PHYS 2211 KMR - Test 1 - Spring 2022

Instructions - Please Read Carefully

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed a copy of the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet except for the proctored video conference and Gradescope until time is called.
  - You must work individually and receive no assistance from any other person or resource.
  - **You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the exam is over.**
- Work through all of the problems first, and then scan/upload your solutions after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

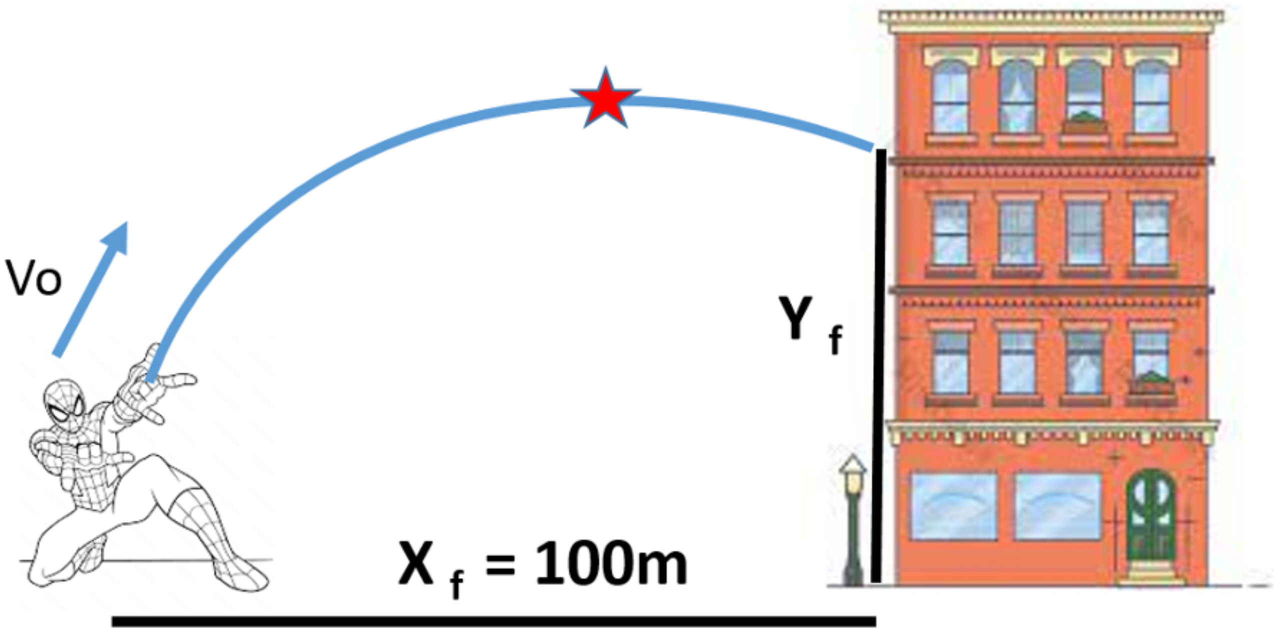
"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

Sign your name on the line above

Spiderman – Q2 in Gradescope [35 pts]

Spiderman shoots his web at a building that is a distance $x_f = 100$ m away, with an initial velocity of $v_0 =< 23, 31, 0 >$ m/s. You can consider Spiderman to be a point at the origin, with the x-axis at the ground. There is no air resistance.



1. [20 pts] What is the **maximum height**, with respect to the ground, of the web on its trajectory to the building? The maximum height is shown in the figure with a red star.

①

$$\vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

$$\overset{0}{\cancel{V_{max,y}}} = V_{i,y} + \frac{-\cancel{m}g}{\cancel{m}}\Delta t_{max}$$

$$0 = V_{i,y} - g\Delta t_{max}$$

$$\Rightarrow \Delta t_{max} = \frac{V_{i,y}}{g}$$

$$= \frac{31\ m/s}{9.8\ m/s^2}$$

$$= 3.16\ s$$

②

$$y_f = y_i + v_{i,y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$y_{max} = V_{i,y}\Delta t_{max} - \frac{1}{2}g\Delta t_{max}^2$$

$$= (31\ m/s)(3.16\ s) - \frac{1}{2}(9.8\ m/s^2)(3.16\ s)^2$$

$$\boxed{y_{max} = 49.03\ m}$$

① 2. [15 pts] At what **height** y_f above the ground does the web hit the building?

