Week 9

Internal Energy & Dissipation

Topics for this week

1. Potential energy of a spring
2. Air drag and dissipation
3. Thermal energy

By the end of the week

1. Predict the change in energy for systems with a spring
2. Model drag
3. Find the change in temperature

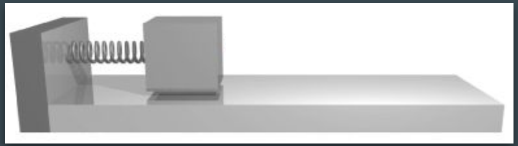
Macroscopic Springs

- What is the potential energy change when a spring is in our system?
  - Hooke's Law provides a modeled for the force
    - The magnitude of the force is equal to the stretch times the spring stiffness
- Consider a spring where x = 0 is at the rest length of the spring

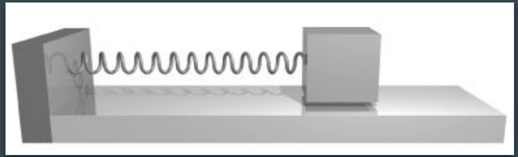
$$\Delta U_{spring} = -\int_i^f -kx\,dx$$

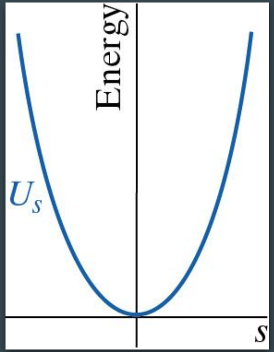
$$\Delta U_{spring} = \frac{1}{2}k\left(s_f^2 - s_i^2\right)$$

Initial State



Final State



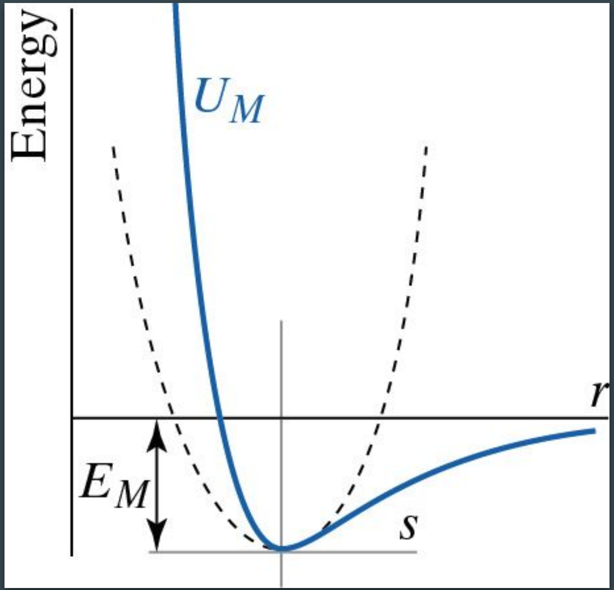


Potential Energy for Neutral Atoms

- Two neutral atoms will interact very little when far apart, attract each other at intermediate distances, and repel each other at very short distances
- Quantum Mechanics gives us a model for the potential energy of two neutral atoms
  - The Morse Potential Energy

$$U_M = E_M\left(1 - e^{-\alpha(r - r_{eq})}\right)^2 - E_m$$

- An idealized potential energy function for a real spring or interatomic bond is a good fit as long as the oscillations about equilibrium are small
  - This is what motivated the ball and spring model for a solid

