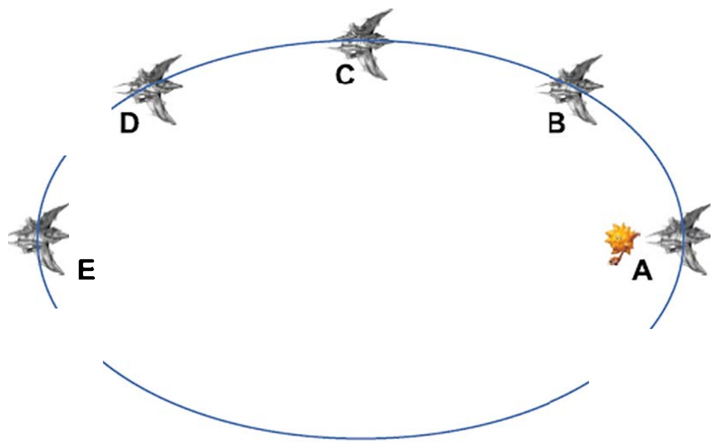
Physics 2211 GPS
Week 9 8

Problem #1

The diagram shows the path of a spacecraft orbiting a star. You will be asked to rank order various quantities in terms of their values at the locations marked on the path, with the largest first. You can use the symbols "<" and "=". For example, if you were asked to rank order the locations in terms of their distance from the star: "A <B <C <D <E"



(a) Rank order the locations on the path in terms of the spacecraft's kinetic energy at each location, starting with the location where the kinetic energy is the largest.

A > B > C > D > E

(b) Consider the system of the comet plus the star. Which of the following statements are correct?

A. As the kinetic energy of the system increases, the gravitational potential energy of the system decreases. $\Delta E = \Delta K + \Delta U = 0$

B. As the comet slows down, the kinetic energy of the system decreases. $K = \frac{1}{2}mv^2$

C. As the comet slows down, energy is lost from the system. closed system

D. External work must be done on the system to speed up the comet. Kepler

E. As the comet's kinetic energy increases, the gravitational potential energy of the system also increases. Not possible here

~~(c) Still considering the system of the comet plus the star, which of the following statements are correct?~~

F ~~A.~~ Along this path the gravitational potential energy of the system is never zero. $U = 0$ @ $r \to \infty$

G ~~B.~~ The sum of the kinetic energy of the system plus the gravitational potential energy of the system is a positive number. bound system, $E < 0$

H ~~C.~~ The gravitational potential energy of the system is inversely proportional to the square of the distance between the comet and star. $U_{grav} = -GMm/r$

I ~~D.~~ The sum of the kinetic energy of the system plus the gravitational potential energy of the system is the same at every location along this path. $\Delta E = 0$

J ~~E.~~ At every location along the comet's path the gravitational potential energy of the system is negative. $U_{grav} = -GMm/r$

~~(d) Rank order the locations on the path in terms of the potential energy of the system at each location, largest (least negative) first.~~

E > D > C > B > A

Problem #2

In the rough approximation that the density of a planet is uniform throughout its interior, the gravitational field strength (force per unit mass) inside the planet at a distance r from the center is $\frac{GM}{R^3}r$, where M is the mass of the planet and R is the radius of the planet.

A. Using the uniform-density approximation, calculate the amount of energy required to move an object of mass m from the center of a planet to the surface.

System: object
Initial: center of planet $(r=0)$
Final: surface of planet $(r=R)$

$$\text{Force per unit mass} = \frac{GMr}{R^3}(-\hat{r})$$

$$\text{Force} = \frac{GMmr}{R^3}(-\hat{r})$$

$$\checkmark\ W_{earth} = \int \vec{F}\cdot d\vec{r} = \int_0^R \frac{-GMmr}{R^3}\,dr = \frac{-GMm}{R^3}\int_0^R r\,dr =$$

$$= \frac{-GMm}{R^3}\left.\frac{r^2}{2}\right|_0^R = \frac{-GMm}{\cancel{R^3}}\cdot\frac{\cancel{R^2}}{2} = \frac{-GMm}{2R}$$

