Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2-(pc)^2=\left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} =< 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}|-L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

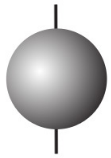
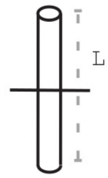
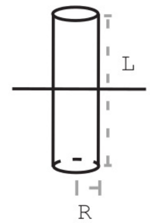
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Test 1 - Summer 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

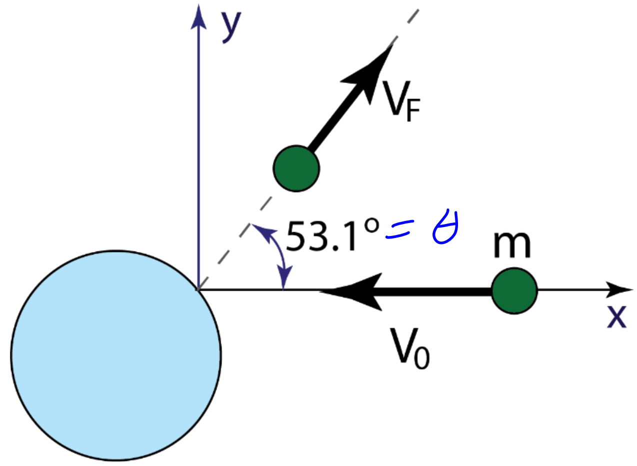
"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

Collision with a surface [30 pts]

As a result of a glancing collision with the surface of the sphere (see the Figure), the velocity of a small ball (mass $m = 305g$, shown in green) changes from $V_0 = 12\ m/s$ directed leftward, to $V_F = 10\ m/s$ directed 53.1° above the horizontal. The ball was in contact with the sphere for a time $0.20s$.



1. [20 pts] What is the magnitude of the change in momentum of the ball?

$$|\Delta \vec{p}| = m|\Delta \vec{v}| = m|\vec{v}_f - \vec{v}_i|$$

$$m = 0.305\ kg$$

$$\vec{v}_i = V_0(-\hat{x}) = \langle -12, 0, 0\rangle\ m/s$$

$$\vec{v}_f = V_f \langle \cos(\theta), \sin(\theta), 0\rangle$$

$$= 10\ m/s\ \langle \cos(53.1°), \sin(53.1°), 0\rangle$$

$$\approx \langle 6, 8, 0\rangle\ m/s$$

$$|\Delta \vec{p}| = (0.305\ kg)|\langle 6, 8, 0\rangle m/s - \langle -12, 0, 0\rangle m/s|$$

$$= (0.305\ kg)|\langle 18, 8, 0\rangle m/s| \approx \boxed{6.01\ kg\ m/s}$$

7:15

2. [10 pts] Find the magnitude of the average force that acted on the ball during the collision.

$$|\vec{F}| = \frac{|\Delta \vec{p}|}{\Delta t}$$

$$|\Delta \vec{p}| = 6.01 \text{ kg m/s}$$

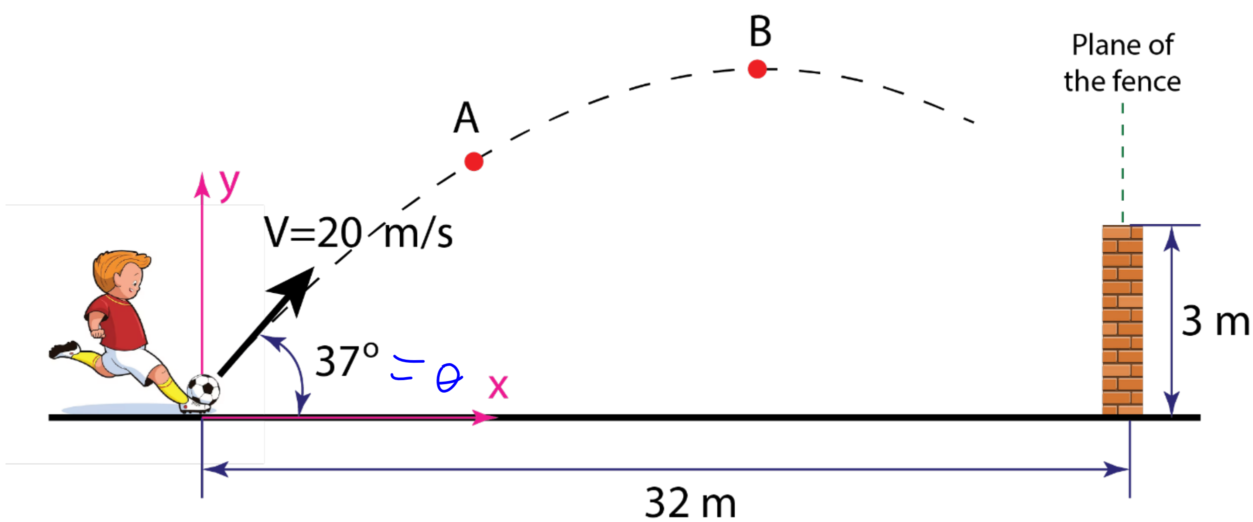
$$\Delta t = 0.20 \text{ s}$$

$$|\vec{F}| = \frac{6.01 \text{ kg m/s}}{0.20 \text{ s}} \approx \boxed{30.04 \text{ N}}$$

