Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

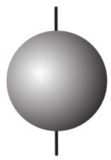
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

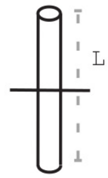
$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis



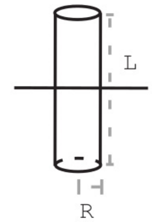
$I = \frac{2}{5}MR^2$

$I = \frac{1}{2}MR^2$



$I = \frac{1}{12}ML^2$

$I = \frac{1}{3}ML^2$



$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}		kilo	k	1×10^{3}
micro	μ	1×10^{-6}		mega	M	1×10^{6}
nano	n	1×10^{-9}		giga	G	1×10^{9}
pico	p	1×10^{-12}		tera	T	1×10^{12}

PHYS 2211 - Final Exam - Summer 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

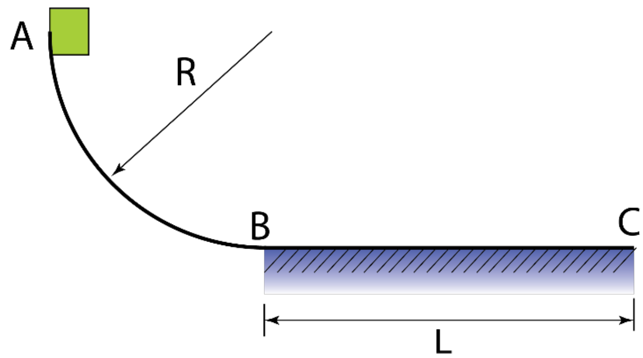
**"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."**

KEY

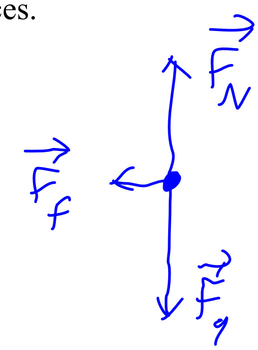
PRINT your name and GTID on the line above

A block on the incline [40 pts]

A block of mass m is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R (point A in the Figure). The block then slides onto a horizontal plane (point B) where it finally comes to rest distance L from the beginning of the plane (point C). The curved incline is frictionless, but there is friction on the block while it slides horizontally between points B and C.



1. [6 pts] Draw a free body diagram for the block as it moves along horizontal plane. Show all forces.



2. [15 pts] Express the work done by the force of friction on the block as it moves along the horizontal plane in terms of m and the coefficient of kinetic friction μ. Determine the coefficient of kinetic friction μ between the block and the horizontal plane. Express your answer for μ in terms of the given quantities R, L.

$$W_f = \vec{F}_f \cdot \Delta\vec{r} = \mu |\vec{F}_N| (-\hat{x}) \cdot L\hat{x}$$

$$= \mu m g (-\hat{x}) \cdot L\hat{x} = \underline{-\mu m g L}$$

$A \to C: \Delta K = 0$

$\Delta E = W \Rightarrow \Delta U_g = W_f$

$\Delta U_g = mg\Delta h = -mgR$

$-mgR = -\mu m g L \Rightarrow \boxed{\mu = \frac{R}{L}}$

3:22

3. [6 pts] Determine the speed of the block at point B. Express your answer in terms of the known quantities (R and g).

$A \to B$:

$\Delta E = W \Rightarrow \Delta K + \Delta U_g = 0 \Rightarrow \Delta K = -\Delta U_g$

$\Delta U_g = mg\Delta h = -mgR$

$\Delta K = \frac{1}{2}m(v_B^2 - v_A^2) = \frac{1}{2}mv_B^2 = mgR \Rightarrow \boxed{v_B = \sqrt{2gR}}$

4. [7 pts] Determine the magnitude of the acceleration of the block while it slides along the horizontal plane. Use your result in part 2 to express your answer in terms of the known quantities (R, L and g).

