StatMeth Exercise Recipe (期中篇)

`statmeth`

Week 2 -> Section 3.2~4.2

Section 3.2 Basic concept

`Q21,Q23 refer to the sample data in the following Table`

Table 1 Pre-Employment Drug Screening Results

	Positive Test Result (Drug Use Is Indicated)	**Negative Test Result** (Drug Use Is Not Indicated)
Subject Uses Drugs	44 (True Positive)	6 (False Negative)
Subject Is Not a Drug User	90 (False Positive)	860 (True Negative)

> Q21) **Pre-Employment Drug Screening.**
> Find the probability of selecting someone who got a result that is a false negative.
> Who would suffer from a false negative result? Why?

`Solution`

(|sample space| = sample size)

$|\Omega|$ = TP + FP + FN +TN = 44 + 90 + 6 + 860 = 1000

$$P(FN) = \frac{6}{1000} = 0.006$$

The probability of selecting someone who got a result that is a false negative is 0.006.

Employer would suffer from a false negative result, because it would be at risk by hiring someone who uses drugs.

> Q23) **Pre-Employment Drug Screening.**
> Find the probability of selecting someone who uses drugs.
> Is the result close to the probability of 0.134 for a positive test result?

`Solution`

|{ Drugs }| = TP + FN = 44 + 6 = 50

$$P(Drugs) = \frac{50}{1000} = 0.05$$

The probability of selecting someone who uses drugs is 0.05. The result is not close to the rpbability of 0.134 for a positive test result.

`Q37,Q38 use the given sample space or construct the required sample space to find the indicated probability`

> Q37)**Three Children.**
> Use this sample space listing the eight simple events that are possible when a couple has three children : {bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}.
> Assume that boys and girls are equally likely, so that the eight simple events are equally likely.
> Find the probability that when a couple has three children, there is exactly one girl.

`Solution`

Ω = { bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg }

A = {one girl} = {bbg, bgb, gbb}

$$P(A) = \frac{3}{8} = 0.375$$

The probability of having one gilr in three children is 0.375.

> Q38)**Three Children.**
> Using the same sample space and assumption from Exercise 37, find the probability that when a couple has three children, there are exactly two girls.

`Solution`

B = {two girls} = {bgg, gbg, ggb}

$$P(A) = \frac{3}{8} = 0.375$$

The probability of having two gilrs in three children is 0.375.

Section 3.4 Multiplication Rule: Basics

`Q5 (a) determine whether events A and B are independent or dependent`

`(b) find P(A and B), the probability that events A and B both occur`

> Q5)
> A: When a month is randomly selected and ripped from a calendar and destroyed, it is July.
> B: When a different month is randomly selected and ripped from a calendar, it is November.

`Solution`

(因为在第一次取样后，日历从12个月变成只剩11个月，事件A和B的sample size不同)

$$P(A) = \frac{1}{12}$$

$$P(B) = \frac{1}{11}$$

If A didnot happen, *P(B)* would be $\frac{1}{12}$.

`Comment`

判断两个事件（event）是否相互独立（independent）有两个标准：

1. $P(A \cap B) = P(A) \cdot P(B)$
2. The occurence of A does not influence *P(B)*.

以上两点满足一个即可证明两个事件是相互独立的。

Section 3.5 Multiplication Rule: Complements and Conditional Probability

`Q5, provide a written description of the complement of the given event, then find the probability of the complement of the given event.`

> Q5)
> Five Girls When a couple has five children, all five are girls. (Assume that boys and girls are equally likely.)

`Solution`

A = { five girls }
$\overline{A}$ = { at least one boy }

$$P(\overline{A}) = 1 - P(A)$$

Ω ={ {ggggg}, {ggggb}, ..., {bbbbb}}

$|\Omega| = 2^5 = 32$ (每个小项是两种选择，boy or girl，五次选择)

$$P(A) = \frac{1}{2^5} = \frac{1}{32}$$

$$P(\overline{A}) = 1 - \frac{1}{32} = \frac{31}{32} \approx 0.969$$

Section 3.8 Baye's Theorem

`Q1 refer to the results summarized in the table below`

	Positive Test Result (Pregnancy Is Indicated)	Negative Test Result (Pregnancy Is Not Indicated)
Subject Is Pregnant	80	5
Subject Is Not Pregnant	3	11

> Q1)
> a). If one of the 99 test subjects is randomly selected, what is the probability of getting a subject who is pregnant?

