Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

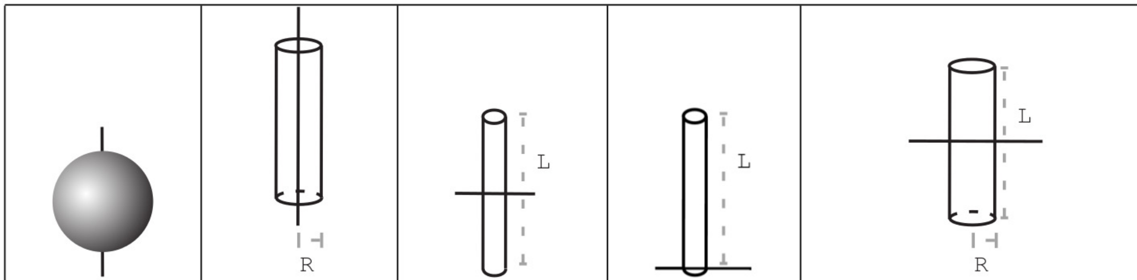
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

Moment of inertia for rotation about indicated axis



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/HP) - Spring 2024 - Test 2

Name: ______________________ GTID: ______________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\frac{a \cdot b}{c \cdot d} = \frac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

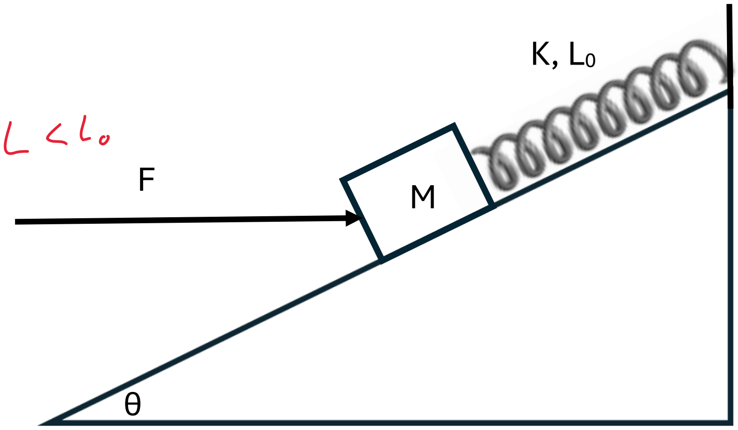
KEY

Sign your name on the line above

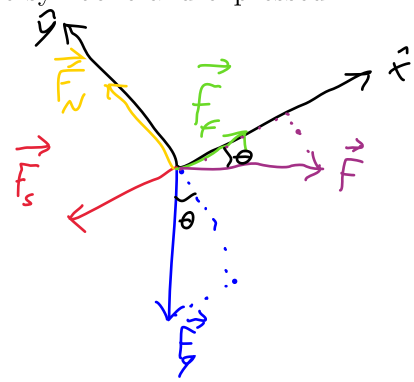
Problem 1: Equilibrium [30 pts]

A block (mass M) is kept on a rough inclined plane with an angle of inclination θ as shown in the diagram. There is a compressed spring (stiffness k and relaxed length L_0) attached to the block. There is also a horizontal force F applied to the block that is just strong enough to prevent the block from sliding down the ramp. The coefficient of static friction between the box and the inclined plane is μ. You can assume that this is a maximum static friction.

$L < L_0$



1.1 [10 pts] What is the **magnitude** of the spring force, F_s? Your answer must be symbolic and expressed in terms of M, g, θ, μ, and F.



x: $F_x + F_f - F_s - F_{g,x} = 0$

$\Rightarrow F\cos(\theta) + F_f - F_s - F_g \sin(\theta) = 0$

y: $F_N - F_y - F_{g,y} = 0$

$\Rightarrow F_N - F\sin(\theta) - F_g\cos(\theta) = 0$

$F_g = Mg \Rightarrow \underline{F_N = F\sin(\theta) + Mg\cos(\theta)}$

$F_f = \mu F_N \Rightarrow \boxed{F_s = F\cos(\theta) + \mu\left(F\sin(\theta) + Mg\cos(\theta)\right) - Mg\sin(\theta)}$