$F_{net} = ma = F_f = \mu F_N = \mu mg = \frac{R}{L}mg$

$\Rightarrow \boxed{a = \frac{R}{L}g}$

5. [6 pts] How much time elapses while the block is sliding horizontally between points B and C? Express your answer in terms of the known quantities (R, L and g). Hint: use your results for the speed at point B (part 3) and the acceleration between points B and C (part 4).

$a = \frac{|\Delta \vec{v}|}{\Delta t} \Rightarrow \Delta t = \frac{|v_f - v_i|}{a} = \frac{|v_C - v_B|}{a}$

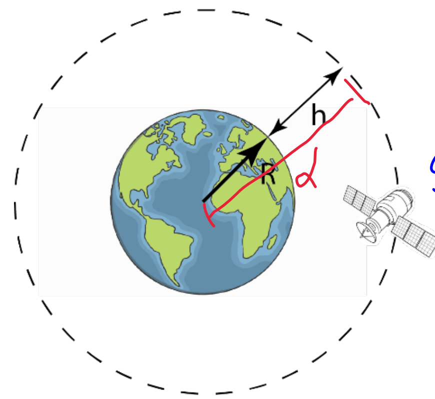
$= \frac{|0 - \sqrt{2gR}|}{\frac{R}{L}g} = \boxed{\frac{L\sqrt{2gR}}{gR} = L\sqrt{\frac{2}{gR}}}$

8:18

A satellite in geostationary orbit [30 pts]

A satellite is said to be in geostationary orbit when it moves in uniform circular motion directly above the Earth's equator, with an orbital period that matches the Earth's rotation rate.

Consider a communications satellite in geostationary orbit. The satellite has mass $m = 4000\ kg$ and has orbital period T equal to the rotation rate of the Earth ($T = 24\ hrs$). What is the height h of this satellite above the surface of the Earth (see the Figure)?



$d = R + h$

Note that the mass of the Earth is $M = 6 \times 10^{24}\ kg$, the radius of the Earth is $R = 6.4 \times 10^{6} m$, and remember that for universal gravitation you should always use the center-to-center distance. You can assume the satellite is a point mass.

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi d}{T} = \frac{2\pi (R+h)}{T}$$

$$F_{cent} = F_g \Rightarrow \frac{mv^2}{d} = \frac{GMm}{d^2} \Rightarrow \frac{\left(\frac{2\pi d}{T}\right)^2}{d} = \frac{GM}{d^2}$$

$$\Rightarrow \frac{4\pi^2 d}{T^2} = \frac{GM}{d^2} \Rightarrow d^3 = \frac{GMT^2}{4\pi^2}$$

$$\Rightarrow d = R+h = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \Rightarrow \underline{h = \sqrt[3]{\frac{GMT^2}{4\pi^2}} - R}$$

$$h = \sqrt[3]{\frac{(6.7 \cdot 10^{-11}\ m^3/kg\ s^2)(6 \cdot 10^{24}\ kg)(86400\ s)^2}{4\pi^2}} - (6.4 \cdot 10^6\ m)$$

$$\approx \boxed{3.60 \cdot 10^7\ m}$$

14:42

Monkey pulls the rope [40 pts]

An object with mass m, on the end of a string, moves in a circle on a horizontal frictionless table as shown in Figure 1. At the initial state the speed of an object is v_i and the radius of the circle is R. A monkey pulls the string very slowly through a small hole in the table (point O).

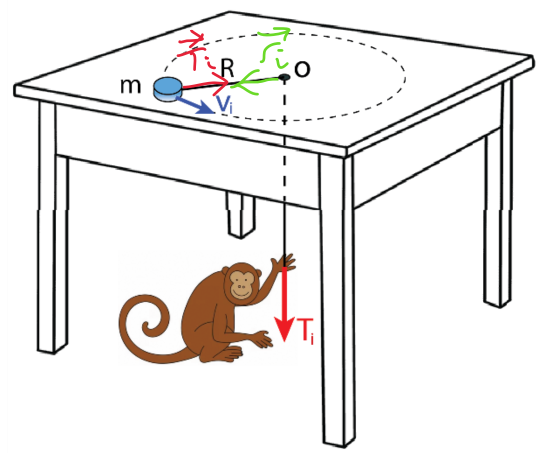


Figure 1 Initial State

1. [6 pts] What is the tension in the string T_i in the initial state? Express your answer in terms of the given quantities m, v_i, R.

