PHYS 2211 - Test 2 - Spring 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

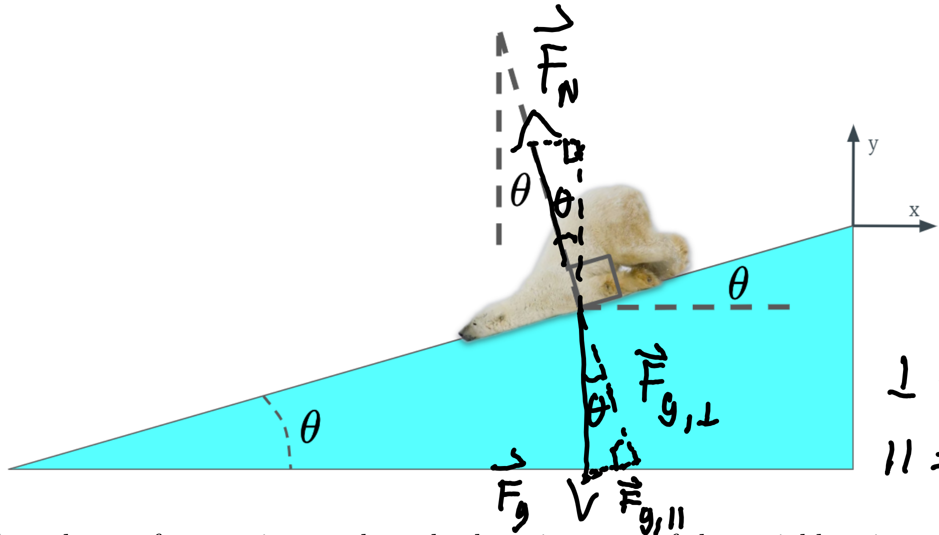
"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

Problem 1 - 30 Points

A polar bear of mass m is observed sliding down a snow ramp with an unknown non-constant velocity. Friction between the bear and the snow is negligible and can be ignored. Take the origin to be at the top of the ramp so that the ramp makes an angle of θ with the flat ground and gravity points in the $-\hat{y}$ direction as indicated in the diagram.



$\perp = \perp$ to ramp
$\parallel = \parallel$ to ramp

1. [10pts] Calculate the net force acting on the polar bear in terms of the variables given. Your answer should be a vector with x and y components. Please note that the bear's velocity is not constant but is parallel to the snow ramp.

Method 1: Working in x-y coordinates

-1 clerical
-20% minor
-40% major
-80% min prog

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_N$$

$$\vec{F}_g = <0, -mg, 0>$$

$$|\vec{F}_N| = |\vec{F}_{g,\perp}| = |\vec{F}_g|\cos\theta = mg\cos\theta$$

$$\vec{F}_N = |\vec{F}_N| < -\sin\theta, \cos\theta, 0> = < -mg\cos\theta\sin\theta, mg\cos^2\theta, 0>$$

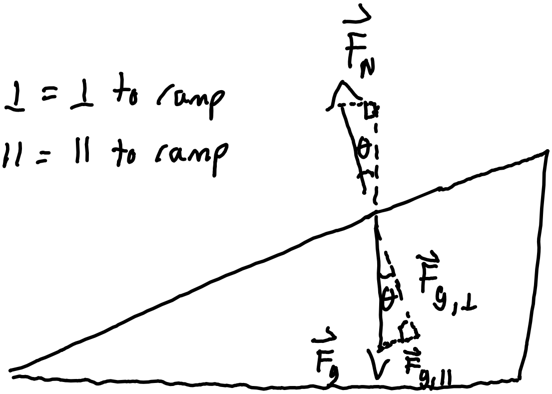
$$\boxed{\vec{F}_{net} = mg < -\cos\theta\sin\theta, \underbrace{-1+\cos^2\theta}_{=-\sin^2\theta}, 0>}$$

