M263A Project2 Report

Zhi Li, Yunchu Zhang

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1.Design the Motion of the robot

Our group decide to use hand to knock off the table tennis. In order to achieve the task without losing creativity, our group decide to knock the ball at the back of the robot by right hand.

In order to make the hand be able to touch the ball, we should first reduce the altitute of the base frame. It means that we should design leg motions.

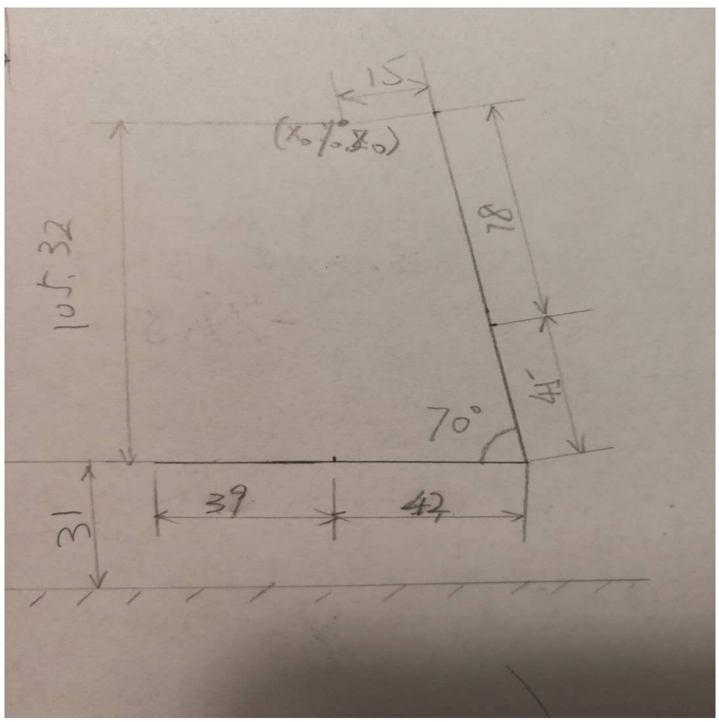

Figure 1

Figure 1 shows the crouch position of the robot and the position of the base frame

The configurations of legs are shown as follows:

θ_7	θ_9	θ_{11}	θ_{13}	θ_{15}
0	5	110	90	0

θ_8	θ_{10}	θ_{12}	θ_{14}	θ_{16}
0	-5	-110	-90	0

From Figure 1, it is easy for us to compute the reachable workspace of right arms and then decide the position of the tee.

The maximam reachable workspace(not the accurate workspace, just the maximum reachable distance of the right arm) and the positon of tee are shown in Figure 2.

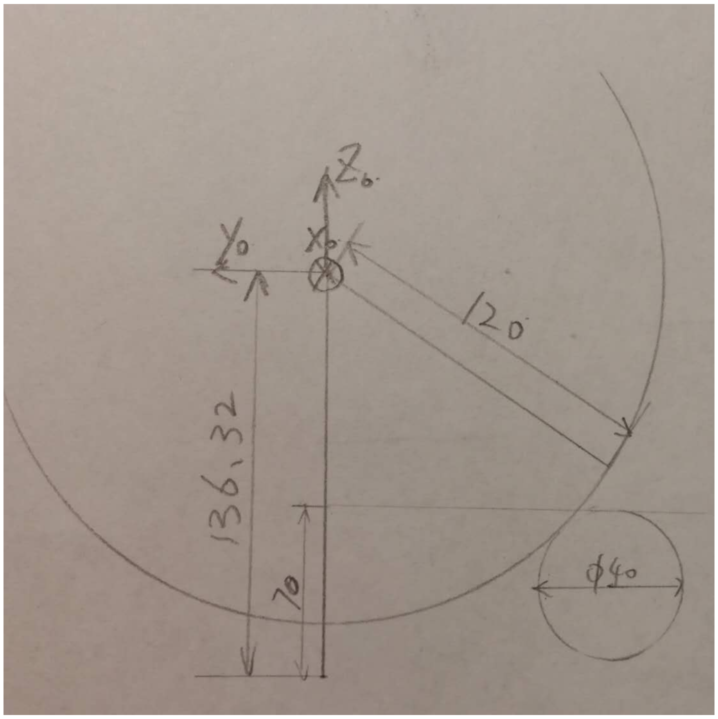

Figure 2

Finaly, we choose $(-80, -33)$ as our tee position with repect to the base frame in initial condition.

