PHYS 2211 Test 2
Fall 2011

Name(print)____

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you don't want us to read!
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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Sign your name on the line above

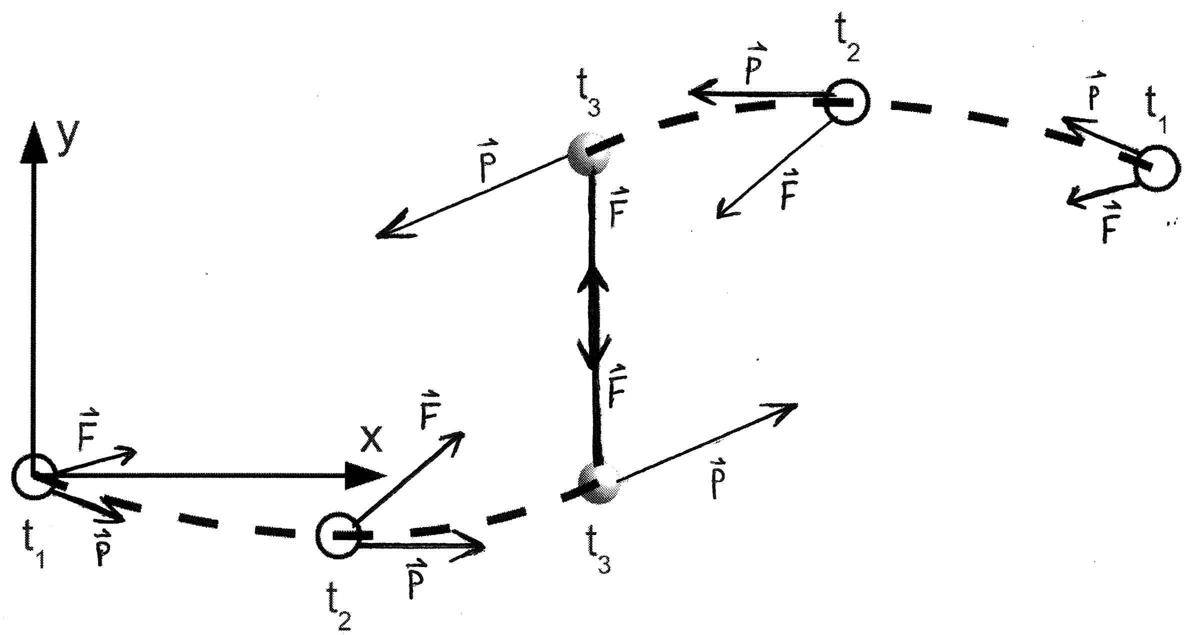
Problem 1 (25 Points)

The starship Enterprise and her crew were exploring deep space when they encountered a large black cube. After attempting to scan the cube for life, a large tractor beam from the cube locked onto the Enterprise and began to pull the starship towards itself with an attractive force with magnitude Kr^{-1}, where K is positive and r is the distance between the mysterious black cube and the Enterprise. Below is an incomplete VPython program to calculate the position of the Enterprise while under the influence of the cube's tractor beam (the only force acting on the ship). Fill in the missing VPython statements below to update the position of the spacecraft. You may assume that the mysterious black cube remains motionless.

```
from visual import *
# Objects
cube = box(pos=vector(0,0,0),color=color.black, length=3.4e6,width=3.4e6,height=3.4e6)
enterprise = sphere(pos=vector(8e5,6e5,0),color=color.gray,radius=6.4e2)

# Constants and Mass
K = 5e30
mEnterprise = 4e8

# Initial values
pEnterprise = mEnterprise*vector(2e3,-7e3,0)
deltat = 1800
t = 0
while = t<5*3600
# (a 15pts) Update the position of the Enterprise
    r = enterprise.pos - cube.pos
    rmag = sqrt(r.x **2 + r.y **2 + r.z **2)
    rhat = r/rmag
    Fnet = -(K/rmag) * rhat
    p_init = mag(pEnterprise)        #added for part (b)
    pEnterprise = pEnterprise + Fnet * deltat
    enterprise.pos = enterprise.pos + (pEnterprise/mEnterprise) * deltat
# (b 10pts) Calculate the components of the net force on the Enterprise.
#           You may need to add a line of code to part (a).
    p_final = mag(pEnterprise)
    Fnet_tangent = (p_final - p_init)/deltat * pEnterprise/p_final

    Fnet_perpendicular = Fnet - Fnet_tangent

    t = t + deltat
```

Problem 2 (25 Points)

Here is a portion of the trajectories of two identical asteroids of mass m interacting gravitationally and far from anything else. The asteroids are moving toward each other, with positions marked at times t_1, t_2, and t_3.



