Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = <0, -mg, 0>$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

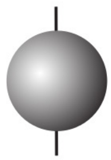
$$\omega = \sqrt{\frac{k_s}{m}}$$

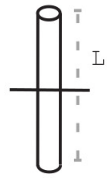
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

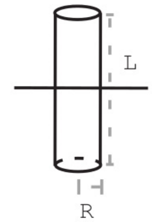
The cross product

$$\vec{A}\times\vec{B} = \langle A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x\rangle$$

Moment of inertia for rotation about indicated axis







$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Summer 2024 - Test 2

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You must join the appropriate proctoring meeting in MS Teams and keep your camera on and microphone muted throughout the exam period.
  - Other than MS Teams and Gradescope (and a PDF annotation app if applicable), you must not access any other app or website during the exam.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically (i.e., symbolically).
  - Numerical solutions should only be evaluated (i.e., plug in numbers) at the last step.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be marked as incorrect.**
  - Incorrect work or explanations mixed in with correct work will be marked as incorrect. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\dfrac{a \cdot b}{c \cdot d} = \dfrac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

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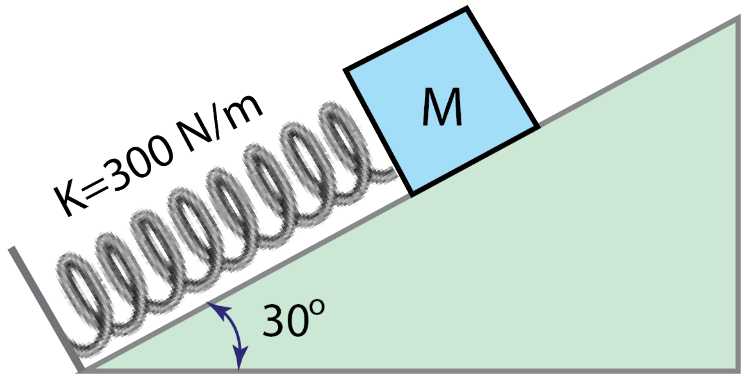
Sign your name on the line above

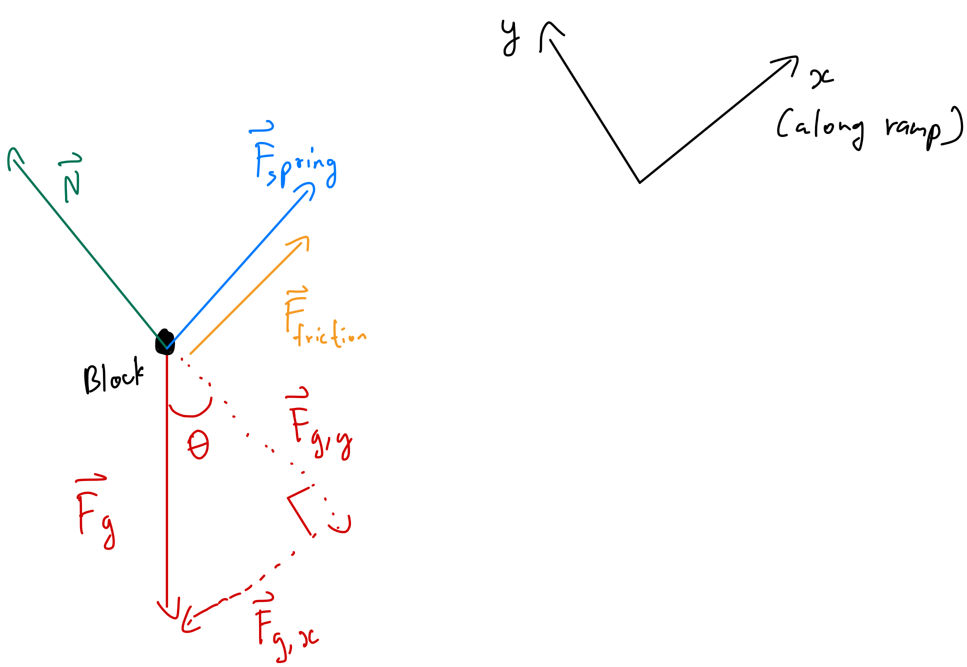
Block at rest on the incline [40 pts]

A block of mass $m = 3.06kg$ is at rest on a plane that makes an angle $\theta = 30°$ above the horizontal. The coefficient of static friction between the block and the plane is $\mu_S = 0.347$. The block is attached to the spring with spring constant $k = 300\ N/m$. The other end of the spring is attached to the wall (see the Figure).



