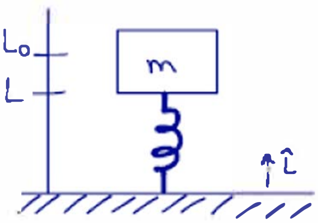
Problem #5

A spring with stiffness k_s and relaxed length L_0 stands vertically on a table. You hold a mass M just barely touching the top of the spring.

(a) You very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. How much did the spring compress? Hint: Use Newton's 2nd law.



(Note: 1D motion)

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring} = 0$$

$$mg(-\hat{y}) + -k(L-L_0)\hat{\imath} = 0$$

$$-mg\hat{y} - k(L-L_0)\hat{y} = 0$$

$$(-mg - ks)\hat{y} = 0$$

$$-ks = mg$$

$$\boxed{s = \frac{-mg}{k}}$$ negative b/c compressed

* OK to say $s = mg/k$ only if student explicitly states that the spring is compressed.

(b) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block to be the system and use the energy principle to determine the work done by the Earth, the spring and your hand. Hint: the spring force is not constant.

System: block

Surroundings: Earth, spring, hand

Initial: block at rest, spring relaxed

Final: block at rest, spring compressed

✓ $W_{grav} = \vec{F}_{grav} \cdot \Delta\vec{r} = mg(-\hat{y}) \cdot \left(\frac{mg}{k}\right)(-\hat{y}) = \frac{m^2g^2}{k}$ → positive

✓ $W_{spring} = \int_i^f \vec{F}_{spring} \cdot d\vec{r} = \int_{s_i}^{s_f} -ks\,ds = -k\int_0^{-mg/k} s\,ds = -k\left(\frac{s^2}{2}\right)_0^{-mg/k} = -\frac{k}{2}\frac{m^2g^2}{k^2} = -\frac{1}{2}\frac{m^2g^2}{k}$

$\Rightarrow \Delta E = \Delta K = W_{total}$

$$\cancel{K_f} - \cancel{K_i} = W_{grav} + W_{spring} + W_{hand}$$

$$0 = W_{grav} + W_{spring} + W_{hand}$$

$$\Rightarrow W_{hand} = -W_{grav} - W_{spring} = -\frac{m^2g^2}{k} - \frac{-1}{2}\frac{m^2g^2}{k} = \frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k}$$

$$\Rightarrow \boxed{W_{hand} = \frac{-1}{2}\frac{m^2g^2}{k}}$$

(c) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block+spring+Earth to be the system and use the energy principle to determine the work done by your hand.

System: block, spring, Earth
Surroundings: hand

Initial: block at rest, spring relaxed, height $= L_0$
Final: block at rest, spring compressed, height $= L$

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = W_{hand}$$

$$\underbrace{\cancel{\tfrac{1}{2}m(v_f^2 - v_i^2)}}_{0} + mg(h_f - h_i) + \tfrac{1}{2}k(s_f^2 - \underbrace{\cancel{s_i^2}}_{0}) = W_{hand}$$

$$mg(L - L_0) + \tfrac{1}{2}k s_f^2 = W_{hand}$$

$$mg\left(-\frac{mg}{k}\right) + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = W_{hand}$$

$$\frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2\cancel{k}}{k^{\cancel{2}}} = W_{hand}$$

$$\Rightarrow W_{hand} = \frac{1}{2}\frac{m^2g^2}{k} - \frac{m^2g^2}{k} = \boxed{-\frac{1}{2}\frac{m^2g^2}{k}}$$

Same as in part (a)

(d) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block
Surroundings: Earth, spring

Initial: block at rest, spring relaxed
Final: block moving, spring compressed

✓ W_{grav} = same as in part (b) b/c moves same distance $= \frac{m^2g^2}{k}$

✓ W_{spring} = same as in part (b) b/c same compression $= -\frac{1}{2}\frac{m^2g^2}{k}$

$$\Rightarrow \Delta E = \Delta K = W_{total}$$

$$\frac{1}{2}m(v_f^2 - \underbrace{\cancel{v_i^2}}_{0}) = W_{grav} + W_{spring} = \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k}$$

$$\cancel{\tfrac{1}{2}}\cancel{m}v_f^2 = \cancel{\tfrac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Rightarrow \boxed{v_f = g\sqrt{m/k}}$$

(e) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block+spring+Earth to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block, spring, Earth
Surroundings: nothing

