

# Generative Adversarial Networks (GANs)

## Introduction to the theory of GANs

Jiashun Yao

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# Outline

- Generative model
- Adversarial loss
- Theory of GANs
- How to train GANs
- Flavours of GANs

# Why GANs



## Quora Session with Yann LeCun

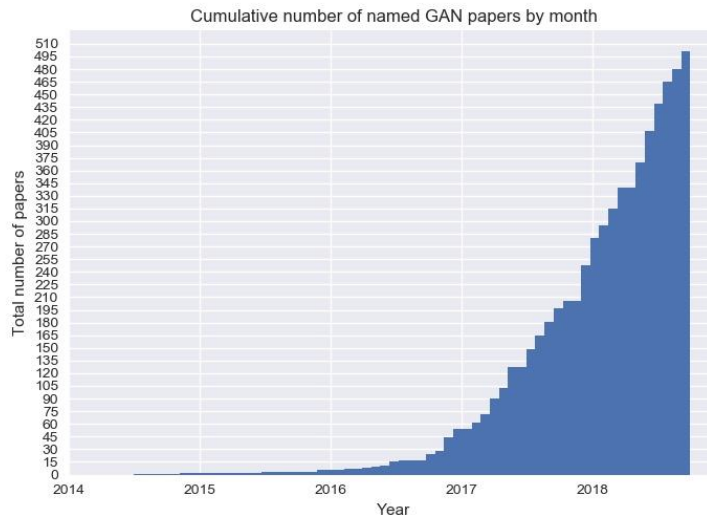
Yann LeCun, Director of AI Research at Facebook and Professor at NYU

Answered July 28, 2016 · Upvoted by Joaquin Quiñero Candela, studied Machine Learning and Gokul Krishnan, M.Sc Computer Science & Machine Learning, ETH Zurich (2018)

There are many interesting recent development in deep learning, probably too many for me to describe them all here. But there are a few ideas that caught my attention enough for me to get personally involved in research projects.

The most important one, in my opinion, is adversarial training (also called GAN for Generative Adversarial Networks). This is an idea that was originally proposed by Ian Goodfellow when he was a student with Yoshua Bengio at the University of Montreal (he since moved to Google Brain and recently to OpenAI).

- GANs has shown tremendous success in the generation of realistic data;
- Can be used to address many kinds of problems (generation, clustering, representation learning, translation ...) for many types of data (image, audio, video, text ...)



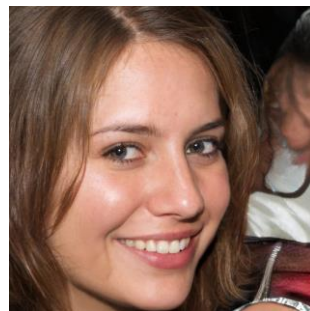
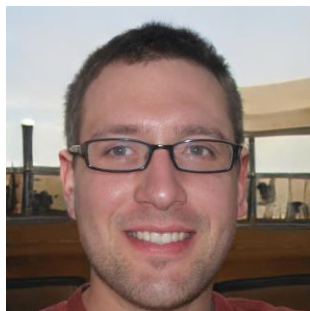
<https://github.com/hindupuravinash/the-gan-zoo>

## Generative adversarial networks

IJ Goodfellow, J Pouget-Abadie, M Mirza, B Xu... - arXiv preprint arXiv ..., 2014 - arxiv.org  
We propose a new framework for estimating **generative** models via an **adversarial** process, in which we simultaneously train two models: a **generative** model G that captures the data distribution, and a discriminative model D that estimates the probability that a sample came ...

☆ 99 Cited by 30182 Related articles All 55 versions »

