Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_s s \qquad U_{spring}=\frac{1}{2}k_s s^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\dots}{m_1+m_2+\dots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\dots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

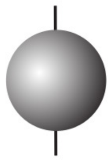
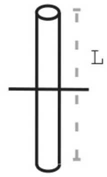
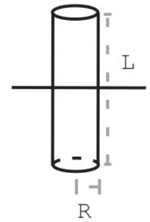
$$\text{prob}(E)\propto\Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\dots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\dots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

				
$I = \frac{2}{5} MR^2$	$I = \frac{1}{2} MR^2$	$I = \frac{1}{12} ML^2$	$I = \frac{1}{3} ML^2$	$I = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 Test 1 - Spring 2019

Please circle your lab section and then clearly print your name & GTID

Sections (M) 10AM, (K) 11AM		
Day	12-3pm	3-6pm
Monday	M01 K01	M02 K02
Tuesday	M03 K03	M04 K04
Wednesday	M05 K05	M06 K06
Thursday	M07 K07	M08 K08

Name: Key

GTID: ______________________

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

____

Sign your name on the line above

Problem 1 [25 pts]

The code below models the motion of the moon orbiting the Sun and Earth. The code is similar to the one you completed in lab except some of the lines of code are missing. Note that in this problem the Sun is assumed to be stationary. **Add in the missing lines of code below to complete the program.**

```
GlowScript 2.7 VPython
G = 6.7e-11
mSun = 2e30 #in kg
mEarth = 6e24 #in kg
mMoon = 7.35e22 #in kg
## OBJECTS with radii are not to scale and are exaggerated
Sun = sphere(pos=vector(0,0,0), radius=7e8*5e1, color=color.yellow)
Earth = sphere(pos=vector(1.5e11,0,0), radius=6.4e6, color=color.blue,make_trail=True)
Moon = sphere(pos=vector(1.5e11,384472282,0), radius=1736482, color=color.green)
t = 0
Earth.p = mEarth*vector(0, 29951.68, 0)
Moon.p = mMoon*vector(-1023.056, 29951.68, 0)
Sun.p = vector(0,0,0)
deltat = 60*60
## CALCULATIONS
while True:
```

A. [13 pts] Add the missing lines of code to calculate the net force on the Moon and Earth.

```
+1    r_earthmoon = Earth.pos - Moon.pos
+1    r_earthsun   = Earth.pos - Sun.pos
+1    r_moonsun   = Moon.pos - Sun.pos

+2    F_earthmoon = -G*mEarth*mMoon/mag(r_earthmoon)**3*r_earthmoon
+2    F_moonearth = -Fearthmoon
+2    F_earthsun    = -G*mEarth*mSun/mag(r_earthsun)**3*r_earthsun
+2    F_moonsun   = -G*mMoon*mSun/mag(r_moonsun)**3*r_moonsun

+1    Fnet_earth  = F_earthmoon + F_earthsun
+1    Fnet_moon = F_moonearth + F_moonsun
```

GRADING

-1 Clerical/Syntax

Points assignments per line are given on the left

B. [6 pts] Add the missing lines of code to update the momentum of the Moon and Earth.

```
+3    Earth.p = Earth.p + Fnet_earth*deltat

+3    Moon.p = Moon.p + Fnet_moon*deltat
```

GRADING

-1 Clerical/Syntax

Points assignments per line are given on the left

C. [6 pts] Add the missing lines of code to update the position of the Moon and Earth.

```
+3    Earth.pos = Earth.pos + Earth.p/mEarth*deltat

+3    Moon.pos = Moon.pos + Moon.p/mMoon*deltat


    t = t + deltat
```

GRADING

-1 Clerical/Syntax

Points assignments per line are given on the left

Problem 2 [25 pts]

A tennis ball, with mass 0.5 kg, flies toward Serena Williams with velocity $< 40, 5, 0 >$ m/s. She hits it; contact with the racket is maintained for 0.01 seconds. After contacting the racket, the ball's velocity is $< -40, -5, 0 >$ m/s, and its position is $\vec{r} =< 0, 1, 0 >$ m. Neglect any air resistance, friction, etc.

A. [10 pts] What is the contact force of the racket on the ball during this short time interval?

Assuming the contact force is constant, over this interval:

$$\Delta \vec{p} = \vec{F}_{contact} \Delta t$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

So

$$\vec{F}_{contact} = \frac{\Delta \vec{p}}{\Delta t}$$

$$= \frac{m \Delta \vec{v}}{\Delta t}$$

$$= \frac{(.5\,kg)}{(0.01\,s)} \left(\langle -40, -5, 0 \rangle \frac{m}{s} - \langle 40, 5, 0 \rangle \frac{m}{s} \right)$$

$$\boxed{\vec{F}_{contact} = \langle -4000, -500, 0 \rangle\ N}$$

B. [15 pts] After contacting the racket, the only force on the ball is gravity (in the direction $-\hat{y}$). Predict the position and velocity of the tennis ball 0.1 seconds after leaving the racket using a single time step.

