Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

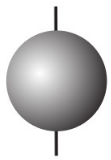
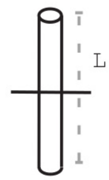
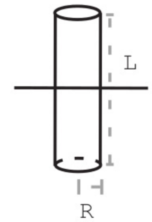
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\dots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis

	 	 	 	
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/K/M/N/HP) - Fall 2023 - Test 2

Please clearly print your name & GTID in the lines below

Name: ______________________________ GTID: ______________________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format, and they must be **readable** in order to be graded. Unreadable files will earn a zero.
  - We recommend you upload a single PDF file for your entire work. You **must** indicate which page corresponds to each problem when you upload and submit.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

KEY
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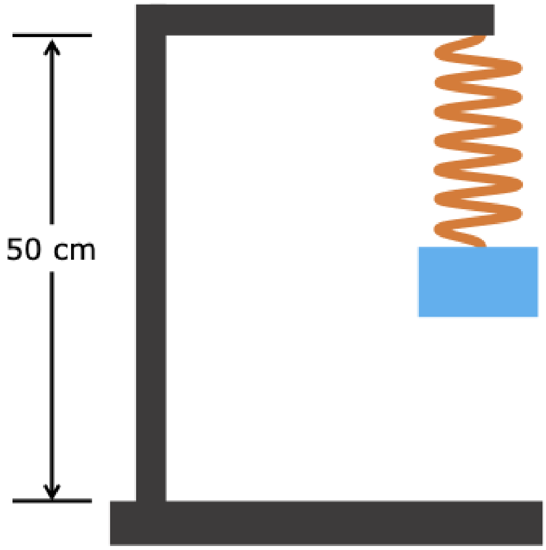
Sign your name on the line above

Problem 1: Springs in Equilibrium [30 pts]

You hang a mass $m = 2.2$ kg on a spring with stiffness $k_1 = 400$ N/m and relaxed length $L_{01} = 30$ cm. The spring is attached to a mount 50 cm above a table, as shown in the figure on the right.



1.1 [15 pts] With the mass hanging motionless, determine what is L_1, the **current length** of the spring.

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{s1} = \vec{0}$$

$$\vec{F}_g = mg\,(-\hat{y})$$

$$\vec{F}_{s1} = -k_1 (L_1 - L_{01})\,\hat{L}_1 = k_1 (L_1 - L_{01})\,\hat{y}$$

$$F_{net,y} = k_1 (L_1 - L_{01}) - mg = 0$$

$$\Rightarrow L_1 - L_{01} = \frac{mg}{k_1}$$

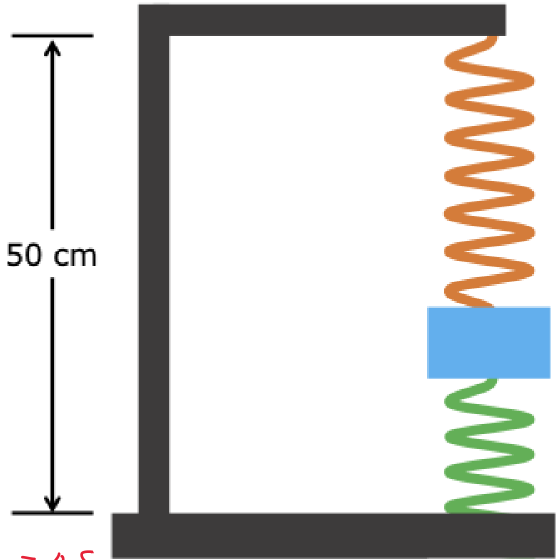
$$\Rightarrow L_1 = \frac{mg}{k_1} + L_{01}$$

$$= \frac{(2.2\text{ kg})(9.8\text{ m/s}^2)}{(400\text{ N/m})} + (0.3\text{ m})$$

$$\approx \boxed{0.354\text{ m}}$$

4:39

1.2 [15 pts] You attach a second spring to the bottom of the mass, with the other end of the second spring attached to the table, resulting in the mass being motionless in a position that is further down than before (see the new figure on this page). What is the **final height of the mass above the table**, assuming it is at rest once it has reached this new position? The stiffness of the second spring is $k_2 = 500$ N/m, and its relaxed length is $L_{02} = 10$ cm.



Hint: You can assume the mass is a point, so the combined length of both springs is 50 cm.

