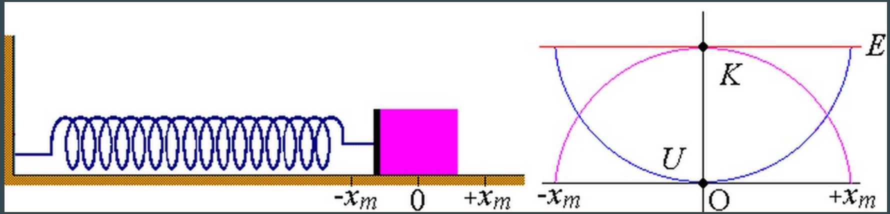

PHYS 2211 K

Week 9, Lecture 1

2022/03/08

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6 clicker questions today

On today's class…

1. Wrapping up energy graphs
2. Spring potential energy
3. Path independence
4. Conservative vs dissipative forces

CLICKER 1: How was the test?

A B C D E

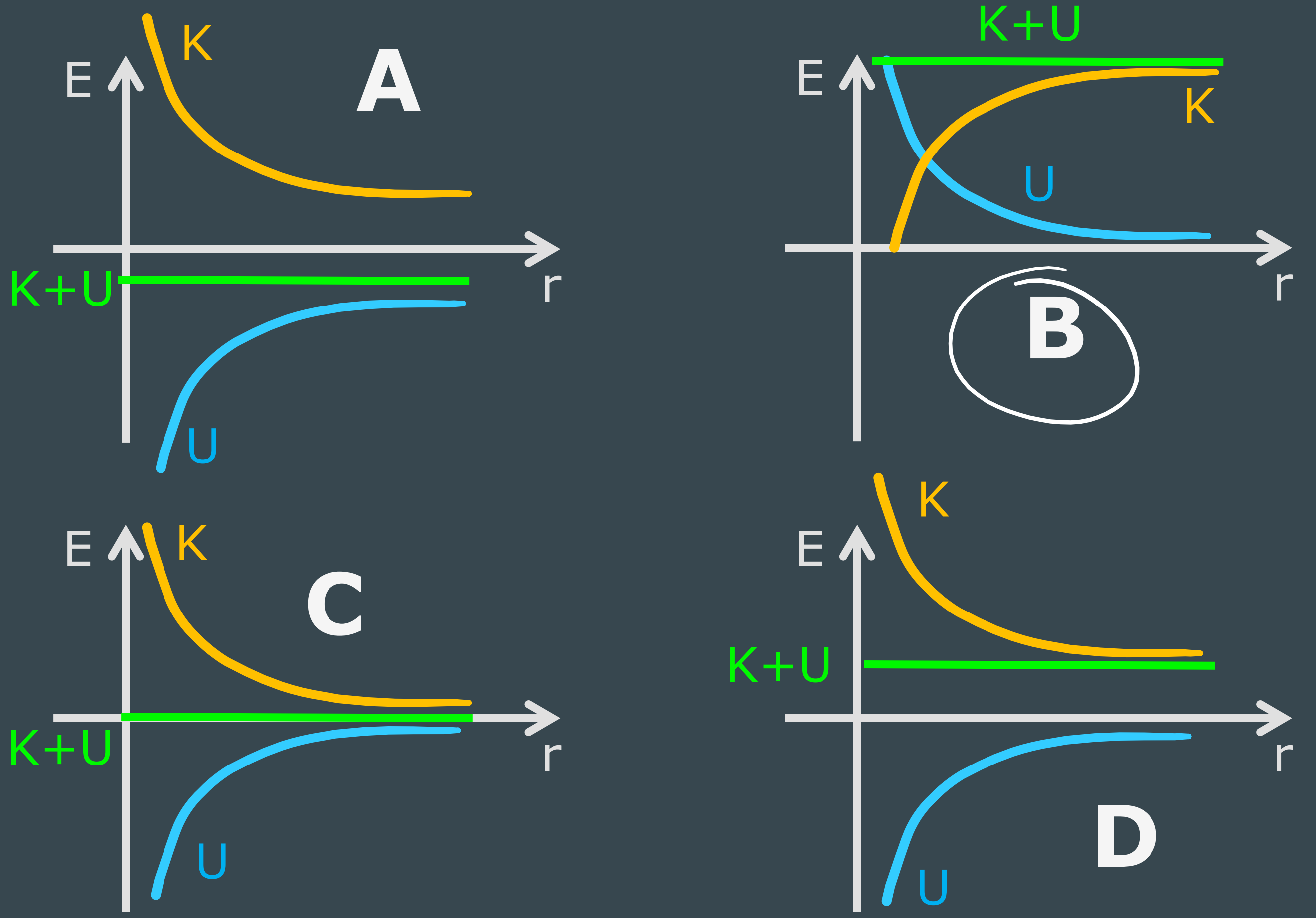
How to draw energy graphs

- Identify if the potential energy is attractive (gravitational, electric for opposite charges) or repulsive (electric for like charges) then draw it in the diagram of energy vs distance

