PHYS 2211 Test 1 - Fall 2018

Please circle your lab section and then clearly print your name & GTID

Sections (M) Parker, (N) Yunker		
Day	12-3pm	3-6pm
Monday	M01 N01	M02 N02
Tuesday	M03 N03	M04 N04
Wednesday	M05 N05	M06 N06
Thursday	M07	N07

Name: Test Key

GTID: ______

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

______

Sign your name on the line above

Problem 1 [25 pts]

A group of Tech students are going from Howey to the student center. Howey is located at $\langle 0, 0, 0\rangle$ and the student center is located at $\langle 50, 0, -400\rangle$ m.

All A. [5 pts] How far must the students walk to get to the student center?

$\bar{r}_i = \bar{0}$

$\bar{r}_f = \langle 50, 0, -400\rangle$

$d = \|\bar{r}_f - r_i\| = \sqrt{50^2 + 0^2 + 400^2}$

$$\boxed{d = \sqrt{162500}\ \text{m} = 50\sqrt{65}\ \text{m} = 403.113\ \text{m}}$$

B. [5 pts] If the trip takes 350 s and a particular student has a mass of 70 kg, what is the average (vector) momentum of the student?

$$\bar{P}_{avg} = m\bar{V}_{avg} = m\frac{\Delta\bar{r}}{\Delta t} = \frac{m}{\Delta t}(\bar{r}_f - \cancel{\bar{r}_i}^{\,\bar{0}})$$

$$\boxed{\bar{P}_{avg} = \frac{70}{350}\langle 50, 0, -400\rangle\ \text{kg}\ \frac{\text{m}}{\text{s}} = \langle 10, 0, -80\rangle\ \text{kg}\ \frac{\text{m}}{\text{s}}}$$

Grading Decision

-3 pts for finding $|\bar{P}_{avg}|$

C. [10 pts] If this student rents a 10 kg scooter instead, and travels at 8 m/s on a straight path toward the student center, what is the (vector) momentum of the scooter (including the rider!).

$m = (70 + 10)$ kg

$V = 8$ m/s

$$\bar{P} = mv\,\hat{\Delta r} = mv\frac{\bar{r}_f}{|\bar{r}_f|} = \frac{80 \cdot 8 \cdot \langle 50, 0, -400\rangle}{\sqrt{162500}}\ \text{kg}\ \frac{\text{m}}{\text{s}}$$

$$\boxed{\bar{P} = \langle 79.38, 0, -635.06\rangle\ \text{kg}\ \frac{\text{m}}{\text{s}}}$$

Grading Rubric	
-1 pt	Clerical
-2 pt	Minor
-4 pt	Major
-8 pt	BTN

Grading Decision

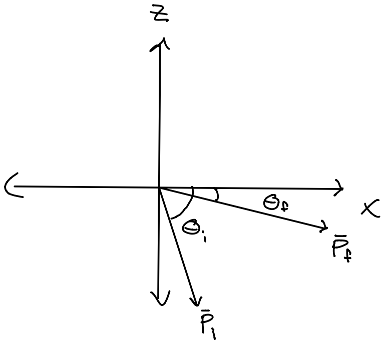
-2 pts for using $m = 10$ kg

D. [5 pts] Finding the student center full, the student steers the scooter off course so that its momentum is now $\langle 639, 0, -38\rangle$ kg m/s. By what angle was the scooter turned (answer in degrees).

Watch out for POE

$$\theta_i = \text{atan}\left(\frac{P_{i,x}}{P_{i,z}}\right) = -\text{atan}\left(\frac{635.06}{79.38}\right) = -82.875\ \text{deg.}$$

$$\theta_f = \text{atan}\left(\frac{P_{f,x}}{P_{f,z}}\right) = -\text{atan}\left(\frac{38}{639}\right) = -3.49\ \text{deg.}$$



$$\boxed{|\Delta\theta| = 79.38\ \text{deg} = 1.39\ \text{rad}}$$

Grading Decision

-1 pt for radians

Problem 2 [25 pts]

The Relativistic Heavy Ion Collider (RHIC) in New York state accelerates gold (Au) ions to very high speeds and is currently the nation's largest operating collider. The mass of a gold ion is 3.27×10^{-25} kg.

All A. [5 pts] If a gold ion travels with a velocity of $\langle -2.90, 0, 0\rangle \times 10^{8}$ m/s, what is its momentum?

$$\bar{P}_i = \gamma m_o V$$

$$\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-1/2} = \left(1 - \left(\frac{2.90}{3}\right)^2\right)^{-1/2} = 3.90$$

So

$$\bar{P}_i = (3.90)\, 3.27 \times 10^{-25} \langle -2.90 \times 10^{8}, 0, 0\rangle \text{ kg } \frac{m}{s}$$

$$\boxed{\bar{P}_i = \langle -3.70 \times 10^{-16}, 0, 0\rangle \text{ kg } \frac{m}{s}}$$

B. [5 pts] Suppose that such an ion collides with a stationary target. During the very brief interaction an impulse of $\langle 6.1, 0, 0\rangle \times 10^{-16}$ N · s is applied to the ion. What is the momentum of the ion after the interaction?