7:06

1.2 [10 pts] Obtain a symbolic expression for the **compressed length** of the spring, L. Your answer must be in terms of M, g, θ, μ, k, and L_0.

$$|\vec{F}_s| = k\,|L - L_0|$$

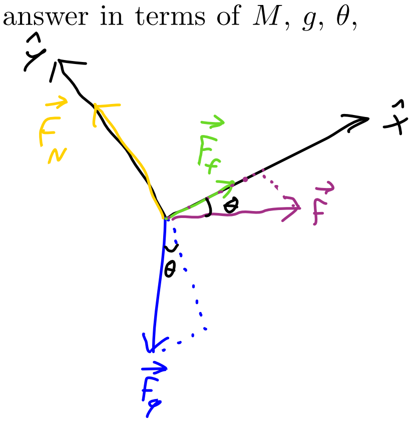
$$L < L_0 \Rightarrow |L - L_0| = (L_0 - L) \Rightarrow |\vec{F}_s| = k\,(L_0 - L)$$

$$F_s = k\,(L_0 - L) = F\cos(\theta) + \mu\,(F\sin(\theta) + Mg\cos(\theta)) - Mg\sin(\theta)$$

$$\Rightarrow \boxed{L = L_0 - \frac{1}{k}\left[F\cos(\theta) + \mu\,(F\sin(\theta) + Mg\cos(\theta)) - Mg\sin(\theta)\right]}$$

9:27

1.3 [10 pts] What must be the **magnitude** of the force F to keep the spring at its relaxed length while also keeping the block at rest? Assume maximum static friction, and provide your answer in terms of M, g, θ, μ, k, and L_0.



$L = L_0 \Rightarrow \underline{F_s = 0}$

$x: F_x + F_f - F_{g,x} = 0$

$\Rightarrow F\cos(\theta) + F_f - F_g\sin(\theta) = 0$

$y: F_N - F_y - F_{g,y} = 0$

$\Rightarrow F_N - F\sin(\theta) - F_g\cos(\theta) = 0$

$F_g = Mg \Rightarrow \underline{F_N = F\sin(\theta) + Mg\cos(\theta)}$

$F_f = \mu F_N \Rightarrow F\cos(\theta) + \mu(F\sin(\theta) + Mg\cos(\theta)) - Mg\sin(\theta) = 0$

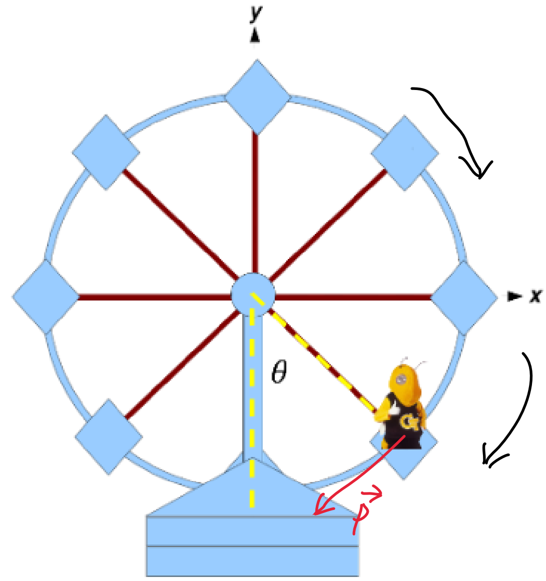
$\Rightarrow F\cos(\theta) + \mu F\sin(\theta) = Mg\sin(\theta) - \mu Mg\cos(\theta)$

$$\Rightarrow \boxed{F = \frac{Mg\sin(\theta) - \mu Mg\cos(\theta)}{\cos(\theta) + \mu\sin(\theta)}}$$

15:09

Problem 2: Curving Motion [30 pts]

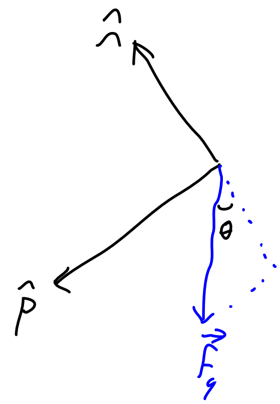
Buzz is riding the SkyView Ferris wheel in Centennial Park. The Ferris wheel is rotating clockwise so that Buzz has a constant speed, v_0. The Ferris wheel has radius R, and Buzz has mass m. Buzz is currently at an angle θ from the vertical, as defined in the figure.