`Solution`

|{ pregnant }| = TP + FN = 80 + 5 = 85

$$P(pregnant) = \frac{85}{99} \approx 0.859$$

> b). A test subject is randomly selected and is given a pregnancy test. What is the probability of getting a subject who is pregnant, given that the test result is positive?

`Solution`

$$\begin{aligned} P(pregnant|positive) &= \frac{P(pregnant \cap positive)}{P(positive)} \\ &= \frac{80/99}{(80+83)/99} = \frac{80}{83} \approx 0.964 \end{aligned}$$

> Q7) Pleas and Sentences In a study of pleas and prison sentences, it is found that 45% of the subjects studied were sent to prison.
> Among those sent to prison, 40% chose to plead guilty.
> Among those not sent to prison, 55% chose to plead guilty.
>
> a). If one of the study subjects is randomly selected, find the probability of getting someone who was not sent to prison.

`Solution`

P -> Prison
G -> Guilt

$P(P) = 0.45$

$P(G|P) = 0.4$

$P(G|\overline{P}) = 0.55$

$$P(\overline{P}) = 1 - P(P) = 1 - 0.45 = 0.55$$

> b). If a study subject is randomly selected and it is then found that the subject entered a guilty plea, find the probability that this person was not sent to prison.

`Solution`

$$
\begin{aligned}
P(\overline{P}|G) &= \frac{P(G|\overline{P})\cdot P(\overline{P})}{P(G|\overline{P})\cdot P(\overline{P}) + P(G|P)\cdot P(P)} \\
&= \frac{0.55 * 0.55}{0.55 * 0.55 + 0.4 * 0.45} \\
&= 0.627
\end{aligned}
$$

Additional Exercise

> Q1.4)
> A consulting firm rents cars from three agencies,
> 20% from agency A, 20% from agency B, and 60% from agency C.
> If 10% of the cars from A, 12% of the cars from B, and 4% of
> the cars from C have bad tires,
> what is the probability that the firm will get a car with bad tires?

`Solution`

$P(A) = 0.2$
$P(B) = 0.2$
$P(C) = 0.6$

$P(BAD|A) = 0.1$
$P(BAD|B) = 0.12$
$P(BAD|C) = 0.04$

⇒ Law of total probability :

$$
\begin{aligned}
P(BAD) &= P(BAD|A)\cdot P(A) + P(BAD|B)\cdot P(B) + P(BAD|C)\cdot P(C) \\
&= 0.1 * 0.2 + 0.12 * 0.2 + 0.04 * 0.6 \\
&= 0.068
\end{aligned}
$$

> Q1.6)
> "Where people turn to for news is different for various age groups."
> Suppose that a study conducted on this issue was based on **300 respondents who were between the ages of 46 and 60** and **300 respondents who were over age 60**. Of the 300 respondents who were **between the ages of 46 and 60, 82 got their news primary from newspapers**. Of the 300 respondents who were **over age 60, 120 got their news primarily from newspapers**.
>
> a) Given that a respondent is over age 60, what then is the probability that he or she gets news primarily from newspapers?

`Solution`

$P(Old \cap News) = \frac{120}{300} = 0.4$

> b) Given that a respondent gets news primarily from newspapers, what is the probability that he or she is over age 60?

`Solution`

|{News}| = 300 + 300 = 600

$$P(News) = \frac{(120+82)}{600} = \frac{202}{600}$$

$$P(Old|News) = \frac{P(Old \cap News)}{P(News)} = \frac{0.4}{202/600} \approx 0.594$$

> c) Explain the difference in the results in parts a and b.

`Solution`

$(Old \cap News)$ and $(Old|News)$ describe different events.

> d) Are the two events, whether the respondent is **over age 60** and whether he or she gets news primarily from **newspapers**, independent?