How long does it take the web to hit the building?

$$x_f = x_i + v_{avg,x} \Delta t_{building}$$

Because $F_{net,x} = 0$, $v_{avg,x} = v_{x,i}$

$$x_f = v_{x,i} \Delta t_{building}$$

$$\Rightarrow \Delta t_{building} = \frac{x_f}{v_{x,i}}$$

$$= \frac{100 \text{ m}}{23 \text{ m/s}}$$

$$\Delta t_{building} = 4.35 \text{ s}$$

② How high is the net when it hits the building?

$$y_f = y_i + v_{y,i} \Delta t_{building} - \frac{1}{2} g \Delta t_{building}^2$$

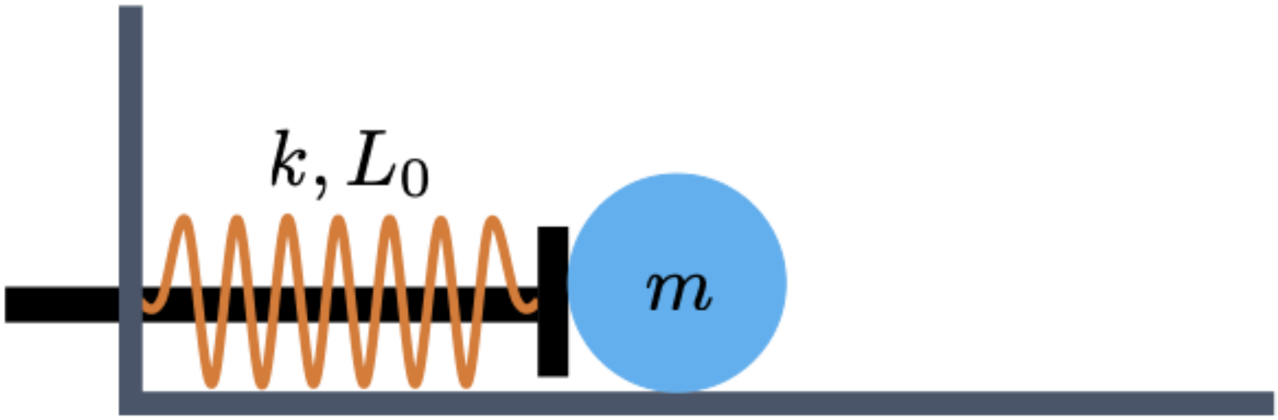
$$= (31 \text{ m/s})(4.35 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(4.35 \text{ s})^2$$

$$\boxed{y_f = 42.13 \text{ m}}$$

Pinball – Q3 in Gradescope [30 pts]

In a pinball machine, you launch a ball of mass $m = 100$ g by pulling on a handle that compresses a horizontal spring. The spring has relaxed length $L_0 = 10$ cm and stiffness $k = 200$ N/m. At $t = 0$ the handle is pulled as far out as it can go, and the spring is maximally compressed to a length of $L = 5$ cm.

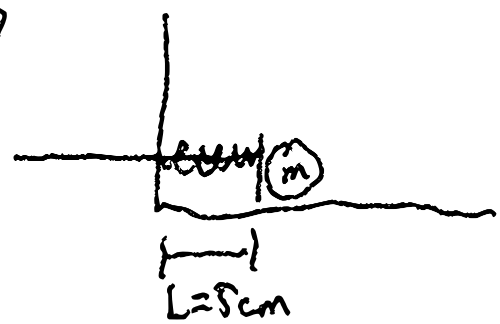
In this problem you can assume that the origin of the coordinate system is located at the fixed end of the spring (on the left side of the image), the $+x$ axis goes to the right, and the $+y$ axis goes upwards. There is no friction or air resistance anywhere in this problem.



1. [10 pts] Determine the **vector net force** on the ball at $t = 0$.

