since only the client incomes and fees matrices are affected, the function returns just a modified version of the client data structure. Not surprisingly, it uses information from the *client*, *market* and *iFAPlockbox* data structures.

The first section uses mortality rates from the client structure to produce three vectors, each with probabilities of payments made in the initial annuity year and all subsequent years. There are three such vectors, based on the personal state at the time the annuity is purchased. If only person 1 is alive at that time, the mortality rates are his or hers. Similarly, if only person 2 is alive, only his or her mortality rates are utilized. If both are alive at the outset, it is easiest to first calculate the probability that each will be dead, multiply these probabilities to determine the probability that both will be dead, and then subtract this product from 1 to find the probability that one or both will be alive at the time.

```
% compute annual real income per dollar depending on personal state
% when annuity is purchased
% find mortality rates in future year (if alive)
FAPyear = iFAPlockbox.yearOfAnnuityPurchase;
mortP1 = client.mortP1;
mortP1 = mortP1( FAPyear+1: length(mortP1) );
mortP2 = client.mortP2;
mortP2 = mortP2( FAPyear+1: length(mortP2) );
% compute probabilities of payment for each initial personal state
% probability of payment each year if only 1 is alive at outset
probP1Alive = cumprod( 1 - mortP1 );
% probability of payment each year if only 2 is alive at outset
probP2Alive = cumprod( 1 - mortP2 );
% probability of payment if both alive at outset and payment is made
% when either or both are alive
probBothDead =( 1-probP1Alive) .* (1-probP2Alive);
probPayment1 = probP1Alive;
probPayment2 = probP2Alive;
probPayment3 = 1 - probBothDead;
% add an initial payment of $1 at the beginning of first year
probPayment1 = [ 1 probPayment1 ];
probPayment2 = [ 1 probPayment2 ];
probPayment3 = [ 1 probPayment3 ];
```

Note that although these vectors are named probabilities of payments, they also indicate the expected annual real amounts that the insurance company should expect to pay (and, if a large number of similar policies are written, the amounts that it will actually pay). As usual, we assume that the annuity will be priced accordingly.

The next section of the function computes annuity costs. Since the payments are in real dollars, this requires discounting each expected payment at the real interest rate (which we optimistically assume is constant and known when the annuity is purchased), then dividing by the value over cost to take the issuer's fees into account.

```
% find discounted sum of payments
n = length( probPayment1 );
dfs = market.rf .^ [ 0: n-1 ];
pvs = 1 ./ dfs;
valuePerDollar1 = sum( probPayment1 .* pvs );
valuePerDollar2 = sum( probPayment2 .* pvs );
valuePerDollar3 = sum( probPayment3 .* pvs );
% find costs of annuities for initial personal states
valOverCost = iFAPlockbox.annuityValueOverCost;
costPerDollar1 = valuePerDollar1 / valOverCost;
costPerDollar2 = valuePerDollar2 / valOverCost;
costPerDollar3 = valuePerDollar3 / valOverCost;
```

For each relevant initial personal state we now have the cost to the annuity issuer of providing a dollar of annuity income until the last survivor dies.

The next section deals with the lockbox values. It provides a matrix of the future values for each scenario and each year through and including the year the annuity is to be purchased. The calculations are straightforward:

```
% create values available to purchase annuity
tipsProp = iFAPlockbox.proportionInTIPS;
if tipsProp > 1; tipsProp = 1; end;
if tipsProp < 0; tipsProp = 0; end;
tipsAmt = tipsProp * iFAPlockbox.investedAmount;
mktAmt = iFAPlockbox.investedAmount - tipsAmt;
mktVals = mktAmt * market.cumRmsM( :, FAPyear );
tipsVals = tipsAmt * market.cumRfsM( :, FAPyear );
totVals = mktVals + tipsVals;
```

Next, annuity payments and the fees paid to the annuity provider are computed. Each set of computations results in a vector with the full number of scenarios in the analysis. All such vectors are set initially to have all values equal to zero. Subsequent statements affect only scenarios in which a beneficiary is alive at the beginning of the year in which the annuity is to be purchased. For each possibility (personal state 1, 2 or 3), annuity payments are computed based on the relevant cost per dollar. The associated fees are also calculated based on the value over cost ratio.

```
% create annuity payments and fees vectors
[ nscen nyrs ] = size( client.incomesM );
annPayments = zeros( nscen, 1 );
feesV = zeros( nscen, 1 );
ii = find( client.pStatesM( :, FAPyear ) == 1 );
    annPayments(ii) = totVals(ii) ./ costPerDollar1;
    feesV(ii) = ( 1 - valOverCost ) * totVals(ii);
ii = find( client.pStatesM( :, FAPyear ) == 2 );
    annPayments(ii) = totVals(ii) ./ costPerDollar2;
    feesV(ii) = ( 1 - valOverCost ) * totVals(ii);
ii = find( client.pStatesM( :, FAPyear ) == 3 );
    annPayments(ii) = totVals(ii) ./ costPerDollar3;
    feesV(ii) = ( 1 - valOverCost ) * totVals(ii);
```

At this point we begin creating an full incomes matrix and fees matrix. To the former we add the annuity payments for the initial annuity year, then add for each subsequent year the applicable elements in the annuity payment vector (for scenarios in which the client personal state is 1, 2 or 3). The fees matrix is simpler. We simply add all the fees earned in the annuity purchase year to a previous matrix of zero entries.

```
% create incomes matrix
incsM = zeros( nscen,nyrs );
% add payments in FAPyear
incsM( :, FAPyear ) = annPayments;
% add payments for years after FAPyear
for yr = FAPyear+1 : nyrs
    ps = client.pStatesM( :, yr );
    v = (ps>0) & (ps<4);
    incsM( :, yr ) = v .* annPayments;
end;

% create fees matrix
feesM = zeros( nscen, nyrs );
feesM( :, FAPyear ) = feesV;
```

The major tasks are almost completed, but one remains. In any scenario in which both beneficiaries are dead before the year in which the annuity is to be purchased, the value of the securities in the lockbox goes to the estate. We need to provide a matrix with such payments.

To do so is relatively straightforward. First we compute a complete matrix of all the possible future values of the market shares in the lockbox. Then we do the same for the Tips. Adding these together gives a matrix of the values of the lockbox in every possible scenario and year. We know that the estate will get nothing after the year in which the annuity is to be purchased, so that section of the matrix can be set to all zeros. Next we create a matrix with a value of 1 in every cell in which an estate is paid (personal state 4) and zero in every other cell. Finally, we simply multiply every element in the lockbox total values matrix by the value in this new matrix. The result is a matrix with the amount paid in each scenario and year in which a lockbox is cashed in to pay the estate.

```
% find payments to estate before FAPyear
marketValsM = mktAmt * market.cumRmsM;
tipsValsM = tipsAmt * market.cumRfsM;
totValsM = marketValsM + tipsValsM;
totValsM( :, FAPyear+1:nyrs ) = 0;
estatePaidM = client.pStatesM == 4;
amtsPdM = totValsM .* estatePaidM;
```

Finally, we add the new matrices to the corresponding existing client matrices and conclude:

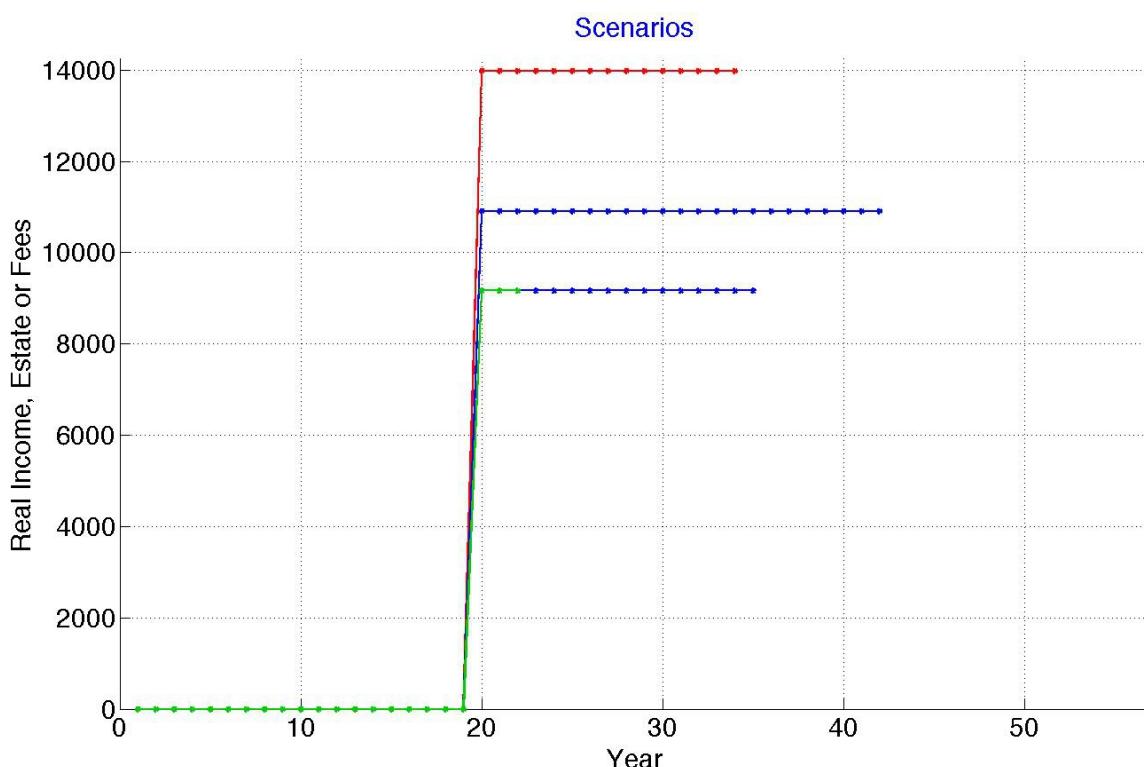
```
% add incomes and amounts paid to client incomes
client.incomesM = client.incomesM + incsM + amtsPdM;
% add fees to client fees
client.feesM = client.feesM + feesM;
```

