PHYS 2211 MNR - Test 2 - Fall 2022

Please clearly print your name & GTID in the lines below

Name: ______________________________ GTID: ______________________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

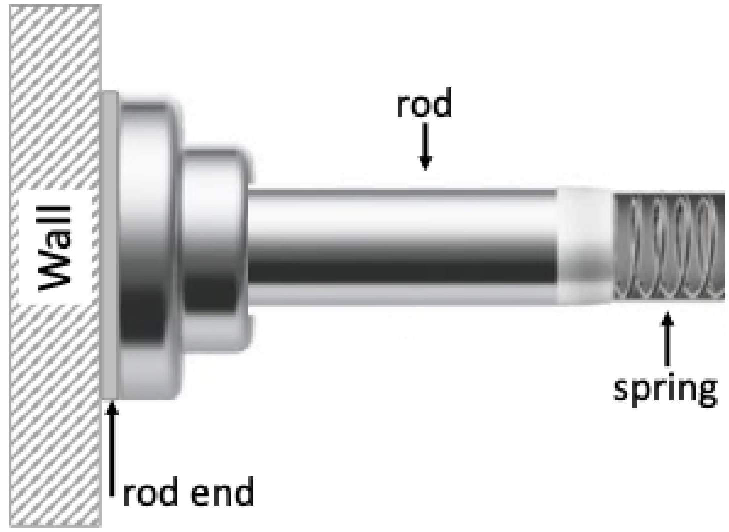
If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

**"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."**

KEY
____
Sign your name on the line above

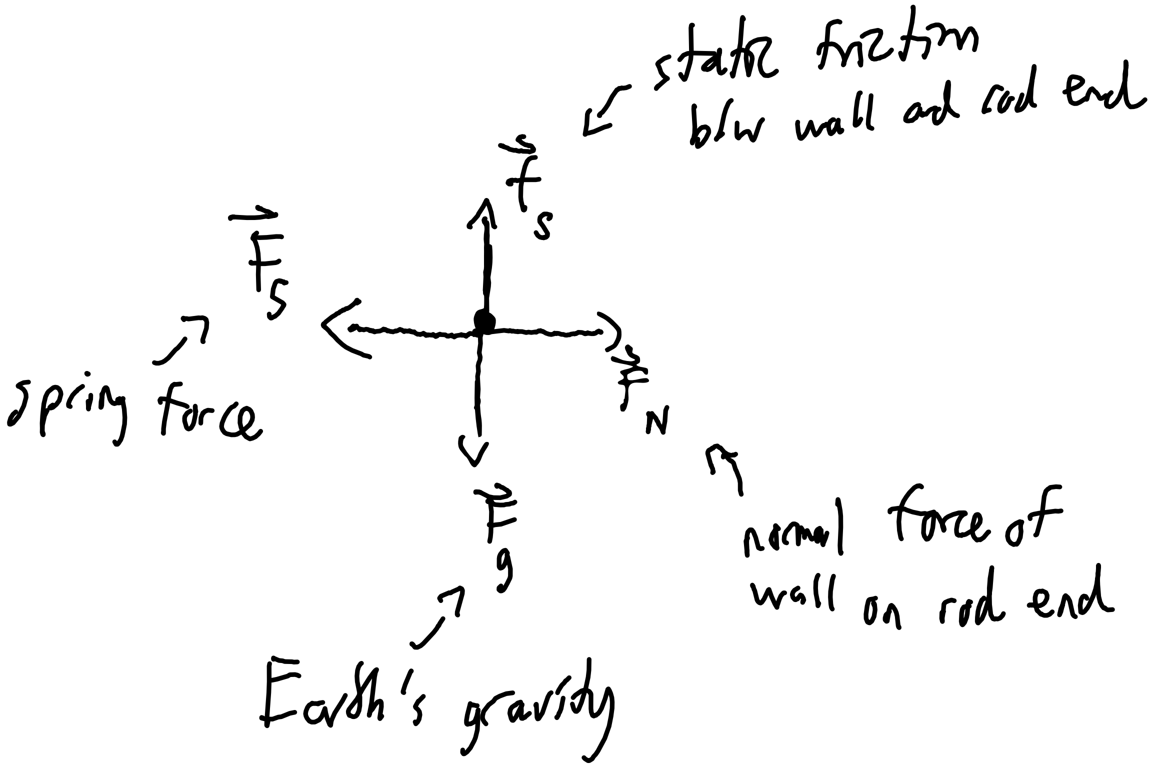
Curtain Rod [40 pts]