# Example – 1 – generation of realistic human faces

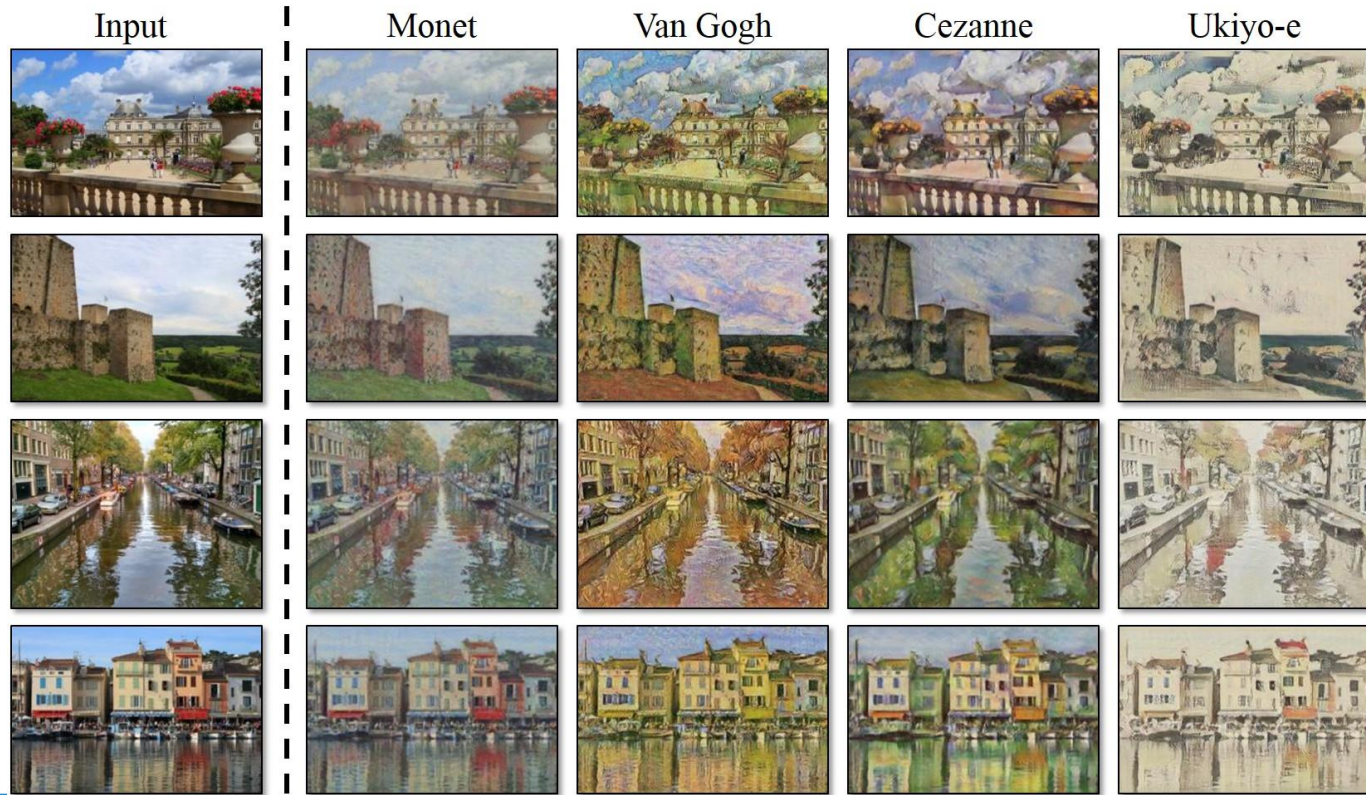


## Example – 2 – style mix of faces





# Example – 3 – photos to arts

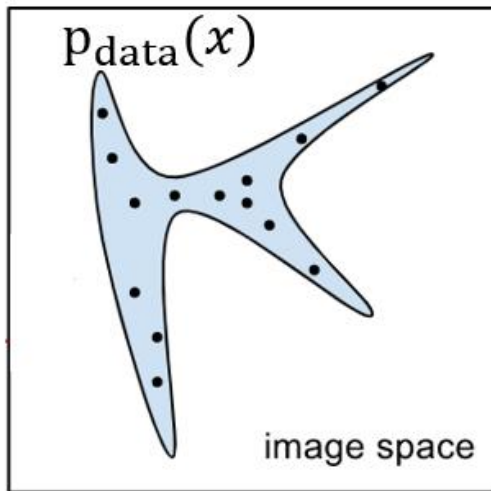


# Generative model

Given training real data

- find the distribution  $p_{\text{data}}(x)$  that explains where the samples are from
- Use  $p_{\text{data}}(x)$  to generate realistic samples.

true data distribution



$x$  – image or other data  
high dimensional vector

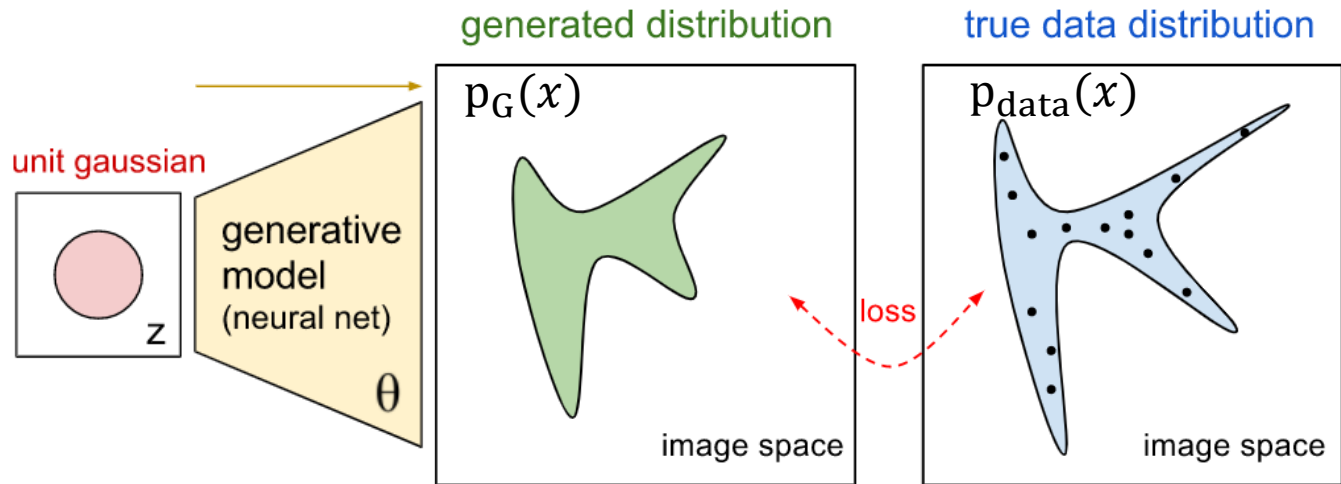
Cannot explicitly know the probability density function that describes of  $p_{\text{data}}(x)$ .

Use neural network  $G$  to learning  $p_{\text{data}}(x)$ .

# Generative model

Given training real data, for a generative network ( $G$ ) :

- $G$  is a mapping function from  $z$  to data space
- True data distribution  $p_{\text{data}}(x)$
- Generated data distribution  $p_G(x)$



We want to find  $G^*$   
that  
 $p_{G^*} \approx p_{\text{data}}$

$x$ : high dimensional vector



# Adversarial loss for the generative model

To find optimal  $G^*$ , update network parameters, to minimise the divergence between  $p_G$  and  $p_{data}$ :

$$G^* = \arg \min_G Div (P_{data}(x) || P_G(x))$$

No simple loss function available to measure this divergence.

VAE trains on evidence lower bound (ELBO), a surrogate of  $Div (P_{data}(x) || P_G(x))$ .

# Adversarial loss for the generative model

## VAE:

1. Sample  $x_i$  from training data
2. Train  $Encoder(x_i) \rightarrow z_i, Decoder(z_i) \rightarrow \hat{x}_i$ 
  - Explicit loss between  $x_i$  and  $\hat{x}_i$  (for images, pixel-wise comparison)
3. Once trained, sample  $z_m$  from  $z$ , to generate  $x_m$

## Issues of VAE:

- Pixel-loss not intelligent enough
- Blurry output

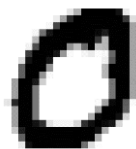
# Adversarial loss for the generative model

## VAE:

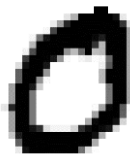
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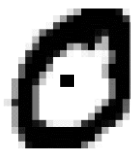
- Pixel-loss not intelligent enough
- Blurry output



*True*



*Sample 1*



*Sample 2*

*Which is better or equally good?*

# Adversarial loss for the generative model

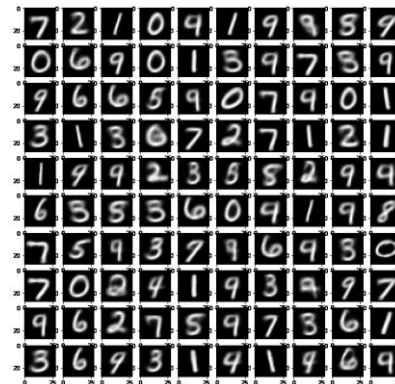
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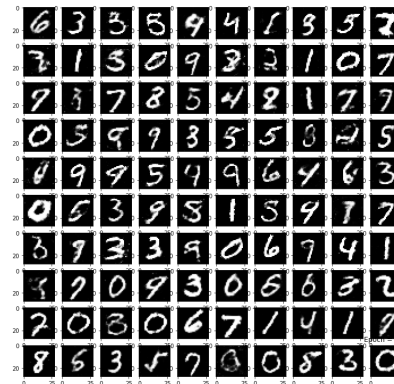
## Issues of VAE:

- Pixel-loss not intelligent enough
- Blurry output

Distance between samples good enough?  
Distance between complex distributions?