2.Description of Knocking ball

The right hand of robot will knock off the ball at the back of the robot's body. The direction the ball would be knocked off is positive y direction.(Shown in Figure 3)

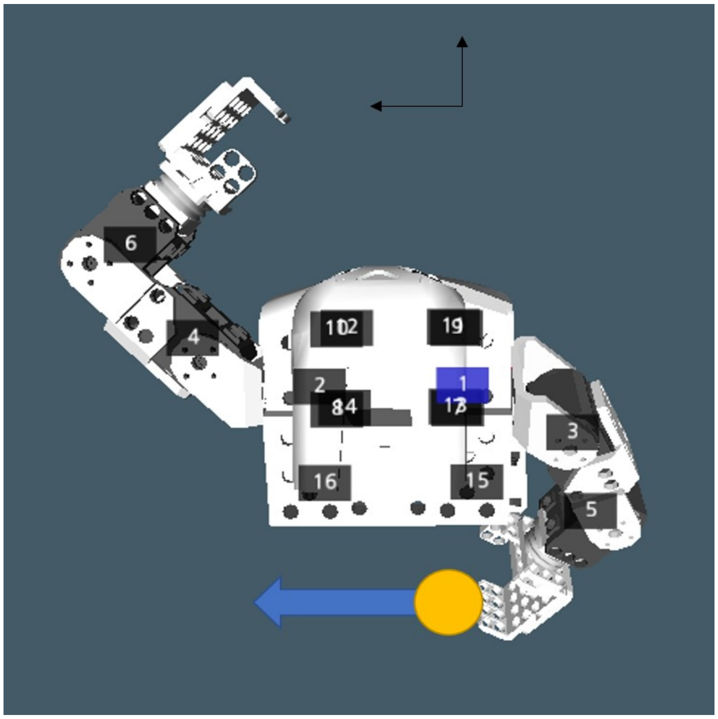

Figure 3

3.Inverse Kinematics of DARWIN-Mini

(1). Forward Kinematics Solution

The forward kinematics results of DARWIN-Mini are as follows:

a.Right arm

The transformation matrix between frame 0 and frame RH ${}_{RH}^{0}T$ is:

$$
{}_{RH}^{0}T = \begin{bmatrix} c\theta_1 c(\theta_3+\theta_5) & s\theta_1 & c\theta_1 s(\theta_3+\theta_5) & 12c\theta_1 + 45c\theta_1 c\theta_3 + 75c\theta_1 s(\theta_3+\theta_5) \\ -s(\theta_3+\theta_5) & 0 & c(\theta_3+\theta_5) & 75c(\theta_3+\theta_5) - 45s\theta_3 - 57 \\ s\theta_1 c(\theta_3+\theta_5) & -c\theta_1 & s\theta_1 s(\theta_3+\theta_5) & 12s\theta_1 + 45c\theta_3 s\theta_1 + 75s\theta_1 s(\theta_3+\theta_5) \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

b.Left arm

The transformation matrix between frame 0 and frame LH ${}_{LH}^{0}T$ is:

$$
{}_{LH}^{0}T = \begin{bmatrix} c\theta_2 c(\theta_4+\theta_6) & s\theta_2 & -c\theta_2 c(\theta_4+\theta_6) & 12c\theta_2 + 45c\theta_2 c\theta_4 - 75c\theta_2 c(\theta_4+\theta_6) \\ -s(\theta_4+\theta_6) & 0 & -c(\theta_4+\theta_6) & -75c(\theta_4+\theta_6) - 45s\theta_4 + 57 \\ -s\theta_2 c(\theta_4+\theta_6) & c\theta_2 & s\theta_2 s(\theta_4+\theta_6) & 75s\theta_2 s(\theta_4+\theta_6) - 12s\theta_2 - 45c\theta_4 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

c.Right leg

The transformation matrix between frame 0 and frame RF ${}_{RF}^{0}T$ is:

$$
{}_{RF}^{0}T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & p_x \\ a_{21} & a_{22} & a_{23} & p_y \\ a_{31} & a_{32} & a_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

