6.2.2 Poles and zeros of z-transform

Use the Matlab function **roots.m** to find the poles and zeros of the following z-transform

(a) $H_1(z) = -1 + 2z^{-1} - 3z^{-2} + 6z^{-3} - 3z^{-4} + 2z^{-5} - z^{-6}$

(b) $G_1(z) = \frac{3z^4 - 2.4z^3 + 15.36z^2 + 3.84z + 9}{5z^4 - 8.5z^3 + 17.6z^2 + 4.7z - 6}$

(c) $G_2(z) = \frac{2z^4 + 0.2z^3 + 6.4z^2 + 4.6z + 2.4}{5z^4 + z^3 + 6.6z^2 + 4.2z + 24}$

```
clear;
close all;
clc;
P1 = [-1 2 -3 6 -3 2 -1];
D1 = [1 zeros(1,6)];
P2 = [3 -2.4 15.36 3.84 9];
D2 = [5 -8.5 17.6 4.7 -6];
P3 = [2 0.2 6.4 4.6 2.4];
D3 = [5 1 6.6 4.2 24];

zeros1 = roots(P1)
```

```
zeros1 = 6×1 complex
   1.8054 + 0.0000i
  -0.1269 + 1.5457i
  -0.1269 - 1.5457i
  -0.0528 + 0.6426i
  -0.0528 - 0.6426i
   0.5539 + 0.0000i
```

```
poles1 = roots(D1)
```

```
poles1 = 6×1
     0
     0
     0
     0
     0
     0
```

```
zeros2 = roots(P2)
```

```
zeros2 = 4×1 complex
   0.6000 + 2.1541i
   0.6000 - 2.1541i
  -0.2000 + 0.7483i
  -0.2000 - 0.7483i
```

```
poles2 = roots(D2)
```

```
poles2 = 4×1 complex
   0.9000 + 1.7861i
   0.9000 - 1.7861i
  -0.6000 + 0.0000i
   0.5000 + 0.0000i
```

```
zeros3 = roots(P3)
```

```
zeros3 = 4×1 complex
   0.3296 + 1.7980i
   0.3296 - 1.7980i
  -0.3796 + 0.4637i
  -0.3796 - 0.4637i
```

```
poles3 = roots(D3)
```

```
poles3 = 4×1 complex
   0.8272 + 1.2744i
   0.8272 - 1.2744i
  -0.9272 + 1.1045i
  -0.9272 - 1.1045i
```

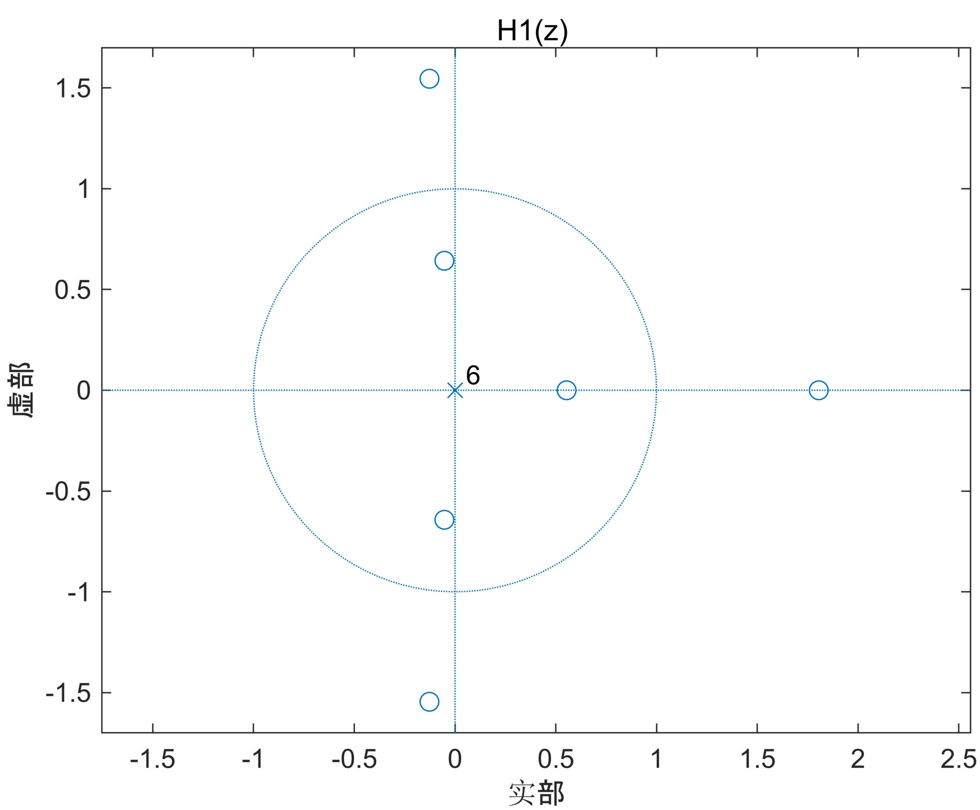
```


display(max(abs(poles1)));
```

```
     0
```

```
display(min(abs(poles1)));
```

```
     0
```

```
figure
zplane(zeros1,poles1);
title('H1(z) ');
```




```
%left-sided ROC: impossible, pole 全 0
```

