PHYS 2211 - Test 1 - Spring 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

Problem 1 - 35 Points

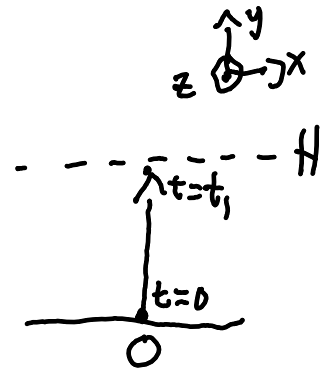
In the 1965 James Bond movie Thunderball, Bond and his partner Lady Domino Del Val are rescued by strapping themselves to a rising Helium balloon. A few moments later, the balloon is caught in the air by a rescue plane and our heroes are spirited away to safety.

1. [5pts] Calculate the magnitude of the minimum force needed to lift Bond and Domino, who together have a mass M.

$$\boxed{|\vec{F}_{lift,min}| = Mg}$$

All or nothing

2. [15pts] Assume Bond and Domino start from rest on the ground at time $t = 0$. The rising balloon exerts an unknown upward vertical force $\langle 0, F, 0\rangle$ on Bond and Domino. The rescue plane flies at an altitude H and needs to intersect the balloon at a known time t_1 after Bond's and Domino's liftoff. Calculate the magnitude of the upward force, $|F|$, required for a successful intersection. You can assume that this force is constant during this time interval.



$$\vec{F}_{net} = \vec{F} + \vec{F}_{grav}$$

$$= F\hat{y} + Mg(-\hat{y})$$

$$\vec{F}_{net} = \langle 0, F - Mg, 0 \rangle$$

constant

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$= \langle 0, 0, 0\rangle + \langle 0, F - Mg, 0\rangle t_1$$

$$= \langle 0, (F - Mg)t_1, 0\rangle$$

$$= M\vec{v}_f$$

-1 clerical

-20% minor

-40% major

-80% min prog

$$\vec{v}_{avg} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)$$

$$\vec{v}_{avg} = \frac{1}{2}\left(<0,0,0> + <0, F-Mg, 0> \frac{t_1}{M} \right)$$

$$= <0, (F-Mg)\frac{t_1}{2M}, 0>$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$<0, H, 0> = <0,0,0> + <0, (F-Mg)\frac{t_1}{2M}, 0> t_1$$

$$H = (F-Mg)\frac{t_1^2}{2M}$$

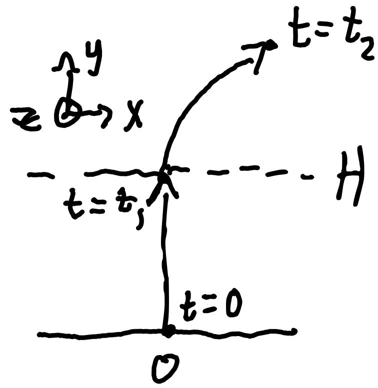
$$\Rightarrow \boxed{|F| = F = \frac{2HM}{t_1^2} + Mg}$$

Note: Kinematic formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

is also acceptable.

3. [15pts] Unfortunately, the plane misses the balloon and has to loop back around for another attempt. At time t_1 (same as in part 2.) and thereafter, a strong wind blows and the force of the balloon on Bond and Domino changes direction to become $\langle F, 0, 0\rangle$. This force is constant and has the same magnitude as earlier. The plane loops back around and rescues them at time $t_2 = 2t_1$. Take the location of Bond and Domino at time $t = 0$ to be the origin and calculate their new position at the time of rescue.



$$\vec{F}_{net} = <F, 0, 0> + \vec{F}_g$$

$$\text{constant} \rightarrow \vec{F}_{net} = <F, -Mg, 0>$$

Note: initial values here are the final values from part 1

-1 clerical

-20% minor

-40% major

-80% min prog

$$\vec{P}_f = \vec{P}_i + \vec{F}_{net} \Delta t \quad , \Delta t = t_2 - t_1 = t_1$$

$$= <0, (F-Mg)t_1, 0> + <F, -Mg, 0> t_1$$

$$= <Ft_1, (F-2Mg)t_1, 0>$$

$$= M\vec{v}_f$$

$$\vec{v}_{avg} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)$$

$$= \frac{1}{2}\left(<0, (F-Mg)\frac{t_1}{M}, 0> + <\frac{Ft_1}{M}, (F-2Mg)\frac{t_1}{M}, 0>\right)$$

$$= <\frac{Ft_1}{2}, (2F-3Mg)\frac{t_1}{2M}, 0>

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$= \langle 0, H, 0 \rangle + \langle \frac{F t_1}{2}, (2F - 3Mg)\frac{t_1}{2M}, 0 \rangle t_1$$

$$\boxed{\vec{r}_f = \langle \frac{F t_1^2}{2M}, H + (2F - 3Mg)\frac{t_1^2}{2M}, 0 \rangle}$$

