# Physics 2211 GPS Week 8

## Problem #1

Ed and Mike are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.5 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Ed exerts a force $\langle -400, 0, 200\rangle$ N and Mike exerts a force $\langle 150, 0, 300\rangle$ N.

(a) How much work does Ed do?

✓ $\Delta\vec{r}_{boat} = \vec{r}_f - \vec{r}_i = \langle 4,0,2\rangle - \langle 2,0,3\rangle = \langle 2,0,-1\rangle$ m

$\Rightarrow W_{Ed} = \vec{F}_{Ed} \cdot \Delta\vec{r}_{boat} = \langle -400, 0, 200\rangle \cdot \langle 2, 0, -1\rangle =$

$= (-400)(2) + (200)(-1) = -800 - 200 = \boxed{-1000 \text{ J}}$

(b) How much work does Mike do?

✓ $\Delta\vec{r}_{boat}$ = same as above = $\langle 2, 0, -1\rangle$ m

$\Rightarrow W_{Mike} = \vec{F}_{Mike} \cdot \Delta\vec{r}_{boat} = \langle 150, 0, 300\rangle \cdot \langle 2, 0, -1\rangle =$

$= (150)(2) + (300)(-1) = 300 - 300 = \boxed{0}$

(c) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

$$\Delta E = W_{total}$$

$$\Delta K = W_{Ed} + \cancel{W_{Mike}}^{0}$$

$$K_f - K_i = W_{Ed}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = W_{Ed}$$

$$v_f^2 - v_i^2 = \frac{2}{m}W_{Ed}$$

$$v_f^2 = \frac{2W}{m} + v_i^2$$

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-1000)}{3000} + (1.5)^2} = \boxed{1.26 \text{ m/s}}$$

(d) What effect does Mike have on the boat's motion?

Steering (changing direction of motion)

Problem #2

A lighthouse keeper spots a sailboat of mass $M$ at location A $\langle x_0, 0, 0\rangle$ moving with speed $v_0$. After dozing off for a quick nap, the lighthouse keeper awakens to find the sailboat at location B $\langle 0, y_0, 0\rangle$. Having no way to measure the passage of time, the keeper decides to use her vast knowledge of the sea to estimate the speed of the sailboat. The keeper estimates that the net force acting on the sailboat is constant and given by $\langle a, b, 0\rangle$ where both $a$ and $b$ are positive constants. What would the lighthouse keeper predict for the speed of the sailboat at location B?

$$\vec{r}_A = \langle x_0, 0, 0 \rangle$$

$$|\vec{v}_A| = v_0$$

$$\vec{r}_B = \langle 0, y_0, 0 \rangle$$

$$\vec{F}_{net} = \langle a, b, 0 \rangle \quad \begin{Bmatrix} a > 0 \\ b > 0 \end{Bmatrix}$$

$$\Delta E = W$$

$$\Delta K = \int \vec{F}_{net} \cdot d\vec{l}$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \vec{F}_{net} \cdot \Delta \vec{L} \qquad \text{where } \Delta \vec{L} = \vec{r}_f - \vec{r}_i$$

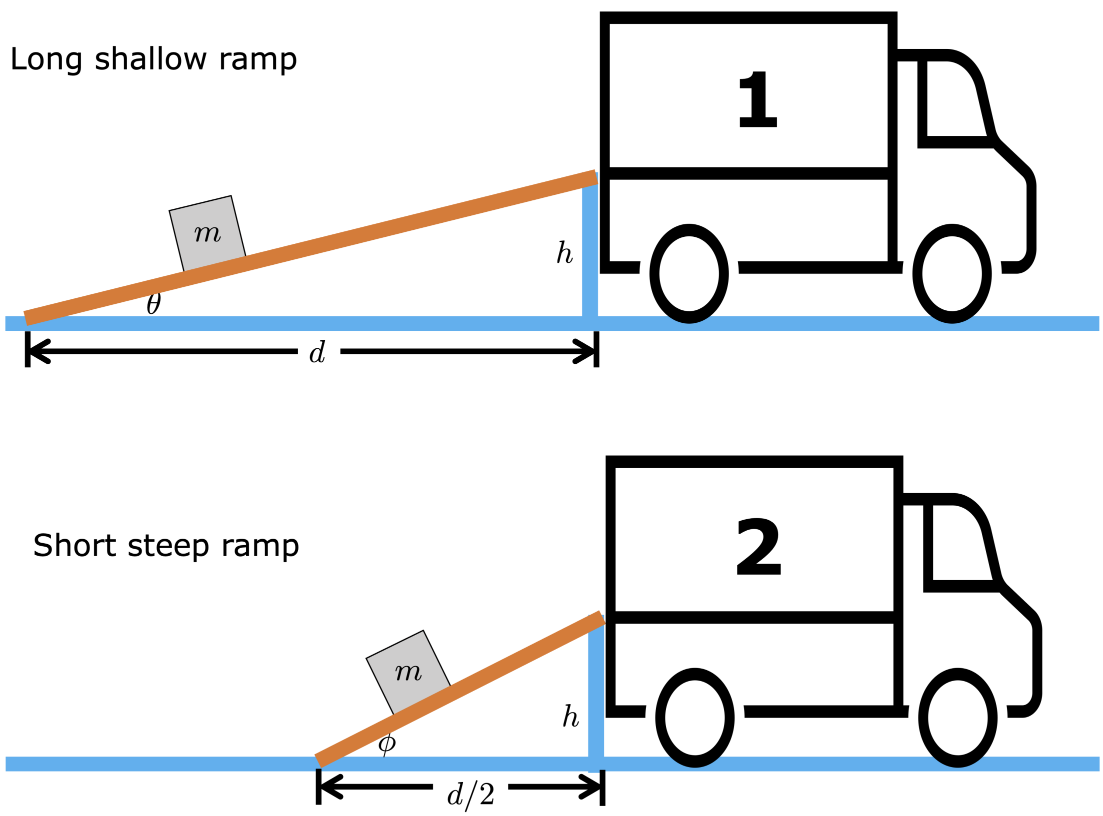
$$= \vec{F}_{net} \cdot \langle -x_0, +y_0, 0 \rangle \qquad = \langle -x_0, +y_0, 0 \rangle$$