Momentum Principle:

$$\bar{P}_f = \bar{P}_i + \bar{F}_{net}\,\Delta t \qquad \text{where} \qquad \bar{F}_{net} = \langle 0, -mg, 0 \rangle$$

So

$$m\bar{v}_f = m\bar{v}_i + \bar{F}_{net}\,\Delta t$$

$$\bar{v}_f = \bar{v}_i + \frac{\bar{F}_{net}}{m}\,\Delta t$$

$$= \langle -40, -5, 0 \rangle \,\tfrac{m}{s} + \langle 0, (-9.81)\,\tfrac{m}{s^2}, 0 \rangle\,(0.1\,s)$$

$$\boxed{\bar{v}_f = \langle -40, -5.981, 0 \rangle \,\tfrac{m}{s}}$$

Position Update:

$$\bar{x}_f = \bar{x}_i + \bar{v}_{avg}\,\Delta t \qquad \text{where constant force} \Rightarrow \qquad \bar{v}_{avg} = \frac{\bar{v}_i + \bar{v}_f}{2}$$

So

$$\bar{x}_f = \bar{x}_i + \left(\frac{\bar{v}_i + \bar{v}_f}{2}\right)\Delta t$$

$$= \langle 0, 1, 0 \rangle\, m + \frac{(0.1\,s)}{2}\left(\langle -40, -5, 0 \rangle + \langle -40, -5.981, 0 \rangle \right)$$

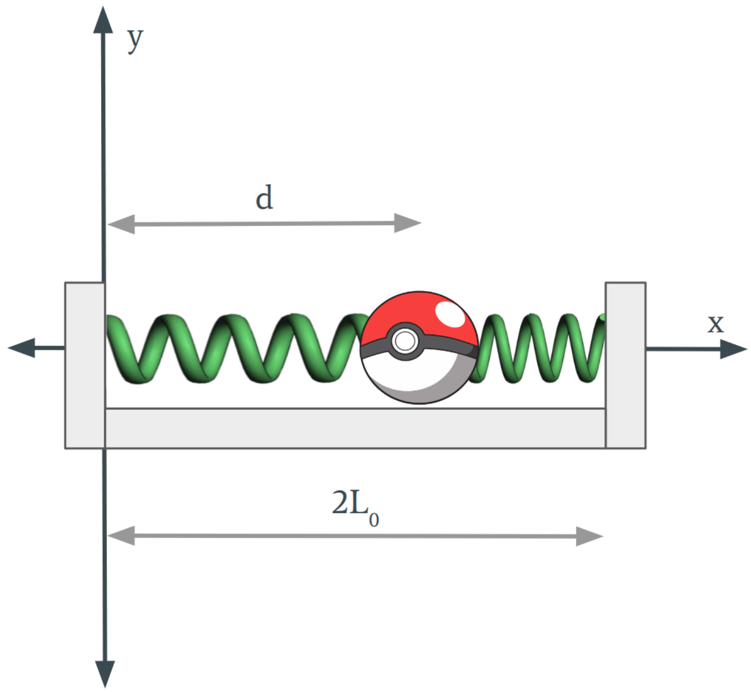
$$\boxed{\bar{x}_f = \langle -4, 0.450, 0 \rangle}$$

GRADING

-1 Clerical
-3 Minor
-6 Major
-12 BTN

Problem 3 [25 pts]

You are playing a game where a ball of mass m, attached to two identical springs, can slide back and forth on a frictionless surface. The base for one spring is located at position $\langle 0, 0, 0\rangle$. The base of the other spring is located at position $\langle 2L_0, 0, 0\rangle$ as indicated in the diagram. The springs are identical with a rest length L_0 and spring constant k_s. Using your hand you move ball to position $\langle d, 0, 0\rangle$ while you hold the ball motionless. This position is such that $d > L_0$.



A. [5 pts] Calculate the net spring force acting on the ball while you hold it motionless. Your answer should be a vector.

$$\vec{F}_{net}(x) = \overbrace{-k(|x|-L_0)\hat{x}}^{+2} \overbrace{-k(|2L_0-x|-L_0)(-\hat{x})}^{+2}$$

$$= -k(x-L_0)\hat{x} + k(2L_0 - x - L_0)(\hat{x}) \quad \text{for } 0<x<2L_0$$

$$= -2k(x-L_0)\hat{x}$$

So

$$\boxed{\vec{F}_{net}(d) = -2k(d-L_0)\hat{x}} \quad +1$$

GRADING

+2 points for each force
+1 for Fnet

B. [10 pts] You release the ball and it begins to move under the influence of the springs. After a short time Δt, determine the new location of the ball. Your answer should be a vector.