VAE



GANs

# Adversarial loss for the generative model

**GAN** provides a method to optimise  $G^* = \arg \min_G \text{Div} (p_{data} || p_G)$  by using another NN (Discriminator) for the loss function – the “adversarial loss”.

**Training GAN → Minimising J-S divergence**  
(other divergences/distances possible too)

- Intelligent & flexible
- “Perceptually better” than sample-wise losses

# Theory of GANs

**GANs:** Generator NN ( $G$ ) + Discriminator NN ( $D$ )

Generator  $G$ :

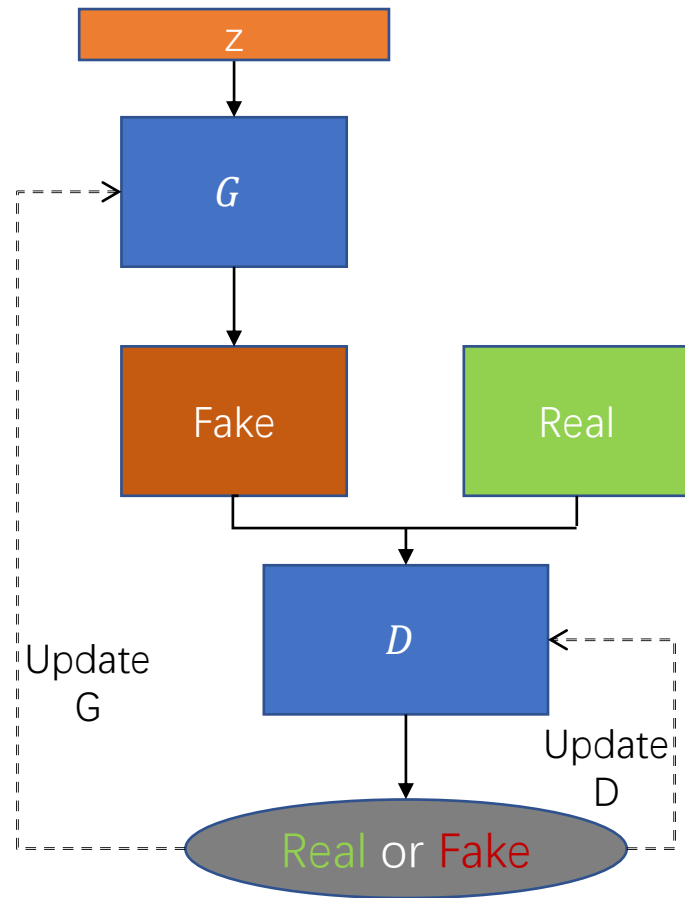
- Input: a vector  $z$  from  $p_{prior}(z)$
- Output: **Fake** data sample  $G(z)$

Discriminator  $D$ :

- Input: a **Real** data sample  $x$  or a **Fake** sample  $G(z)$
- Output: a scalar (possibility of input to be **Real**)
  - $D$  is a binary classifier

**Real:** training set data

**Fake:** generator output





# Theory of GANs

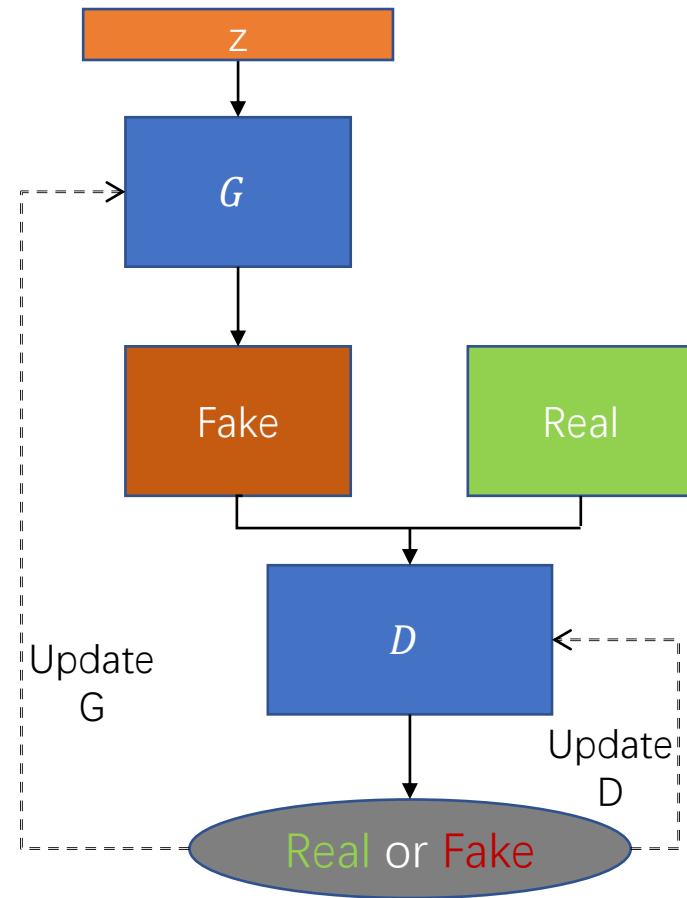
To make  $G(z) \rightarrow$  Realistic, we play a zero-sum competition game:

- train  $D$  to correctly label its input to be **Real** or **Fake**;
- train  $G$  to “fool”  $D$  to label  $G(z)$  to be **Real**.

So that both  $D$  and  $G$  improve as during training:

- $G$  can generate more and more realistic data
- $D$  can tell more and more detailed differences between Fake and Real data

Optimal  $G$ :  $p_G = p_{data}$



# Theory of GANs

To play a zero-sum game, we use a min-max loss function:

$$\min_G \max_D V_{GAN}(G, D) = \mathbb{E}_{x \sim p_{data}} [\log D(x)] + \mathbb{E}_{z \sim p_z} [\log (1 - D(G(z)))]$$

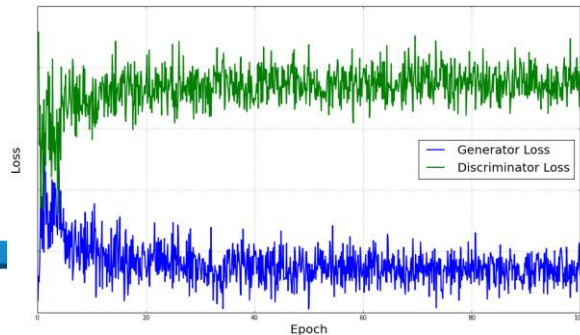
Train  $D$  to maximise the loss, so that:

- Input is **Real**  $\rightarrow D(x) \rightarrow 1$
- Input is **Fake**  $\rightarrow D(G(z)) \rightarrow 0$

