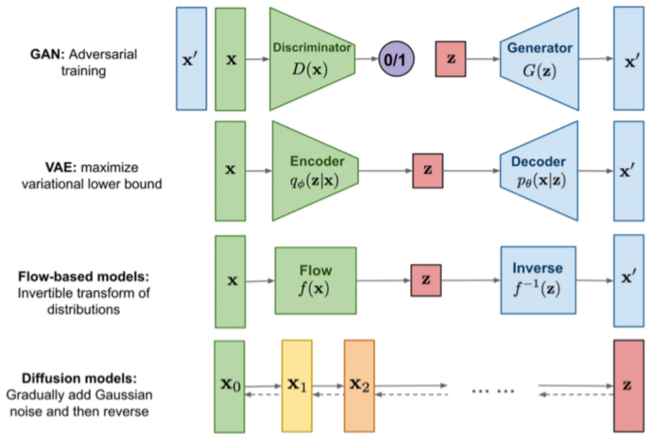
MC $n \rightarrow n-1 \rightarrow n-2$

x_0 x_1 x_2



t

$P(x_t \mid x_{t-1})$

FC: $F = AX + B$

$$\frac{\partial F}{\partial X} = A$$

$$\frac{\partial F}{\partial A} =$$

$$\frac{\partial F}{\partial B} = 1$$

MLP

FCs

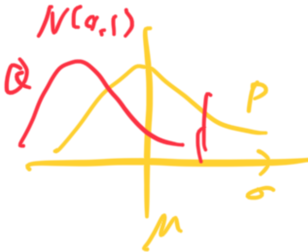
MC

MC

Random sampling

$z \sim N(\mu, \sigma)$

Reparameterlization trick



$z \sim N(\mu, \sigma)$

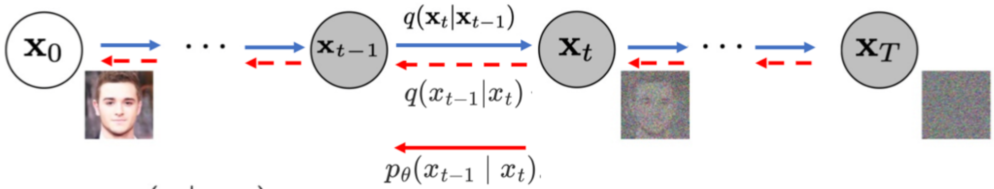
$z := N(z; \mu, \sigma I)$

$0 \rightarrow \mu$

$1 \rightarrow \sigma$

$z = \underbrace{\mu + \sigma \odot \varepsilon, \quad \varepsilon \sim N(0,1)}$

Key concept / difinations



Diffuse : $q(\mathbf{x}_t|\mathbf{x}_{t-1})$
add noise (dim dim dim + mask mask mask)

Reverse-Diffuse : $q(x_{t-1}|x_t)$
The reverse process of diffuse. (can't be calculated but can be estimated)

Denoise : $p_\theta(x_{t-1} \mid x_t)$
The estimated process projecting denoised samples.

$q: \; X_t \sim N(\underbrace{\sqrt{1-\beta_t}\, X_{t-1}}_{\mu}, \; \underbrace{\sqrt{\beta_t}}_{\sigma}) \qquad \underbrace{0 < \beta_t < 1}$

$\Rightarrow X_t = N(X_t; \sqrt{1-\beta_t}\, X_{t-1}, \sqrt{\beta_t} \cdot I)$

$X_t = \underbrace{\sqrt{1-\beta_t}}_{0<\,<1}\, \underbrace{X_{t-1}} + \underbrace{\sqrt{\beta_t} \odot \varepsilon, \quad \varepsilon \sim N(0,1)}_{\text{mask mask mask}}$

q: Diffuse $\overset{t-1}{O} \xrightarrow[q]{} \overset{t}{O}$

Dim Dim Dim

$$\begin{cases} \text{Diffuse}: \; q(X_t \mid X_{t-1}) = N(X_t; \sqrt{1-\beta_t}\, X_{t-1}, \beta_t I) \\ \text{prob.} \quad q(X_{1:T} \mid X_0) = \prod_{t=1}^{T} q(X_t \mid X_{t-1}) \end{cases}$$