$t=0$

What forces are acting on the ball? (at moment of launch)



cancel out:
- Earth's gravity
- Normal force of Floor

- Spring force $\Rightarrow \vec{F}_{net} = \vec{F}_{spring}$

$$\vec{F}_{spring} = -k_s(|\vec{L}| - L_0)\hat{L}, \quad \vec{L} = L\hat{x}$$

$$= -k_s(L - L_0)\hat{x} \qquad |\vec{L}| = L = 0.05\,m$$

$$\hat{L} = \frac{\vec{L}}{||\vec{L}||} = \hat{x}$$

$$= -(200\ N/m)(0.05m - 0.10\,m)\hat{x}$$

$$= 10\ N\ \hat{x}$$

$$\Rightarrow \boxed{\vec{F}_{net}(t=0) = \langle 10, 0, 0 \rangle\ N}$$

$\to \Delta t = 0.1\,s$

2. [15 pts] Determine the **position** of the ball at $t = 0.1$ s after launching it.

Update the velocity:

$$\vec{v}(t=0.1s) = \cancel{\vec{v}(t=0s)}^{\to 0} + \frac{\vec{F}_{net}(t=0s)}{m}\Delta t$$

$$= \frac{0.1s}{0.1\,kg} < 10, 0, 0 > N$$

$$= < 10, 0, 0 > m/s$$

Update the position:

$$\vec{r}(t=0.1s) = \vec{r}(t=0s) + \vec{v}_{avg}\,\Delta t$$

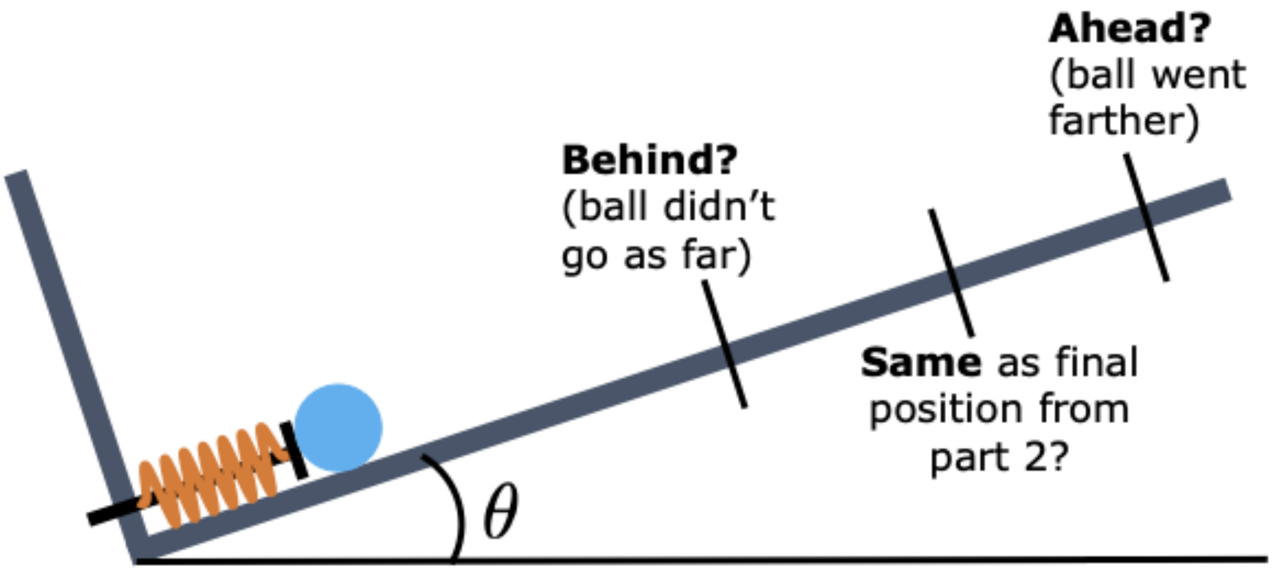
non-constant force

$$\approx \vec{r}(t=0s) + \vec{v}(t=0.1s)\Delta t$$

$$= <0.05, 0, 0> m + (0.1s) <10, 0, 0> m/s$$

$$\boxed{\vec{r}(t=0.1s) = <1.05, 0, 0> m}$$

3. [5 pts] In a real pinball machine, the "floor" in which the ball rests is actually tilted upwards (the ball is higher than the handle) by an angle of $\theta = 7°$. If you were to repeat your calculations but this time taking into account the elevated floor, would the ball's position be ahead, behind, or the same as the position you calculated in the previous part? **Explain your reasoning.** You do not have to redo the calculations.



