PHYS 2211 Test 2 - Fall 2019

Please circle your lab section and then clearly print your name & GTID

Day	12-3pm	3-6pm
Monday	W01 W08	W02 W09
Tuesday	W03 W10	W04 W11
Wednesday	W05 W12	W06 W13
Thursday	W07	W14

Name: Key

GTID:

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

____

Sign your name on the line above

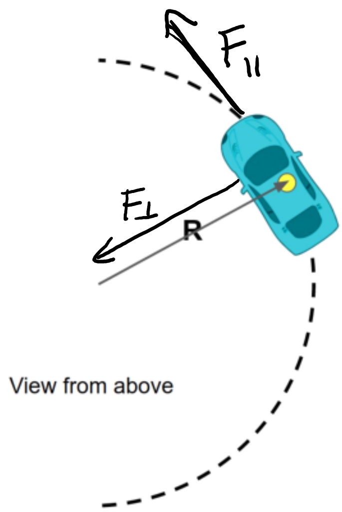
Problem 1 [25 pts]

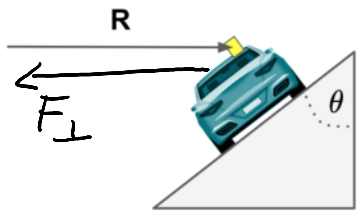
Below is an incomplete program that calculates the motion of a ball attached to a spring. The ball experience two forces: gravitational and spring. The spring has been damaged by overzealous students in lab so that the magnitude of the spring force is proportional to the fifth power of the stretch $|\vec{F}_s| = k * s^5$. Fill in the missing lines of code below.

```
GlowScript 2.8 VPython
g = 9.81 ## acceleration from gravity (in N/kg)
mball = .1524    ## Mass of the ball (in kg)
L0 = 0.3      ## Relaxed length of the spring (in m)
k = 12    ## Spring constant you measured (in N/m^5)
ceiling = box(pos=vector(0,0,0), size=vector(0.2, 0.01, 0.2))
ball = sphere(pos=vector(-0.1,-0.2,-0.3),radius=0.025, color=color.orange)
spring = helix(pos=ceiling.pos, color=color.cyan, thickness=.003, coils=40, radius=0.015)
ball.vel = vector(0.024,0.084,-0.522)
ball.p = mball*ball.vel
deltat = 0.001  ## timestep (in s)
t = 0        ## start counting time at zero
while t < 10
    ## A. [10 pts] Add statements to calculate Fnet on the ball
     s = ball.pos - ceiling.pos     #or just ball.pos
     smag = mag(s) - L0                          -2 Wrong s or smag

    Fs = -k * smag**5 * norm(s)                  -2 Wrong spring force

    Fgrav = mball * g * vector(0,-1,0)           -4 Wrong gravitational force

    Fnet = Fs + Fgrav                            -2 Wrong net force

    ## B. [9 pts] Add statements to update the momentum, velocity and position of the ball
      pmag = mag(ball.p)  #Need to store for Fparallel

    ball.p =  ball.p + Fnet*deltat               -3 Wrong p update

    ball.vel = ball.p/mball                      -3 Wrong v update

    ball.pos = ball.pos + ball.vel*deltat        -3 Wrong position update

    ## C. [6pts] Add statements to calculate the parallel and perpendicular components
          of the net force

    F_parallel = norm(ball.p)*(mag(ball.p) - pmag)/deltat
                     #or F_parallel = dot(Fnet, norm(ball.p))*norm(ball.p)

    F_perpendicular = Fnet - F_parallel          -3 Wrong F∥

    t = t+deltat                                 -3 Wrong F⊥

                     -1 for Python syntax errors (TAs discuss)
```

Problem 2 [25 pts]

You accidentally leave your favorite GT cup of mass m on the roof of your friends car. Your friend decides to tease you and drive the car in a circle of radius R at a constant speed $|\vec{v}|$ in the hopes your cup will slide off the top of the car. Thankfully, at this speed, the cup is not sliding. The road itself is banked so that the surface of the road and the force of gravity make an angle θ. A view of the shenanigans as seen from above and from behind is shown in the diagram.





A. [5 pts] Calculate the magnitude of the component of the net force, acting on the cup, that is parallel to the cups momentum. Briefly describe how you determined this.

Since $\frac{d}{dt}(|\vec{p}|) = 0$, $|\vec{F}_{\parallel}| = 0$.

