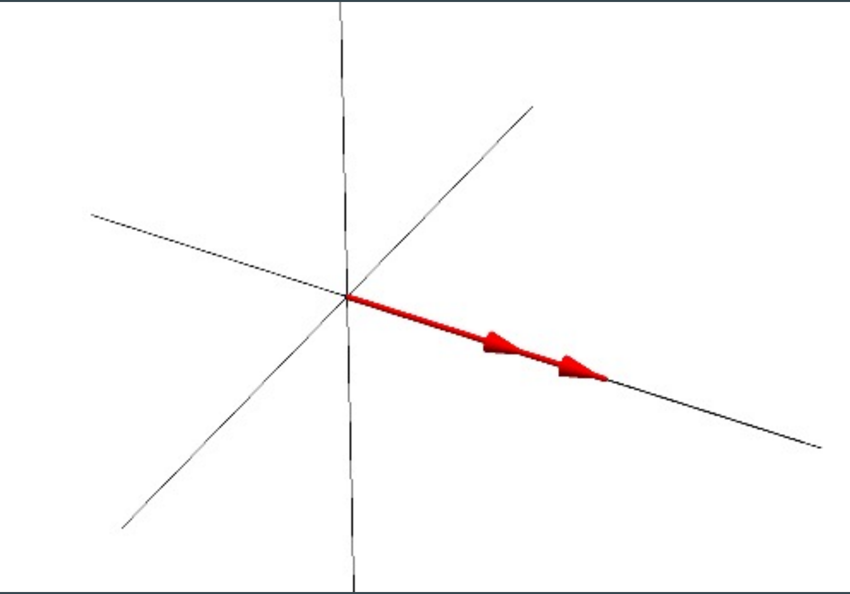

PHYS 2211 K

Week 13, Lecture 1

2022/04/05

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6 clicker questions today

On today's class…

1. Introduction to angular momentum and torque
2. The cross product

Road map for the rest of the semester

- Week 13 ← you are here
  - Lectures topics: Cross product, Torque, Angular momentum
  - Lab 5 submission due on Sunday April 10
- Week 14
  - Test 3 on Monday April 11 (coverage: weeks 9, 10, 12)
  - Lecture topics: Angular momentum principle, multiparticle angular momentum, angular momentum of rigid systems
  - Lab 5 peer evals due at the end of the week (Sunday April 17)
- Week 15
  - Lecture topics: Wrapping up angular momentum; Quantum stuff
  - Hard deadline for everything (edx, etc) on Sunday April 24
- Week 16
  - (Optional) review session on Tuesday’s class period (April 26)
  - Final exam on Friday April 29

CLICKER 1: If you're happy and you know it clap your hands…

A. CLAP CLAP!!   ^_^

B. I'm a T-Rex   T_T

Intro: Angular Momentum and Torque

- The angular momentum of a point mass is a measure of its rotational motion relative to some specific reference point A

$$\vec{L}_A = \vec{r}_A \times \vec{p}$$

- For multiparticle systems, angular momentum can be separated into translational and rotational (we'll see about this next week)

- Torque is the rotational equivalent of a linear force; it's a force applied at some (perpendicular) distance from an axis that causes a rotation to happen about that axis