Train  $G$  to minimise the loss:

- Fools  $D$  to give high score when input is **Fake**:  $D(G(z)) \rightarrow 1$

Loss curve oscillates rather than decreases:



# Theory of GANs

$$\min_G \max_D V_{GAN}(G, D) = \mathbb{E}_{x \sim p_{data}} [\log D(x)] + \mathbb{E}_{z \sim p_z} [\log (1 - D(G(z)))]$$

Given  $x$ :

$$D^* = \arg \max_D [p_{data}(x) \log D(x) + p_G(x) \log (1 - D(x))]$$

It gives:

$$D^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$

Substitute into the objective function:

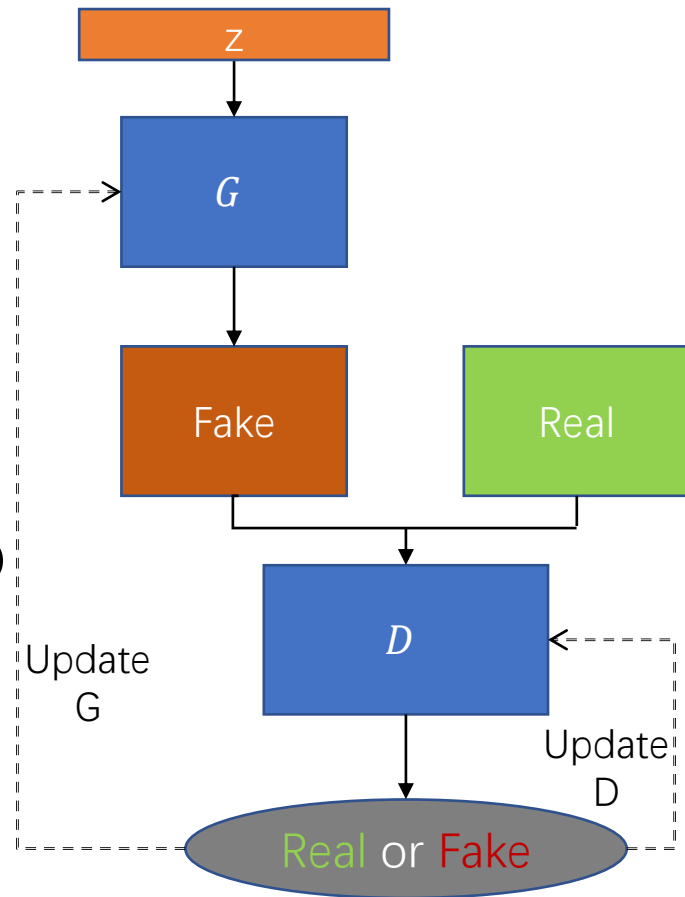
$$\begin{aligned} V_{GAN}(G, D^*) &= \mathbb{E}_{x \sim p_{data}} \left[ \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + \mathbb{E}_{z \sim p_z} \left[ \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} \right] \\ &= -\log 4 + 2 \text{JSD}(p_{data} || p_G) \end{aligned}$$

**Jensen-Shannon divergence**

# Train GANs – will implement tomorrow

In each iteration:

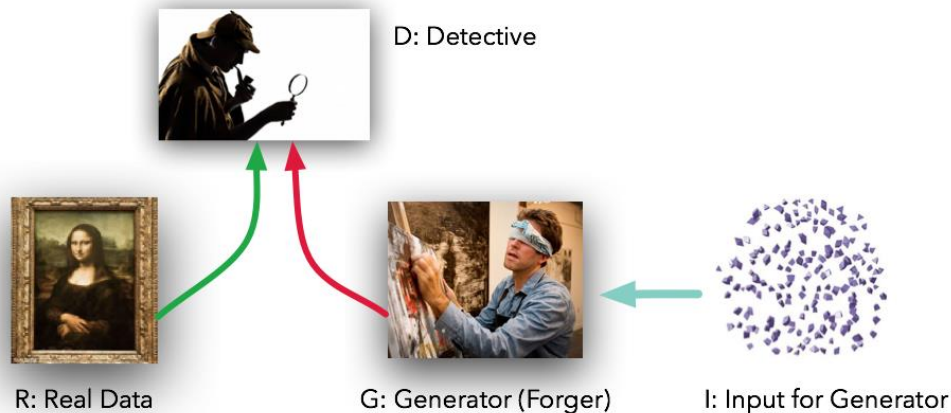
1. Fix  $G$ , train  $D$  to distinguish Real or Fake data:
  - $D(x) \rightarrow 1$ 
    1. Sample real samples  $x$
    2. Input  $x$  into  $D$
    3. Train  $D$ :  $D(x) \rightarrow 1$
  - $D(G(z)) \rightarrow 0$ 
    1. Sample vectors of  $z$  from  $p_{prior}(z)$
    2. Input  $z$  into  $G$  to generate fake samples  $G(z)$
    3. Train  $D$ :  $D(G(z)) \rightarrow 0$
2. Fix  $D$ , train  $G$  to “fool”  $D$ :
  1. Sample vectors  $z$  from  $p_{prior}(z)$
  2. Input  $z$  into  $G$  to generate fake samples  $G(z)$
  3. Train  $G$ :  $D(G(z)) \rightarrow 1$



# Theory of GANs

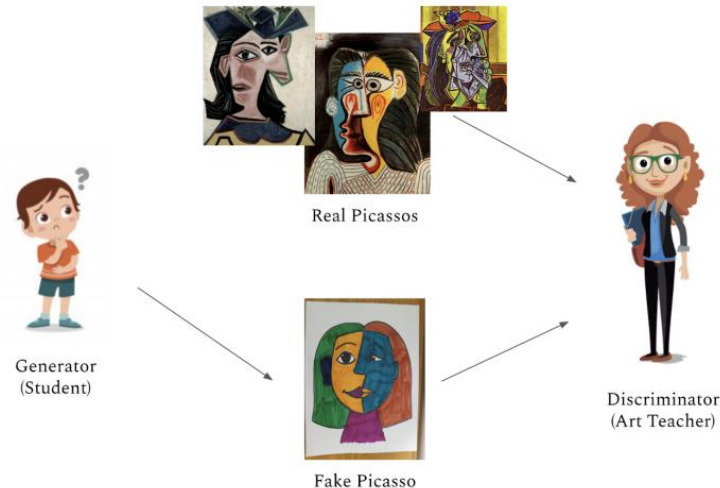
Relationship between  $G$  and  $D$ ?

