PHYS 2211 K

Week 2, Lecture 1
2022/01/18
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6 clicker questions today

On today's class…

1. Momentum and Forces
2. Newton's 2nd Law (The Momentum Principle)
3. Constant velocity motion
4. Constant force motion

Last week: Vectors!

$$\vec{r} = < r_x, r_y, r_z >$$

Component form

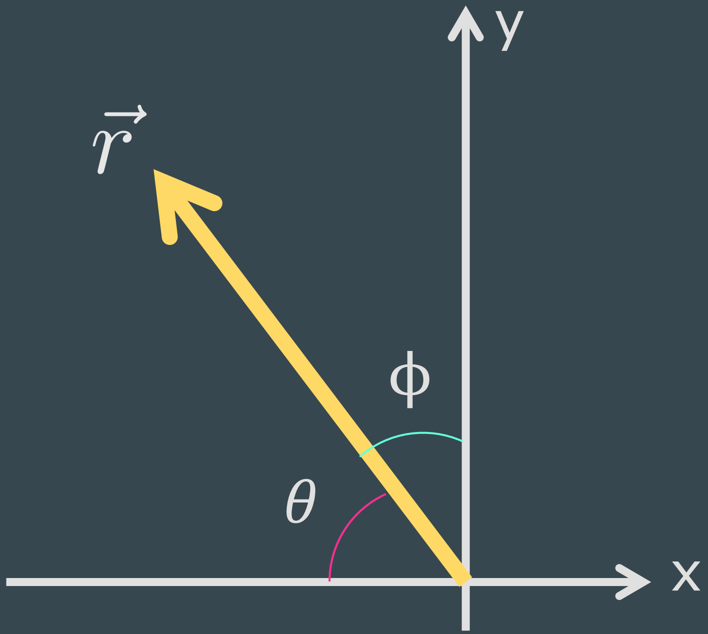
$$|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

Magnitude

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{< r_x, r_y, r_z >}{\sqrt{r_x^2 + r_y^2 + r_z^2}}$$

Unit vector

Last week: Vectors!