1. The block is at rest and is about to start moving down the incline. In this state, the spring has the maximum length (i.e., minimal compression) and exerts the minimal force on the block that still prevents the block from sliding down. How much is the spring compressed relative to its relaxed length? Find the spring compression s_{min} (relative to the spring's relaxed length) for which the block is at rest and is about to start moving down the incline.

   a) [5 pts] Draw a free-body diagram of all the forces acting on the block. Show x- and y-axes.



b) [15 pts] Solve for the minimal spring compression s_{min} that ensures the block is at rest.

$$x: \quad F_{spring} + F_{friction} - F_{g,x} = 0$$

$$\Rightarrow \quad k \underbrace{s_{min}}_{|L - L_0|} + \mu_s N - mg\sin\theta = 0$$

$$y: \quad N - F_{g,y} = N - mg\cos\theta = 0$$

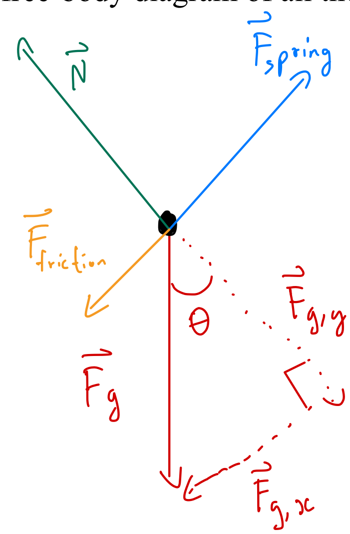
$$\Rightarrow \quad N = mg\cos\theta$$

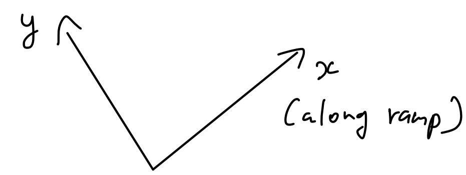
$$\Rightarrow \quad k s_{min} + \mu_s mg\cos\theta - mg\sin\theta = 0$$

$$\Rightarrow \quad s_{min} = \frac{mg(\sin\theta - \mu_s\cos\theta)}{k} = 0.02 \text{ m}$$

1. Find the maximum spring compression s_{max} (relative to the spring's relaxed length) for which the block is at rest – i.e. about to move up the plane. In this state the spring has the minimal length (i.e. maximum compression) and exerts the maximum force on the block at which the block is still not sliding up.

   a) [5 pts] Draw a free-body diagram of all the forces acting on the block. Show x- and y- axes.





   b) [15 pts] Solve for the maximum spring compression s_{max} for which block is still at rest.

$$x: \quad F_{spring} - F_{friction} - F_{g,x} = 0$$

$$\Rightarrow \quad k \underbrace{s_{max}}_{|L-L_0|} - \mu_s N - mg\sin\theta = 0$$

$$y: \quad N - F_{g,y} = N - mg\cos\theta = 0$$

$$\Rightarrow \quad N = mg\cos\theta$$

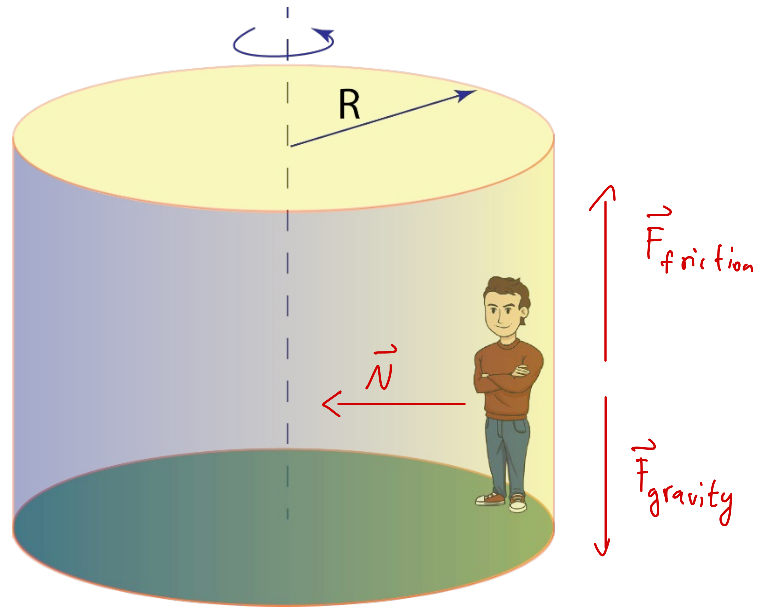
$$\Rightarrow \quad k s_{max} - \mu_s mg\cos\theta - mg\sin\theta = 0$$

$$\Rightarrow \quad s_{max} = \frac{mg(\sin\theta + \mu_s\cos\theta)}{k} = 0.08\ m$$

Amusement park ride [30 pts]

An amusement park ride consists of a rotating vertical cylinder of radius R with rough canvas walls. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass M and the period of rotation (i.e. the time for one complete revolution) of the cylinder is T.