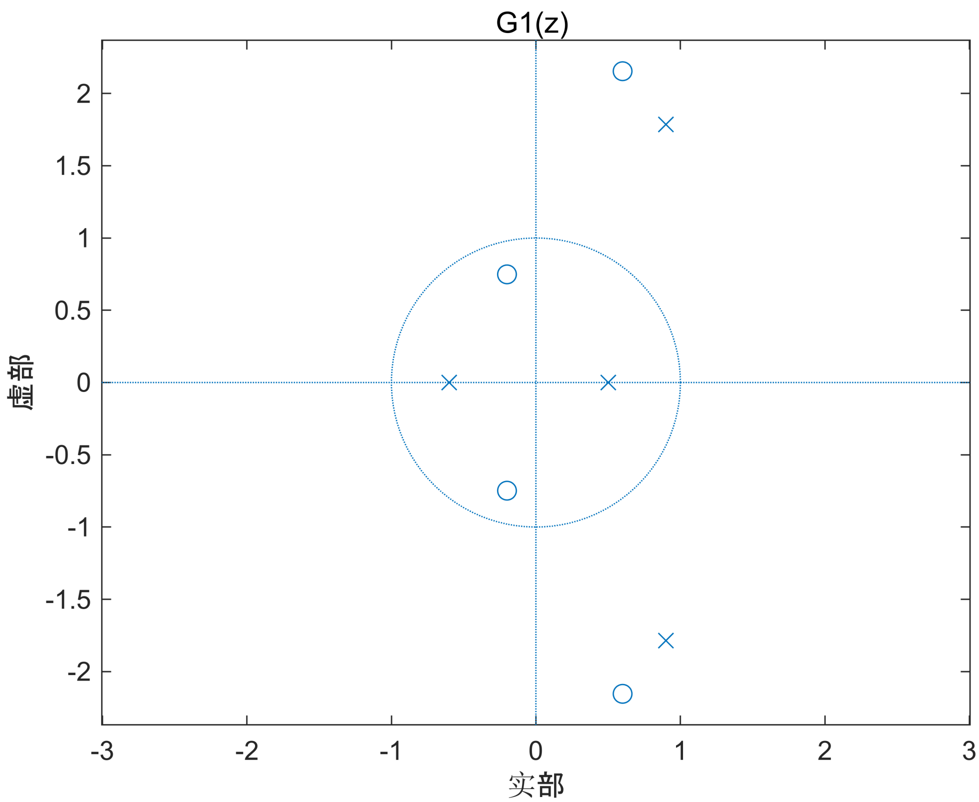
```
%right-sided ROC:|�| > 0
%two-sided ROC:impossible


display(max(abs(poles2)));
```

```
2.0000
```

```
display(min(abs(poles2)));
```

```
0.5000
```

```
figure
zplane(zeros2,poles2);
title('G1(z) ');
```



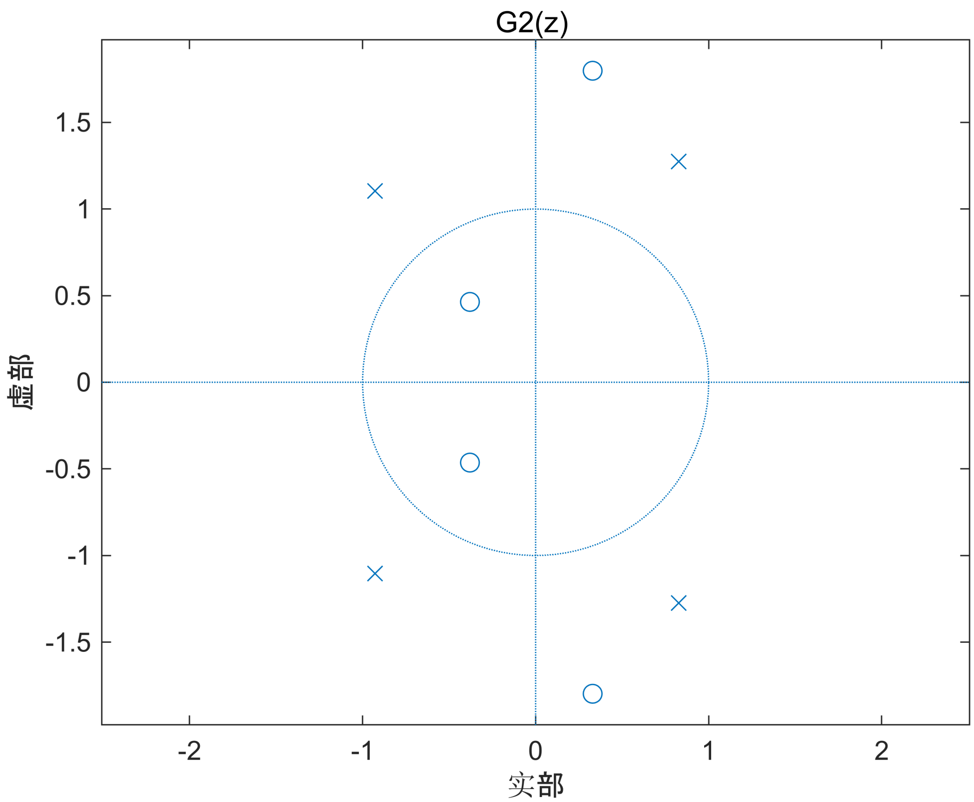

```
%left-sided ROC:|z|<0.5
%right-sided ROC:|z|>2
%two-sided ROC:0.6<|z|<2


display(max(abs(poles3)));
```

```
1.5193
```

```
display(min(abs(poles3)));
```

```
1.4421
```

```
figure
zplane(zeros3,poles3);
title('G2(z) ');
```




```
%left-sided ROC:|z|<1.4421
%right-sided ROC:|z|>1.5193
%two-sided ROC:1.4421<|z|<1.5193
```

For right-sided sequences: ROC extends outward from the outermost pole to infinity.

For left-sided: ROC inwards from the innermost pole to the original point.

For two-sided: ROC either is a ring or do not exist.

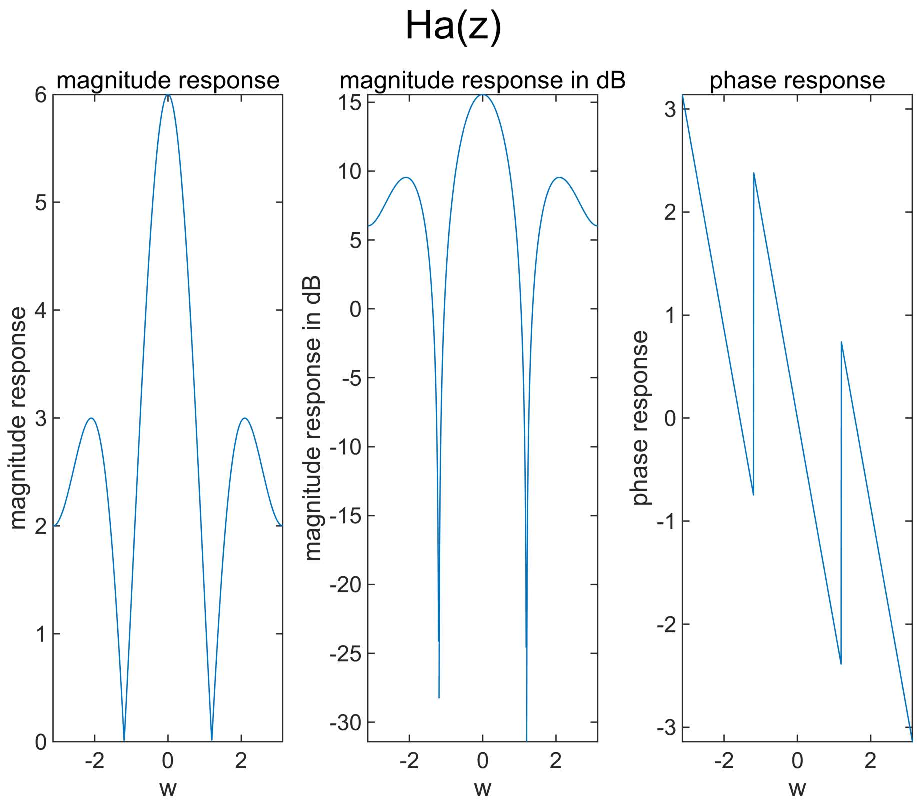
6.2.3 z-transform and frequency response

$$H\left(e^{j\omega}\right) = H(z)|_{z=e^{j\omega}}$$

For the digital filters characterized by the following transfer functions:

(a) $H(z) = z^{-4} + 2z^{-3} + 2z^{-1} + 1,$ (b) $H(z) = \frac{z^2-1}{z^2-1.2z+0.95}$

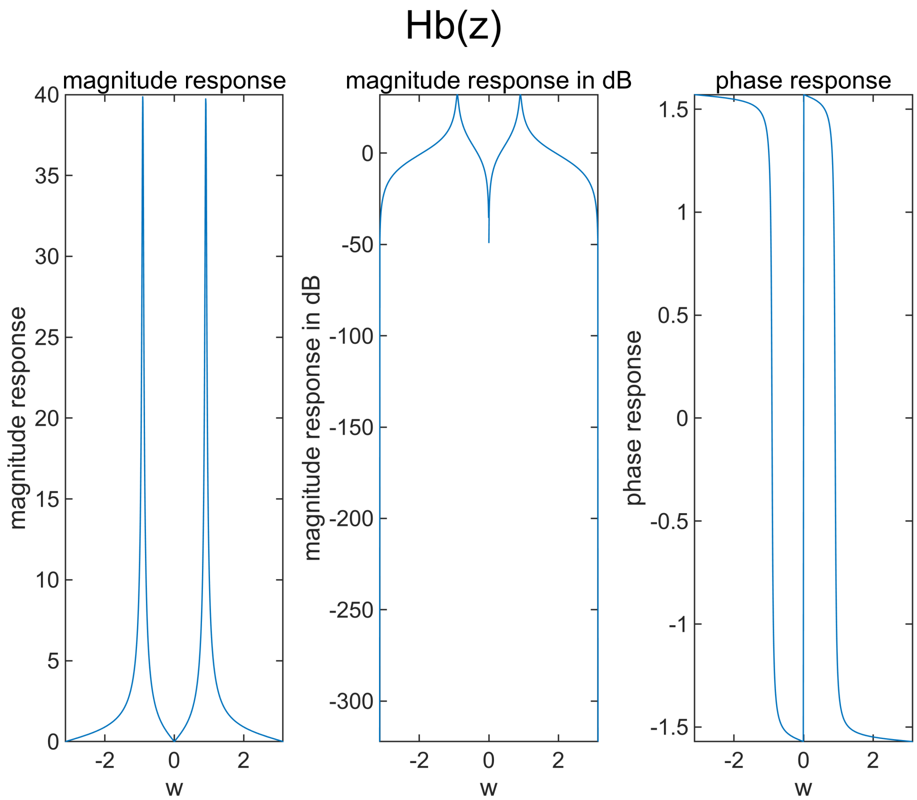
```
P1 = [1 2 0 2 1];
D1 = [1 0 0 0 0];

P2 = [1 0 -1];
D2 = [1 -1.2 0.95];

[mag1,phase1]=FreRes(P1,D1);
[mag2,phase2]=FreRes(P2,D2);

syms w;
figure()
sgtitle('Ha(z) ');
subplot(1,3,1);
fplot(w,mag1,[-pi,pi]);
xlabel('w');
ylabel('magnitude response');
title('magnitude response');

subplot(1,3,2);
fplot(w,20*log10(mag1),[-pi,pi]);
xlabel('w');
ylabel('magnitude response in dB');
title('magnitude response in dB');

subplot(1,3,3);
fplot(w,phase1,[-pi,pi]);
xlabel('w');
ylabel('phase response');
title('phase response');
```




```
figure()
sgtitle('Hb(z) ');
subplot(1,3,1);
fplot(w,mag2,[-pi,pi]);66666
```

```
ans =
66666
```

```
xlabel('w');
ylabel('magnitude response');
title('magnitude response');

subplot(1,3,2);
fplot(w,20*log10(mag2),[-pi,pi]);
xlabel('w');
ylabel('magnitude response in dB');
title('magnitude response in dB');

subplot(1,3,3);
fplot(w,phase2,[-pi,pi]);
xlabel('w');
ylabel('phase response');
title('phase response');
```



6.2.4 Inverse z-transform

$$X(z) = \frac{P(z)}{D(z)} = \frac{\sum_{i=0}^{M} p_i z^{-i}}{\sum_{i=0}^{N} d_i z^{-i}}$$

is first written to a integer polynomial $K(z)$ plus a proper fraction, i.e.,

$$X(z) = K(z) + \frac{P_1(z)}{D(z)}$$

where the degree of $P_1(z)$ is less than *N*. Then, the proper fraction is expanded to partial fractions in the forms of

$$\frac{P_1(z)}{D(z)} = \sum_{l=1}^{N} \left(\frac{\rho_l}{1 - \lambda_l z^{-l}} \right)$$

where the $z = \lambda_k$, $1 \leq k \leq N$ are the poles of the proper fraction. In Matlab, the integer polynomial and the partial fractions together with the residues, ρ_l, maybe found by using function **residue.m**.