A video of a case utilizing AMD2 lockboxes, a bequest utility ratio of 0.5, and purchase of an annuity with payments deferred for 20 years is available at:

www.stanford.edu/~wfsharpe/RISMAT/SmithCase_Chapter20_DFA.mp4

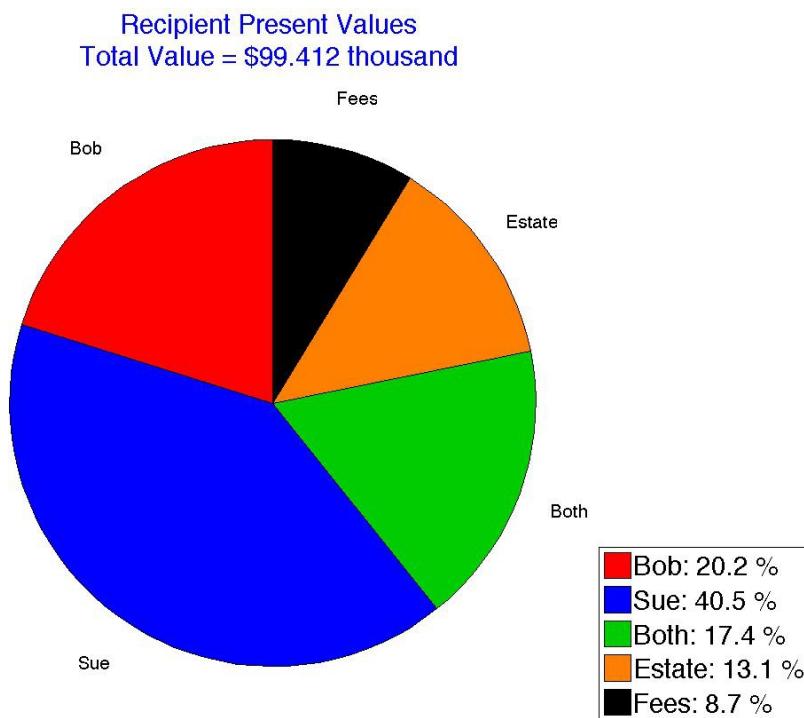
Incomes from Future Purchase of an Annuity

We are now ready to explore some of the properties of lockboxes designed to purchase an annuity in a future year. To begin, consider a lockbox investing only in TIPS. Here are some scenarios for future incomes:



There is of course no income before the year in which the annuity is purchased. The red curve shows that if only Bob is alive at that time, he will receive roughly \$14,000 a year as long as he lives. The blue curve shows that if only Sue is alive in year 20, she will receive close to \$11,000 per year as long as she lives. The difference results entirely from Sue's greater chances for a long life, which the annuity company has taken into account. The lowest incomes are those for scenarios in which both Bob and Sue are alive in year 20 (the green curve). In such cases, the annuity will pay slightly over \$9,000 per year as long as anyone is alive. Several such scenarios are shown in the figure. In some, both live many years, In others, Sue outlives Bob, and in yet others he outlives her. But in every case, annuity income remains constant as long as someone is there to receive it.

The present values of all the prospective incomes and fees are shown below.



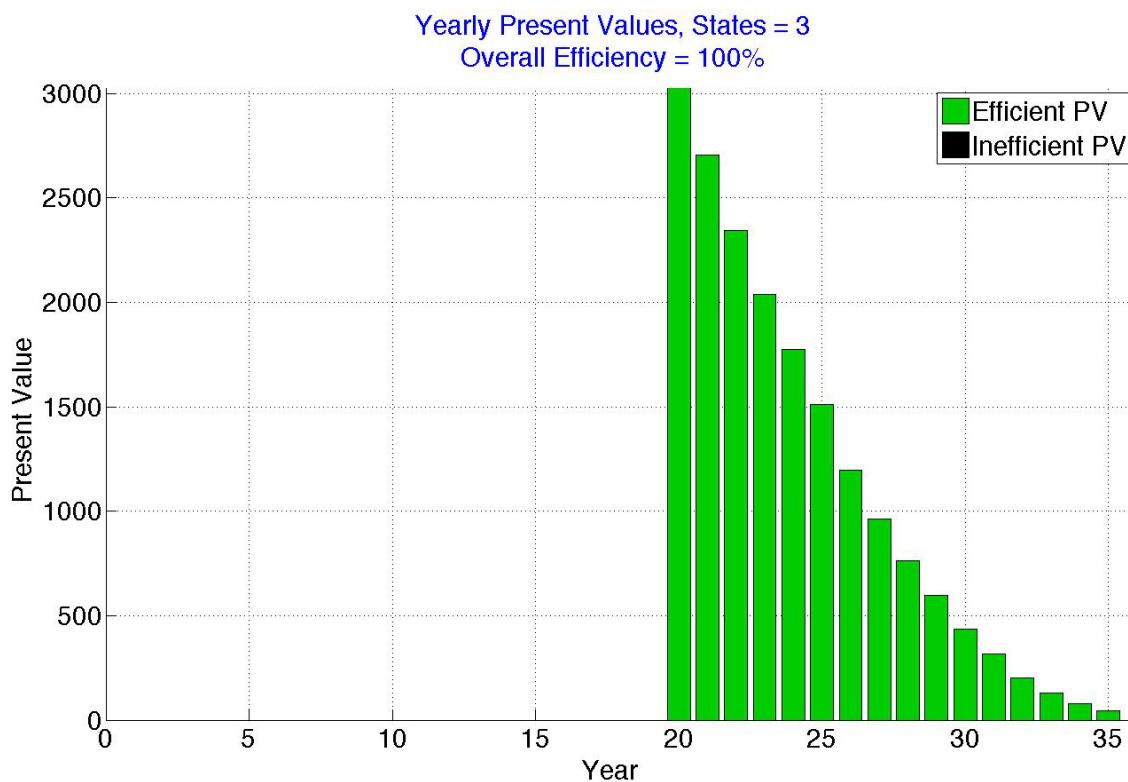
As usual, the sum will not precisely equal the amount invested (in this case, \$100 thousand), due to sampling error. Here the difference is relatively small but it can be larger, especially in cases (such as this) with investment wholly in Tips.

Not surprisingly, Sue's prospective incomes are worth more than Bob's. Being younger and female, she is more likely to survive to buy an annuity, and if they both are alive when it is purchased, she is likely to get payments for a longer time.

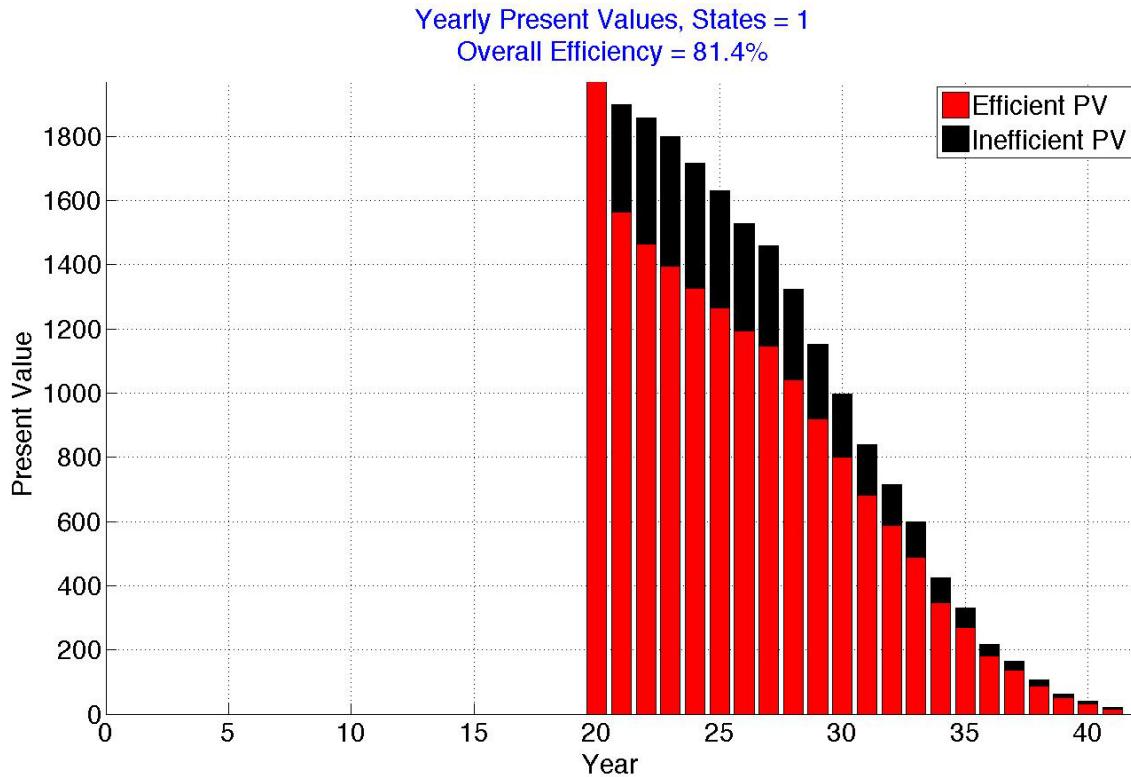
Note that the estate is may well receive income, since neither Bob nor Sue may live to purchase the annuity. The present value of such possibilities is slightly over 13% of the initial amount. This contrasts with the immediate purchase of a deferred annuity, in which all of the present value after annuity fees goes to prospective payments for Bob and/or Sue.

Finally, there is the matter of annuity fees. These are not 10% of the total present value, since the insurance company only makes money if Bob and/or Sue survive until the chosen purchase date. Again, this contrasts with buying a deferred annuity, in which the provider receives an initial fee worth the full percentage of the initial value.