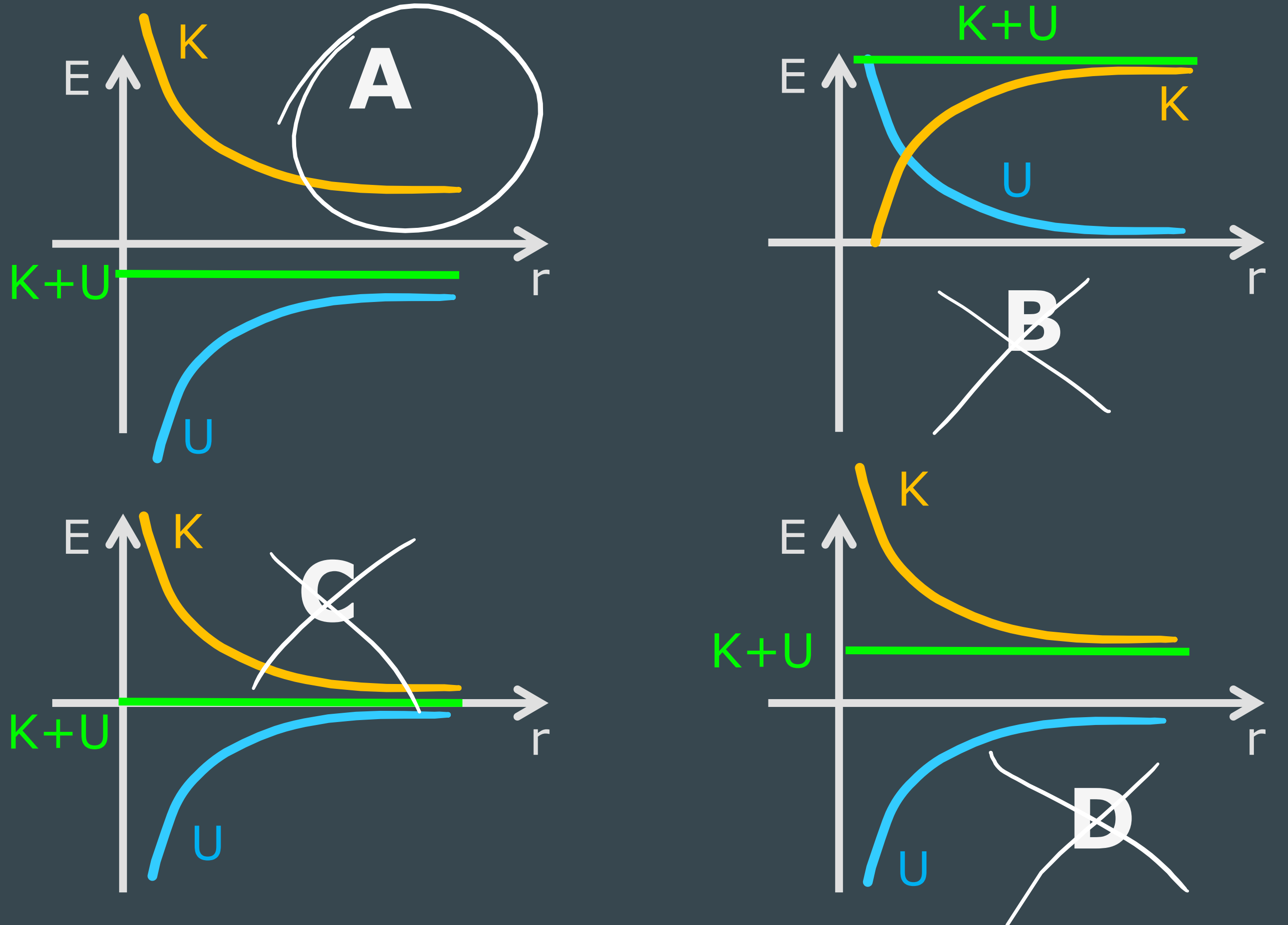
- Determine if the system is bound ($E < 0$), unbound ($E > 0$), or at escape speed ($E = 0$), then draw the total energy as a horizontal line