展开分子次数高于分母次数的分式

当分子次数大于分母次数时，输出 k 为代表 s 中多项式系数的向量。

使用 `residue` 执行 $F(s)$ 的以下部分分式展开式。

$$F(s)=\frac{b(s)}{a(s)}=\frac{2s^4+s}{s^2+1}=\frac{0.5-1i}{s-1i}+\frac{0.5+1i}{s+1i}+2s^2-2.$$

```
b = [2 0 0 1 0];
a = [1 0 1];
[r,p,k] = residue(b,a)
```

```
r = 2×1 complex

   0.5000 - 1.0000i
   0.5000 + 1.0000i


p = 2×1 complex

   0.0000 + 1.0000i
   0.0000 - 1.0000i


k = 1×3

     2     0    -2
```

k 代表多项式 $2s^2-2$。

find the partial fraction expansion of the following two z-transforms: 令 s=z^-1

(a) $X(z)=\dfrac{3-7.8z^{-1}}{(1-0.7z^{-1})(1+1.6z^{-1})}$, (b) $Y(z)=\dfrac{3z^2+1.8z+1.28}{(z-0.5)(z+0.4)}$

```
P1 = [-7.8 3];
D1 = [-1.12 0.9 1];
P2 = [1.28 1.8 3];
D2 = [-0.2 -0.1 1];
[r, p, k] = residue(P1,D1);
[rou1,p1,k1] = residue_expansion(P1,D1);
[rou2,p2,k2] = residue_expansion(P2,D2);

display(rou1);
```

```
rou1 = 2×1
   -2.4783
    5.4783
```

```
display(p1);
```

```
p1 = 2×1
    0.7000
   -1.6000
```

```
display(k1);
```

```
k1 =

     []
```

```
display(rou2);
```

```
rou2 = 2×1
    2.8889
    6.5111
```

```
display(p2);
```

```
p2 = 2×1
   -0.4000
    0.5000
```

```
display(k2);
```

```
k2 =
-6.4000
```

6.2.5 Stability Conditions

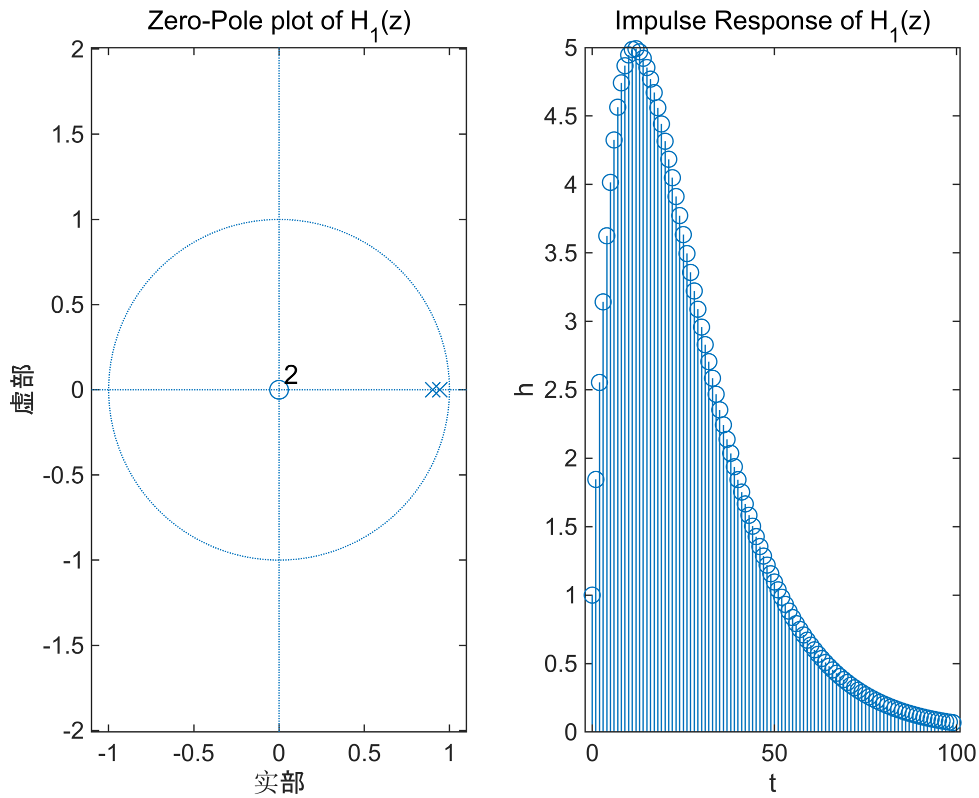
A stable causal LTI system has all poles inside unit circle.

$$H(z) = \frac{1}{1 - 1.845z^{-1} + 0.850586z^{-2}}$$

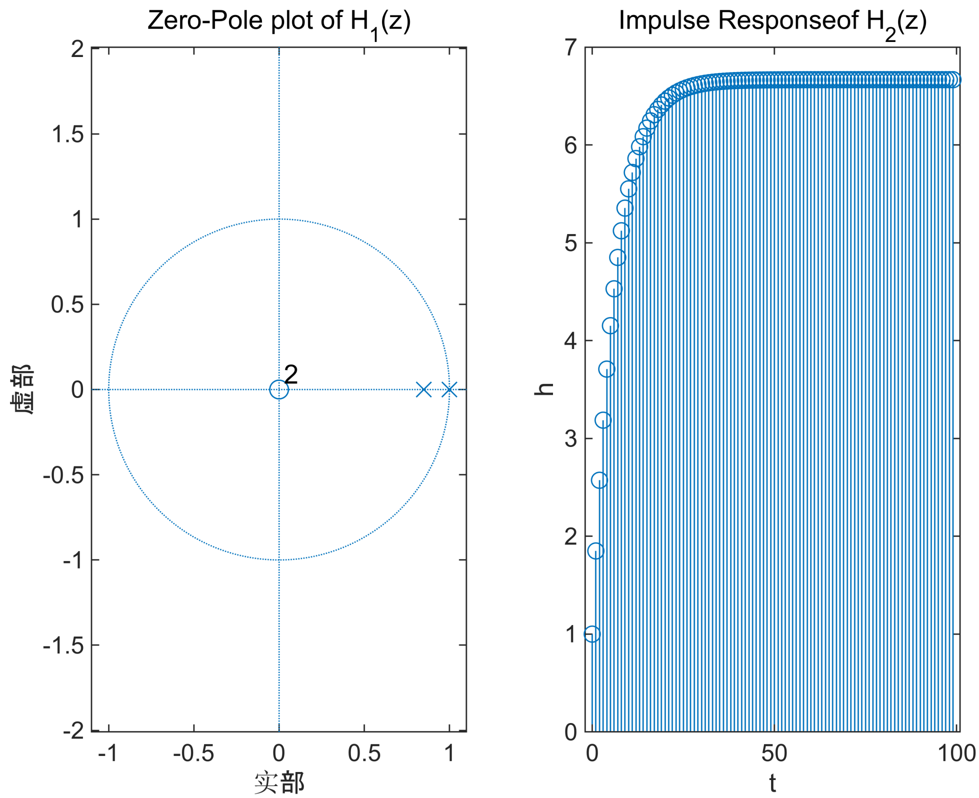
$$H(z) = \frac{1}{1 - 1.85z^{-1} + 0.85z^{-2}}$$

```
poles1 = [0.943 0.902]; zeros1 = [0 0];
b1 = [1]; a1 = [1 -1.845 0.850586];
[h1,t1] = impz(b1,a1,100);
poles2 = [1 0.85]; zeros2 = [0 0];
b2 = [1]; a2 = [1 -1.85 0.85];
[h2,t2] = impz(b2,a2,100);

figure();
subplot(1,2,1);
zplane(zeros1',poles1');
title('Zero-Pole plot of H_1(z)');

subplot(1,2,2);
```

```
stem(t1,h1);
xlabel('t');
ylabel('h');
title('Impulse Response of H_1(z)');
```




```
figure();
subplot(1,2,1);
zplane(zeros2',poles2');
title('Zero-Pole plot of H_1(z)');

subplot(1,2,2);
stem(t2,h2);
xlabel('t');
ylabel('h');
title('Impulse Responseof H_2(z)');
```




```
%Poles of H1(z) are all in unit circle, it's stable
%One pole of H2(z) is  on unit circle, it's unstable
```