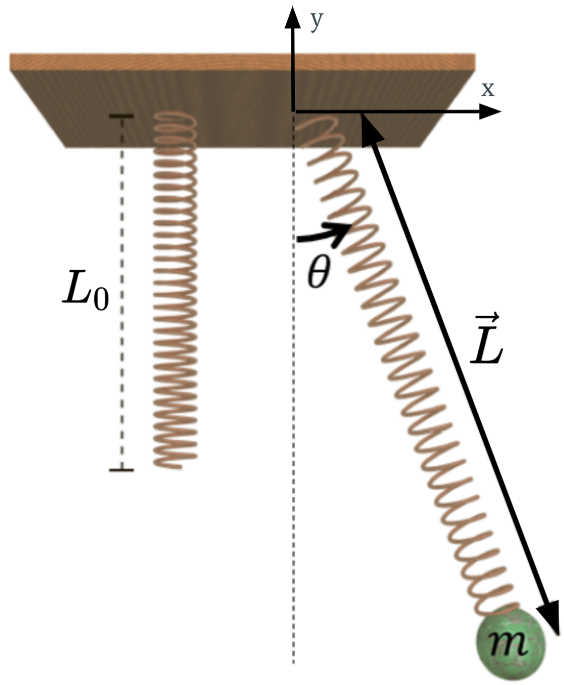
Note: Kinematic formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

is also acceptable.

Problem 2 - 35 Points

A mass m is attached to a spring with spring constant k and a relaxed length of L_0 hanging from the ceiling. The mass is then pulled at an angle θ from the vertical direction until the spring is extended to length L. The mass is then released. Take the point where the spring attaches to the ceiling as the origin so that the gravitational force acting on the ball is $\langle 0, -mg, 0 \rangle$



1. [15pts] Calculate the net force (a vector) acting on the mass at this moment. Express your answer in terms of the variables and constants given in the problem statement.

$$\vec{L} = L \langle \sin\theta, -\cos\theta, 0 \rangle$$

$$|\vec{L}| = L$$

$$\hat{L} = \frac{\vec{L}}{|\vec{L}|} = \langle \sin\theta, -\cos\theta, 0 \rangle$$

$$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{grav}$$

$$= -k(L-L_0)\langle \sin\theta, -\cos\theta, 0 \rangle + mg(-\hat{y})$$

$$\boxed{\vec{F}_{net} = \langle -k(L-L_0)\sin\theta,\ k(L-L_0)\cos\theta - mg,\ 0 \rangle}$$

-1 clerical

-20% minor

-40% major

-80% min prog

$\nearrow F_{net,y} = 0$

2. [20pts] Upon release, the mass starts from rest, and is observed to initially move only in the horizontal direction. Determine the horizontal distance $|\Delta x|$ the mass travels during a short time interval Δt immediately after release. **Your final answer should only depend on θ, g, and Δt.**

$\swarrow$ non-constant

Note: ok to replace $F_{net,y}$ with 0

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$= <0,0,0> + <-k(L-L_0)\sin\theta, \underbrace{k(L-L_0)\cos\theta - mg}_{=0}, 0>\Delta t$$

$$= <-k(L-L_0)\sin\theta, k(L-L_0)\cos\theta - mg, 0>\Delta t$$

$$= m\vec{v}_f$$

$$\vec{v}_{avg} \approx \vec{v}_f$$

$$= <-k(L-L_0)\sin\theta, k(L-L_0)\cos\theta - mg, 0>\frac{\Delta t}{m}$$

$$\Delta\vec{r} = \vec{v}_{avg}\Delta t$$

$$= <-k(L-L_0)\sin\theta, k(L-L_0)\cos\theta - mg, 0>\frac{(\Delta t)^2}{m}$$

$$|\Delta x| = |-k(L-L_0)\sin\theta\frac{(\Delta t)^2}{m}|$$

$$= k(L-L_0)\sin\theta\frac{(\Delta t)^2}{m}$$

-1 clerical

-20% minor

-40% major

-80% min prog

But $F_{net,y} = 0$. This implies

$$F_{net,y} = k(L-L_0)\cos\theta - mg = 0$$

$$\Rightarrow \left(k(L-L_0)\cos\theta = mg \right) \cdot \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \underline{k(L-L_0)\sin\theta = mg\tan\theta} \quad (*)$$