- Draw the kinetic energy, remembering that it's always positive and making sure that $K + U = E$

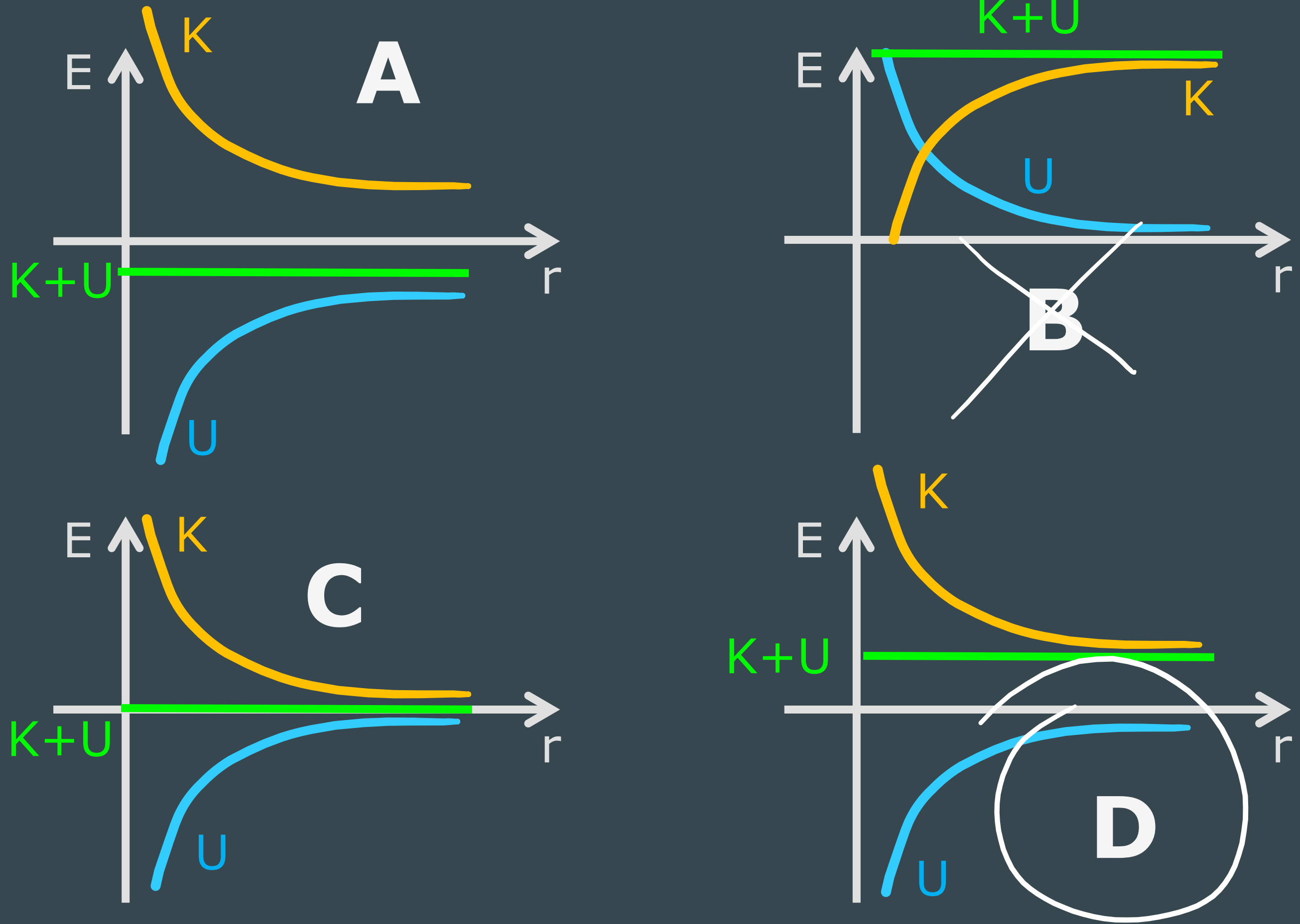
CLICKER 2: Match the energy graph! Two electrons are held at rest close together and then are let go.



CLICKER 3: Match the energy graph! Halley's Comet orbits the Sun once every 76 years.