`Solution`

$P(Old) = \frac{300}{600} = 0.5$

$P(News) = \frac{202}{600} = \frac{101}{300}$

$P(Old \cap News) = \frac{120}{300} = 0.4$

Check if : $P(Old) \cdot P(News = P(Old \cap News)$

$P(Old) \cdot P(News) = 0.5 * \frac{101}{300} \approx 0.168 \neq 0.4$

$\Rightarrow$ not independent

Section 4.2 Discrete Probability function : Basic Skills and Concepts

`Q5, identify the given values as a discrete random variable, continuous random variable, or not a random variable.`

> Q5)
> a. Exact weights of quarters now in circulation in the United States

`Solution`

$\Rightarrow$ Continous random variable

> b. Numbers of tosses of quarters required to get heads

`Solution`

$\Rightarrow$ Discrete random variable

> c. Responses to the survey question "Did you smoke at least one cigarette in the last week?"

`Solution`

⇒ Not a random variable

> d. Numbers of spins of roulette wheels required to get the number 7

`Solution`

⇒ Discrete random variable

> e. Exact foot lengths of humans

`Solution`

⇒

Continuous random variable

> f. Shoe sizes (such as 8 or 8½) of humans

`Solution`

⇒ Discrete random variable

`Q7, determine whether a probability distribution is given.`

`If a probability distribution is given, find its mean`

`and standard deviation.`

`If a probability distribution is not given, identify the requirements that are not satisfied.`

> Q7) **Genetic Disorder**
> Four males with an X-linked genetic disorder have one child each. The random variable x is the number of children among the four who inherit the X-linked genetic disorder.

x	***P*(x)**
0	0.0625
1	0.2500
2	0.3750
3	0.2500
4	0.0625

Check : $\sum P(x) = 0.0625 + 0.2500 + 0.3750 + 0.2500 + 0.0625 = 1$

$$\begin{aligned}\mu = E(X) &= \sum_{i=1}^{k} x_i \cdot P(X = x_i) \\ &= 0 * 0.0625 + 1 * 0.2500 + 2 * 0.3750 + 3 * 0.2500 + 4 * 0.0625 \\ &= 2\end{aligned}$$

$$
\begin{aligned}
\sigma = \sqrt{Var(X)} &= \sqrt{\sum_{i=1}^{k} x_i^2 \cdot P(X = x_i) - \mu^2} \\
&= \sqrt{0^2 * 0.0625 + 1^2 * 0.2500 + 2^2 * 0.3750 + 3^2 * 0.2500 + 4^2 * 0.0625 - 2^2} \\
&= \sqrt{1} \\
&= 1
\end{aligned}
$$

Probability distribution with $\mu = 2.2, \sigma = 1.0$.

Additional Exercise

> Q2.9)
> Someone tosses a biased coin twice.
> The probability of head is 0.4, the probability of tail is 0.6.
> In the items below, you may leave a ratio or product in your answer.
>
> a) What are the sample space Ω and the probability measure P for this experiment?

`Solution`

$P(H) = 0.4$

$P(T) = 0.6$

Ω = {HH, HT, TH, TT}

$$
\begin{aligned}
P(HH) &= P(H)^2 = 0.4 * 0.4 = 0.16 \\
P(HT) &= P(TH) = 0.4 * 0.6 = 0.24 \\
P(TT) &= P(T)^2 = 0.6 * 0.6 = 0.36
\end{aligned}
$$

`Comment`

两个独立事件一同出现的概率是 $P(A \cap B) = P(A) \cdot P(B)$

`If head comes up the person who throws receives 1 euro,`

`if tail comes up she receives nothing.`

> b) Which outcomes belong to the event that the number of received euros of the second throw is the same as that of the first throw, and what is the probability of this event?

`Solution`

A = { 2nd throw is the same as 1st throw } = { HH, TT }

$P(A) = P(HH) + P(TT) = 0.16 + 0.36 = 0.52$

`Comment`

一个事件发生的概率是事件池里所有单独项的和。

c) Consider the random variable *X* which is the total amount of euros received in the experiment. Construct for all possible values *x* of *X* the probability *P(X = x)* based on the formal definition.

`Solution`

x	*X = x*	*P(X=x)*
0	{ TT }	0.36
1	{ TH, HT }	0.48 (0.24 *2)
2	{ HH }	0.16

d) Calculate the expectation *E(X)* of *X*; do not only give the result, but also show how this was obtained.