All or nothing

B. [5 pts] Calculate the magnitude of the component of the net force, acting on the cup, that is perpendicular to the cups momentum.

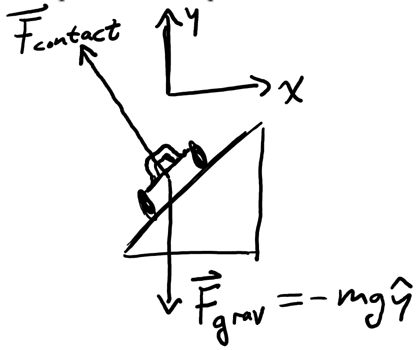
Because the cup is constrained to move in a plane, $\vec{F}_{\perp} = -\frac{mv^2}{R}\hat{r}$ (entirely in the radial direction).

$\vec{F}_{net} = \vec{F}_{\perp} + \cancel{\vec{F}_{\parallel}}^{\,0} \Rightarrow |\vec{F}_{net}| = \frac{mv^2}{R}$

All or nothing

C. [10 pts] Determine the vector contact force (normal and frictional) acting on the cup from the top of the car.

From (B), $\vec{F}_{net} = -\frac{mv^2}{R}\hat{r} = -\frac{mv^2}{R}\hat{x}$.



Also, $\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{contact}$

$$\Rightarrow \vec{F}_{contact} = \vec{F}_{net} - \vec{F}_{grav}$$

$$= -\frac{mv^2}{R}\hat{x} + mg\hat{y}$$

$$\boxed{\vec{F}_{contact} = \left\langle -\frac{mv^2}{R}, mg, 0 \right\rangle}$$

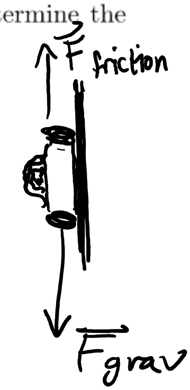
−1 Clerical
−2 Minor
−4 Major
−8 Minimal progress

D. [5 pts] The coefficient of friction between the cup and car is μ. In the limit where $\theta = 0$, determine the minimum speed so that the cup will not slide on the top of the car.

In the $\theta = 0$ limit, the criteria for no sliding is

$$|\vec{F}_{friction}| \geq |\vec{F}_{grav}|$$

$$\Rightarrow \mu\left(\frac{mv_{min}^2}{R}\right) \geq mg \quad \Rightarrow \boxed{|v_{min}| = \sqrt{\frac{gR}{\mu}}}$$



All or nothing

Problem 3 [30 pts]

The Earth (mass 6×10^{24} kg) is in an elliptic orbit around the sun (mass 2×10^{30} kg). The sun is so much heavier that it can be considered fixed (i.e. motionless).

A. [5 pts] The point of closest approach is known as the perihelion, and will next occur on January 5, 2020. On this date, what will be the magnitude of the component of the Sun's gravitational pull that is parallel to the Earth's momentum? briefly explain how you determined this.

At the perihelion, $\frac{d}{dt}(|\vec{p}|) = 0$, so $|\vec{F}_{grav,\parallel}| = 0$.

All or nothing

B. [5 pts] The distance to the sun during the perihelion is 1.5×10^{11} m. What will be the magnitude of the component of the Sun's gravitational pull that is perpendicular to earth's momentum at this point?

$$\vec{F}_{grav} = \vec{F}_{grav,\perp} + \cancel{\vec{F}_{grav,\parallel}}^{\,0}$$

$$\Rightarrow \vec{F}_{grav,\perp} = \frac{mMG}{d^2} = \frac{(6 \times 10^{24}\,kg)(2 \times 10^{30}\,kg)(6.7 \times 10^{-11}\,N \cdot m^2/kg^2)}{(1.5 \times 10^{11}\,m)^2}$$

$$= \boxed{3.57 \times 10^{22}\,N}$$

All or nothing (−1 Clerical)

C. [10 pts] The kinetic energy of the earth during the perihelion is 2.8×10^{33} J. What is the radius of the kissing circle which meets the Earth's elliptic orbit at the perihelion?

At perihelion, $\frac{mv^2}{R_{kiss}} = |\vec{F}_{grav,\perp}| = \frac{mMG}{d^2}$

Also, $K = \frac{1}{2}mv_i^2 \Rightarrow mv^2 = 2K_i$

$$\Rightarrow R_{kiss} = \frac{2K_i}{|\vec{F}_{grav,\perp}|} = \frac{2 \times (2.8 \times 10^{33} J)}{(3.57 \times 10^{22} N)} = \boxed{1.57 \times 10^{11} m}$$

-1 Clerical

-2 Minor

-4 Major

-8 Minimal progress.