$L_1 + L_2 = L_{tot} = 0.5\ m$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{s1} + \vec{F}_{s2} = \vec{0}$$

$$\vec{F}_g = mg\,(-\hat{y})$$

$$\vec{F}_{s1} = -k_1(L_1 - L_{01})\,\hat{L}_1 = k_1(L_{tot} - L_2 - L_{01})\,\hat{y}$$

$$\vec{F}_{s2} = -k_2(L_2 - L_{02})\,\hat{L}_2 = k_2(L_2 - L_{02})\,(-\hat{y})$$

$$F_{net,y} = k_1(L_{tot} - L_2 - L_{01}) - k_2(L_2 - L_{02}) - mg = 0$$

$$\Rightarrow k_1(L_{tot} - L_{01}) - k_1 L_2 - k_2 L_2 + k_2 L_{02} - mg = 0$$

$$\Rightarrow L_2(k_1 + k_2) = k_1(L_{tot} - L_{01}) + k_2 L_{02} - mg$$

$$\Rightarrow L_2 = \frac{k_1(L_{tot} - L_{01}) + k_2 L_{02} - mg}{k_1 + k_2}$$

$$= \frac{(400\ N/m)(0.5\ m - 0.3\ m) + (500\ N/m)(0.1\ m) - (2.2\ kg)(9.8\ m/s^2)}{(400\ N/m) + (500\ N/m)}$$

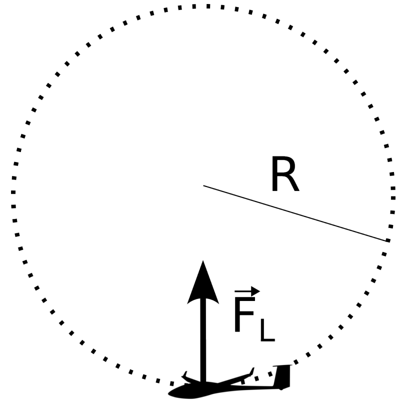
$$\approx \boxed{0.12\ m}$$

13:10

Problem 2: Curving Motion [25 pts]

A glider is an aircraft with no engine. An aircraft flies because of the lifting force produced by the aircraft's wings as it moves through the air. The lifting force points in the direction perpendicular to the wings.

The pilot of a glider wants to fly a loop-the-loop. However, they must be careful not to fly a too tight loop. If the loop is too tight (radius too small), the glider's structural limits are exceeded, and the wings will separate from the glider, resulting in a very bad day for the pilot.



2.1 [10 pts] How much **lifting force** $|\vec{F}_L|$ must the glider's wings produce at the bottom of a loop with radius R? The glider has mass m (which includes the pilot), speed v, and gravity points in the downward direction. Your answer should be a symbolic expression.

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_L = \vec{F}_{cent} = \frac{mv^2}{R}\hat{n}$$

$$\vec{F}_g = mg(-\hat{y}) = mg(-\hat{n})$$

$$\vec{F}_g + \vec{F}_L = \vec{F}_{cent}$$

$$\Rightarrow mg(-\hat{n}) + \vec{F}_L = \frac{mv^2}{R}$$

$$\Rightarrow \vec{F}_L = \frac{mv^2}{R}\hat{n} + mg\,\hat{n} = \left(\frac{mv^2}{R} + mg\right)\hat{n}$$

$$\Rightarrow \boxed{|\vec{F}_L| = \frac{mv^2}{R} + mg}$$

3:00

2.2 [10 pts] Gliders are certified for a lifting force up to and not exceeding $|\vec{F}_L| = 6mg$, where m is the max gross mass of the glider, including the pilot. What is the **smallest radius** R_{min} of the loop that the pilot can fly at speed v before exceeding the structural limit of the glider?

$$|\vec{F}_L| \leq 6mg$$

$$\Rightarrow \frac{mv^2}{R} + mg \leq 6mg$$

$$\Rightarrow \frac{mv^2}{R_{min}} = 5mg$$

$$\Rightarrow \boxed{R_{min} = \frac{v^2}{5g}}$$

4:07

2.3 [5 pts] Will a glider that has speed v at the bottom of a loop of radius R_{min} **make it to the top of the loop**? Ignore drag and use the energy principle to justify your answer. Show your work.

System: glider + Earth

$$\Delta E = W \Rightarrow \Delta K + \Delta U_g = 0$$

$$\Delta U_g = mg\,\Delta y = mg(2R_{min}) = 2mgR_{min}$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv^2 + 2mgR_{min} = 0$$

$$\Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv^2 - 2mgR_{min}$$

$$\Rightarrow v_f = \sqrt{v^2 - 4gR_{min}} = \sqrt{v^2 - 4g\left(\frac{v^2}{5g}\right)}$$

$$= \sqrt{v^2 - \frac{4}{5}v^2} = \sqrt{\frac{1}{5}v^2} = \boxed{\frac{v}{\sqrt{5}} > 0}$$

Yes, the glider makes it to the top of the loop with non-negative speed.

2.3 [5 pts] Will a glider that has speed v at the bottom of a loop of radius R_{min} **make it to the top of the loop**? Ignore drag and use the energy principle to justify your answer. Show your work.

