Test 2 Review

Things from Test 1 that you should remember

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$ (Newton's 2nd)

$$\vec{F}_s = -k(L - L_0)\hat{L}$$ (spring force)

$$\vec{F}_g = \langle 0, -mg, 0 \rangle$$ (near surface of Earth)

$$\vec{F}_g = \frac{GMm}{r^2}(-\hat{r})$$ (in general)

from these two, $g = \frac{GM_E}{R_E^2}$

$$\vec{F}_e = \frac{kq_1q_2}{r^2}(\hat{r})$$ (electrostatic force)

$$\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$$ (Newton's 3rd)

Static & Dynamic Equilibrium

✓ Equilibrium $\Rightarrow$ $\vec{F}_{net} = 0$

✓ Static $\Rightarrow$ $\vec{F}_{net} = 0$ & $\vec{v} = 0$

✓ Dynamic $\Rightarrow$ $\vec{F}_{net} = 0$ & $\vec{v} \neq 0$ (constant)

✓ $\vec{F}_{net}$ = vector sum of all the forces acting on the system

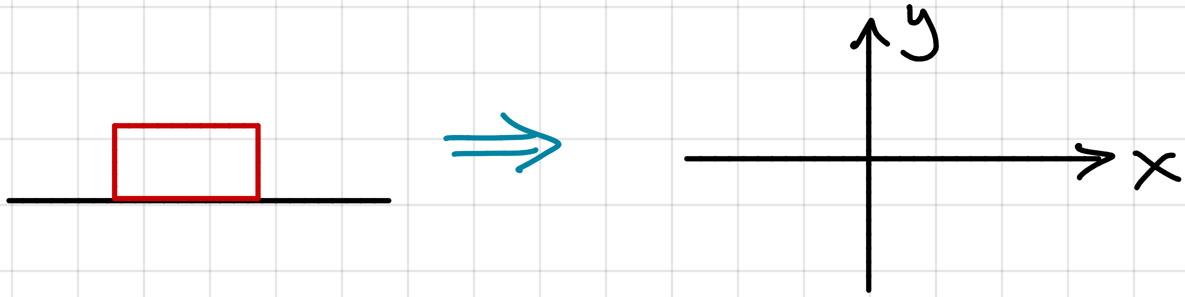
✓ $\vec{F}_{net_x}$ = vector sum of all the x-components of all the forces acting on the system

✓ $\vec{F}_{net_y}$ = vector sum of all the y-components of all the forces acting on the system

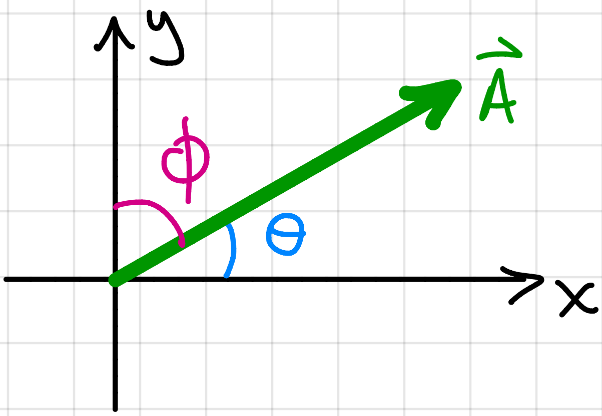
✓ If $\vec{F}_{net} = 0$ then $\vec{F}_{net_x} = 0$ & $\vec{F}_{net_y} = 0$