where

$$a_{11} = c\theta_{15} s(\theta_{13} - \theta_9 - \theta_{11})$$

$$a_{12} = s\theta_{15} s(\theta_{13} - \theta_9 - \theta_{11})$$

$$a_{13} = c(\theta_{13} - \theta_9 - \theta_{11})$$

$$p_x = 31c\theta_{15} s(\theta_{13} - \theta_9 - \theta_{11}) - 42s(\theta_9 + \theta_{11}) - 45s\theta_9 + 15$$

$$a_{21} = -c\theta_7 s\theta_{15} + c\theta_{15} s\theta_7 c(\theta_{13} - \theta_9 - \theta_{11})$$

$$a_{22} = c\theta_7 c\theta_{15} + s\theta_{15} s\theta_7 c(\theta_{13} - \theta_9 - \theta_{11})$$

$$a_{23} = -s\theta_7 s(\theta_{13} - \theta_9 - \theta_{11})$$

$$p_y = 6s\theta_7 + 45c\theta_9 s\theta_7 - 31c\theta_7 s\theta_{15} + 31c\theta_{15} s\theta_7 c(\theta_{13} - \theta_9 - \theta_{11}) + 42s\theta_7 c(\theta_9 + \theta_{11}) - 24$$

$$a_{31} = -c\theta_{15} c\theta_7 c(\theta_{13} - \theta_9 - \theta_{11}) - s\theta_7 s\theta_{15}$$

$$a_{32} = -s\theta_{15} c\theta_7 c(\theta_{13} - \theta_9 - \theta_{11}) + s\theta_7 c\theta_{15}$$

$$a_{33} = c\theta_7 s(\theta_{13} - \theta_9 - \theta_{11})$$

$$p_z = -31c\theta_{15} c\theta_7 c(\theta_{13} - \theta_9 - \theta_{11}) - 45c\theta_7 c\theta_9 - 31s\theta_7 s\theta_{15} - 6c\theta_7 - 42c\theta_7 c(\theta_9 + \theta_{11}) - 72$$

d.Left leg

The transformation matrix between frame 0 and frame LF ${}_{LF}^{0}T$ is:

$$
{}_{LF}^{0}T = \begin{bmatrix} b_{11} & b_{12} & b_{13} & p_x \\ b_{21} & b_{22} & b_{23} & p_y \\ b_{31} & b_{32} & b_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

where

$$
\begin{gathered}
b_{11} = -c\theta_{16} s\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
b_{12} = -s\theta_{16} s\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
b_{13} = c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
p_x = -31c\theta_{16} s\left(\theta_{14} - \theta_{10} - \theta_{12}\right) + 42s\left(\theta_{10} + \theta_{12}\right) + 45s\theta_{10} + 15 \\
b_{21} = -c\theta_8 s\theta_{16} + c\theta_{16} s\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
b_{22} = c\theta_8 c\theta_{16} + s\theta_{16} s\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
b_{23} = s\theta_8 s\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
p_y = 6s\theta_8 + 45c\theta_{10} s\theta_8 - 31c\theta_8 s\theta_{16} + 31c\theta_{16} s\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) + 42s\theta_8 c\left(\theta_{10} + \theta_{12}\right) + 24 \\
b_{31} = -c\theta_{16} c\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) - s\theta_8 s\theta_{16} \\
b_{32} = -s\theta_{16} c\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) + s\theta_8 c\theta_{16} \\
b_{33} = -c\theta_8 s\left(\theta_{14} - \theta_{10} - \theta_{12}\right) \\
p_z = -31c\theta_{16} c\theta_8 c\left(\theta_{14} - \theta_{10} - \theta_{12}\right) - 45c\theta_8 c\theta_{10} - 31s\theta_8 s\theta_{16} - 6c\theta_8 - 42c\theta_8 c\left(\theta_{10} + \theta_{12}\right) - 72
\end{gathered}
$$

(2). Inverse Kinematics Solution

a.Solution for Right Arm

Denote the elements in ${}_{RH}^{0}T_{goal}$ as,

$$
{}_{RH}^{0}T_{goal} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Therefore, we can deduce the joint angle:

solve for θ_1:

$$
\theta_1 = Atan2(p_z, p_x)
$$

solve for θ_3:

$$
\theta_3 + \theta_5 = Atan2(r_{33}, r_{31})
$$

$$
75c\left(\theta_3 + \theta_5\right) - 45s\theta_3 - 57 = p_y \tag{1}
$$

$$
12s\theta_1 + 45c\theta_3 s\theta_1 + 75s\theta_1 s\left(\theta_3 + \theta_5\right) = p_z \tag{2}
$$