CLICKER 1: Choose your chosen one

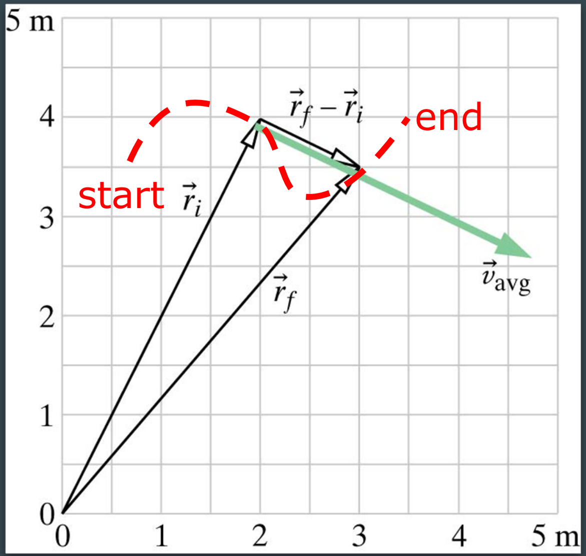
A

B

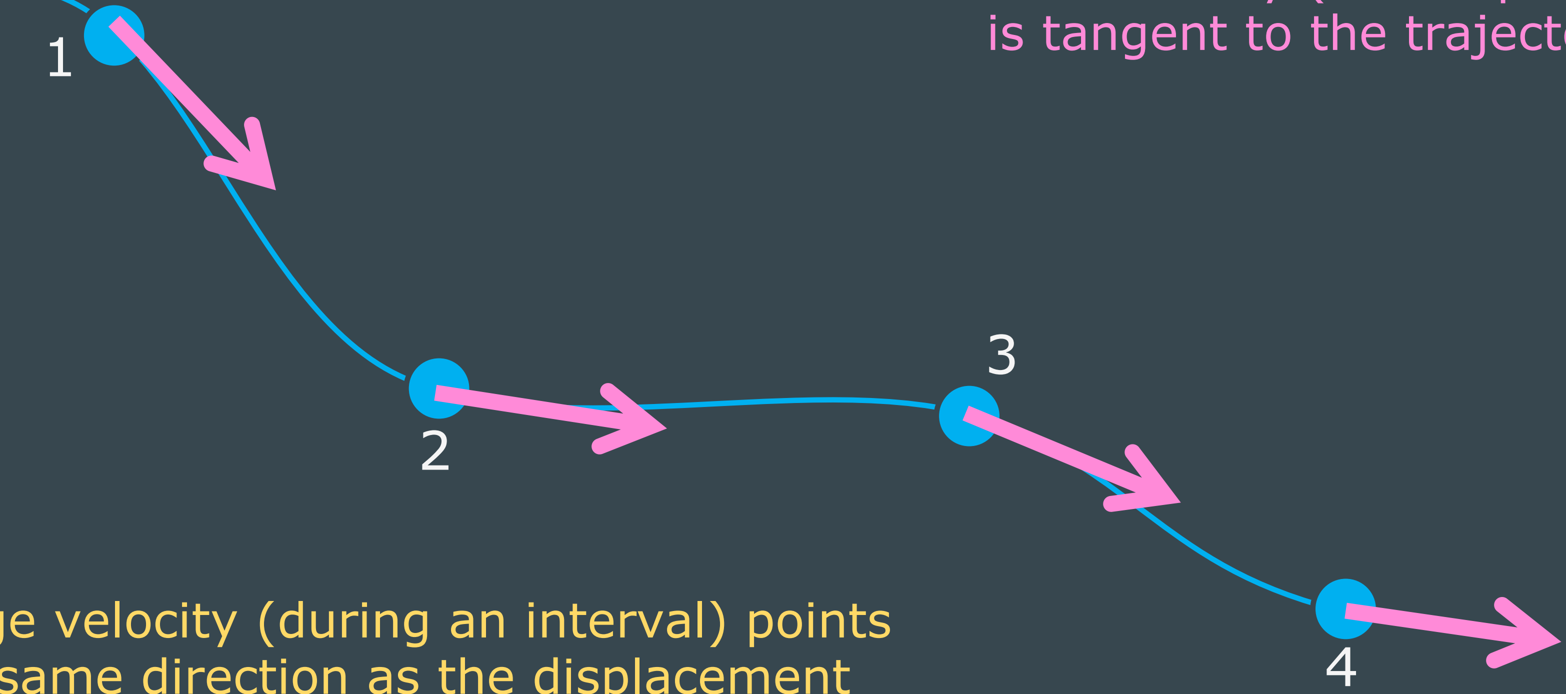
C

Position, Displacement, Velocity

- Position $\vec{r} = < r_x, r_y, r_z >$
- Displacement $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
- Instantaneous velocity (at a single point) $\vec{v} = \frac{d\vec{r}}{dt}$
- Average velocity (during a time interval) $\vec{v}_{avg} \equiv \frac{\Delta\vec{r}}{\Delta t}$