From the objective function



<https://medium.com/@devnag/generative-adversarial-networks-gans-in-50-lines-of-code-pytorch-e81b79659e3f>

From the process of interaction

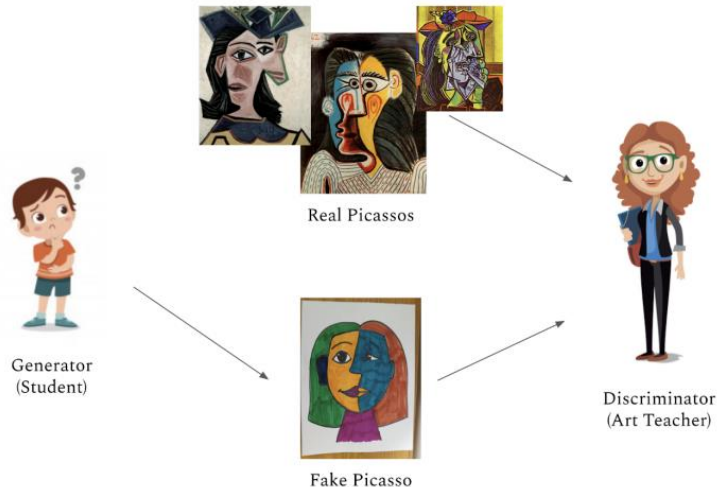


[http://robotic.media.mit.edu/wp-content/uploads/sites/7/2021/03/EAAI-What-are-GANs\\_.pdf](http://robotic.media.mit.edu/wp-content/uploads/sites/7/2021/03/EAAI-What-are-GANs_.pdf)

# Theory of GANs

Interaction between  $G$  and  $D$  :

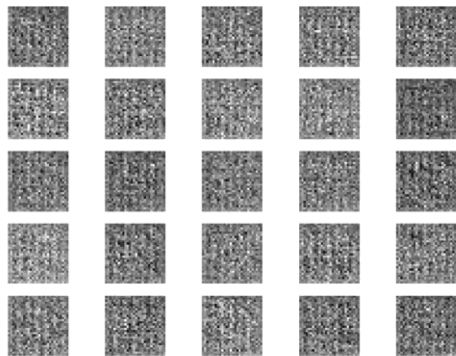
- $D$  is leading the  $G$ 
  - $D$  is trained first
  - $D$  provide “knowledge” to update  $G$
- $D$  needs to “teach” according to the current level of  $G$ 
  - Measure the distance between current  $p_G$  and  $p_{data}$



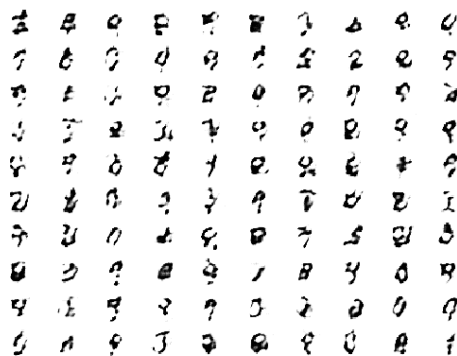
[http://robotic.media.mit.edu/wp-content/uploads/sites/7/2021/03/EAAI-What-are-GANs\\_.pdf](http://robotic.media.mit.edu/wp-content/uploads/sites/7/2021/03/EAAI-What-are-GANs_.pdf)



# Theory of GANs



0<sup>th</sup> Epoch



10<sup>th</sup> Epoch



100<sup>th</sup> Epoch

<https://machinelearningmastery.com/how-to-develop-a-generative-adversarial-network-for-an-mnist-handwritten-digits-from-scratch-in-keras/>

# Pros and Cons

- GANs trained to minimise the JS divergence between  $p_G$  and  $p_{data}$
- GANs produce “sharper” and more “perceptually realistic” results
- VAEs are stable in training, and converge faster
- GANs are hard to train, and have unclear stopping criteria
- VAEs provide generative model and inference model
  - Learn an encoder decoder pair
- GANs only has generative model
- VAEs are more theoretically justified

# Many different flavours of GANs:

## Different objective functions

GAN (JS divergence)

Wasserstein GAN (WGAN)

WGAN GP (Gradient Penalty)

f-GAN

LSGAN

Energy-based GAN (EBGAN)

BEGAN

Fisher GAN (Chi-square)

...

## Different architectures

Conditional GAN (cGAN)

VAE-GAN

Cycle-GAN

BiGAN

BicycleGAN

Style-GAN

Self-attention GAN

BigGAN

Disco-GAN

Countless more ...

# Many different flavours of GANs:

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Energy-based GAN (EBGAN)

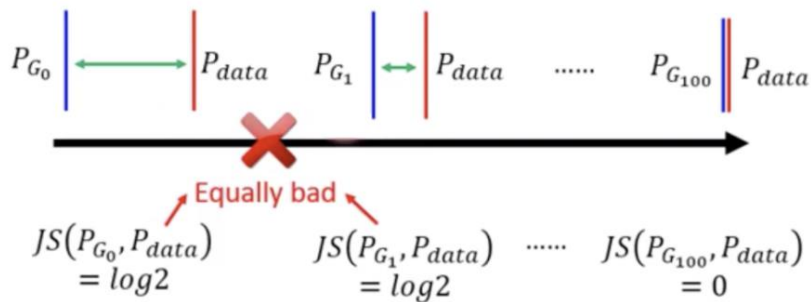
BEGAN

Fisher GAN (Chi-square)

...

Works for initial  $p_G$  and  $p_{data}$  are not overlapped (most of the cases).

### What is the problem of JS divergence?



JS divergence is  $\log 2$  if two distributions do not overlap.

Intuition: If two distributions do not overlap, binary classifier achieves 100% accuracy

# Many different flavours of GANs:

## Different objective functions

GAN (JS divergence)

Wasserstein GAN (WGAN)

WGAN GP (Gradient Penalty)

