Week 3 Lecture 1

Constant Forces

Topics for today

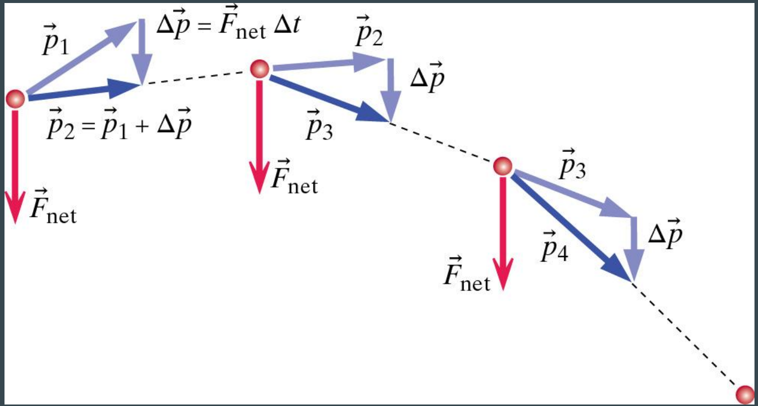
1. Constant forces
2. Integration

By the end of class you will

1. Determine the position, for all time, of any system with a constant force

Predicting the future iteratively

- Divide up the total time into smaller intervals and iteratively apply our update procedure
  - Apply the Momentum Principle
    - Idealize: Identify the most important interactions and ignore the rest
    - Calculate the net force
  - Update Momentum
  - Estimate the average velocity
    - Arithmetic for constant forces
    - Final velocity for non-constant forces
  - Update the Position of the system
  - Repeat!




```
t = 0
deltat=1e-4
while t < t_final
    Fnet = vector(0,-ball.m*g,0)
    ball.p = ball.p + Fnet*deltat
    v_avg = (ball.p + p_init_ball)/(2*ball.m)
    ball.pos = ball.pos + v_avg*deltat
    t = t + deltat
```

Week 3 Lecture 2

Non-Constant Forces

Topics for today

1. Non-constant forces
2. Hooke's Law
3. Convergence

By the end of class you will

1. Calculate the vector spring force
2. Prediction motion with a spring force
3. Know how to choose a time step

Predicting the motion of the Earth

- Suppose that we are given information about the position, velocity and net force on the Earth at a given instant.
  - Using this info, calculate the new position of the Earth three months later
- Given initial conditions

$$\vec{r}_i = \langle 1.5 \times 10^{11}, 0, 0 \rangle \ m$$

$$\vec{v}_i = \langle 0, 3 \times 10^4, 0, 0 \rangle \ m/s$$

$$\vec{F}_{net} = \langle -3.6 \times 10^{22}, 0, 0 \rangle \ N$$

Prediction the motion of the Earth cont.

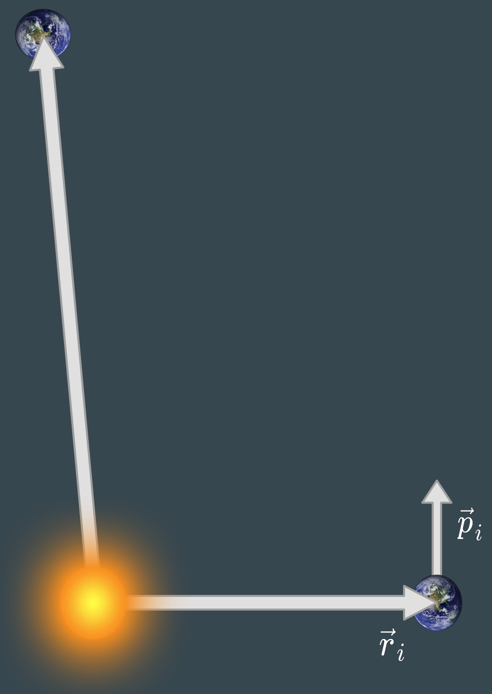
- System: The Earth, Surroundings: The Sun, Time Interval: 7.8e6 seconds

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\vec{v}_{avg} = \frac{\vec{p}_f + \vec{p}_i}{2m}$$