From $(1)^2 + (2)^2$ we can derive

$\Longrightarrow$

$$
\sin\theta_3 = \frac{75\cos\left(\theta_3 + \theta_5\right) - 57 - p_y}{45}
$$

$$
\cos\theta_3 = \frac{p_z - 12\sin\theta_1 - 75\cos\left(\theta_3 + \theta_5\right)}{45\sin\theta_1}
$$

$\Longrightarrow$

$$\theta_3 = Atan2(\sin\theta_1, \cos\theta_1)$$

Solve for θ_5:

$$\theta_5 = (\theta_5 + \theta_3) - \theta_3 = Atan2(r_{33}, r_{31}) - Atan2(\sin\theta_1, \cos\theta_1)$$

summary

$$\theta_1 = Atan2(p_z, p_x)$$
$$\theta_3 = Atan2(\sin\theta_1, \cos\theta_1)$$
$$\theta_5 = Atan2(r_{33}, r_{31}) - Atan2(\sin\theta_1, \cos\theta_1)$$

b.Solution for Right Leg

Similarly denote the elements in ${}^{0}_{RF}T_{goal}$ as,

$${}^{0}_{RF}T_{goal} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, we can derive the joint angles:
Solve for θ_{15}:

$$\theta_{15} = Atan2(r_{12}, r_{11})$$

Solve for θ_7:

$$\theta_7 = Atan2(-r_{33}, r_{23})$$

Solve for θ_{11}:

$$\theta_{13} - \theta_9 - \theta_{11} = Atan2(\frac{r_{33}}{\cos\theta_7}, r_{13})$$

$${}^{0}_{7}T^{-1}_{goal}\,{}^{0}_{RF}T_{goal}\,{}^{13}_{RF}T^{-1}_{goal} = {}^{0}_{7}T^{-1}_{goal}\,{}^{0}_{RF}T\,\,{}^{13}_{RF}T^{-1}_{goal}$$

$\Longrightarrow$

$${}^{0}_{7}T^{-1}_{goal}\,{}^{0}_{RF}T_{goal}\,{}^{13}_{RF}T^{-1}_{goal} = {}^{7}_{13}T$$

$$Left = \begin{bmatrix} m_{11} & m_{12} & m_{13} & p_x \\ m_{21} & m_{22} & m_{23} & p_y \\ m_{31} & m_{32} & m_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$m_{11} = r_{21}c_{15}s_7 - r_{31}c_7c_{15} - r_{32}c_7s_{15} + r_{22}s_7s_{15}$$
$$m_{12} = -r_{23}s_7 - r_{33}c_7$$
$$m_{13} = r_{32}c_7c_{15} - r_{22}s_7c_{15} - r_{31}c_7s_{15} + r_{21}s_7s_{15}$$
$$p_x = 24s_7 - 72c_7 + 31r_{31}c_7 - p_zc_7 - 31r_{21}s_7 + p_ys_7$$
$$m_{21} = r_{21}c_7c_{15} + r_{22}c_7s_{15} + r_{31}s_7c_{15} + r_{32}s_7s_{15}$$
$$m_{22} = r_{23}c_7 + r_{33}s_7$$
$$m_{23} = r_{21}c_7s_{15} - r_{22}c_7c_{15} - r_{32}s_7c_{15} + r_{31}s_7s_{15}$$
$$p_y = 24c_7 + 72s_7 - 31r_{21}c_7 + p_zs_7 - 31r_{31}s_7 + p_yc_7$$
$$m_{31} = r_{11}c_{15} + r_{12}s_{15}$$
$$m_{32} = r_{13}$$
$$m_{33} = r_{11}s_{15} - r_{12}c_{15}$$

$$p_z = p_x - 31r_{11} - 15$$

$$Right = \begin{bmatrix} c_{13-9-11} & -s_{13-9-11} & 0 & 42c_{9+11} + 45c_9 + 6 \\ 0 & 0 & -1 & 0 \\ s_{13-9-11} & c_{13-9-11} & 0 & -42s_{9+11} - 45s_9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, we can get two equations:

$$42c_{9+11} + 45c_9 + 6 = 24s_7 - 72c_7 + 31r_{31}c_7 - p_z c_7 - 31r_{21}s_7 + p_y s_7$$

