PHYS 2211 Test 1 - Fall 2019

Please circle your lab section and then clearly print your name & GTID

Day	12-3pm	3-6pm
Monday	W01 W08	W02 W09
Tuesday	W03 W10	W04 W11
Wednesday	W05 W12	W06 W13
Thursday	W07	W14

Name: Key

GTID: ______________________

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a\cdot b}{c\cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code, I have not given or received unauthorized aid on this test."

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Sign your name on the line above

Problem 1 [25 pts]

The *Rosetta* space probe has a mass of 3000 kg. On November 12th 2014, *Rosetta* landed on Comet 67P/C-G which has a mass of $M = 9.98 \times 10^{12}$ kg. The comet is a solid sphere with a radius of 2000 meters. In the code below, the motion of the space probe is modeled as it blasts off from the surface of the comet. There are two forces acting on the probe: the gravitational interaction with the comet, the thrust from the probe's propulsion system. This force has a constant magnitude of 24 N and always points in the same direction as the probe's velocity. **Add in the missing lines of code below to complete the program.**

```
GlowScript 2.9 VPython
G = 6.7e-11
mProbe = 3000 #in kg
mComet = 9.98e12 #in kg
Comet = sphere(pos=vector(0,0,0), radius=2000, color=color.green)
Probe = sphere(pos=vector(0,2000,0), radius=10, color=color.blue,make_trail=True)
t = 0
Probe.p = mProbe*vector(1, 2, 0) initial momentum of probe in kg*m/s
deltat = 10
while True:
```

A. [15 pts] Add the missing lines of code to calculate the net force on the probe. You can assume that the comet is stationary for all time.

```
r = Probe.pos - Comet.pos
rhat = norm(r)
Fgrav = -mProbe * mComet * G / mag2(r) * rhat
vhat = norm(Probe.p)
Fthrust = 24 * vhat # in Newtons
Fnet = Fgrav + Fthrust
```

) -3 Wrong r

)-5 Wrong Fgrav

) -5 Wrong Fthrust

) -2 Wrong or missing Fnet

B. [5 pts] Add the missing lines of code to update the momentum of the probe.

```
Probe.p = Probe.p + Fnet * deltat
```

All or nothing

C. [5 pts] Add the missing lines of code to update the position of the probe.

```
Probe.pos = Probe.pos + Probe.p/m * deltat
```

All or nothing

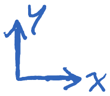
```
t = t + deltat
```

All parts : -1 for Python syntax (watch for acceptable coding alternatives)

Problem 2 [25 pts]

A vacuum capsule system is designed to pull capsules through tubes. A capsule with mass m is traveling straight upward through a vertical section of tube with momentum $\vec{p} = \langle 0, p_0, 0 \rangle$. The capsule is at the origin when $t = 0$.

A. [5 pts] At time $t = 0$, the vacuum system fails, so the capsule is acted on by only two forces: gravity, and the force from the tube wall. The tube wall exerts a force whose magnitude is always W, where $W < mg$, and the direction of this force is always opposite to the capsule's momentum. How long will the capsule take to reach its maximum height? Answer in terms of p_0, m, g, and W.



While the capsule is travelling upwards, the tube force opposes the momentum

$$\Rightarrow \vec{F}_{net} = \vec{F}_g + \vec{F}_{tube} = (-mg - W)\hat{y}.$$

For a constant force,

$$V_f = V_i + \frac{F_{net}}{m} \Delta t$$

$$\Rightarrow \cancel{V_{f,y}}^{0} = V_{i,y} + \frac{F_{net}}{m} \Delta t = \frac{p_0}{m} + \frac{F_{net}}{m} \Delta t.$$

$$\Rightarrow \frac{p_0}{m} = -\frac{F_{net}}{m} \Delta t = \frac{mg + W}{m} \Delta t$$

$$\Rightarrow \boxed{\Delta t = \frac{p_0}{mg + W}}$$

−1 Minor
−2 Major
−4 Minimal progress

B. [15 pts] After the maximum height is reached, the capsule falls back down. How long will the capsule take to reach the original position?

Because the capsule's momentum is now negative, the sign of W switches:

$$F_{net} = (-mg + W)\hat{y}.$$

What is the maximum height?