`Solution`

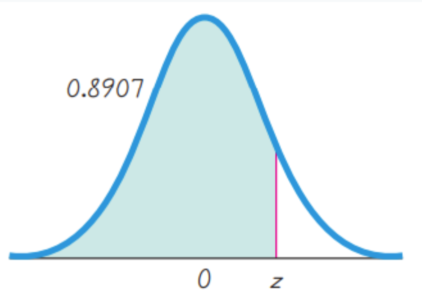
$$\mu = E(X) = 0 * 0.36 + 1 * 0.48 + 2 * 0.16 = 0.8$$

Week3 -> Section 5.2, 5.3, 5.5

Section 5.2 The Standard Normal Distribution

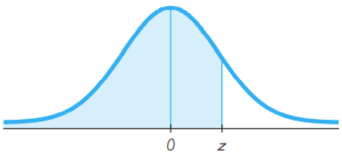
`In Q13,Q15, find the indicated z score.`
`The graph depicts the standard normal distribution of bone density scores with mean 0 and standard deviation 1.`

Q13)



`Solution`

$P(X < z) = 0.8907$



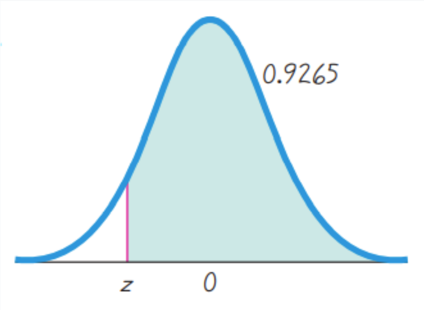
POSITIVE z Scores

Table 2 *(continued)* Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

$$Z_{0.8907} = 1.23$$

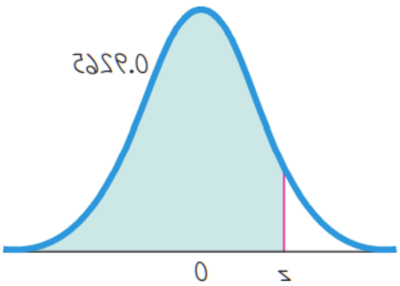
Q15)



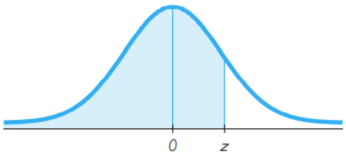
Solution

$$P(X > z) = 0.9265$$

方法一：
因为Normal Distribution的对称性，想象将图镜像处理，找到z的相反值，而小于-(z)的面积等于大于z的面积。



我们可以先在表格中找到 -(z)的值，然后再取它的负。



POSITIVE *z* Scores

Table 2 *(continued)* Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

（后面不再展示z-score Table的使用）

$$-Z_{0.8907} = 1.45$$

$$\Rightarrow Z = -1.45$$

方法二：

先求出白色区域面积（$P(X < z)$），根据白色面积的值在表格中找到对应的z-score.

$$P(X < z) = 1 - P(X > z) = 1 - 0.9265 = 0.0735$$

$$Z_{0.0735} = -1.45$$

`In Q17~25, assume that a randomly selected subject is given a bone density test.`

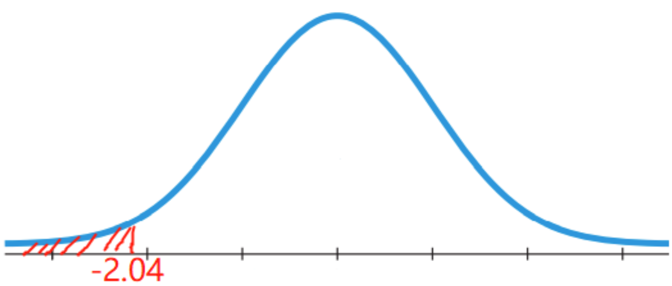
`Those test scores are normally distributed`

`with a mean of 0 and a standard deviation of 1.`

`In each case, **draw a graph** and **find the probability** of the given scores.`

> Q17) Less than -2.04

`Solution`

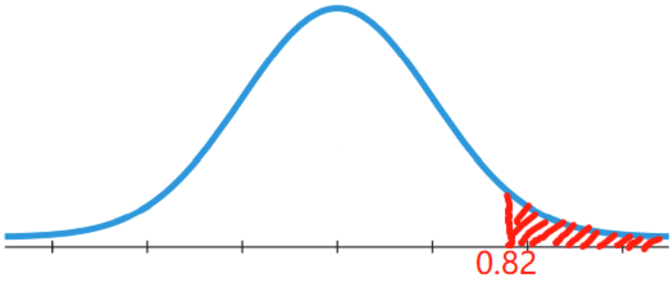


Look for the value in Table directly.