<u>Watch for POE!!</u>

D. [10 pts] On July 4, 2020, the earth will be at its farthest point from the sun, known as the aphelion. It is a geometric fact that at this point, the radius of the kissing circle meeting the orbit is identical to that at the perihelion (if you did not solve the previous part you may assume the answer was 1.5×10^{11} m). What is the farthest distance from the sun reached by the earth? (You may find the following relation helpful $x = [-b \pm \sqrt{b^2 - 4ac}]/2a$).

At the aphelion, $\frac{mv_f^2}{R} = \frac{mMG}{L^2}$, where R is the kissing radius found in (C) and L is the farthest point from the sun.

Also, since $\Delta E = 0$, $K_f + U_f = K_i + U_i$

$$\Rightarrow \frac{1}{2}mv_f^2 + \left(-\frac{GMm}{L}\right) = \frac{1}{2}mv_i^2 + \left(-\frac{GMm}{d}\right)$$

$$\Rightarrow mv_f^2 = 2\Big(\underbrace{\tfrac{1}{2}mv_i^2}_{K_i} + \frac{GMm}{L} - \frac{GMm}{d}\Big)$$

$$\Rightarrow \frac{mv_f^2}{R} = \frac{mMG}{L^2} = \frac{2}{R}\left(K_i + \frac{GMm}{L} - \frac{GMm}{d}\right)$$

$$\Rightarrow \frac{mMGR}{2} = \left(K_i - \frac{GMm}{d}\right)L^2 + GMmL$$

$$\Rightarrow \boxed{\left(K_i - \frac{GMm}{d}\right)L^2 + (GMm)L - \frac{mMGR}{2} = 0}$$

$a = -2.56 \times 10^{33}$

$b = 8.04 \times 10^{44}$

$c = -6.314 \times 10^{55}$

$$\Rightarrow \boxed{L = 1.59 \times 10^{11}\ m}$$

−1 Clerical

−2 Minor

−4 Major

−8 Minimal progress

Students are not required to solve the quadratic. Watch for POE!!

Problem 4 [25 pts]

An alpha particle with mass 6.8×10^{-27} kg is moving with a speed of 2.85×10^8 m/s.

A. [5 pts] Calculate the rest energy of the alpha particle.

$$E_{rest} = mc^2$$
$$= (6.8 \times 10^{-27}\ kg) \times (3 \times 10^8\ m/s)^2$$
$$= \boxed{6.12 \times 10^{-10}\ J}$$

All or nothing

B. [10 pts] Calculate the kinetic energy of the alpha particle.

$$K = E_{total} - E_{rest} = \gamma mc^2 - mc^2.$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 3.20256$$

$$\Rightarrow K = (\gamma - 1)mc^2 = (3.203 - 1)(6.12 \times 10^{-10}\ J)$$
$$= \boxed{1.34797 \times 10^{-9}\ J}$$

-1 Clerical

-2 Minor

-4 Major

-8 Minimal progress

Watch for round-off error and POE!!

C. [10 pts] The same alpha particle enters a new region of space at location $\vec{r}_i = \langle 1, 0, 0 \rangle$ m and moves to location $\vec{r}_f = \langle 1.5, -0.1, 0 \rangle$ m in a short amount of time. During this time, the alpha particle experiences a constant net force $\vec{F}_{net} = \langle -1.5 \times 10^{-9}, -0.5 \times 10^{-9}, 0 \rangle$ N. Determine the new kinetic energy of the alpha particle at $\vec{r}_f$.

$$\Delta E = W \Rightarrow \cancel{\Delta E_{rest}}^{0} + \Delta K = \vec{F} \cdot \Delta \vec{r}$$

$$\Rightarrow K_f = K_i + \vec{F} \cdot \Delta \vec{r}$$

$$= 1.34797 \times 10^{-9}\,\text{J} + \langle -1.5 \times 10^{-9}, -0.5 \times 10^{-9}, 0 \rangle \langle 1.5 - 1, -0.1 - 0, 0 \rangle\ \text{N} \cdot \text{m}$$

$$= \boxed{6.4797 \times 10^{-10}\,\text{J}}$$

−1 Clerical

−2 Minor

−4 Major

−8 Minimal progress

Watch for round-off error and POE!!