Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

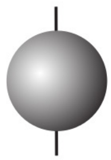
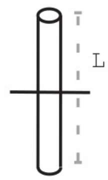
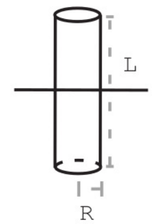
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Test 2 - Summer 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

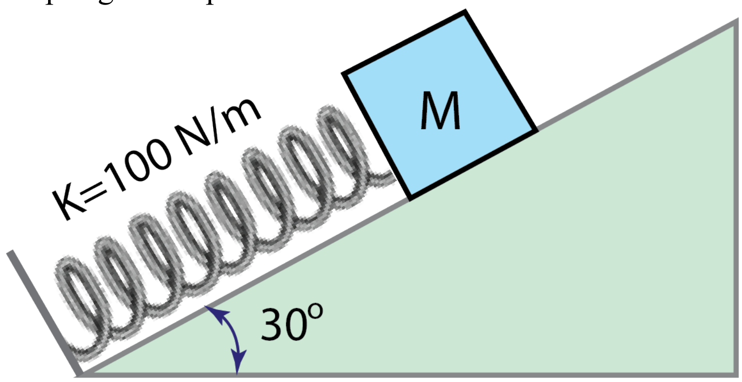
"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

Block at rest on the incline [40 pts]

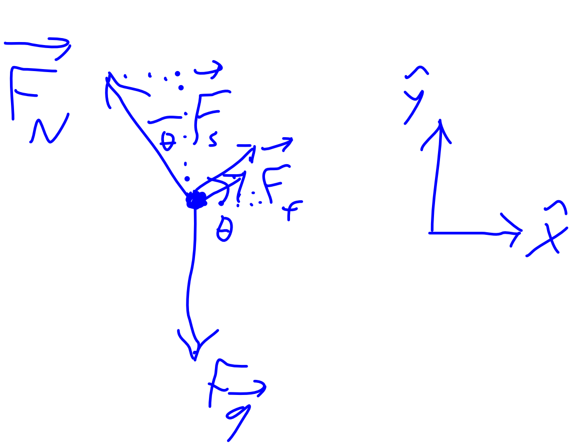
A block of mass $m = 2.04kg$ is at rest on a plane that makes an angle $\theta = 30°$ above the horizontal. The coefficient of static friction between the block and the plane is $\mu_S = 0.462$. The block is attached to the spring with spring constant $k = 100\ N/m$. The other end of the spring is attached to the wall (see the Figure). The spring is compressed in both Part 1 and Part 2.



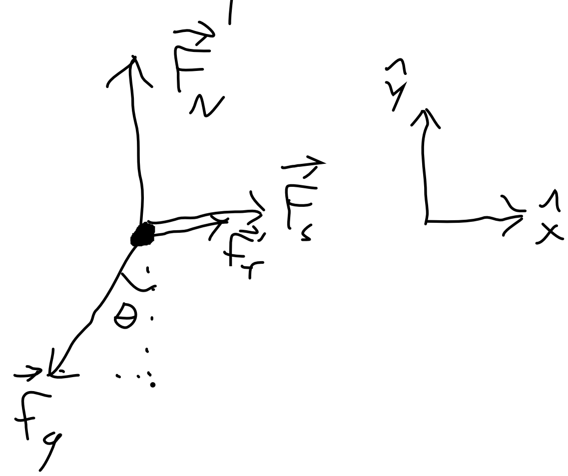
1. Find the minimal spring compression s_{min} (relative to the spring's relaxed length) for which the block is at rest – i.e. the block is prevented from sliding down the plane. In this state the spring has the maximum length (i.e. minimal compression) that still prevents the block from sliding down.

a) [5 pts] Draw a free body diagram of all the forces acting on the block. Show x- and y-axes.

Standard



Ramp



2:42

b) [15 pts] Write appropriate equations and solve for the minimal spring compression s_{min} that ensures the block is at rest.

Ramp $\hat{y}$: $\vec{F}_{net,y} = \vec{F}_N + \vec{F}_{g,y} = \vec{0}$

$$\Rightarrow F_N - mg\cos(\theta) = 0 \Rightarrow \underline{F_N = mg\cos(\theta)}$$

Ramp $\hat{x}$: $\vec{F}_{net,x} = \vec{F}_s + \vec{F}_f + \vec{F}_{g,x} = \vec{0}$

$$\Rightarrow k s_{min} + \mu_s F_N - mg\sin(\theta) = 0$$

$$\Rightarrow s_{min} = \frac{mg}{k}\left[\sin(\theta) - \mu_s \cos(\theta)\right]$$

$$= \frac{(2.04\ kg)(9.81\ m/s^2)}{(100\ N/m)}\left[\sin(30^\circ) - 0.462\cos(30^\circ)\right]$$

