PHYS 2211 MNR - Test 3 - Fall 2022

Please clearly print your name & GTID in the lines below

Name: ______________________ GTID: ______________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code, I have not given or received unauthorized aid on this test."

KEY

Sign your name on the line above

Thermal Energy [20 pts]

A car with mass $M = 1500$ kg travels at a speed of $v_i = 90$ km/hr, then it brakes to a stop. In the process of braking, the brakes' steel drums get hotter. If we need the brakes' temperature to not increase by more than 120°C, how much mass m should the brakes have? The heat capacity of steel is $C = 0.47$ J/(g°C). You can assume that the temperature of the rest of the car and the air around it does not change. You can ignore the spinning of the car's wheels and any friction from the pavement.

System: car (including brakes)
surr: air, pavement

$$\Delta E_{sys} = W_{ext} = 0 = \Delta KE_{trans} + \Delta E_{thermal,\,brakes}$$

$$\Rightarrow \quad \Delta E_{thermal} = -\Delta KE_{trans}$$

$$mC\Delta T = -\left[\frac{1}{2}M\left(\cancel{v_f^2}^{\,0} - v_i^2\right)\right]$$

$$m = \frac{\frac{1}{2}Mv_i^2}{C\Delta T}$$

$$\Delta T \leq 120°C \quad \Rightarrow \quad \boxed{m \geq \frac{\frac{1}{2}Mv_i^2}{C\Delta T} = 8.31\text{ kg}}$$

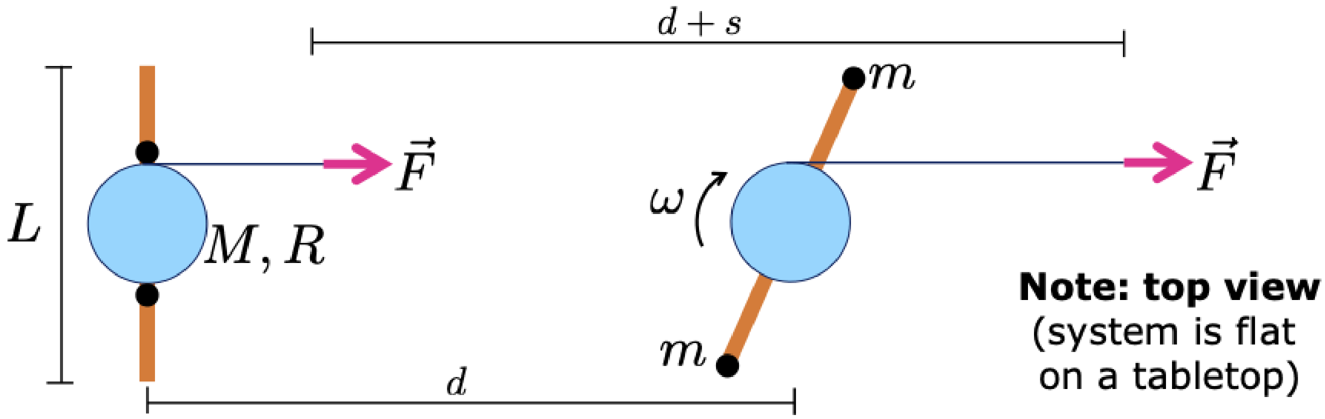
$$v_i = \frac{90\,\cancel{km}}{\cancel{hr}}\bigg|\frac{1000\text{ m}}{1\,\cancel{km}}\bigg|\frac{1\,\cancel{hr}}{3600\text{ s}} = 25\text{ m/s}$$

$$C = \frac{0.47\text{ J}}{\cancel{g}°C}\bigg|\frac{1000\,\cancel{g}}{1\text{ kg}} = 470\frac{\text{J}}{\text{kg}°C}$$

Rotations [30 pts]

A system consists of a uniform solid cylinder of mass M, radius R, and negligible height, attached to the middle of a massless rod of length L. Two small point masses m rest on the rod next to the cylinder. The entire apparatus sits at rest on top of a frictionless table (see the left side of the figure).