f-GAN

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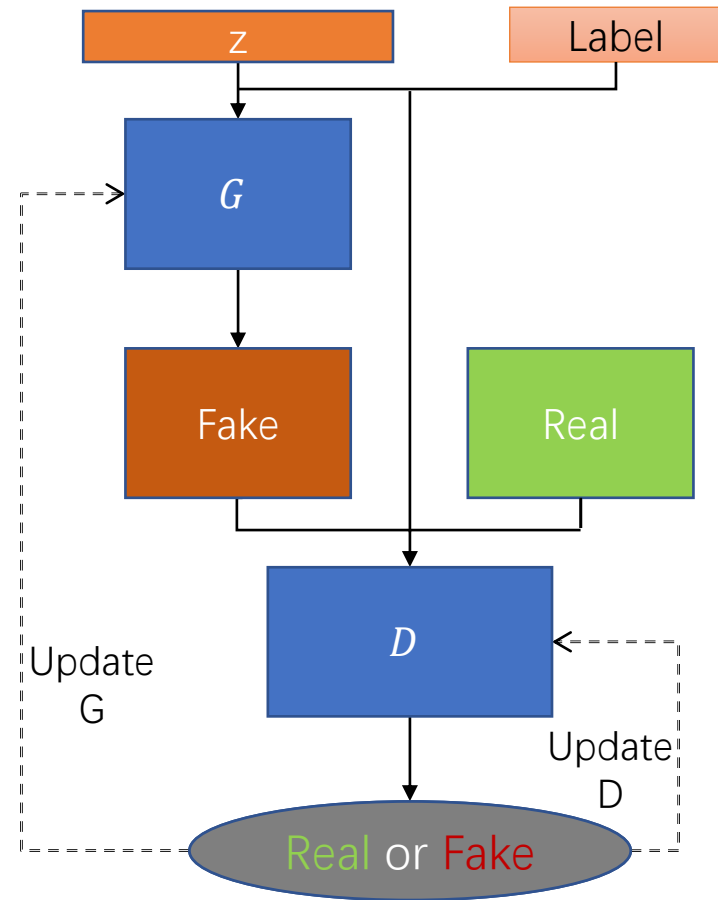
Countless more ...

# Conditional GAN (cGAN)

Vanilla GAN has limited implementations – unsupervised generation of unlabelled data.

cGAN solves this by providing the label, so that the output of  $G$  should be:

- Realistic
- Matching to the given label



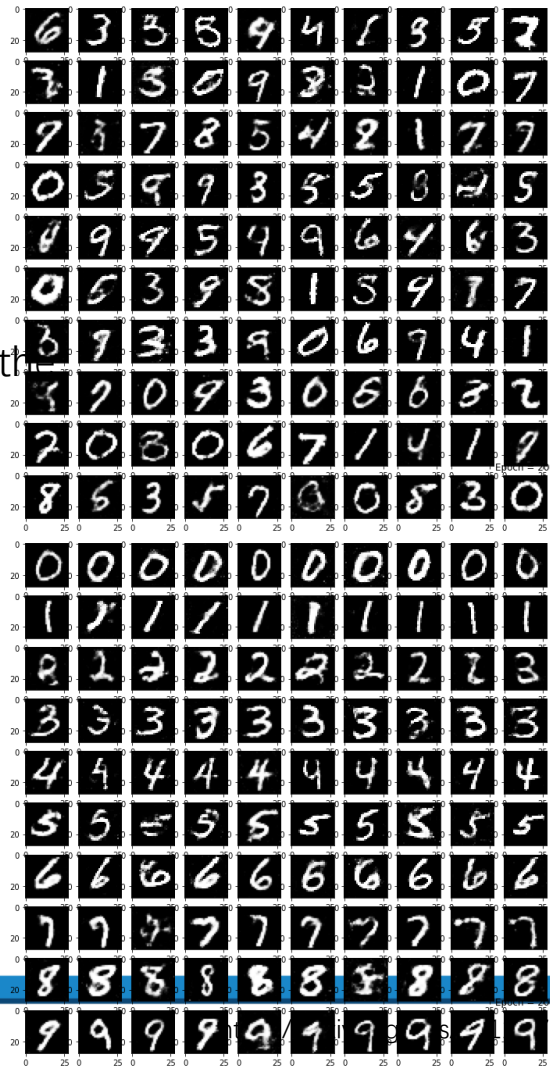


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Vanilla GAN

cGAN

# Text Conditioned Auxiliary Classifier GAN

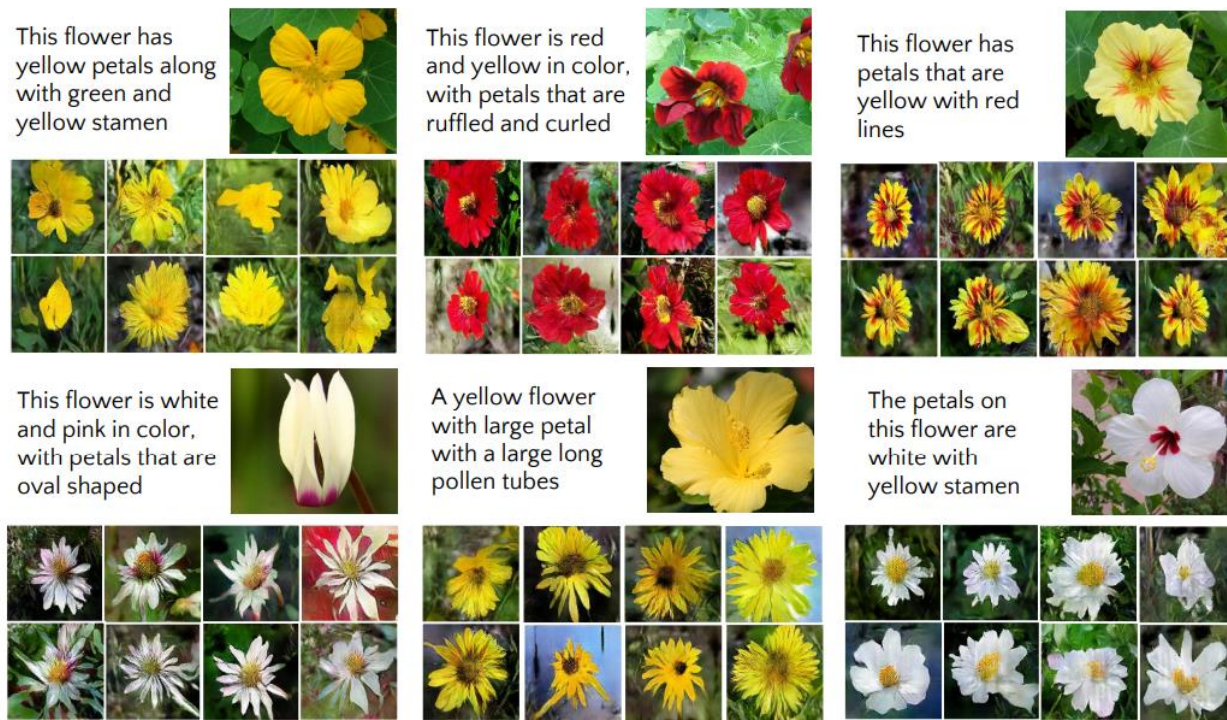


Figure 3. Images synthesized from text descriptions using different noise vectors. In each block, the images at the bottom are generated from the text embeddings of the image description and a noise vector. The image on the top of each block are real images corresponding to the text description.