Free Body Diagrams

✓ Align axes in a way that is convenient for the problem



✓ Remember SOHCAHTOA



Using θ

$A_x = A\cos\theta$

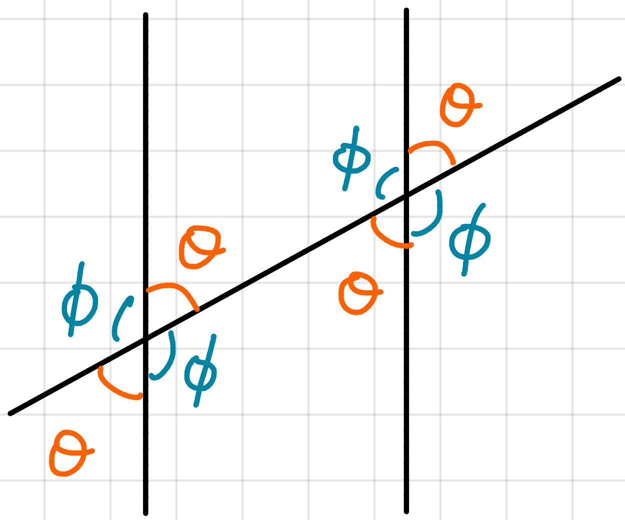
$A_y = A\sin\theta$

Using ϕ

$A_x = A\sin\phi$

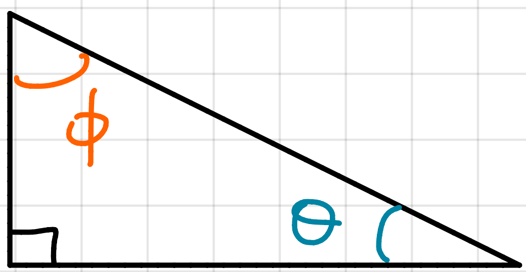
$A_y = A\cos\phi$

Geometry Things

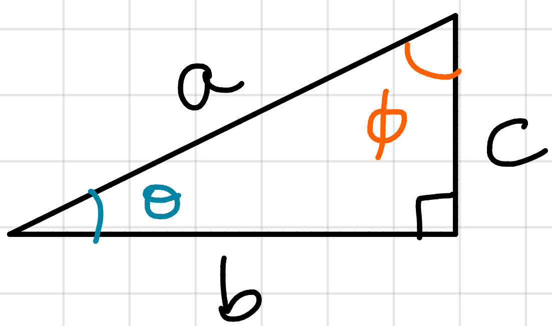


All angles θ here are the same

All angles ϕ here are the same



$$\theta + \phi = 90^\circ$$



$$a^2 = b^2 + c^2$$

$$b = a\cos\theta = a\sin\phi$$

$$c = a\sin\theta = a\cos\phi$$

Friction & Normal Forces

✓ Normal ($\vec{N}$) = force that acts perpendicular to a surface

✓ friction ($\vec{f}$) = force that acts parallel to a surface and wants to prevent sliding

✓ static friction ($\vec{f}_s$) = when the system is not moving; its magnitude increases up to a maximum value right before the system begins to slide

✓ Kinetic friction ($\vec{f}_k$) = when the system is sliding; magnitude is less than $|\vec{f}_{s\,max}|$

✓ $|\vec{f}| = \mu|\vec{N}|$

coefficient of friction

Non-Equilibrium: Curving Motion

- ✓ $\vec{F}_{net} = \vec{F}_{net_{\parallel}} + \vec{F}_{net_{\perp}}$

- ✓ $\vec{F}_{net_{\parallel}} = \left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \frac{|\vec{p}_f| - |\vec{p}_i|}{\Delta t}\hat{p}$

- ✓ $\vec{F}_{net_{\perp}} = \left(\frac{d\vec{p}}{dt}\right)_{\perp} = \frac{mv^2}{R}\hat{n}$

- ✓ $\hat{p}$ = unit vector in the direction of the motion of the system

- ✓ $\hat{n}$ = unit vector perpendicular to the direction of the motion, and pointing towards the center of the turning circle

- ✓ $\vec{F}_{net_{\parallel}}$ is responsible for changing the speed of the system (speeding up or slowing down)

- ✓ $\vec{F}_{net_{\perp}}$ is responsible for changing the direction of the motion of the system