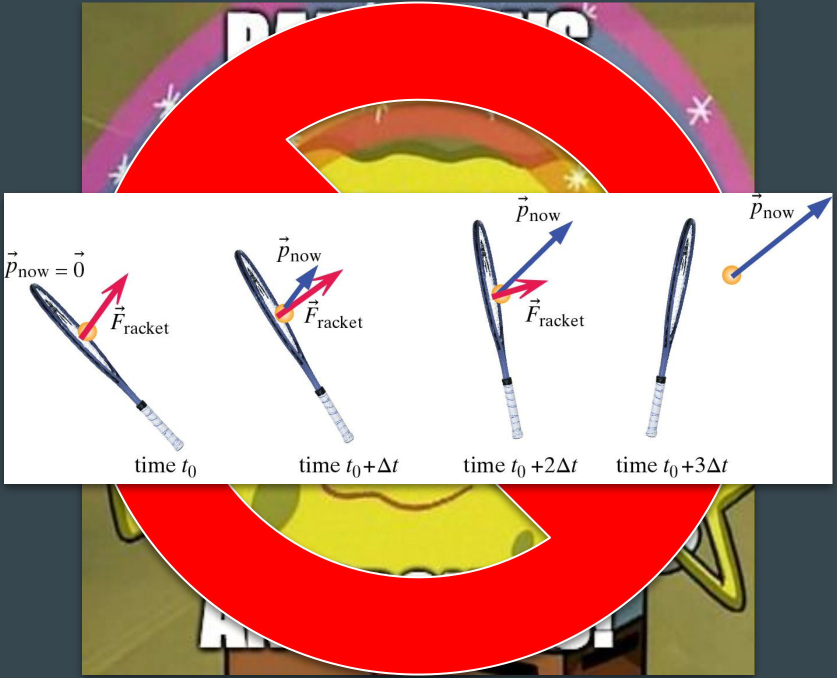
$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{r}_f = \langle -3.1 \times 10^{10}, 2.3 \times 10^{11}, 0 \rangle \ m$$



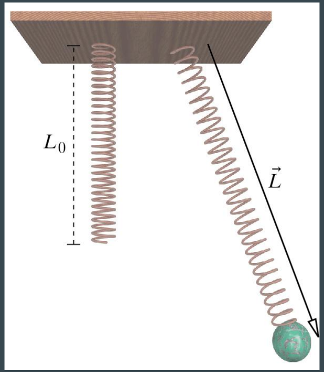
Updating the force - physics isn't all sunshine and ponies

- Iteratively apply our update procedure
  - Apply the Momentum Principle
    - Idealize: Identify the most important interactions and ignore the rest
    - Update the net force!
  - Update Momentum
  - Estimate the average velocity
  - Update the Position of the system
  - Repeat!
- For the iterative prediction to work we need a model to quantify forces given information about our system and surroundings



A simple non-constant force

- Before modelling gravity start with a simple spring
- Robert Hooke (1678 AD) and the ideal spring force
  - Published as an anagram: ceiiinosssttuv - "Ut tensio, sic vis" meaning "As the extension, so the force."

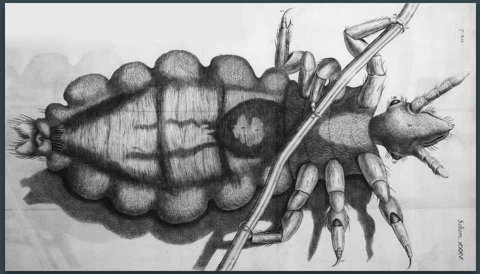


$$\vec{F} = -k_s s \hat{L}$$

$$s = |\vec{L}| - L_0$$

1. L is the current length of spring and L_0 is the relaxed spring
2. The stiffness k_s is a property of the spring

- Considered the Leonardo of England
  - Made an enemy of Newton!

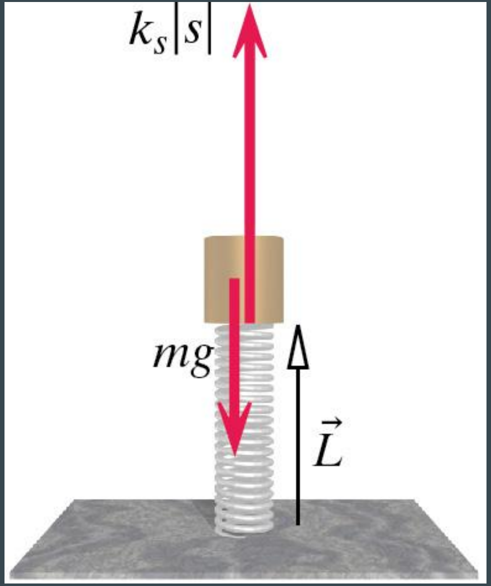


Example: Spring Motion

- Calculate iteratively (three steps), the position of a block attached to a compressed spring after 0.3 seconds. The relaxed length of the spring is 20 cm, the spring stiffness is 8 N/m, the initial length is 10 cm and the mass of the block is 0.06 kg.
  - System: Block
  - Surroundings: Earth + Spring
  - Time step: 0.1 s

