# Physics 2211 GPS Week 12

Problem #1

A thin box in outer space contains a large ball of clay of mass M, connected to an initially relaxed spring of stiffness k_s. The mass of the box is negligible compared to M. The apparatus is initally at rest. Then a force of constant magnitude F is applied to the box. When the box has moved a distance b, the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount s. See the diagram for distances.
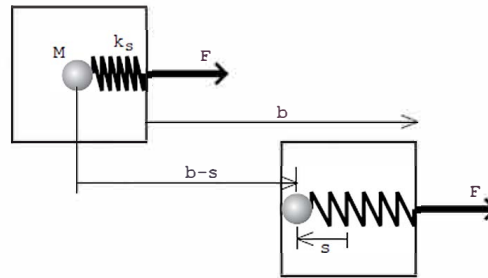
(a) Immediately after the clay sticks to the box, how fast is the box moving?

System : the whole thing is a point

$$\Delta E = \Delta K_{trans} = W = \vec{F} \cdot \Delta \vec{r}_{cm}$$

$$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}^{0}) = F(b-s)$$

$$\frac{1}{2}mv_f^2 = F(b-s)$$

$$mv_f^2 = 2F(b-s)$$

$$v_f^2 = \frac{2F}{m}(b-s)$$

$$\boxed{v_f = \sqrt{\frac{2F}{m}(b-s)}}$$

(b) What is the increase in thermal energy of the clay?

System : real (clay + spring)

$$\Delta E = \Delta K_{trans} + \Delta U_s + \Delta E_{int} = W = \vec{F} \cdot \Delta \vec{r}$$

$$F(b-s) + \frac{1}{2}k(s_f^2 - \cancel{s_i^2}^{0}) + \Delta E_{int} = Fb$$

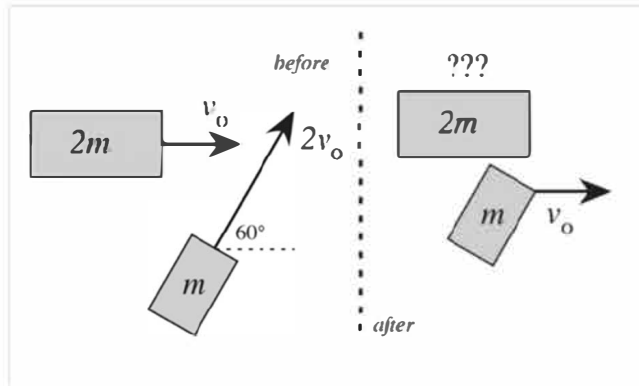$$\cancel{Fb} - Fs + \frac{1}{2}ks^2 + \Delta E_{int} = \cancel{Fb}$$

$$-Fs + \frac{1}{2}ks^2 + \Delta E_{int} = 0$$

$$\boxed{\Delta E_{int} = Fs - \frac{1}{2}ks^2}$$

Problem #2

A cruise ship is moving due east with some speed v_0 when it collides with a boat moving $60°$ north of east with a speed $2v_0$. The boat's mass is m and the cruise ship's mass is $2m$. Immediately after the collision, the boat is observed to be floating due east with a speed v_0.

(a) Determine the velocity of the cruise ship just after the collision. You need to express your answer in terms of the parameter v_0.

$$\vec{V}_{ci} = \langle v_0, 0, 0 \rangle \qquad \vec{V}_{bi} = 2v_0 \langle \cos 60°, \sin 60°, 0 \rangle$$

$$\vec{V}_{cf} = ? \qquad \vec{V}_{bf} = \langle v_0, 0, 0 \rangle$$

$$m_c = 2m \qquad m_b = m$$

$$\vec{P}_{ci} + \vec{P}_{bi} = \vec{P}_{cf} + \vec{P}_{bf}$$

$$m_c \vec{V}_{ci} + m_b \vec{V}_{bi} = m_c \vec{V}_{cf} + m_b \vec{V}_{bf}$$

$$m_c \vec{V}_{ci} + m_b (\vec{V}_{bi} - \vec{V}_{bf}) = m_c \vec{V}_{cf}$$

$$\vec{V}_{cf} = \frac{m_c \vec{V}_{ci}}{m_c} + \frac{m_b}{m_c}(\vec{V}_{bi} - \vec{V}_{bf}) = \vec{V}_{ci} + \frac{m}{2m}(\vec{V}_{bi} - \vec{V}_{bf}) =$$

