PHYS 2211 GPS Topics

Brandon Pries

August 2023

Fall/Summer Week 2, Spring Week 3:

- Quantities
  - Scalars vs. vectors
  - Symbolic vs. numeric
    - Units
- Vector algebra
  - Addition/subtraction, scalar multiplication/division
    - Graphical ("tip-to-tail")
  - Magnitudes
- Physics variables
  - Position, velocity, acceleration, momentum, force
  - Mass, time
- Position/velocity update equations and Newton's Second Law
  - $\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = m\vec{a}$
  - $\vec{v} = \frac{d\vec{r}}{dt} \approx \frac{\Delta \vec{r}}{\Delta t} \implies \vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\Delta t$
  - $\vec{a} = \frac{d\vec{v}}{dt} \approx \frac{\Delta \vec{v}}{\Delta t} \implies \vec{v}_f = \vec{v}_i + \vec{a}\Delta t = \vec{v}_i + \frac{\vec{F}_{\text{net}}}{m}\Delta t$

Fall/Spring Week 4, Summer Week 3:

- Constant forces
  - Gravity near Earth's surface $\vec{F}_g = -mg\,\hat{y}$
  - Contact forces
  - $\vec{v}_{\text{avg}} = \frac{\vec{v}_i + \vec{v}_f}{2}$
- Non-constant forces
  - Spring force $\vec{F}_s = -k\left(\left|\vec{L}\right| - L_0\right)\hat{L}$
  - $\vec{v}_{\text{avg}} \approx \vec{v}_f$

Fall/Spring Week 5, Summer Week 4:

- Non-constant forces
  - Spring force $\vec{F}_s = -k\left(\left|\vec{L}\right| - L_0\right)\hat{L}$
  - Gravitational force $\vec{F}_g = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$
  - Electric force $\vec{F}_e = k\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$
- Reciprocity/Newton's Third Law
  - $\vec{F}_{12} = -\vec{F}_{21}$

Fall/Spring Week 6, Summer Week 5:

- Equilibrium, force balancing, force decomposition
  - $\vec{F}_{\text{net}} = \vec{0} \implies F_{\text{net},x} = F_{\text{net},y} = 0$
- Contact forces, friction
  - Normal force $\vec{F}_N$
  - Friction $\left|\vec{F}_f\right| = \mu\left|\vec{F}_N\right|$
    - Static friction μ_s, kinetic friction μ_k
- Tension $\vec{T}$
- Rotated coordinate systems

Fall/Spring Week 7, Summer Week 6:

- Circular/elliptical/orbital motion
- Momentum coordinates $\hat{p}$, $\hat{n}$
- Forces in momentum coordinates
  - $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\,\frac{d\hat{p}}{dt}\hat{n}$
    - $F_{\text{net},\parallel} = \left(\frac{\Delta\vec{p}}{\Delta t}\right)_{\parallel} = \frac{\Delta\vec{p}}{\Delta t}\cdot\hat{p} = \frac{\Delta|\vec{p}|}{\Delta t}\hat{p}$
    - $F_{\text{net},\perp} = \left(\frac{\Delta\vec{p}}{\Delta t}\right)_{\perp} = \frac{\Delta\vec{p}}{\Delta t}\cdot\hat{n} = |\vec{p}|\,\frac{\Delta\hat{p}}{\Delta t}$
- Centripetal force
  - $\vec{F}_{\text{centripetal}} = \frac{mv^2}{R}\hat{n}$

Fall/Spring Week 8, Summer Week 7:

- Dot product
  - Component formula $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$
  - Magnitude formula $\vec{a}\cdot\vec{b} = |\vec{a}|\left|\vec{b}\right|\cos(\theta)$
    - θ is angle measured from $\vec{a}$ to $\vec{b}$
- Work-energy principle $W = \Delta E$
  - Work done by surroundings W
  - Change in energy in system ΔE
- Work $W = \int_C \vec{F}(\vec{r})\cdot d\vec{r}$
  - $\vec{F}$ constant $\implies W = \vec{F}\cdot\Delta\vec{r}$

Fall/Spring Week 9, Summer Week 7:

- Types of energy
  - Kinetic energy $K = \frac{1}{2}mv^2$
  - Gravitational potential energy $U_g = mgh, -\frac{GMm}{r}$
  - Spring potential energy $U_s = \frac{1}{2}ks^2$
  - Electric potential energy $U_e = \frac{kq_1q_2}{r}$
- Total system energy
  - $E < 0 \implies$ bound
  - $E > 0 \implies$ unbound
- Energy graphs

Fall/Spring Week 10, Summer Week 8:

- Types of energy
  - Spring potential energy $U_s = \frac{1}{2}ks^2$
  - Thermal energy $E_{\text{therm}} = mcT$
  - Internal energy E_{int}
- Heat $Q = \Delta E_{\text{therm}}$

Fall Week 11, Spring Week 12, Summer Week 8:

- Center of mass $\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$
- Extended objects, real systems
  - Point particle system: $W = \vec{F} \cdot \Delta\vec{r}_{CM}$ (center of mass)
  - Real system: $W = \vec{F} \cdot \Delta\vec{r}_{PC}$ (point of contact)
- System decomposition, relative motion
  - $K_{\text{trans}} (= K_{CM})$, K_{rel}
- Rotational motion
  - Angular velocity $\vec{\omega}$
  - Moment of inertia I
    - Parallel axis theorem $I_{\parallel} = I_{CM} + md^2$
  - Rotational kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$

Fall Week 12, Spring Week 13, Summer Week 9:

- Conservation of momentum $\vec{p}_i = \vec{p}_f$
- Collisions
  - (Perfectly) elastic collisions $\Delta K = 0$
  - Inelastic collisions $\Delta K < 0$
  - Maximally inelastic collisions $\Delta K < 0, \vec{v}_{f,1} = \vec{v}_{f,2}$

Fall Week 13, Spring Week 14, Summer Week 9:

- Cross product
  - Component formula $\vec{a} \times \vec{b} = \langle a_y b_z - a_z b_y,\, a_z b_x - a_x b_z,\, a_x b_y - a_y b_x \rangle$
  - Magnitude formula $\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \left|\vec{b}\right| \sin(\theta)$
    - θ is angle measured from $\vec{a}$ to $\vec{b}$
  - Right hand rule
- Translational angular momentum $\vec{L}_{\text{trans}} = \vec{r} \times \vec{p}$

Fall Week 13, Spring Week 14, Summer Week 10:

- Angular momentum $\vec{L}$
  - Translational angular momentum $\vec{L}_{\text{trans}} = \vec{r} \times \vec{p}$ (point particle)
  - Rotational angular momentum $\vec{L}_{\text{rot}} = I\vec{\omega}$ (real system)
- Torque $\vec{\tau} = \frac{d\vec{L}}{dt}$
  - Point particle $\vec{\tau} = \vec{r} \times \vec{F}$
  - Real system $\vec{\tau} = I\vec{\alpha}$
- Conservation of angular momentum $\vec{L}_i = \vec{L}_f$

Fall/Spring Week 15, Summer Week 10:

- Conservation of angular momentum $\vec{L}_i = \vec{L}_f$
- Angular momentum update formula
  - $\vec{\tau} = \frac{d\vec{L}}{dt} \approx \frac{\Delta\vec{L}}{\Delta t} \implies \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$
- Tidal locking