(a 12pts) At each of these positions, draw vectors of appropriate lengths and directions for the forces acting on each of the asteroids at that location. Label these vectors $\vec{F}$. At the same locations draw vectors of appropriate lengths and directions for the momenta of each of the asteroids at that location, and label them $\vec{p}$. ("Appropriate lengths" means that larger magnitudes are represented by longer vectors.)

(b 10pts) At time t_3, asteroid 1 is located at $\vec{r}_1 = \langle d, 0, 0\rangle$ and asteroid 2 is located at $\vec{r}_2 = \langle d, w, 0\rangle$. Calculate the gravitational force (magnitude and direction) acting on asteroid 2.

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = \langle d, \omega, 0\rangle - \langle d, 0, 0\rangle = \langle 0, \omega, 0\rangle$$

$$|r| = \sqrt{0^2 + \omega^2 + 0^2} = \omega$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \langle 0, 1, 0\rangle$$

$$\vec{F}_g = -G\frac{m_1 m_2}{|r|^2}\hat{r}$$

$$\vec{F}_g = -G\frac{m^2}{\omega^2}\langle 0, 1, 0\rangle$$

$$\vec{F}_g = \boxed{\left\langle 0, -\frac{Gm^2}{\omega^2}, 0\right\rangle}$$

(c 3pts) The total momentum of the two asteroid system was zero at time t_1. Determine the center of mass $\vec{r}_{CM}$ and momentum of the center of mass $\vec{p}_{CM}$ of the two asteroid system at time t_3.

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M_{total}} = \frac{m\vec{r}_1 + m\vec{r}_2}{2m} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$\vec{r}_{CM} = \frac{\langle d, 0, 0\rangle + \langle d, \omega, 0\rangle}{2} = \boxed{\left\langle d, \frac{\omega}{2}, 0\right\rangle}$$

Conservation of Momentum:

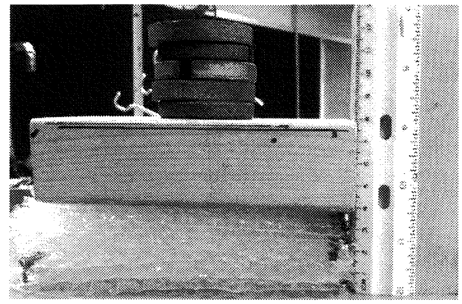
$$\vec{p}_{CM,i} = \vec{p}_{CM,f}$$

If $\vec{p}_{CM,t_1} = \langle 0, 0, 0\rangle$

then $\boxed{\vec{p}_{CM,t_3} = \langle 0, 0, 0\rangle}$

Problem 3 (25 Points)

You and a friend decide that physics really is amazing and set out to measure the speed of sound in bubble wrap based on a previous lab you both completed. To do this you collect a large quantity of bubble wrap and cut it up into sheets with a cross section area of 125.8 cm^2. You stack several of these sheets on top of one another until your stack is 6.0 cm high. By measuring the volume and mass of your bubble stack you approximate the density of bubble wrap to be 0.017 g/cm^3.



(a 5pts) The bubble wrap is made of plastic which is mostly carbon. Carbon has an atomic mass of 12.011 g/mol. If we approximate our bubble stack as a ball-and-spring collection of carbon atoms, determine the center-to-center distance between two of the carbon atoms in our stack.

$$d \approx V^{1/3} = \left[\left(\frac{1}{D}\right)(M)\left(\frac{1}{N_A}\right)\right]^{1/3}$$