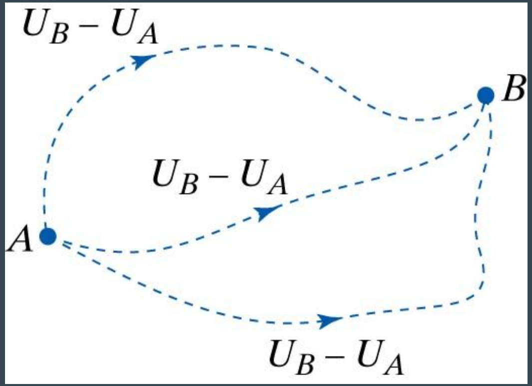


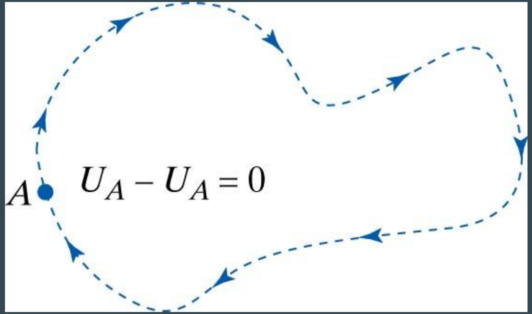
Example: The Bungee Jumper

After earning an 'A' in 2211 you land a job with Guinness World Records! For your first project, they ask you to estimate the spring stiffness k_s required for the cords on their new jump challenge. The requirements are that a jumper of mass 200 kg will only fall 73 m. The standard length of an un-stretched cord is 20 m

Conservative Forces vs dissipation

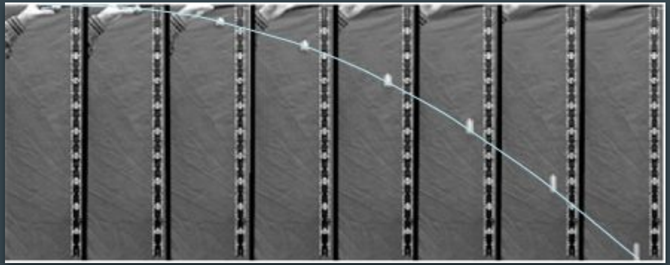
- For a spring mass system the internal interaction depended only on relative position not on velocity
  - We say that the force doing internal work is conservative (not time dependent)
  - As a consequence, the change in potential energy does not depend on the path taken but only on the relative position of objects within the system
- What if the interaction is time dependent?
  - Such as sliding friction or air resistance
- Instead of using potential energy we describe the flow of energy into and out of the system using the concept of work
  - The surroundings do negative work on the system causing irreversible energy dissipation





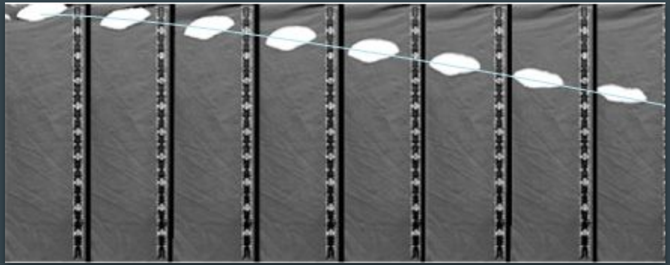
Snapshots of dropped objects

- Consider two different experiments in which we record the position of a falling mass as a function of time
  - A small heavy ball



Equal spacing in time does not result in equal changes in position

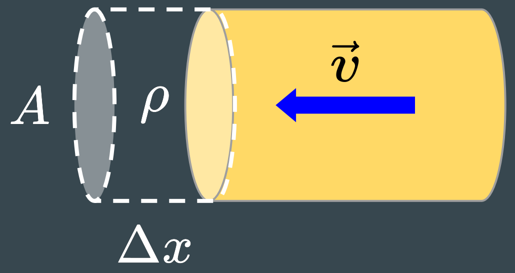
  - A large piece of tissue paper



Equal spacing in time result in equal changes in position

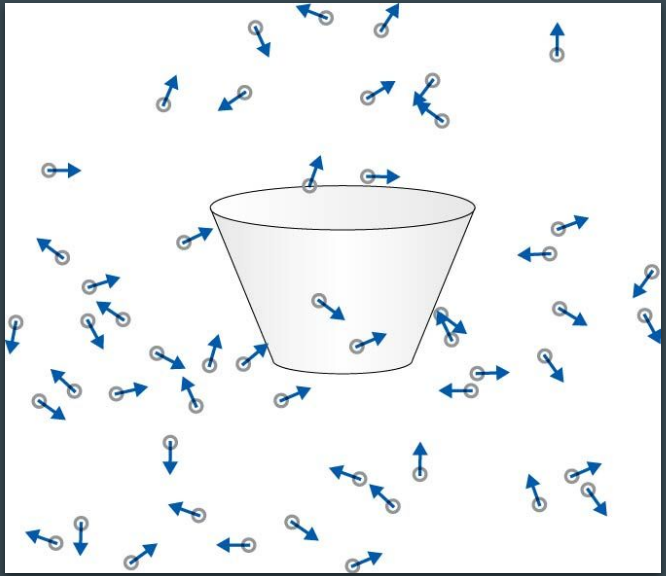
Air resistance

- When an object moves through the air it collides with more air molecules from the front than from behind
  - There is a change of momentum for the air in front of the object