Initial: block at rest, spring relaxed, height $= L_0$
Final: block moving, spring compressed, height $= L$

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = 0$$

$$\frac{1}{2}m(v_f^2 - \underbrace{v_i^2}_{0}) + mg(h_f - h_i) + \frac{1}{2}k(s_f^2 - \underbrace{s_i^2}_{0}) = 0$$

$$\frac{1}{2}mv_f^2 + mg(L - L_0) + \frac{1}{2}ks_f^2 = 0$$

$$\frac{1}{2}mv_f^2 + mg\left(\frac{-mg}{k}\right) + \frac{1}{2}\cancel{k}\left(\frac{m^2g^2}{k^{\cancel{2}}}\right) = 0$$

$$\frac{1}{2}mv_f^2 - \frac{m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\cancel{\frac{1}{2}}\cancel{m}v_f^2 = \cancel{\frac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Rightarrow \boxed{v_f = g\sqrt{m/k}}$$ same as part (d)

(f) Compare your answers in parts (b) and (c), and the answers in parts (d) and (e). What does that tell you about your choices when you have an energy principle problem?

(b) and (c) are the same

(d) and (e) are the same

You can choose what to put in your system and what to count as part of the surroundings when solving an energy principle problem, so pick whatever makes the problem easier.

Physics 2211 GPS Week 10

Problem #1

After watching "The Big Lebowski" for the first time this summer, you and a friend get into an argument about how much ice to add when making the perfect white russian cocktail. You both agree that, for optimum taste, the cocktail should be enjoyed at 10 degrees Celsius. The two ingredients for the cocktail, cream and a "vodka & kahlua" mix, both leave the fridge at 15 degrees Celsius. Ice from a standard freezer is at a temperature of -10 degrees Celsius. If typical white russian calls for 0.06 L of cream and 0.14 L of the "vodka & kahlua" mix, how much ice is needed to bring the drink down to its optimum temperature?

Ice: density = 0.91 kg/L, C = 4.18 J/(Cg)
Mix: density = 0.8 kg/L, C = 2.44 J/(Cg)
Cream: density = 1 kg/L, C = 3.77 J/(Cg)

We assume that the ingredients are sufficiently isolated from their surroundings when they are mixed, so that a negligible amount of heat transfers to the surroundings. This means that the system is isolated and the energy principle predicts that

$$\Delta E_{sys} = 0.$$

Taking the initial and final state to be before the mixing process and after the combination has reached equilibrium, respectively, there are then no appreciable changes in kinetic or potential energy, and so

$$\Delta E_{therm,sys} = 0.$$

Each ingredient contributes to the total change in thermal energy:

$$\Delta E_{therm,sys} = \Delta E_{therm,mix} + \Delta E_{therm,cream} + \Delta E_{therm,\ ice}$$

$$= m_{mix} C_{mix} \Delta T_{mix} + m_{cream} C_{cream} \Delta T_{cream} + m_{ice} C_{ice} \Delta T_{ice} = 0.$$

Solving for the mass of the ice we find that

$$m_{ice} = -\frac{m_{mix} C_{mix} \Delta T_{mix} + m_{cream} C_{cream} \Delta T_{cream}}{C_{ice} \Delta T_{ice}}$$

$$= -\frac{\left(\left(\left(0.8\frac{kg}{L}\right)(0.14L)\left(2.44\frac{J}{gC^\circ}\right)\right)(10°C - 15°C) + \left(\left(1.0\frac{kg}{L}\right)(0.06L)\left(3.77\frac{J}{gC^\circ}\right)\right)(10°C - 15°C)\right)}{\left(4.18\,\frac{J}{g\,C^\circ}\right)(10°C - (-10°C))}$$

$$\approx 0.030\ kg = 30\ g$$

Note: The heat capacities and the densities do not use the same mass units, so in principle we could convert first. However, in this case there's no need because the same conversion factor appears in the numerator and the denominator so they cancel out.

Problem #2

During 3 hours one winter afternoon, when the outside temperature was 11° C, a house heated by electricity was kept at 25° C with the expenditure of 58 kwh (kilowatt·hours) of electric energy.

(a) What was the average energy leakage in joules per second (watts) through the walls of the house to the environment (the outside air and ground)?