Next we consider the cost efficiency of this approach, retaining the assumption that the lockbox used for the future purchase of the annuity is invested entirely in TIPS. The first point to make is that the usual analysis is fully applicable only to personal state 3. Since there is no variation at all in the amount of income received across all scenarios in which both are alive at the time the annuity is purchased, the income distribution is 100% cost efficient, as shown by the graph of present values of the incomes in each year:

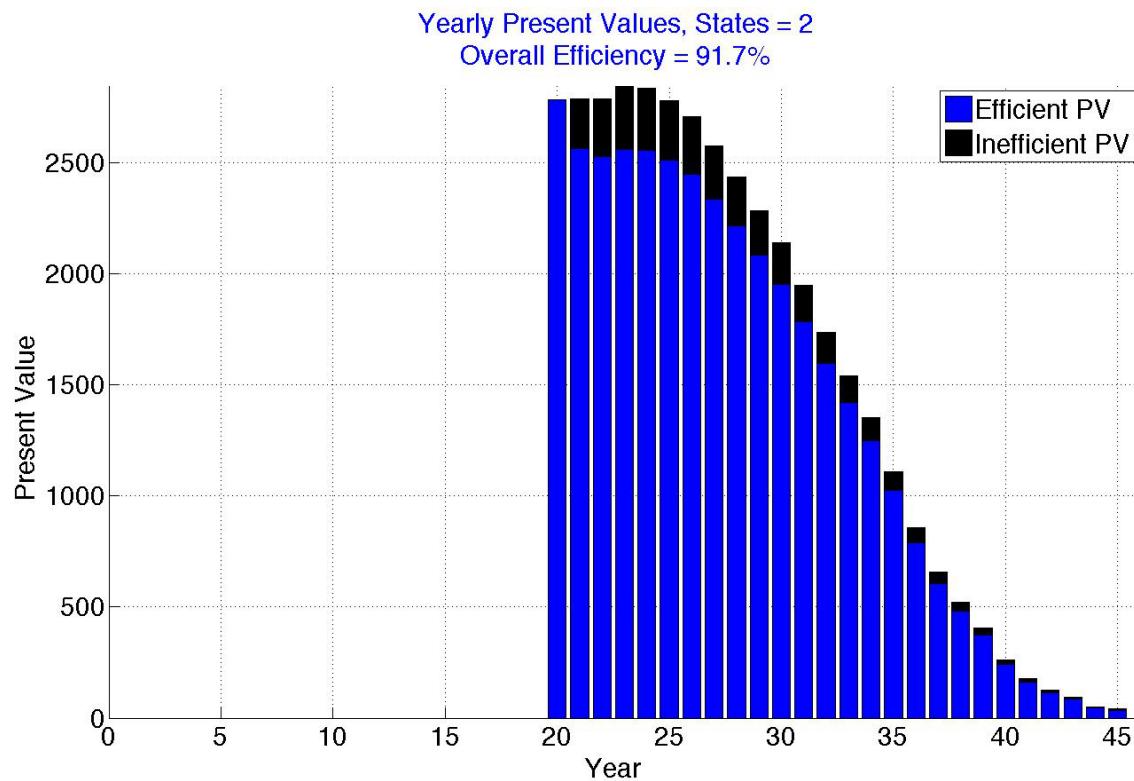


Contrast this with graph for personal state 1:



For each year but the initial payment year, some scenarios (those in which only Bob was alive when the annuity was purchased) have higher incomes than others (those in which both were alive in that year). But if it had been possible, the same distribution of incomes could have been obtained at a significantly lower cost (by arranging to receive the lower income in more expensive scenarios and the higher income in less expensive scenarios). Of course this is not possible, so nothing can be done about it. But this is one cost of deferring the annuity purchase.

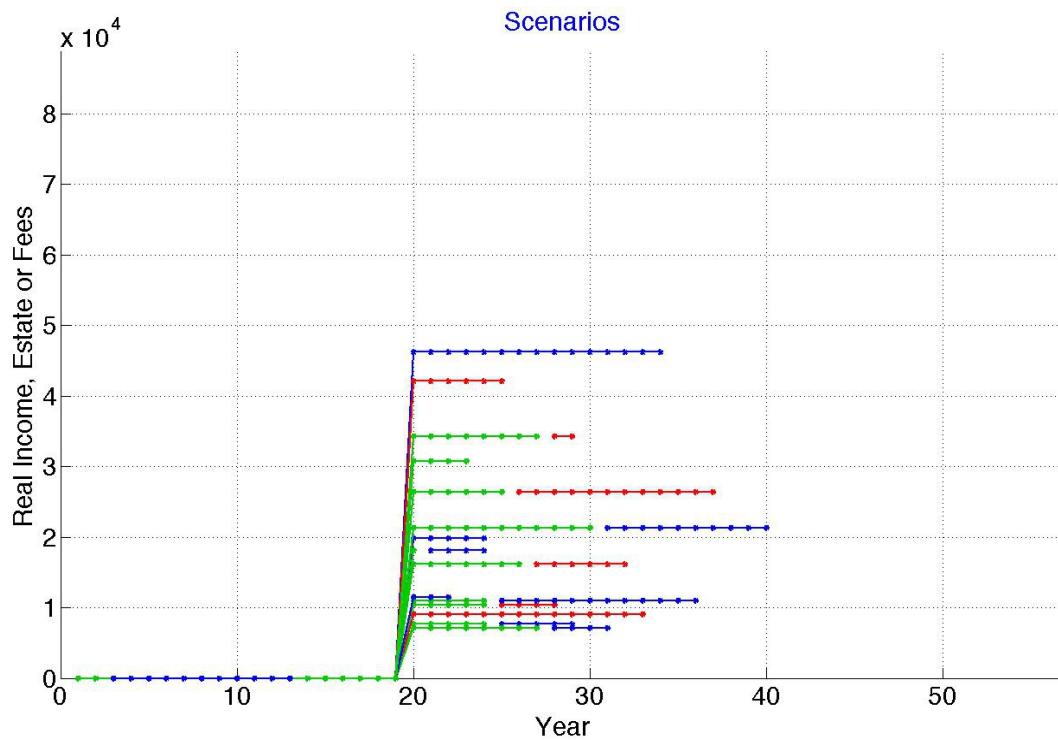
The efficiency is somewhat greater for cases in which Sue is alone:



This is due to the smaller difference between the amount Sue receives in the scenarios in which she was alone at the outset and the amount she receives if Bob was also alive at that time.

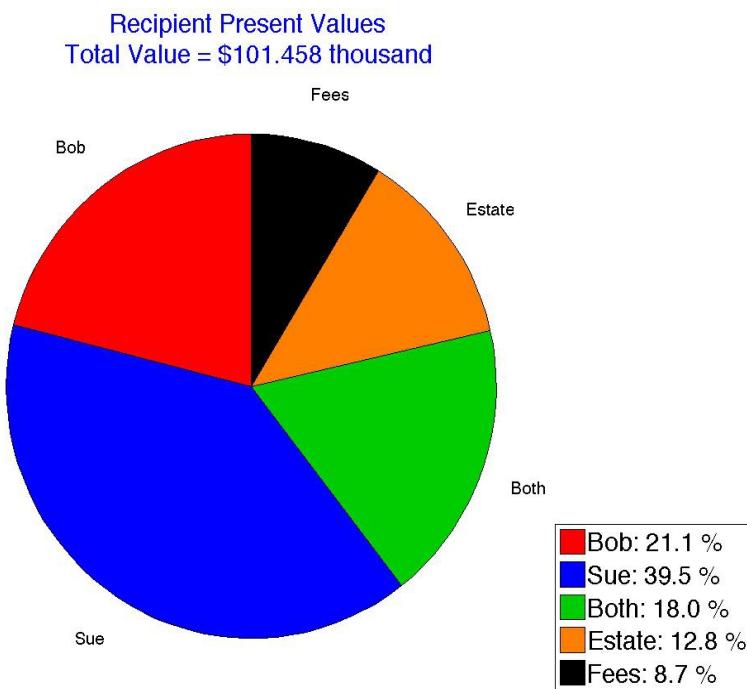
While there is nothing one can do about these cost inefficiencies, they are a negative factor to consider before adopting a strategy that defers purchase of an annuity, even one financed entirely by Tips.

Now consider a case in which the lockbox designed to purchase the annuity in year 20 is invested entirely in the market portfolio. The results follow.

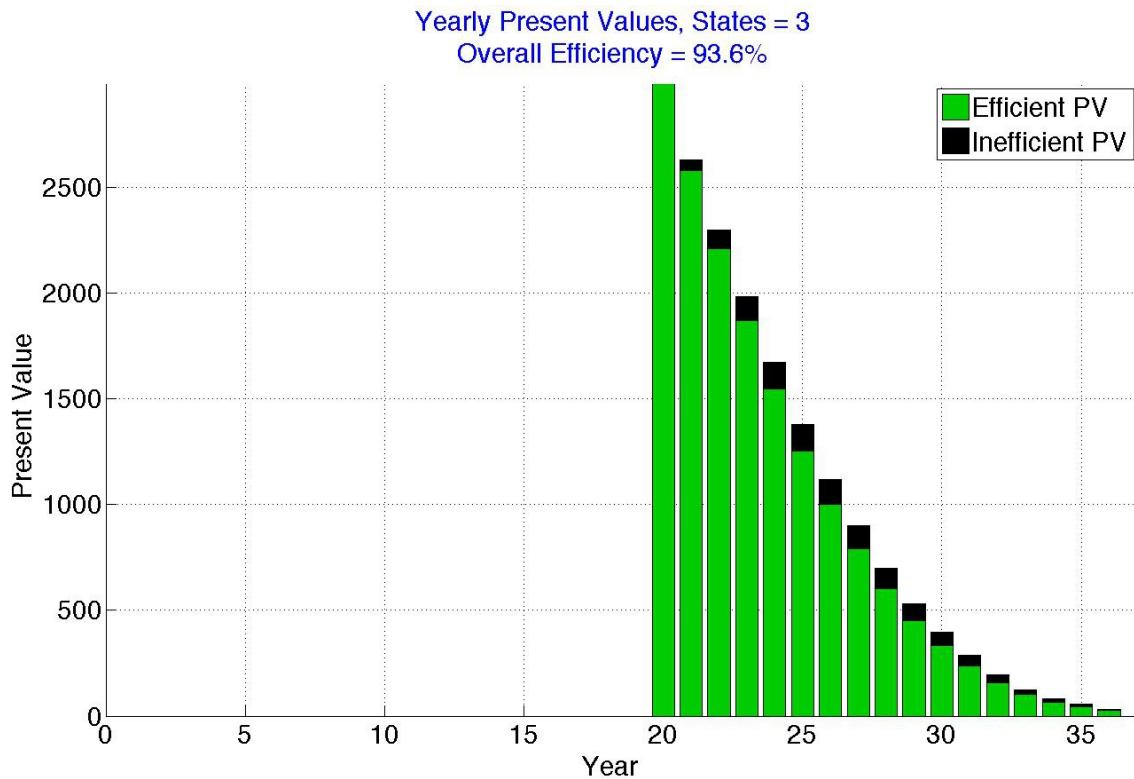


Not surprisingly, there is considerable variation in the income provided by the annuity, even for scenarios with the same initial personal state at the time when the annuity is purchased.

The distribution of present values among the relevant parties is very similar to that when only Tips were used. In this case, the sampling error was larger, overstating total value by almost 1.5%, but another analysis using the same inputs could give different total values.