$x_0 \to x_1 \to \cdots \to x_T$

$$\underset{\epsilon_1}{N(0,\sigma_1^2 I)} + \underset{\epsilon_2}{N(0,\sigma_2^2 I)} \sim \underset{\tilde{\epsilon}}{N(0,(\sigma_1^2+\sigma_2^2) I)}$$

$$\underset{x_0}{O} \qquad \underset{t-1}{O} \xrightarrow{\beta_t} \underset{t}{O} \qquad x_t \to x_0,\ \beta = (\beta_1 \dots \beta_t)$$

let $\alpha_t = 1-\beta_t$, $\bar{\alpha}_t = \prod_{i=1}^{t} \alpha_i$

$$x_t = \sqrt{\alpha_t}\, x_{t-1} + \sqrt{1-\alpha_t}\, \epsilon_{t-1}$$

$$\Rightarrow x_{t-1} = \sqrt{\alpha_{t-1}}\, x_{t-2} + \sqrt{1-\alpha_{t-1}}\, \epsilon_{t-2}$$

$$\Rightarrow x_t = \sqrt{\alpha_t}\left(\sqrt{\alpha_{t-1}} \cdot x_{t-2}\right) + \sqrt{\alpha_t} \cdot \sqrt{1-\alpha_{t-1}}\, \epsilon_{t-2} + \sqrt{1-\alpha_t}\, \epsilon_{t-1}$$

$\epsilon \sim N(0,1)$

$$= \sqrt{\alpha_t \cdot \alpha_{t-1}}\, x_{t-2} + \sqrt{\alpha_t \cdot (1-\alpha_{t-1}) + 1 - \alpha_t}\; \tilde{\epsilon}_{t-2}$$

$$= \sqrt{\alpha_t \cdot \alpha_{t-1}}\, x_{t-2} + \sqrt{1-\alpha_t \cdot \alpha_{t-1}}\; \tilde{\epsilon}_{t-2}$$

$$\Rightarrow x_t = \sqrt{\alpha_t \cdot \alpha_{t-1} \cdots \alpha_1}\, x_0 + \sqrt{1-\alpha_t \cdot \alpha_{t-1} \cdots \alpha_1}\; \tilde{\epsilon}_0$$

$$\Rightarrow x_t = \sqrt{\bar{\alpha}_t}\, x_0 + \sqrt{1-\bar{\alpha}_t}\, \epsilon, \quad \epsilon \sim N(0,1)$$

$$\beta \in (0,1) \quad , \boxed{T}$$

$$\lim_{T\sim\infty} X_T = \sqrt{\bar{\alpha}_T}\, X_0 + \sqrt{1-\bar{\alpha}_T}\ \varepsilon$$

$$= 0 + 1 \cdot \varepsilon$$

$$X_T \sim N(0,1)$$

$$\underset{x_0}{0} \rightarrow 0 \rightarrow 0 \rightarrow 0 \xrightarrow{q} \underset{}{\overset{x_T}{0}} \sim z \sim N(0,1)$$

$\beta \downarrow$ reverse q $\beta \uparrow$

$$\beta = (0.0001 \ , \ 0.02)$$

$$q(x_t \mid x_{t-1}) \rightarrow N$$

$$\underset{t-1}{0} \underset{\theta}{\rightleftarrows} \underset{t}{0}$$

reverse diffuse

$$\neg q(x_{t-1} \mid x_t) \qquad \neg q \rightarrow N$$

$$P_\theta(X_{t-1} \mid X_t)$$

$$P_\theta(X_{t-1} \mid (X_t, X_0))$$

$$= N(X_{t-1};\ \tilde{\mu}(X_t, X_0),\ \tilde{\beta}_t I)$$

bayesmban

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)} \qquad (b\text{-}0)$$

$$P(AB) = P(A)\,P(B \mid A) \qquad (b\text{-}1)$$

$$P(ABC) = P(A)\,P(B|A)\,P(C|AB) \quad (b\text{-}2)$$

$$(b\text{-}0) \leftarrow (b\text{-}1) \Rightarrow P(A|B) = \frac{P(AB)}{P(B)} \quad (b\text{-}3)$$

transmission matrix

$$\Rightarrow q(x_{t-1}|(x_t, x_0)) \overset{(b\text{-}3)}{=} \frac{q(x_t, x_0, x_{t-1})}{q(x_t, x_0)}$$