$$\checkmark\ \Delta E = \overset{0}{\cancel{\Delta K}} = W_{earth} + W_{\substack{\text{moving}\\\text{the object}}} \implies W_{\substack{\text{moving}\\\text{the object}}} = -W_{earth} = -\left(\frac{-GMm}{2R}\right) = \frac{GMm}{2R}$$

$$\implies \Delta E_{\substack{\text{moving the}\\\text{object}}} = W_{\substack{\text{moving the}\\\text{object}}} = \boxed{\frac{GMm}{2R}}$$

(b) For comparison, how much energy would be required to move the mass from the surface of the planet to a very large distance away?

System: object + planet
Initial: surface of planet $(r=R)$
Final: far far away $(r\to\infty)$

$$\Delta E = \overset{0}{\cancel{\Delta K}} + \Delta U = U_f - U_i = \overset{0}{\cancel{\frac{-GMm}{r_f}}} - \frac{-GMm}{r_i} = \boxed{\frac{GMm}{R}}$$

(c) Imagine that a small hole is drilled through the center of the Earth from one side to the other. Determine the speed of an object of mass m, dropped into this hole, when it reaches the center of the planet.

System: object

Initial: surface of planet $(r = R)$, $v = 0$

Final: center of planet $(r = 0)$, $v = ?$

$$\Delta E = \Delta K = W$$

$$\frac{1}{2} m (v_f^2 - \cancel{v_i^2}) = \frac{GMm}{2R}$$

from part (a), work done by Earth, but opposite sign b/c the object moves in the opposite direction

$$\cancel{\tfrac{1}{2}} \cancel{m} v_f^2 = \frac{GM\cancel{m}}{\cancel{2}R}$$

$$v_f^2 = \frac{GM}{R}$$

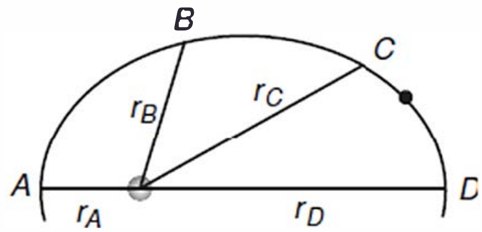
$$\boxed{v_f = \sqrt{\frac{GM}{R}}}$$

Numbers: $G = 6.7e{-}11 \ \mathrm{Nm^2/kg^2}$, $M = 6e24 \ \mathrm{kg}$, $R = 6.4e6 \ \mathrm{m}$

