# Week 14

Quantization of Angular Momentum

## Topics for this week

1. Quantization of Angular Momentum

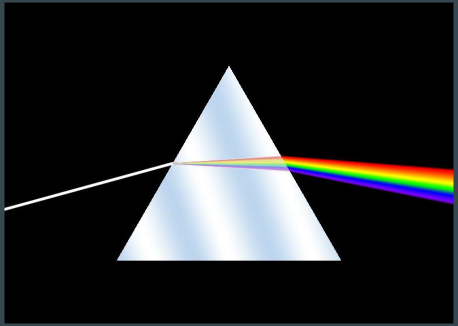
## By the end of the week

1. Be able to predict the spectral lines of hydrogen

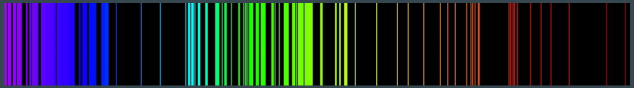
# Spectral Lines

- In the 1670 Isaac Newton discovers you can separate a beam of visible light into its component energies by passing a narrow beam of light through a prism
  - Splits light into 7 colors with a prism and coins the word “spectrum”
- In 1835 Charles Wheatstone found that different metals could be easily distinguished by the different bright lines in the emission spectra of their sparks
- In 1854 David Alter and Anders Jonas Ångström observed the same result for different gases

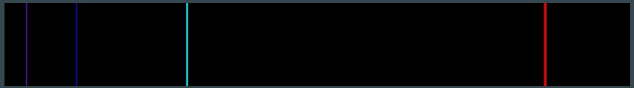
White Light



Iron



Hydrogen Gas



# An atomic puzzle

- Why does the emission spectra of hydrogen look so different from the spectra of white light?
- In 1913 Niels Bohr explained the emission spectra of hydrogen by improving on the Rutherford model of the atom
    - Rutherford's planetary model predicted a continuous spectrum of light from hydrogen
    - Bohr corrected for this by proposing that the translational angular momentum of the electron be quantized
        - N is an integer 1, 2, 3, etc.
        - $\hbar$ is planck constant $1.05\text{x}10^{-34}$ Js

$$|\vec{L}| = N\hbar$$

# The radius and energy of hydrogen

- The net force on the electron is the electric force
  - The electron is only allowed to line in specific orbits
  - Louis de Broglie interpreted this to mean that the electron had a wavelength that must fit within the circumference of the orbit

$$\frac{N^2\hbar^2}{mr^3} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \quad \Longrightarrow \quad r = \frac{4\pi\epsilon_0\hbar^2}{me^2}N^2$$

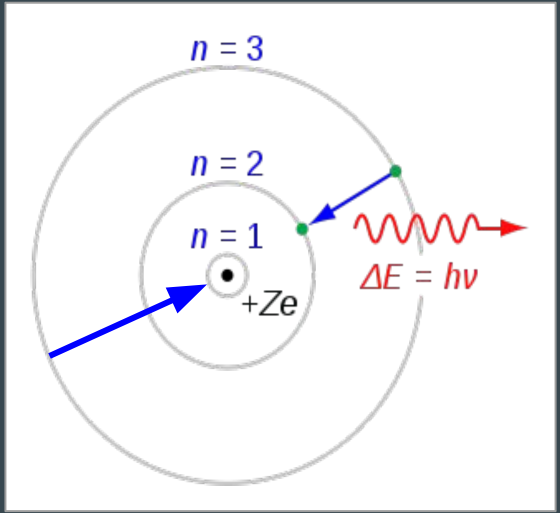
- The total energy of the atom is the sum of kinetic and potential
  - The total energy is quantized!

$$E_{tot} = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \quad \Longrightarrow \quad E_{tot} = -\frac{\left(\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{me^2}{\hbar}\right)}{N^2} = \frac{-13.6\ eV}{N^2}$$