$$-42s_{9+11} - 45s_9 = p_x - 31r_{11} - 15$$

Denote

$$a = 24s_7 - 72c_7 + 31r_{31}c_7 - p_z c_7 - 31r_{21}s_7 + p_y s_7 - 6$$

$$b = p_x - 31r_{11} - 15$$

Thus

$$42c_{9+11} + 45c_9 = a$$

$$-42s_{9+11} - 45s_9 = b$$

Square the two equations and add them up:

$$\Longrightarrow 1764 + 3780\cos\theta_{11} + 2025 = a^2 + b^2$$

Thus

$$\cos\theta_{11} = \frac{a^2 + b^2 - 3798}{3780}$$

$$\sin\theta_{11} = \pm\sqrt{1 - \cos\theta_{11}^2}$$

$$\theta_{11} = Atan2(\sin\theta_{11}, \cos\theta_{11})$$

Solve For θ_9:

By operating the two equations:

$$42c_{9+11} + 45c_9 = a$$

$$-42s_{9+11} - 45s_9 = b$$

we can get:

$$42c_9c_{11} - 42c_9c_{11} + 45c_9 = a \tag{3}$$

$$42s_9c_{11} + 42c_9s_{11} + 45s_9 = -b \tag{4}$$

Multiply s_9 on (3) equation and c_9 on (4) equation and then, (3) − (4)

$$\Longrightarrow bc_9 + as_9 = -42s_{11}$$

Thus,

$$\theta_9 = Atan2(\pm\sqrt{a^2 + b^2 - c^2}, c) + Atan2(a, b), c = -42s_{11}$$

Solve for θ_{13}

$$\theta_{13} = (\theta_{13} - \theta_9 - \theta_{11}) + \theta_9 + \theta_{11}$$

Summary:

$$\theta_{15} = Atan2(r_{12}, r_{11})$$

$$\theta_7 = Atan2(-r_{33}, r_{23})$$

$$\theta_{11} = Atan2(\pm\sqrt{1 - \cos\theta_{11}^2}, \frac{a^2 + b^2 - 3798}{3780})$$

$$\theta_9 = Atan2(\pm\sqrt{a^2 + b^2 - c^2}, c) + Atan2(a, b)$$

where

$$a = 24s_7 - 72c_7 + 31r_{31}c_7 - p_zc_7 - 31r_{21}s_7 + p_ys_7 - 6$$
$$b = p_x - 31r_{11} - 15$$
$$c = -42s_{11}$$

and finally

$$\theta_{13} = (\theta_{13} - \theta_9 - \theta_{11}) + \theta_9 + \theta_{11}$$

Another solution we also want to show(calculate by different methods):

$$\theta_{15} = Atan2(-\frac{r_{12}}{s_{9+11-13}}, -\frac{r_{11}}{s_{9+11-13}})$$

$$\theta_7 = Atan2(\pm\sqrt{k_1^2 + k_2^2 - r_{31}^2}, r_{31}) + Atan2(k_2, k_1)$$

where

$$k_1 = -c_{15}c_{9+11-13}$$
$$k_2 = -s_{15}$$
$$\theta_{11} = Atan2(\pm\sqrt{1-c^2}, c)$$

where

$$c = \frac{a^2 + b^2}{3780}$$

,

$$a = \frac{p_y - 6s\theta_7 + 31c\theta_7 s\theta_{15} - 31c\theta_{15}s\theta_7 c\left(\theta_{13} - \theta_9 - \theta_{11}\right) + 24}{s_7}$$
$$b = -p_x + 31c\theta_{15}s\left(\theta_{13} - \theta_9 - \theta_{11}\right) + 15$$

$$\theta_9 = Atan2(\pm\sqrt{a^2 + b^2 - c^2}, c) + Atan2(a, b)$$

where

$$a = 42c_{11} - 45$$
$$b = s_{11}$$
$$c = 42s_9c_{11} + 42c_9s_{11} - 45s_9$$
$$\theta_{13} = \theta_9 + \theta_{11} - \theta_{9+11-13}$$

4.Design of Robot Motion

The motion process can be simply described as:
Kneel Down⇒Moving Right Arm to knock⇒Stand up⇒Celebrating motions