<https://arxiv.org/abs/1703.06412>

# Conditional GAN + L1 in paired image translation – pix2pix

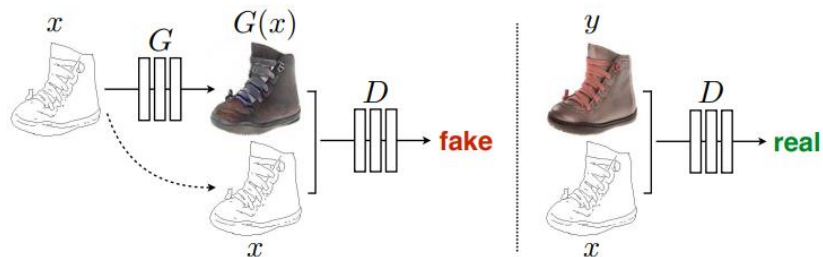
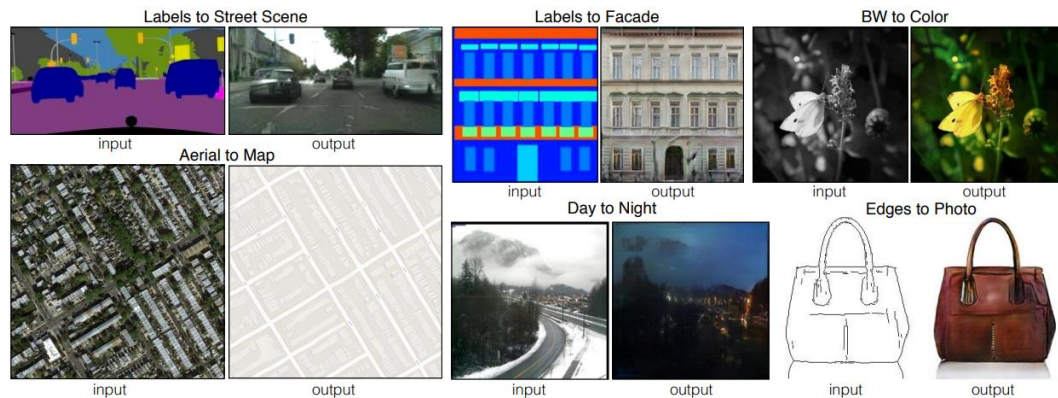


Image as  $G$  input, and as condition label.

