1 Workspace

$$\begin{aligned} &\neg(\exists x P(x)) \\ &\equiv \neg(P(x_1) \vee P(x_2) \vee ... \vee P(x_n)) \\ &\equiv \neg P(x_1) \wedge \neg P(x_2) \wedge ... \wedge \neg P(x_n) \\ &\equiv \forall x \neg P(x) \end{aligned}$$

1.1 sample01

We want to show that $-2 = 2$.

Proof.
$$\begin{array}{r|l} -2 = 2 & \text{assuming the conclusion} \\ (-2)^2 = 2^2 & \text{square both sides} \\ 4 = 4 & \text{as desired} \end{array}$$
■

1.2 sample02

We want to show that $-2 = 2$.

Proof.

$$\begin{array}{r|l} -2 = 2 & \text{assuming the conclusion} \\ (-2)^2 = 2^2 & \text{square both sides} \\ 4 = 4 & \text{as desired} \end{array}$$

■

1.3 Equation and Theorem

$$\begin{aligned} A &= \frac{\pi r^2}{2} \\ &= \frac{1}{2}\pi r^2 \end{aligned} \tag{1}$$

$$\begin{aligned} A \cup (B \cup A) &= B \cup (A \cup A) \\ &= B \cup A \end{aligned} \tag{2}$$

Theorem 1.1. *Let f be a function whose derivative exists in every point, then f is a continuous function.*

1.4 Proof

Step	Statement	Reasoning
(1)	$n = 2k + 1$ for some $k \in \mathbf{Z}$	definition of odd
(2)	$n^3 = (2k + 1)^3$	cube both sides of (1)
(3)	$n^3 = 8k^3 + 12k^2 + 6k + 1$	expand (2)
(4)	$n^3 + 12 = 8k^3 + 12k^2 + 6k + 13$	add 12 to both sides
(5)	$n^3 = 2(4k^3 + 6k^2 + 3k + 6) + 1$	factor out 2 from RHS
(6)	$t = 4k^3 + 6k^2 + 3k + 6t \in \mathbf{Z}$	Closure of multiplication and addition of $\mathbf{Z}$
(7)	$n^3 + 12 = 2t + 1$	Substitution of (6) into (5)
(8)	$n^3 + 12$ is odd	Definition of odd

1.5 Theorem Examples

Theorems can easily be defined:

Theorem 1.2. *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Theorem 1.3 (Pythagorean theorem)**.** *This is a theorem about right triangles and can be summarised in the next equation*

$$x^2 + y^2 = z^2$$

And a consequence of theorem 1.3 is the statement in the next corollary.

Corollary 1.3.1. *There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.*

You can reference theorems such as 1.3 when a label is assigned.

Lemma 1.4. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

1.6 Definition Examples

Unnumbered theorem-like environments are also possible.

Remark. This statement is true, I guess.

And the next is a somewhat informal definition

Definition 1.1 (Fibration)**.** A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

1.7 Proof

Lemma 1.5. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Proof. To prove it by contradiction try and assume that the statement is false, proceed from there and at some point you will arrive to a contradiction. ■

1.8 Quick Reference

1.8.1 Fraction

The binomial coefficient, $\binom{n}{k}$, is defined by the expression:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Fractions can be used inline within the paragraph text, for example $\frac{1}{2}$, or displayed on their own line, such as this:

$$\frac{1}{2}$$

We use the `amsmath` package command `\text{...}` to create text-only fractions like this:

$$\frac{\text{numerator}}{\text{denominator}}$$

Without the `\text{...}` command the result looks like this:

$$\frac{numerator}{denominator}$$

1.8.2 Fraction within a Paragraph

Fractions typeset within a paragraph typically look like this: $\frac{3x}{2}$. You can force LaTeX to use the larger display style, such as $\displaystyle\frac{3x}{2}$, which also has an effect on line spacing. The size of maths in a paragraph can also be reduced: $\scriptstyle\frac{3x}{2}$ or $\scriptscriptstyle\frac{3x}{2}$. For the `\scriptscriptstyle` example note the reduction in spacing: characters are moved closer to the *vinculum* (the line separating numerator and denominator).

Equally, you can change the style of mathematics normally typeset in display style:

$$f(x) = \frac{P(x)}{Q(x)} \quad \text{and} \quad \textstyle f(x) = \frac{P(x)}{Q(x)} \quad \text{and} \quad \scriptstyle f(x) = \frac{P(x)}{Q(x)}$$

1.8.3 Nested Fraction

Fractions can be nested but, in this example, note how the default math styles, as used in the denominator, don't produce ideal results...

$$\frac{1 + \frac{a}{b}}{1 + \frac{1}{1 + \frac{1}{a}}}$$

...so we use `\displaystyle` to improve typesetting:

$$\frac{1+\frac{a}{b}}{1+\frac{1}{1+\frac{1}{a}}}$$

Here is an example which uses the `amsmath` `\cfrac` command:

$$a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+\cdots}}}$$

Here is another example, derived from the `amsmath` documentation, which demonstrates left and right placement of the numerator using `\cfrac[l]` and `\cfrac[r]` respectively:

$$\cfrac[l]{1}{\sqrt{2}+\cfrac[r]{1}{\sqrt{2}+\cfrac{1}{\sqrt{2}+\cdots}}}$$