(1).Kneel Down

The legs configuration have been shown in the first part of the report. The results restate as follows:

time	θ_7	θ_9	θ_{11}	θ_{13}	θ_{15}
0	0	-26	29	13	0
2s	0	5	110	90	0

time	θ_8	θ_{10}	θ_{12}	θ_{14}	θ_{16}
0	0	26	-29	-13	0
2s	0	-5	-110	-90	0

Therefore, the configuration of the robot at 2 second is:

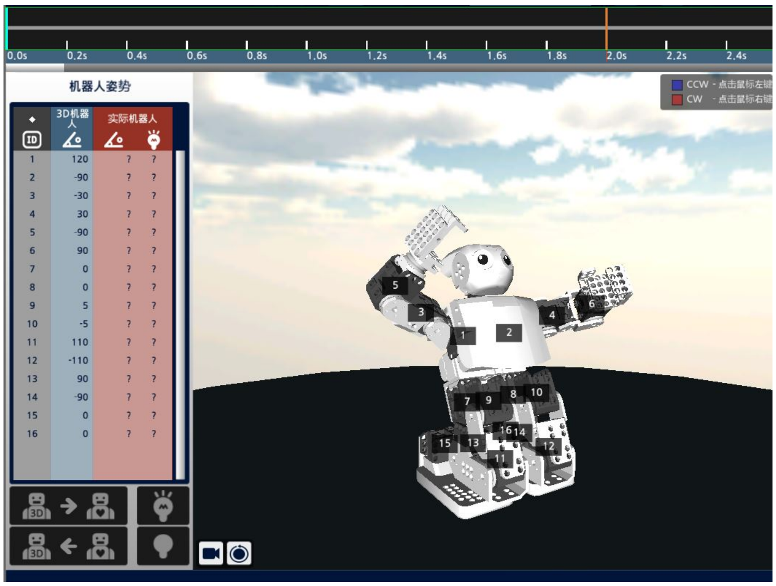

Figure 4

We just use the forward kinematics to design the hand motions of the robot in 2 seconds. The configuration is shown in Figure 4

(2).Moving Right Arm to Knock

According to the position of the tee and the analysis of inverse kinematics, we derive the joint angles for the arm. The results are compared with joint angle limits. Sign change was conducted on some joint angles if necessary.

Figure 5 shows the mathematic model of the robot and the geometric relations between base frame and tee position. according to the tee position we have chosen in part 1 of this report, tee$(-80, -33)$ with respect to the base frame of the initial condition.

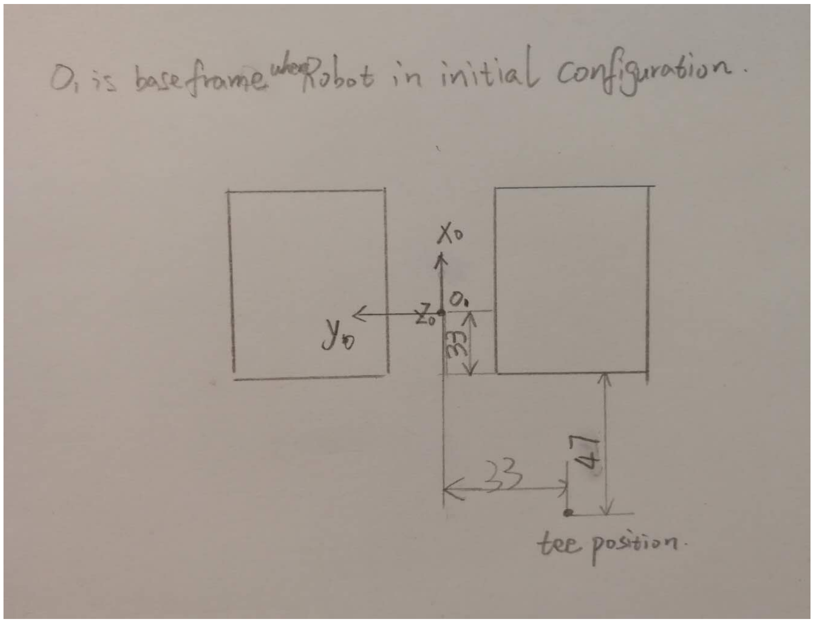

Figure 5

According to the geometric analysis in Figure 6 we can derive that the position of the finger tip which is $(-98.9, -33, -50)$.