Answer all questions in this problem in terms of known quantities R, M, T, and g.



1. [5 pts] Draw a free-body diagram of all the forces acting on the rider when the system is rotating and the floor has dropped down for the instant shown in the figure.

2. [5 pts] Find the speed and the magnitude of the centripetal acceleration of the rider when the cylinder is rotating. State what force provides that acceleration.

$$\text{speed} = |\vec{v}| = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T}$$

$$\Rightarrow |\vec{a}| = \frac{|\vec{v}|^2}{R} = \frac{4\pi^2 R}{T^2}$$

The normal force of the wall provides the acceleration.

3. [5 pts] Find the upward force that keeps the rider from sliding down when the floor is dropped down and state what provides that force.

$$F_{friction} - F_{gravity} = 0$$

$$\Rightarrow \quad F_{friction} = Mg .$$

Friction provides that force.

4. [10 pts] What must be the minimum coefficient of static friction between the rider and the wall of the cylinder μ so that the rider does not slide down?

$$F_{friction} = \mu_s N = \mu_s \cancel{M} |\vec{a}| = \cancel{M} g$$

$$\Rightarrow \quad \mu_s \left(\frac{4\pi^2 R}{T^2} \right) = g$$

$$\Rightarrow \quad \mu_s = \frac{gT^2}{4\pi^2 R}$$

5. [5 pts] At the same park ride, would a rider of twice the mass slide down the wall? Explain your answer.

No, the mass of the rider does not matter since both gravity and friction scale with mass M.

Sliding Block [30 pts]

A block of mass M starts from rest and slides down a frictionless plane inclined at angle θ with the horizontal as shown in Figure 1. After sliding a distance d, the block strikes a spring. The block comes to rest after compressing the spring distance s (see Figure 2).

Answer all questions in this problem in terms of given known quantities M, θ, d, s, and g.

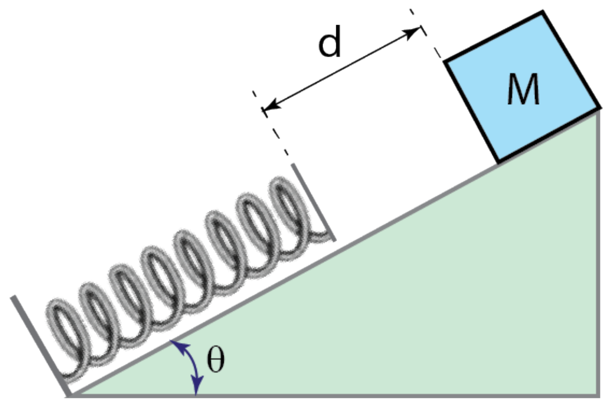


Figure 1

1. [8 pts] What is the total energy of the system at the initial state in Figure 1? Choose and properly identify (in words or sketch) the appropriate reference level for the gravitational potential energy.

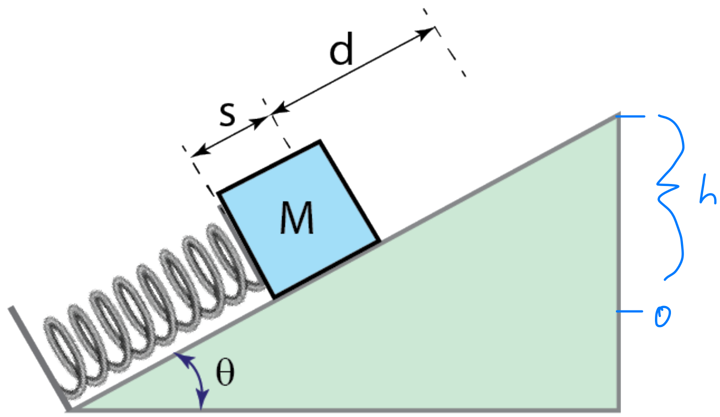


Figure 2

$E_i = U_g + K + U_s$

$v = 0$ initially, $\Rightarrow K = 0$.

No spring compression initially, so $U_s = 0$.

$U_g = mgh$, $h = (d+s)\sin\theta \Rightarrow U_g = mg(d+s)\sin\theta$

2. [8 pts] What is the total energy of the system at the final state in Figure 2 when the spring is compressed distance s?

$$E_f = U_g + K + U_s.$$

We set $U_g = 0$ at maximum spring compression.

$K = 0$ because mass stops.

$$E_f = U_s = \frac{1}{2} k s^2.$$

3. [14 pts] Find the spring force constant k.

$$E_i = E_f$$

$$\Rightarrow \quad Mg(d+s)\sin\theta = \frac{1}{2} k s^2$$

$$\Rightarrow \quad k = \frac{2Mg(d+s)\sin\theta}{s^2}$$