where $D = 0.017\, g/cm^3 = 1.7\times10^4\, g/m^3$

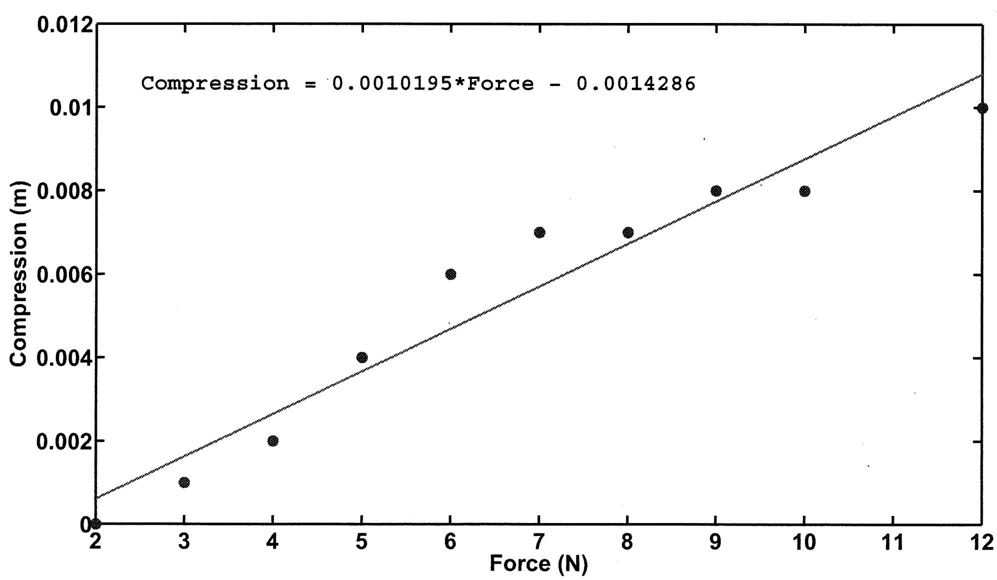
$M = 12.011\, g/mol$

$N_A = 6.02\times10^{23}$ atoms/mol

$$d \approx \left[\frac{m^3}{1.7\times10^4 g} \cdot \frac{12.011 g}{mol} \cdot \frac{mol}{6.02\times10^{23} atoms}\right]^{1/3}$$

$$\boxed{d \approx 1.05\times10^{-9}\ m}$$

Next you stack weights on your stack of bubble wrap (see above) and measure how much the stack compresses. You record this data in excel and find a linear fit of your data just as you did in lab for the stretch of a copper wire. This data is given below along with a best fit.



(b 5pts) What is the spring stiffness k_s of the entire bubble stack, considered as a single macroscopic (large scale) spring?

$$k_s = \frac{1}{m} = \frac{1}{0.0010195} = \boxed{980.9\ N/m}$$

(c 5pts) Calculate Young's modulus Y for this bubble stack.

$$Y = \frac{F/A}{\Delta L/L} = \frac{k_s L}{A} = \frac{(980.9\ N/m)(0.06m)}{(0.01258m^2)} = \boxed{4678\ N/m^2}$$

Use $k_s = \frac{F}{\Delta L}$ as in lab. Do not use a data point from the chart.

(d 5pts) How tall of a stack of bubble wrap, with the same cross sectional area of 125.8 cm^2, would you need so that 100 kgs placed on top of the stack compresses 1 meter?

$$Y = \frac{F/A}{\Delta L/L} = \frac{F \cdot L}{\Delta L \cdot A} \Rightarrow L = \frac{Y \Delta L A}{F} = \frac{Y \Delta L A}{mg}$$

$$L = \frac{(4678\ N/m^2)(0.01258\ m^2)(1m)}{(100kg)(9.8\ m/s^2)} = \boxed{0.06\ m}$$

(Unphysical)

It would pop!

(e 5pts) Determine the speed of sound in the bubble stack?

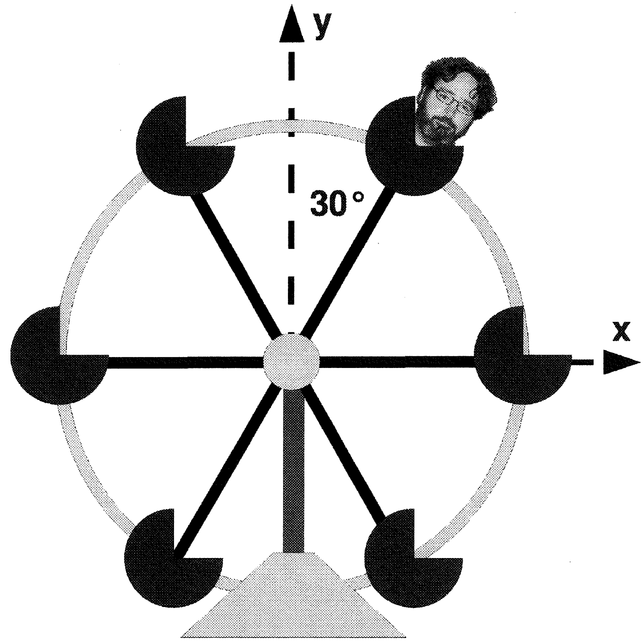
$$v = d\sqrt{\frac{k_{si}}{m_a}} \quad \text{where } k_{si} = Yd = (4678\ N/m^2)(1.05 \times 10^{-9} m) = 4.91 \times 10^{-6}\ N/m$$

$$m_a = \frac{0.012011\ kg}{mol} \cdot \frac{mol}{6.02 \times 10^{23} atoms} = 2.00 \times 10^{-26}\ kg/atom$$

$$v = (1.05 \times 10^{-9} m)\sqrt{\frac{(4.91 \times 10^{-6}\ N/m)}{(2.00 \times 10^{-26} kg)}} = \boxed{16.5\ m/s}$$