$$P(X < -2.04) = 0.0207$$

> Q21) Greater than 0.82

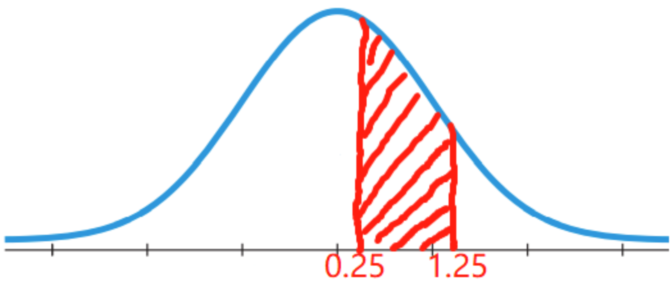
`Solution`



$$P(X > 0.82) = 1 - P(X < 0.82) = 1 - 0.7939 = 0.2061$$

> Q25) Between 0.25 and 1.25

`Solution`



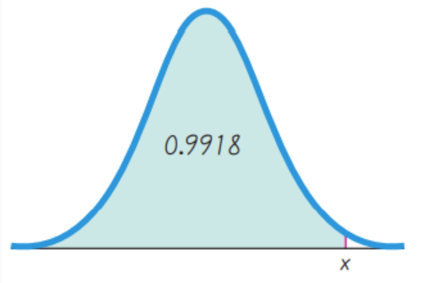
$$\begin{aligned} P(0.25 < X < 1.25) &= P(X < 1.25) - P(X < 0.25) \\ &= 0.8944 - 0.5987 \\ &= 0.2957 \end{aligned}$$

Section 5.3 Applications of Normal Distributions

`In Q9&Q11, find the indicated IQ score, and round to the nearest whole number.`

`The graphs depict IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard dev`

> Q9)



`Solution`

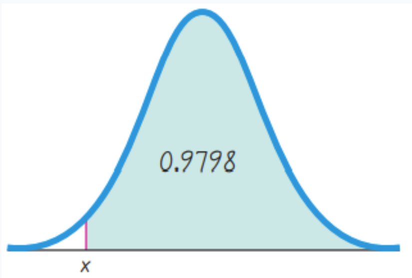
$X \sim N(100, 15^2)$

$P(X < Z) = 0.9918$

$Z_{0.9918} = 2.4$

$$z = \frac{x - \mu}{\sigma}$$
$$\Rightarrow x = z \cdot \sigma + \mu = 2.4 * 1.5 + 100 = 136$$

Q11)



`Solution`

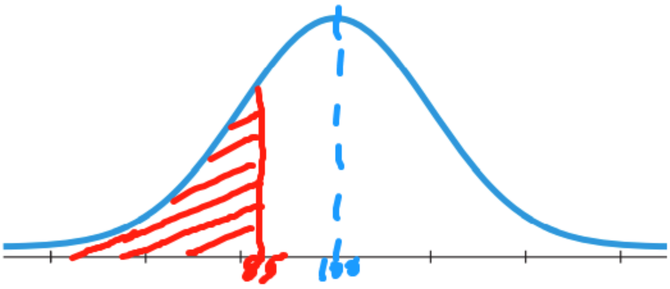
$P(X > z) = 0.9798$
$-z_{0.9798} = 2.05 \Rightarrow z = -2.05$

$$x = z \cdot \sigma + \mu = -2.05 * 15 + 100 = 69.25$$

`In Q13~17, assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviatio`
`For a randomly selected adult, find the indicated probability or IQ score.`
`Round IQ scores to the nearest whole number. (Hint: Draw a graph in each case.)`

Q13) Find the probability of an IQ less than 85.

`Solution`



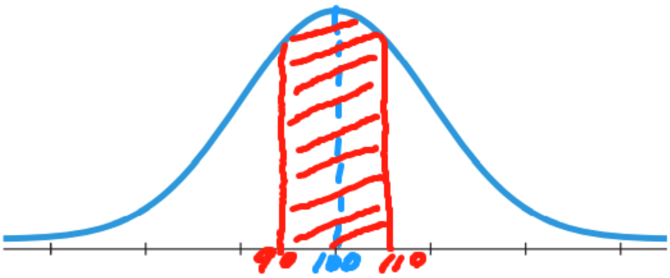
$X \sim N(100, 15^2)$

$z = \frac{x-\mu}{\sigma} = \frac{85-100}{15} = -1$

$$P(z < -1) = 0.1587$$

> Q15) Find the probability that a randomly selected adult has an IQ between 90 and 110 (referred to as the normal range).