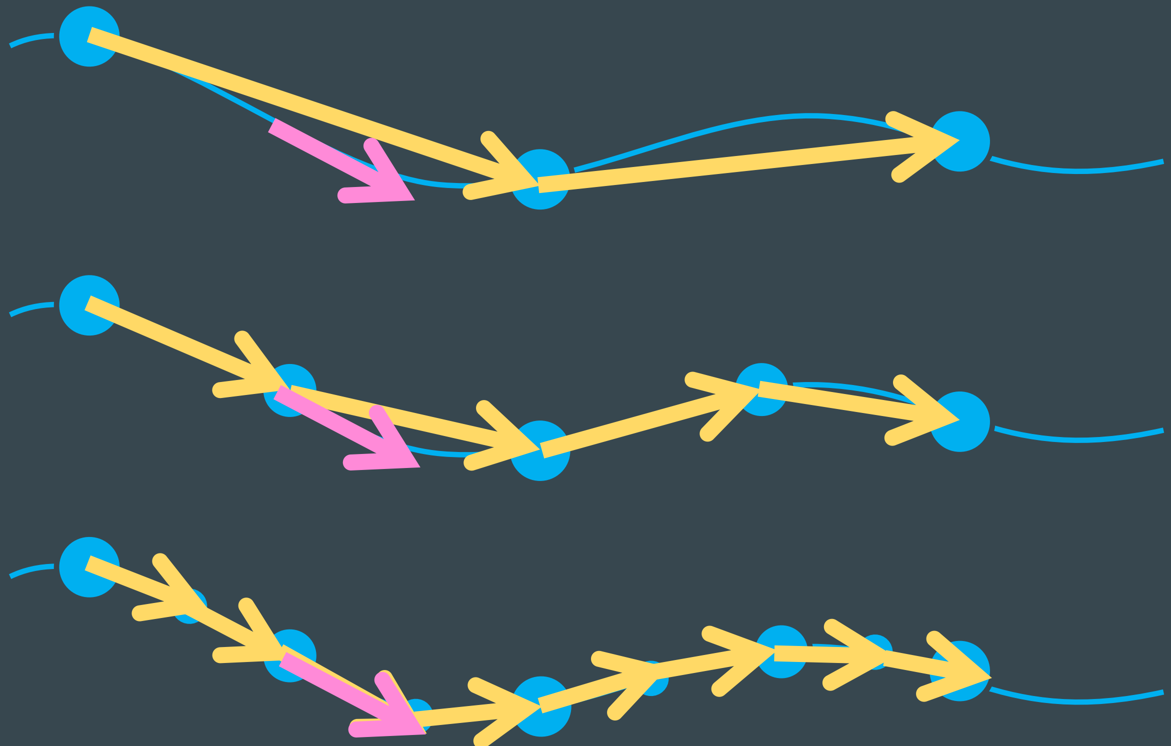
Instantaneous vs Average Velocity



Instantaneous velocity (at one point) is tangent to the trajectory

Average velocity (during an interval) points in the same direction as the displacement

As Δt gets smaller, $\vec{v}_{\mathrm{avg}}$ gets closer to $\vec{v}$



CLICKER 2: The position of an object as a function of time is given by: $\vec{r} = < at^2, bt + 2, c/t^3 >$

What is the instantaneous velocity of this object as a function of time? (note that "a", "b", and "c" are constants)

A. $\vec{v} = < 2at, b + 2, -3ct^{-2} >$

B. $\vec{v} = < 2at, b, -3ct^{-4} >$

C. $\vec{v} = < a/t, bt, 3c/t^2 >$

D. $\vec{v} = < at, b, ct^3 >$

The Position Update Formula

- Measuring two positions (at two different times) allows us to determine the average velocity for that object during that time interval

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

- If we know the initial position and the average velocity, we can predict the final position of the object at a future time

$$\vec{v}_{\text{avg}} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \Rightarrow \boxed{\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}} \Delta t}$$

(smaller deltat = more accurate)

Example: An object is moving in a straight line at constant speed.

At $t=0$, the object is located at position $<2,4,0>$ m.

At $t=0.5$ sec, the object is located at position $<3,3.5,0>$ m.

How much time will pass, from this moment, until the object reaches the ground, at $y=0$?

Solution:

Start by calculating the average velocity

Example: An object is moving in a straight line at constant speed.

At t=0, the object is located at position <2,4,0> m.

At t=0.5 sec, the object is located at position <3,3.5,0> m.

How much time will pass, from this moment, until the object reaches the ground, at y=0?