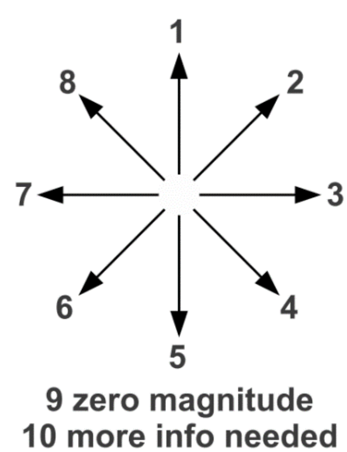
9:02

Soccer Ball [40 pts]

A ball, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown in the Figure. The velocity of the ball as it leaves the kicker's foot is 20 m/s at an angle of 37° above the horizontal. The top of the fence is 3 meters high. The ball hits nothing while in flight and air resistance is negligible. The Figure (not up to scale) also shows a part of the ball trajectory.
In your calculations use $\cos(37°) = 0.80$, $\sin(37°) = 0.60$, $g = 10.0\ m/s^2$.



1. [10 pts] Using the numbered directions shown by the rosette, indicate (by number) which arrow best represents the direction of the quantities listed below. If a quantity has zero magnitude or cannot be determined, indicate that using the corresponding number.



a. [2 pts] __2__ The velocity of the ball at point A.

b. [2 pts] __5__ The acceleration of the ball at point A.

c. [2 pts] __3__ The velocity of the ball at point B.

~9:30

d. [2 pts] ___S___ The acceleration of the ball at point B.

e. [2 pts] ___S___ The change in velocity of the ball between points A and B.

2. [5 pts] Determine the time it takes for the ball to reach the plane of the fence.

$$\vec{V} = \frac{\Delta \vec{r}}{\Delta t} \Rightarrow V_x = \frac{\Delta r_x}{\Delta t} \Rightarrow \Delta t_{fence} = \frac{\Delta r_x}{V_x}$$