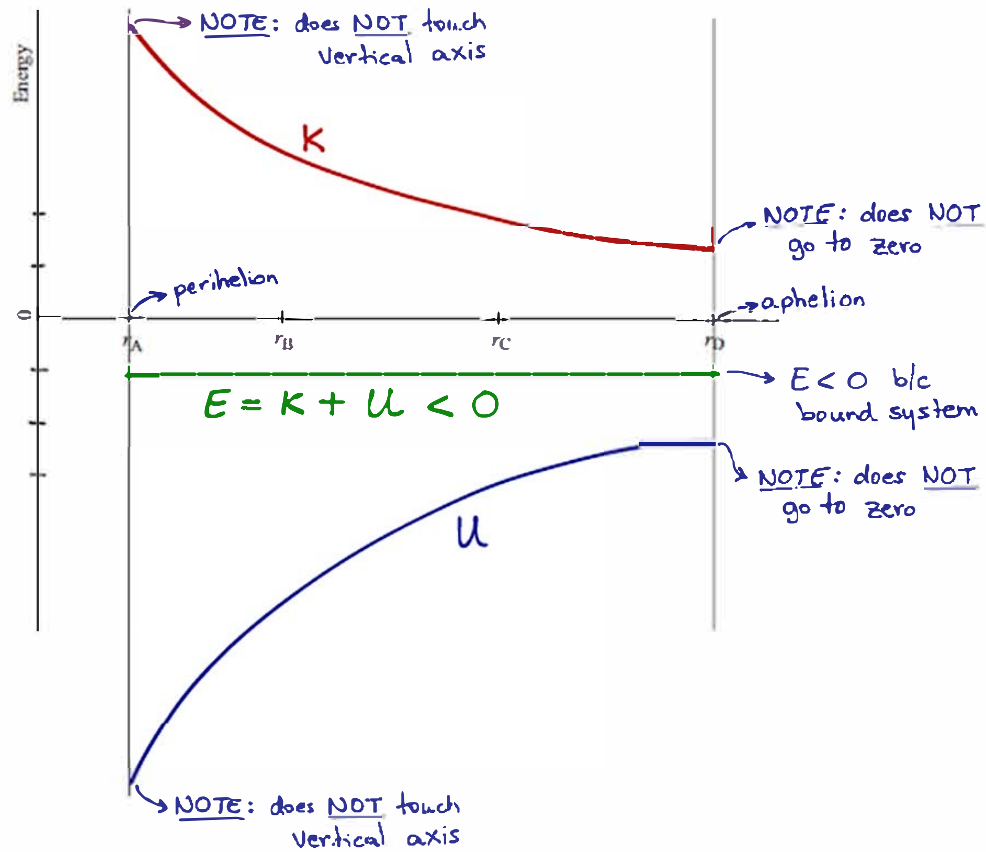
$$\Rightarrow v_f = 7925 \ \mathrm{m/s}$$

Problem #3

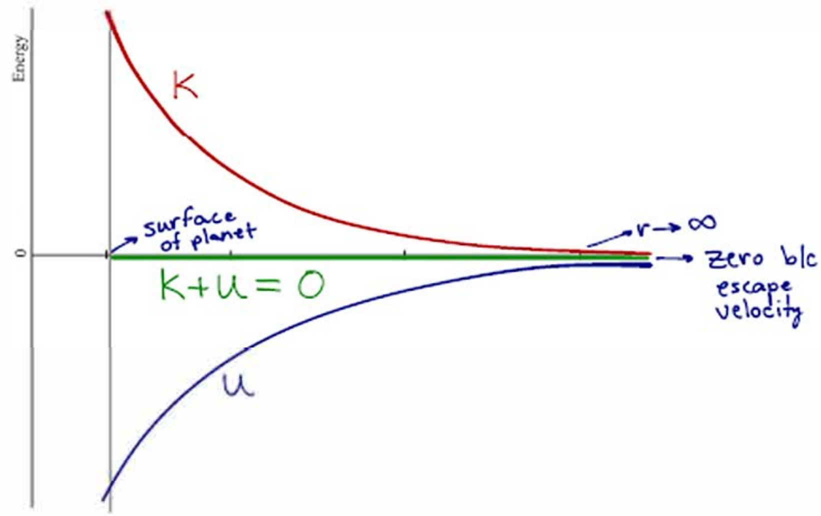
Here is a portion of the orbit of an asteroid around the Sun in an elliptical orbit, moving from A to B to C to D.



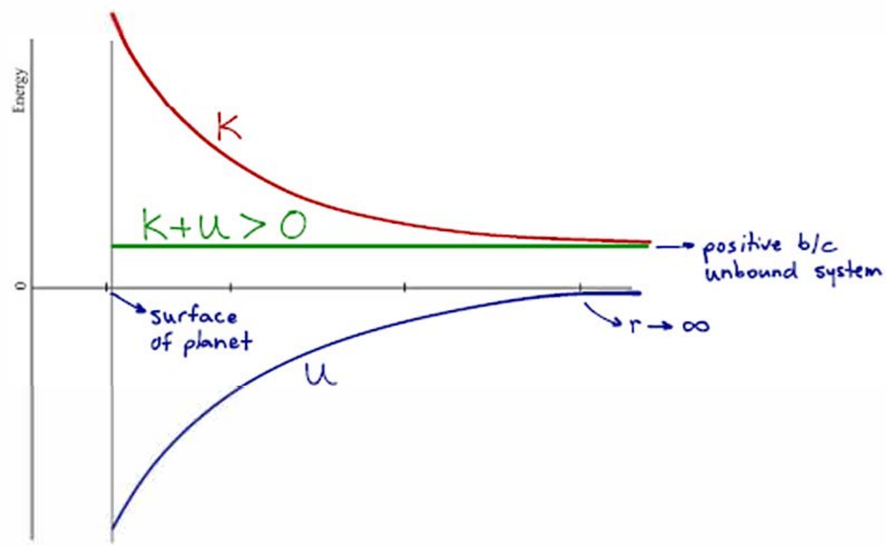
(a) For the system consisting of the Sun plus the asteroid, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between Sun and asteroid. ***Label each curve.*** Along the r axis are shown the various distances between Sun and asteroid.