$$\vec{\tau}_A = \vec{r}_A \times \vec{F}$$

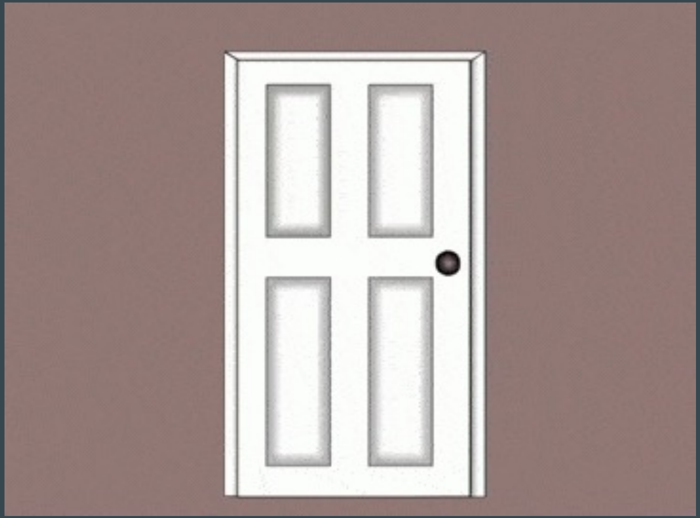
Intro: Angular Momentum and Torque

- The angular momentum principle (which we'll see in detail next week) is analogous to the momentum principle (Newton's 2nd Law), in that it connects a change in motion of a system to the net interaction between the system and the surroundings

$$\vec{\tau}_{\mathrm{net,A}} = \frac{d\vec{L}_A}{dt} \qquad \vec{F}_{\mathrm{net}} = \frac{d\vec{p}}{dt}$$

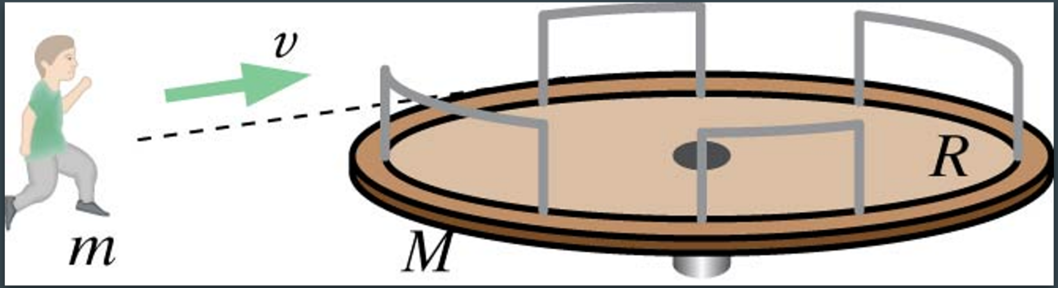
Intro: Angular Momentum and Torque

- Visualize pushing open a door
- The door is rotating about the axis defined by its hinges
- If you push in at the opposite side (where the doorknob is), the door will open easily
- If you push in at a distance halfway between the hinge and the knob, it'll be harder to open the door
- If you push in at the hinge, the door will not move at all



Intro: Angular Momentum and Torque

- Example: a kid running at constant speed v who then jumps onto a stationary merry-go-round, therefore making it spin