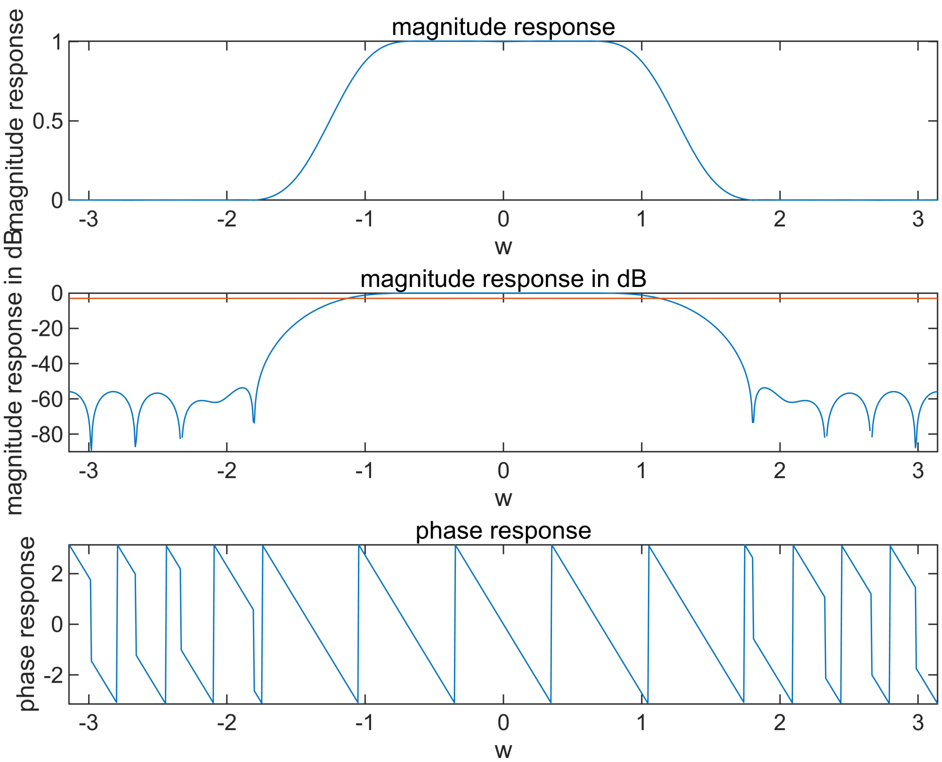
即使原始设计中所有极点都位于单位圆内（确保了滤波器的稳定性），量化后极点的这种微小移动可能会导致它们穿过单位圆，落入单位圆外部。当极点位于单位圆外时，滤波器的某些部分的响应会随着时间的推移而指数增长，导致系统不稳定。

6.3 Linear Phase FIR Filters

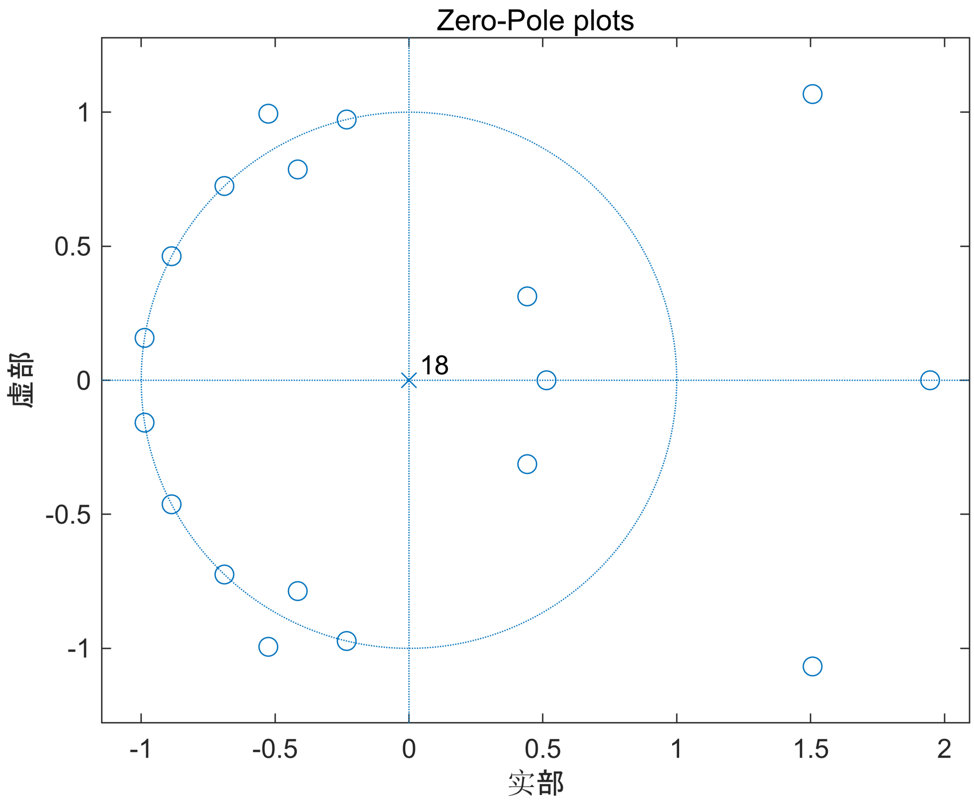
6.3.1 Four Types of Linear Phase FIR Filters

```
h=[-0.0035,-0.0039,0.0072,0.0201,-0.0000,-0.0517,-0.0506,0.0855,0.2965,0.4008,0.2965
,0.0855,-0.0506,-0.0517,-0.0000,0.0201,0.0072,-0.0039,-0.0035];

figure();
[mag,phase]=FreRes(h,1);
syms w;
figure()

subplot(3,1,1);
fplot(w,mag,[-pi,pi]);
xlabel('w');
ylabel('magnitude response');
```

```
title('magnitude response');

subplot(3,1,2);
fplot(w,20*log10(mag),[-pi,pi]);
xlabel('w');
ylabel('magnitude response in dB');
title('magnitude response in dB');
hold on;
plot(linspace(-pi,pi,1000),-3*ones(1,1000));

subplot(3,1,3);
fplot(w,phase,[-pi,pi]);
xlabel('w');
ylabel('phase response');
title('phase response');
```




```
figure();
P=h;
D=[1, zeros(1,18)];
Zeros = roots(P);
poles = roots(D);
zplane(Zeros,poles);
title('Zero-Pole plots');
```