A string is wound up around the cylinder, and you start pulling on the string with a constant force of magnitude F. The apparatus slides to the right along the tabletop. When the center of the cylinder has moved a distance d, a length of string s has come off the cylinder, the cylinder+rod is spinning, and the two point masses have slid along the rod and got stuck at the ends of the rod (see the right side of the figure).



1. [10 pts] Use the point-particle system to find what is the speed v of the apparatus in the final state (right side of the figure).

$$\Delta E_{sys,pt} = \Delta KE_{trans} = W_F$$

$$\vec{F} \cdot \vec{d}_{CM} = F\hat{x} \cdot d\hat{x} = Fd$$

$$\frac{1}{2}(M+2m)(v^2 - \cancel{v_i^2}^{\,0}) = Fd$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2Fd}{M+2m}}}$$

2. [20 pts] Use the extended system to find what is the angular speed ω of the apparatus in the final state (right side of the figure).

$$\Delta E_{sys,\,extended} = \Delta KE_{trans} + \Delta KE_{rot} = W_F$$

$$\frac{1}{2}(M+2m)v^2 + \frac{1}{2}I_f\omega^2 = F(d+s)$$

$$\vec{F}\cdot\vec{d}_{\text{pt of application}} = F\hat{x}\cdot(d+s)\hat{x} = F(d+s)$$

$$\Rightarrow \omega = \sqrt{\frac{F(d+s) - \frac{1}{2}(M+2m)v^2}{\frac{1}{2}I_f}}$$

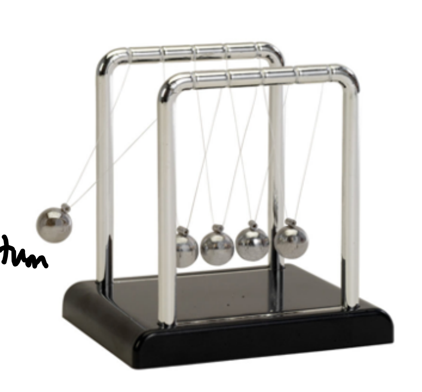
$$I_f = \underbrace{\frac{1}{2}MR^2}_{\text{disk/cyl. w/ ~0 height}} + \underbrace{2m\left(\frac{L}{2}\right)^2}_{\text{2 pt masses}}$$

$$= \sqrt{\frac{2F(d+s) - \cancel{(M+2m)}\left(\frac{2Fd}{\cancel{M+2m}}\right)}{\frac{1}{2}MR^2 + 2m\left(\frac{L}{2}\right)^2}}$$

$$\boxed{\omega = \sqrt{\frac{4Fs}{MR^2 + mL^2}}}$$

Newton's Cradle [20 pts]

A Newton's Cradle is a device where 5 steel balls of the same mass m are suspended by massless strings next to each other (see pictures). When balls on one side get lifted and released, they hit the others and then the outside balls on the other side get kicked up, while the inner balls stay in place. The physical mechanism that causes this is a shock wave that travels through the balls.



conservation of momentum

Given that the collisions between the balls can be considered elastic, and there is no air drag and no friction, which of the following scenarios are possible? Briefly explain using what you know about elastic collisions.

conservation of kinetic energy

1. [5 pts] One ball with speed v_0 hits the others, then one ball gets kicked up on the other side. Possible OR not possible? Briefly explain.

Possible. The kicked up ball leaves with the same velocity as the initially moving ball when it makes contact with the other balls. This satisfies both conservation of momentum and kinetic energy.

2. [5 pts] One ball with speed v_0 hits the others, then two balls get kicked up on the other side. Possible OR not possible? Briefly explain.

Not possible. Conservation of momentum and kinetic energy cannot be jointly satisfied. See next page for proof.

3. [5 pts] Two balls each with speed v_0 hit the others, then one ball gets kicked up on the other side. Possible OR not possible? Briefly explain.

Not possible. Same reason as #2.

4. [5 pts] Two balls each with speed v_0 hit the others, then two balls get kicked up on the other side. Possible OR not possible? Briefly explain.

Possible. The kicked up balls leave with the same velocities as the initially moving balls.