Note: trig identity optional

Method 2: Working in ramp coordinates

$\perp$ = $\perp$ to ramp

$\parallel$ = $\parallel$ to ramp



$$\hat{x}' = \cos\theta(\hat{x}) + \sin\theta(\hat{y})$$

$$\hat{y}' = -\sin\theta(\hat{x}) + \cos\theta(\hat{y})$$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_N$$

In ramp coordinates,

$$\vec{F}_g = mg\left[-\sin\theta(\hat{x}') - \cos\theta(\hat{y}')\right]$$

$$= -mg(\hat{y})$$

$$\Sigma \vec{F}_{y'} = 0$$

$$\Rightarrow \vec{F}_{g,y'} + \vec{F}_N = 0$$

$$\vec{F}_N = -\vec{F}_{g,y'} = mg\left[\cos\theta(\hat{y}')\right]$$

$$= mg\left[-\cos\theta\sin\theta(\hat{x}) + \cos^2\theta(\hat{y})\right)$$

$$\vec{F}_{net} = \left[-mg\sin\theta\right](\hat{x}')$$

$$= \left[-mg\sin\theta\right]\left[\cos\theta(\hat{x}) + \sin\theta(\hat{y})\right]$$

$$\boxed{\vec{F}_{net} = mg\langle -\sin\theta\cos\theta, -\sin^2\theta, 0\rangle}$$

-1 clerical

-20% minor

-40% major

-80% min prog

2. [20pts] Below is an incomplete Glowscript program to update the velocity and position of the polar bear using the force you just calculated. The polar bear is assumed to start at the origin at time $t = 0$. Add the missing python code below to calculate the net force (that you found in part 1) on the polar bear and update the velocity and position of the bear.

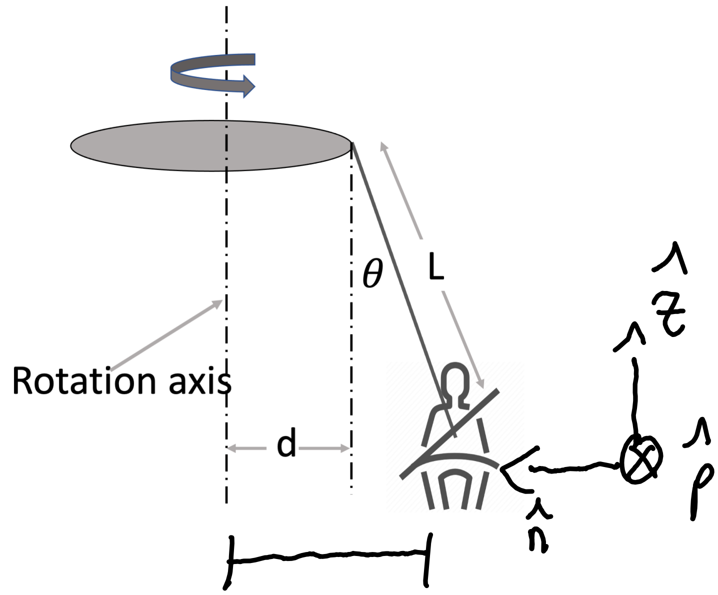
```
Web VPython 3.2
scene.background = color.white

g = 9.81 #acceleration (m/s^2) near surface of the Earth
theta = pi/5 #angle of ramp with ground in radians
bear = sphere(color=color.yellow, pos=vector(0,0,0), radius=3,make_trail=True)
bear.vel = vector(0,0,0) #bear starts from rest
bear.m = 600 #mass in kg

deltat = 1e-1 #timestep in seconds
t = 0
while t <1000: # iterate for 1000 seconds
    rate(100)
    Fgrav = bear.m * g * vector(0,-1,0)
    Fnormal = bear.m * g * vector(-cos(theta)*sin(theta),
                                  cos(theta)*cos(theta))
-5  Fnet = Fgrav + Fnormal
```

Note: can write Fnet in one line

```
    vel_init = bear.vel  # store initial velocity
-5  bear.vel = bear.vel + (Fnet/bear.m)*deltat



-5  vel_avg = (vel_init + bear.vel)/2  # constant force so
                                       use arithmetic average

-5  bear.pos = bear.pos + vel_avg*deltat


    t = t + deltat  # Advance the clock
print("Run!")
```

Problem 2 - 35 Points

In an amusement park, riders sit on a small seat suspended by ropes of length L that are attached to a rotating disk of radius d as indicated in the diagram. As the disk spins, the rope, initially vertical at rest, makes an angle θ with the vertical, as shown in the second diagram. The speed of the rider v is constant but unknown.



