Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

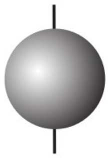
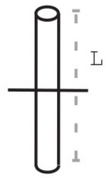
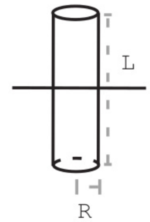
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Test 3 - Summer 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

Rotating Reel [30 pts]

A reel consists of a cylinder of radius R and mass $2M$ with 4 very small (i.e. point) masses M attached at the outer rim of the cylinder (see Figure 1). A reel can freely rotate around a fixed axis through its center. A light rope is wound around the cylinder. At the initial state the reel is motionless. Then a force of constant magnitude F is applied to the rope. At the final state the rope is unwound distance b while the reel acquires angular speed ω (see Figure 2).

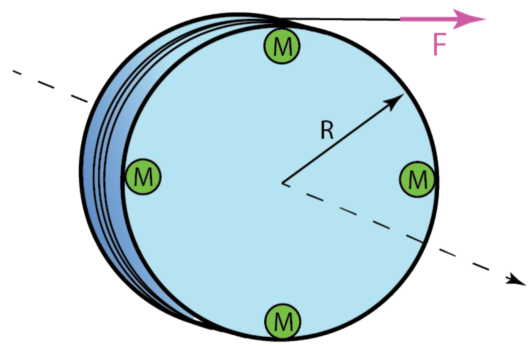


Figure 1. Initial state

Answer all questions in this problem in terms of known quantities R, M, F, b.

1. [10 pts] Determine the total moment of inertia I of the reel.

$$I = I_{reel} + 4 I_{points}$$
$$= \frac{1}{2}(2M)R^2 + 4(0 + MR^2)$$
$$= \boxed{5MR^2}$$

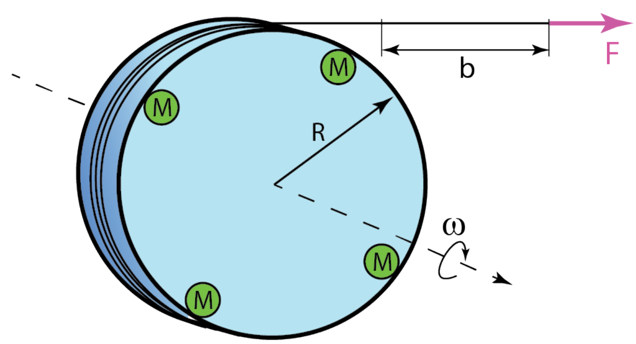


Figure 2. Final state

2. [20 pts] Determine the angular speed ω of the reel at the final state.

<u>Real system</u>

$$\Delta E = \Delta K_{rot} = \frac{1}{2} I_{tot} (w_f^2 - w_i^2) = \frac{5}{2} MR^2 w^2$$

$$W = \vec{F} \cdot \Delta \vec{r}_{poc} = Fb$$

$$\Delta E = W \Rightarrow \frac{5}{2} MR^2 w^2 = Fb \Rightarrow \boxed{w = \sqrt{\frac{2}{5} \frac{Fb}{MR^2}}}$$

S : S8

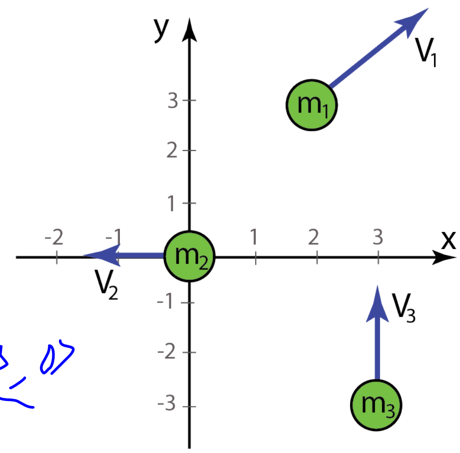
Center of Mass [30 pts]

Three small particles have masses $m_1 = 6.0\ kg$, $m_2 = 4.0\ kg$, and $m_3 = 2.0\ kg$ and are located at

$\vec{r}_1 = \langle 2.0, 3.0, 0.0 \rangle m$, $\vec{r}_2 = \langle 0.0, 0.0, 0.0 \rangle m$, and $\vec{r}_3 = \langle 3.0, -3.0, 0.0 \rangle m$.