An example of a type I linear phase FIR filter $H(z)$ has impulse response given by

$$\{h[n]\} = \{-0.0035, -0.0039, 0.0072, 0.0201, -0.0000, -0.0517, -0.0506, 0.0855, 0.2965, 0.4008, 0.2965, 0.0855, -0.0506, -0.0517, -0.0000, 0.0201, 0.0072, -0.0039, -0.0035\}$$

for $0 \le n \le 18$.

Using the program developed in Section 6.2.3, plot the frequency response of the filter. What is the magnitude characteristic of the filter? Show as well the zero-pole plot of the filter.

> INLAB REPORT: Submit the frequency response and zero-pole plots of the filter. Answer (1) the magnitude characteristic of the filter, and (2) if a type III filter may be designed to be a lowpass filter, and justify your answer.

(1):低通滤波器,

Additionally, it does not exhibit a zero phase across all frequencies

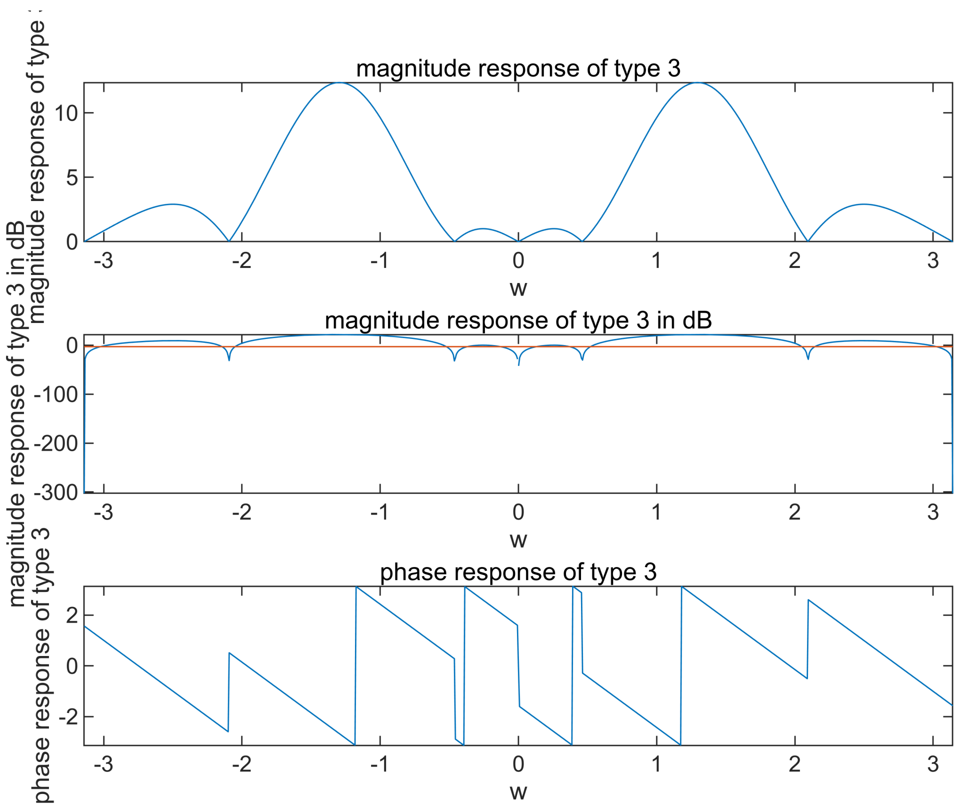
Cutoff frequency is about -1.13 and +1.13

????This type of filter has either an even number or no zeros at � = 1 and � =−1.

(2)type3 不可能作为低通，构造反对称序列

```
h_type3 = [-1 -2 2 3 0 -3 -2 2 1];
[mag2,phase2]=FreRes(h_type3,1);
syms w;
figure();
```

```
subplot(3,1,1);
fplot(w,mag2,[-pi,pi]);
xlabel('w');
ylabel('magnitude response of type 3');
title('magnitude response of type 3');

subplot(3,1,2);
fplot(w,20*log10(mag2),[-pi,pi]);
xlabel('w');
ylabel('magnitude response of type 3 in dB');
title('magnitude response of type 3 in dB');
hold on;
plot(linspace(-pi,pi,1000),-3*ones(1,1000));

subplot(3,1,3);
fplot(w,phase2,[-pi,pi]);
xlabel('w');
ylabel('phase response of type 3');
title('phase response of type 3');
```




```
% (2):No, it can be a bandpass filter,频谱在 0 是小, 反例
```