Well, $x_f = x_i + V_{avg}\Delta t$ and $V_{avg} = \frac{V_i + V_f}{2}$

for a constant force.

$$\Rightarrow d \equiv x_f - x_i = V_{avg}\Delta t_{up} = \frac{V_i + \cancel{V_f}^{0}}{2}\Delta t_{up} = \frac{p_0}{2m}\Delta t_{up}$$

From (1), $\Delta t_{up} = \frac{p_0}{mg + W} \Rightarrow d = \frac{p_0^2}{2m(mg + W)}$.

-1 Clerical

-3 Minor

-6 Major

-12 Minimal progress

Watch for POE!

Deriving the kinematic equations,

$$x_f = x_i + V_{avg}\Delta t = x_i + \frac{v_i + (v_i + \frac{F}{m}\Delta t)}{2}\Delta t = \underbrace{x_i + v_i\Delta t + \frac{F}{2m}(\Delta t)^2}.$$

$$\Rightarrow x_f - x_i = \cancel{v_i}^{0}\Delta t_{down} + \frac{F_{net}}{2m}(\Delta t_{down})^2 = -d.$$

$$\Rightarrow \boxed{\Delta t_{down} = \sqrt{\frac{2md}{mg - W}}} = \sqrt{\frac{2m}{mg - W}\left(\frac{p_0^2}{2m(mg + W)}\right)}$$

$$= \sqrt{\frac{p_0^2}{(mg)^2 - W^2}}$$

C. [5 pts] Compare your answers to the previous two questions. Briefly explain why these two times are or are not the same. (If you didn't find the answers, just explain if you think they should be the same).

The time going up should be different than the time going down since the net forces are different. (The sign of W changes, similar to friction in the fan cart lab.)

All or nothing. TAs discuss.

Problem 3 [25 pts]

A particle accelerator uses the electric force to accelerate tiny charged particles.

A. [5 pts] Suppose a force of $\langle 0, -1.0 \times 10^{-12}, 0\rangle$ N is applied to an initially stationary electron for 1.0×10^{-9} s. What will be the (vector) momentum of the electron afterwards?

$$\Delta \vec{p} = \vec{p}_f - \cancel{\vec{p}_i}^{\,0} = \vec{F}_{net} \Delta t$$

$$\Rightarrow \vec{p}_f = \langle 0, -1.0 \times 10^{-12}, 0 \rangle \text{ N} \times (1.0 \times 10^{-9} \text{ sec})$$

$$= \boxed{\langle 0, -1.0 \times 10^{-21}, 0 \rangle \text{ N}\cdot\text{s}}$$

All or nothing

B. [10 pts] Will the electron (mass 9.11×10^{-31} kg) reach 90% or more of the speed of light as a result of this process? Justify your answer.

$|\vec{p}| = \gamma m |\vec{v}| = \frac{1}{\sqrt{1 - (\frac{|\vec{v}|}{c})^2}} m |\vec{v}|$. Let's solve for $|\vec{v}|$:

$\frac{|\vec{p}|}{m} = \frac{1}{\sqrt{1 - (\frac{|\vec{v}|}{c})^2}} |\vec{v}|$. Squaring, $\frac{|\vec{p}|^2}{m^2} = \frac{|\vec{v}|^2}{1 - (\frac{|\vec{v}|}{c})^2}$.

$$\Rightarrow \left(1 - \left(\frac{|\vec{v}|}{c}\right)^2\right)\left(\frac{|\vec{p}|}{m}\right)^2 = |\vec{v}|^2 \Rightarrow \left(\frac{|\vec{p}|}{m}\right)^2 = |\vec{v}|^2\left(1 + \left(\frac{|\vec{p}|}{m}\right)^2\right)$$

$$\Rightarrow |\vec{v}| = \frac{|\vec{p}|/m}{\sqrt{1 + \left(\frac{|\vec{p}|}{mc}\right)^2}}. \quad \underline{c = 3 \times 10^8 \text{ m/s.}}$$

$$\Rightarrow |\vec{v}| \approx 2.89 \times 10^8 \text{ m/s} = \boxed{0.96c}$$

Therefore, greater than 90% of c.