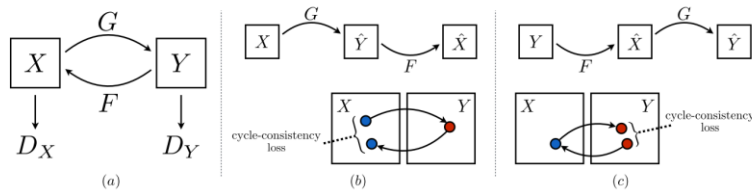


Interactive Demo: <https://affinelayer.com/pixsrv/>

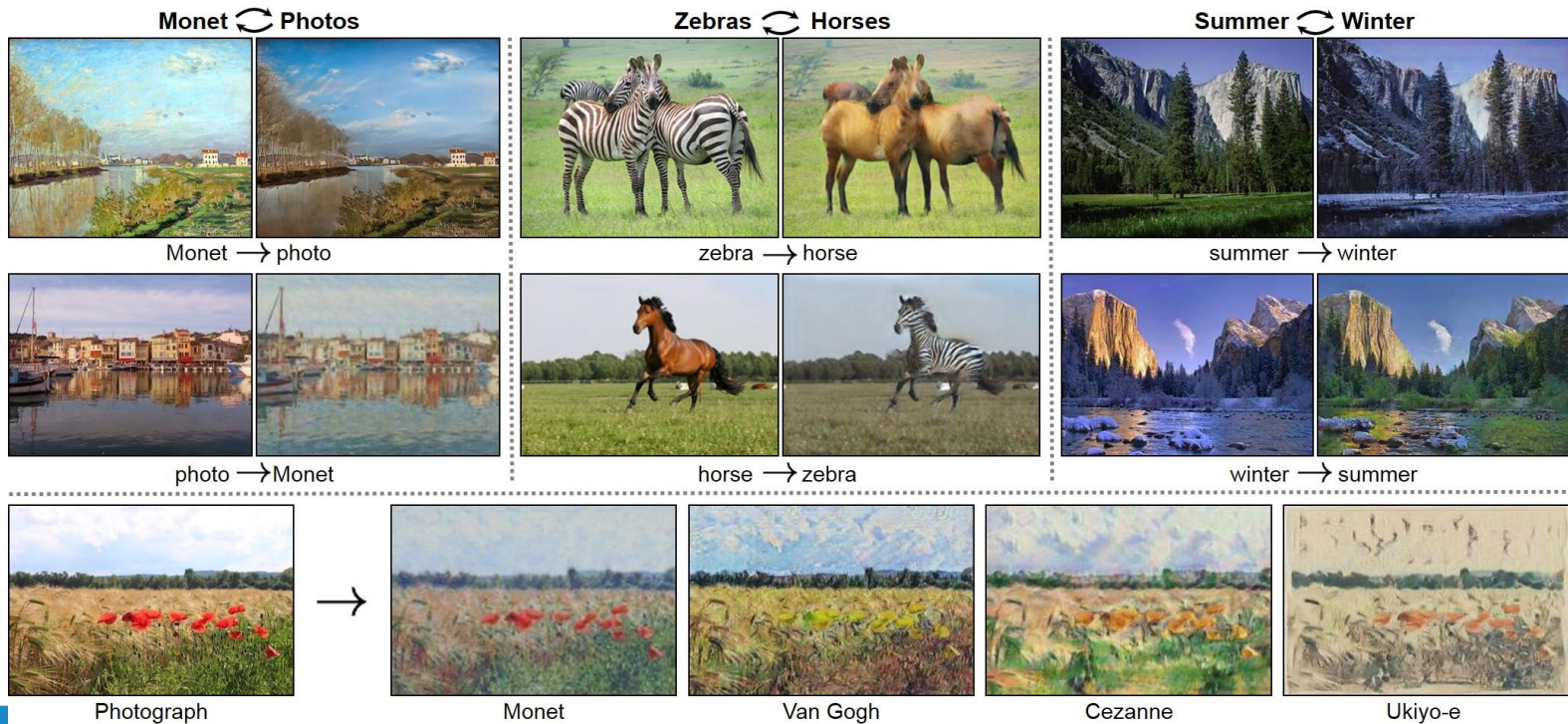


# Cycle-GAN

## unpaired image translation

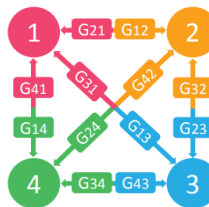


<https://junyanz.github.io/CycleGAN/>

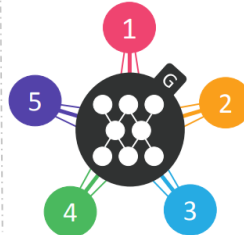


# StarGAN

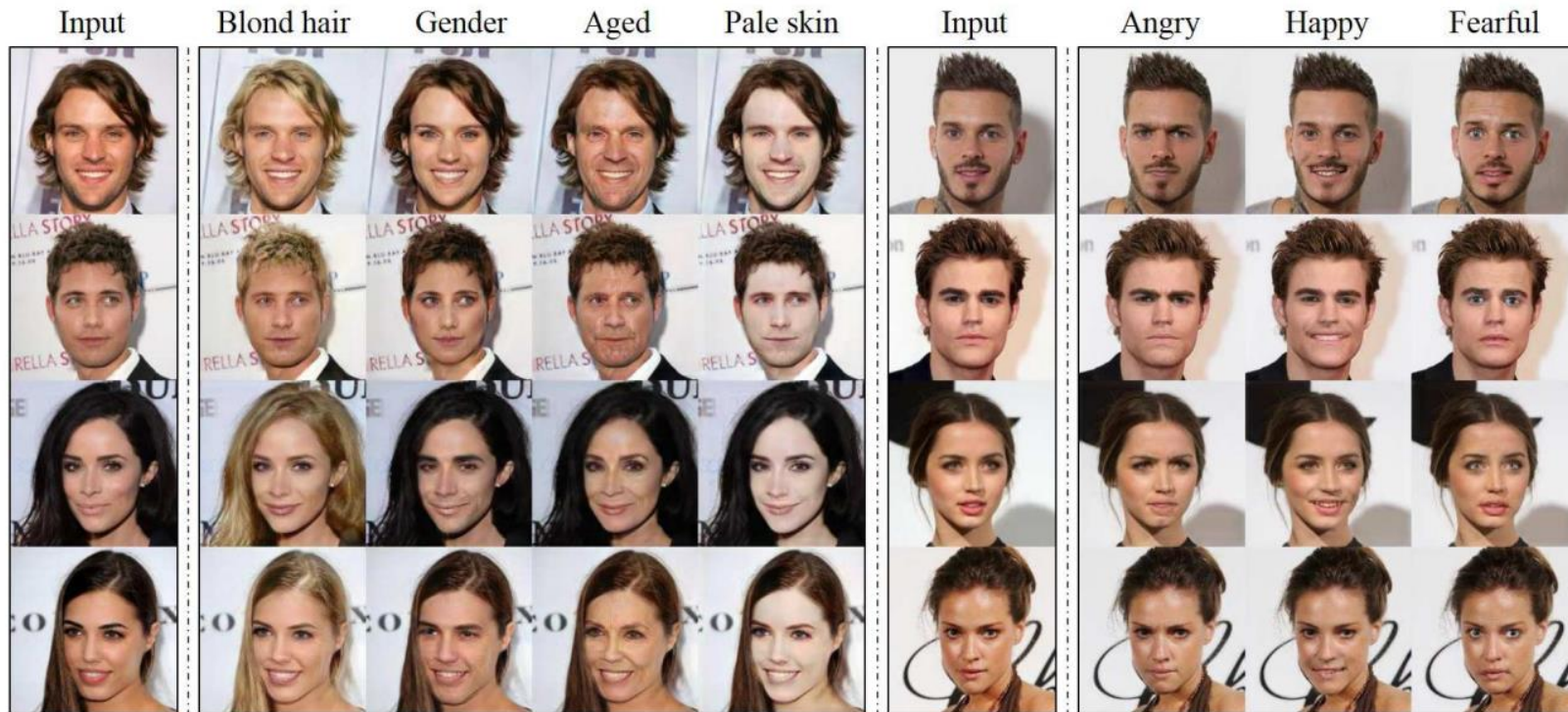
(a) Cross-domain models



(b) StarGAN



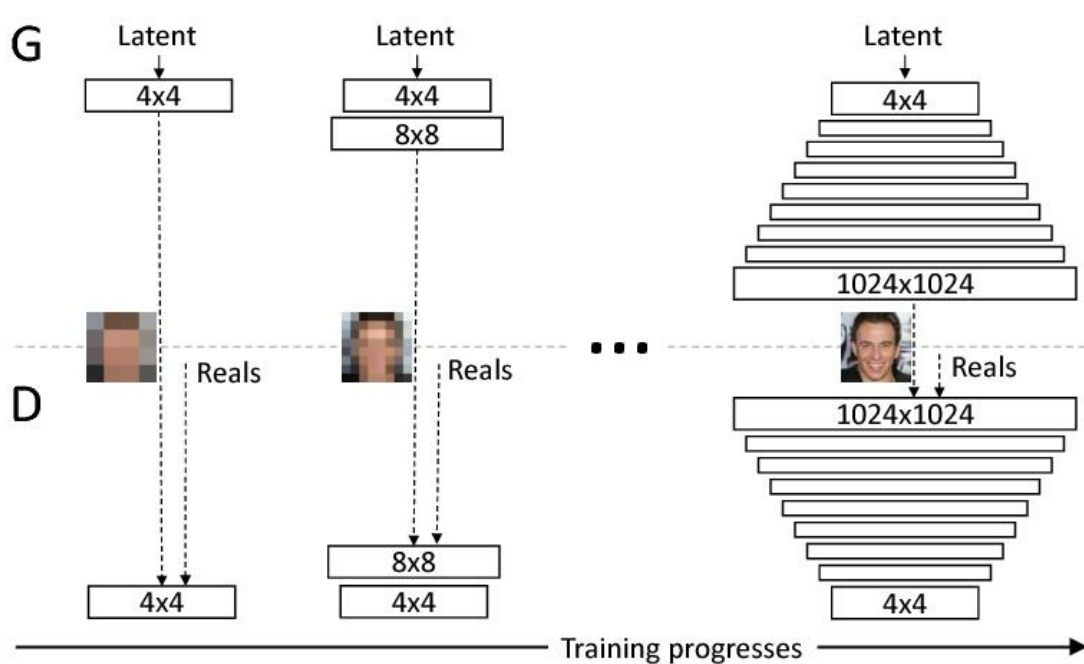
[https://openaccess.thecvf.com/content\\_cvpr\\_2018/papers/Choi\\_StarGAN\\_Unified\\_Generative\\_CVPR\\_2018\\_paper.pdf](https://openaccess.thecvf.com/content_cvpr_2018/papers/Choi_StarGAN_Unified_Generative_CVPR_2018_paper.pdf)



# Progressively growing of GANs

<https://arxiv.org/abs/1710.10196>

Logic of multi-scale optimisations





# Summary

- Generative model
- Adversarial loss
- Theory of GANs
- How to train GANs
- Flavours of GANs

Thanks for your  
attention.