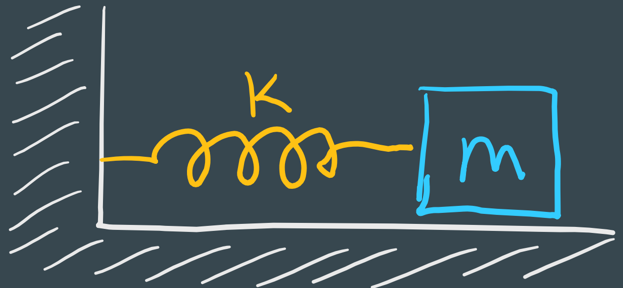
CLICKER 4: Match the energy graph! Voyager 1 is very, very far away from the Sun and is moving with speed 17 km/s.



Spring Potential Energy

- Remember the spring force equation:
  (where $s = L - L_0$)

$$\vec{F}_s = -ks\hat{L}$$

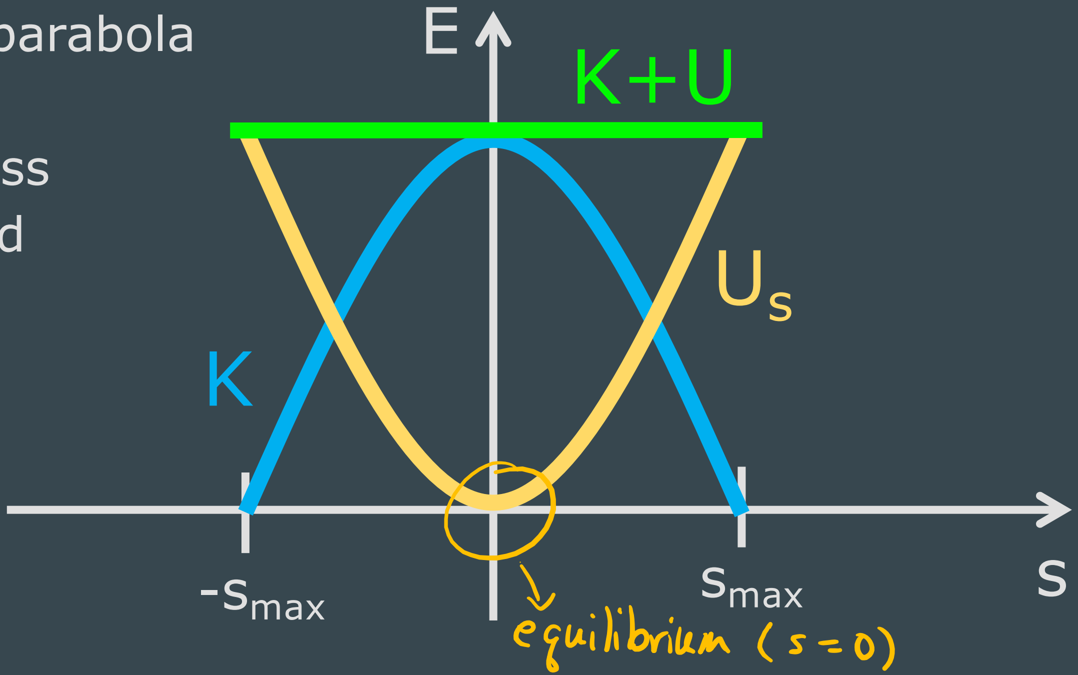


- We can use $\Delta U = -W_{int}$ to get the spring potential energy:

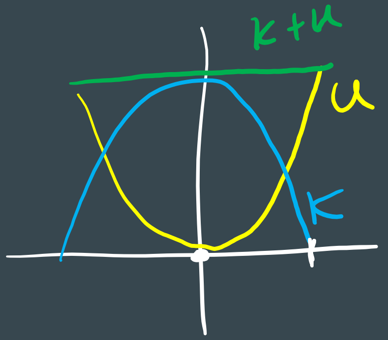
$$U_s = \frac{1}{2}ks^2 \qquad \Delta U_s = \frac{1}{2}k(s_f^2 - s_i^2)$$

Spring Potential Energy

- If you include the spring in your system, then you can use spring potential energy → no need to calculate work done by spring
- The graph of U_s vs s is a parabola
- For an isolated spring-mass system, $E = K+U > 0$, and it is constant



CLICKER 5: A horizontal spring has a mass attached which can move with negligible friction. You stretch the spring and release the mass from rest. For the resulting motion, which of the following statements ~~are~~ is TRUE?