Problem 4 (25 Points)

The Singapore Flyer (the world's largest Ferris wheel) has a diameter of 165 m. The wheel rotates counterclockwise at a *constant rate*, completing a full rotation in 37 minutes. Dr. Greco, a 100 kg passenger, is riding in the gondola at an angle of 30 degrees from the vertical.



(a 3pts) Choosing Dr. Greco as your system, which objects in the surroundings exert a force on him.

The Earth & the seat of the Ferris wheel

(b 2pts) What is the parallel component of the rate of change of Dr. Greco's momentum? Explain how you know this. Your answer should be a vector.

$$\boxed{\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \langle 0, 0, 0 \rangle}$$

because the wheel rotates at a constant rate, meaning $|v| = \text{constant} \Rightarrow |p| = \text{constant} \Rightarrow \left|\frac{dp}{dt}\right|_{\parallel} = 0$

(c 5pts) What is the perpendicular component of the rate of change of Dr. Greco's momentum? Your answer should be a vector.

$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = \vec{F}_{net,\perp} = \frac{|p||v|}{R}\hat{n}$$

where $|v| = \frac{2\pi R}{T} = \frac{\pi(165\,m)}{(37 \cdot 60\,s)} = 0.2335\,m/s$

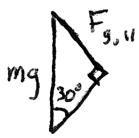
$$= \frac{(100\,kg)(0.2335\,m/s)^2}{(82.5\,m)}\hat{n}$$

$$= \boxed{(0.0661\,N)\,\hat{n}}$$

(d 5pts) What is the magnitude of the parallel component of the contact force exerted by the Ferris wheel on Dr. Greco?

From (b), $\vec{F}_{net,\parallel} = \langle 0,0,0 \rangle$. But $\vec{F}_{net,\parallel} = \vec{F}_{FW,\parallel} + \vec{F}_{g,\parallel}$

So, $\vec{F}_{FW,\parallel} = -\vec{F}_{g,\parallel} \Rightarrow |F_{FW,\parallel}| = |F_{g,\parallel}|$

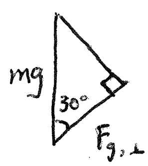


Thus, $|F_{FW,\parallel}| = |F_{g,\parallel}| = mg \sin(30°) = (100kg)(9.8\,m/s^2)\sin(30°)$

$= \boxed{490\ N}$

(e 5pts) What is the magnitude of the perpendicular component of the contact force exerted by the Ferris wheel on Dr. Greco?

$\vec{F}_{net,\perp} = \vec{F}_{FW,\perp} + \vec{F}_{g,\perp}$



$$\frac{m|v|^2}{R}\hat{n} = F_{FW,\perp}\hat{n} + mg\cos(30°)\hat{n}$$

$$F_{FW,\perp}\hat{n} = \left(mg\cos(30°) - \frac{m|v|^2}{R}\right)\hat{n} = \left((100kg)(9.8\,m/s^2)\left(\tfrac{\sqrt{3}}{2}\right) - \frac{(100kg)(0.2335\,m/s)^2}{(82.5m)}\right)\hat{n}$$

$$F_{FW,\perp}\hat{n} = (-848.6\,N)\hat{n}$$

$$\boxed{|F_{FW,\perp}| = 848.6\ N}$$

(f 5pts) Now consider the instant where Dr. Greco is momentarily at the top of the Ferris wheel. With what speed must Dr. Greco be traveling so that he feels weightless?

$\vec{F}_{net,\perp} = \vec{F}_{FW,\perp} + \vec{F}_{g,\perp}$

To feel weightless, the contact force $F_{FW,\perp}$ must be zero. So:

$$|F_{net,\perp}| = |F_{g,\perp}| \Rightarrow \frac{m|v|^2}{R} = mg \Rightarrow |v| = \sqrt{gR}$$

$$\Rightarrow |v| = \sqrt{(9.8\,m/s^2)(82.5m)} = \boxed{28.4\ m/s}$$