$R = d + L\sin\theta$

1. [5pts] Calculate the parallel component of the rate of change of momentum for the rider.

$\boxed{0}$ (constant speed)

All or nothing

2. [10pts] For a rider and seat of total mass m, Determine the magnitude of the tension in the rope. Your answer should not depend on the unknown velocity of the rider.

$$\vec{F}_{net} = \vec{F}_g + \vec{T}$$

$$= mg(-\hat{z}) + |\vec{T}|\sin\theta\,(\hat{n}) + |\vec{T}|\cos\theta\,(\hat{z})$$

$$\vec{F}_{net,z} = 0$$

$$\Rightarrow \boxed{|\vec{T}| = \frac{mg}{\cos\theta}}$$

-1 clerical

-20% minor

-40% major

-80% min prog

3. [20] Calculate the speed of the rider in terms of the known quantities and universal constants.

$$\vec{F}_{net,n} = \frac{mv^2}{R}\hat{n} = |\vec{T}|\sin\theta\,\hat{n}$$

$$\Rightarrow \frac{\cancel{m}v^2}{d+L\sin\theta} = \left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta$$

$$\boxed{v = \sqrt{g(d+L\sin\theta)\tan\theta}}$$

-1 clerical

-20% minor

-40% major

-80% min prog

4. [5pts extra credit] For small angle θ (so that $sin\theta \approx \theta$), determine the tension T as a function of the speed v. Your answer should not contain the angle θ. You may find the following identity useful: Use $1/cos^2\theta = 1 + tan^2\theta$.

For small angle θ, $R \equiv d + L\sin\theta \approx d$

z-eq'n (part 2)

$$|\vec{T}| = \frac{mg}{\cos\theta}$$

n-eq'n (part 3)

$$v = \sqrt{g(d+L\sin\theta)\tan\theta}$$

$$|\vec{T}|^2 = \frac{m^2g^2}{\cos^2\theta} = m^2g^2(1+\tan^2\theta) \qquad v^2 \approx g d \tan\theta$$

$$\Rightarrow \tan\theta = \sqrt{\frac{|\vec{T}|^2}{m^2g^2} - 1} \qquad \tan\theta \approx \frac{v^2}{gd} << 1$$

$$|\vec{T}|^2 \approx m^2g^2\left(1 + \frac{v^4}{g^2d^2}\right)$$

-2 wrong v^4 constants

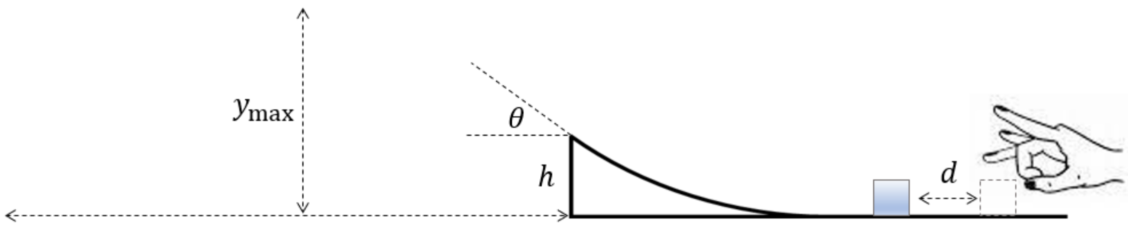
-5 no v^4

$$\boxed{|\vec{T}| \approx mg\sqrt{1 + \frac{v^4}{g^2d^2}}} \quad \xrightarrow{\frac{v^4}{g^2d^2} << 1} \quad \boxed{|\vec{T}| \approx mg\left(1 + \frac{v^4}{2g^2d^2}\right)}$$

OR

Problem 3 - 35 Points

An object is launched from a ramp using a finger flick. Assume that the surface of the launch pad and the air have no friction, and the gravitational force is constant. The mass of the object is m, and the height of the launch point is h.