A. When the spring is (momentarily) fully compressed, K has its largest value

B. When the spring (momentarily) has its relaxed length, U has its largest value

C. When the spring (momentarily) has its relaxed length, K has its smallest value

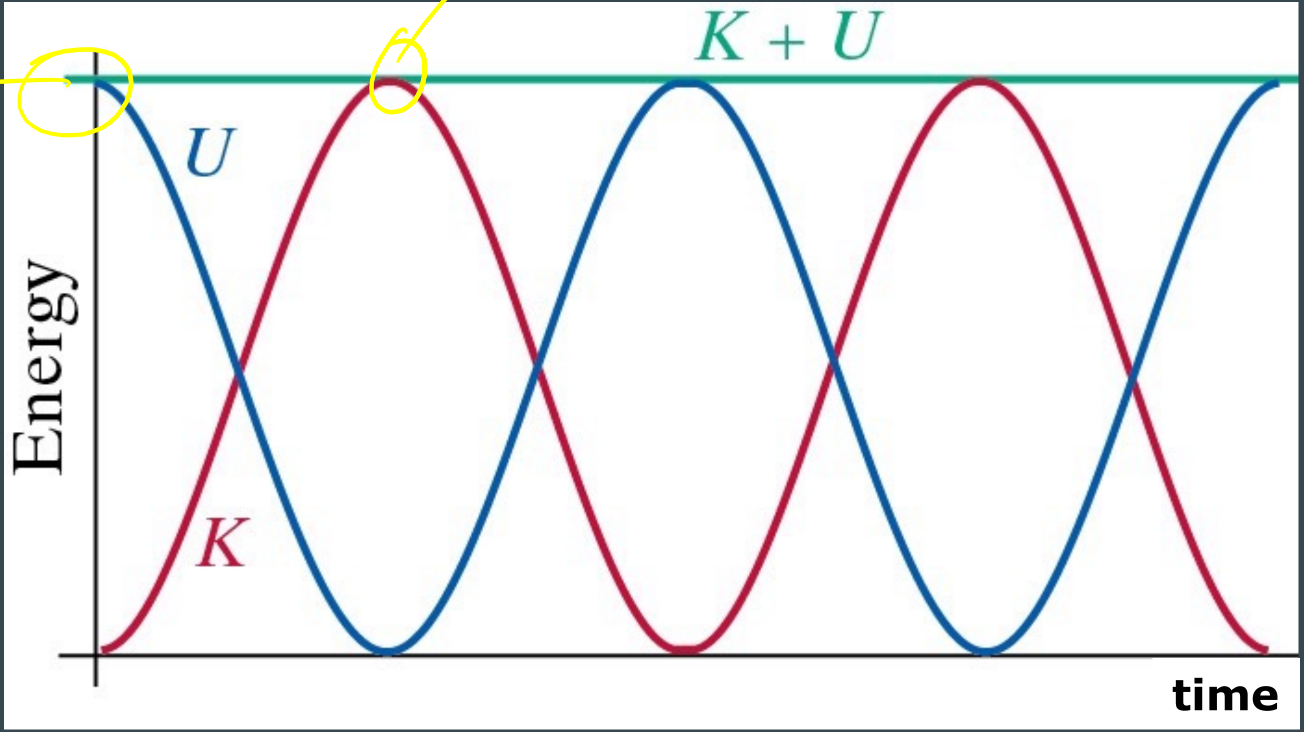
D. When K is large, U is small, and viceversa