Velocities of these particles are:

$\vec{v}_1 = \langle 4.0, 5.0, 0.0 \rangle m/s$, $\vec{v}_2 = \langle -3.0, 0.0, 0.0 \rangle m/s$, and $\vec{v}_3 = \langle 0.0, 3.0, 0.0 \rangle m/s$.



1. [8 pts] Find the position $\vec{r}_{CM}$ of the center of mass of this system.

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

$$= \frac{6\langle 2,3,0\rangle + 4\langle 0,0,0\rangle + 2\langle 3,-3,0\rangle}{6+4+2}$$

$$= \boxed{\langle 1.5, 1.0, 0.0\rangle\ m}$$

2. [8 pts] Find the velocity $\vec{V}_{CM}$ of the center of mass of this system.

$$\vec{V}_{cm} = \frac{\vec{P}_{total}}{M_{total}} = \frac{\sum_i m_i \vec{V}_i}{\sum_i m_i}$$

$$= \frac{6\langle 4,5,0\rangle + 4\langle -3,0,0\rangle + 2\langle 0,3,0\rangle}{6+4+2}$$

$$= \boxed{\langle 1.0, 3.0, 0.0\rangle\ m/s}$$

~12:00

3. [4 pts] Find the translational kinetic energy K_{trans} of this system.

$$K_{trans} = \frac{1}{2} M_{total} |\vec{V}_{cm}|^2$$
$$= \frac{1}{2}(12\ kg)(10\ m^2/s^2)$$
$$= \boxed{60.0\ J}$$

4. [8 pts] Find the total kinetic energy K_{tot} of this system.

$$K_{tot} = K_1 + K_2 + K_3$$

$$K_1 = \frac{1}{2} m_1 |\vec{V}_1|^2 = \frac{1}{2}(6\ kg)(41\ m^2/s^2) = \underline{123.0\ J}$$

$$K_2 = \frac{1}{2} m_2 |\vec{V}_2|^2 = \frac{1}{2}(4\ kg)(9\ m^2/s^2) = \underline{18.0\ J}$$

$$K_3 = \frac{1}{2} m_3 |\vec{V}_3|^2 = \frac{1}{2}(2\ kg)(9\ m^2/s^2) = \underline{9.0\ J}$$

$$K_{tot} = K_1 + K_2 + K_3$$
$$= 123.0\ J + 18.0\ J + 9.0\ J = \boxed{150.0\ J}$$

5. [2 pts] Find the kinetic energy of this system relative to the center of mass K_{rel}.

$$K_{tot} = K_{trans} + K_{rel}$$

$$\Rightarrow K_{rel} = K_{tot} - K_{trans}$$
$$= 150.0\ J - 60.0\ J = \boxed{90.0\ J}$$

18:38

Projectile Launch [40 pts]

A projectile (rocket) of mass m is launched from the surface of the Earth with the initial speed $V_i = \sqrt{\frac{3GM}{2R}}$ where G is the universal gravitational constant, M is the mass of the Earth, and R is its radius (see Figure 1).

1. [10 pts] Determine the total energy of the projectile in the initial state (at the launch time, Figure 1).

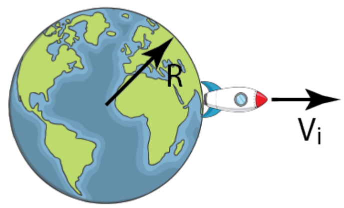


Figure 1. Initial state

$$E_i = K_i + U_{g,i}$$
$$= \frac{1}{2} m V_i^2 - \frac{GMm}{R}$$
$$= \frac{1}{2} m \left(\frac{3GM}{2R}\right) - \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R} - \frac{GMm}{R}$$
$$= \boxed{-\frac{1}{4}\frac{GMm}{R}}$$

2. [10 pts] At the final state the projectile is at the maximum height h relative to the Earth's surface and is momentarily at rest (see Figure 2). Express the total energy of the projectile in the final state in terms of given quantities and h.