The impulse is given:

$$\Delta t\, \bar{F}_{net} = \langle 6.1, 0, 0\rangle \times 10^{-16} \text{ N}\cdot\text{s}$$

So:

$$\Delta \bar{P} = \Delta t\, \bar{F}_{net}$$

$$\bar{P}_f = \bar{P}_i + \Delta t\, \bar{F}_{net}$$

watch out for PGE

$$\bar{P}_f = \langle -3.70 \times 10^{-16}, 0, 0\rangle + \langle 6.1 \times 10^{-16}, 0, 0\rangle \text{ N}\cdot\text{s}$$

$$\boxed{\bar{P}_f = \langle 2.40 \times 10^{-16}, 0, 0\rangle \text{ N}\cdot\text{s}}$$

C. [15 pts] Following the collision, the same ion is located at the origin at time $t = 0$, and no forces are acting upon it. What will be its position after a time of 1 μs? (1 $\mu\text{s} = 1 \times 10^{-6}$ s)

No net force means $\bar{V}_{avg} = \bar{V}_i = \bar{V}_f$, so

$$\bar{r}_f = \bar{r}_i + \bar{V}_{avg} \Delta t$$

$$= \bar{r}_i + \bar{V}_i \Delta t$$

Grading Rubric

- -1 pt Clerical
- -3 pt Minor
- -6 pt Major
- -12 pt BTN

For $\bar{V}_i = V_i \hat{x}$,

from part B (POE)

$$\gamma m_0 V_i = \frac{m_0 V_i}{\left(1 - \left(\frac{V_i}{c}\right)^2\right)^{1/2}} = P_i \rightarrow V_i = \frac{cP_i}{(c^2 m^2 + P_i^2)^{1/2}}$$

$$= \frac{(3\times10^8)(2.4\times10^{-16})}{\left[(3\times10^8)^2(3.27\times10^{-25})^2 + (2.4\times10^{-16})^2\right]^{1/2}} \ \text{m/s}$$

$$= 2.78 \times 10^8 \ \text{m/s}$$

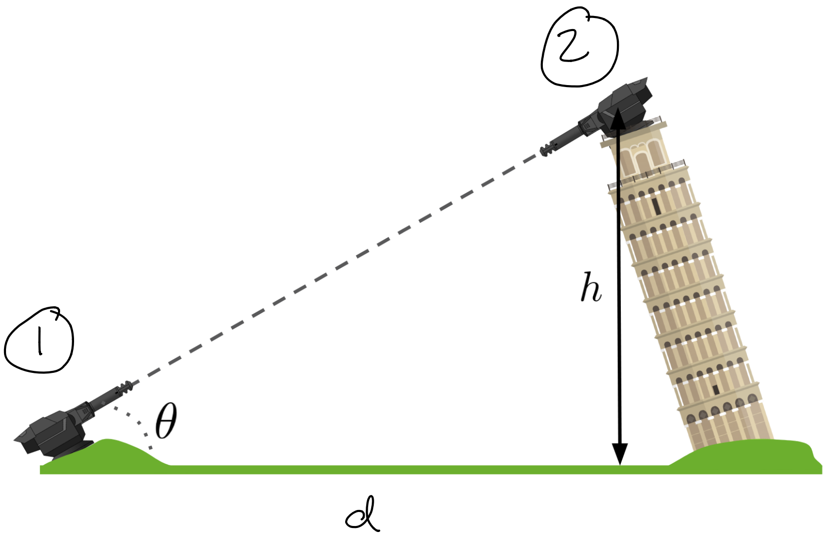
Therefore,

$$\bar{r}_f = \bar{0} + 10^{-6}\ \text{s}\ \langle 2.78\times10^8, 0, 0\rangle \ \frac{\text{m}}{\text{s}}$$

$$\boxed{\bar{r}_f = \langle 278, 0, 0 \rangle \ \text{m}}$$

Problem 3 [25 pts]

Two identical cannons both fire projectiles of mass m at speed v_0. One cannon is placed on the ground and the other at the top of a tower of height h above the other cannon. The two cannons are aimed directly at each other so that the lower cannon makes and angle of θ with the ground as seen in the diagram. Surprisingly, if the cannons are fired simultaneously, the projectiles will collide!



A. [10 pts] How much time Δt passes before the collision. Your answer should be in terms of the known variables stated in the problem and physical constants.