E. When K is large, U is also large

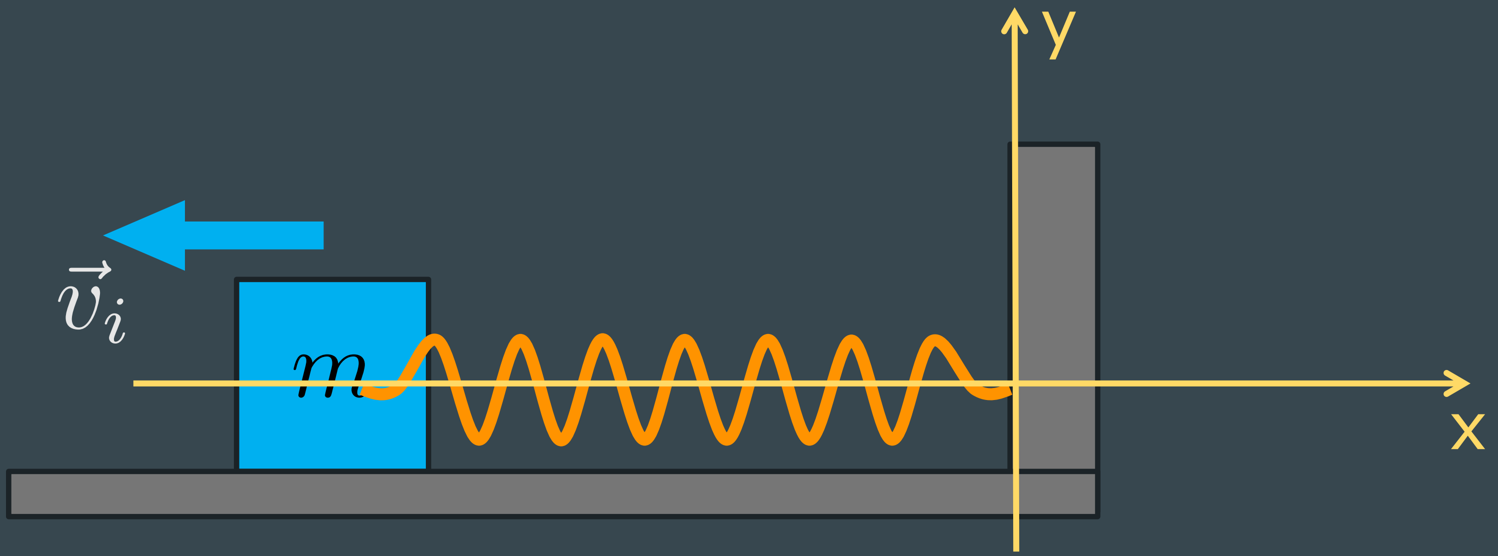
Uspring as function of time

Kmax, Umin ⇒ equilibrium

Umax
Kmin
⇒ Smax



Example: A horizontal spring with stiffness $k = 15$ N/m and relaxed length $L_0 = 4$ m is fixed to a wall and attached to a block of mass $m = 7$ kg on the other end. Right now, the spring is compressed to a length $L = 1.8$ m and the block moves to the left with an initial speed of 2 m/s. How fast will the block move when the spring is relaxed?



Solution: A horizontal spring with stiffness k = 15 N/m and relaxed length L_0 = 4 m is fixed to a wall and attached to a block of mass m = 7 kg on the other end. Right now, the spring is compressed to a length L = 1.8 m and the block moves to the left with an initial speed of 2 m/s. How fast will the block move when the spring is relaxed?

System: block + spring
surr: Nothing

Initial: $s_i = L - L_0 = 1.8 - 4 = -2.2\,m$
$v_i = 2\,m/s$

Find: $s_f = 0$
$v_f = ?$

$$\Delta E = \cancel{W}^{\,0}$$

$$\Delta K + \Delta U_s = 0$$

$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) + \frac{1}{2}k\left(\cancel{s_f^2}^{\,0} - s_i^2\right) = 0$$

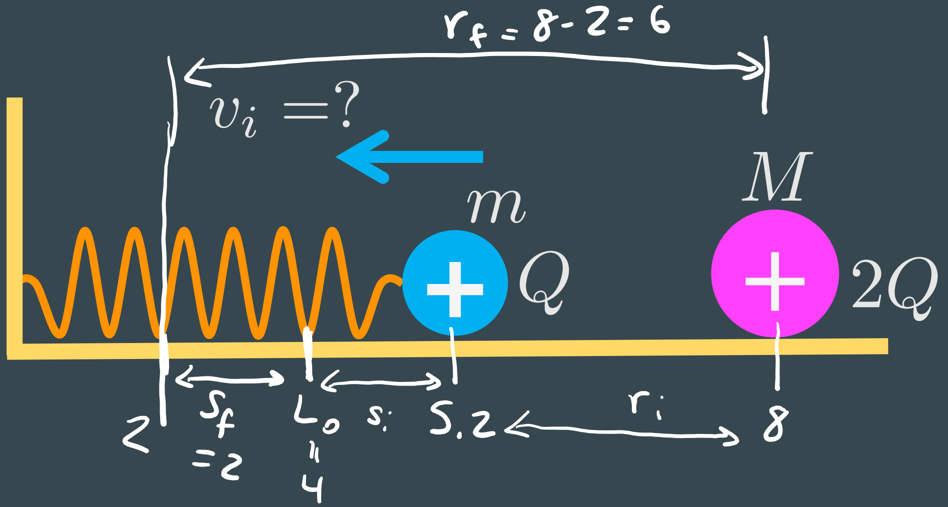
$$\cancel{\frac{1}{2}}m\left(v_f^2 - v_i^2\right) - \cancel{\frac{1}{2}}k s_i^2 = 0$$

$$v_f^2 - v_i^2 = \frac{k}{m}s_i^2$$

$$v_f = \sqrt{\frac{k}{m}s_i^2 + v_i^2} = \text{plug in numbers}$$

$$= \boxed{3.79\,m/s}$$

CLICKER 6: A ball with mass $m = 2$ kg and charge $Q = 3\times10^{-4}$ C is attached to a spring with stiffness $k = 300$ N/m and relaxed length $L_0 = 4$m. The ball is currently at position <5.2, 0, 0> m and moves to the left with unknown speed. A second ball with mass $M = 5$ kg and charge $+2Q$ is fixed at location <8, 0, 0> m. Sometime later, the m ball is momentarily at rest when the spring is compressed by an amount 2m. What is the unknown initial speed?