Solution:

Then use the position update formula, in the y-component only, to find deltat

Momentum

- The momentum of an object contains information about how easy or difficult it is to change the motion of the object (think: stopping a baseball vs stopping a train)
- At low speeds, momentum is defined as $$\boxed{\vec{p} = m\vec{v}}$$
- Note that this means that $\vec{v} = \dfrac{\vec{p}}{m}$
- Units of momentum: kg m/s

Relativistic Momentum

- When an object moves close to the speed of light, the momentum needs a relativistic correction (Lorentz factor; "gamma")

$$\vec{p} = \gamma m \vec{v} \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - (v^2/c^2)}}$$

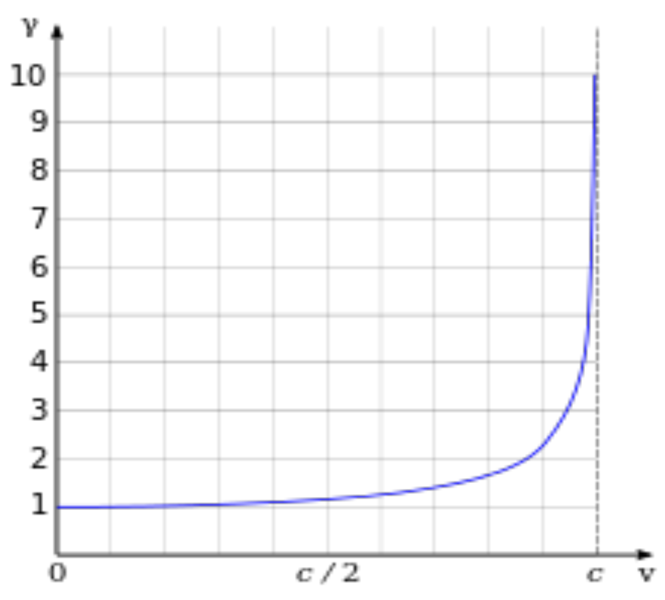
- The (exact!) speed of light is $c = 299792458$ m/s

  but you can just use $c = 3 \times 10^8$ m/s in all calculations

  (note that 3×10^8 can also be written as 3e8)

Relativistic Momentum

- When do you need relativistic momentum?
  Answer: Only at very VERY high speeds!



|v|	|v|/c	γ
0	0	1
3	1e-8	1.0000
300	1e-6	1.0000
3e6	1e-2	1.0001
3e7	0.1	1.005
1.5e8	0.5	1.1547
2.997e8	0.999	22.36
2.9997e8	0.9999	70.7124
3e8	1	∞

Newton's 2nd = The Momentum Principle

- Relates the net interaction between system and surroundings to the observed motion (changes in momentum) of the system

$$\boxed{\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}}$$

without derivatives, $\vec{F}_{\text{net}} = \dfrac{\Delta\vec{p}}{\Delta t}$

But isn’t Newton’s 2nd law $\vec{F} = m\vec{a}$???

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

The Momentum Update Formula

Start here: $\vec{F}_{\text{net}} = \dfrac{d\vec{p}}{dt}$

The Velocity Update Formula

Start here:

$$\vec{p}_f = \vec{p}_i + \vec{F}_{\mathrm{net}}\Delta t$$

The Net Force

- The vector sum of all the forces acting on the system
- $\vec{F}_{\text{net}} = 0$ means that all the forces acting on the system cancel out
  - System is in equilibrium
  - Results in constant velocity motion
  - Does not necessarily mean there are NO FORCES AT ALL acting on the system!

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$ (velocity update formula)

CLICKER 3: You push a book across a table. In order to keep the book moving with constant velocity, you have to keep pushing with a constant force. Which of these statements explains this?