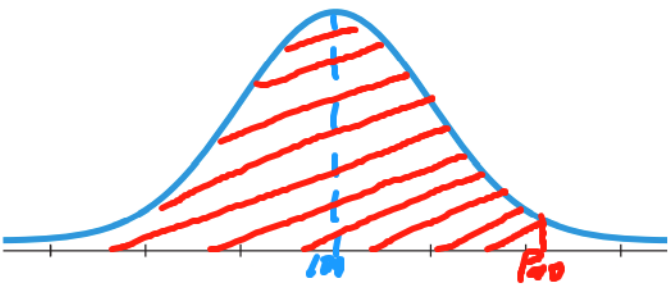
`Solution`



$$\begin{aligned} P(90 < X < 110) &= P(X < 110) - P(X < 90) \\ &= P(z < \frac{110-100}{15}) - P(z < \frac{90-100}{15}) \\ &= 0.7486 - 0.2514 \\ &= 0.4972 \end{aligned}$$

> Q17) Find P_{90}, which is the IQ score separating the bottom 90% from the top 10%.

`Solution`



P_{90} Separats the 90% from the top 10%.

$P(X < z) = 0.9 \Rightarrow z = 1.28$

$$x = 1.28 * 15 + 100 = 119.2$$

Section 5.5 The Central Limit Theorem

`In Q5,Q7, use this information about the overhead reach distances of adult females: Mean = 205.5 cm, SD = 8.6 cm, and overhead reach distances are normally distributed (based on data from the Federal Aviation Administration). The overhead reach distances are used in planning assembly work stations`

> Q5) a. If 1 adult female is randomly selected, find the probability that her overhead reach is less than 222.7 cm.

`Solution`

$\mu = 205.5$, $\sigma = 8.6$, $X \sim N(205.5, 8.6^2)$

$$\begin{aligned} &P(x < 222.7) \\ &= P(z < \frac{222.7 - 205.5}{8.6}) \\ &= P(z < 2) \\ &= 0.9772 \end{aligned}$$

> b. If 49 adult females are randomly selected, find the probability that they have a mean overhead reach less than 207.0 cm.

`Solution`

Apply the Central Limit Theorem :

$$\overline{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

Note that, because we take samples from a normal distributed population, therefore, no matter what the sample size is, the Central Limit Theorem applies.

$$\begin{aligned} &P(x < 207.0) \\ &= P(z < \frac{207 - 205.5}{8.6/\sqrt{49}}) \\ &= P(z < 1.22) \\ &= 0.8888 \end{aligned}$$

> Q7) 7. a. If 1 adult female is randomly selected, find the probability that her overhead reach is greater than 218.4 cm.

`Solution`

$$
\begin{aligned}
&P(x>218.4)\\
&=1-P(x<218.4)\\
&=1-P(z<\frac{218.5-205.5}{8.6})\\
&=1-P(z<1.5)\\
&=1-0.9332\\
&=0.0668
\end{aligned}
$$

> b. If 9 adult females are randomly selected, find the probability that they have **a mean** overhead reach greater than 204.0 cm.

`Solution`

Apply CLT :

$$
\begin{aligned}
&P(x>204)\\
&=1-P(x<204)\\
&=1-P(z<\frac{204-205.5}{8.6/\sqrt{9}})\\
&=1-P(z<-0.52)\\
&=1-0.3015\\
&=0.6985
\end{aligned}
$$

> c. Why can the normal distribution be used in part (b), even though the sample size does not exceed 30?

`Solution`

Because the sample came from a population with normal distribution, the sample size can be any.

> Q11) **Elevator Safety.**
> Example 2 referred to an Ohio elevator with a maximum capacity of 2500 lb. When rating elevators, it is common to use a 25% safety factor, so the elevator should actually be able to carry a load that is 25% greater than the stated limit.
>
> The maximum capacity of 2500 lb becomes 3125 lb after it is increased by 25%, so 16 male passengers can have a mean weight of up to 195.3 lb.
>
> If the elevator is loaded with 16 male passengers, find
> the probability that it is overloaded because they have a mean weight greater than 195.3 lb.
>
> (As in Example 2, assume that weights of males are normally distributed with a mean of 182.9 lb and a standard deviation of 40.8 lb.)
>
> Does this elevator appear to be safe?