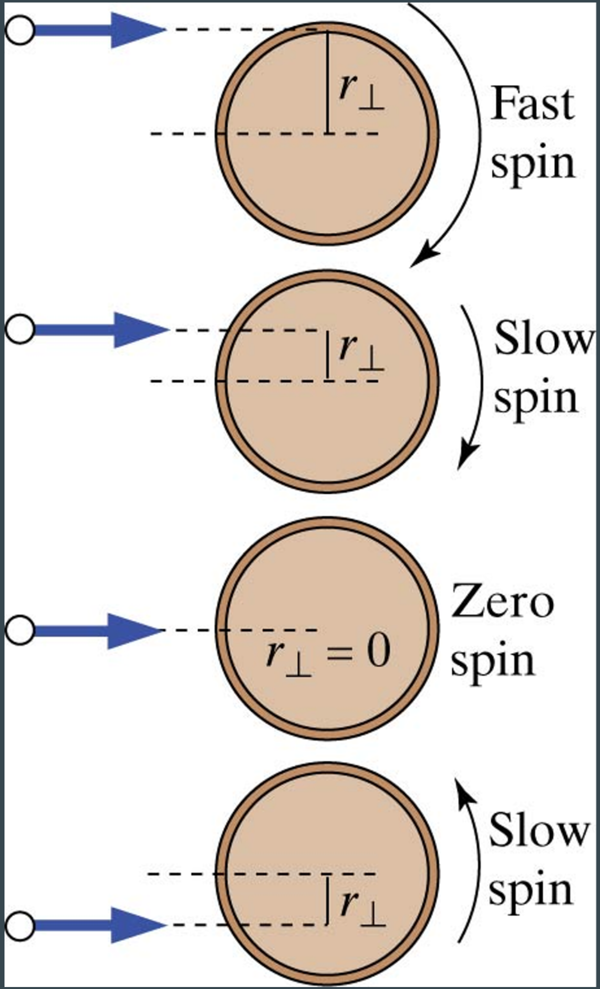


- The rate of spin of the merry-go-round will depend on the mass of the kid, how fast the kid was moving, and where the kid lands on the merry-go-round (how far away from the axis of rotation that passes through the center of it)

Intro: Angular Momentum and Torque

Possibilities (top view):

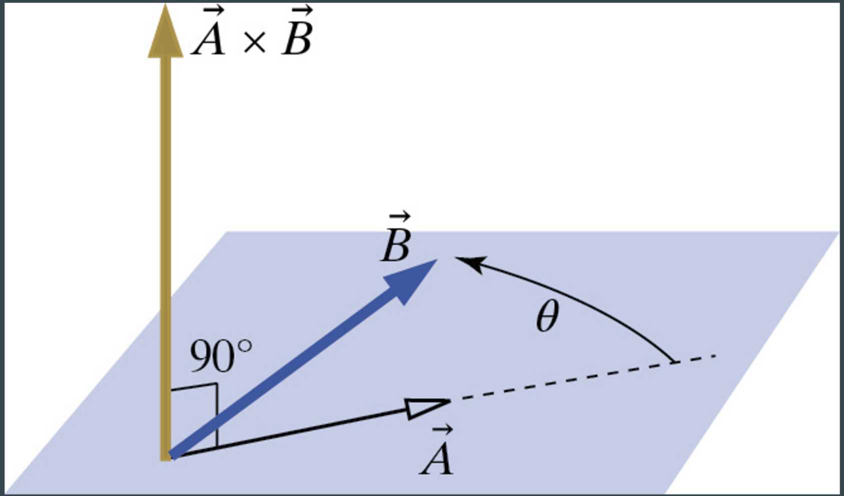
- Kid lands on the edge at the top, so the merry-go-round spins clockwise and fast
- Kid lands between the edge and the center, so the merry-go-round spins clockwise but slower
- Kid lands directly in front of the central axis, so the merry-go-round doesn’t spin at all
- Kid lands below the central axis, so the merry-go-round spins counterclockwise



Multiplying vectors: the cross product

- The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ that have an angle θ between them is defined as

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$



- The result of a cross product is a vector whose direction is given by the right-hand-rule

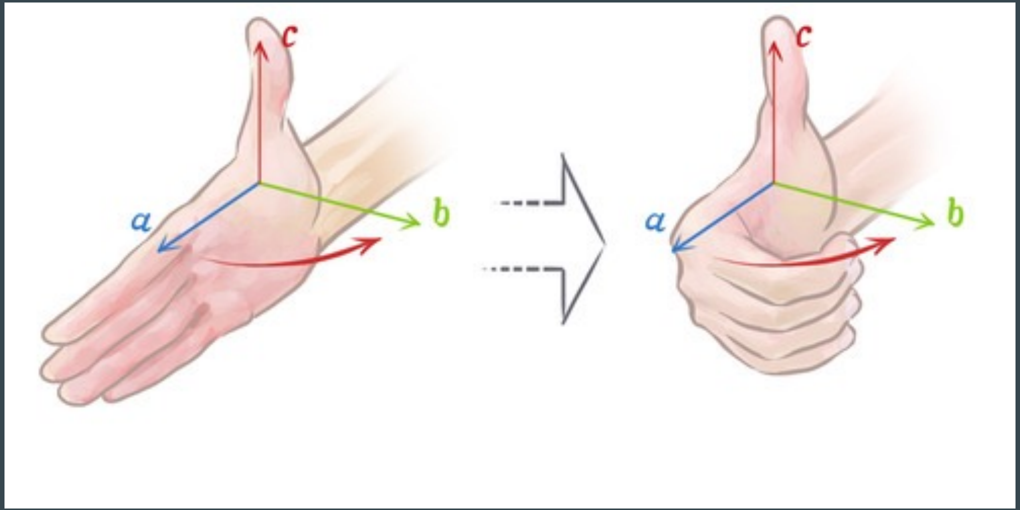
- The resulting vector is perpendicular to the plane defined by the original vectors

