[ECE 6707] Mobile Robot Mapping
Problem Set for Week #03

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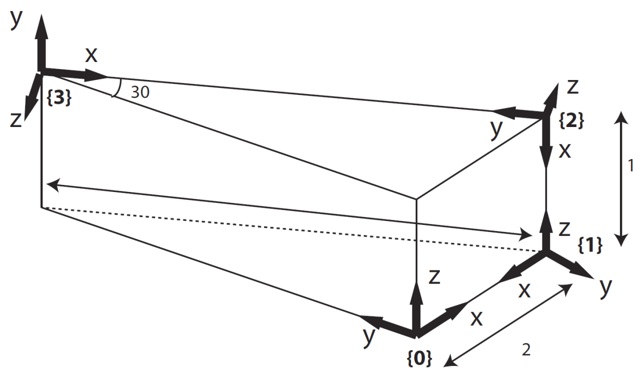
Problem 1. (a) Find SE(3) between {0} and {2}, T_{02}.



(b) What is SE(3) between {2} and {3}, T_{23}?

(c) What is SE(3) between $\{0\}$ and $\{3\}$, T_{03}?

(d) Check if $T_{03} = T_{02}T_{23}$ by multiplying them.

Problem 2. Let's consider the mobile robot that are equipped with encoder on each wheel.

(a) Derive distance from each wheel d_R, d_L where the encoder counter is C_R, C_L and coefficient α is determined by encoder resolution(=2048) and wheel diameter(=0.64).

(b) From the equation below,

$$d = \alpha C + \omega$$

Derive your uncertainty of distance (d_R,d_L) when noise has normal distribution $\sim \mathcal{N}(0, \sigma_\omega^2)$.

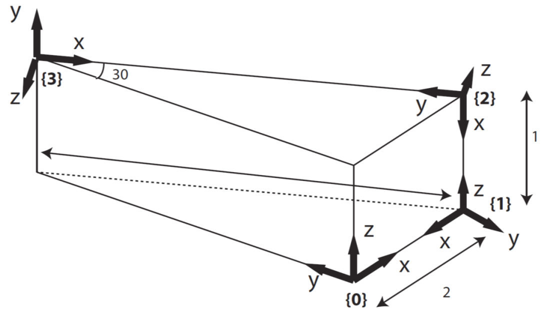
(c) From the equation below,

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{B} & -\frac{1}{B} \end{bmatrix} \begin{bmatrix} d_R \\ d_L \end{bmatrix}$$

Derive your uncertainty of odometry (Δx,Δy,$\Delta\theta$).

(d) Fill in Matlab script `ps_wheel_odometry.m` and plot trajectory.

Problem 1. (a) Find SE(3) between {0} and {2}, T_{02}.



Use Z-Y-X Euler angle!

$$R = Rot(z,\alpha)\, Rot(y,\beta)\, Rot(x,\gamma)$$

$$= \begin{bmatrix} c(\alpha)c(\beta) & c(\alpha)s(\beta)s(\gamma) - s(\alpha)c(\gamma) & c(\alpha)s(\beta)c(\gamma) + s(\alpha)s(\gamma) \\ s(\alpha)c(\beta) & s(\alpha)s(\beta)s(\gamma) + c(\alpha)c(\gamma) & s(\alpha)s(\beta)c(\gamma) - c(\alpha)s(\gamma) \\ -s(\beta) & c(\beta)s(\gamma) & c(\beta)c(\gamma) \end{bmatrix}$$

$$R_{01} = Rot(z,\pi)\, Rot(y,0)\, Rot(x,0)$$

$$t_{01} = [2, 0, 0]^T$$

$$R_{12} = Rot(z,0)\, Rot(y,\tfrac{\pi}{2})\, Rot(x,\tfrac{5\pi}{6})$$

$$t_{12} = [0, 0, 1]^T$$

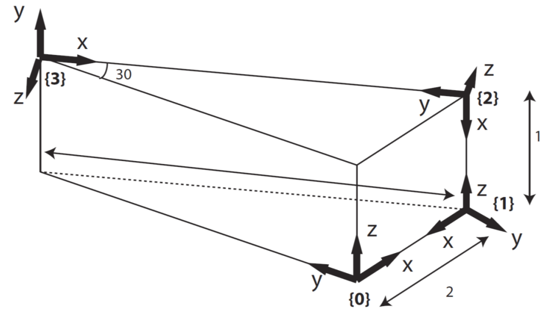
$$T_{01} = \begin{bmatrix} R_{01\,3\times3} & t_{01\,3\times1} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12\,3\times3} & t_{12\,3\times1} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$T_{02} = T_{01} \times T_{12} = \begin{bmatrix} 0 & -0.5 & 0.866 & 2 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