The working principle of a tension curtain rod is that the rod is attached to a spring pressing the rod's end against the wall (see diagram). The spring has stiffness $k = 500$ N/m and is compressed by an amount $s = 5$ cm from its relaxed length. The rod and spring are horizontal, and encased in an external tube (not shown).



1. [10 pts] Draw a free body diagram of all the forces acting on the rod end. Consider that the end carries the total weight of the rod, and the rod is held in place so it doesn't slip down.

Static equilibrium – All forces balanced



2. [10 pts] What is the maximum mass m_0 that the rod can have without slipping down on the wall? The coefficient of static friction of the rod end on the wall is $\mu_{s1} = 0.6$.

(1)

$$\underline{\sum F_y = 0}$$

$$\vec{f}_{s,max} + \vec{F}_g = \vec{0}$$

$$|\vec{f}_{s,max}|\hat{y} + |\vec{F}_g|(-\hat{y}) = 0\hat{y}$$

$$|\vec{f}_{s,max}| - |\vec{F}_g| = 0$$

$$|\vec{f}_{s,max}| = |\vec{F}_g|$$

$$\mu_{s1}|\vec{F}_N| = m_0 g$$

(2)

$$\underline{\sum f_x = 0}$$

$$\vec{F}_N + \vec{F}_s = \vec{0}$$

$$|\vec{F}_N|\hat{x} + |\vec{F}_s|(-\hat{x}) = 0\hat{x}$$

$$|\vec{F}_N| - |\vec{F}_s| = 0$$

$$\boxed{|\vec{F}_N| = |\vec{F}_s| = k|s|}$$

(3)

$$\mu_{s1}(k|s|) = m_0 g$$

$$\Rightarrow \boxed{m_0 = \frac{\mu_{s1} k|s|}{g} \approx 1.53\ \text{kg}}$$

3. [10 pts] After months in place, the rod end wears off and the rod starts to slide down. The rod's mass is m_0 (calculated in Part 2), ~~and the new coefficient of static friction is $\mu_{s2} = 0.4$~~. If the rod falls at constant speed, what is the value of the coefficient of kinetic friction μ_k of the rod end on the wall?

constant speed (+ dir.) => forces balanced

(but w/ $\vec{f}_s \rightarrow \vec{f}_k$)

$\underline{\Sigma F_x = 0}$ ⇐ same eq'n as before, i.e.

$$|\vec{F}_N| = |\vec{F}_s| = k|s|$$

$\underline{\Sigma F_y = 0}$

$$\vec{f}_k + \vec{F}_g = 0$$

$$\Rightarrow |\vec{f}_k| = |\vec{F}_g|$$

$$\mu_k k s = m_0 g$$

$$\boxed{\mu_k = \frac{m_0 g}{k|s|} = 0.6}$$

4. [10 pts] To fix the slippage problem, the owner decides to insert a wooden block between the rod end and the wall to compress the spring further. What should be the minimum width w of the block to prevent sliding? The block is glued to the rod and its coefficient of friction on the wall is $\mu_{s3} = 0.5$.

Static equilibrium -> forces balanced

(but $\vec{f}_k \rightarrow \vec{f}_s$)

$$|\vec{f}_{s,max}| = \mu_{s3} \, k(|s| + w) = m_0 g$$

$$\Rightarrow \boxed{w = \frac{m_0 g - \mu_{s3} k |s|}{\mu_{s3} k} = 0.01 \text{ m}}$$

Supermassive Black Hole [40 pts]

A star of mass $m_s = 2 \times 10^{31}$ kg moves in uniform circular motion in its orbit around a supermassive black hole. The radius of the orbit is $r = 1.5 \times 10^{14}$ m, and it takes the star 16 Earth-years to complete one full orbit around the black hole.