2.1 [10 pts] What is the magnitude of the **parallel component** of the contact force of the Ferris wheel on Buzz?

$$F_{net,\parallel} = F_{g,\parallel} - F_{c,\parallel} = 0$$

$$\Rightarrow F_{c,\parallel} = F_{g,\parallel} = \boxed{mg\sin(\theta)}$$



4:35

2.2 [10 pts] What is the magnitude of the **perpendicular component** of the contact force of the Ferris wheel on Buzz?

$$F_{net,\perp} = F_{c,\perp} - F_{g,\perp} = F_{cent} = \frac{m v_0^2}{R}$$

$$\Rightarrow F_{c,\perp} = F_{g,\perp} + \frac{m v_0^2}{R} = \boxed{mg\cos(\theta) + \frac{m v_0^2}{R}}$$

2.3 [10 pts] The Ferris wheel has a sudden but brief acceleration so that Buzz now has a new constant speed $2v_0$. What is the **ratio** of the net force in the perpendicular direction of the Ferris wheel now compared to before? In other words, determine:

$$\frac{F_{\text{net}\perp}(v = 2v_0)}{F_{\text{net}\perp}(v = v_0)}$$

$$F_{net,\perp}(v_0) = \frac{m v_0^2}{R}$$

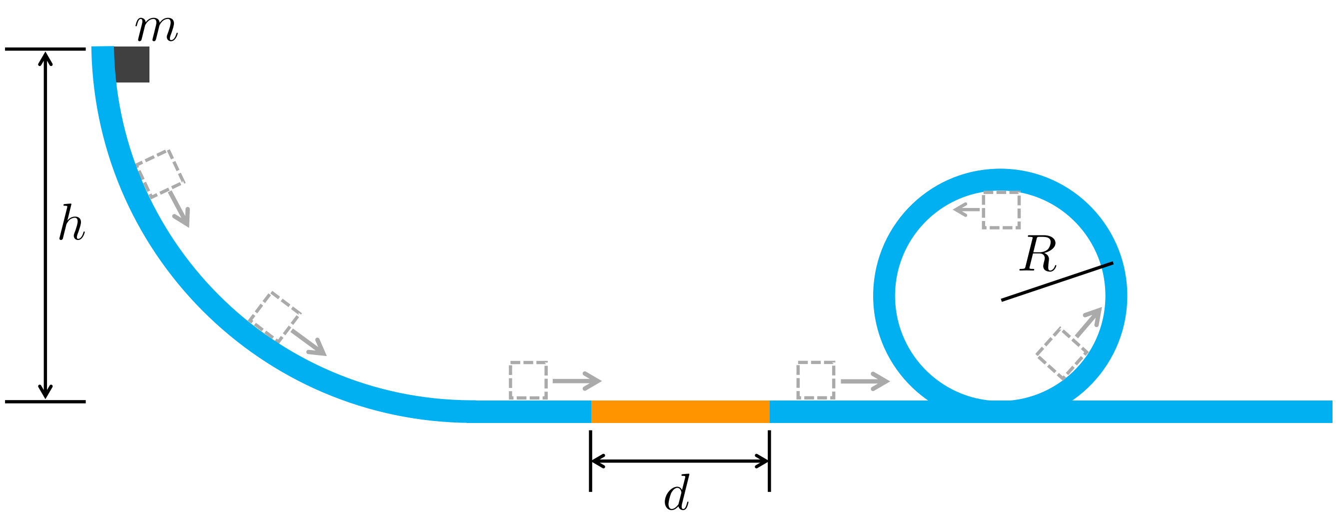
$$F_{net,\perp}(2v_0) = \frac{m(2v_0)^2}{R} = \frac{4 m v_0^2}{R}$$

$$\frac{F_{net,\perp}(2v_0)}{F_{net,\perp}(v_0)} = \frac{\frac{4 m v_0^2}{R}}{\frac{m v_0^2}{R}} = \boxed{4}$$