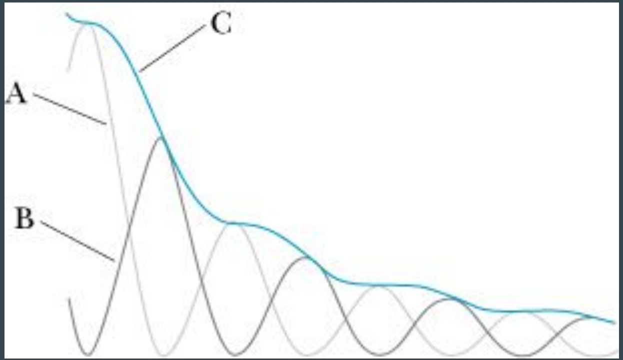
$$|\Delta\vec{p}| = (A\Delta x\rho)\,|\vec{v}|$$

$$\vec{f}_{air} = -\frac{1}{2}CA\rho|\vec{v}|^2\,\hat{v}$$



A real spring mass system

- When we include a force of air resistance on the mass the mass eventually stops moving!
    - https://www.glowscript.org/#/user/ed/folder/My_Programs/program/3DSpring/edit
- Where did the energy go?
    - Some of the “lost” energy goes into the surroundings, the rest goes into increasing the atomic motion of the system
    - This increases the total energy for each atom
    - It is not realistic to think that we can measure these energies one by one



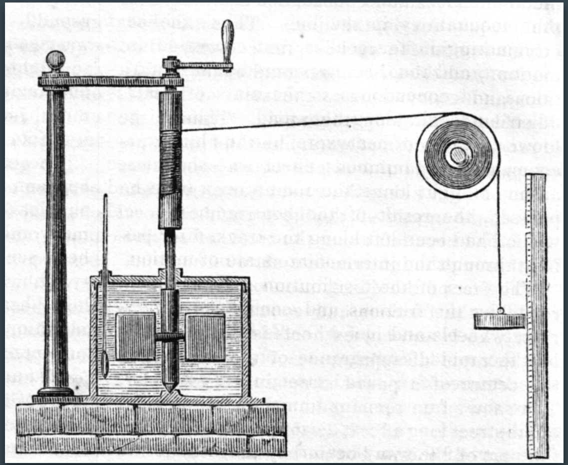
A. The Spring Potential Energy
B. The Kinetic Energy of the block
C. The Total Energy of the system

Temperature

- Instead of measuring the energy of each atom we measure the temperature of the system
    - Related to the average kinetic energy due to the random motion of atoms in the system
    - A thermometer measures thermal expansion measured in degrees C, F, or K
- A change in energy is proportional to the change in temperature
    - C is the specific heat of the material

$$\Delta E_{therm} = mC\Delta T$$

- James Prescott Joule (1818 – 1889)
    - Physicist and Master Brewer who demonstrated the mechanical equivalent of heat

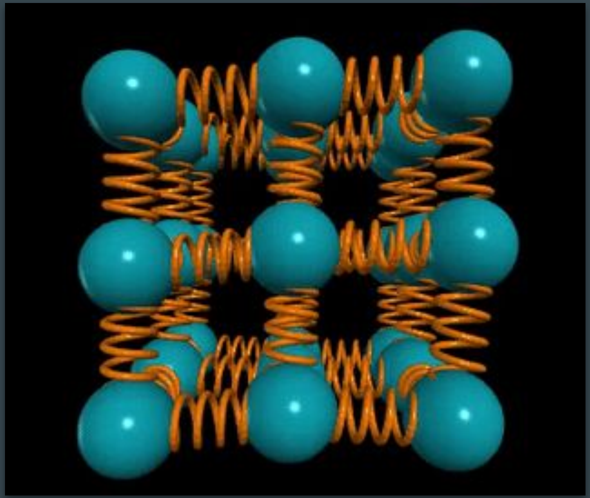


Fundamental energy changes

- There are only three times of fundamental energy in our multiparticle system
  - Rest Energy, Kinetic Energy, Potential Energy