`Solution`

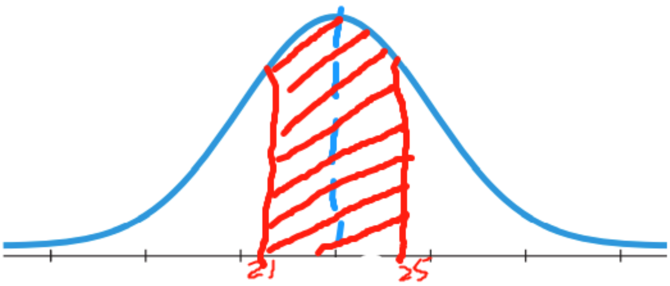
$X \sim N(182.9, 40.8^2)$ -distribution

$$\begin{aligned}P(X > 195.3) &= 1 - P(195.3)\\&= 1 - P(z < \frac{195.3 - 182.9}{40.8/\sqrt{16}})\\&= 1 - P(z < \frac{12.4}{10.2})\\&= 1 - P(z < 1.22)\\&= 1 - 0.8888\\&= 0.1112\end{aligned}$$

There is about 10% chance that the elevator overloads.
Since 10% is less than the safety factor 25%, the elevator is safe.

> Q13) **Designing Hats**.
> Women have head circumferences that are normally distributed with a mean of 22.65 in. and a standard deviation of 0.80 in. (based on data from the National Health and Nutrition Examination Survey).
>
> a. If the Hats by Leko company produces women' s hats so that they fit head circumferences between 21.00 in. and 25.00 in.,
> what percentage of women can fit into these hats?

`Solution`

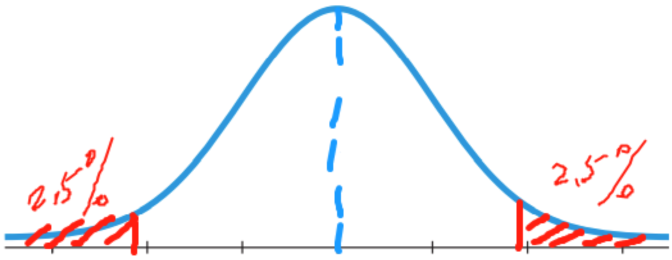


$X \sim N(22.65, 0.8^2)$

$$\begin{aligned}P(21 < X < 25) &= P(X < 25) - P(X < 21)\\&= P(z < \frac{25 - 22.65}{0.8}) - P(z < \frac{21 - 22.65}{0.8})\\&= P(z < 2.94) - P(z < -2.06)\\&= 0.9984 - 0.0197\\&= 0.9787\end{aligned}$$

> b. If the company wants to produce hats to fit all women except for those with the smallest 2.5% and the largest 2.5% head circumferences, what head circumferences should be accommodated?

`Solution`



$z_{0.0025} = -1.96$
$z_{1-0.0025} = 1.96$

$$x_1 = -1.96 * 0.8 + 22.65 = 21.08$$

$$x_2 = 1.96 * 0.8 + 22.65 = 24.2$$

> c. If 64 women are randomly selected, what is the probability that their mean head circumference is between 22.00 in. and 23.00 in.?
> If this probability is high, does it suggest that an order for 64 hats will very likely fit each of 64 randomly selected women?
> Why or why not?

`Solution`

n = 64

$$\begin{aligned} P(22 < x < 23) &= P(x < 23) - P(x < 21) \\ &= P(z < \frac{23 - 22.65}{0.8/\sqrt{64}}) - P(z < \frac{21 - 22.65}{0.8/\sqrt{64}}) \\ &= P(z < 3.5) - P(z < -6.5) \\ &= 0.9999 - 0.0001 \\ &= 0.9998 \end{aligned}$$

Even though the result is almost 100%, it does not suggest that an order for 64 hats will very like fit each of 64 randomly selected women. Because the result represent the distribution of means, there is a big popultion be excluded between 21 and 23.

Good luck with the exam !