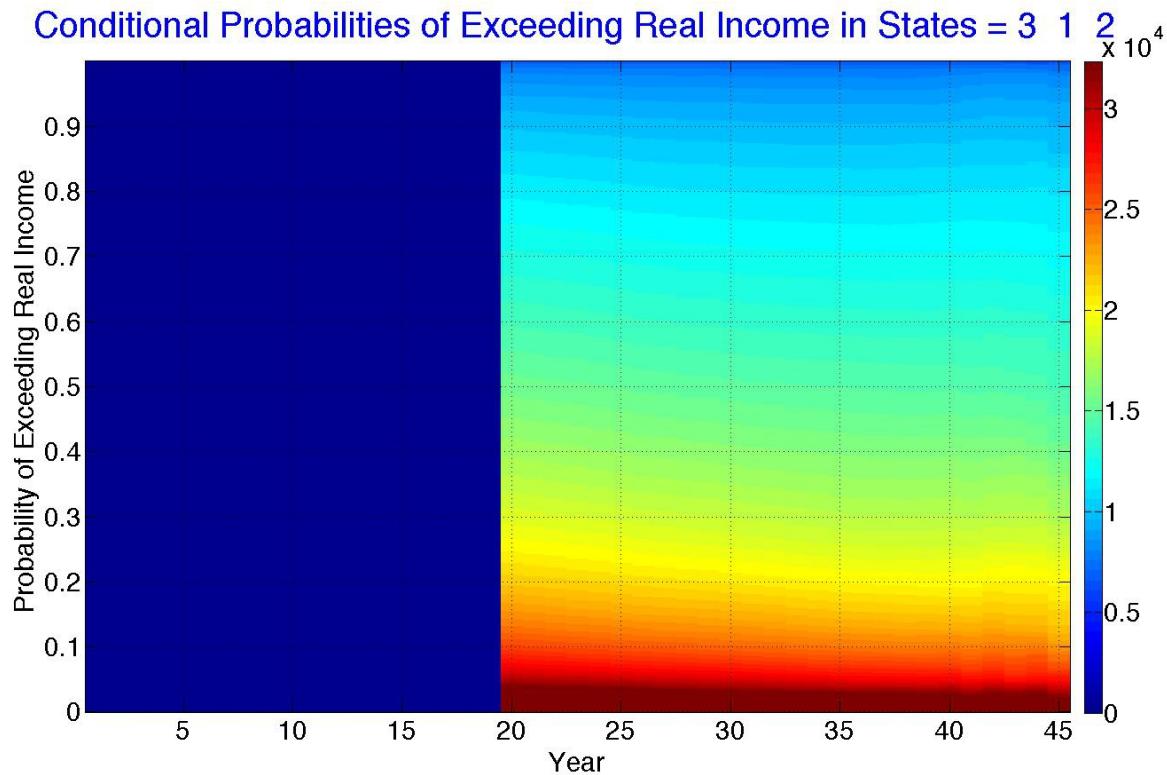
In this case there is cost inefficiency even for personal state 3:



Why? Because in all but the first year, the income received equals (a) the cumulative market return up to the date the annuity is purchased times (b) a constant based on mortality estimates. But the present value of a payment in any subsequent year will depend on the cumulative market return up to the date the payment is made. Hence each income distribution could be obtained more cheaply if lower incomes could have been obtained in less expensive scenarios.

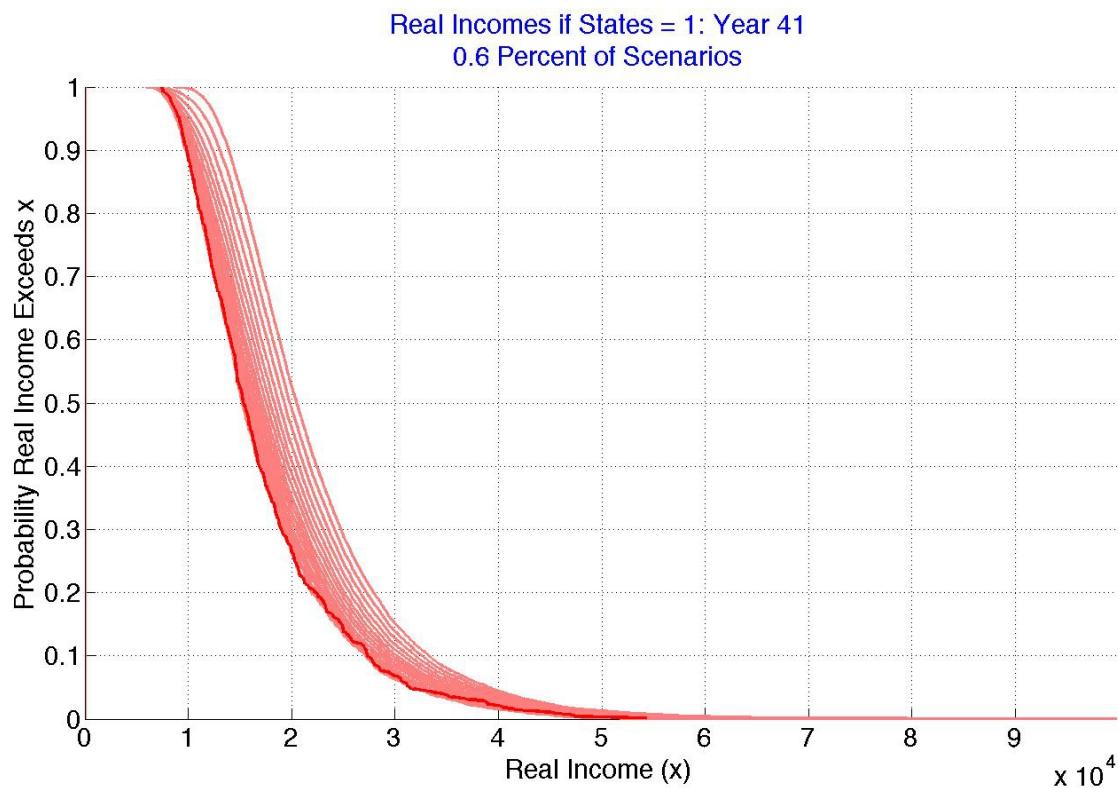
As before, the distributions for personal states 1 and 2 are also cost-inefficient, but now for two reasons: the variation in initial annuity purchase values and differences in the personal state when the annuity is purchased.

To complete the analysis of a strategy that only includes the future purchase of an annuity, we show income distributions for a case in which the lockbox is initiated with 50% of the amount invested in Tips and the other 50% in the market portfolio. First an income map showing incomes for all personal states:



The large blue area reflects the fact that this strategy produces no income in the first 19 years (and thus needs to be supplemented with one that will cover those years, which we will provide shortly). As can be seen, in the subsequent years the range of incomes is considerable, even though both Tips and the market portfolio were utilized.

The income distributions for scenarios in which only Bob is alive in year 20 show this:



The range of incomes for year 20 is shown by the curve farthest to the right. That for year 21 lies just to its left. And each subsequent curve is to the left of its predecessor. Why? Remember that in some scenarios Bob receives income from an annuity purchased when both he and Sue were alive, and in other scenarios he receives income from an annuity purchased when only he was alive. For any given cumulative market return, the former amount would be less than the latter. The distribution shown for year 20 includes only scenarios in which Bob is the only survivor at the time. The distribution for year 21 includes a few scenarios in which Sue and Bob were both alive when the annuity was purchased and the amount paid per dollar in the lockbox was smaller. That for year 22 includes more scenarios in which the amount paid per dollar was smaller, and so on.

It is important to emphasize that these distributions indicate the ranges of possible future real incomes in each year as viewed from the present – that is 20 years hence, 21 years hence, etc.. clearly the distributions viewed from later years will differ. For example, in any given scenario and personal state, the income for year 21 would be the same as that for the personal state in the prior year. Thus for any personal state, every point on a year-over-year graph would lie on the 45-degree line. Once the annuity is purchased, all future uncertainty is resolved.

Lockbox Spending plus Future Purchase of an Annuity

The *iFAPlockbox* functions provide incomes that start in some future year. But our protagonists need income in the prior years. They need to couple a spending approach for those years with a plan to purchase an annuity thereafter. In the remainder of this chapter we will do so using lockbox spending for the initial years.

This can be done with the functions we already have. Funds available for the overall strategy would be allocated in some manner between the two components (a set of lockboxes for spending for a fixed number of years and a single lockbox to be used to purchase an annuity in the year after the last spending lockbox is employed). But experimentation would undoubtedly be needed to determine a desirable allocation between the two income sources. To obviate this, we provide a function that uses the two earlier functions to achieve an allocation that meets some pre-specified condition on incomes produced by the two strategies.

We use the name *iLBSplusFAP* to signify a combination of lockbox spending and future annuity purchase. The function for creating the requisite data structure is:

```

function iLBSplusFAP = iLBSplusFAP_create()
    % creates a data structure for a combination of lockbox spending
    % and future purchase of an annuity

    % lockbox proportions (matrix with TIPS in top row, market in bottom row
    iLBSplusFAP.lockboxProportions = [ ];

    % lockbox spending bequest utility ratio for spending
    iLBSplusFAP.bequestUtilityRatio = 0.50;

    % year in which annuity is to be purchased
    iLBSplusFAP.annuitizationYear = 20;

    % set initial proportion in TIPS for lockbox to be used to purchase annuity
    iLBSplusFAP.FAPlockboxProportionInTIPS = 0.50;

    % annuity ratio of value to initial cost
    iLBSplusFAP.annuityValueOverCost = 0.90;

    % percentile of income distribution to match for FAP and last
    % spending lockbox (0 to 100)
    iLBSplusFAP.incomePercentileToMatch = 50;

    % total amount invested
    iLBSplusFAP.amountInvested = 100000;

end

```

The first element is designed to contain a matrix of lockbox proportions of TIPS and the market portfolio for at least the years in which spending is to come from such lockboxes. The next indicates the bequest utility ratio for adjusting these proportions. Next is the future year in which the annuity is to be purchased (so only spending lockboxes for prior years will be included). The next two elements indicate the initial proportion of TIPS in the lockbox to be used to purchase the annuity and the ratio of the value of the incomes to the cost for the annuity..

The next element indicates the condition to be met by allocating funds between the spending lockboxes and the one designed to purchase the annuity. The goal is to have income at a given percentile of the distribution of income in the last spending year equal that in the first annuity year. This parameter specifies the percentile to be utilized . The last element indicates the total amount to be invested in all the lockboxes.