Momentum Principle:

$$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\,\Delta t$$

$$= \langle 0, 0, 0\rangle + \Delta t\langle -2k(d-L_0), 0, 0\rangle$$

Position Update (force is not constant)

$$\vec{x}_f = \vec{x}_i + \vec{v}_{avg}\,\Delta t$$

$$= \vec{x}_i + \frac{\vec{P}_f}{m}\Delta t$$

$$= \langle d, 0, 0\rangle + \frac{\Delta t}{m}\langle -2\Delta t\, k(d-L_0), 0, 0\rangle$$

$$\boxed{\vec{x}_f = \langle d - \tfrac{2k}{m}\Delta t^2(d-L_0), 0, 0\rangle}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

C. [10 pts] Determine the net force acting on the ball at the new location you found in the previous part of this problem. Your answer should be a vector.

From part A

$$\bar{F}(x) = -2k(x - L_o)\hat{x} \quad \text{for} \quad 0 < x < 2L_o$$

So at $\bar{x}_f$

$$\boxed{\bar{F} = -2K\left[\left(d - \frac{2K}{m}\Delta t^2 (d - L_o)\right) - L_o\right]\hat{x}}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

Problem 4 [25 pts]

The US Penny is made of zinc and has a mass of 2.5 g, a diameter of 1.905 cm, and an average thickness of 1.228 mm. The density of zinc is 7140 $\frac{\text{kg}}{\text{m}^3}$ and its atomic mass is 65.4 amu = 65.4 $\frac{\text{g}}{\text{mol}}$.

A. [5 pts] Calculate the mass of single zinc atom.

$$65.4 \frac{g}{mol} \left(\frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} \right) \left(\frac{1 kg}{1000 g} \right) = 10^{-25} \frac{kg}{atom}$$

GRADING

-1 Clerical
-2 Unit Conversion

$$\boxed{m_{atom} = 10^{-25} \text{ kg}}$$

B. [5 pts] In the cubic lattice of our ball and spring model, determine the diameter of zinc atom.

A volume V of zinc weighs ρV. Therefore, there are $\frac{\rho V}{m_{atom}}$ zinc atoms in a volume V. Therefore, the Volume of one atom is

$$V_{atom} = \frac{V}{\left(\frac{\rho V}{m_{atom}}\right)} = \frac{m_{atom}}{\rho} = \frac{10^{-25} \text{ kg}}{7140 \frac{kg}{m^3}} = 1.4 \times 10^{-29} \text{ m}^3$$

watch for POE

In our cubic lattice model,

$$V_{atom} = d^3$$

So

$$\boxed{d = (V_{atom})^{1/3} = 2.41 \times 10^{-10} \text{ m}}$$

GRADING

-1 Clerical
-3 for getting wrong V_atom

C. [5 pts] In the cubic lattice of our ball and spring model, determine the stiffness $k_{s,i}$ of the bond between two zinc atoms.

From Eqn Sheet: $Y = \frac{k_{si}}{d}$

So $k_{si} = Yd$

$= \left(1.08 \times 10^{11} \frac{N}{m^2}\right)\left(2.41 \times 10^{-10} m\right)$

$\boxed{k_{si} = 26 \frac{N}{m}}$

GRADING

-1 Clerical
-2 Unit Conversion

D. [5 pts] An African bush elephant with a mass of 5,900 kg steps on the zinc penny. The Young's modulus for zinc is 1.08×10^{11} N/m^2. By what percentage does the penny compress if the elephant presses down on the penny with all of his weight?

given, $Y = \frac{F/A}{\Delta L/L}$ solve for $\frac{\Delta L}{L}$.

$\frac{\Delta L}{L} = \frac{F/A}{Y} = \frac{mg}{YA} = \frac{(5900\,kg)(9.81\,\frac{m}{s^2})}{(1.08 \times 10^{11}\,\frac{N}{m^2})(\frac{\pi}{4}(1.905 \times 10^{-2} m)^2)} = 0.00188$ ← dimensionless

So the penny compresses $\boxed{.188 \text{ percent}}$

GRADING

-1 Clerical
-1 Solved for ΔL
-2 Wrong Area

E. [5 pts] Ten pennies are stacked on top of each other (face to face) and the elephant steps on this stack. By what percentage does the stack of pennies compress if the elephant presses down on them with all of his weight?

The percentage compressed does not change.

GRADING

All or Nothing

This page is for extra work, if needed.

This page is for extra work, if needed.