# Changes in energy for the atom

- What is the change in energy as the electron moves from one state to another?
  - If the atom moves from a larger orbit to a smaller it loses energy in the form of a photon
  - Moving from a smaller orbit to a larger requires a specific gain in energy

$$E_3 - E_1 = \frac{-13.6\ eV}{3^2} - \frac{-13.6\ eV}{1^2} = 12.09\ eV$$

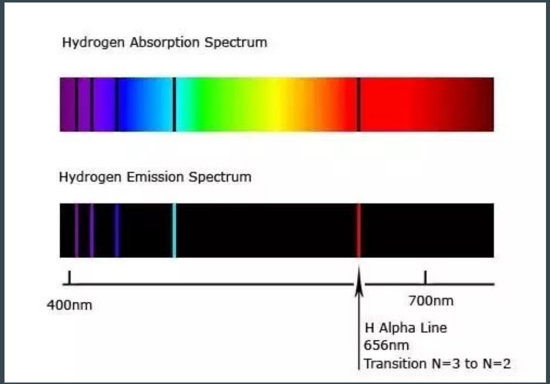


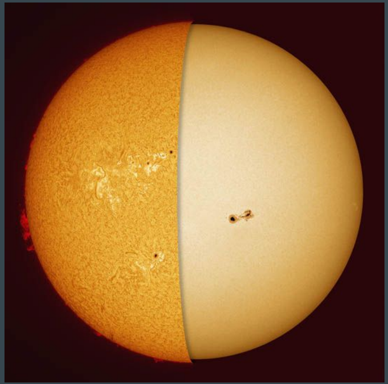

- The energy of the emitted photon is related to planck's constant

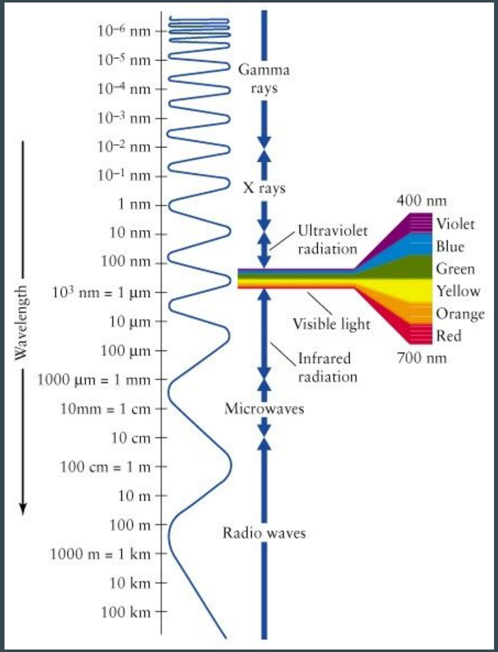
$$E = \frac{2\pi\hbar c}{\lambda} \Rightarrow \lambda_{12.09\ eV} = 102\ nm$$

# The hydrogen spectrum

- We can stimulate the emission of photos by exciting the hydrogen gas with electron current
  - The electron current gives up some of its kinetic energy to the electron in the hydrogen atom
  - When this electron falls back to the ground state it gives up a photon
  - 410 nm, 434 nm, 486 nm, 656 nm
  - 656 nm is the H alpha line