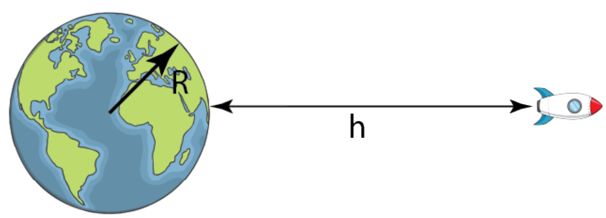


Figure 2. Final state

$$E_f = K_f + U_{g,f} = 0 - \frac{GMm}{R+h}$$
$$= \boxed{-\frac{GMm}{R+h}}$$

22:55

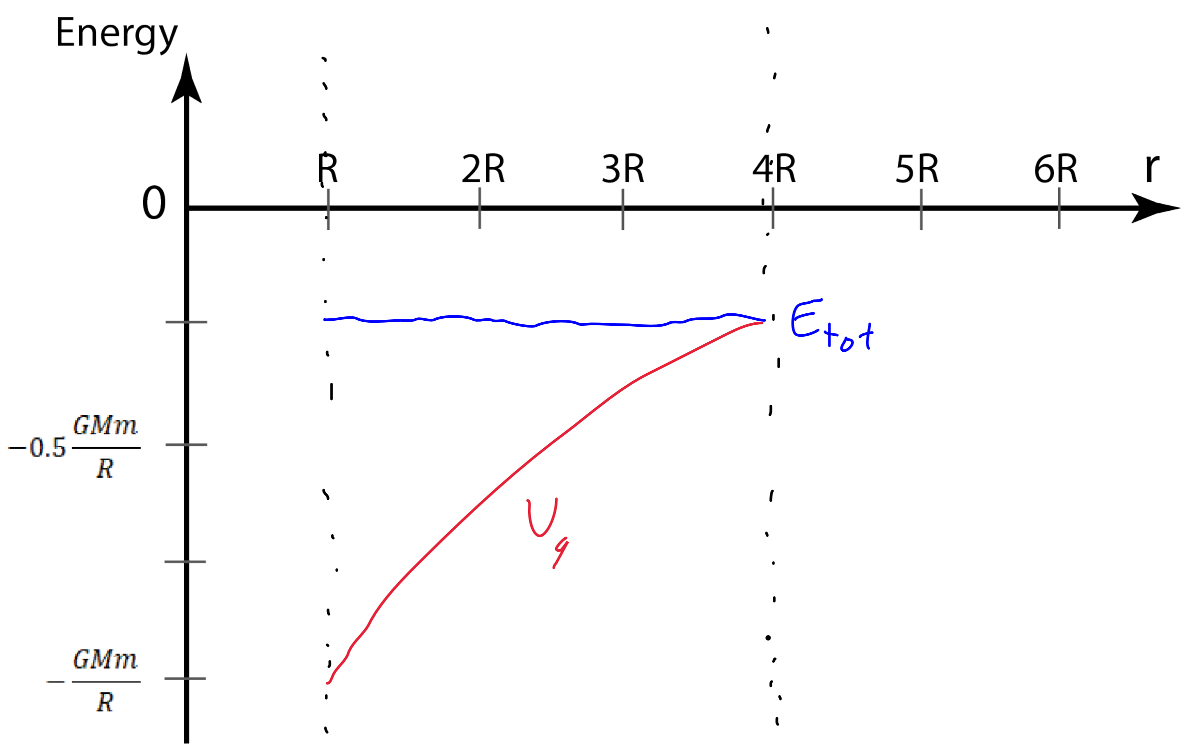
3. [10 pts] Determine the maximum height h of the projectile relative to the Earth's surface in terms of R.

Energy conservation: $\Delta E = 0 \Rightarrow E_i = E_f$

$$\Rightarrow -\frac{1}{4}\frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow \frac{1}{4R} = \frac{1}{R+h} \Rightarrow R+h = 4R \Rightarrow \boxed{h = 3R}$$

4. [10 pts] Sketch the gravitational potential energy and the total energy of the projectile between initial and final states as a function *of the distance to the center of the Earth* r on the provided graph.



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