Non-Equilibrium: Curving Motion

- ✓ The $\hat{p}$ and $\hat{n}$ axes move with the system

- ✓ $\vec{F}_{net}$ = vector sum of all the forces acting on the system

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

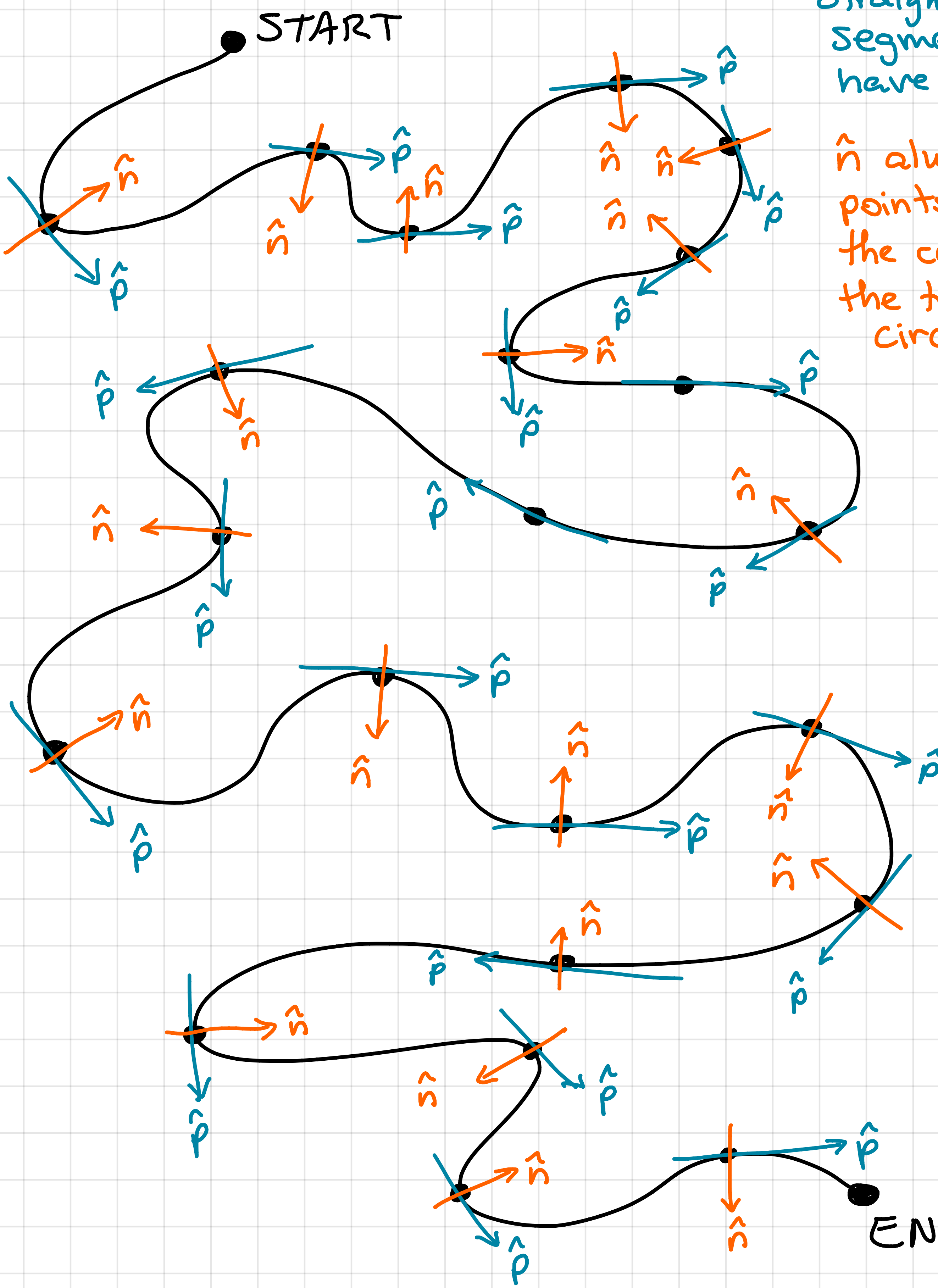
- ✓ $\vec{F}_{net_{\parallel}}$ = vector sum of the parallel components of all the forces acting on the system

$$\vec{F}_{net_{\parallel}} = \vec{F}_{1_{\parallel}} + \vec{F}_{2_{\parallel}} + \vec{F}_{3_{\parallel}} + \ldots$$

- ✓ $\vec{F}_{net_{\perp}}$ = vector sum of all the perpendicular components of all the forces acting on the system

$$\vec{F}_{net_{\perp}} = \vec{F}_{1_{\perp}} + \vec{F}_{2_{\perp}} + \vec{F}_{3_{\perp}} + \ldots$$