1. [15pts] During the flick, assume that the finger exerts a constant force $\vec{f} = \langle -f_x, -f_y, 0 >\rangle$ as the object is displaced an amount $\langle -d, 0, 0\rangle$. What is the maximum speed $v_{\max}$ of the object during the launch from the ramp?

system: object

surr: hand, ramp

$\Delta K_{object} = W_{hand},\ 0$

$\frac{1}{2} m(v_f^2 - \cancel{v_i^2}) = \langle -f_x, -f_y, 0 \rangle \cdot \langle -d, 0, 0 \rangle$

$= f_x d$

$\Rightarrow \boxed{V_{max} = \sqrt{\frac{2 f_x d}{m}}}$

(no more positive work done while on ramp)

-1 clerical

-20% minor

-40% major

-80% min prog

2. [10pts] Calculate the minimum force ($|\vec{f}_{\min}|$) the finger must exert during the flick for the object to leave the launch pad.

system: object + Earth

surr: ramp

initial: right after W_{hand}

final: leaving ramp

$\Delta E_{sys} = 0 = \Delta K_{obj} + \Delta U_{grav}$

$= \frac{1}{2} m(\cancel{v_f^2}^{\,0} - v_i^2) + mgh$

v_{max} from part a

min force is when $v_f = 0$ $\Rightarrow \frac{1}{2} m\left(\frac{2 f_x d}{m}\right) = mgh$

$f_x = \frac{mgh}{d}$, $f_y = 0$ $\Rightarrow \boxed{|\vec{f}_{min}| = \frac{mgh}{d}}$

(f_y doesn't affect v_{max} so set it to 0)

-1 clerical

-20% minor

-40% major

-80% min prog

2. Alternative method: initial: before W_{hand}

final: leaving ramp

$$\Delta E_{sys} = \Delta K_{obj} + \Delta U_g = W_{hand}$$

At $\vec{f}_{min}$, $\Delta K_{obj} = 0$.

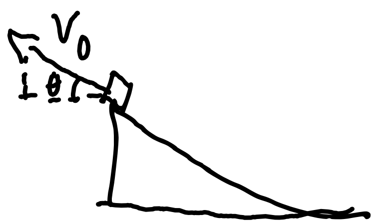
$$\Delta U_g = mgh$$

$$W_{hand} = f_{min,x}\, d$$

$$\Rightarrow f_{min,x} = \frac{mgh}{d} \Rightarrow \boxed{|\vec{f}_{min}| = \frac{mgh}{d}}$$

$$f_{min,y} = 0$$

3. [10pts] The object leaves the pad at an angle θ with speed v_0. Using the energy principle to calculate the maximum height $y_{\max}$ from the ground that the object will reach.



-1 clerical

-20% minor

-40% major

-80% min prog

$$\Delta K_{obj} + \Delta U_g = 0$$

$$\frac{1}{2} m (\underbrace{v_f^2}_{= v_{x,i}^2} - \underbrace{v_i^2}_{= v_0^2}) + mg(y_{max} - h) = 0$$

$$v_{x,i} = v_0 \cos\theta$$

$$\frac{1}{2} \cancel{m} (v_0^2 \cos^2\theta - v_0^2) = -\cancel{m}g(y_{max} - h)$$

$$\boxed{y_{max} = -\frac{v_0^2}{2g} \underbrace{(\cos^2\theta - 1)}_{= -\sin^2\theta} + h}$$

Note: trig relationship optional