$\boxed{\text{Behind}}$, When doing iterations to find the new position of the ball, the net force along the direction of motion, i.e. along the tilted floor, is reduced by part of Earth's gravity which no longer cancels with the normal force.

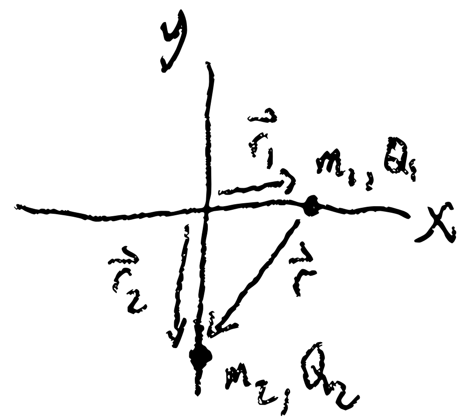
ElectroGrav – Q4 in Gradescope [35 pts]

Two objects are located in the xy plane. They both have the same mass ($m_1 = m_2 = 1000$ kg) and the same positive electric charge ($Q_1 = Q_2 = 1 \times 10^{-8}$ C). Object 1 is located at $\vec{r}_1 =< 3, 0, 0 >$ m and Object 2 is located at $\vec{r}_2 =< 0, -4, 0 >$ m.

1. [10 pts] Find the position vector $\vec{r}$ that points from Object 1 to Object 2, its magnitude $|\vec{r}|$, and its unit vector, $\hat{r}$.

$\vec{r} = \vec{r}_2 - \vec{r}_1$ (see picture)

$= <0, -4, 0> m - <3, 0, 0> m$



$$\boxed{\begin{aligned} \vec{r} &= <-3, -4, 0> m \\ |\vec{r}| &= \sqrt{(-3m)^2 + (-4m)^2} = 5\ m \\ \hat{r} &= \frac{\vec{r}}{|\vec{r}|} = <-\tfrac{3}{5}, -\tfrac{4}{5}, 0> \end{aligned}}$$

2. [10 pts] What is the **vector gravitational force on Object 2** due to Object 1?

$$\vec{F}_{grav,1 on 2} = -\frac{G m_1 m_2}{|\vec{r}|^2} \hat{r}$$

$$= -\frac{(6.7 \times 10^{-11}\ N\ m^2/kg^2)(1000\ kg)(1000\ kg)}{(5m)^2} <-\tfrac{3}{5}, -\tfrac{4}{5}, 0>$$

$$\boxed{\vec{F}_{grav,1 on 2} = <1.61 \times 10^{-6}, 2.14 \times 10^{-6}, 0> N}$$

3. [10 pts] What is the **vector electric force on Object 2** due to Object 1?

$$\vec{F}_{electric,1on2} = \frac{KQ_1 Q_2}{|\vec{r}|^2}\hat{r}$$

$$= \frac{(9\times10^9\ Nm^2/C^2)(1\times10^{-8}\ C)(1\times10^{-8}C)}{(5m)^2}\langle -\tfrac{3}{5}, -\tfrac{4}{5}, 0\rangle$$

$$\boxed{\vec{F}_{electric,1on2} = \langle -2.16\times10^{-8}, -2.88\times10^{-8}, 0\rangle\ N}$$

4. [5 pts] If nothing else is interacting with the two objects, will they move towards each other or away from each other? **Explain your reasoning**. You do not have to do any new calculations.

They will move towards each other, because the two forces acting on each of the particles, gravity and electric force due to the other particle, act in opposite directions (gravity is attractive and like charges repel). But since gravity is stronger (ie has a greater magnitude), the net force on each particle points toward the other. (The reason we can say the same thing is happening to object 1 is because it is identical to object 2 OR Newton's 3rd Law.)