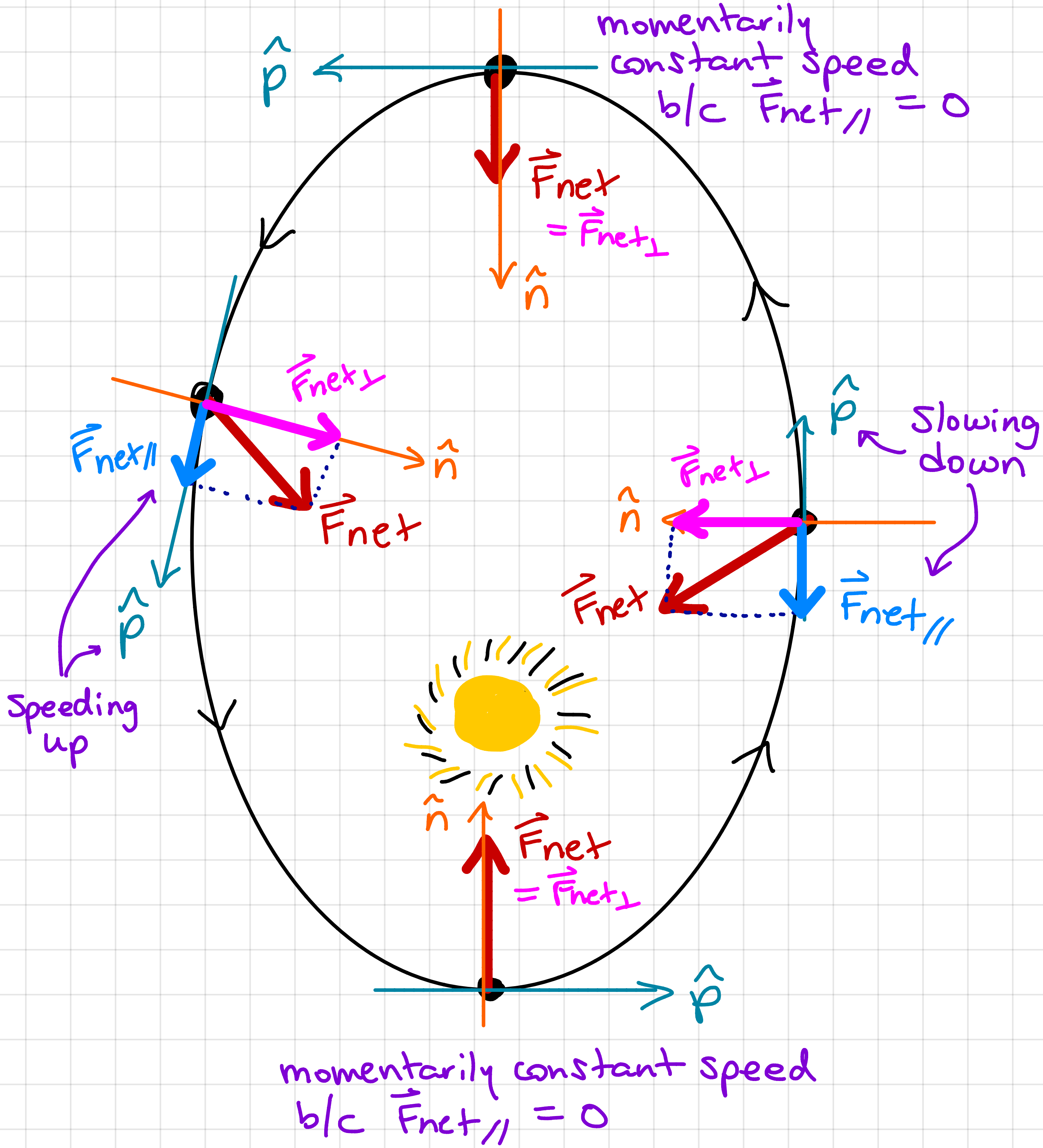
Non-Equilibrium: Curving Motion



Straight line segments only have $\hat{p}$

$\hat{n}$ always points towards the center of the turning circle

Non-Equilibrium: Curving Motion



The Energy Principle

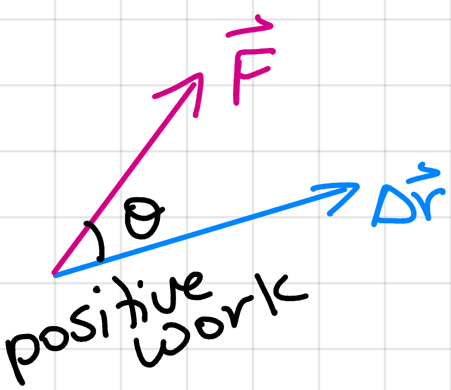
$$\Delta E_{system} = W_{surroundings}$$

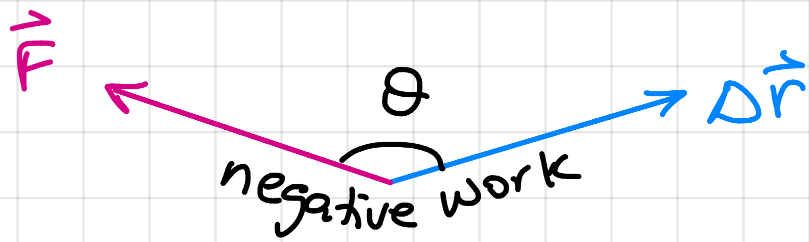
✓ Work (W) = transfer of energy between system and surroundings due to the application of an external force

