Q 5

"energy conservation" "momentum conservation"

Elastic Collision means $\sum \Delta E = 0$ and $\sum \Delta \vec{p} = 0$

⟨Initial⟩

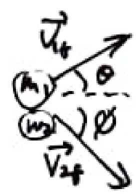
$m_1 \longrightarrow$ $\quad m_2$

$V = \langle V_{1i}, 0, 0 \rangle$ $\quad V = \langle 0, 0, 0 \rangle$

Total momentum $= \langle m_1 \cdot V_{1i}, 0, 0 \rangle$

Total kinetic energy $= \frac{1}{2} m_1 V_{1i}^2$

⟨Final⟩



$V = \langle V_{1f} \cos\theta, V_{1f} \sin\theta, 0 \rangle$

$V = \langle V_{2f} \cos\phi, -V_{2f} \sin\phi, 0 \rangle$

Total momentum $= \langle m_1 \cdot V_{1f} \cos\theta, m_1 \cdot V_{1f} \sin\theta, 0 \rangle + \langle m_2 \cdot V_{2f} \cos\phi, -m_2 V_{2f} \sin\phi, 0 \rangle$

$= \langle m_1 V_{1f} \cos\theta + m_2 V_{2f} \cos\phi, m_1 \cdot V_{1f} \sin\theta - m_2 V_{2f} \sin\phi, 0 \rangle$

Total Energy $= \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

Therefore, now we have the following equations,

$$\begin{cases} m_1 V_{1i} = m_1 V_{1f} \cos\theta + m_2 V_{2f} \cos\phi \\ 0 = m_1 V_{1f} \sin\theta - m_2 V_{2f} \sin\phi \\ \frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 \end{cases}$$

this will be usful.

$m_1 V_{1f} \sin\theta = m_2 V_{2f} \sin\phi$

$$\boxed{\frac{m_1 V_{1f}}{m_2 V_{2f}} \sin\theta = \sin\phi} \quad \cdots (1)$$

These will be usful.

$$\begin{cases} m_1^2 V_{1i}^2 = m_1^2 V_{1f}^2 \cos^2\theta + m_2^2 V_{2f}^2 \cos^2\phi + 2 m_1 m_2 V_{1f} V_{2f} \cos\theta \cos\phi \\ m_1^2 V_{1i}^2 = m_1^2 V_{1f}^2 + m_1 m_2 V_{2f}^2 \end{cases}$$

(Subtraction)

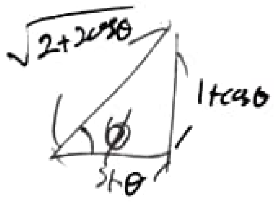
$$0 = m_1^2 V_{1f}^2 \sin^2\theta + V_{2f}^2 (m_1 m_2 - m_2^2 \cos^2\phi) - 2 m_1 m_2 V_{1f} V_{2f} \cos\theta \cos\phi$$

$$
\begin{aligned}
&\left(m_1 V_{1f} = m_1 V_{1f} \cos\theta + m_2 V_{2f} \cos\phi\right) \sin\phi \\
+ &\left(0 = m_1 V_{1f} \sin\theta - m_2 V_{2f} \sin\phi\right) \cos\phi
\end{aligned}
$$

$$\cancel{m_1 V_{1f}} \sin\phi = \cancel{m_1 V_{1f}} \cos\theta \cdot \sin\phi + \cancel{m_1 V_{1f}} \sin\theta \cdot \cos\phi$$

$$\Leftrightarrow \quad (1-\cos\theta)\cdot\sin\phi = \sin\theta\cdot\cos\phi .$$

$$\Leftrightarrow \quad \tan\phi = \frac{1+\cos\theta}{\sin\theta}$$



which means, $\boxed{\sin\phi = \dfrac{1+\cos\theta}{\sqrt{2+2\cos\theta}}}$ $\cdots(2)$

Use (1) and (2),

$$\frac{m_1 V_{1f}}{m_2 V_{2f}} = \frac{1+\cos\theta}{\sin\theta\sqrt{2+2\cos\theta}} = \frac{\sqrt{1+\cos\theta}}{\sqrt{2}\cdot\sin\theta} = \frac{1}{\sqrt{2}\cdot\sqrt{1-\cos\theta}}$$

Thus, $\sqrt{1-\cos\theta} = \dfrac{1}{\sqrt{2}}\cdot\dfrac{m_2 V_{2f}}{m_1 V_{1f}} \Leftrightarrow 1-\cos\theta = \dfrac{1}{2}\cdot\dfrac{m_2^2 V_{2f}^2}{m_1^2 V_{1f}^2}$

$$\therefore \boxed{\cos\theta = 1 - \frac{1}{2}\cdot\frac{m_2^2 V_{2f}^2}{m_1^2 V_{1f}^2}} \cdots(3)$$

Now, it's time to get the answers, using (3), $\boxed{\theta = \cos^{-1}\left(1 - \dfrac{1}{2}\cdot\dfrac{m_2^2 V_{2f}^2}{m_1^2 \cdot V_{1f}^2}\right)}$

Using (1) and the value of θ, $\boxed{\phi = \sin^{-1}\left(\dfrac{m_1 V_{1f}}{m_2 V_{2f}} \sin\left(\cos^{-1}\left(1 - \dfrac{1}{2}\dfrac{m_2^2 V_{2f}^2}{m_1^2 \cdot V_{1f}^2}\right)\right)\right)}$ $\therefore$ answers!!