PHYS 2211, Summer 2021

Week 6: The Energy Principle

total change in energy for the system

$$\Delta E_{sys} = W_{surr} + Q$$

total work done on system by surroundings

exchange in energy between system & surroundings due to a difference in temperature

In this video:

- ✓ vector dot product
- ✓ work
- ✓ kinetic energy
- ✓ potential energy (U_{grav}, $U_{electric}$)
- ✓ energy graphs

Multiplying vectors: The Dot product

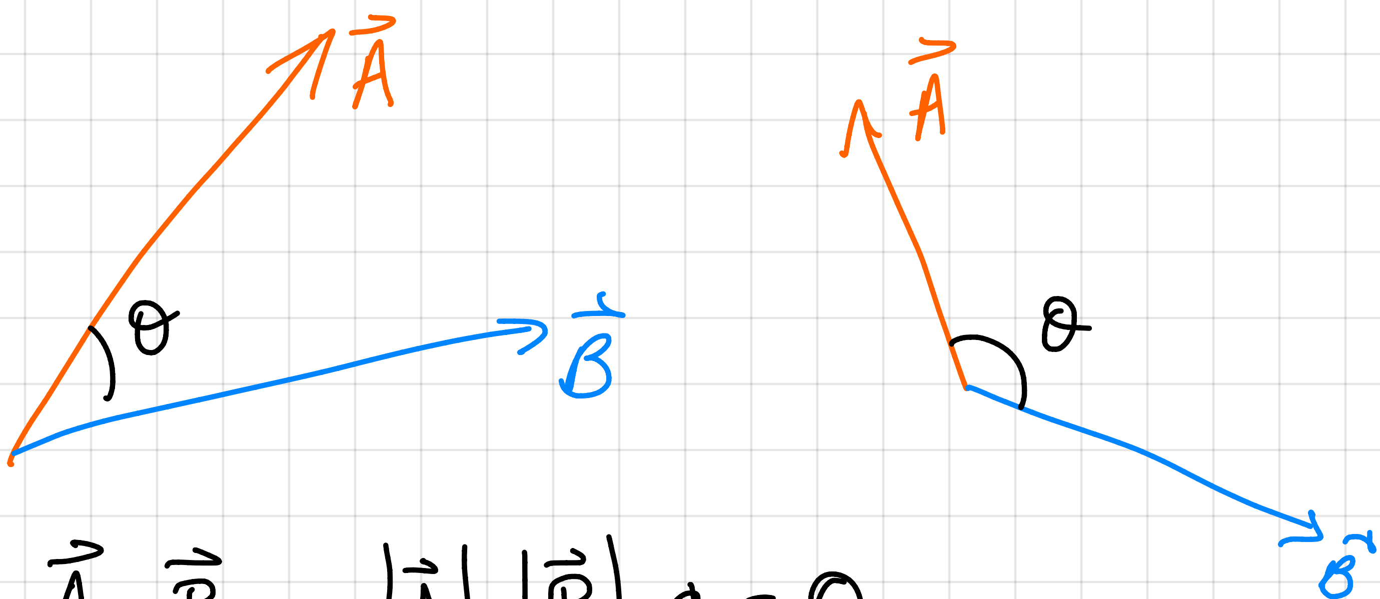
"Scalar product"

$\vec{A} = \langle A_x, A_y, A_z \rangle$

$\vec{B} = \langle B_x, B_y, B_z \rangle$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The result of the dot product is a scalar



$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

If $\theta < 90° \Rightarrow \vec{A} \cdot \vec{B} > 0$

If $90° < \theta < 180° \Rightarrow \vec{A} \cdot \vec{B} < 0$

If $\theta = 90° \Rightarrow \vec{A} \cdot \vec{B} = 0$

Work

$$W = \int \vec{F} \cdot d\vec{r} \quad \text{(general form)}$$

If $\vec{F} = \text{constant}$ e.g. $\vec{F}_g = mg\,(-\hat{y})$

$$W = \vec{F} \cdot \Delta\vec{r}$$

Work is a scalar & has units of:

$\vec{F} \Rightarrow N \Rightarrow kg\, m/s^2$

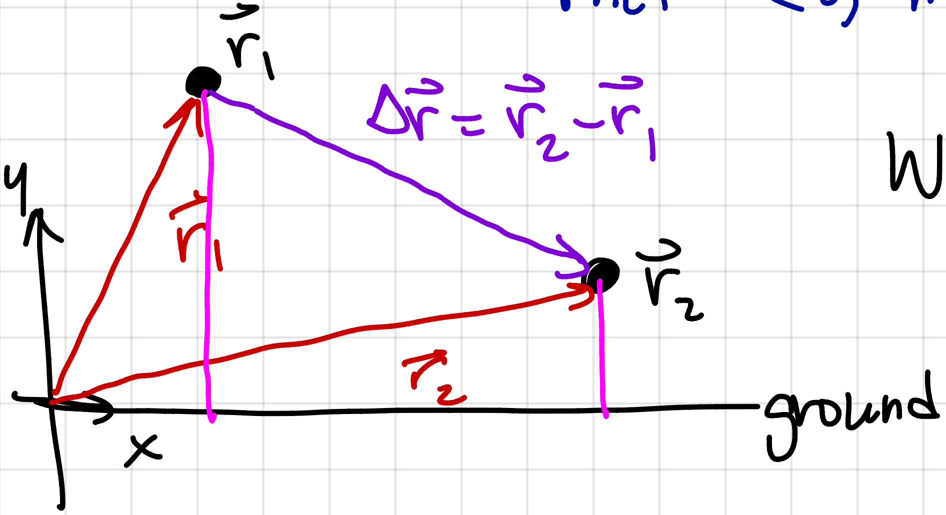
$\Delta\vec{r} \Rightarrow m$

$\vec{F} \cdot \Delta\vec{r} \Rightarrow kg\, m^2/s^2 = Nm = \text{Joule}$

Unit of work is $\underline{\underline{J}}$

(also the unit of energy)

$$\vec{F}_{net} = \langle 0, -mg, 0 \rangle$$



$$W = \vec{F}_{net} \cdot \Delta\vec{r}$$

$$= \langle 0, -mg, 0 \rangle \cdot (\vec{r}_2 - \vec{r}_1)$$

$$= \cancel{(0)(\Delta r_x)} + (-mg)(\Delta r_y) + \cancel{(0)(\Delta r_z)}$$

$$W = (-mg)\,(r_{2y} - r_{1y})$$

capital W

$$\underbrace{(-mg)}_{(-)} \qquad \underbrace{r_{2y} < r_{1y}}_{(-)}$$

$$\underline{\underline{W = \text{positive}}}$$

Kinetic Energy = energy of motion

$$K = \frac{1}{2} m v^2$$

capital K (K), mass (m), speed (v)

Change in kinetic energy

$$\Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\Delta K = \frac{1}{2} m \left(v_f^2 - v_i^2\right)$$

Not the same as $\cancel{(v_f - v_i)^2}$

Potential Energy ⇒ due to interactions between objects in the system

- ✓ gravitational potential energy
- ✓ electric potential energy

~~✓ spring potential energy → week 7~~

Potential Energy U

$$\vec{F}_{grav} = mg\,(-\hat{y})$$ (near surface of Earth)

$$U_{grav} = mgy$$ ← vertical distance from the ground

$$\Delta U_{grav} = mg\,\Delta y = mg\,(y_f - y_i)$$

$$\vec{F}_{grav} = \frac{Gm_1m_2}{r^2}\,(-\hat{r})$$

$$U_{grav} = \frac{-Gm_1m_2}{r}$$

$$\Delta U_{grav} = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\vec{F}_{electric} = \frac{kq_1q_2}{r^2}\,\hat{r}$$

$$U_{electric} = \frac{kq_1q_2}{r}$$

When solving energy principle problems

① Identify system & surroundings

system → things in system contribute energy

surroundings → things in surroundings do work on the system

② Identify types of energy involved

③ Clearly define the initial state & the final state of the system

④ Put things together in the energy principle

$$\Delta E_{sys} = W_{surr}$$

$$\underbrace{\Delta K + \Delta U_g + \Delta U_e + \dots}_{\text{total change in energy of the system}} = \underbrace{\vec{F}_{net} \cdot \Delta \vec{r}_{system}}_{\text{total work done on the system}}$$

Energy Graphs

(look in Fall 2020 lectures)

making line plots of energy vs distance

① Identify the potential energy

② Determine total energy of system

✓ If $E = K + U > 0 \Rightarrow$ unbound

✓ If $E = K + U < 0 \Rightarrow$ bound

✓ If $E = K + U = 0 \Rightarrow$ system @ escape speed

③ Put the kinetic energy in graph in a way that ensures $K + U \stackrel{?}{=} E$