$$= \langle v_0, 0, 0 \rangle + \frac{1}{2}\left[2v_0 \langle \cos 60°, \sin 60°, 0 \rangle - \langle v_0, 0, 0 \rangle \right] =$$

$$= \langle v_0, 0, 0 \rangle + v_0 \langle \cos 60°, \sin 60°, 0 \rangle - \langle \tfrac{v_0}{2}, 0, 0 \rangle =$$

$$= \langle \tfrac{v_0}{2}, 0, 0 \rangle + v_0 \langle \cos 60°, \sin 60°, 0 \rangle =$$

$$= \langle \tfrac{v_0}{2} + v_0 \cos 60°, \; v_0 \sin 60°, 0 \rangle = \langle \tfrac{v_0}{2} + \tfrac{v_0}{2}, \; 0.866\, v_0, 0 \rangle =$$

$$= \langle v_0, \; 0.866\, v_0, 0 \rangle = \boxed{v_0 \langle 1, 0.866, 0 \rangle}$$

(b) What percentage of the original kinetic energy was lost in the collision?

$\checkmark$ $K_i = K_{ci} + K_{bi} = \frac{1}{2}m_c v_{ci}^2 + \frac{1}{2}m_b v_{bi}^2 = \frac{1}{2}(2m)v_0^2 + \frac{1}{2}m(2v_0)^2 =$

$\qquad = mv_0^2 + \frac{1}{2}m \cdot 4 v_0^2 = mv_0^2 + 2mv_0^2 = 3mv_0^2$

$\checkmark$ $K_f = K_{cf} + K_{bf} = \frac{1}{2}m_c v_{cf}^2 + \frac{1}{2}m_b v_{bf}^2 = \frac{1}{2}(2m)v_{cf}^2 + \frac{1}{2}mv_0^2 =$

$\qquad\qquad\qquad v_{cf}^2 = v_0^2(1 + 0.866^2) = v_0^2(1 + 0.75) = 1.75v_0^2$

$\qquad = 1.75mv_0^2 + \frac{1}{2}mv_0^2 = (1.75 + 0.5)mv_0^2 = 2.25mv_0^2$

$\checkmark$ $|\Delta k| = |K_f - K_i| = |2.25mv_0^2 - 3mv_0^2| = |-0.75mv_0^2| = 0.75mv_0^2$

$\Rightarrow \dfrac{K_f}{K_i} = \dfrac{2.25mv_0^2}{3mv_0^2} = \dfrac{2.25}{3} = 0.75$

$\Rightarrow$ % lost $= (1 - 0.75) \times 100\% = \boxed{25\%}$

Problem #3

Two cars of identical mass m are involved in a collision. Both cars are moving at the same speed v_0. One of the cars is initially moving due north, and the other is initially moving south of west at an angle θ (where $45° < \theta < 90°$). The collision between the two vehicles is maximally inelastic; in other words, the vehicles stick together after the collision.
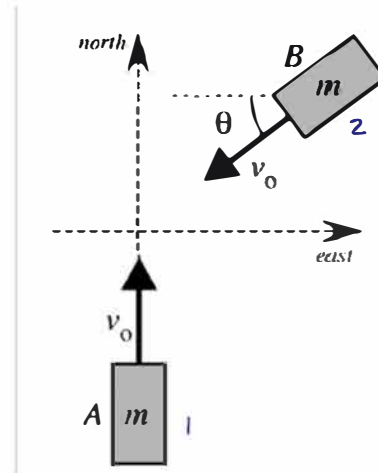
(a) Determine the magnitude of the final velocity of each car after the collision, in terms of the quantities m, v_0, and θ.

$$m_1 \vec{v_1} + m_2 \vec{v_2} = (m_1 + m_2) \vec{v_f} \implies \vec{v_f} = \frac{m_1}{m_1+m_2} \vec{v_1} + \frac{m_2}{m_1+m_2} \vec{v_2}$$

$$\vec{v_f} = \frac{m}{2m} \langle 0, v_0, 0 \rangle + \frac{m}{2m} \langle -v_0 \cos\theta, -v_0 \sin\theta, 0 \rangle = \frac{v_0}{2} \langle 0, 1, 0 \rangle + \frac{v_0}{2} \langle -\cos\theta, -\sin\theta, 0 \rangle =$$