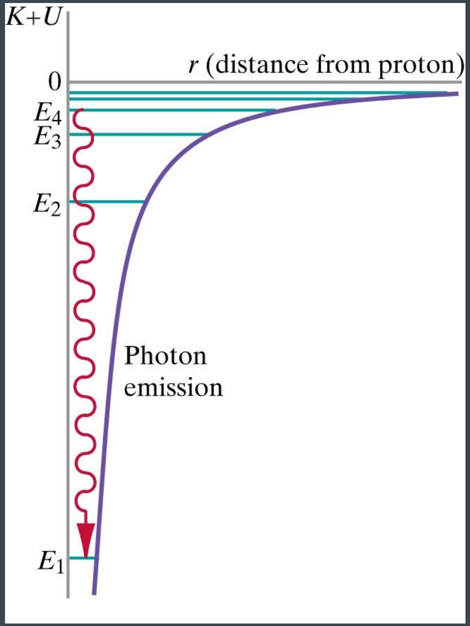


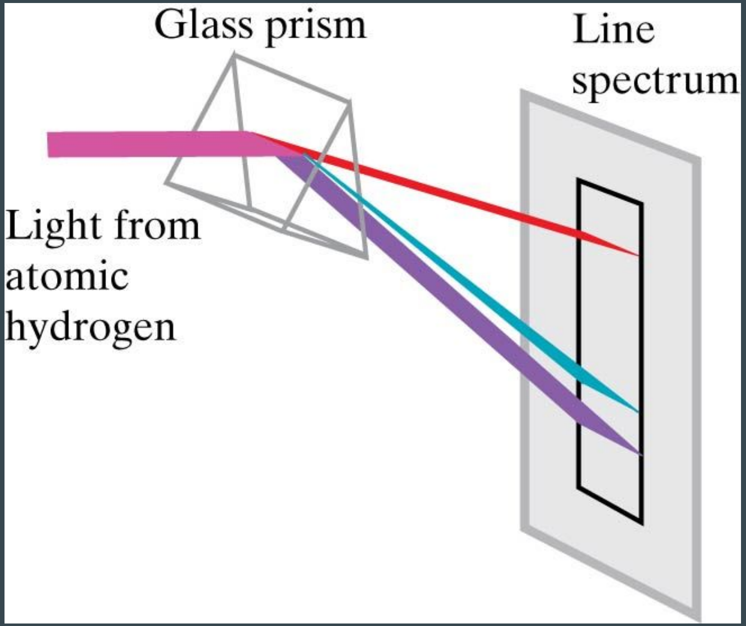





# Emission

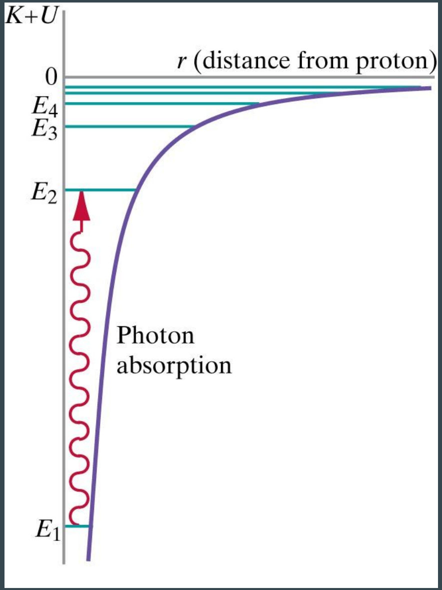
- Atoms in excited states will emit photons as the atoms drop to lower energy levels
  - The emitted photon has the same energy as the difference between energy levels
  - The photons from the atom are called the emission spectrum

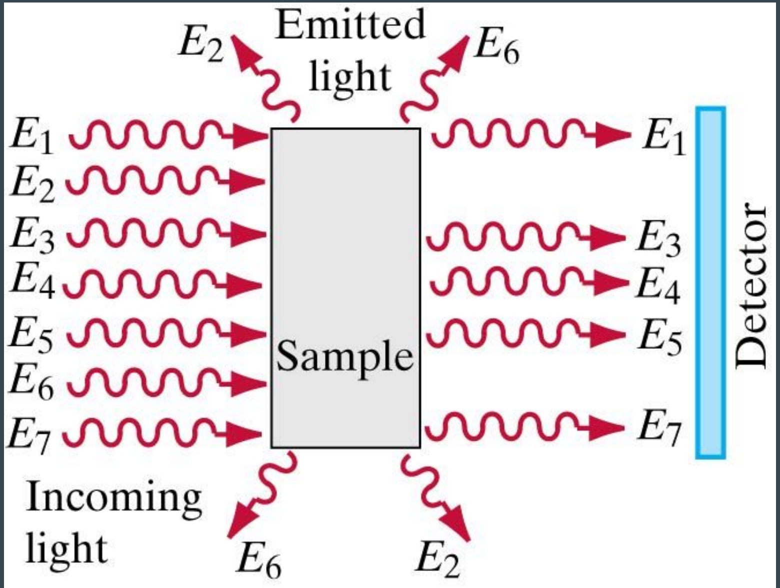


# Absorption

- Atoms can absorb photons and go from the ground state to an excited state
  - Only if the photons energy exactly matches the energy difference between the two state
  - This leads to missing photons; a “darkline” spectrum



# Quantum Mechanics