As indicated earlier, the *iLBSplusFAP_process* function uses the functions described in the previous sections of this chapter. And it does so twice. First, equal amounts are invested in the spending and FAP lockboxes with a temporary client data structure in order to find the distributions of income produced in the last lockbox spending year and the first annuity year. Next, these results are used to find the allocation of funds between the two sources that can achieve the desired relationship between the distributions. Finally, given this allocation, the analyses are repeated to produce new income and fee matrices to be added to the corresponding elements in the actual client data structure.

Here are the first and last statements in the function:

```
function [client, iLBSplusFAP] = iLBSplusFAP_process(client, iLBSplusFAP, market );  
....  
end
```

Note that the function returns a revised version of the *iLBSplusFAP* data structure. Why? In order to add two new elements indicating the dollar amounts actually invested in the two components (the set of lockboxes for spending and the lockbox to be used for the future purchase of the annuity).

The first set of statements creates a data structure called *clientTemp* that has the same data as the actual client. An *iLockboxSpending* data structure is then created and assigned the intended proportions of TIPS and the market portfolio, but only up to and including the year before the annuity is to be purchased. The amount invested is set to half the total available, then the lockbox spending strategy is processed.

```
% create a temporary client
clientTemp = client;
% process lockbox spending with 0.5 of the total amount invested
iLockboxSpending = iLockboxSpending_create;
% set lockbox proportions
iLockboxSpending.lockboxProportions = iLBSplusFAP.lockboxProportions;
% use lockbox spending up to and including the year before annuity purchase
lastSpendingYr = iLBSplusFAP.annuitizationYear - 1;
iLockboxSpending.lockboxProportions =...
    iLockboxSpending.lockboxProportions( :, 1:lastSpendingYr );
% set bequest utility ratio
iLockboxSpending.bequestUtilityRatio = iLBSplusFAP.bequestUtilityRatio;
% do not show lockbox proportions
iLockboxSpending.showLockboxAmounts = 'n';
% amount invested for lockbox spending
iLockboxSpending.investedAmount = 0.50 * iLBSplusFAP.amountInvested;
% process lockbox spending
clientTemp = client;
[ nscen nyrs ] = size( client.incomesM );
clientTemp.incomesM = zeros( nscen, nyrs );
clientTemp = iLockboxSpending_process( iLockboxSpending, clientTemp, market );
```

The next statements rather tediously analyze the distribution of income in the last year funded by lockbox spending. Only scenarios in which someone was alive at the time (personal states 1, 2 and 3) are considered. The incomes are sorted and the one corresponding to the desired percentile (*iLBSplusFAP.incomePercentileToMatch*) determined. This is then saved as *pctlIncSpending*.

```
% find percentile income in last lockbox spending year for matching states
ps = clientTemp.pStatesM( :, lastSpendingYr );
ii = find( (ps>0) & (ps<4) );
incs = clientTemp.incomesM( ii, lastSpendingYr );
sortincs = sort( incs, 'descend' );
matchPctl = iLBSplusFAP.incomePercentileToMatch;
matchPctl = matchPctl / 100;
if matchPctl >1 ; matchPctl = 1; end;
if matchPctl <0 ; matchPctl = 0; end;
n = matchPctl * length(sortincs);
n = round(n);
if n > length(sortincs); n = length(sortincs); end;
if n < 1; n = 1; end;
pctlIncSpending = sortincs(n);
```

The next two sections repeat the process for the first year in which income is to be provided by the annuity. As before, the income at the desired percentile (*pctlIncAnnuity*) is found. This is saved in the variable *pctlIncAnnuity*:

```
% create lockbox for future annuity purchase
iFAPlockbox = iFAPlockbox_create( );
% set year annuity is to be purchased
iFAPlockbox.yearOfAnnuityPurchase = iLBSplusFAP.annuitizationYear;
% set initial proportion in TIPS in the FAPlockbox
propTIPS = iLBSplusFAP.FAPlockboxProportionInTIPS;
iFAPlockbox.proportionInTIPS = propTIPS;
% set initial amount ($) in the lockbox
iFAPlockbox.investedAmount = 0.5 * iLBSplusFAP.amountInvested;
% process FAP lockbox with temporary client
clientTemp = client;
[ nscen nyrs ] = size( client.incomesM );
clientTemp.incomesM = zeros( nscen, nyrs );
clientTemp = iFAPlockbox_process( clientTemp, iFAPlockbox, market );
% find percentile amount spent in first annuity year matching states
ps = clientTemp.pStatesM( :, lastSpendingYr+1 );
incs = clientTemp.incomesM( ii, lastSpendingYr+1 );
sortincs = sort( incs, 'descend' );
n = matchPctl * length(sortincs);
n = round(n);
if n > length(sortincs); n = length(sortincs); end;
if n < 1; n = 1; end;
pctlIncAnnuity = sortincs(n);
```

All information needed to compute the amounts to be actually invested in the two strategies is now available. First we compute the incomes per dollar invested at the chosen percentile of the distributions. Given these values, it is straightforward to compute the proportions of total investment that should be allocated to the two income sources in order to make incomes the same in the adjacent years at the chosen percentile. These are used to revise the corresponding parameters in our data structures and the results added to the *iLBSplusFAP* data structure:

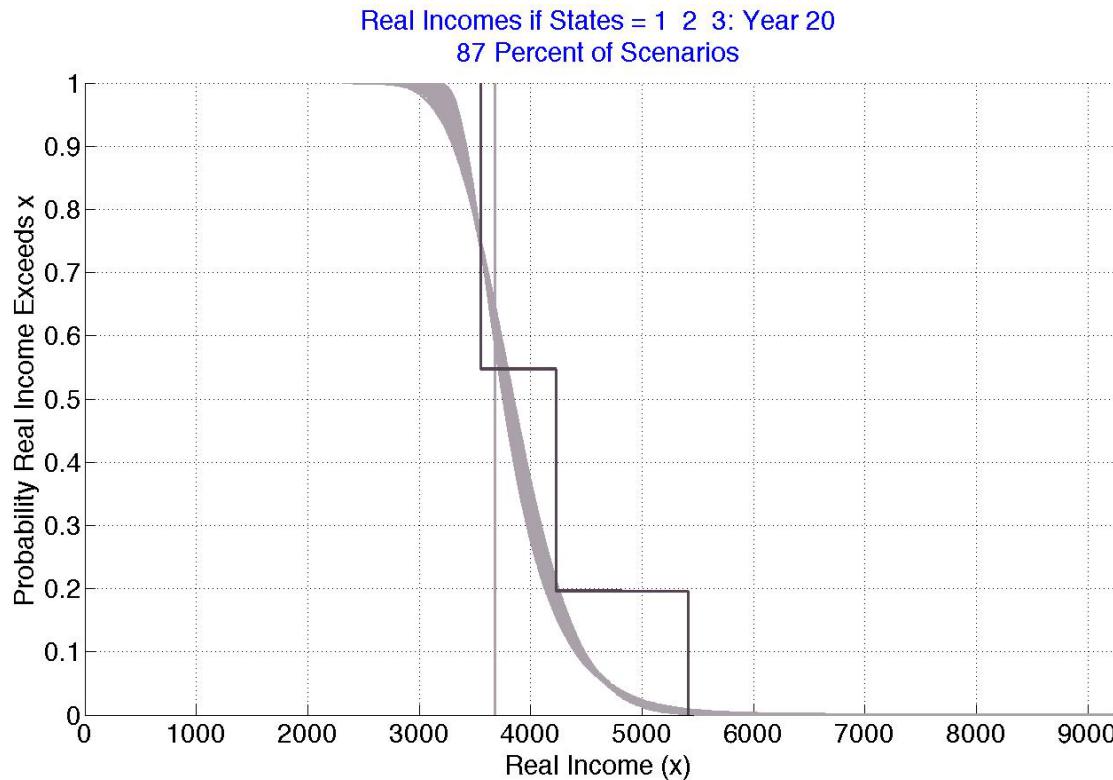
```
% compute revised amounts to be invested
% find incomes per dollar
incomePerDollarSpending = pctlIncSpending / iLockboxSpending.investedAmount;
incomePerDollarAnnuity = pctlIncAnnuity / iFAPlockbox.investedAmount;
% find proportions of total investment
sum = incomePerDollarSpending + incomePerDollarAnnuity;
propSpending = incomePerDollarAnnuity / sum;
propAnnuity = incomePerDollarSpending /sum;
% find total amount invested
totAmountInvested = ...
    iLockboxSpending.investedAmount + iFAPlockbox.investedAmount;
% put amounts to be invested in data structures
iLockboxSpending.investedAmount = propSpending * totAmountInvested;
iFAPlockbox.investedAmount = propAnnuity * totAmountInvested;
% add to iLBSplusFAP data structure
iLBSplusFAP.spendingAmountInvested = iLockboxSpending.investedAmount;
iLBSplusFAP.FAPAmountInvested = iFAPlockbox.investedAmount;
```

Finally, all is in place to generate incomes. We create another temporary client with zero incomes, create incomes from the spending lockboxes, then add the incomes and fees from the future annuity purchase. Finally, we add the incomes and fees for this strategy to the amounts in the respective client matrices:

```
% create incomes from lockbox spending
clientTemp = client;
[ nsцен nyrs ] = size( clientTemp.incomesM );
clientTemp.incomesM = zeros( nsцен, nyrs );
[clientTemp,iLockboxSpending] = ...
    iLockboxSpending_process( iLockboxSpending, clientTemp, market );
% add incomes and fees from FAP
clientTemp = iFAPlockbox_process( clientTemp, iFAPlockbox, market );
% add incomes to client income matrix
client.incomesM = client.incomesM + clientTemp.incomesM;
% add fees to client fee matrix
client.feesM = client.feesM + clientTemp.feesM;
```

And the tasks are done.