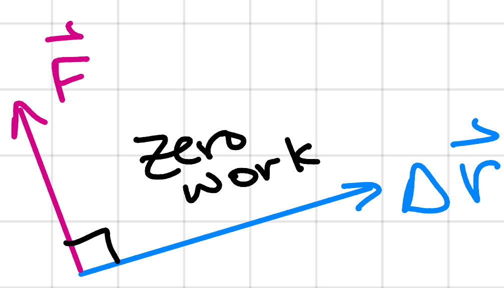
✓ $W = \vec{F} \cdot \Delta\vec{r}$ (constant force)

✓ $W = \int_i^f \vec{F} \cdot d\vec{r}$ (non-constant force)

✓ $\vec{A} \cdot \vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$ dot product







The Energy Principle

$$\Delta E_{system} = W_{surroundings}$$

✓ Kinetic Energy = energy associated with motion

$$K = \frac{1}{2}mv^2$$

✓ Gravitational potential energy = energy associated with the gravitational interaction between two objects in the system

$$U_g = mgh \quad \text{(at surface of Earth)}$$

$$U_g = \frac{-GMm}{r} \quad \text{(in general)}$$

✓ Electric potential energy = energy associated with the electrostatic interaction between charged objects in the system

$$U_e = \frac{k q_1 q_2}{r} \quad \left(k = 9e9 \ \frac{Nm^2}{C^2}\right)$$

The Energy Principle

$$\Delta E_{system} = W_{surroundings}$$

✓ Things that can go into ΔE_{system}:

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\Delta U_g = mg(h_f - h_i)$$

$$\Delta U_g = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\Delta U_e = kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

✓ Other things (BUT NOT IN THIS TEST)

$$\Delta U_s = \frac{1}{2}k(s_f^2 - s_i^2)$$

$$\Delta E_{th} = mC\,\Delta T = mC(T_f - T_i)$$

$$\Delta K_{rot} = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$$

$$\Delta E_{int}$$

Procedure for solving energy principle problems

✓ Identify the objects that are in the system and in the surroundings

- If there's nothing in the surroundings, then $W = 0$
- If there's stuff in the surroundings, then identify all the forces exerted by the surroundings on the system, the displacement over which each force acts, and then calculate the work done by each force

✓ Identify the initial state and the final state of the system

✓ Determine the kinds of energies involved

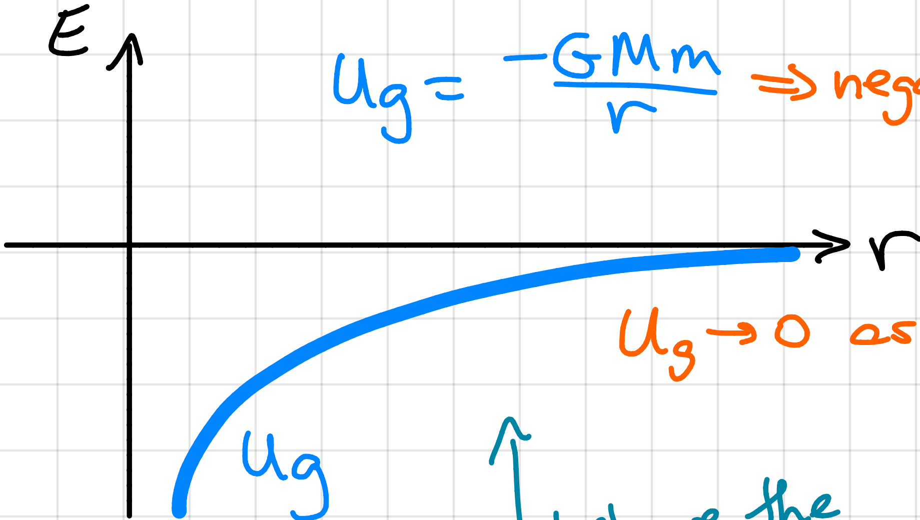
- motion? ΔK
- gravitation? ΔU_g (two possibilities)
- electrostatic? ΔU_e

✓ Apply $\Delta E = \Delta K + \Delta U_g + \Delta U_e = W_{total}$ and solve as needed