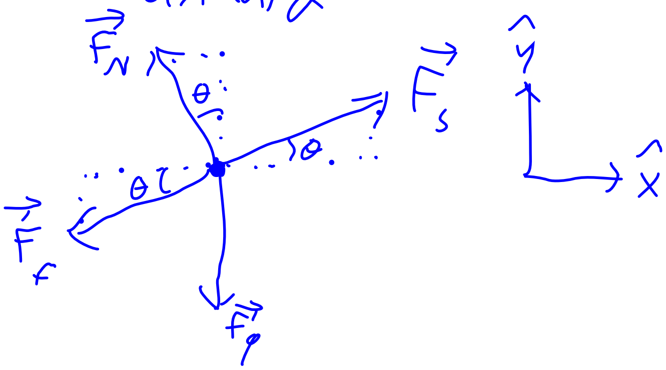
$$\boxed{\approx 0.02\ m = 2\ cm}$$

7:00

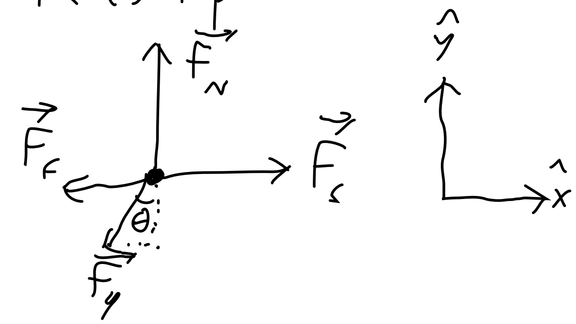
2. Find the maximum spring compression s_{max} (relative to the spring's relaxed length) for which the block is at rest – i.e. about to move up the plane. In this state the spring has the minimal length (i.e. maximum compression) at which the block is still not sliding up.

a) [5 pts] Draw a free body diagram of all the forces acting on the block. Show x- and y- axes.

Standard



Ramp



b) [15 pts] Write appropriate equations and solve for the maximum spring compression s_{max} for which block is still at rest.

Ramp $\hat{y}$: $\vec{F}_{net,y} = \vec{F}_N + \vec{F}_g = \vec{0}$

$\Rightarrow F_N - mg\cos(\theta) = 0 \Rightarrow \underline{F_N = mg\cos(\theta)}$

Ramp $\hat{x}$: $\vec{F}_{net,x} = \vec{F}_s + \vec{F}_f + \vec{F}_{g,x} = \vec{0}$

$\Rightarrow k s_{max} - \mu_s F_N - mg\sin(\theta) = 0$

$\Rightarrow s_{max} = \frac{mg}{k}\left[\sin(\theta) + \mu_s\cos(\theta)\right]$

$= \frac{(2.04\ kg)(9.81\ m/s^2)}{(100\ N/m)}\left[\sin(30°) + 0.462\cos(30°)\right]$

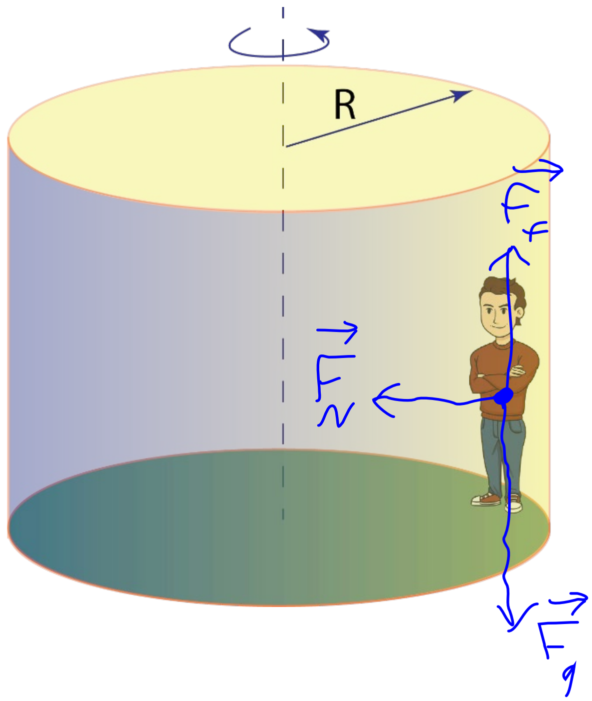
$\boxed{\approx 0.18\ m = 18\ cm}$

12:24

Amusement park ride [30 pts]

An amusement park ride consists of a rotating vertical cylinder of radius R with rough canvas walls. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass M and the period of rotation (i.e. the time for one complete revolution) of the cylinder is T.

Answer all questions in this problem in terms of known quantities R, M, T, and g.



1. [5 pts] On the diagram above, draw and identify the forces on the rider when the system is rotating, and the floor has dropped down.

