Some applications of Deep Learning in Finance

AI for Global Goals
OxML 2023

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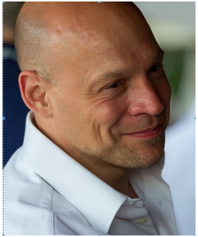
University of Oxford, Oxford Man Institute &
The Alan Turing Institute

Part II: Market Generation

10th July 2023, Oxford

The talk is based on joint work with

Hans Bühler, Terry Lyons, Imanol Perez Arribas, Ben Wood

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(1) Classical and Neo-classical stochastic market models:

$$dS_t = rS_t dt + \sigma S_t dW_t, \quad S_0 = s.$$

Black-Scholes, diffusions, models with jumps, stochastic volatility, path-dependent models, LSV, multifactor models, Rough Volatility, ...

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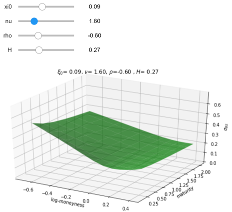
(1.5) Augmenting and extending currently prevalent stochastic models
models that are adaptive to market environments by mixing of (neo-)classical models

- **Mixture of expert models**: A first step towards this was demonstrated in the Deep Learning Volatility framework: Take a mixture of two (or more) stochastic volatility models and calibrate it to data including the mixture parameter a.
  $a \times$ **Heston** $+ (1-a) \times$ **rBergomi**



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- **OT approach:** From Optimal Transport to Robust Pricing and Hedging via Neural Networks (see J. Obłój et al., 2018), a framework that interpolates between model-specific and model independent settings.
- **Non-parametric mixtures:** Signature-based interpolation and approx. of models (I. Perez-Arribas and C. Salvi, 2020) or directly mixing classical with data driven models (C.A. Lehalle).

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- In this work we take an approach via **Rough Paths** using **Signatures**.

Motivation for our Market Generators

The concept of "Model" (here, in form of a numerical program):

- Classical: (Program; Data) $\Rightarrow$ Output
- **Now: (Architecture, ObjF; TrainData) $\Rightarrow$ Program**
  (Program, TestData) $\Rightarrow$ Output

$\Rightarrow$ This may redefine the concept of model governance too:

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Rough Paths Approach to Generative Modelling of Markets

Levin, Lyons, & Ni. (2013) firstly proposed the signature of a path as a basis of functions for a functional on path space.

Definition (Signature of a path)

Let $X : [0, T] \to \mathbb{R}^d$ be a continuous path of bounded variation. The signature of X is then defined by the sequence of iterated integrals given by

$$\mathbb{X}_T^{\leq\infty} := (1, \mathbb{X}_t^1, \ldots, \mathbb{X}_T^n, \ldots), \quad \text{where}$$

$$\mathbb{X}_T^n := \int_{0<u_1<\ldots<u_k<T} dX_{u_1} \otimes \ldots \otimes dX_{u_k} \in (\mathbb{R}^d)^{\otimes n}$$

with $\otimes$ the tensor product. Similarly, given $N \in \mathbb{N}$, the truncated signature of order N is defined by

$$\mathbb{X}_T^{\leq N} := (1, \mathbb{X}_T^1, \ldots, \mathbb{X}_T^N).$$

If the path X has bounded variation – which is the case of discrete data – the integrals above can be defined using Riemann-Stieltjes integrals.

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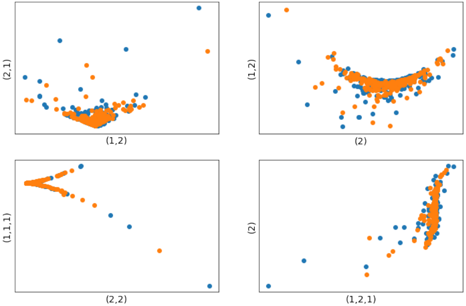
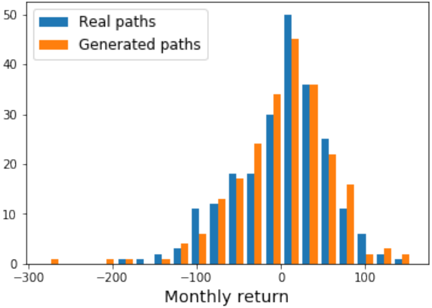
- Robustness to missing data and irregular sampling and towards highly oscillatory data, and invariance under time re-parametrisation. **Liao, Lyons, Ni, Yang (2019)**
- Signatures provide the right framework for performance evaluation metrics on pathspace, one of the difficulties being that the pathspace $C([0,1], R^d)$ is infinite-dimensional and not locally compact. **Chevyrev, Oberhauser (2018)**
- In fact, **Liao, Lyons, Ni, Yang (2019)** demonstrated the advantages of log-signatures (with all positive properties listed above, but lower dimensionality)

Numerical Results

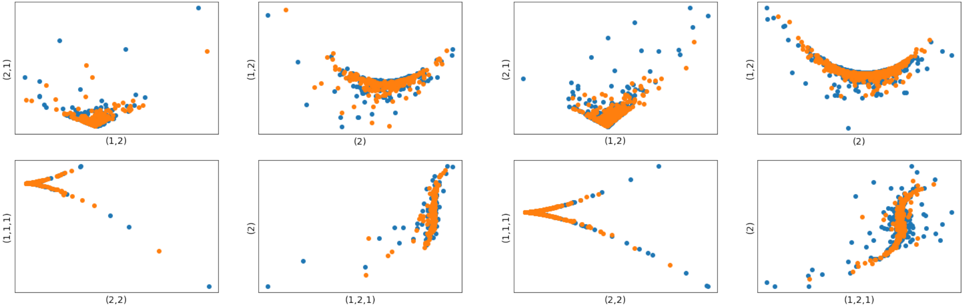
- Signature based training (ii) at least as accurate as returns-based training (i).