It is time to exercise all these functions. To begin, here is a graph in progress for a case in which (1) AMD2 lockboxes are used for spending and (2) the lockbox created to purchase the future annuity is invested entirely in Tips:

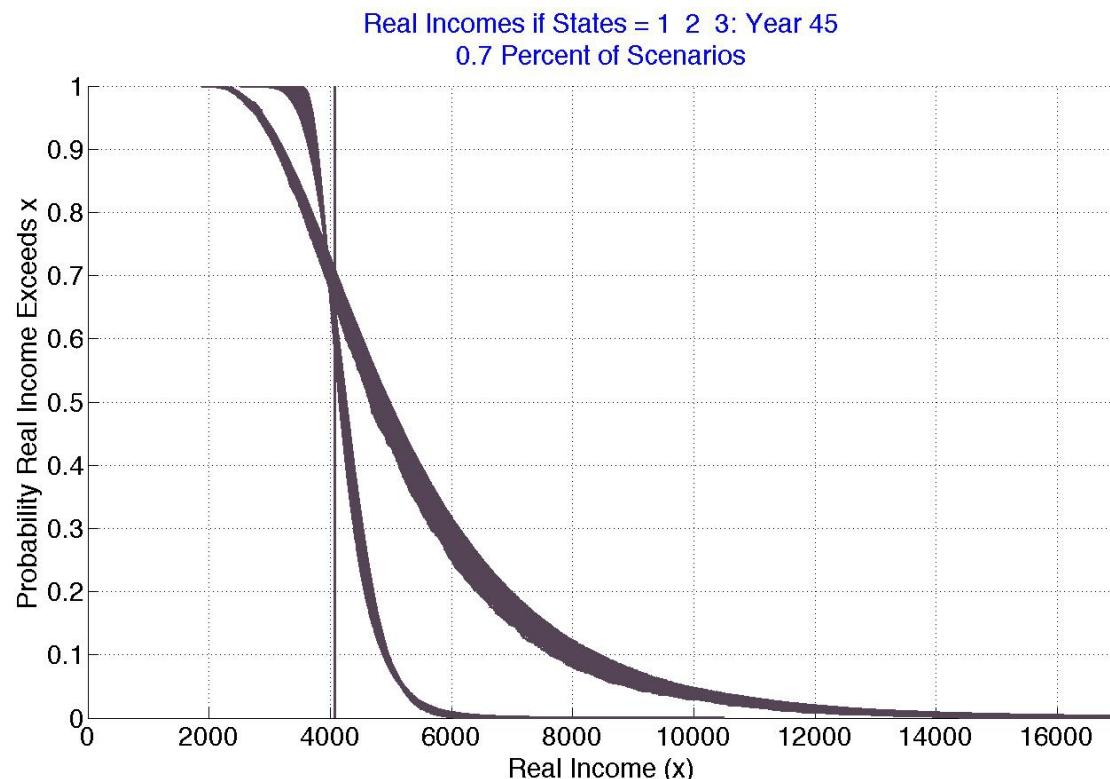


The distribution of income in year 20 is shown by the step function. The left-most section shows incomes produced in scenarios in which both Bob and Sue were alive in year 20. In each such case real income was the same – roughly \$3,300. The next section shows incomes produced in scenarios in which only Sue was alive at the time – roughly \$4,100. And the last section shows incomes when only Bob was alive – over \$5,000.

This is, of course, due to the fact that the insurance company will pay less per year when the number of years over which it might have to make payments is larger. The likely number of years is largest if both Bob and Sue are alive, somewhat lower if only Sue is alive, and even lower if Bob is alive, and the annuity purchase terms reflect such projections.

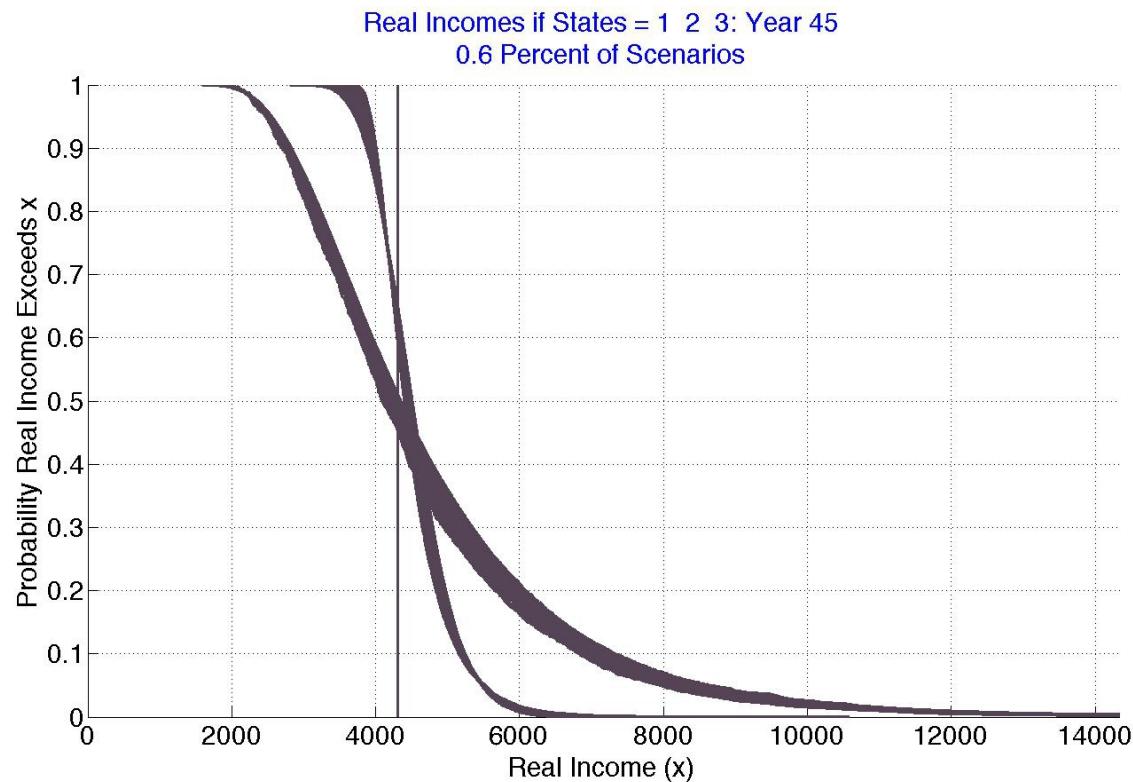
This relationship shows clearly an undesirable feature of a plan to purchase an annuity in some future year. The amount received per year is likely to be greater in personal states in which the need (somehow defined) may actually be less. When purchasing an immediate annuity, couples often choose to have lower payments when only one is alive (for example, with a 50% joint survivor benefit), on the grounds that expenses will be lower. But here, payments will be greater if only one is alive when the annuity is purchased. This will hold as well if the lockbox used to purchase the future annuity is invested in a combination of Tips and the market portfolio or even just the latter.

Consider now a case in which AMD2 lockboxes are again used for spending, with the FAP lockbox split equally between TIPS and the market portfolio at the outset:



The steeper curves showing incomes from the spending lockboxes are the same as before. The other curves show the distributions of incomes produced by the lockbox used to purchase the future annuity. They reflect not only the range of initial values at the time the annuity is purchased, but also the number of scenarios in which the income was set when both Bob and Sue were alive when the annuity was purchased, the number in which only Bob was alive, and the number in which only Sue was alive. And as we have seen, these proportions differ in different future years.

In the previous case, we used the default setting of *iLBS.incomePercentileToMatch* so the curves for years 20 and 21 cross at the 75'th percentile (0.75 on the vertical axis). Here is one in which the desired percentile was set to 50:

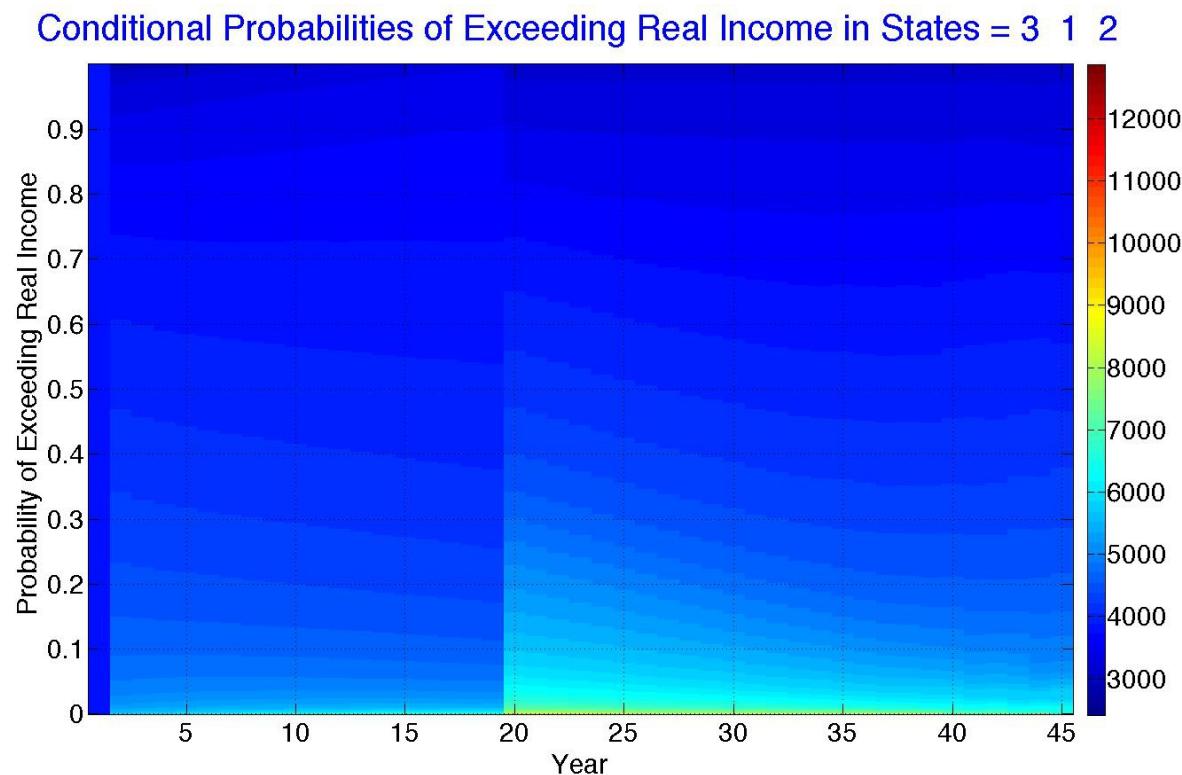


As desired, the curves for the last spending year and the first annuity year cross at the point at which probability equals 0.50. This shows how the parameter can be used to change the relationship between the distributions of income before and after the annuity takes over. To find most appropriate result for a given client may, of course, be difficult.

