Week 2 Lecture 1

Momentum Change

Topics for today

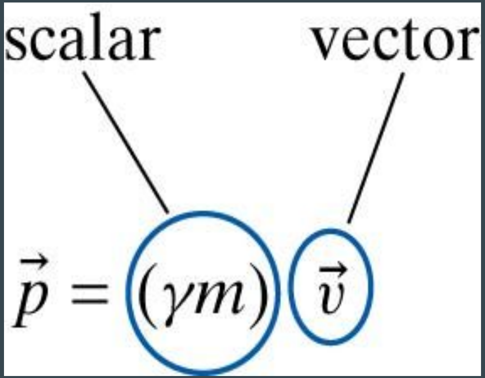
1. Momentum
2. Changes in Momentum

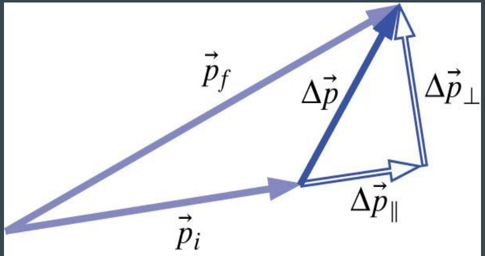
By the end of class you will

1. Calculate a change in momentum

Changes in momentum indicate an interaction

- There are many way different ways to observe a change in momentum
  - Total change $$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
  - Magnitude of the change $$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i|$$
  - Changes in magnitude $$\Delta|\vec{p}| = |\vec{p}_f| - |\vec{p}_i|$$
  - Changes in direction $$\Delta\hat{p} = \hat{p}_f - \hat{p}_i$$
- Commons mistake when calculating change?





Week 2 Lecture 2

The Momentum Principle

Topics for today

1. The Momentum Principle
2. System vs Surroundings

By the end of class you will

1. Predict a change in momentum for a given force

The Momentum Principle

- The change in momentum of a system is equal to the net interaction (force) acting on the system by the surroundings multiplied by the duration of the interaction
  - System: One or more objects of interest
  - Surroundings: Everything else in the Universe
  - Net Force: The sum of all the interaction (forces) and measured in Newtons (N)
- Called Newton's Second Law by some
  - "The change of momentum of a body is proportional to the impulse impressed on the body, and happens along the straight line on which that impulse is impressed." - Newton
  - True for all forces not just gravity!
  - Not valid for systems with non-constant mass.

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

or

$$\Delta\vec{p} \approx \vec{F}_{net}\Delta t$$

Week 2 Lecture 3

Predicting motion

Topics for today

1. The momentum principle
2. Estimating the average velocity
3. Iterative prediction

By the end of class you will

1. Calculating a force from a change in momentum
2. Be able to decide which estimate to use for the average velocity
3. Predict the future?

Applying the momentum principle

1. Chose your system
   a. Everything else belongs in the surroundings
2. Draw a force diagram
   a. All forces must be due to interactions with surroundings
   b. No internal forces
3. Chose your time interval
   a. Given or estimated from motion data
4. Substitute known values and solve for the unknowns
   a. Is the system over or under determined?
5. Check the units and reasonableness of your answer
   a. Does your answer pass the sniff test?

What Would Newton Do?

Example

A baseball of mass m = 0.145 kg is travelling at a speed of v = 67 m/s when it bounce off a wooden bat. Determine the magnitude of the force by the bat on the ball. Assume that the baseball bounces off the bat with the same speed but in the opposite direction and that the force is constant. High speed video indicates that the baseball is crushed by 0.025 m during the collision with the bat.

Example - Solution

- Chose your system
  - System: Baseball
  - Surroundings: Bat and the the Earth
- Draw a force diagram
- Chose your time interval

$$\Delta t = \frac{|\Delta\vec{r}|}{|\vec{v}_{avg}|} \Rightarrow \Delta t = \frac{2\Delta x}{v}$$

- Substitute known values and solve for the unknowns

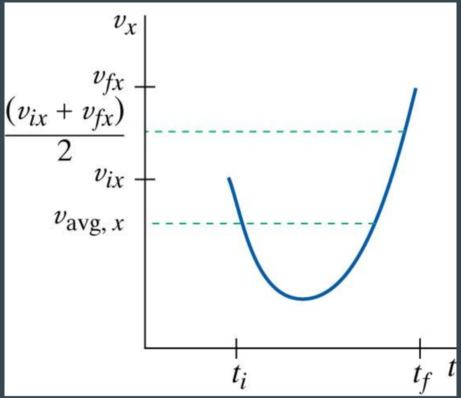
$$\Delta\vec{p} = \vec{F}_{bat}\Delta t \Rightarrow |\vec{F}_{bat}| = \frac{mv^2}{2\Delta x} = 13018\ N$$

- Check the units and reasonableness of your answer

Estimating the average velocity

- Whenever there is a force, the velocity is constantly changing
    - Without position information we must approximate!
- We will use one of two different approximations
    - The final velocity
    - The arithmetic average
- How do we know which approximation is best?
    - It turns out that when the force being applied to the system is constant the arithmetic average is the actual average velocity
    - When the force is non-constant, as most forces are, both choices are approximations and we use the final velocity
    - What really matters is that we pick a time interval small enough that both choices would be good approximations!