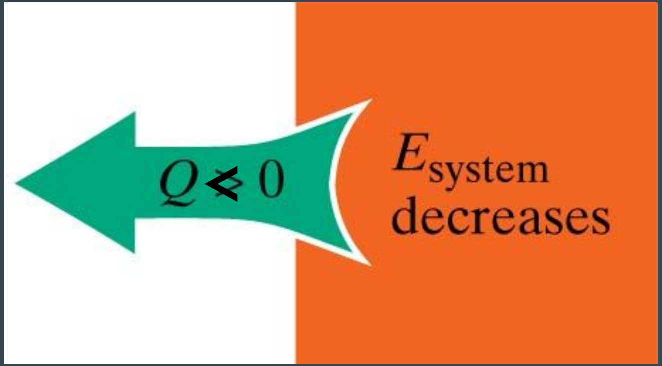
$$\Delta E_{sys} = \Delta E_{rest} + \Delta K_{total} + \Delta U_{total}$$

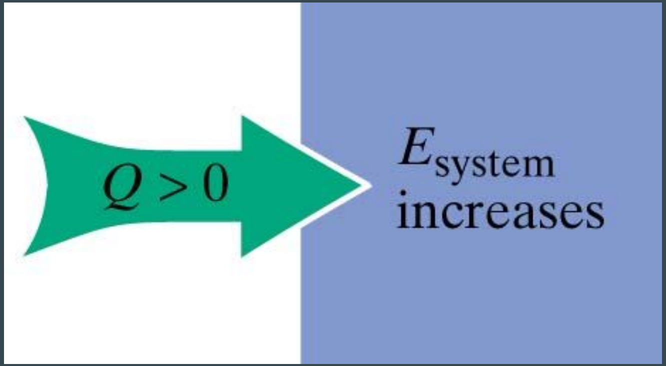
- Internal energy is a way of describing the rest energy of a multiparticle system
  - We can model some of the internal energy changes to make predictions when convenient
    - Rotational, Vibrational, Chemical, Thermal, etc.
  - Thermal energy is a measure of the random microscopic energy of the atoms and their bonds
    - We associate this energy with a temperature!



Energy Transfer Q

- An energy transfer Q can flow between a system and the surroundings when they are in contact and at a different temperature
    - This is microscopic work at the atomic level
        - The fast moving atoms (hot) collide with the slow moving atoms (cold) and transfer energy
        - We don't have a simple way of modelling this
        - If we wait long enough the system and surroundings will reach an equilibrium temperature





Example: The White Russian

The Dude and Walter have gotten into an argument about how much ice to add to a white russian cocktail. They agree that a white russian taste best at a cool 10 degrees Celsius and that they keep there cream, vodka and kahlua in the fridge at 15 degrees Celsius. Ice from a standard freezer is at a temperature of -10 degrees Celsius. If a white russian calls for 0.06 L of cream and 0.14 L of a vodka/kahlua mix, how much ice is needed to bring the drink down to its optimum temperature without melting the ice?

Example: The White Russian Soln.

- Assume the system of liquids is isolated from the environment and at steady state

$$\Delta E_{therm}^{cream} + \Delta E_{therm}^{ethanol} + \Delta E_{therm}^{ice} = 0$$

	Density	Specific Heat
Ice	0.91 kg/L	4.18 J/Cg
Ethanol	0.80 kg/L	2.44 J/Cg
Cream	1 kg/L	3.77 J/Cg

- Calculate the mass of the liquid by using density
  - Note that each liquid has the same final temperature

$$\Delta E_{therm}^{liquid} = (\rho V C)_{liquid}(T_f - T_i)$$

- Solve for the volume of ice

$$-1.132\ J - 1.366\ J + V_{ice} * 38.04\ J/L = 0 \quad \Rightarrow \quad V_{ice} = 0.066\ L$$

- Use approximately three ice cubes and wait until just before the ice melts