The Right Hand Rule (RHR)

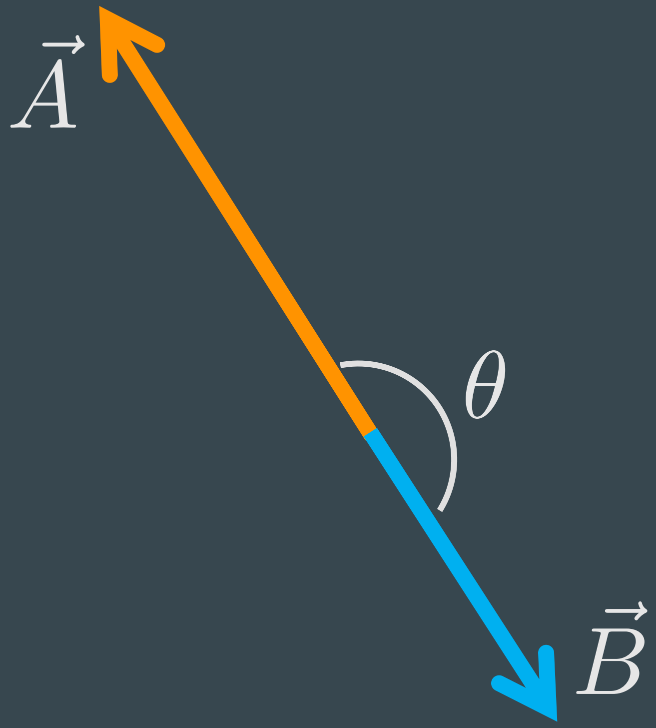
To find the direction of $\vec{A} \times \vec{B}$ using the right-hand-rule:

- Align the fingers of your right hand with the vector A
- Rotate your hand such that your palm faces vector B
- Curl your fingers from A towards B
- Stick out your thumb

The direction of the cross product is the direction in which your thumb is pointing



CLICKER 2: If $|\vec{A}|$ = 35 and $|\vec{B}|$ = 12, what is $\vec{A} \times \vec{B}$?



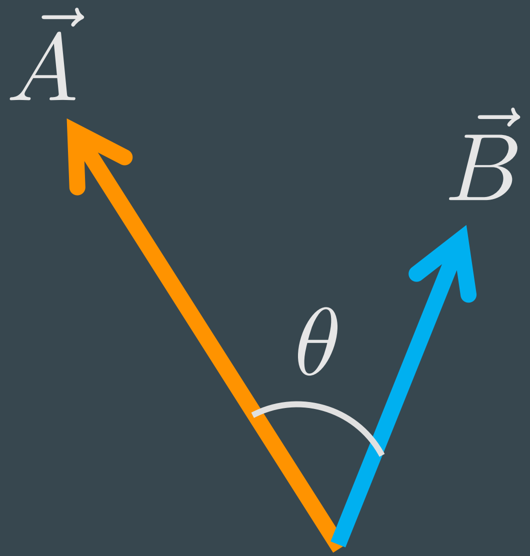
$\theta = 180^{\circ}$

A. 420 into the page

B. 420 out of the page

C. Zero

CLICKER 3: If $|\vec{A}|$ = 35 and $|\vec{B}|$ = 12, what is $\vec{A} \times \vec{B}$?