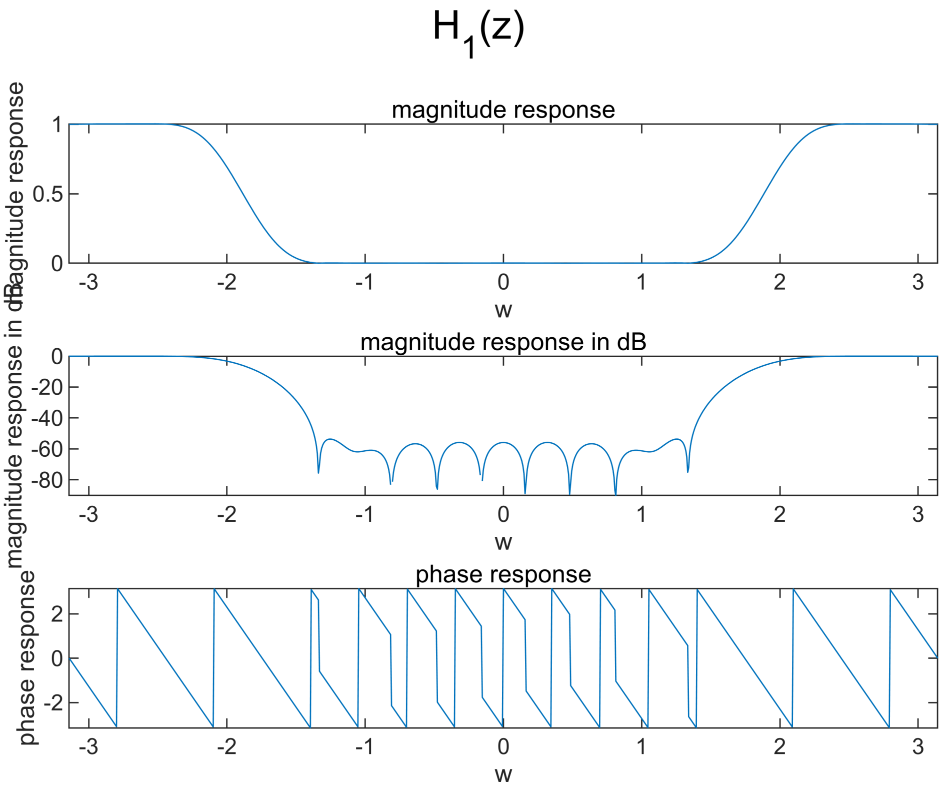
The impulse response of the $h_1[n]$ and $h_2[n]$ is obtained by

$$h_1[n] = \begin{cases} h[n], & \text{for even } n \\ -h[n], & \text{for odd } n \end{cases}$$

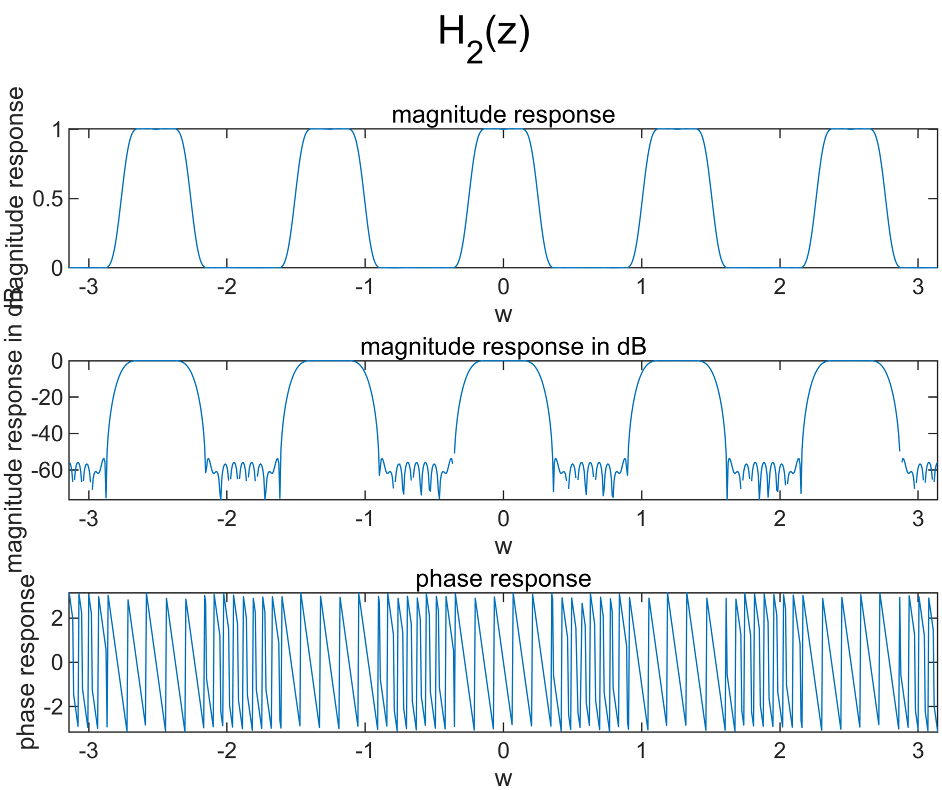
$$h_2[n] = \begin{cases} h[n/5], & \text{for } n = 5k \text{ for interger } k \\ 0, & \text{otherwise} \end{cases}$$

<u>INLAB REPORT: Express the z-transform of $h_1[n]$ and $h_2[n]$ in terms of $H(z)$, and express the frequency response of $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ in terms of $H(e^{j\omega})$. Verify the frequency response expressions by plotting the frequency response of $h_1[n]$ and $h_2[n]$. Describe the magnitude characteristic of the filter $H_1(z)$ and $H_2(z)$ with respect to that of $H(z)$.</u>

```
n=0:18;
h=[-0.0035,-0.0039,0.0072,0.0201,-0.0000,-0.0517,-0.0506,0.0855,0.2965,0.4008,0.2965
,0.0855,-0.0506,-0.0517,-0.0000,0.0201,0.0072,-0.0039,-0.0035];
h1=h;
h2=zeros(1,91);
h1(2:2:19)=-h1(2:2:19);
h2(1:5:91)=h(1:1:19);
syms w;

[mag1,phase1]=FreRes(h1,1);
[mag2,phase2]=FreRes(h2,1);

figure();
sgtitle('H_1(z) ');
subplot(3,1,1);
fplot(w,mag1,[-pi,pi]);
xlabel('w');
ylabel('magnitude response');
title('magnitude response');

subplot(3,1,2);
fplot(w,20*log10(mag1),[-pi,pi]);
xlabel('w');
ylabel('magnitude response in dB');
title('magnitude response in dB');

subplot(3,1,3);
fplot(w,phase1,[-pi,pi]);
xlabel('w');
ylabel('phase response');
title('phase response');
```




```
figure();
sgtitle('H_2(z)');
subplot(3,1,1);
fplot(w,mag2,[-pi,pi]);
xlabel('w');
ylabel('magnitude response');
title('magnitude response');

subplot(3,1,2);
fplot(w,20*log10(mag2),[-pi,pi]);
xlabel('w');
ylabel('magnitude response in dB');
title('magnitude response in dB');

subplot(3,1,3);
fplot(w,phase2,[-pi,pi]);
xlabel('w');
ylabel('phase response');
title('phase response');
```