$$F_{cent} = \boxed{T_i = \frac{mv_i^2}{R}}$$

2. [6 pts] What is the torque of the tension force in the horizontal string about point O? Note: as the object moves the tension force in the string is always directed along the string towards point O.

$$\vec{\tau} = \vec{r}_i \times \vec{T}_i = \vec{0} \Rightarrow \boxed{\tau = 0}$$

$\vec{r}_i$ and $\vec{T}_i$ are anti-parallel

3. [8 pts] Which of the following is correct as the monkey pulls the rope and the radius of the circle decreases (circle all the true statements):

(A) The angular momentum of an object about point O remains constant. $\tau = 0$

(B) The angular momentum of an object about point O decreases.

(C) The angular momentum of an object about point O increases.

(D) The kinetic energy of an object remains constant.

(E) The kinetic energy of an object decreases.

(F) The kinetic energy of an object increases. $W > 0$

~18:00

4. [10 pts]. The monkey has pulled the string such that the radius of the circle is half of its initial value: $R_f = R/2$ (see Figure 2). What is the speed of an object v_f at this final state? Express your answer in terms of the given quantity v_i.

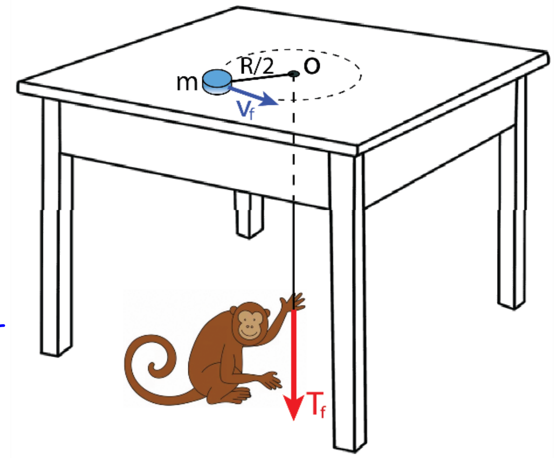


Figure 2 Final State

$\tau = 0 \Rightarrow \Delta L = 0 \Rightarrow L_i = L_f$

$L_i = |\vec{r}_i||\vec{p}_i| = R m v_i$

$L_f = |\vec{r}_f||\vec{p}_f| = \frac{R}{2} m v_f$

$R m v_i = \frac{R}{2} m v_f \Rightarrow \boxed{v_f = 2 v_i}$

5. [2 pts] Is the tension in the string T_f in the final state larger or smaller than that in the initial state?

$T_f = \frac{m v_f^2}{r_f} = \frac{m(2v_i)^2}{R/2} = 8 \frac{m v_i^2}{R} \Rightarrow \boxed{T_f > T_i}$

6. [8 pts] What is the work done by the monkey as it pulls the string (from the initial to the final state). Express your answer in terms of the given quantities m, v_i, R.

$\Delta E = W \Rightarrow \Delta K = W_{monkey}$

$W_{monkey} = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} m (4v_i^2 - v_i^2)$

$= \boxed{\frac{3}{2} m v_i^2}$

21:28

Bullet and the block [50 pts]

A rubber bullet of mass m is moving horizontally with speed v_0 when it hits a wooden block of mass $M = 6m$ that is hanging at rest at the end of a thin rod of negligible mass (see Figure 1). After the collision, the bullet recoils back with the speed $v_f = v_0/2$ and the wooden block acquires velocity v_b (see Figure 2).