$$\Delta r_x = r_{f,x} - r_{i,x} = (32\ m) - (0\ m) = 32\ m$$

$$V_x = V_0 \cos(\theta) = 20\ m/s \cos(37°)$$

$$= 20\ m/s\ (0.8) = 16\ m/s$$

$$\Delta t_{fence} = \frac{32\ m}{16\ m/s} = \boxed{2\ s}$$

13:25

3. [15 pts] Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

Kinematics: assumes $\vec{a}$ constant

$$\vec{r}_f = \vec{r}_i + \vec{v}_0 \Delta t + \frac{\vec{a}}{2}(\Delta t)^2$$

$$h_f = h_i + V_{i,y}\,\Delta t_{fence} - \frac{g}{2}(\Delta t_{fence})^2$$

$h_i = 0\ m$

$V_{i,y} = V_0 \sin(37^\circ) = 20\ m/s\ (0.6) = 12\ m/s$

$\Delta t_{fence} = 2\ s$

$g = 10.0\ m/s^2$

$$h_f = 0\ m + (12\ m/s)(2\ s) - \frac{(10.0\ m/s^2)}{2}(2\ s)^2$$

$$= 0\ m + 24\ m - 20\ m = \boxed{4\ m}$$

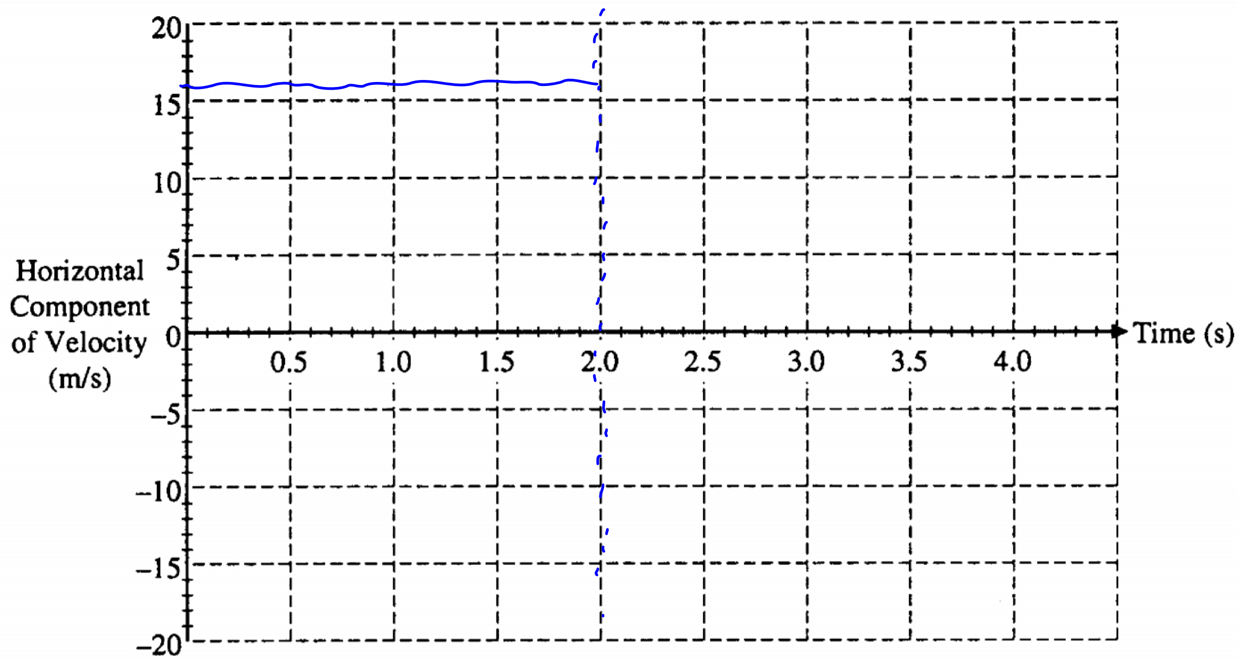
$h_{fence} = 3\ m < h_f$

Ball goes over fence by 1 m.

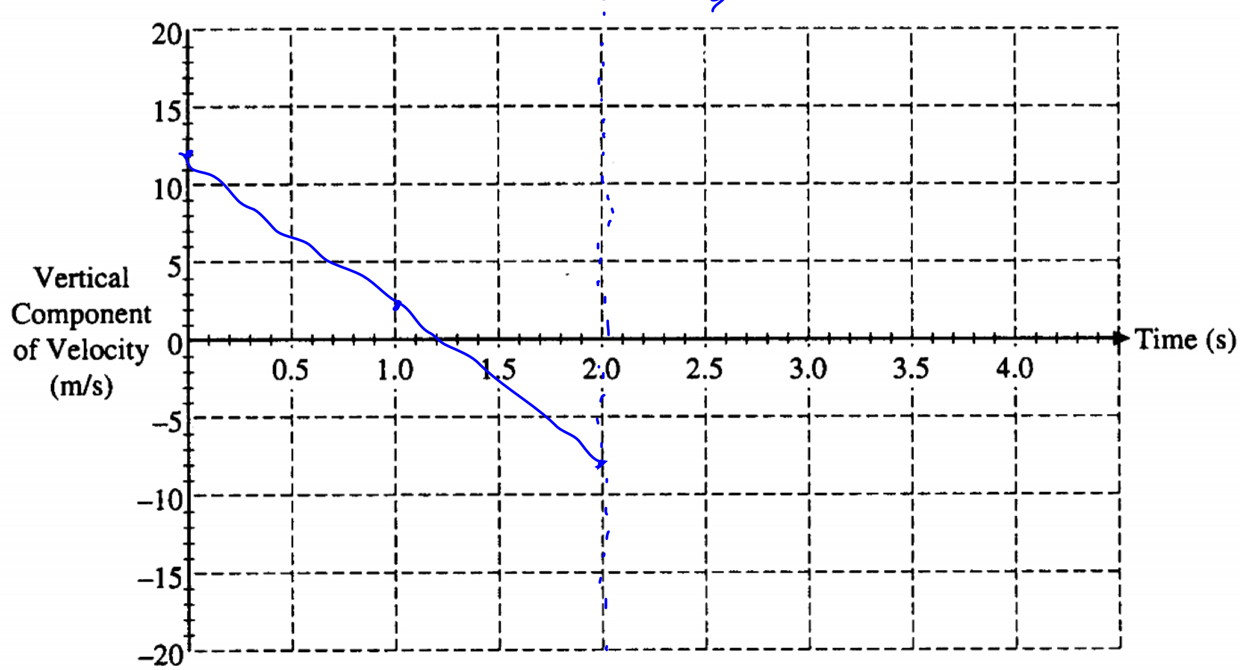
17:46

4. [10 pts] On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence. If you are not using this template sketch these graphs on clear paper.