1. [20 pts] What is the mass M_{bh} of the supermassive black hole? Hint: remember Lab 3.

0

uniform circular motion => constant speed => $\vec{F}_{net} = \vec{F}_{net,\perp} + \cancel{\vec{F}_{net,\parallel}}$

$$|\vec{F}_{net,\perp}| = |\vec{F}_{g,BH\text{ on star}}|$$

$$\frac{\cancel{m_s} v^2}{r} = \frac{G M_{bh} \cancel{m_s}}{r^2}$$

$$\Rightarrow M_{bh} = \frac{v^2 r}{G}$$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2 r}{G}$$

$\boxed{T = 16\text{ yr} = 5.05 \times 10^8\text{ s}}$

$$\boxed{M_{bh} = \frac{4\pi^2 r^3}{G T^2} = 7.8 \times 10^{36}\text{ kg}}$$

2. [20 pts] The star that orbits the black hole has several planets orbiting it (like the solar system). One of those planets, which we'll call Aliceum, has a mass of $M_a = 6 \times 10^{23}$ kg and a radius of $R_a = 4 \times 10^6$ m.

The inhabitants of Aliceum launch their satellites using an extra-large horizontal spring of relaxed length $L_0 = 50$ m and stiffness $k = 1.2 \times 10^6$ N/m. The spring is compressed to a length L and whatever is being launched is placed at rest in front of the spring. When the spring is released it pushes out the package, launching it into space once the package is no longer in contact with the spring.

If a satellite of mass $m = 50$ kg needs to be launched with speed $v_0 = 4500$ m/s, what should be the compressed length L of the spring when getting it ready for launch? For this calculation you can ignore the star, the black hole, and all other planets and moons and atmospheric drag.

system: satellite + spring + Aliceum

surr: (nothing; ignore star & BH)

(horizontal spring)

$$\Delta E_{sys} = 0 = \Delta K_{sat} + \Delta U_{spring-sat} \underbrace{\left(+ \cancelto{0}{\Delta U_{grav, Ali-sat}}\right)}$$

Note: optional!
no penalty if omitted

$$\frac{1}{2} m \left(v_f^2 - \cancelto{0}{v_i^2}\right) + \frac{1}{2} k \left(\cancelto{0}{s_f^2} - s_i^2\right) = 0$$