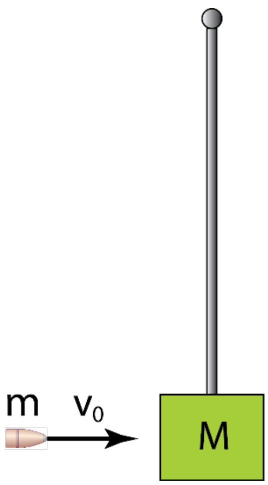


Figure 1 Initial state before the collision

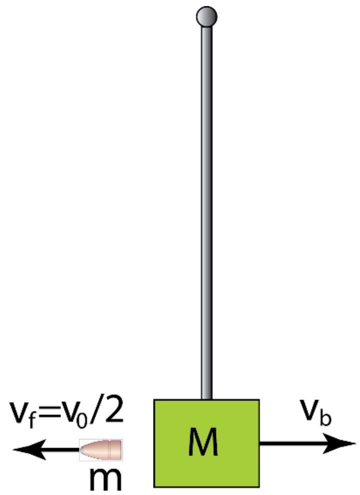


Figure 3 The state after the collision

1. [18 pts] Find the speed of the wooden block v_b immediately after the bullet recoils back (immediately after the collision). Express your answer in terms of the given quantity v_0.

Conservation of momentum: $\vec{p}_i = \vec{p}_f$

$$\vec{p}_i = m v_0 \, \hat{x}$$

$$\vec{p}_f = m \frac{v_0}{2} (-\hat{x}) + 6 m v_b \, \hat{x}$$

$$\vec{p}_i = \vec{p}_f \Rightarrow m v_0 = -m \frac{v_0}{2} + 6 m v_b \Rightarrow 6 m v_b = \frac{3}{2} m v_0$$

$$\Rightarrow \boxed{v_b = \frac{1}{4} v_0}$$

23:31

2. [16 pts] After the collision the block swings upward (see Figure 3). What is the maximum height h (relative to the original position) reached by the block? Express your answer in terms of the given quantities m, v_0.

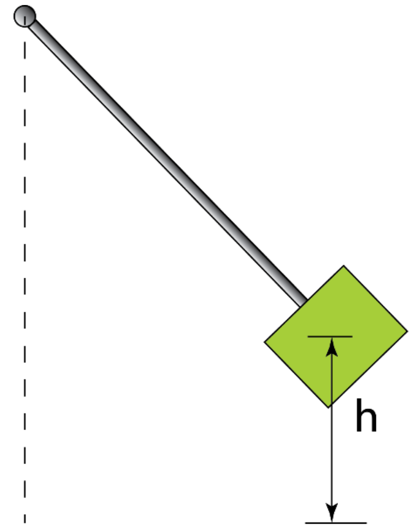


Figure 3 The final state

$$\Delta E = W \Rightarrow \Delta K + \Delta U_g = 0$$

$$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$
$$= \frac{1}{2}(6m)(-v_b^2) = -3m\left(\frac{v_0}{4}\right)^2$$
$$= -\frac{3}{16} m v_0^2$$

$$\Delta U_g = M g \Delta h = 6mgh$$

$$\Delta U_g = -\Delta K \Rightarrow 6mgh = \frac{3}{16} m v_0^2 \Rightarrow \boxed{h = \frac{1}{32}\frac{v_0^2}{g}}$$

3. [16 points] How much of the initial kinetic energy of the bullet is converted to thermal energy during the collision? Express your answer in terms of the given quantities m, v_0.

$$\Delta E = W \Rightarrow \Delta K + \Delta E_{therm} = 0 \Rightarrow \Delta E_{therm} = -\Delta K$$