$$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{Earth}$$

$$\vec{F}_{net} = \langle 0, -mg - k_s(|\vec{L}| - L_0), 0 \rangle$$



Example: Spring Motion - First time step

- Update momentum and position

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

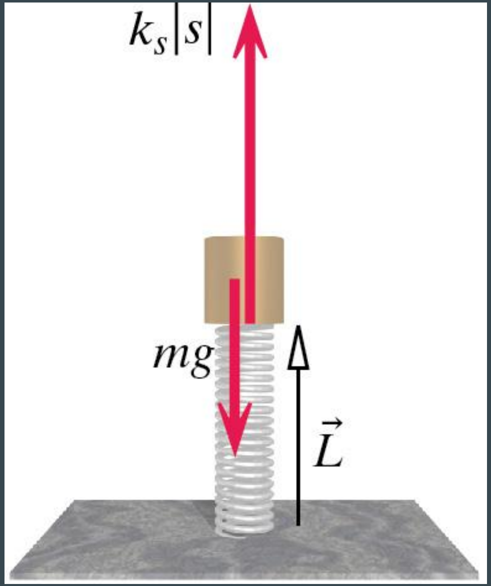
$$\vec{v}_{avg} = \frac{\vec{p}_f}{m} \Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{r}_f = \langle 0, 0.135, 0 \rangle \ m$$

- Start next time step: update force

$$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{Earth}$$

$$\vec{F}_{net} = \langle 0, -mg - k_s(|\vec{r}_f| - L_0), 0 \rangle$$



Example: Spring Motion - Second time step

- Update momentum and position

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

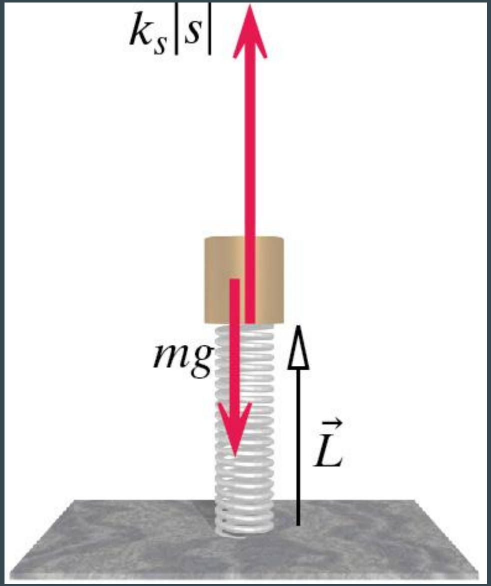
$$\vec{v}_{avg} = \frac{\vec{p}_f}{m} \Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{r}_f = \langle 0, 0.159, 0 \rangle \ m$$

- Start next time step: update force

$$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{Earth}$$

$$\vec{F}_{net} = \langle 0, -mg - k_s(|\vec{r}_f| - L_0), 0 \rangle$$



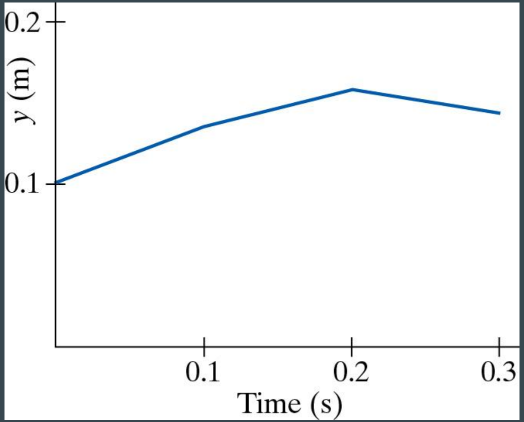
Example: Spring Motion - Third time step

- Update momentum and position

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\vec{v}_{avg} = \frac{\vec{p}_f}{m} \quad \Rightarrow \quad \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{r}_f = \langle 0, 0.139, 0 \rangle \ m$$



- This would be much faster in Glowscript
  - http://www.glowscript.org/#/user/ed/folder/My_Programs/program/CH2-Block-spring/edit
  - As we decrease the time step how does our solution change?
    - Convergence!
  - How can we pick a good first guess for a time step?

How to pick a good time step

- For the vertical spring example we found that the solution didn't change much once we picked a small enough time step
  - http://www.glowscript.org/#/user/ed/folder/My_Programs/program/CH2-Block-spring/edit
  - This is called "convergence"
- Why not just start with a very small time step?
- How to make a decent first guess?
  - Pick a time step so that the impulse is of the same order as the initial momentum
    - Or just the inverse of the net force

$$\Delta t \approx |\vec{p}_i|/|\vec{F}_{net}|$$

- What about the average velocity estimate?