Here is another case. In this client script, after the lockbox proportions were set (in this case to *AMD2*), the proportion of TIPS in the lockbox to be used to purchase the annuity was set to equal the proportion in the last spending lockbox for the prior year:

```
% set proportions in FAP lockbox to those in the last spending lockbox  
yr = iLBSplusFAP.annuitizationYear;  
props = iLBSplusFAP.lockboxProportions( :, yr-1 );  
iLBSplusFAP.FAPlockboxProportionInTIPS = props(1)/(props(1)+props(2));
```

The resulting income map is colorful:



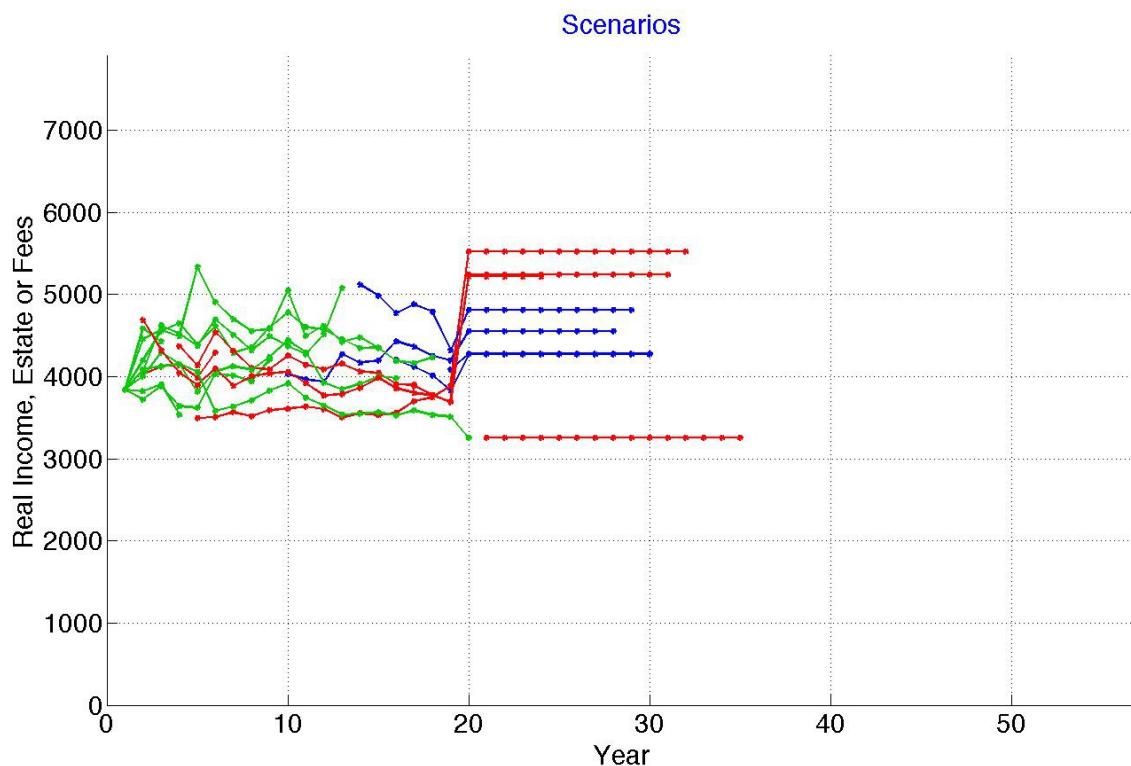
Note: to make the range of colors more informative, we set:

```
analysis.plotIncomeMapsPctMaxIncome = 95;
```

As can be seen, the range becomes larger when the annuity starts providing payments.

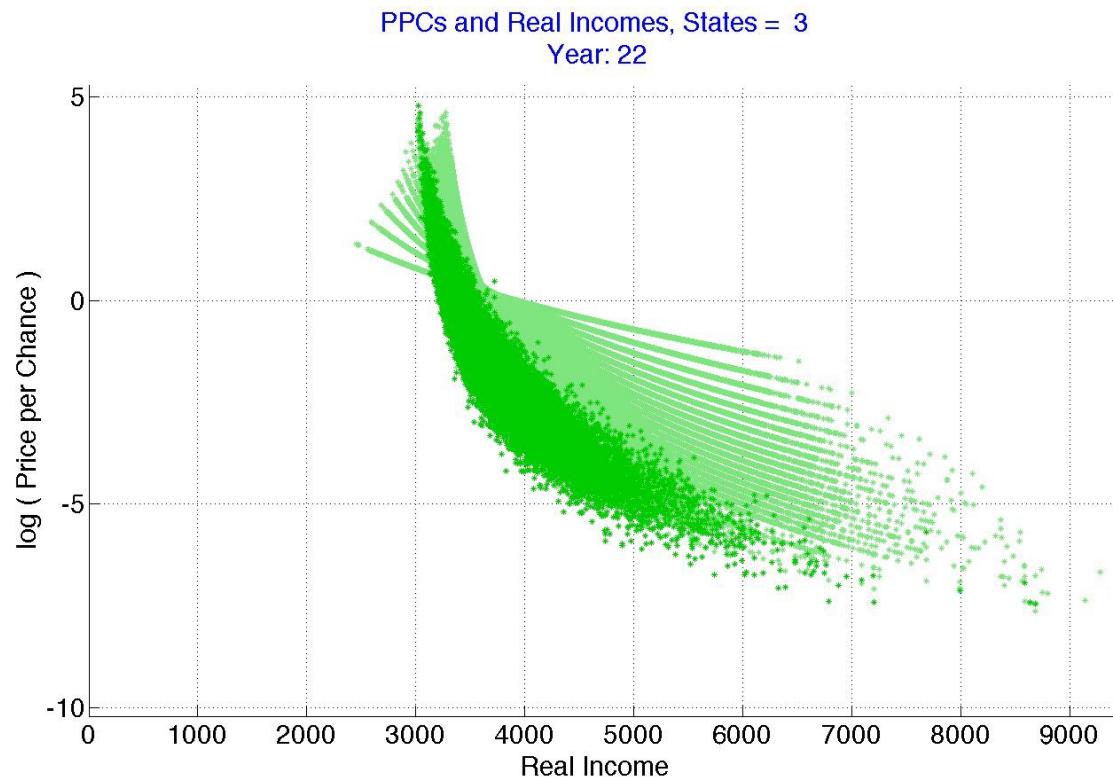
This income map could be misinterpreted. Before year 20, the range of incomes is the result of different market returns in each prior year. For each subsequent year it depends on the market returns up to year 20 and the person or persons who were alive at that time.

This can be seen clearly when a few scenarios are plotted:



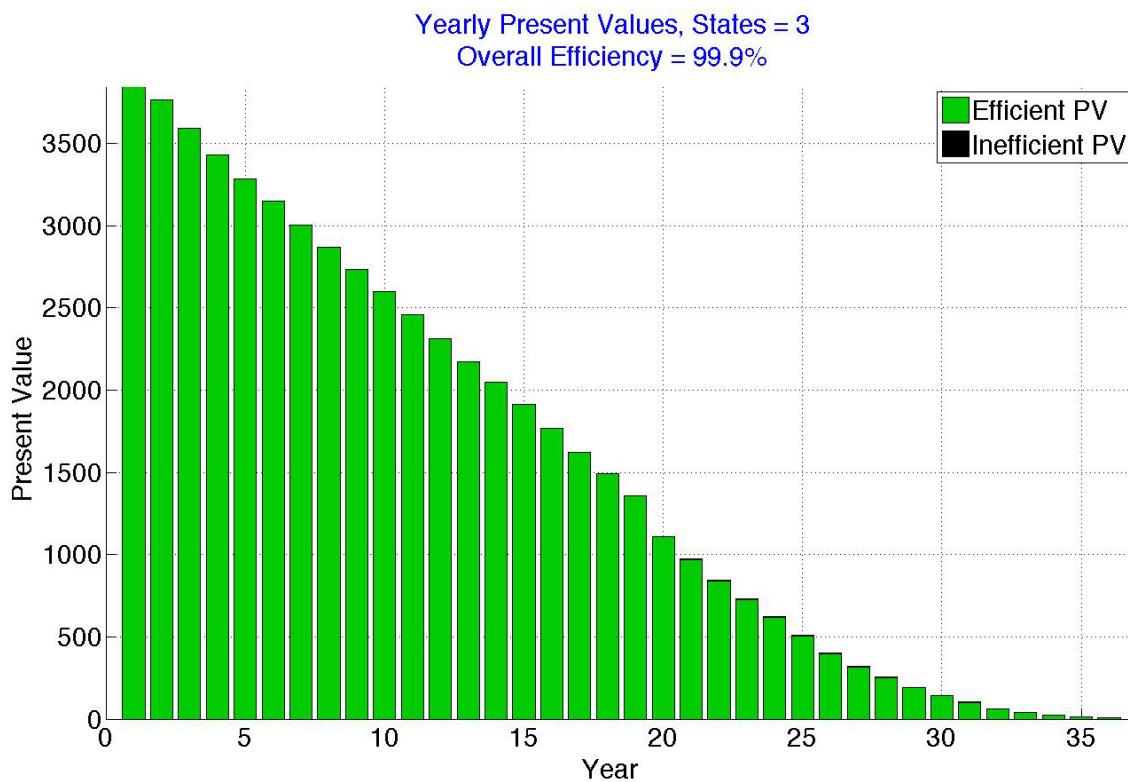
Looked at from today, there is uncertainty about incomes in, say, year 30. But once year 20 has occurred and the annuity has been purchased, all future incomes paid while someone is alive are known with complete certainty.

Next, the relationships between PPCs and real incomes. The following figure shows that the years with income from the spending lockboxes are completely consistent with marginal utility curves that become steeper each year, as we would expect from an AMD2 lockbox strategy. But once the annuity takes over, there is a scatter of points for each year, rather than a clearly defined curve, as illustrated by the points for year 22.



Why? Because the incomes in year 22 depend on cumulative market returns through year 19 times a constant, rather than on the cumulative market returns through year 22. And the prices for incomes year 22 are related to the cumulative market returns through that year. This strategy is cost-efficient only for years in which income is provided by the spending lockboxes and the first year that income is provided by the annuity.