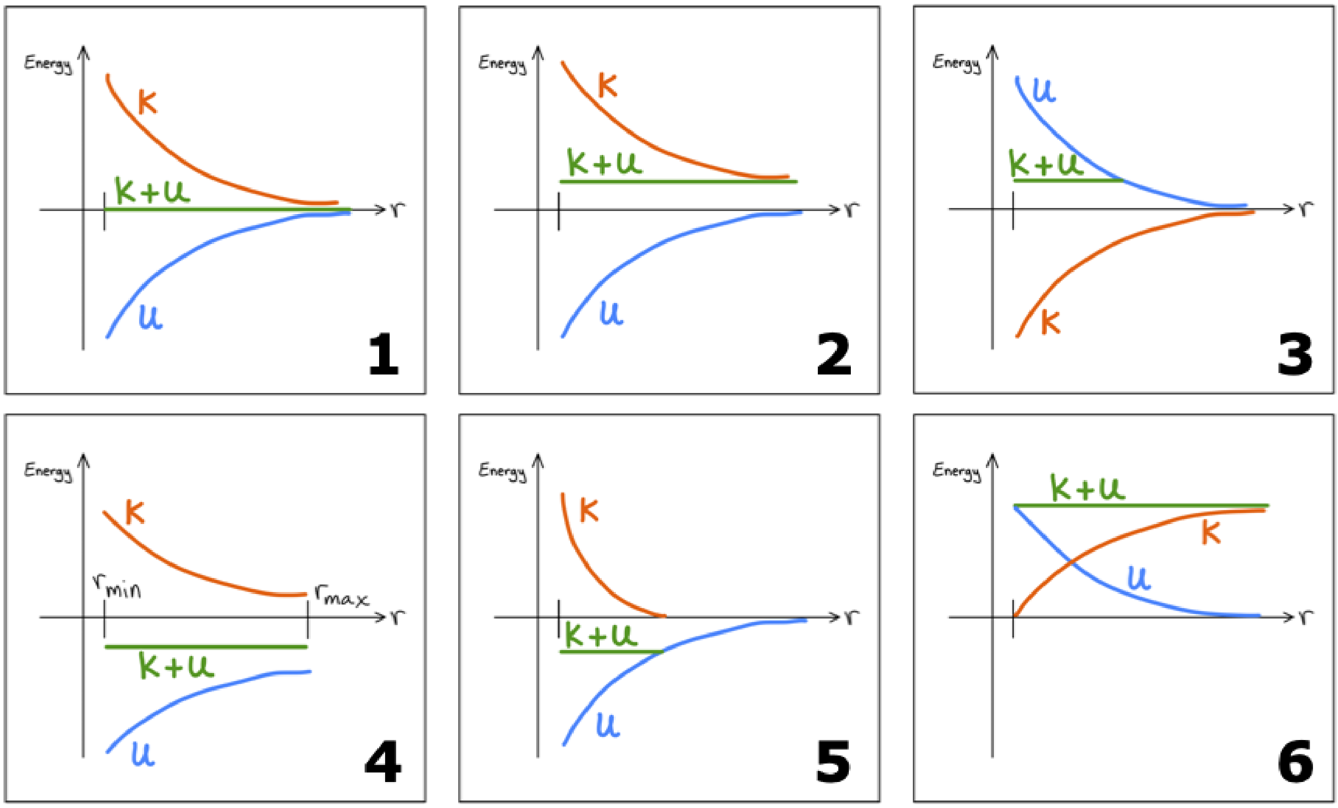
$$\Rightarrow \boxed{|s_i| = \sqrt{\frac{m}{k}}\, v_f = 29 \text{ m}} \leftarrow \text{OR}$$

Compression length $\boxed{L = L_0 - |s_i| = 21 \text{ m}}$

Note to graders: Accept $|s_i| = 29$ m compressed OR compressed length of spring $L = 21$ m. Make sure answer is explicit.

Energy Graphs [20 pts]

The following image shows six graphs of Energy versus distance (r). In each graph, the orange curve labeled K represents kinetic energy, the blue curve labeled U represents the potential energy, and the green line labeled $K+U$ represents the total energy. Indicate which graph (by number) best represents each of the scenarios described below. Each graph can represent several scenarios.



1. 2 [2 pts] A proton and an electron are moving away from each other with high initial speeds. When they are infinitely far away from each other, they are still moving.
2. 4 [2 pts] A star in an elliptical orbit around a black hole.
3. 6 [2 pts] Two protons are moving straight towards each other. When they get very close, they momentarily stop and then start moving away from each other.
4. 4 [2 pts] Pluto and Charon, its biggest moon, orbiting each other.
5. 1 [2 pts] An electron breaks free from its companion proton in a hydrogen atom. When the electron is infinitely far away from the proton, its speed is zero.
6. 6 [2 pts] Two electrons are held near each other at rest and then are let go.
7. 1 [2 pts] A comet swings by close to the Sun and then moves away. The comet's speed is zero when it is very, very far away from the Sun.
8. 5 [2 pts] A proton and an electron are held at rest at some distance away from each other and then are let go.
9. 2 [2 pts] Voyager II is currently very, very far away from the Sun, and moves with a speed of 15.3 km/s.
10. 3 [2 pts] An impossible set of energy graphs.