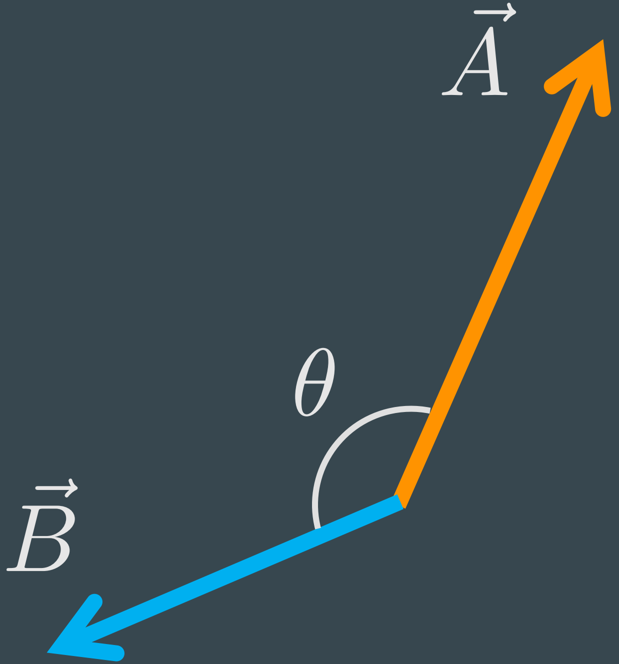
A. 177.5 into the page

B. 177.5 out of the page

C. Zero

$$\theta = 25^\circ$$

CLICKER 4: If $|\vec{A}| = 35$ and $|\vec{B}| = 12$, what is $\vec{A} \times \vec{B}$?



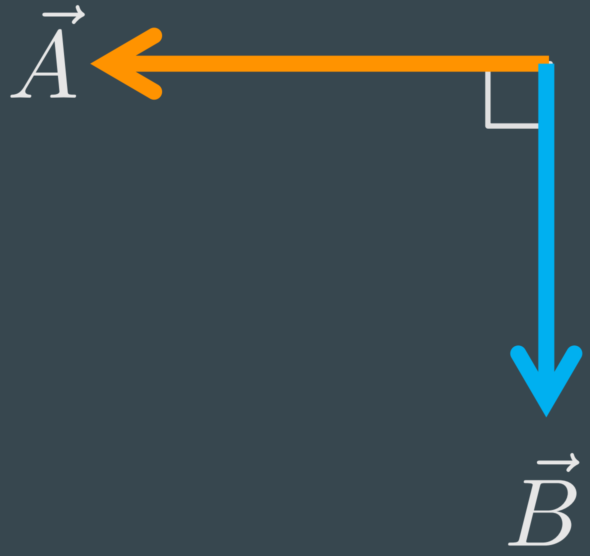
A. 321.7 into the page

B. 321.7 out of the page

C. Zero

$$\theta = 130^\circ$$

CLICKER 5: If $|\vec{A}|$ = 35 and $|\vec{B}|$ = 12, what is $\vec{B} \times \vec{A}$?



A. 420 into the page

B. 420 out of the page

C. Zero

$$\theta = 90^\circ$$

Symbols for indicating the direction

- Into the page $\otimes$

  (tail of an arrow; thumb points away from you)

- Out of the page $\odot$

  (head of an arrow; thumb points towards you)

Properties of the cross product

- The cross product takes as input two vectors and outputs a vector that is orthogonal to the plane defined by the vectors being crossed

- The cross product DOES NOT commute $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

- The cross product is distributive over addition

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

- The cross product of a vector with itself is zero $\vec{A} \times \vec{A} = 0$

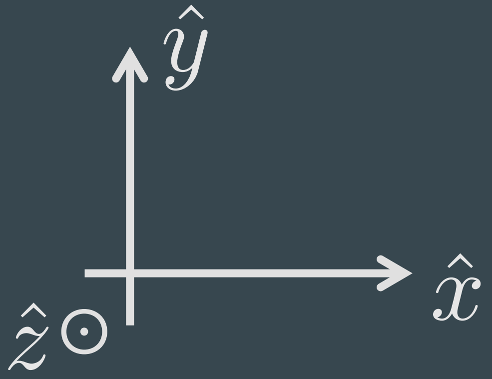
Right-handed coordinates

- The usual cartesian coordinate system (+x to the right, +y upwards, +z out of the page) is called a right-handed coordinate system because of the cross products of the unit vectors