(b) A spacecraft leaves the surface of a planet at exactly the escape speed. For the system consisting of a planet and a spacecraft, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between planet and the spacecraft. ***Label each curve.***

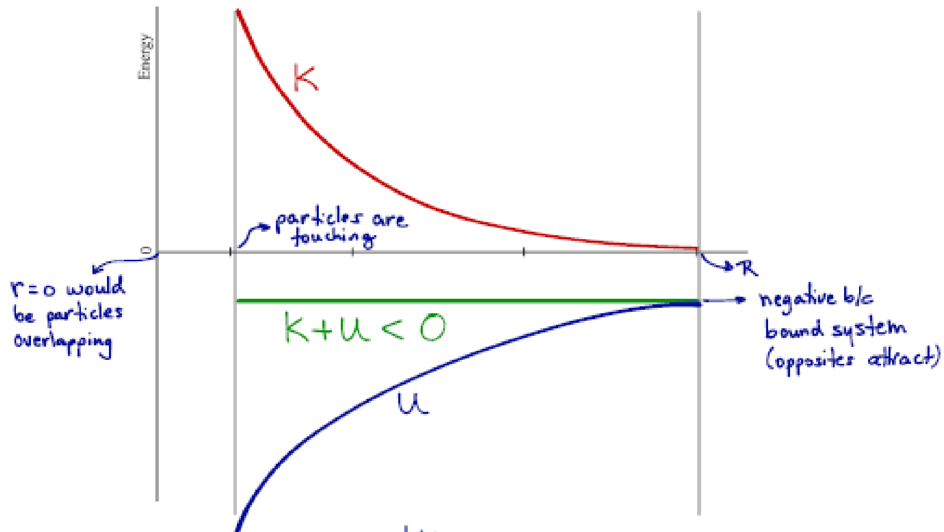


(c) A spacecraft leaves the surface of a planet with a velocity that is twice the escape speed. For the system consisting of a planet and a spacecraft, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between planet and the spacecraft. ***Label each curve.***



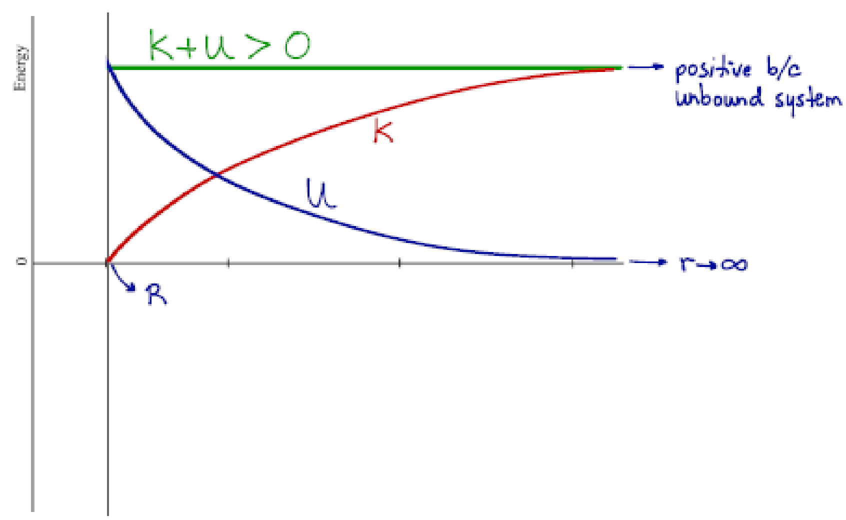
attractive

(d) Two charged particle with opposite charge and identical mass are released from rest a distance R from each other. For the system consisting of the two charges, graph the electric potential energy U, the kinetic energy K, and the sum $K+U$, as a function of the separation distance between the two charges. ***Label each curve.***