Solve for d: $\tan\theta = h/d \rightarrow d = \dfrac{h\cos\theta}{\sin\theta}$

Cannonball 1: $x_f = 0 + V_0\cos\theta\,\Delta t$

Cannonball 2: $x_f = d - V_0\cos\theta\,\Delta t$

They collide when $V_0\cos\theta\,\Delta t = d - V_0\cos\theta\,\Delta t$

So, $\boxed{\Delta t = \dfrac{d}{2V_0\cos\theta} = \dfrac{h}{2V_0\sin\theta}}$

Grading Rubric

- -1 pt Clerical
- -2 pt Minor
- -4 pt Major
- -8 pt BTN

B. [15 pts] Determine the height of the projectiles, relative to the ground, at the time of the collision. If you were unable to complete part A. please leave your answer in terms of Δt.

For a constant force, $\bar{V}_{avg} = \dfrac{\bar{V}_f + \bar{V}_i}{2}$. So,

$$\left.\begin{aligned} \bar{r}_f &= \bar{r}_i + \bar{V}_{avg}\Delta t \\ \bar{V}_f &= \bar{V}_i + \frac{\bar{F}_{net}}{m}\Delta t \end{aligned}\right\} \rightarrow \bar{r}_f = \bar{r}_i + \bar{V}_i\Delta t + \frac{1}{2}\frac{\bar{F}_{net}}{m}\Delta t^2$$

Grading Rubric

- -1 pt Clerical
- -3 pt Minor
- -6 pt Major
- -12 pt BTN

Looking only along the y axis,

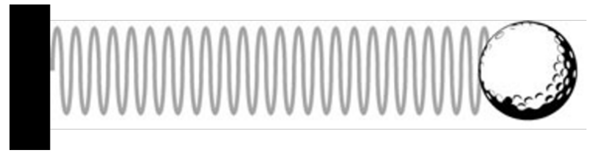
$$y_f = y_i + V_i\Delta t + \frac{F_{net,y}}{2m}\Delta t^2 \quad \text{for} \quad \bar{F}_{net} = -mg\,\hat{y}$$

Cannonball 1: $y_f = 0 + V_0\sin\theta\,\Delta t - \dfrac{g}{2}\Delta t^2$

$$\boxed{y_f = \frac{h}{2} - \frac{g}{8}\left(\frac{h}{V_0\sin\theta}\right)^2}$$

Problem 4 [25 pts]

Consider a horizontal spring attached to a wall, the other end is free to move and has a ball with mass 2 kg attached to it. The ball slides in and out of a frictionless tube so that all motion takes place in the horizontal direction and the net force acting on the ball is just the spring force.



All A. [5 pts] The relaxed length of the spring is 0.3 meters. A constant force of 6 N is applied to the ball and the spring is compressed to a length of 0.22 m while the ball is held motionless. What is the stiffness of this spring?

Equilibrium: $\vec{F}_{net} = \vec{F}_s + \vec{F} = \vec{0} \rightarrow F_s - F = 0$

So,

$$ks - F = 0 \rightarrow k = \frac{F}{s}$$

$$\boxed{K = \frac{6}{(.3-.22)} \frac{N}{m} = 75 \frac{N}{m}}$$

note: $s = |\vec{L}| - L_o$

All B. [5 pts] The spring is now compressed so that its length is 0.15 m. Calculate the magnitude of the force required to hold the ball motionless at this new length.

$|F_s| = |ks|$

POE from part A.

$= (75)(.3-.15)$ N

$$\boxed{F_s = 11.25 \text{ N}}$$

C. [15 pts] The spring is compressed to a length of 0.15 m and the ball is released from rest at time $t = 0$ s. Determine the new force on the the ball 0.05 seconds later? Solve this problem iteratively using a value of $\Delta t = 0.05$ s.

Step :

$\vec{F}_{net} = 11.25 \ \hat{x}$ ← POE from part a.

$$v_f = v_i + \frac{F_{net}}{m}\Delta t$$

$$= 0 + \frac{1}{2}(11.25)(0.05) \ \frac{m}{s}$$

$$= .281 \ m/s$$

$$r_f = r_i + v_f \Delta t$$ ← constant force, so we assume $v_{avg} \approx v_f$

$$= .15 + .281(0.05) \ m$$

$$= .164 \ m$$

Calculate $\vec{F}_s$:

$$\vec{F}_s = -k(x - L_0)\hat{x}$$

$$= -75(.164 - .3)\hat{x} \ N$$

$$= 10.19 \ \hat{x} \ N$$

Grading Rubric	
-1 pt	Clerical
-3 pt	Minor
-6 pt	Major
-12 pt	BTN

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\text{prob}(E) \propto \Omega\,(E)\,e^{-\frac{E}{kT}}$$

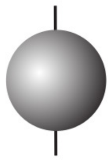
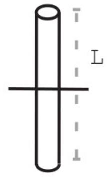
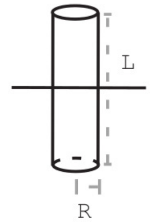
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}