// Ans

Problem 1. (b) What is SE(3) between {2} and {3}, T_{23}?



Use Z-Y-X Euler angle!

$$R = Rot(z,\alpha)\, Rot(y,\beta)\, Rot(x,\gamma)$$

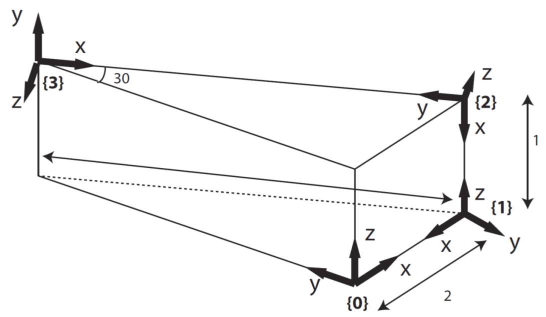
$$= \begin{bmatrix} c(\alpha)c(\beta) & c(\alpha)s(\beta)s(\gamma) - s(\alpha)c(\gamma) & c(\alpha)s(\beta)c(\gamma) + s(\alpha)s(\gamma) \\ s(\alpha)c(\beta) & s(\alpha)s(\beta)s(\gamma) + c(\alpha)c(\gamma) & s(\alpha)s(\beta)c(\gamma) - c(\alpha)s(\gamma) \\ -s(\beta) & c(\beta)s(\gamma) & c(\beta)c(\gamma) \end{bmatrix}$$

$$R_{23} = Rot(z,\tfrac{\pi}{2})\, Rot(y,\pi)\, Rot(x,0)$$

$$t_{23} = [0,4,0]$$

$$T_{23} = \begin{bmatrix} R_{23\,3\times3} & t_{23\,3\times1} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ // Ans

Problem 1. (c) What is SE(3) between {0} and {3}, T_{03}?



Use X−Y−Z Euler angle!

$$R = Rot(x,\alpha)\,Rot(y,\beta)\,Rot(z,\gamma)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = \frac{\pi}{2},\ \beta = -\frac{\pi}{3},\ \gamma = 0$$

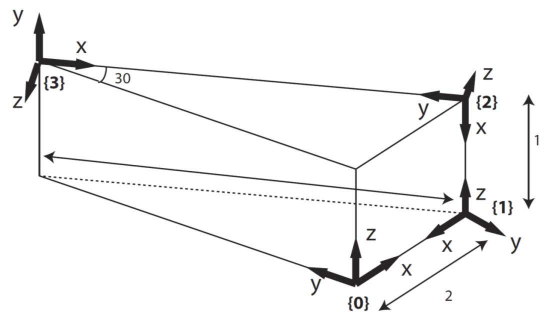
$$R_{03} = Rot(x, \tfrac{\pi}{2})\,Rot(y, -\tfrac{\pi}{3})\,Rot(z, 0)$$

$$t_{03} = (0, 2\sqrt{3}, 1)$$

$$T_{03} = \begin{bmatrix} R_{03\ 3\times3} & t_{03\ 3\times1} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & -0.866 & 0 \\ -0.866 & 0 & -0.5 & 3.4641 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

// Ans

Problem 1. (d) Check if $T_{03} = T_{02}T_{23}$ by multiplying them.