$$\hat{x} \times \hat{y} = \hat{z}$$

$$\hat{y} \times \hat{z} = \hat{x}$$

$$\hat{z} \times \hat{x} = \hat{y}$$



- Notice how we cycle in order through x, y, z to get these

Component form

- If you have the vectors expressed in component form,

$$\vec{A} = \langle A_x, A_y, A_z \rangle$$

$$\vec{B} = \langle B_x, B_y, B_z \rangle$$

- Then the cross product $\vec{A} \times \vec{B}$ is:

$$\vec{A} \times \vec{B} = \langle (A_y B_z - A_z B_y), (A_z B_x - A_x B_z), (A_x B_y - A_y B_x) \rangle$$

Component form: Determinant method

- A good way of remembering how to do the component form cross product is by arranging the vectors to cross in a 3 x 3 matrix and calculating its determinant

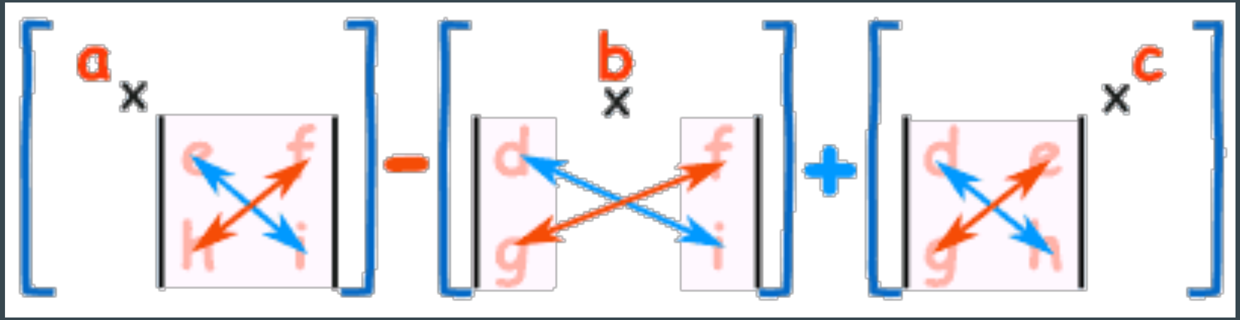
- Math Stuff: the determinant of a 2 x 2 matrix M is defined as:

$$\det(M) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Component form: Determinant method

- Math Stuff: the determinant of a 3 x 3 matrix M is defined as:

$$\det(M) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$



What is the determinant of this matrix?

$$\begin{vmatrix} 2 & 1 & 5 \\ 6 & 3 & 4 \\ 1 & 3 & 2 \end{vmatrix}$$

Component form: Determinant method

- So, to get the cross product $\vec{A} \times \vec{B}$ ...

$$\vec{A} = \langle A_x, A_y, A_z \rangle \quad \text{and} \quad \vec{B} = \langle B_x, B_y, B_z \rangle$$

- We calculate the determinant of this 3 x 3 matrix:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{array}{l} \leftarrow \text{First row: unit vectors} \\ \leftarrow \text{Second row: first vector} \\ \leftarrow \text{Third row: second vector} \end{array}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Component form: Determinant method

$$\vec{A} \times \vec{B} = \langle (A_y B_z - A_z B_y), (A_z B_x - A_x B_z), (A_x B_y - A_y B_x) \rangle$$

CLICKER 6: Determine the cross product $\vec{A} \times \vec{B}$

$$\vec{A} = < 3, 0, -1 >$$

$$\vec{B} = < -4, -2, 2 >$$

A. <2, 2, 6>
B. <-2, -2, 6>
C. <2, -10, 6>
D. <2, 10, 6>
E. -24
F. 24