7:42

Problem 3: Energy Principle [30 pts]

A small box of mass m, small enough to be considered a point mass, is held motionless at the top of a frictionless ramp. The ramp is shaped like a quarter of a circle. The top of the ramp is at a height h above the ground, and the bottom of the ramp connects with a track that lays horizontally on the ground. The track is frictionless everywhere except for a short section of length d that has a rough surface. Further ahead, the track curves upwards in a frictionles vertical loop of radius R.



3.1 [10 pts] The box is let go and it slides down the ramp. What is the **speed** v of the box when it reaches the bottom of the ramp?

System: box, Earth

Surroundings: ramp

Initial state: $v_i = 0$, $y_i = h$

Final state: $v_f = v$ $y_f = 0$

$\Delta E = W$ Frictionless $\Rightarrow W = 0$

$\Delta K + \Delta U_g = 0$

$\frac{1}{2} m (v_f^2 - v_i^2) + mg(y_f - y_i) = 0$

$\frac{1}{2} m v^2 - mgh = 0$

$\boxed{v = \sqrt{2gh}}$

S:03

3.2 [20 pts] After exiting the ramp, the box slides along the frictionless track, then across the short rough section, and then up the vertical loop. The speed of the box at the top of the loop, v_{top}, is half of the speed that the box had when it first entered the loop, v_{loop}. Under these conditions, determine what must be the **coefficient of friction** μ in the rough section of the track.

System: box

Surroundings: track

Initial state: $v_i = v$

Final state: $v_f = v_{loop}$

$$\Delta E = W$$

$$\Delta K = W_f$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = \vec{F}_f \cdot \Delta\vec{r}$$

$$\frac{1}{2}m(v_{loop}^2 - v^2) = (-\mu m g)(d) = -\mu m g d$$

$$\frac{1}{2}m(v_{loop}^2 - 2gh) = -\mu m g d$$

$$v_{loop}^2 - 2gh = -2\mu g d$$

$$\underline{v_{loop} = \sqrt{2gh - 2\mu g d}}$$

System: box, Earth

Surroundings: loop

Initial state: $v_i = v_{loop}$, $y_i = 0$

Final state: $v_f = v_{top}$, $y_f = 2R$

$\Delta E = W$ frictionless $\Rightarrow W = 0$

$$\Delta K + \Delta U_g = 0$$

$$\frac{1}{2}m(v_f^2 - v_i^2) + mg(y_f - y_i) = 0$$

$$\frac{1}{2}m(v_{top}^2 - v_{loop}^2) + mg(2R) = 0 \qquad v_{top} = \frac{v_{loop}}{2}$$