repulsive

(e) Two charged particle with identical charge and mass are released from rest a distance R from each other. For the system consisting of the two charges, graph the electric potential energy U, the kinetic energy K, and the sum $K+U$, as a function of the separation distance between the two charges. ***Label each curve.***



Problem #4

During the spring semester at MIT, residents of the parallel buildings of the East Campus Dorms battle one another with large sling-shots made from surgical hose mounted to window frames. Water balloons (with a mass of about 0.5 kg) are placed in a pouch attached to the hose, which is then stretched nearly the width of the room (about 3.5 meters). If the hose obeys Hooke's Law, with a spring constant of 100 N/m, how fast is the balloon traveling when it leaves the dorm room window?

System: water ballon

Surroundings: hose

Initial: max stretch, balloon at rest

Final: no stretch, balloon released

Assumption: no vertical displacement between stretch and release. (horizontal spring)

$$\Delta K_{sys} = W_{surr}$$

$$K_f - \cancel{K_i}^{0} = \int_{s_i}^{f} \vec{F} \cdot d\vec{r}$$

$$\frac{1}{2} m v_f^2 = \int_{s_i}^{s_f} -ks \, ds = -k \int_{s_i}^{0} s \, ds$$

$$\cancel{\tfrac{1}{2}} m v_f^2 = -k \left(\frac{s^2}{2}\right)\Big|_{s_i}^{0} = -k\left(0 - \frac{1}{2} s_i^2\right) = \cancel{\tfrac{1}{2}} k s_i^2$$

$$m v_f^2 = k s_i^2$$

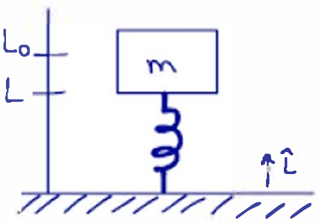
$$v_f^2 = \frac{k}{m} s_i^2$$

$$v_f = \sqrt{\frac{k}{m}}\, s_i = \sqrt{\frac{100}{0.5}}\,(3.5) = \boxed{49.5 \text{ m/s}}$$

Problem #5

A spring with stiffness k_s and relaxed length L_0 stands vertically on a table. You hold a mass M just barely touching the top of the spring.

(a) You very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. How much did the spring compress? Hint: Use Newton's 2nd law.



(Note: 1D motion)

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring} = 0$$

$$mg(-\hat{y}) + -k(L-L_0)\hat{\imath} = 0$$

$$-mg\hat{y} - k(L-L_0)\hat{y} = 0$$

$$(-mg - ks)\hat{y} = 0$$

$$-ks = mg$$

$$\boxed{s = \frac{-mg}{k}}$$ negative b/c compressed

* OK to say $s = mg/k$ only if student explicitly states that the spring is compressed.

(b) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block to be the system and use the energy principle to determine the work done by the Earth, the spring and your hand. Hint: the spring force is not constant.