A. $v_i = 26.32$ m/s

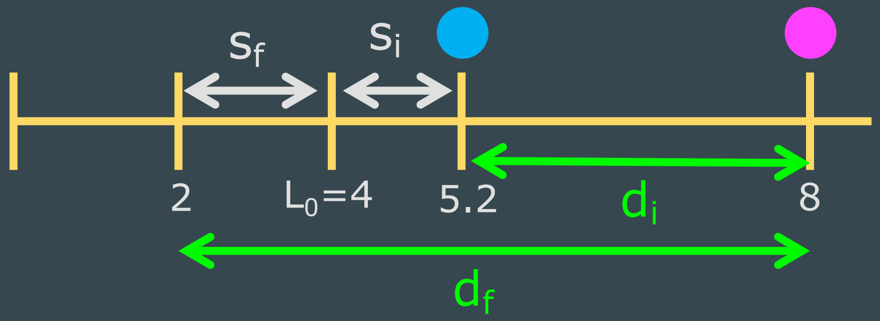
B. $v_i = 24.81$ m/s

C. $v_i = 21.24$ m/s

D. $v_i = 8.66$ m/s

$s_i = 5.2-4 = 1.2$ $\quad r_i = 8-5.2 = 2.8$

Solution: A ball with mass $m = 2$ kg and charge $Q = 3\text{x}10^{-4}$ C is attached to a spring with stiffness $k = 300$ N/m and relaxed length $L_0 = 4$m. The ball is currently at position <5.2, 0, 0> m and moves to the left with unknown speed. A second ball with mass $M = 5$ kg and charge $+2Q$ is fixed at location <8, 0, 0> m. Sometime later, the m ball is momentarily at rest when the spring is compressed by an amount 2m. What is the unknown initial speed?



System: m + M + spring

Initial: $v_i = ?,\ s_i = 1.2,\ d_i = 2.8$

Final: $v_f = 0,\ s_f = 2,\ d_f = 6$

$\Delta E = \cancel{W}^{0}$

$$\Delta K + \Delta U_s + \Delta U_e = 0$$

$$\frac{1}{2}m(\cancel{v_f^2}^{\,0} - v_i^2) + \frac{1}{2}k(s_f^2 - s_i^2) + k_e Q\, 2Q\left(\frac{1}{d_f} - \frac{1}{d_i}\right) = 0$$

$$\frac{1}{2}m v_i^2 = \frac{1}{2}k(s_f^2 - s_i^2) + 2Q^2 k_e\left(\frac{1}{d_f} - \frac{1}{d_i}\right)$$

$$v_i^2 = \cancel{\frac{2}{m}}\cancel{\frac{1}{2}}k(s_f^2 - s_i^2) + \frac{2}{m}2Q^2 k_e\left(\frac{1}{d_f} - \frac{1}{d_i}\right)$$

$$v_i^2 = \frac{k}{m}(s_f^2 - s_i^2) + \frac{4}{m}Q^2 k_e\left(\frac{1}{d_f} - \frac{1}{d_i}\right)$$

$$v_i = \sqrt{\frac{k}{m}(s_f^2 - s_i^2) + \frac{4}{m}Q^2 k_e\left(\frac{1}{d_f} - \frac{1}{d_i}\right)}$$

$$= \boxed{8.66 \text{ m/s}}$$

Force and Potential Energy

- Remember how we derived ΔU_g, ΔU_e, and ΔU_s from internal work? This involved integration

- If you have a force, you can integrate to find potential energy

$$\Delta U = -W_{\mathrm{int}} = -\int_i^f \vec{F} \cdot d\vec{r}$$

- Inversely, if you have a potential energy, you can differentiate to find the force that is responsible for that potential energy

Force and Potential Energy

- Force is a vector but potential energy is a scalar

- How can you get a vector from differentiating a scalar?
  By using the gradient vector operator $\vec{\nabla}$ on a scalar function:

$$\vec{\nabla} f(x, y, z) = < \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} >$$