$$\overset{(b\text{-}2)}{=} \frac{q(x_0)\,q(x_{t-1}|x_0)\,q(x_t|(x_{t-1}, x_0))}{q(x_0)\,q(x_t|x_0)} \quad \underline{(b\text{-}1)}$$

$$= \underbrace{q(x_t|(x_{t-1}, x_0))}_{0 \to 0} \cdot \frac{q_\theta(x_{t-1}|x_0)}{q_\theta(x_t|x_0)}$$

$$= q(x_t|x_{t-1}) \frac{q_\theta(x_{t-1}|x_0)}{q_\theta(x_t|x_0)}$$

$x \sim N(\mu, \sigma)$ Gaussian exp

$$\propto \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$= \exp\left(-\frac{1}{2}\left(\underset{\uparrow_2}{\frac{1}{\sigma^2}x^2} - \underset{\uparrow_1}{\frac{2\mu}{\sigma^2}x} + \underset{\uparrow_0}{\frac{\mu^2}{\sigma^2}}\right)\right)$$

$$\propto \exp\left[-\frac{1}{2}\left(\frac{(x_t - \sqrt{\alpha_t}\,x_{t-1})^2}{\beta_t} + \frac{(x_{t-1} - \sqrt{\bar{\alpha}_{t-1}}\,x_0)^2}{1-\bar{\alpha}_{t-1}} - \frac{(x_t - \sqrt{\bar{\alpha}_t}\,x_0)^2}{1-\bar{\alpha}_t}\right)\right)$$

$$= \exp\left(-\frac{1}{2}\left(\left(\frac{\alpha_t}{\beta_t}+\frac{1}{1-\bar{\alpha}_{t-1}}\right)\underset{\uparrow\ \text{sqr}}{x_{t-1}^2}-\left(\frac{2\sqrt{\alpha_t}}{\beta_t}x_t+\frac{2\sqrt{\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_{t-1}}x_0\right)\underset{\uparrow\ 1}{x_{t-1}}+\underbrace{C(x_t,x_0)}_{0}\right)\right)$$

$\Rightarrow$ Based on matching the component:

$$\tilde{\beta}_t = \frac{1}{\frac{\alpha_t}{\beta_t}+\frac{1}{1-\bar{\alpha}_{t-1}}} = 1\Big/\left(\frac{\alpha_t-\bar{\alpha}_t+\beta_t}{\beta_t(1-\bar{\alpha}_{t-1})}\right) = \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\cdot\beta_t$$

$$\tilde{\mu}_t(x_t,x_0) = \left(\frac{\sqrt{\alpha_t}}{\beta_t}x_t+\frac{\sqrt{\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_{t-1}}x_0\right)\Big/\left(\frac{\alpha_t}{\beta_t}+\frac{1}{1-\bar{\alpha}_{t-1}}\right)$$

$$= \left(\frac{\sqrt{\alpha_t}}{\beta_t}x_t+\frac{\sqrt{\bar{\alpha}_{t-1}}}{1-\alpha_{t-1}}x_0\right)\cdot\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\cdot\beta_t$$

$$= \frac{\sqrt{\alpha_t}\,(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t}x_t+\frac{\sqrt{\bar{\alpha}_{t-1}}\,\beta_t}{1-\bar{\alpha}_t}x_0$$

and because of

$$x_t = \sqrt{\bar{\alpha}_t}\, x_0 + \sqrt{1-\bar{\alpha}_t}\, \varepsilon \leftarrow \varepsilon \sim N(0,1)$$

$$\Rightarrow x_0 = \frac{1}{\sqrt{\bar{\alpha}_t}} \left(x_t - \sqrt{1-\bar{\alpha}_t}\, \varepsilon_t \right)$$

put it

$$\Rightarrow \tilde{\mu}_t = \frac{1}{\sqrt{\alpha_t}} \left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \varepsilon_t \right), \quad \varepsilon_t \sim N(0,1)$$

therefore ~~then~~ $q(x_{t-1} | (x_t, x_0))$ can be calcul-

$$p : x_{t-1} = N(x_{t-1}; \tilde{\mu}_t, \tilde{\sigma}_t)$$

$$N\left(x_{t-1}; \frac{1}{\sqrt{\alpha_t}} \left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \varepsilon_t \right), \underline{\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t} \cdot \beta_t} \right)$$