Using this condition, we get

$$|\Delta x| = mg\tan\theta \, \frac{(\Delta t)^2}{m}$$

$$\boxed{|\Delta x| = g\tan\theta \, (\Delta t)^2}$$

Problem 3 - 30 Points

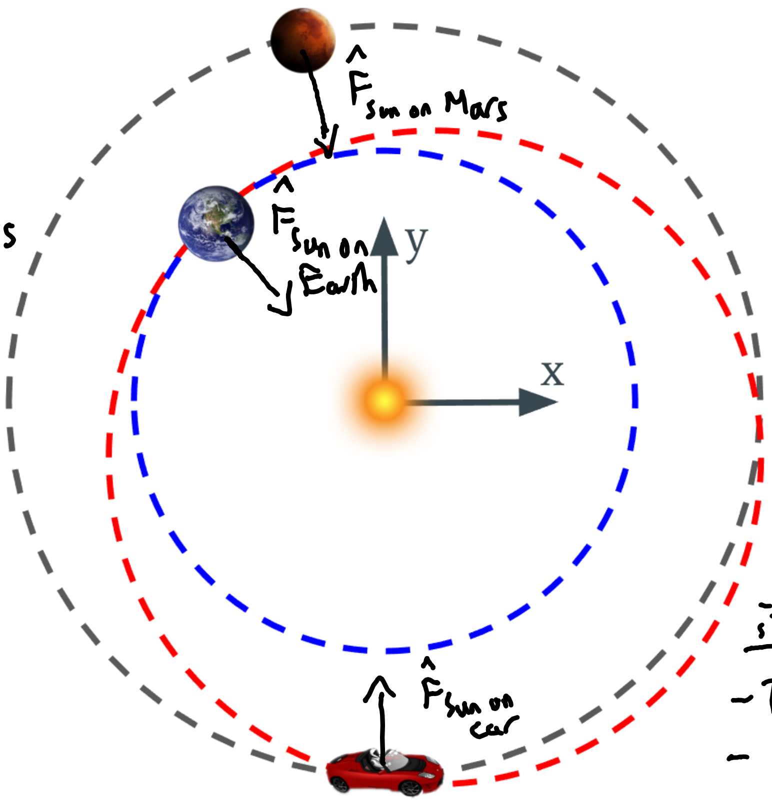
In February of 2018, Elon Musk launched his personal car (mass m_T) into orbit around our Sun (true fact!) m_S Mars (mass m_M) on February 13th 2023 is shown in the diagram. At this instant in time, the position of the Earth and the car relative to the Sun are given by:

m_E

$\vec{r}_E = \langle -\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\rangle$

$\vec{r}_T = \langle 0, -2R, 0\rangle$

Where R is the radius of the circular orbit of the Earth.



if wrong
-2 Mars
-2 Earth
-1 car

1. [5pts] On the diagram sketch three unit vectors (arrows) and label them, each indicating the direction of the gravitational force of the Sun acting on Mars, Earth, and the car.

2. [10pts] Calculate the gravitational force (a vector) acting on the car due to the Sun.

-1 clerical

-20% minor

-40% major

-80% min prog

$$|\vec{r}_T| = [(-2R)^2]^{1/2} = 2R$$

$$\vec{F}_{\text{Sun on Car}} = -\frac{G m_S m_T}{|\vec{r}_T|^3}\vec{r}_T$$

$$= -\frac{G m_S m_T}{(2R)^3}\langle 0, -2R, 0\rangle$$

$$\boxed{\vec{F}_{\text{Sun on car}} = \frac{G m_S m_T}{4R^2}\hat{y}}$$

3. [15pts] Calculate the gravitational force (a vector) acting on the car due to the Earth.

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{source}$$

$$= \vec{r}_T - \vec{r}_E$$

$$\boxed{\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \left\langle \frac{1}{[10+4\sqrt{2}]^{1/2}}, \frac{-2\sqrt{2}-1}{[10+4\sqrt{2}]^{1/2}}, 0 \right\rangle = \frac{1}{[10+4\sqrt{2}]^{1/2}} \langle 1, -2\sqrt{2}-1, 0 \rangle}$$

$$= \langle 0, -2R, 0 \rangle - \left\langle -\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0 \right\rangle$$

$$= \left\langle \frac{R}{\sqrt{2}}, -2R - \frac{R}{\sqrt{2}}, 0 \right\rangle$$

$$|\vec{r}| = \left[\left(\frac{R}{\sqrt{2}}\right)^2 + \left(-2R - \frac{R}{\sqrt{2}}\right)^2 \right]^{1/2}$$

$$= \left[\frac{R^2}{2} + 4R^2 + \frac{4R^2}{\sqrt{2}} + \frac{R^2}{2} \right]^{1/2}$$

$$= R\left[5 + 2\sqrt{2}\right]^{1/2}$$

-1 clerical

-20% minor

-40% major

-80% min prog

$$\vec{F}_{\text{Earth on car}} = -\frac{G m_E m_T}{|\vec{r}|^2} \hat{r} = -\frac{G m_E m_T}{|\vec{r}|^3} \vec{r}$$

$$\boxed{\vec{F}_{\substack{\text{Earth}\\\text{on car}}} = -\frac{G m_E m_T}{R^2 \sqrt{2} [5 + 2\sqrt{2}]^{3/2}} \langle 1, -2\sqrt{2} - 1, 0 \rangle}$$