$$K_i = \frac{1}{2} m v_0^2$$

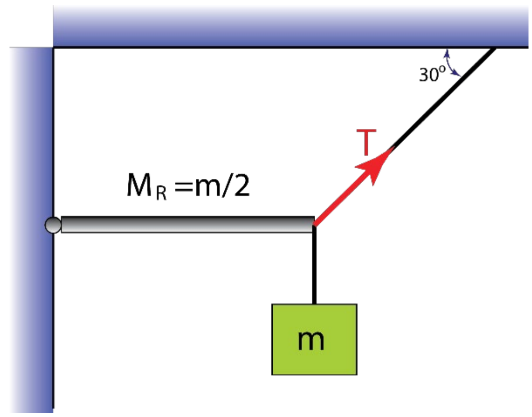
$$K_f = \frac{1}{2} m \left(\frac{v_0}{2}\right)^2 + \frac{1}{2}(6m)\left(\frac{v_0}{4}\right)^2 = \frac{5}{16} m v_0^2$$

$$\Delta E_{therm} = -\Delta K = K_i - K_f = \frac{1}{2} m v_0^2 - \frac{5}{16} m v_0^2$$
$$= \boxed{\frac{3}{16} m v_0^2}$$

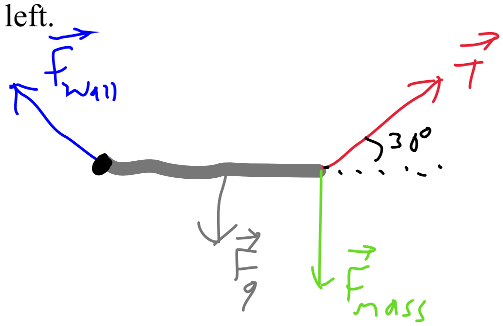
31:10

A rod in equilibrium [40 pts]

A horizontal rod of mass $M_R = m/2$ and length L is attached on a pivot point to a wall. A block of mass m is hanging at the end of the rod. To prevent the rod from tilting a rope is also attached to the end of the rod with the other end of the rope tied to the ceiling. This rope makes an angle 30° with the ceiling (see the Figure).



1. [4 pts] Show all the forces that act on the rod. Sketch appropriate vectors at points where the forces are applied. You may use the provided figure or sketch the rod separately. Note: the wall exerts a force on the rod at the pivot point. This force is directed up and left.



2. [6 pts] Determine the torque of the force by the wall about the pivot point.

$$\vec{\tau}_{wall} = \vec{r}_{wall} \times \vec{F}_{wall}$$

$$\vec{r}_{wall} = \vec{0} \Rightarrow \boxed{\vec{\tau}_{wall} = \vec{0}}$$

3. [10 pts] Determine the torque of the weight of the block about the pivot point. Determine the torque of the weight of the rod about the pivot point. What are the directions of these torques?

$$\vec{\tau}_{mass} = \vec{r}_{mass} \times \vec{F}_{mass} = (L)(mg)(-\hat{z}) = \boxed{-Lmg\,\hat{z}}$$

$$\vec{\tau}_g = \vec{r}_g \times \vec{F}_g = \left(\frac{L}{2}\right)\left(\frac{m}{2}g\right)(-\hat{z}) = \boxed{-\frac{1}{4}Lmg\,\hat{z}}$$

Direction is $-\hat{z}$ / clockwise

37:09

4. [10 pts] Express the torque of the tension force T about the pivot point in terms of given quantities and (still unknown) magnitude of T. What is the direction of this torque?

$$\vec{\tau}_T = \vec{r}_T \times \vec{T} = (L)(T)\sin(30°)\,\hat{z}$$

$$= \boxed{\frac{1}{2} L T\,\hat{z}}$$

Direction is $+\hat{z}$/counterclockwise

5. [10 pts] Find the magnitude of the tension force T. Express your answer in terms of the given quantity m and g.

Equilibrium: $\vec{\tau}_{net} = \vec{0}$

$$\vec{\tau}_T + \vec{\tau}_{wall} + \vec{\tau}_{mass} + \vec{\tau}_g = \vec{0}$$

$$\Rightarrow \frac{1}{2}LT + 0 - Lmg - \frac{1}{4}Lmg = 0$$

$$\Rightarrow \frac{1}{2}LT = \frac{5}{4}Lmg \Rightarrow \boxed{T = \frac{5}{2}mg}$$

41:14