Energy Graphs

U_{grav}

$U_g = \frac{-GMm}{r}$ ⇒ negative b/c attractive

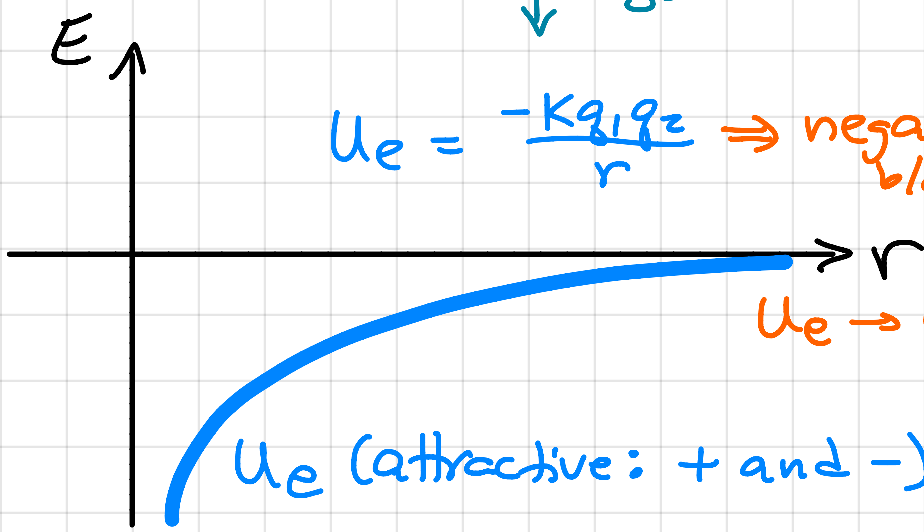


$U_g \to 0$ as $r \to \infty$

behave the same way

U_{elec} $(+-)$

$U_e = \frac{-kq_1q_2}{r}$ ⇒ negative b/c attractive



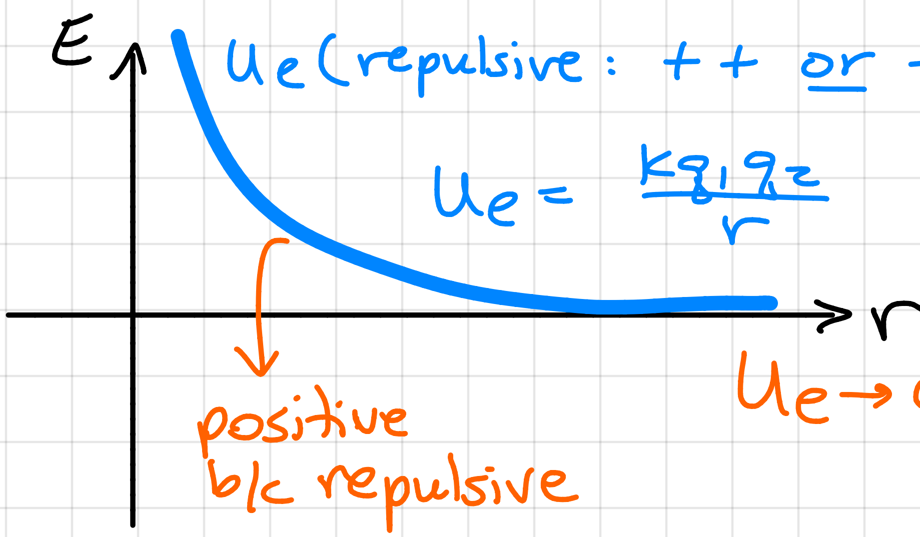
$U_e \to 0$ as $r \to \infty$

U_e (attractive: + and −)

U_{elec} $(++, --)$

U_e (repulsive: + + <u>or</u> − −)