- So, if $\Delta U = -W$, then:

$$\vec{F} = -\vec{\nabla} U$$

Force and Potential Energy

- Example: gravity

$$U_g = -\frac{GMm}{r}$$

$$\vec{F}_g = -\vec{\nabla}U_g = -\frac{d}{dr}\left(-\frac{GMm}{r}\right) = GMm\frac{d}{dr}\left(r^{-1}\right) =$$

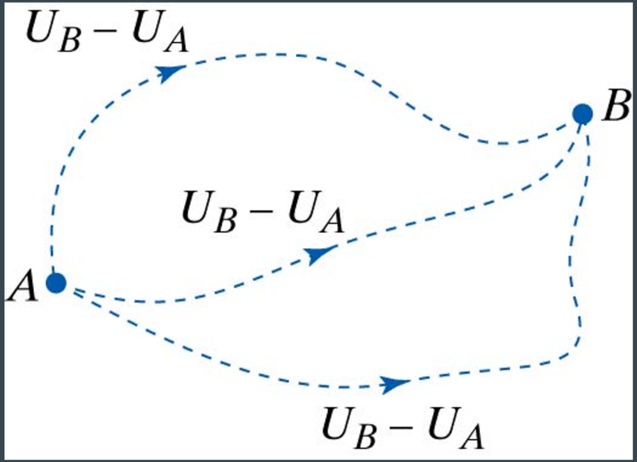
$$= GMm(-1)r^{-2} = \boxed{-\frac{GMm}{r^2}\hat{r}}$$

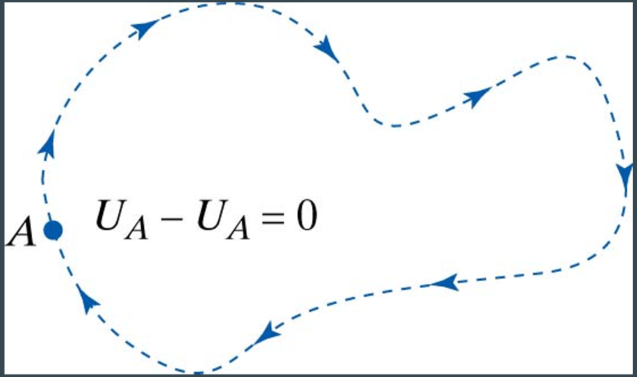
Path Independence

- Potential energy depends on the relative positions of the objects within the system, not their absolute positions

- Changes in potential energy also only depend on the initial and final state of the system, we don't care about what happens in between

- This is called path independence, and it means that for a round trip, $\Delta U = 0$





Conservative vs Dissipative Forces

- A force is **conservative** if:
  - it can be derived as the negative gradient of a potential energy
  - its potential energy exhibits path independence

- When a force is conservative, it means that its associated potential energy can be converted into other types of energies (e.g., you can convert potential into kinetic energy and make the system move)
  - The interaction represented by a conservative force can be included in the left side of the energy principle as a ΔU

- Examples of conservative forces: gravity, electric, springs

Conservative vs Dissipative Forces

- A force is **dissipative** if:
  - it depends on time or velocity
  - it is not associated with any kind of potential energy

- When a force is dissipative, it irreversibly dissipates energy away from the system and into the surroundings
  - This energy cannot be recovered by the system, so it cannot be converted into other types of energy
  - This type of interaction can only be on the right side of the energy principle, as work done on the system

- Examples of dissipative forces: kinetic friction, air resistance

Conservative vs Dissipative Forces

Horizontal springs

https://www.glowscript.org/#/user/ealicea/folder/Public/program/dissipation1

$$\Delta K + \Delta U_s = W_{\mathrm{drag}}$$

Vertical springs

https://www.glowscript.org/#/user/ealicea/folder/Public/program/dissipation2

$$\Delta K + \Delta U_s + \Delta U_g = W_{\mathrm{drag}}$$

When $W_{drag} = 0$, energy is conserved ($\Delta E = 0$)

Where does the energy go?

- Some of the energy dissipated goes into the surroundings
- Some of the energy dissipated goes into increasing the temperature of the system

- Temperature is a measure of the average kinetic energy of the atoms/molecules that make up the system

- Remember this from chemistry? PV = nRT (ideal gas law)
- From there, we can obtain:

$$\langle K \rangle = \frac{3}{2} k_B T$$

Boltzmann constant

(don't worry about the details - that belongs in a chemistry class or a statistical mechanics class; what matters here is that temperature is a measure of microscopic kinetic energy)