✓ $\Delta t = (3\,\text{hrs})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 10\,800\ \text{sec}$

✓ $\Delta E = Q = (58\ \text{kWh})(1000\,\text{W}/1\,\text{kW})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 2.088e8\ \text{W}\cdot\text{s}$

(same as Joules)

$$\Rightarrow \text{energy leakage} = \frac{\Delta E}{\Delta t} = \frac{2.088e8\ \text{W}\cdot\text{s}}{10800\ \text{sec}} = \boxed{1.93e4\ \text{Watts}}$$

$$\text{Watt} = \frac{\text{Joule}}{\text{second}}$$

$$\underline{\text{Alternative}}: \frac{\Delta E}{\Delta t} = \frac{58\ \text{kWh}}{3\text{h}} = \frac{19.3\ \text{kW} \mid 1000\ \text{W}}{\mid 1\ \text{kW}} = \boxed{1.93e4\ \text{Watts}}$$

(b) The rate at which energy is transferred between two systems due to a temperature difference is often proportional to their temperature difference. Assuming this to hold in this case, if the house temperature had been kept at 28° C (82.4° F), how many kwh of electricity would have been consumed?

$$\text{energy transfer} \propto \text{temperature difference} \Longrightarrow \frac{Q_1}{\Delta T_1} = \frac{Q_2}{\Delta T_2}$$

$$\Longrightarrow \frac{58\ \text{kWh}}{(25-11)^\circ\text{C}} = \frac{Q_2}{(28-11)^\circ\text{C}}$$

$$\frac{58}{14} = \frac{Q_2}{17}$$

$$14\,Q_2 = (58)(17)$$

$$Q_2 = \frac{(58)(17)}{(14)} = \boxed{70.4\ \text{kWh}}$$

Physics 2211 GPS Week 12

Problem #1

Consider a system consisting of two particles connected by a spring of negligible mass:
$m_1 = 5$ kg, vector $\vec{v}_1 = \langle 5, \ -10, \ 15\rangle$ m/s
$m_2 = 10$ kg, vector $\vec{v}_2 = \langle -10, \ 0, \ -5\rangle$ m/s

(a) What is the total momentum $\vec{p}_{total}$ of this system?

$$\vec{p}_1 = m_1\vec{v}_1 = (5)<5,-10,15> = <25,-50,75> \text{ kg m/s}$$

$$\vec{p}_2 = m_2\vec{v}_2 = (10)<-10,0,-5> = <-100,0,-50> \text{ kg m/s}$$

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = <25,-50,75> + <-100,0,-50> =$$

$$= \boxed{<-75,-50,25> \text{ kg m/s}}$$

(b) What is $\vec{V}_{CM}$, the velocity of the center of mass of this system?

$$\vec{p}_{total} = M_{total}\vec{V}_{cm} \implies \vec{V}_{cm} = \frac{\vec{p}_{total}}{M_{total}}$$

$$\vec{V}_{cm} = \frac{<-75,-50,25>}{5+10} = \boxed{<-5,-3.3,1.7> \text{ m/s}}$$

(c) What is K_{trans}, the translational kinetic energy of this system?

$$K_{trans} = \frac{1}{2} M_{total} V_{cm}^2$$

$$\checkmark \; V_{cm}^2 = (-5)^2 + (-3.3)^2 + (1.7)^2 = 38.68$$

$$\Rightarrow K_{trans} = \frac{1}{2}(15)(38.68) = \boxed{290 \text{ J}}$$

(d) What is K_{total}, the total kinetic energy of this system?

$$K_{total} = K_1 + K_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\checkmark \; v_1^2 = (5)^2 + (-10)^2 + (15)^2 = 350$$

$$\checkmark \; v_2^2 = (-10)^2 + (-5)^2 = 125$$

$$\Rightarrow K_{total} = \frac{1}{2}(5)(350) + \frac{1}{2}(10)(125) = 875 + 625 = \boxed{1500 \text{ J}}$$