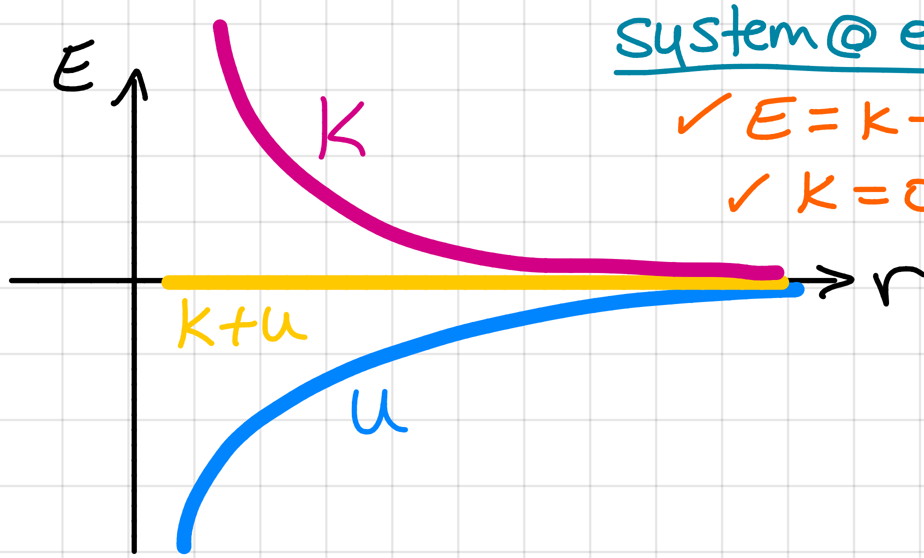
$U_e = \frac{kq_1q_2}{r}$



$U_e \to 0$ as $r \to \infty$

positive b/c repulsive

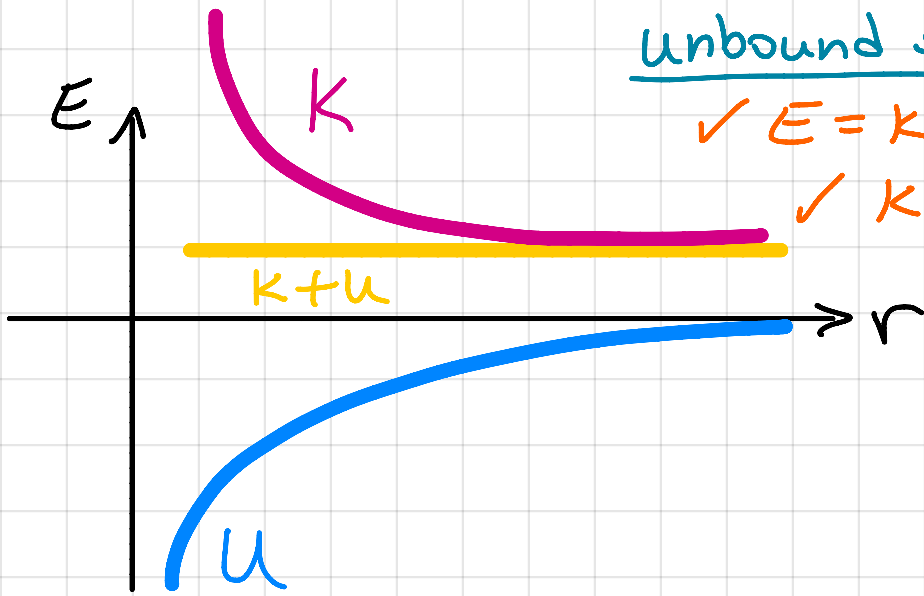
Energy Graphs



system @ escape speed

✓ $E = K + U = 0$

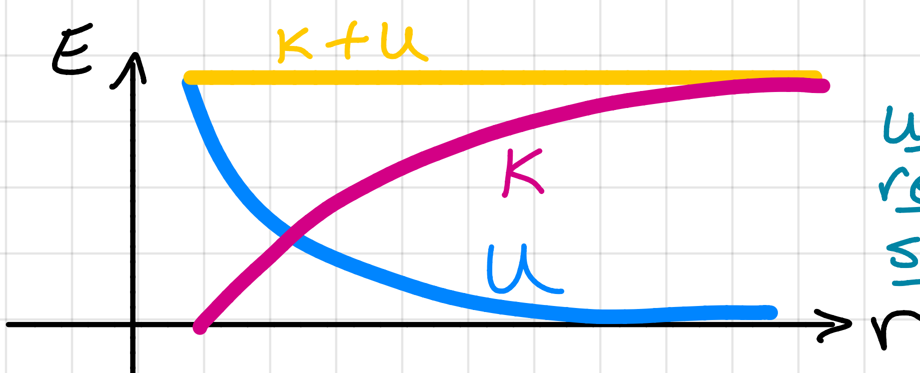
✓ $K = 0$ @ $r \to \infty$



unbound system

✓ $E = K + U > 0$

✓ $K \neq 0$ @ $r \to \infty$

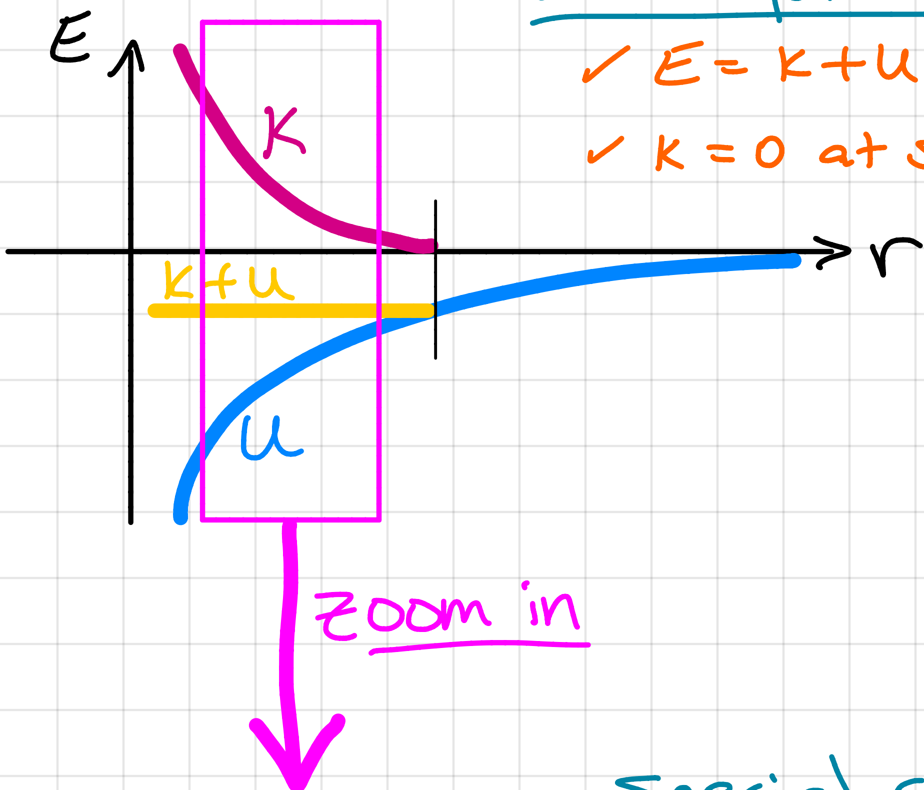


unbound repulsive system

✓ $E = K + U > 0$

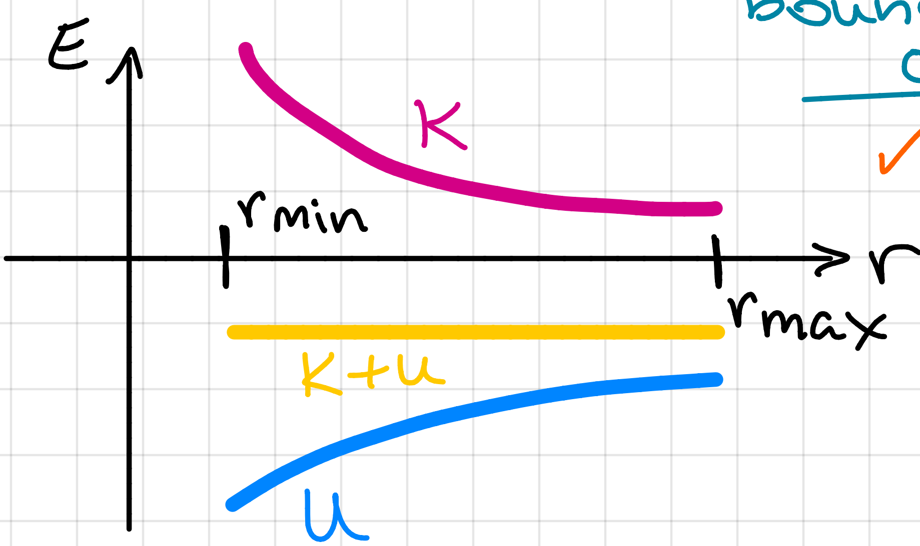
✓ $K \neq 0$ @ $r \to \infty$

Energy Graphs



bound system

- ✓ $E = K + U < 0$
- ✓ $K = 0$ at some point



Special case of bound system: orbits

- ✓ $E = K + U < 0$
- ✓ $\underline{\underline{K \neq 0}}$
- ✓ motion is restricted between a minimum and maximum separation distance

Procedure for drawing energy graphs

✓ Identify/draw the potential energy

- If gravitational interaction, then U_g and it's negative/attractive
- If electric interaction between opposite charges (+ −), then U_e and it's negative/attractive, and behaves the same way as U_g
- If electric interaction between like charges (+ + or − −), then U_e and it's positive/repulsive

✓ Identify/draw the total energy as a straight horizontal line

- $E > 0$ if system is unbound
- $E < 0$ if system is bound
- $E = 0$ if system is at escape speed

✓ Draw the kinetic energy such that $K + U = E$ at every point in the graph.