In the case of personal state 3, the inefficiency is very small, as the yearly present value graph shows:



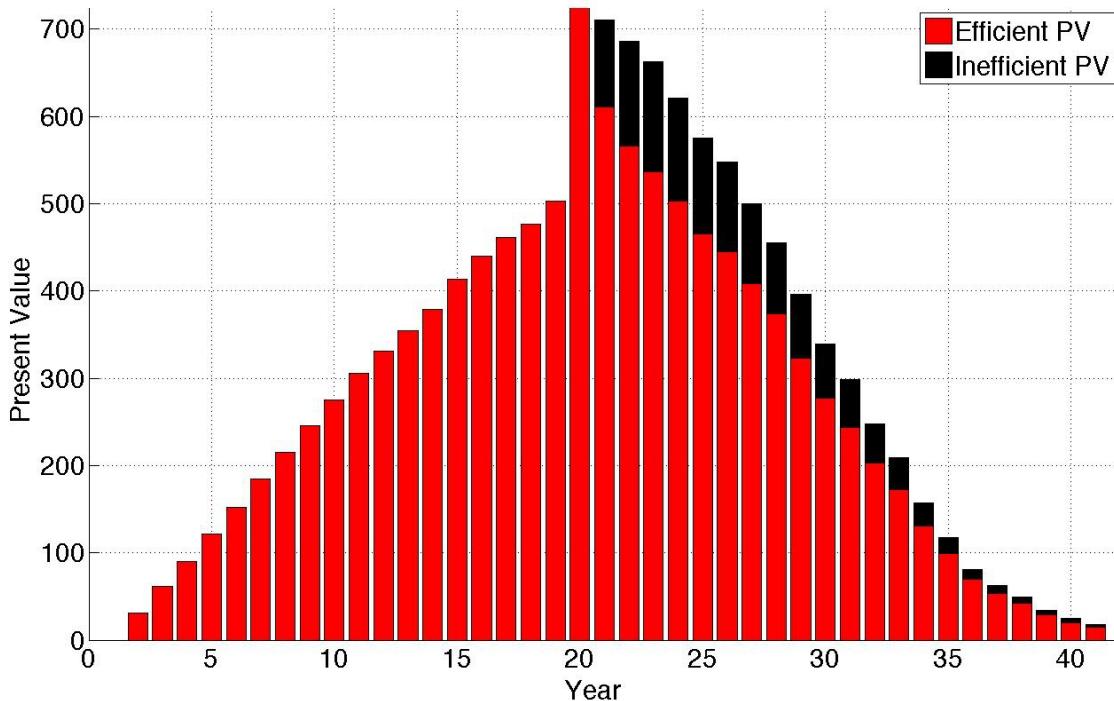
The black portions at the tops of the bars are hardly visible and the overall cost-efficiency is 99.9%. While the prior graph showed a wide scatter of points, the vast majority lie along a monotonic curve.

This is, however, not the case for personal states 1 and 2. As we know, in any given year in which only Bob is alive (state 1), there will be scenarios with income from an annuity purchased when only he was alive and others from an annuity purchased when both he and Sue were alive. And these incomes can be very different. The result is a wide scatter of points in a PPC/Income diagram for each year, with a resulting decrease in cost efficiency.

The figures on the next page show this for personal states 1 and 2.

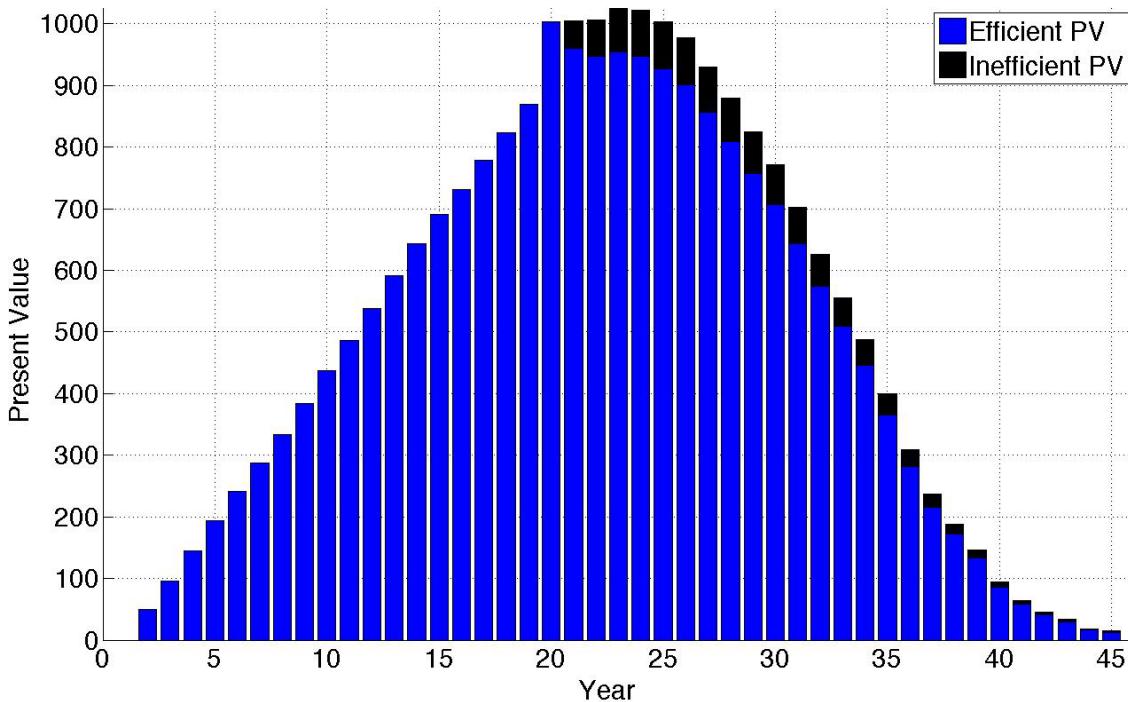
Yearly Present Values, States = 1

Overall Efficiency = 90.4%



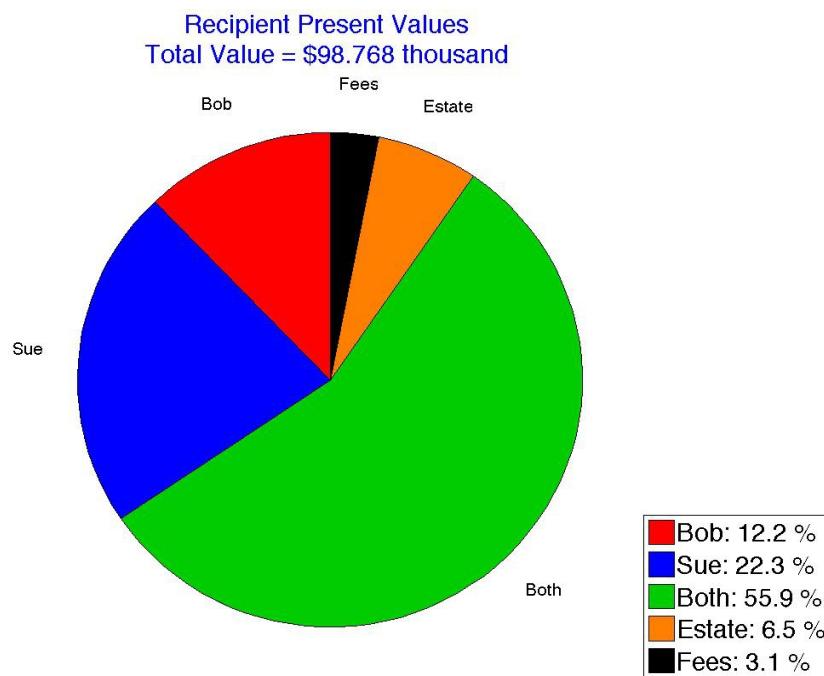
Yearly Present Values, States = 2

Overall Efficiency = 95.5%



The efficiencies are substantial: the minimum cost for incomes in scenarios when Bob is alone is 90.4% of the actual amount, and that for incomes in scenarios when Sue is the survivor is 95.5% of the actual cost. Yet another way in which the future purchase of a fixed annuity may be less than ideal.

Finally, the present values of the prospective incomes and fees for the interested parties:



As usual, the total will not precisely equal the amount invested, due to sampling error. Incomes in scenarios in which both Bob and Sue are alive are worth well over half the total amount invested (55.9%). Sues prospects are next (22.3%) with poor Bob third (12.2%) due to his sex (male) and age (67, compared to Sue at 65).

The estate's prospects are worth 6.5% of the total, due to the possibilities that both Bob and Sue will die before the annuity is purchased. In such situations, the estate will receive the contents of all unused lockboxes, including any intended for future spending as well as the one dedicated to the purchase of the future annuity.

In the event that at least someone lives to purchase the annuity, fees will of course be paid to the issuer. Although the cost will equal 10% of the total paid, this will only happen in scenarios in which an annuity is actually purchased (in other cases, the lockbox contents will go to the estate). Here, the present value is only 3.1% of the total initial investment.

The amounts invested in the two types of lockbox are found in the data structure:

iLBSplusFAP.spendingAmountInvested = 6.3959e+04
iLBSplusFAP.FAPAmountInvested = 3.6040e+04

so roughly 64% of the original amount is used to finance spending in years 1 through 19 and 36% to provide for purchase of an annuity to provide income thereafter.

A video of a case using AMD2 lockboxes, a bequest utility ratio of 0.50 and the future purchase of an annuity in year 20 can be found at:

www.stanford.edu/~wfsharpe/RISMAT/SmithCase_Chapter20_FAP.mp4

Other Spending Strategies plus Future Annuity Purchases

It is, of course possible to provide sequential incomes with other spending approaches, followed by the purchase of an immediate annuity in a future year. One could, for example, couple a proportional spending rule designed to exhaust all remaining funds in, say, year 19 with a lockbox intended to fund an annuity purchase in year 20. Another possibility would use a constant spending rule for income up to a designated year, then invest the remaining amount in an annuity at the time.

Each such approach involves two (or more) strategies that provide incomes *sequentially*, as does our *iLBSplusFAP* combination. Such procedures could substitute for, or complement, other income sources operating in parallel. Most retirees will have at least two such sources of income, one from Social Security, and one or more other sources from the types of strategies covered in this book, and/or possibly some not covered.

An important feature of most spending strategies is the option to change the rules for converting assets into income at future dates in response to unanticipated changes in circumstances. Lockbox spending approaches offer such *optionality* and can also provide cost-efficient income. Lockboxes designed to purchase annuities at a future date also offer optionality up to the time when such an annuity is purchased, although they may not provide subsequent income in a fully cost-efficient manner.

The next and final chapter will have more to say about such issues. Suffice it to say here that for some retirees, lockbox spending strategies can play a useful role in an overall strategy for providing future income.