$$= \frac{v_0}{2} \langle -\cos\theta, 1-\sin\theta, 0 \rangle$$

$$|\vec{v_f}| = \sqrt{\left(\frac{v_0}{2}\right)^2} \sqrt{(-\cos\theta)^2 + (1-\sin\theta)^2} = \frac{v_0}{2} \sqrt{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta} =$$

$$= \frac{v_0}{2} \sqrt{\cos^2\theta + \sin^2\theta + 1 - 2\sin\theta} = \frac{v_0}{2} \sqrt{1 + 1 - 2\sin\theta} =$$

$$= \boxed{\frac{v_0}{2} \sqrt{2 - 2\sin\theta}}$$

Problem #4

A person of mass m_1 is walking at constant velocity $\vec{v_1} = \langle v_1, 0, 0 \rangle$ along the sidewalk towards an intersection, looking down at their phone. At the same time, another person riding a scooter (total mass m_2) approaches the intersection with unknown velocity $\vec{v_2} = \langle 0, v_2, 0 \rangle$. Neither the person walking nor the person on the scooter notice each other, and so they crash.

The two people and the scooter stick together after the crash, moving together with an **unknown final speed** at an angle θ above the x-axis.

1. [15 pts] How fast was the scooter moving before the crash, i.e., what was the initial speed v_2? **Your answer must only use variables that represent known quantities (m_1, m_2, v_1, θ).**

Initial :

Final :
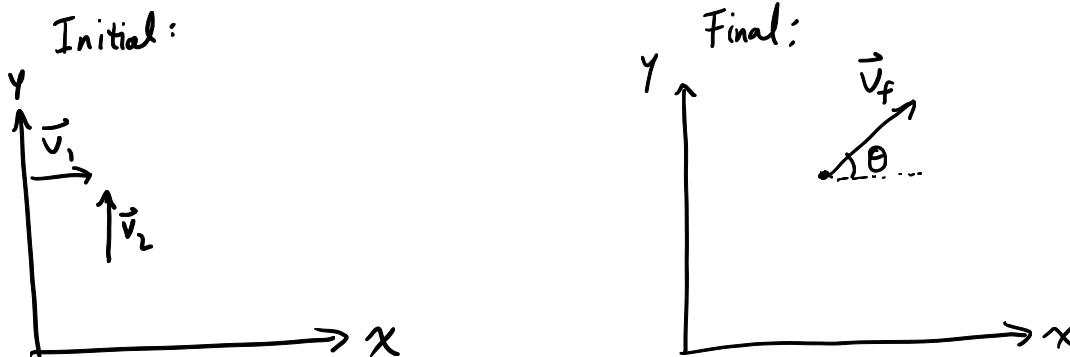
Conservation of momentum : $\vec{P_f} = \vec{P_i}$

$$\Rightarrow \vec{P_i} = m_1 \vec{v_1} + m_2 \vec{v_2} = \langle m_1 v_1, m_2 v_2, 0 \rangle$$

$$\vec{P_f} = (m_1 + m_2) \vec{v_f} = (m_1 + m_2) \langle v_f \cos\theta, v_f \sin\theta, 0 \rangle$$

Comparing $P_{i,x}$ and $P_{f,x}$,

$$m_1 v_1 = (m_1 + m_2) v_f \cos\theta \Rightarrow v_f = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta}$$

Comparing $P_{i,y}$ and $P_{f,y}$,

$$m_2 v_2 = (m_1 + m_2) v_f \sin\theta = \frac{(m_1 + m_2) m_1 v_1 \sin\theta}{(m_1 + m_2)\cos\theta}$$

$$= m_1 v_1 \tan\theta$$

$$\Rightarrow \boxed{v_2 = \frac{m_1 v_1 \tan\theta}{m_2}}$$

2. [5 pts] What was the change in internal energy of the system during the crash? Again, your answer must only use variables that represent known quantities in the problem.

Energy principle:

$$\Delta K + \Delta E_{int} = \overset{0}{\cancel{W_{ext}}}$$

$$\Rightarrow \Delta E_{int} = -\Delta K = K_i - K_f$$

$$= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}(m_1+m_2)v_f^2$$

$$\boxed{= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2\left(\frac{m_1 v_1 \tan\theta}{m_2}\right)^2 - \frac{1}{2}(m_1+m_2)\left(\frac{m_1 v_1}{(m_1+m_2)\cos\theta}\right)^2}$$