See diagram

2. [5 pts] Find the speed and the centripetal acceleration of the rider when the cylinder is rotating. State what force provides that acceleration.

$$v = \frac{\text{distance}}{\text{time}} = \boxed{\frac{2\pi R}{T}}$$

$$F_{cent} = M a_{cent} = \frac{M v^2}{R}$$

$$\Rightarrow a_{cent} = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \boxed{\frac{4\pi^2 R}{T^2}}$$

Normal force provides centripetal acceleration

15:32

3. [5 pts] Find the upward force that keeps the rider from sliding down when the floor is dropped down and state what provides that force.

Friction provides upward force

$$\vec{F}_{net,y} = \vec{F}_f + \vec{F}_g = 0$$

$$\Rightarrow F_f - Mg = 0 \Rightarrow \boxed{F_f = Mg}$$

4. [10 pts] What must be the minimum coefficient of static friction between the rider and the wall of the cylinder μ so that the rider does not slide down?

$$F_f = \mu F_N$$

$$\Rightarrow \mu = \frac{F_f}{F_N} = \frac{Mg}{M\left(\frac{4\pi^2 R}{T^2}\right)} = \boxed{\frac{g T^2}{4\pi^2 R}}$$

18:52

5. [5 pts] At the same park ride, would a rider of twice the mass slide down the wall? Explain your answer.

No, a rider of twice the mass would not slide down because they experience the same accelerations.

20:14

Sliding Block [30 pts]

A block of mass M starts from rest and slides down a frictionless plane inclined at angle θ with the horizontal as shown in Figure 1. After sliding a distance L, the block strikes a spring. The block comes to rest after compressing the spring distance s (see Figure 2).

Answer all questions in this problem in terms of given known quantities M, θ, L, s, and g.

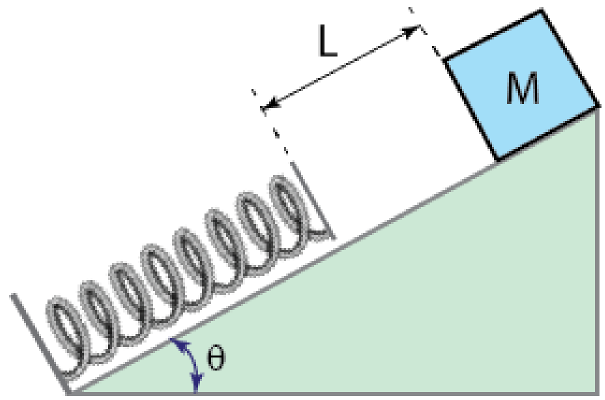


Figure 1

1. [8 pts] What is the total energy of the system at the initial state in Figure 1? Choose appropriate reference level for the gravitational potential energy.

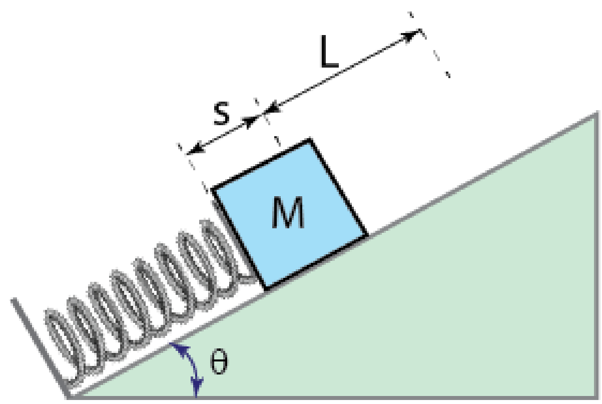


Figure 2

Reference point is final position

Block starts at rest $\Rightarrow$ $K_i = 0$

Spring relaxed $\Rightarrow$ $U_{s,i} = 0$

Block starts at height $(s+L)\cos(\theta) \Rightarrow$ $U_{g,i} = Mg(s+L)\sin(\theta)$

$E_i = K_i + U_{s,i} + U_{g,i}$

$\boxed{= Mg(s+L)\sin(\theta)}$

25:08

2. [8 pts] What is the total energy of the system at the final state in Figure 2?

Block ends at rest $\Rightarrow$ $K_f = 0$

Block ends at zero point $\Rightarrow$ $U_{g,f} = 0$

Spring compressed by s $\Rightarrow$ $U_{s,f} = \frac{k}{2} s^2$

$$E_f = K_f + U_{g,f} + U_{s,f} = \boxed{\frac{k}{2} s^2}$$

3. [14 pts] Find the spring force constant k.

No external work / energy conservation:

$$\Delta E = W = 0 \Rightarrow E_i = E_f$$

$$\Rightarrow Mg(s+L)\sin(\theta) = \frac{k}{2} s^2$$

$$\Rightarrow \boxed{k = \frac{2Mg}{s^2}(s+L)\sin(\theta)}$$

29:38