Week 4 Lecture 1

Gravitation

Topics for this week

1. Universal Gravitation
2. Reciprocity

By the end of the week

1. Be able to compute the force of gravity for any two masses
2. Use Newton's 3rd Law
3. Approximate gravity

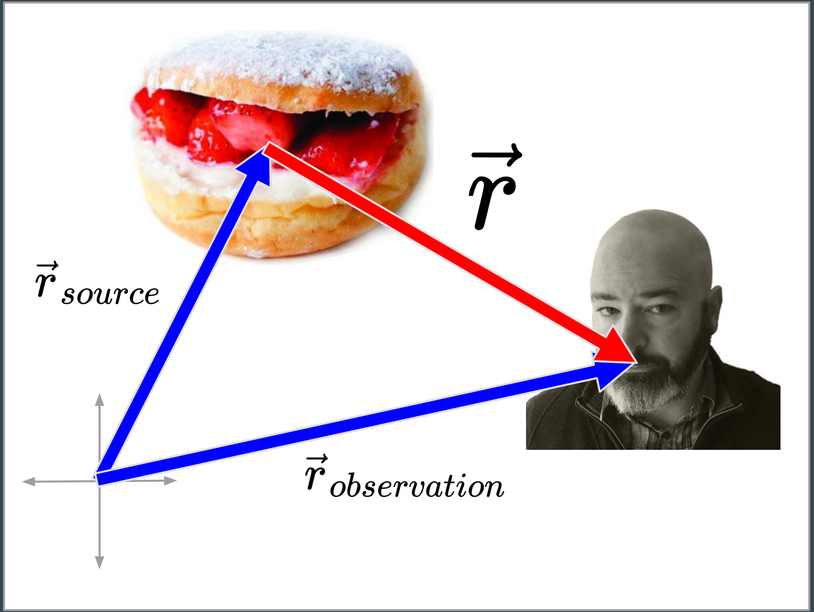

Gravitation
MovieHDWallpapers.com

How do we know so much about Planet X?

- The case for a new planet http://science.sciencemag.org/content/351/6271/330

The Gravitational force law

- Discovered/Invented by Newton in 1687 AD
  - https://www.nationaltrust.org.uk/woolsthorpe-manor/features/the-story-of-our-apple-tree-at-woolsthorpe-manor
- Objects attract each other along a line connecting their centers
  - Fits with the observations of Kepler's Laws of Planetary motion (1609 AD)



$$\vec{F}_{grav} = -G\frac{m_{obs}\,m_{source}}{|\vec{r}|^2}\hat{r}$$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{source}$$

$$G = 6.7 \times 10^{-11}\,\frac{N \cdot m^2}{kg}$$

A procedure for calculating the force of gravity

1. Draw a picture that includes position arrows for each object
2. Calculate the relative position vector
    a. Points to where you want to know the force
3. Calculate the distance between the objects
    a. The magnitude of the relative position vector
4. Calculate the magnitude of the force
    a. Newton's Law of Gravitation
5. Calculate the direction of the force
    a. Don't forget the negative
6. Combine the magnitude and direction
7. Check against your picture

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{r}| = |\vec{r}_2 - \vec{r}_1|$$

$$|\vec{F}_{grav}| = G\frac{m_2 m_1}{|\vec{r}|^2}$$

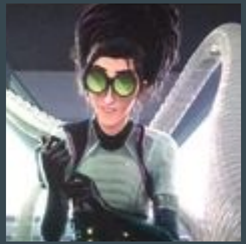
$$\hat{F}_{grav} = -\frac{\vec{r}}{|\vec{r}|}$$

$$\vec{F}_{grav} = |\vec{F}_{grav}|\hat{F}_{grav}$$

Example: Astrology 101

Can astronomical objects determine our destiny? If they can, then Newton tells us there must be an interaction! What types of interaction can work over such large distances? Lets compare the forces exerted on a baby by the planet Venus and the delivery Doctor.

Mass of Venus: 4.9e24 kg
Mass of Doctor: 70 kg
Distance to Venus: 3.8e10 m
Distance to Doctor: 0.3 m



Example: Solution

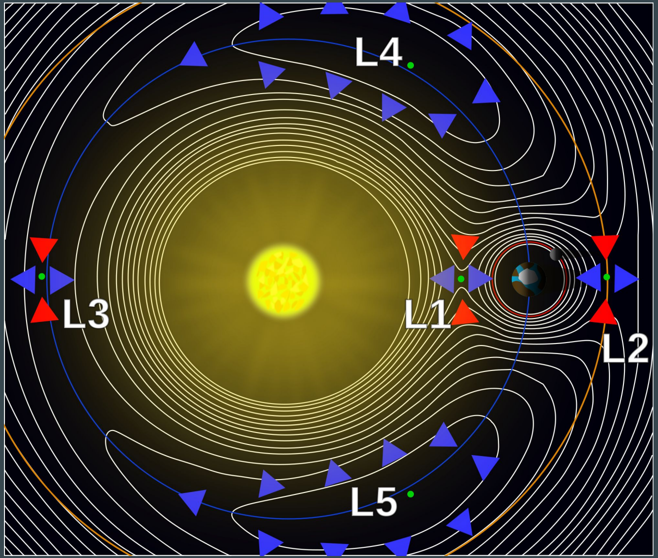
- Ignore directions and compare magnitudes
  - Compute the ratio of the forces