System: glider + Earth

$$\Delta E = W \Rightarrow \Delta K + \Delta U_g = 0$$

$$v_f = 0 \Rightarrow \Delta K = -\frac{1}{2}mv^2 = -\frac{1}{2}m(5gR_{min}) = -\frac{5}{2}gR_{min}$$

$$-\frac{5}{2}mgR_{min} + mg\Delta y = 0$$

$$\Rightarrow mg\Delta y = \frac{5}{2}gR_{min}$$

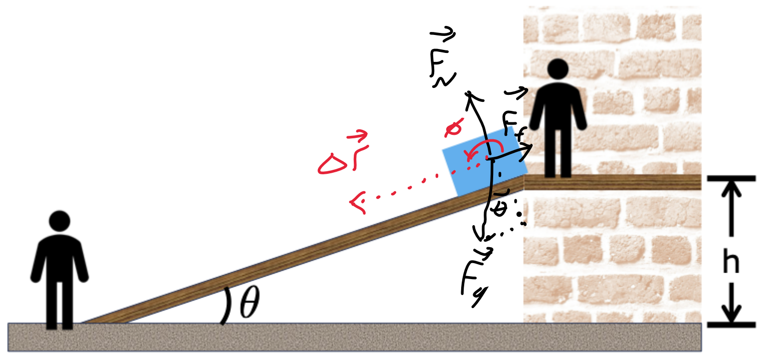
$$\Rightarrow \boxed{\Delta y = \frac{5}{2}R_{min} > 2R_{min}}$$

Yes, the glider has enough kinetic energy to cover the height of the loop.

8:25

Problem 3: Energy Principle [45 pts]

A construction worker places a box of nails (mass m) at the top of a wooden plank, which is at height h above the ground. The plank is leaning against a building, at an angle θ from the ground. The box of nails slides down the plank.

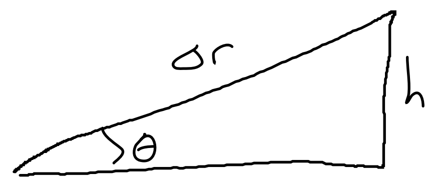


3.1 [15 pts] The coefficient of kinetic friction between the box and the plank is μ_k. What is the **work done by friction** on the box as it slides down the entire length of the plank? Your answer should be a symbolic expression with variables given in the problem (m, h, θ, μ_k) and fundamental constants such as g.

$$F_{net,y} = F_N - F_{g,y} = 0$$

$$\Rightarrow F_N - mg\cos(\theta) = 0$$

$$\Rightarrow \underline{F_N = mg\cos(\theta)}$$



$$\sin(\theta) = \frac{h}{\Delta r}$$

$$|\vec{F}_f| = \mu_k\,|\vec{F}_N| = \mu_k\, mg\cos(\theta)$$

$$W_f = \vec{F}_f \cdot \Delta\vec{r} = |\vec{F}_f|\,|\Delta\vec{r}|\cos(\phi)$$

$$= (\mu_k mg\cos(\theta))\left(\frac{h}{\sin(\theta)}\right)\cos(180°)$$

$$= \boxed{-\mu_k mgh\cot(\theta)}$$

6:47

3.2 [10 pts] What is the speed v_f of the box **when it reaches the bottom of the plank**? You can consider the box to have negligible initial speed. Your final answer should be a symbolic expression.

System: box + Earth

$$\Delta E = W \Rightarrow \Delta K + \Delta U_g = W_f$$

$$\Delta U_g = mg\,\Delta y = mg(-h) = -mgh$$

$$\frac{1}{2}m(v_f^2 - v_i^2) - mgh = -\mu_k mgh\cot(\theta)$$

$$\Rightarrow \frac{1}{2}m v_f^2 = mgh - \mu_k mgh\cot(\theta)$$

$$\Rightarrow v_f = \boxed{\sqrt{2gh - 2\mu_k gh\cot(\theta)} = \sqrt{2gh(1-\mu_k\cot(\theta))}}$$

10:07

3.3 [15 pts] Another construction worker is waiting on the ground. She stops the box when it reaches the end of the plank, then kicks it back up the plank. She wants the box to just reach back up to the original initial height h, and stop moving once it gets there. What **initial speed** v_i does she need to impart on the box when she kicks it?

System: box + Earth

$$\Delta E = W \Rightarrow \Delta K + \Delta U_g = W_f$$

$$\Delta U_g = mg\,\Delta y = mgh$$

$$\frac{1}{2}m(v_f^2 - v_i^2) + mgh = -\mu_k mgh\cot(\theta)$$

$$\Rightarrow \frac{1}{2}mv_i^2 = mgh + \mu_k mgh\cot(\theta)$$

$$\Rightarrow v_i = \boxed{\sqrt{2gh + 2\mu_k gh\cot(\theta)} = \sqrt{2gh(1 + \mu_k\cot(\theta))}}$$

12:48

3.4 [5 pts] Must the speed v_i in Part 3 be **more, the same, or less** than the speed v_f in Part 2 from when the box arrived at the bottom of the plank? Explain your reasoning. Answers without explanations will be marked as incorrect.

Must have reasoning resembling one of the following

1. $V_f = \sqrt{2gh(1-\mu_k \cot(\theta))}$

$V_i = \sqrt{2gh(1+\mu_k \cot(\theta))}$

$0° < \theta < 90° \Rightarrow \cot(\theta) > 0 \Rightarrow \underline{V_i > V_f}$

2. Gravitational potential energy adds to kinetic energy on the way down, but takes away from kinetic energy on the way up

3 More kinetic energy is removed on the way up because gravity and friction both act against the displacement.

15:27

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