$$\frac{1}{2} m (v_f^2 - v_0^2) = -a x_0 + b y_0$$

$$\rightarrow \quad v_f = \sqrt{\frac{2}{m}(b y_0 - a x_0) + {v_0}^2}$$

**Problem #3**

You are moving and want to use your knowledge from PHYS 2211 to help you decide which truck to rent out of two options. Truck number one has a **long ramp** at a shallow angle $\theta$. Truck number two has a **short ramp** at a steep angle $\phi$. You start with the simple problem of pushing a box of mass $m$ up to the height $h$ of the truck. You can assume both trucks have **frictionless** ramps.




Consider the **box** to be the system, the **initial state** to be when the box is motionless at the bottom of the ramp, and the **final state** to be when the box is at the top of the ramp.

(a) What is the **work done by gravity** as the system goes from its initial state to its final state?

$$W_{grav} = \int_i^f \vec{F}_g \cdot d\vec{r}$$

$$= \vec{F}_g \cdot \vec{d} \quad (\text{b/c } \vec{F}_g \text{ is constant})$$

$$= mg(-\hat{y}) \cdot h(\hat{y})$$

$$\boxed{W_{grav} = -mgh}$$

(b) If you push the box with a force of magnitude $F$ that is **parallel to the ramp**, how much work $W_{long}$ do you do on the box if you use the **LONG** ramp?

$$W_{long} = \vec{F}_{long} \cdot \vec{d}_{long}$$

$$|\vec{d}_{long}| = \sqrt{d^2 + h^2}$$

$$\vec{d}_{long} = \langle d, h, 0 \rangle$$

$$\boxed{W_{long} = F\sqrt{d^2 + h^2}}$$

$$(\vec{F}_{long} \parallel \vec{d}_{long})$$

(c) If you push the box with a force of magnitude $F$ that is **parallel to the ramp**, how much work $W_{short}$ do you do on the box if you use the **SHORT** ramp?

$$W_{long} = \vec{F}_{short} \cdot \vec{d}_{short}$$

$$= F\sqrt{\frac{d^2}{4} + h^2}$$

$$|\vec{d}_{short}| = \sqrt{\left(\frac{d}{2}\right)^2 + h^2}$$

$$= \sqrt{\frac{d^2}{4} + h^2}$$

$$\vec{d}_{short} = \langle \frac{d}{2}, h, 0 \rangle$$

$$(\vec{F}_{short} \parallel \vec{d}_{short})$$

(d) Which ramp should you use if you want the box to move slower when it reaches the top of the ramp? Use the energy principle to determine the speed of the box at the top of each ramp.

$$\Delta E_{box} = W_{net\ on\ box}$$

Short

$$\Delta E_{box} = W_{short} + W_{grav}$$

$$= F\sqrt{\frac{d^2}{4}+h^2} - mgh$$

$$= \Delta KE_{box} = K_f - \cancel{K_i}^{0}$$

$$= \frac{1}{2}mv_f^2$$

$$\Rightarrow \boxed{v_{f,short} = \sqrt{\frac{2F}{m}\sqrt{\frac{d^2}{4}+h^2} - 2gh}}$$

Long

$$\Delta E_{box} = W_{long} + W_{grav}$$

$$= F\sqrt{d^2+h^2} - mgh$$

$$= \Delta KE_{box} = K_f - \cancel{K_i}^{0}$$

$$= \frac{1}{2}mv_f^2$$

$$\Rightarrow \boxed{v_{f,long} = \sqrt{\frac{2F}{m}\sqrt{d^2+h^2} - 2gh}}$$

Use the short ramp. Less work is done to move it to the top due to F being constant, so the box has less KE, and hence, a lower speed at the top.