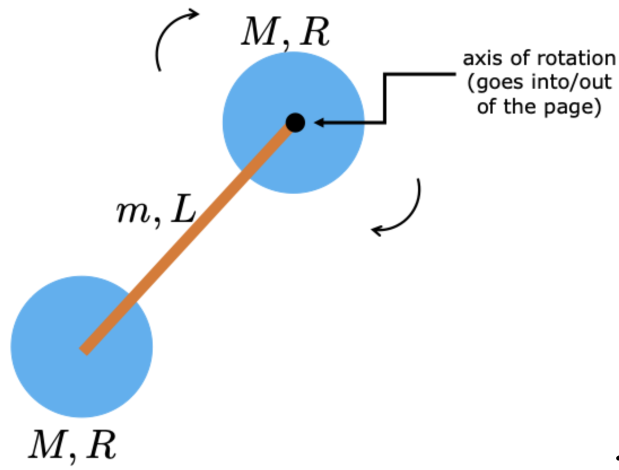
(e) What is K_{rel}, the kinetic energy of this system relative to the center of mass?

$$K_{total} = K_{trans} + K_{rel} \Longrightarrow K_{rel} = K_{total} - K_{trans}$$

$$K_{rel} = 1500 - 290 = \boxed{1210 \text{ J}}$$

Problem #2

A barbell is made up of two solid spheres of mass M and radius R whose centers are attached to the ends of a thin rod that has mass m and length L. The entire thing rotates about an axis that goes through the center of sphere 1. Determine the total moment of inertia of the barbell about this axis of rotation. Hint: remember the parallel axis theorem.



$\parallel$-axis theorem: $I_{\parallel\text{-axis}} = I_{CM} + Md^2$

(d = dist. b/w axis through CM and the $\parallel$-axis)

$$I_{CM,\text{solid sphere}} = \frac{2}{5}MR^2$$

$$I_{CM,\text{thin rod}} = \frac{1}{12}mL^2$$

$$I_{\text{system}} = I_{\text{axis, sphere 1}} + I_{\text{axis, rod}} + I_{\text{axis, sphere 2}}$$

$$= \left[I_{CM,\text{ sphere 1}} + I_{\parallel\text{-axis, sphere 1}}\right] + \left[I_{CM,\text{ rod}} + I_{\parallel\text{-axis, sphere 2}}\right]$$

$$+ \left[I_{CM,\text{ sphere 2}} + I_{\parallel\text{-axis, sphere 2}}\right]$$

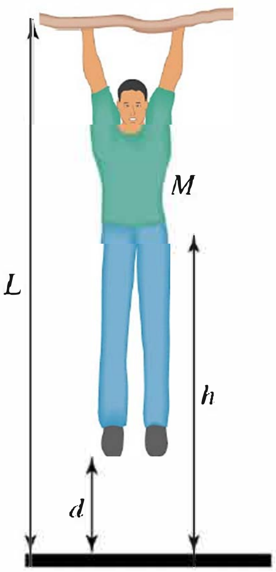
$$= \left[\frac{2}{5}MR^2 + M(0)^2\right] + \left[\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right]$$

$$+ \left[\frac{2}{5}MR^2 + M(L)^2\right]$$

$$\boxed{I_{\text{system}} = \frac{4}{5}MR^2 + \left(\frac{1}{3}m + M\right)L^2}$$

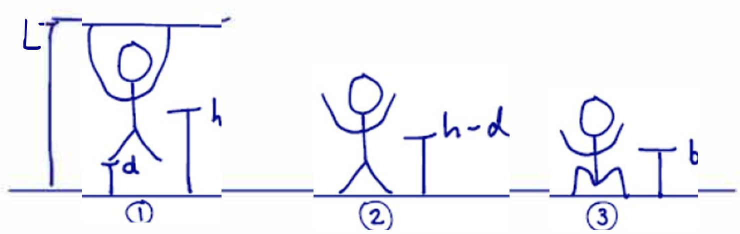
Problem #3

You hang by your hands from a tree limb that is a height $L = 6$ m above the ground, with your center of mass a height $h = 5$ m above the ground and your feet a height $d = 4$ m above the ground, as shown in the figure (not to scale). You then let yourself fall. You absorb the shock by bending your knees, ending up momentarily at rest in a crouched position with your center of mass a height $b = 0.25$ m above the ground. Your mass is $M = 110$ kg.



(a) Starting from the energy principle, find your speed just before your feet touch the ground.

System : point particle
Initial: ①
Final: ②



$$\Delta E = \Delta K = W_{grav}$$

$$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}) = \vec{F}_{grav} \cdot \Delta\vec{r}_{cm} = mg(-\hat{y}) \cdot d(-\hat{y})$$

$$\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}gd$$

$$v_f^2 = 2gd \Rightarrow v_f = \sqrt{2gd} = \sqrt{(2)(9.8)(4)} = \boxed{8.85 \text{ m/s}}$$

(b) Starting from the energy principle (point particle model) and assuming that the contact force of the ground on your feet is constant, find the magnitude of the contact force during your landing.

