Physics 2211 GPS Week 10

Problem #1

After watching "The Big Lebowski" for the first time this summer, you and a friend get into an argument about how much ice to add when making the perfect white russian cocktail. You both agree that, for optimum taste, the cocktail should be enjoyed at 10 degrees Celsius. The two ingredients for the cocktail, cream and a "vodka & kahlua" mix, both leave the fridge at 15 degrees Celsius. Ice from a standard freezer is at a temperature of -10 degrees Celsius. If typical white russian calls for 0.06 L of cream and 0.14 L of the "vodka & kahlua" mix, how much ice is needed to bring the drink down to its optimum temperature?

Ice: density = 0.91 kg/L, C = 4.18 J/(Cg)
Mix: density = 0.8 kg/L, C = 2.44 J/(Cg)
Cream: density = 1 kg/L, C = 3.77 J/(Cg)

We assume that the ingredients are sufficiently isolated from their surroundings when they are mixed, so that a negligible amount of heat transfers to the surroundings. This means that the system is isolated and the energy principle predicts that

$$\Delta E_{sys} = 0.$$

Taking the initial and final state to be before the mixing process and after the combination has reached equilibrium, respectively, there are then no appreciable changes in kinetic or potential energy, and so

$$\Delta E_{therm,sys} = 0.$$

Each ingredient contributes to the total change in thermal energy:

$$\Delta E_{therm,sys} = \Delta E_{therm,mix} + \Delta E_{therm,cream} + \Delta E_{therm,\ ice}$$

$$= m_{mix}C_{mix}\Delta T_{mix} + m_{cream}C_{cream}\Delta T_{cream} + m_{ice}C_{ice}\Delta T_{ice} = 0.$$

Solving for the mass of the ice we find that

$$m_{ice} = -\frac{m_{mix}C_{mix}\Delta T_{mix} + m_{cream}C_{cream}\Delta T_{cream}}{C_{ice}\Delta T_{ice}}$$

$$= -\frac{\left(\left(\left(0.8\frac{kg}{L}\right)(0.14L)\left(2.44\frac{J}{gC^\circ}\right)\right)(10^\circ C - 15^\circ C) + \left(\left(1.0\frac{kg}{L}\right)(0.06L)\left(3.77\frac{J}{gC^\circ}\right)\right)(10^\circ C - 15^\circ C)\right)}{\left(4.18\ \frac{J}{g\,C^\circ}\right)(10^\circ C - (-10^\circ C))}$$

$$\approx 0.030\ kg = 30\ g$$

Note: The heat capacities and the densities do not use the same mass units, so in principle we could convert first. However, in this case there's no need because the same conversion factor appears in the numerator and the denominator so they cancel out.

Problem #2

During 3 hours one winter afternoon, when the outside temperature was 11° C, a house heated by electricity was kept at 25° C with the expenditure of 58 kwh (kilowatt·hours) of electric energy.

(a) What was the average energy leakage in joules per second (watts) through the walls of the house to the environment (the outside air and ground)?

✓ $\Delta t = (3\,\text{hrs})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 10800\ \text{sec}$

✓ $\Delta E = Q = (58\ \text{kWh})(1000\,\text{W}/1\,\text{kW})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 2.088e8\ \text{W}\cdot\text{s}$

(same as Joules)

$\Rightarrow$ energy leakage $= \dfrac{\Delta E}{\Delta t} = \dfrac{2.088e8\ \text{W}\cdot\text{s}}{10800\ \text{sec}} = \boxed{1.93e4\ \text{Watts}}$

$$\text{Watt} = \frac{\text{Joule}}{\text{second}}$$

Alternative: $\dfrac{\Delta E}{\Delta t} = \dfrac{58\ \text{kWh}}{3\,\text{h}} = \dfrac{19.3\ \text{kW} \mid 1000\ \text{W}}{\mid 1\ \text{kW}} = \boxed{1.93e4\ \text{Watts}}$

(b) The rate at which energy is transferred between two systems due to a temperature difference is often proportional to their temperature difference. Assuming this to hold in this case, if the house temperature had been kept at 28° C (82.4° F), how many kwh of electricity would have been consumed?

energy transfer $\propto$ temperature difference $\Longrightarrow \dfrac{Q_1}{\Delta T_1} = \dfrac{Q_2}{\Delta T_2}$

$$\Longrightarrow \frac{58\ \text{kWh}}{(25-11)^\circ\text{C}} = \frac{Q_2}{(28-11)^\circ\text{C}}$$

$$\frac{58}{14} = \frac{Q_2}{17}$$

$$14\,Q_2 = (58)(17)$$

$$Q_2 = \frac{(58)(17)}{(14)} = \boxed{70.4\ \text{kWh}}$$