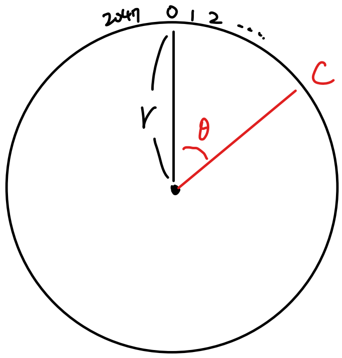
$$T_{02} \cdot T_{23} = \begin{bmatrix} 0 & -0.5 & 0.866 & 2 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & -0.866 & 0 \\ -0.866 & 0 & -0.5 & 3.4641 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \underline{T_{02} T_{23} = T_{03}} \; /\!/ \text{Ans}$$

Problem 2. Let's consider the mobile robot that are equipped with encoder on each wheel.

(a) Derive distance from each wheel d_R, d_L where the encoder counter is C_R, C_L and coefficient α is determined by encoder resolution(=2048) and wheel diameter(=0.64).



Assumption : no slip!

distance = wheel radius (r) × turning angle (θ)

$$= 0.32 \times \left(\frac{C}{2048} \times 2\pi \right)$$

$$\underline{d_L = \frac{\pi}{3200} \cdot C_L} \ // \text{Ans}$$

$$\underline{d_R = \frac{\pi}{3200} \cdot C_R} \ // \text{Ans}$$

Problem 2. Let's consider the mobile robot that are equipped with encoder on each wheel.

(b) From the equation below,

$$d = \alpha C + \omega$$

Derive your uncertainty of distance (d_R,d_L) when noise has normal distribution $\sim \mathcal{N}(0, \sigma_\omega^2)$.

$$d_R = \alpha C_R + W_R$$
$$d_L = \alpha C_L + W_L$$
$$\Rightarrow \begin{bmatrix} d_R \\ d_L \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 1 & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix} \begin{bmatrix} C_R \\ C_L \\ W_R \\ W_L \end{bmatrix}$$

$C_R \sim N(0, 0)$

$C_L \sim N(0, 0)$

$W_R \sim N(0, \sigma_w^2)$

$W_L \sim N(0, \sigma_w^2)$

Since $[C_R \ C_L \ W_R \ W_L]^T$ is gaussian distribution, $[d_R \ d_L]^T$ is also gaussian distribution

$$\Sigma_d = \begin{bmatrix} \alpha & 0 & 1 & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_w & 0 \\ 0 & 0 & 0 & \sigma_w \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \underline{\begin{bmatrix} \sigma_w & 0 \\ 0 & \sigma_w \end{bmatrix}} \text{// Ans}$$

Problem 2. Let's consider the mobile robot that are equipped with encoder on each wheel.

(c) From the equation below,

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{B} & -\frac{1}{B} \end{bmatrix} \begin{bmatrix} d_R \\ d_L \end{bmatrix}$$

Derive your uncertainty of odometry (Δx,Δy,$\Delta\theta$).

$$\Sigma_0 = \begin{bmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ \frac{1}{B} & -\frac{1}{B} \end{bmatrix} \begin{bmatrix} \sigma_w & 0 \\ 0 & \sigma_w \end{bmatrix} \begin{bmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\sin\theta & \frac{1}{B} \\ \frac{1}{2}\cos\theta & \frac{1}{2}\sin\theta & -\frac{1}{B} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\cos\theta\cdot\sigma_w & \frac{1}{2}\cos\theta\cdot\sigma_w \\ \frac{1}{2}\sin\theta\cdot\sigma_w & \frac{1}{2}\sin\theta\,\sigma_w \\ \frac{1}{B}\cdot\sigma_w & -\frac{1}{B}\cdot\sigma_w \end{bmatrix} \begin{bmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\sin\theta & \frac{1}{B} \\ \frac{1}{2}\cos\theta & \frac{1}{2}\sin\theta & -\frac{1}{B} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\cos^2\theta\cdot\sigma_w & \frac{1}{2}\sin\theta\cos\theta\,\sigma_w & 0 \\ \frac{1}{2}\sin\theta\cos\theta\,\sigma_w & \frac{1}{2}\sin^2\theta\,\sigma_w & 0 \\ 0 & 0 & \frac{2}{B^2}\cdot\sigma_w \end{bmatrix}$$

// Ans

(d) Fill in Matlab script `ps_wheel_odometry.m` and plot trajectory.