See extra sheet for more notes.

-1 Clerical
-2 Minor
-4 Major
-8 Minimal progress

C. [5 pts] If the accelerating force were caused by another charged object 1 m away, what would be the magnitude of the charge on that object?

$$|\vec{F}_e| = k\frac{q_1 q_2}{|\vec{r}|^2} \Rightarrow q_2 = \frac{|\vec{F}_e||\vec{r}|^2}{kq_1}$$

$$\Rightarrow q_2 = \frac{(1.0 \times 10^{-12}\ N)(1\ m)^2}{\left(9 \times 10^9\ N\cdot\frac{m^2}{C^2}\right)(1.6 \times 10^{-19}\ C)} = 6.94 \times 10^{-4}\ C$$

-1 Minor

-2 Major

-4 Minimal progress

Watch for POE!

D. [5 pts] Based on your knowledge of the electric force, and the charges on elementary particles, what would be the (vector) force on a proton if it were in the same situation as the electron?

Because $q_{proton} = -q_{electron}$, the sign of the force would change while the magnitude remains the same:

$$\boxed{\vec{F} = (1.0 \times 10^{-12}\ N)\hat{y}}$$

All or nothing.

Problem 4 [25 pts]

One mole of platinum (6×10^{23} atoms) has a mass of 0.195 kg. The density of platinum metal is $21.4 \times 10^{3}\ kg \cdot m^{-3}$.

A. [10 pts] In the cubic lattice of our ball and spring model, determine the diameter of a platinum atom. Show your work.

$$m_{plat} = \frac{(0.195\ kg)}{6 \times 10^{23}\ atoms} = 3.25 \times 10^{-25}\ kg.$$

$$\rho_{plat} = \frac{m_{plat}}{V_{plat}} = \frac{m_{plat}}{(d_{plat})^3}$$

$$\Rightarrow d_{plat} = \left(\frac{m_{plat}}{\rho_{plat}}\right)^{1/3}$$

$$= \left(\frac{3.25 \times 10^{-25}\ kg}{21.4 \times 10^{3}\ kg \cdot m^{-3}}\right)^{1/3}$$

$$\approx \boxed{2.48 \times 10^{-10}\ m}$$

-1 Clerical
-2 Minor
-4 Major
-8 Minimal progress

B. [10 pts] Four thin bars of platinum hang side by side, vertically, from the ceiling. Each bar is 4 m long with a cross section 1 mm by 1 mm, so the cross-sectional area of each bar is 1×10^{-6} m^2. When a 100 kg mass is attached to the bottom of the bars, so that the bars are connected in parallel, each bar stretches 5.8 mm. As measured by this experiment, what is Young's modulus for the bar?

$$Y = \frac{F/A}{\Delta L/L} = \frac{(9.81 \times 100\ N)/(4 \times 10^{-6}\ m^2)}{(5.8 \times 10^{-3}\ m)/(4\ m)}$$

$$\approx 1.69 \times 10^{11}\ Pa$$

$$= \boxed{169\ GPa}.$$

Can also solve by considering a single wire with 1/4 the force.

-1 Clerical
-2 Minor
-4 Major
-8 Minimal progress

C. [5 pts] A single platinum bar that is 4 m long has a cross section of 1 mm by 3 mm is hung from the ceiling. This bar is connected to a 20 kg mass. Determine the amount of stretch you would measure in the bar.

$$Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{F/A}{Y/L}$$

$$\Rightarrow \Delta L = \frac{(20 \times 9.81\ N)/(3 \times 10^{-6}\ m^2)}{(1.6914 \times 10^{11}\ Pa)/(4\ m)}$$

$$\approx \boxed{1.5 \times 10^{-3}\ m}$$

-1 Minor
-2 Major
-4 Minimal progress
Watch for POE!

This page is for extra work, if needed.

For 3B, a few additional notes:

1) Full credit if the momentum of 0.9 c is calculated and compared.

2) Minor error if γ is miscalculated but otherwise used correctly

3) Major error if no γ is used but there is a clear understanding that the problem requires relativistic corrections.

4) -80% if no mention of relativity or γ at all.