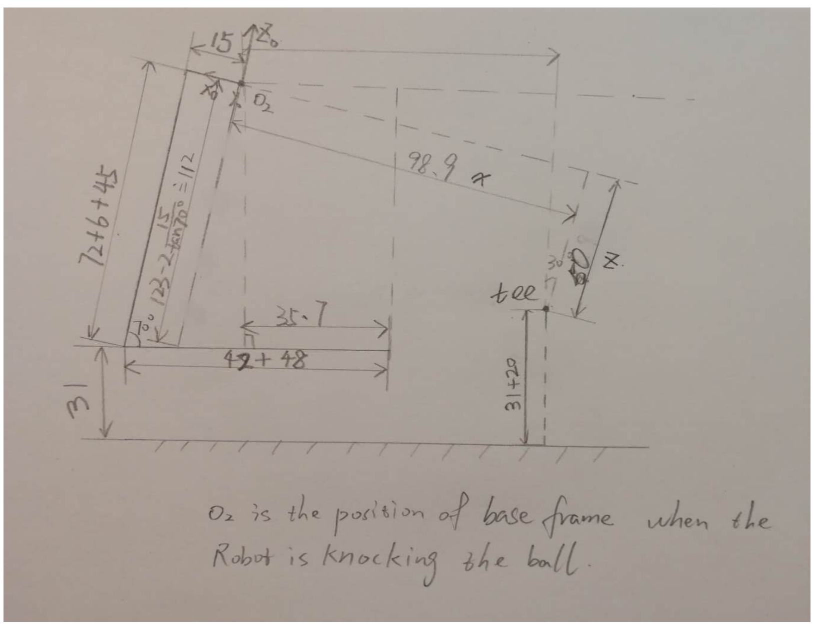

Figure 6

The goal forward matrix is

$$
{}_{RH}^{0}T_{goal} = \begin{bmatrix} -0.5 & -0.5 & -0.7 & -98.9 \\ -0.82 & 0 & 0.574 & -33 \\ -0.29 & 0.866 & -0.41 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

By calculating the inverse kinematic of right arm, we finally derive the joint angle for the right arm. The results are:

$$
\theta_1 \approx -150^\circ
$$

$$\theta_3 \approx 25^\circ$$

$$\theta_5 \approx 30^\circ$$

Since we do not build the frames in robot's initial configuration. So those angle we derive is just the DH joint angles. We should correct the error(which are 90° for θ_1, -90° for θ_3 and θ_5). Therefore, we derive the motor angle for robot to complete the knocking task.

The robots willl knock off the ball at 4 seconds, the configuration of right arm is shown as follow:

time	θ_1	θ_3	θ_5
4s	-60	-65	-60

The motion of the robot is shown in Figure 7

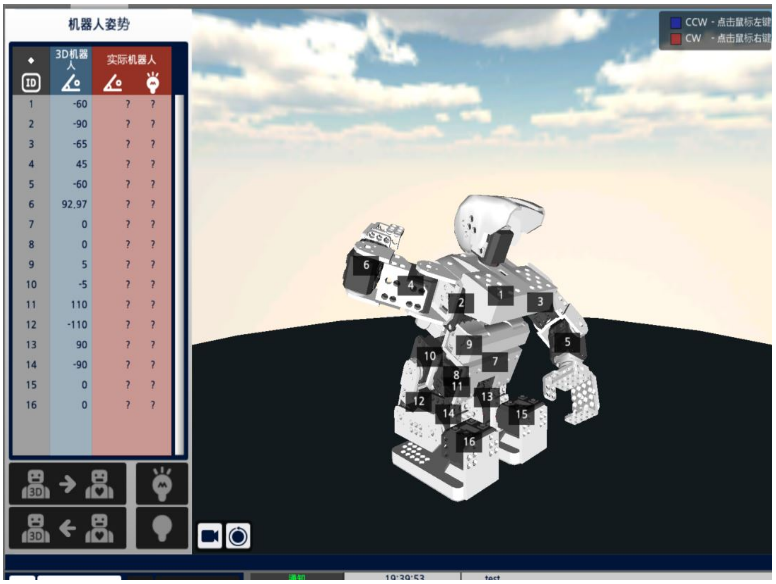

Figure 7

(3).Stand Up

After knocking off the ball,the robot will stand up and resume its initial configuration.

(4).Celebrating

The design of celebrating motion was inspired by the cheering motion of Cristiano Ronaldo, a famous soccer player. The motions include waving hands, twist the waist, taking a bow and so on.