$$|\vec{v}_{avg}| = \frac{1}{\Delta t}\int_{t_i}^{t_f} \vec{v}(t)dt$$

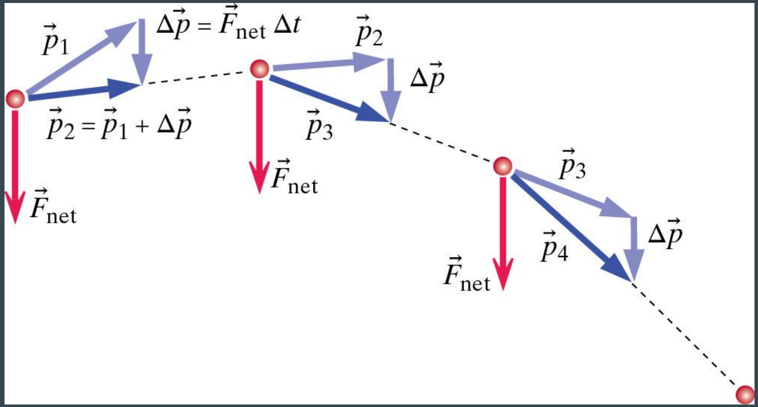


$$\vec{v}_{avg} \approx \frac{\vec{v}_f + \vec{v}_i}{2}$$

$$\vec{v}_{avg} \approx \vec{v}_f$$

Predicting the future iteratively

- Divide up the total time into smaller intervals and iteratively apply our update procedure
  - Apply the Momentum Principle
    - Idealize: Identify the most important interactions and ignore the rest
    - Calculate the net force
  - Update Momentum
  - Estimate the average velocity
    - Arithmetic for constant forces
    - Final velocity for non-constant forces
  - Update the Position of the system
  - Repeat!




```
t = 0
deltat=1e-4
while t < t_final
    Fnet = vector(0,-ball.m*g,0)
    ball.p = ball.p + Fnet*deltat
    v_avg = (ball.p + p_init_ball)/(2*ball.m)
    ball.pos = ball.pos + v_avg*deltat
    t = t + deltat
```

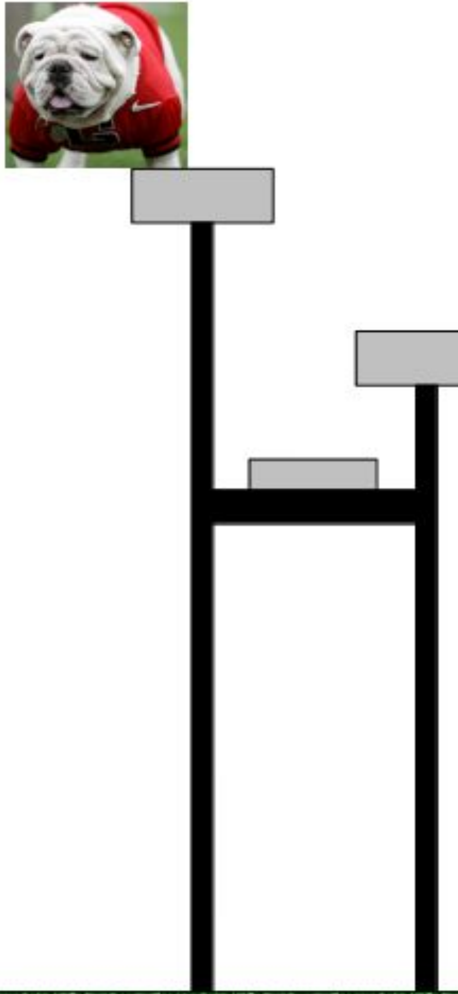
Constant force motion - a special case

- If the net force is always constant.
- If the speed is much less than c.
- We can use Calculus to determine an exact solution to the momentum principle
  - Often called the kinematic equations

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\vec{r}(t) = \left(\frac{1}{2m}\vec{F}_{net}\right)t^2 + \vec{v}_0 t + \vec{r}_0$$

Example

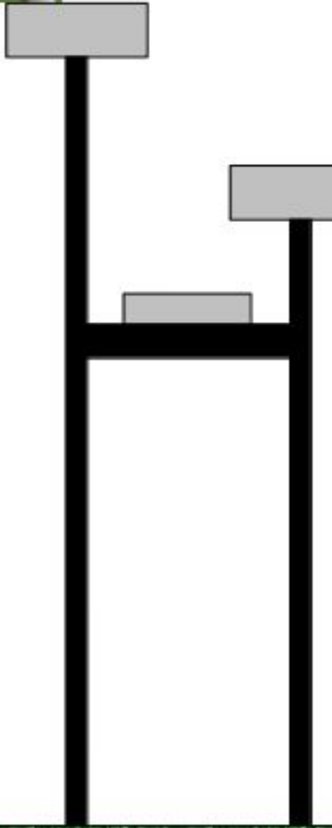


UH-guh is at the top of a set of bleachers when you spot him with your t-shirt cannon. You decide to give him a scare and shoot a balled up t-shirt at him. The instant you fire the gun he drops from the bleachers hoping to avoid your shot.

Where should you aim to be sure and hit your target?

Example - solution

- Assume the cannon fires with speed v_0 at an angle theta
  - UH-guh is a distance D over and H up
- A collision will take place if the position of the two objects occupy the same place at the same time
  - Find the position of Uh-ga as a function of time
  - Find the position of the t-shirt as a function of time



Computational Example

http://www.glowscript.org/#/user/ed/folder/Public/program/MonkeyCH2/edit