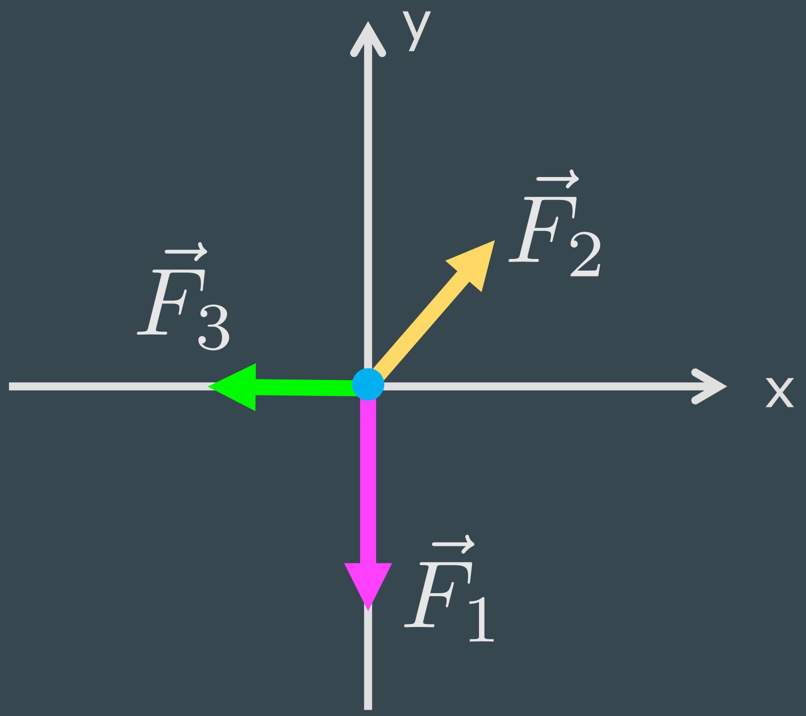
A. A net force is necessary to keep an object moving

B. To make the net force on the book zero, you must push with a force equal and opposite to the friction force on the book

C. The force you exert must be slightly larger than the friction force or the book will stop moving

The Net Force: Force Diagrams

- Represent system as a point, and put it at the origin of the coordinate system
- Represent each force with an arrow, pointing in the direction of the force (angles are important!
- The relative lengths of the arrows represent the relative strengths of the forces



CLICKER 4: An object is acted on by these three forces:

F_1 = 8.3 N pointing down (-yhat),
F_2 = <2,4,0> N, and
F_3 = 1 N pointing left (-xhat).

What is the net force on the object?

A. Fnet = 0
B. Fnet = <10.3, 5, 0> N
C. Fnet = <1, -4.3, 0> N
D. Fnet = <9.3, 4, 0> N
E. Fnet = <-3, 4.3, 0> N

Applying Newton's 2nd Law

1. Chose your system
   - Everything else belongs in the surroundings and exerts forces
2. Draw a force diagram and find the net force
   - All forces must be due to interactions with surroundings
3. Chose your time interval
   - Given in the problem or estimated from motion data
4. Substitute known values and solve for the unknowns
   - You usually need to update the velocity, then update the position, then update the net force again if needed (iteration)
5. Check the units and reasonableness of your answer
   - Does your answer pass the sniff test?

Applying Newton's 2nd Computationally

1. Divide the total time into smaller timesteps

2. At each timestep (meaning, inside the **`while`** loop):
    - Compute the net force: **`Fnet`**
    - Update the velocity:
      `ball.vel = ball.vel + (Fnet/ball.m)*deltat`
    - Update the position:
      `ball.pos = ball.pos + ball.vel*deltat`
    - Update time: **`t = t + deltat`**

3. Repeat!

CLICKER 5: Given a net force of <1,0,3> N and an initial velocity of <0,-2,1> m/s, determine the final velocity after 0.3 sec for an object of mass 10 kg.

A. $\vec{v}_f$ = <0.03, -2, 1.09> m/s

B. $\vec{v}_f$ = <0.3, -2, 1.9> m/s

C. $\vec{v}_f$ = <-0.03, 2, 0.1> m/s

D. $\vec{v}_f$ = <1.1, -2, 1.3> m/s

E. $\vec{v}_f$ = <0.3, -2, 1.3> m/s

Given a net force of <1,0,3> N and an initial velocity of <0,-2,1> m/s, determine the final velocity after 0.3 sec for an object of mass 10 kg.