Numerical Results

- Signature-based generative model works well already with the low number of training samples available in data (1000 samples weekly-unconditioned **left image**, 250 samples monthly **right image** unconditioned).
- Increasing the number of samples in numerically generated data does not lead to better performance.



Numerical Results

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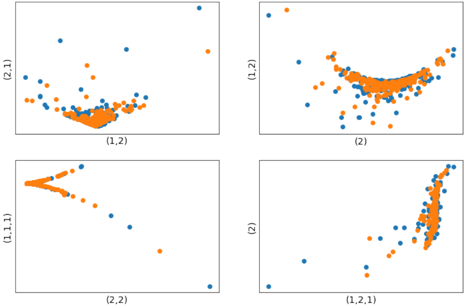
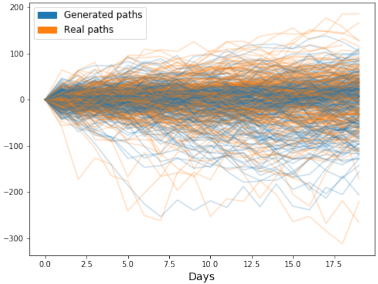
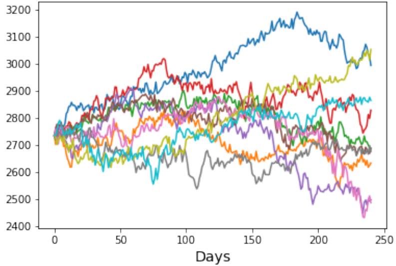




Conditional VAE and Postprocessing

- Conditional Variational Autoencoder is learned to condition VAE on current market conditions (a) current level of the index (b) instantaneous volatility (c) signature of the previous path segment.
- By conditioning, the number of available training samples is even smaller $\Rightarrow$ powerful parsimonious generative model crucial.
- By conditioning on the signature of the previous path segment, one can generate + build paths far longer than the direct output of the generative model. Numerical results consistent between weekly vs. monthly paths. **Below: Yearly paths**



Performance evaluation metrics

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- For classical models and continuous time stochastic processes popular performance evaluation metrics were whether the process reflects **stylised facts** of financial markets (many of these concern the returns) or provides **close fits to option prices** etc.
- If returns of the target distribution are only observable on a time-grid with pre-defined mesh size and a generative model is fitted to observed distributions of returns marginal by marginal this can become problematic between $\mathbb{P}$ and $\mathbb{Q}$ measures. See **Brigo (2019)** and **Armstrong, Bellani, Brigo, Cass (2019)** underline the necessity of pathwise evaluation metrics.
- While in the finite dimensional case it is well-understood how moments describe the law of a random variable this question is more subtle in the infinite-dimensional case of path-valued random variables. **Chevyrev, Oberhauser (2018)** prove an analogous result for path-valued random variables by using the normalized sequence of signature moments.

Issues and Solutions offerend by of Market Generators

Important concerns regarding training data:

Issue 1 Data availability (some data is naturally scarce & not sufficient for training of neural networks)

Issue 2 Data privacy (data often proprietory and not accessible, difficult to compare/regulate models' performance consistently across the industry)

Issue 3 Data quality (noise, biasses, outliers, expressiveness)

$\Rightarrow$ We need standardised and widely accessible testing datasets for innovation and benchmarking of DNN models.

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$\Rightarrow$ We need standardised and widely accessible testing datasets for innovation and benchmarking of DNN models.

With this in mind we demonstrate in a joint work with A. Kondratyev and C. Schwarz the potential of generative models towards **data anonymisation, outlier detection & combating overfitting**, in several experiments **Kondratyev, Schwarz, H. (2020)**.

Thank you for your attention.

Rough Paths Approach to Generative Modelling

In what follows we work instead with the log-signatures

Definition (Log-signature)

Let $X : [0, T] \to \mathbb{R}^d$ be a path such that its signature $\mathbb{X}_{0,T}^{<\infty}$ is well-defined. The log-signature is then defined by

$$\log \mathbb{X}_T^{\leq\infty} := -\mathbb{X}_T^{\leq\infty} + \frac{1}{2}(\mathbb{X}_T^{\leq\infty})^{\otimes 2} - \frac{1}{3}(\mathbb{X}_T^{\leq\infty})^{\otimes 3} + \ldots + (-1)^n \frac{1}{n}(\mathbb{X}_T^{\leq\infty})^{\otimes n} + \ldots,$$

which can be shown to be well-defined.

- There is a one-to-one map between signatures and log-signatures.
- Log-signatures have all positive properties listed above.
- They allow for lower dimensional representation and are better suited to VAE.