Note: If #1 and #4 are correctly shown, it can be argued that Newton's Cradle being a deterministic system thus forbids #2 and #3 from occurring.

Proofs of #1 – #4 on previous page

1. (1) = initially moving ball ; (2) = kicked up ball

$$\vec{p}_i = \underbrace{m\vec{v}_i}_{(1)} + \underbrace{\vec{0}}_{(2)} = \underbrace{\vec{0}}_{(1)} + \underbrace{m\vec{v}_i}_{(2)} = \vec{p}_f \quad \checkmark$$

$$E_i = \frac{1}{2}mv_0^2 + 0 = 0 + \frac{1}{2}mv_0^2 = E_f \quad \checkmark$$

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2. $\vec{p}_i = mv_0\hat{x} + \vec{0} = mv_{f,1}\hat{x} + mv_{f,2}\hat{x} = \vec{p}_f$

$$v_0 = v_{f,1} + v_{f,2}$$

needs to be 0

$$v_0^2 = v_{f,1}^2 + v_{f,2}^2 + 2v_{f,1}v_{f,2} \quad (*)$$

$$E_i = \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}m\left(v_{f,1}^2 + v_{f,2}^2\right) = E_f$$

$$v_0^2 = v_{f,1}^2 + v_{f,2}^2 \quad (**)$$

(*) and (**) true requires $v_{f,1} = 0$ or $v_{f,2} = 0$, i.e. only 1 ball leaves

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3. $\vec{p}_i = \vec{p}_f \Rightarrow 2(mv_0) = mv_f \Rightarrow v_0^2 = \frac{1}{4}v_f^2 \quad (*)$

$E_i = E_f \Rightarrow 2\left(\frac{1}{2}mv_0^2\right) = \frac{1}{2}mv_f^2 \Rightarrow v_0^2 = \frac{1}{2}v_f^2 \quad (**)$

only sol'n is $v_0 = v_f = 0$, ie no motion

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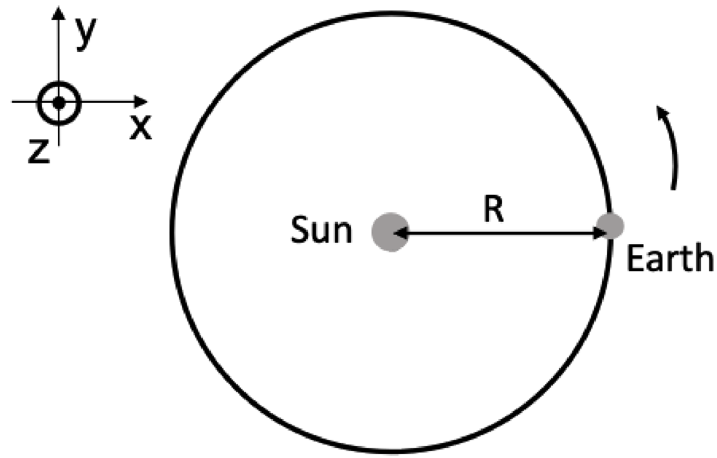
4. same as #1 but with 2 balls ; $\vec{p}_i = \vec{p}_f = 2m\vec{v}_i$, $E_i = E_f = 2\left(\frac{1}{2}mv_0^2\right)$

Earth's Orbit [30 pts]

1. [10 pts] Calculate the translational angular momentum of the Earth, considered as a point particle, relative to the center of the Sun.

   The mass of the Earth is $m = 6 \times 10^{24}$ kg, the mass of the Sun is $M_s = 2 \times 10^{30}$ kg, and the distance between the Earth and the Sun is $R = 1.5 \times 10^{11}$ m.