Estimating $\vec{v}_{\text{avg}}$ for non-zero net force

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t \qquad\qquad \vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\Delta t$$

- When $\vec{F}_{\text{net}}$ is constant, we can approximate $\vec{v}_{\text{avg}}$ as: $\vec{v}_{\text{avg}} = \dfrac{\vec{v}_i + \vec{v}_f}{2}$

- When $\vec{F}_{\text{net}}$ is not constant, we approximate $\vec{v}_{\text{avg}}$ as: $\vec{v}_{\text{avg}} \approx \vec{v}_f$

We'll come back to this next week when we first encounter a non-constant force (springs)

Example of a constant force: Gravity near the surface of Earth ("weight")

$$\vec{F}_g = < 0, -mg, 0 >$$

- g is the acceleration due to gravity at the surface of Earth
- $g = 9.8\ m/s^2$
- Gravity at the surface of Earth is a constant force, and always points down (towards the ground)

CLICKER 6: An object of mass m falls from rest straight down from a height h. Determine the object's velocity when it reaches the ground t seconds later.

A. $\vec{v}_f = gt$

B. $\vec{v}_f = -gt$

C. $\vec{v}_f =< 0, gt, 0 >$

D. $\vec{v}_f =< 0, -gt, 0 >$

What if the problem instead was:

An object of mass m falls from rest straight down from a height h. Determine the object’s velocity when it reaches the ground.

We don’t know the time! What now??

A useful kinematic equation

$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

We can derive this from Newton's 2nd law and the position update formula

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\mathrm{net}}/m)\Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\mathrm{avg}}\Delta t$$

A useful kinematic equation

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Back to the problem:

An object of mass m falls from rest straight down from a height h. Determine the object's velocity when it reaches the ground.

Solution:

First find the time (kinematic), then find the velocity (Newton's 2^{nd})

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An object of mass m falls from rest straight down from a height h. Determine the object's velocity when it reaches the ground.

Solution:
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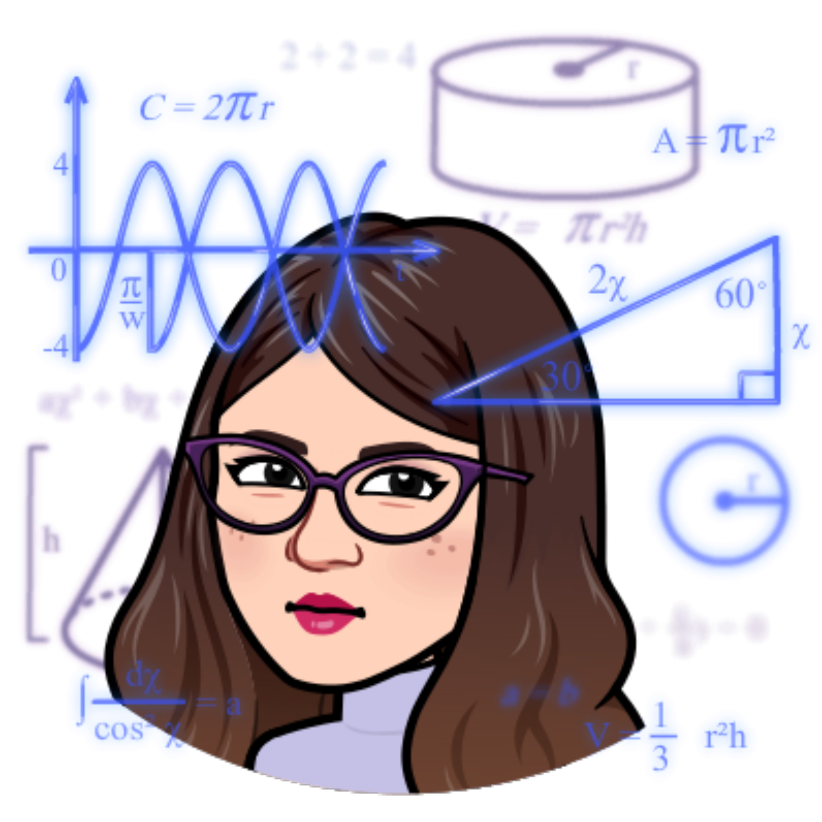
Let’s stop here!



On Thursday:

Problems!
Lots and lots of problems!!