System: block
Surroundings: Earth, spring, hand

Initial: block at rest, spring relaxed
Final: block at rest, spring compressed

✓ $W_{grav} = \vec{F}_{grav} \cdot \Delta\vec{r} = mg(-\hat{y}) \cdot \left(\frac{mg}{k}\right)(-\hat{y}) = \frac{m^2g^2}{k}$ ⟶ positive

✓ $W_{spring} = \int_i^f \vec{F}_{spring} \cdot d\vec{r} = \int_{s_i}^{s_f} -ks\,ds = -k\int_0^{-mg/k} s\,ds = -k\left(\frac{s^2}{2}\right)_0^{-mg/k} = \frac{-\cancel{k}}{2}\frac{m^2g^2}{k^{\cancel{2}}} = \frac{-1}{2}\frac{m^2g^2}{k}$

$\Rightarrow \Delta E = \Delta K = W_{total}$

$$\cancel{K_f} - \cancel{K_i} = W_{grav} + W_{spring} + W_{hand}$$

$$0 = W_{grav} + W_{spring} + W_{hand}$$

$$\Rightarrow W_{hand} = -W_{grav} - W_{spring} = -\frac{m^2g^2}{k} - \frac{-1}{2}\frac{m^2g^2}{k} = \frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k}$$

$$\Rightarrow \boxed{W_{hand} = \frac{-1}{2}\frac{m^2g^2}{k}}$$

(c) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block+spring+Earth to be the system and use the energy principle to determine the work done by your hand.

System: block, spring, Earth
Surroundings: hand

Initial: block at rest, spring relaxed, height = L_0
Final: block at rest, spring compressed, height = L

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = W_{hand}$$

$$\cancel{\tfrac{1}{2}m(v_f^2 - v_i^2)} + mg(h_f - h_i) + \tfrac{1}{2}k(s_f^2 - \cancel{s_i^2}) = W_{hand}$$

$$mg(L - L_0) + \tfrac{1}{2}k s_f^2 = W_{hand}$$

$$mg\left(-\frac{mg}{k}\right) + \tfrac{1}{2}k\left(\frac{mg}{k}\right)^2 = W_{hand}$$

$$\frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2\cancel{k}}{k^{\cancel{2}}} = W_{hand}$$

$$\Rightarrow W_{hand} = \frac{1}{2}\frac{m^2g^2}{k} - \frac{m^2g^2}{k} = \boxed{-\frac{1}{2}\frac{m^2g^2}{k}}$$

Same as in part (a)

(d) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block
Surroundings: Earth, spring

Initial: block at rest, spring relaxed
Final: block moving, spring compressed

✓ W_{grav} = same as in part (b) b/c moves same distance $= \frac{m^2g^2}{k}$

✓ W_{spring} = same as in part (b) b/c same compression $= -\frac{1}{2}\frac{m^2g^2}{k}$

$$\Rightarrow \Delta E = \Delta K = W_{total}$$

$$\tfrac{1}{2}m(v_f^2 - \cancel{v_i^2}) = W_{grav} + W_{spring} = \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k}$$

$$\cancel{\tfrac{1}{2}}\cancel{m}v_f^2 = \cancel{\tfrac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Rightarrow \boxed{v_f = g\sqrt{m/k}}$$

(e) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block+spring+Earth to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block, spring, Earth
Surroundings: nothing

Initial: block at rest, spring relaxed, height = L_0
Final: block moving, spring compressed, height = L

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = 0$$

$$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}) + mg(h_f - h_i) + \frac{1}{2}k(s_f^2 - \cancel{s_i^2}) = 0$$

$$\frac{1}{2}mv_f^2 + mg(L - L_0) + \frac{1}{2}ks_f^2 = 0$$

$$\frac{1}{2}mv_f^2 + mg\left(\frac{-mg}{k}\right) + \frac{1}{2}\cancel{k}\left(\frac{m^2g^2}{k^{\cancel{2}}}\right) = 0$$

$$\frac{1}{2}mv_f^2 - \frac{m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\cancel{\frac{1}{2}}\cancel{m}v_f^2 = \cancel{\frac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Rightarrow \boxed{v_f = g\sqrt{m/k}}$$ same as part (d)

(f) Compare your answers in parts (b) and (c), and the answers in parts (d) and (e). What does that tell you about your choices when you have an energy principle problem?

(b) and (c) are the same

(d) and (e) are the same

You can choose what to put in your system and what to count as part of the surroundings when solving an energy principle problem, so pick whatever makes the problem easier.