   You can assume that the Earth's orbit is circular in the xy plane, and the Earth moves counterclockwise in its orbit as shown in the diagram. Your answer should be a vector with correct units.



$$\vec{L}_{\text{trans, Earth rel to Sun}} = \vec{r}_{\text{Sun to Earth}} \times \vec{p}_{\text{Earth, com}}$$

(see bottom of page)

$$= R\hat{x} \times m\, v_{\text{Earth}}\, \hat{y}$$

$$= R m \sqrt{\frac{GM_s}{R}}\, \hat{z}$$

$$\boxed{\vec{L}_{\text{trans, Earth rel to Sun}} = m\sqrt{GM_s R}\, \hat{z} = 2.69 \times 10^{40}\ \text{kg m}^2/\text{s}\ \hat{z}}$$

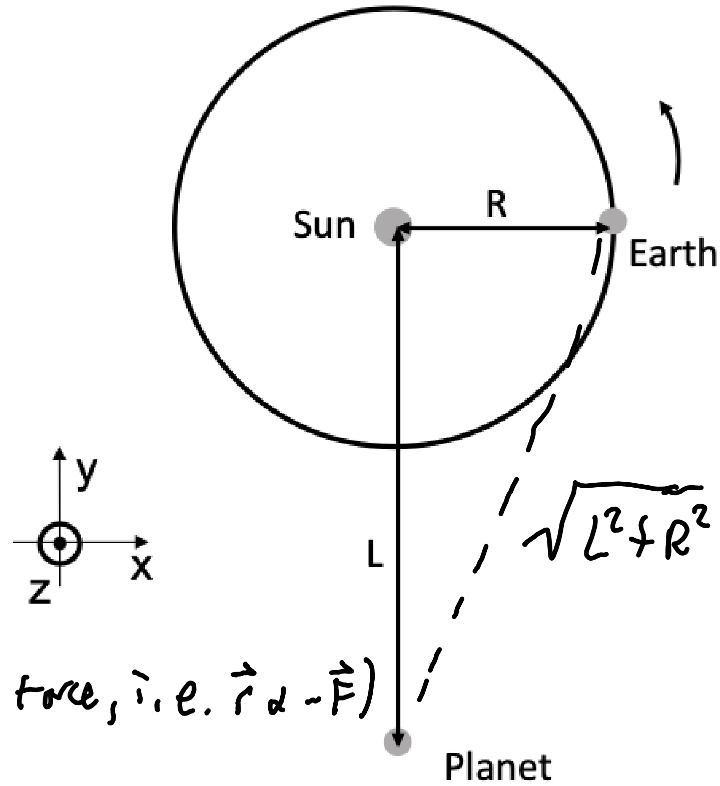
To Find v_{Earth}

$$|\vec{F}_{net,\perp}| = \frac{\cancel{m} v_{\text{Earth}}^2}{\cancel{R}} = \frac{GM_s \cancel{m}}{R^{\cancel{2}}} = |\vec{F}_g|$$

$$\Rightarrow v_{\text{Earth}} = \sqrt{\frac{GM_s}{R}}$$

2. [20 pts] A giant rogue planet with mass M_p approaches the Solar System. At one particular instant, the Earth is at distance R from the sun and the rogue planet is at distance L from the Sun in a different direction. The Sun, Earth, and rogue planet make a right triangle, as shown in the diagram.

What is the total (vector) torque exerted on Earth by the Sun and the rogue planet, with respect to the center of the Sun? Since this is a symbolic calculation, use the variables given in the text (parts 1 and 2) for the necessary masses and distances. You can consider the Earth to be a point mass.



(central force, i.e. $\vec{r} \propto -\vec{F}$)

$$\vec{\tau}_{\text{net, on Earth}} = \cancel{\vec{\tau}_{\text{Sun on Earth}}}^{\,0} + \vec{\tau}_{\text{rogue planet on Earth}}$$

$$= \vec{r}_{\text{origin/Sun to Earth}} \times \vec{F}_{\text{rogue planet on Earth}}$$

$$= R\hat{x} \times \left(- \frac{G M_p m}{(L^2+R^2)^{3/2}} \vec{r}_{\text{rogue planet to Earth}} \right)$$

$$\boxed{\vec{\tau}_{\text{net, on Earth}} = - \frac{R G M_p m L}{(L^2+R^2)^{3/2}} \hat{z}}$$

$\vec{r}_{\text{rogue planet to Earth}} = \langle R, L, 0 \rangle$

$= R\hat{x} + L\hat{y}$

$\hat{x} \times \hat{x} = 0$

$\hat{x} \times \hat{y} = \hat{z}$