$v_x = 16$ m/s

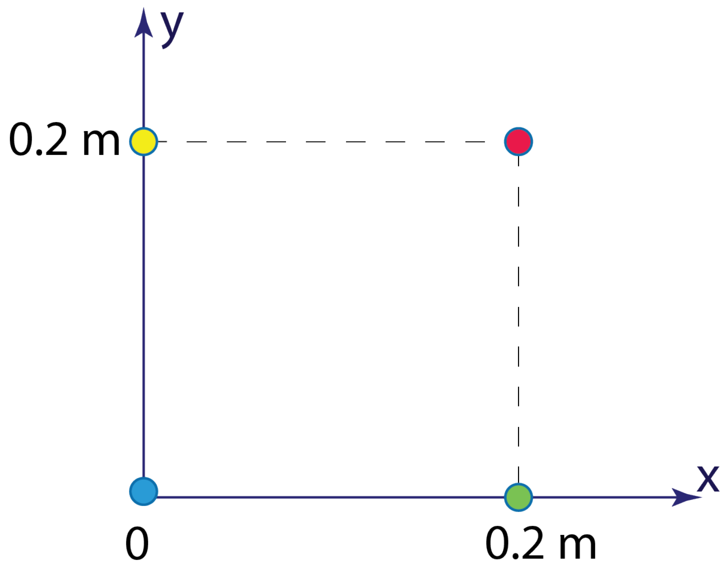


$v_y = 12$ m/s $- (10.0$ m/s$^2) t$



19:17

Force of Gravity [30 pts]



Four identical 13.4 kg masses are located at the corners of the square in the *x*-*y* plane as shown in the figure.

$$G = 6.674 \times 10^{-11} Nm^2/kg^2$$

1. [5 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the upper left corner (in yellow). Your answer must be a vector with appropriate units.

$$\vec{F}_{yellow} = -G \frac{m_y m_r}{|\vec{r}_{yr}|^2} \hat{r}_{yr} = -G \frac{m_y m_r}{|\vec{r}_{yr}|^3} \vec{r}_{yr}$$

$$\vec{r}_{yr} = \langle 0.2, 0, 0 \rangle \, m \Rightarrow |\vec{r}_{yr}| = 0.2 \, m$$

$$\vec{F}_y = -6.674 \cdot 10^{-11} \, Nm^2/kg^2 \frac{(13.4 \, kg)^2}{(0.2 \, m)^2} \langle 1, 0, 0 \rangle \approx \boxed{\langle -3 \cdot 10^{-7}, 0, 0 \rangle \, N}$$

2. [5 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the lower right corner (in green). Your answer must be a vector with appropriate units.

$$|\vec{r}_{gr}| = |\vec{r}_{yr}| \Rightarrow |\vec{F}_g| = |\vec{F}_y|$$

$$\hat{r}_{gr} = \langle 0, 1, 0 \rangle \Rightarrow \boxed{\vec{F}_g = \langle 0, -3 \cdot 10^{-7}, 0 \rangle \, N}$$

25:40

3. [15 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the lower left corner (in blue). Your answer must be a vector with appropriate units.

$$\vec{F}_b = -G \frac{m_b m_r}{|\vec{r}_{br}|^3} \vec{r}_{br}$$

$$G = 6.674 \cdot 10^{-11}\ N\, m^2/kg^2$$

$$m_b = m_r = 13.4\ kg$$

$$\vec{r}_{br} = \langle 0.2, 0.2, 0 \rangle\ m \Rightarrow |\vec{r}_{br}| = 0.2\sqrt{2}\ m \approx 0.28\ m$$

$$\vec{F}_b = -6.674 \cdot 10^{-11}\ N\, m^2/kg^2\ \frac{(13.4\ kg)^2}{(0.28\ m)^3} \langle 0.2, 0.2, 0 \rangle\ m$$

$$\approx \boxed{\langle -1.06 \cdot 10^{-7}, -1.06 \cdot 10^{-7}, 0 \rangle\ N}$$

4. [5 pts] Find the net force on the mass at the upper right corner (in red) caused by the other three masses. Your answer must be a vector with appropriate units.

$$\vec{F}_{net} = \vec{F}_y + \vec{F}_g + \vec{F}_b$$

$$= \langle -3 \cdot 10^{-7}, 0, 0 \rangle\ N + \langle 0, -3 \cdot 10^{-7}, 0 \rangle\ N$$

$$+ \langle -1.06 \cdot 10^{-7}, -1.06 \cdot 10^{-7}, 0 \rangle\ N$$

$$\approx \boxed{\langle -4.06 \cdot 10^{-7}, -4.06 \cdot 10^{-7}, 0 \rangle\ N}$$

32:05