Initial ②, final ③

$$\Delta E = \Delta K = W_{total} = \vec{F}_{net} \cdot \Delta \vec{r}_{cm}$$

$$\frac{1}{2}m(\cancel{v_f^2}^{\,0} - v_i^2) = (F_c - mg)[b - (h-d)]$$

$$-\frac{1}{\cancel{2}}m(\cancel{2}gd) = (F_c - mg)(b-h+d)$$

$$-mgd = (F_c - mg)(b-h+d)$$

$$F_c - mg = \frac{-mgd}{b-h+d}$$

$$F_c = mg - \frac{mgd}{b-h+d} = (110)(9.8) - \frac{(110)(9.8)(4)}{0.25-5+4} = \boxed{6827\text{ N}}$$

(c) What is the (real) work done by the contact force?

$$W_c = \vec{F}_c \cdot \Delta\vec{r} = 0 \quad \text{b/c the floor doesn't move}$$

(d) Starting from the energy principle (real model), find the change in your internal energy during landing.

Method #1
Initial ①, final ③

$$\Delta E = \cancel{\Delta K}^{\,0} + \Delta E_{int} = W_{grav} + \cancel{W_c}^{\,0}$$

$$\Delta E_{int} = -mg(b-h) = -(110)(9.8)(0.25-5) = \boxed{5121\text{ J}}$$

Method #2
Initial ②, final ③

$$\Delta E = \Delta K + \Delta E_{int} = W_{grav} + \cancel{W_c}^{\,0}$$

$$\frac{1}{2}m(\cancel{v_f^2}^{\,0} - v_i^2) + \Delta E_{int} = -mg[b-(h-d)] = -mg(b-h+d)$$

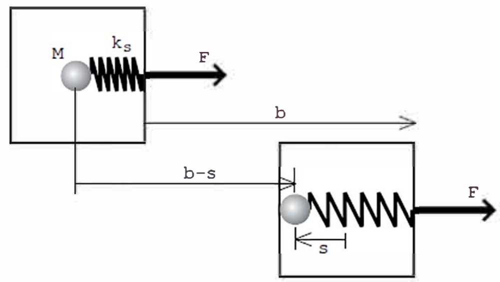
$$-\frac{1}{\cancel{2}}m(\cancel{2}gd) + \Delta E_{int} = -mg(b-h) - mgd$$

$$\cancel{-mgd} + \Delta E_{int} = -mg(b-h) - \cancel{mgd}$$

$$\Delta E_{int} = -mg(b-h) = \boxed{5121\text{ J}}$$

Problem #4

A thin box in outer space contains a large ball of clay of mass M, connected to an initially relaxed spring of stiffness k_s. The mass of the box is negligible compared to M. The apparatus is initally at rest. Then a force of constant magnitude F is applied to the box. When the box has moved a distance b, the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount s. See the diagram for distances.



(a) Immediately after the clay sticks to the box, how fast is the box moving?

System : the whole thing is a point

$$\Delta E = \Delta K_{trans} = W = \vec{F}\cdot\Delta\vec{r}_{cm}$$

$$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}) = F(b-s)$$

$$\frac{1}{2}mv_f^2 = F(b-s)$$

$$mv_f^2 = 2F(b-s)$$

$$v_f^2 = \frac{2F}{m}(b-s)$$

$$\boxed{v_f = \sqrt{\frac{2F}{m}(b-s)}}$$

(b) What is the increase in thermal energy of the clay?

System : real (clay + spring)

$$\Delta E = \Delta K_{trans} + \Delta U_s + \Delta E_{int} = W = \vec{F}\cdot\Delta\vec{r}$$

$$F(b-s) + \frac{1}{2}k(s_f^2 - \cancel{s_i^2}) + \Delta E_{int} = Fb$$

$$\cancel{Fb} - Fs + \frac{1}{2}ks^2 + \Delta E_{int} = \cancel{Fb}$$

$$-Fs + \frac{1}{2}ks^2 + \Delta E_{int} = 0$$

$$\boxed{\Delta E_{int} = Fs - \frac{1}{2}ks^2}$$