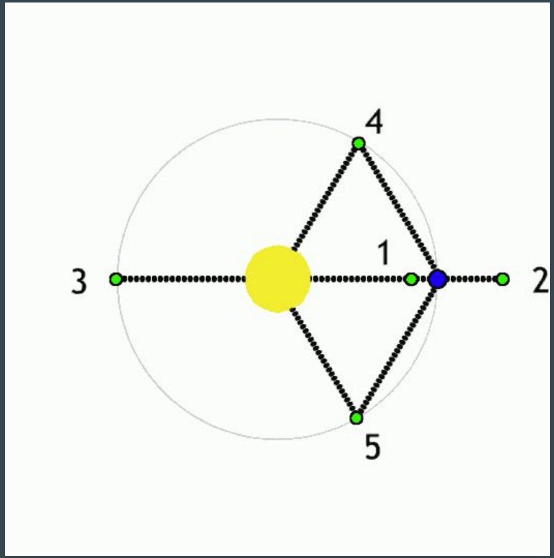
$$\left|\vec{F}_{grav}\right| = G\frac{m_2 m_1}{\left|\vec{r}\right|^2} \rightarrow \frac{\left|\vec{F}_{baby,doc}\right|}{\left|\vec{F}_{baby,venus}\right|}$$

$$\frac{m_{doc}\left|\vec{r}_{baby,venus}\right|^2}{m_{venus}\left|\vec{r}_{baby,doc}\right|^2} \rightarrow \frac{(70\ kg)\ (3.8e10\ m)^2}{(4.9e24\ kg)\ (0.3\ m)^2} = 0.23$$

The Lagrange Points

- Positions in an orbital configuration of two large bodies where a small object affected only by gravity can maintain a stable position relative to the large bodies.
  - We will model these points numerically with Glowscript!

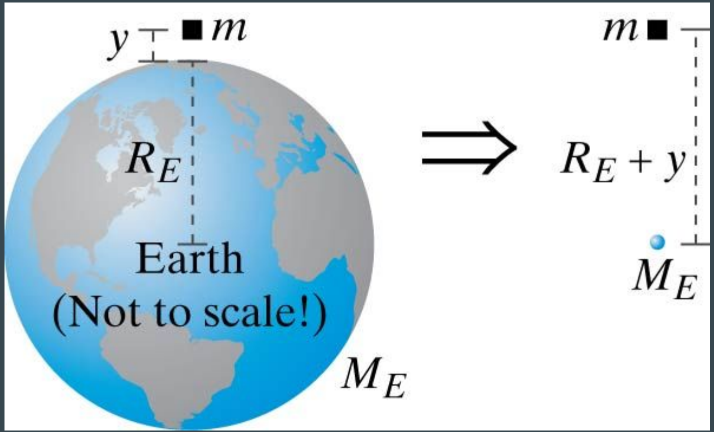




The Gravitational near Earth

- Consider the force of gravity a distance y from the surface of the earth

$$|\vec{F}_{grav}| = G\frac{(m)m_{Earth}}{(R_{Earth}+y)^2}$$



- By Newton's generalized binomial theorem

$$(1+\frac{y}{R_{Earth}})^{-2} = 1 - 2\left(\frac{y}{R_{Earth}}\right) + 3\left(\frac{y}{R_{Earth}}\right)^3 + \ldots$$

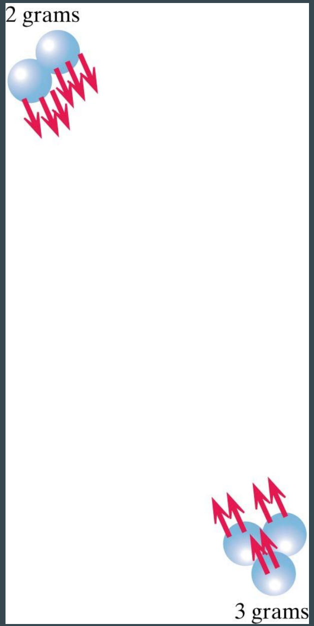
- Near the surface of the earth (y << R_{Earth})

$$|\vec{F}_{grav}| \approx G\frac{(m)m_{Earth}}{(R_{Earth})^2} = m\frac{Gm_{Earth}}{(R_{Earth})^2} = mg$$

Reciprocity

- Newton's 3rd law: The gravitational force on one object is equal and opposite the gravitational force on the other object.
  - A mathematical consequence of the symmetry in the force law
    - Not strictly true for all forces but true for contact forces which are electrostatic!
  - You can't pick yourself by your bootstraps!
- Qualitatively consider two objects of different mass but comprised of the same basic building blocks (e.g. atoms) each with the same mass
  - The net force is the sum of all forces which is the same for the two systems

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$



Prediction motion due to gravity

- Armed with an analytic expression for the gravitational force law we can now solve the Sun + Earth problem iteratively in Vpython/Glowscript
    - Define Constants
    - Specify initial conditions
        - Position, Momentum, Δt, etc...
    - Create a loop that will repeatedly
        - Calculate The Net Force
        - Update Momentum
        - Update Position
- http://www.glowscript.org/#/user/ed/folder/My_Programs/program/EarthSun/edit
- http://www.glowscript.org/#/user/ed/folder/My_Programs/program/EarthSunMoon/edit