$$\frac{1}{2}m\left(\frac{v_{loop}^2}{4} - v_{loop}^2\right) + 2mgR = 0$$

$$\frac{1}{2}v_{loop}^2\left(\frac{1}{4} - 1\right) + 2gR = 0$$

$$-\frac{3}{8}v_{loop}^2 = -2gR$$

$$v_{loop}^2 = \frac{16}{3}gR$$

$$2gh - 2\mu g d = \frac{16}{3}gR$$

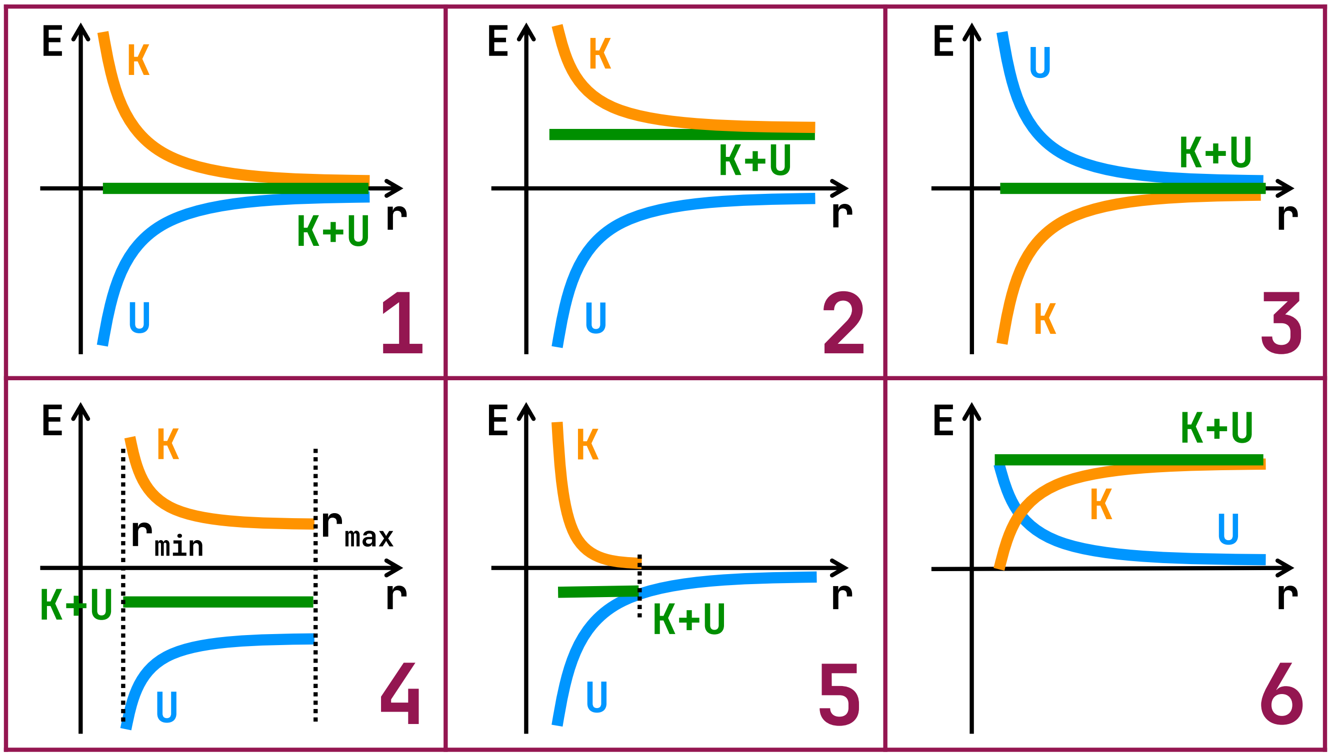
$$2\mu g d = 2gh - \frac{16}{3}gR$$

$$\boxed{\mu = \frac{h - \frac{8}{3}R}{d}}$$

~17:00

Problem 4: Energy Graphs [10 pts]

The following image shows six graphs of Energy versus distance (r). In each graph, the orange curve labeled K represents kinetic energy, the blue curve labeled U represents the potential energy, and the green line labeled $K+U$ represents the total energy. **Indicate which graph (by number) best represents each of the scenarios described below.**



4.1 __3__ [1 pt] An impossible set of energy graphs.

4.2 __4__ [1 pt] Mars moving in an elliptical orbit around the Sun.

4.3 __6__ [1 pt] Two protons are held near each other at rest and then are let go.

4.4 __4__ [1 pt] Pluto and Charon, its biggest moon, in bound orbits around each other.

4.5 __5__ [1 pt] A proton and an electron are held at rest at a short distance away from each other, then they are let go.

4.6 __2__ [1 pt] Voyager II taking off from Earth with an initial speed that is just a little bit higher than the Earth's escape speed.

4.7 __1__ [1 pt] A rock is ejected from a volcano on Io (one of Jupiter's moons) with a speed that is exactly equal to Io's escape speed.

4.8 __1__ [1 pt] A comet swings by close to the Sun and then moves away. The comet's speed is zero when it is very, very far away from the Sun.

4.9 __2__ [1 pt] An electron and a proton are moving away from each other with high initial speeds. When they are infinitely far away from each other, they are still moving.

4.10 __6__ [1 pt] Two electrons are a large distance away from each other and moving straight towards each other with nonzero initial speeds. When they get very close, they momentarily stop and then start moving away from each other.

0:58

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