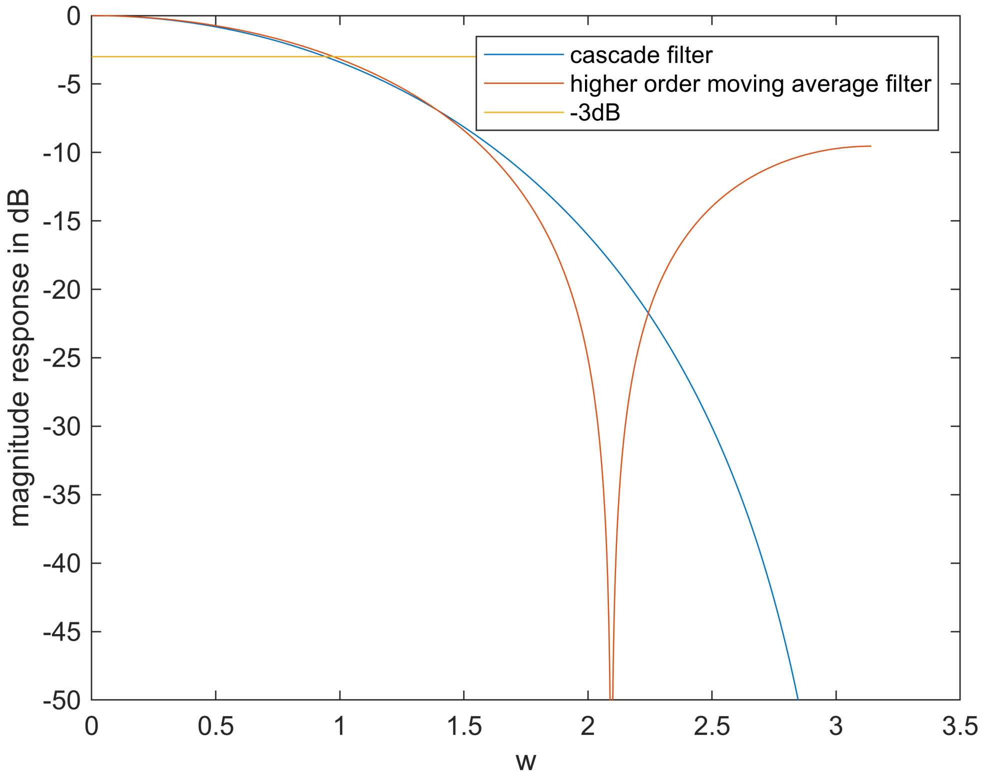
6.3.2 Design of Simple FIR Filters

$$H_0(z) = \frac{1 + z^{-1}}{2}.$$

Design a linear phase lowpass FIR filter with a 3-dB cutoff frequency at 0.3�

```
w = linspace(0,pi,4000);
% M = -1/2*log2(cos(wc/2))
M = 3;
H_cascade = exp(-j*3*w/2).*(cos(w/2)).^3;
H_moving_average = (1/M)*sin(M.*w/2)./sin(w/2).*exp(-1j*(M-1).*w./2);
figure
plot(w,20*log10(abs(H_cascade)));
hold on;
plot(w,20*log10(abs(H_moving_average)));
plot(w,-3*ones(1,4000));
ylim([-50,0]);
xlabel('w');
ylabel('magnitude response in dB');
```

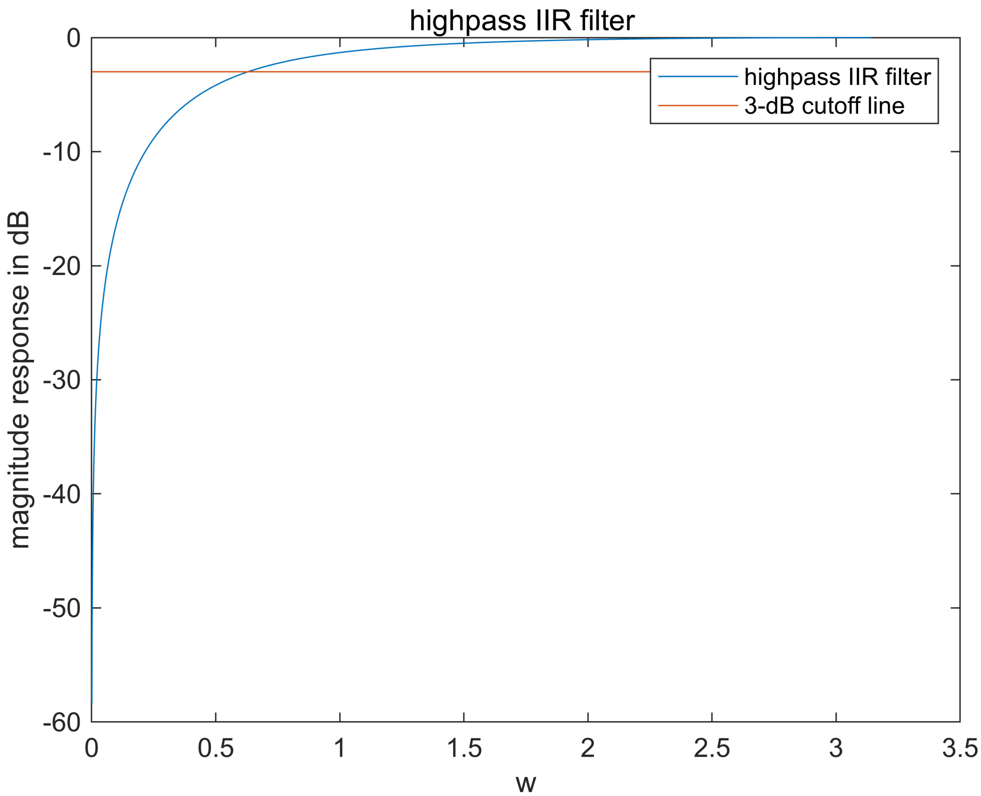
```
legend('cascade filter','higher order moving average filter','-3dB ');
hold off
```



6.4 IIR Filters

Design a first order highpass IIR filter with a 3-dB cutoff frequency of 0.2 �.

```
a = (1-sin(0.2*pi))/cos(0.2*pi);
w = linspace(0,pi,4000);
H = sqrt(((1+a)^2*(1-cos(w)))./(2*(1+a^2-2*a*cos(w))));
figure
plot(w,20*log10(abs(H)));
hold on;
plot(w,-3*ones(1,4000));
xlabel('w');
ylabel('magnitude response in dB');
title('highpass IIR filter ');
legend('highpass IIR filter','3-dB cutoff line');
```


highpass IIR filter
highpass IIR filter
3-dB cutoff line
magnitude response in dB
w
0
-10
-20
-30
-40
-50
-60
0
0.5
1
1.5
2
2.5
3
3.5