Physics 2211 GPS Week 8

Problem #1

Ed and Mike are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.5 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Ed exerts a force $\langle -400, 0, 200\rangle$ N and Mike exerts a force $\langle 150, 0, 300\rangle$ N.

(a) How much work does Ed do?

✓ $\Delta\vec{r}_{boat} = \vec{r}_f - \vec{r}_i = \langle 4, 0, 2\rangle - \langle 2, 0, 3\rangle = \langle 2, 0, -1\rangle$ m

$$\Rightarrow W_{Ed} = \vec{F}_{Ed} \cdot \Delta\vec{r}_{boat} = \langle -400, 0, 200\rangle \cdot \langle 2, 0, -1\rangle =$$

$$= (-400)(2) + (200)(-1) = -800 - 200 = \boxed{-1000 \text{ J}}$$

(b) How much work does Mike do?

✓ $\Delta\vec{r}_{boat}$ = same as above = $\langle 2, 0, -1\rangle$ m

$$\Rightarrow W_{Mike} = \vec{F}_{Mike} \cdot \Delta\vec{r}_{boat} = \langle 150, 0, 300\rangle \cdot \langle 2, 0, -1\rangle =$$

$$= (150)(2) + (300)(-1) = 300 - 300 = \boxed{0}$$

(c) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

$$\Delta E = W_{total}$$

$$\Delta K = W_{Ed} + \cancel{W_{Mike}}^{\,0}$$

$$K_f - K_i = W_{Ed}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = W_{Ed}$$

$$v_f^2 - v_i^2 = \frac{2}{m}W_{Ed}$$

$$v_f^2 = \frac{2W}{m} + v_i^2$$

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-1000)}{3000} + (1.5)^2} = \boxed{1.26 \text{ m/s}}$$

(d) What effect does Mike have on the boat's motion?

Steering (changing direction of motion)

Problem #2

A lighthouse keeper spots a sailboat of mass M at location A $\langle x_0, 0, 0\rangle$ moving with speed v_0. After dozing off for a quick nap, the lighthouse keeper awakens to find the sailboat at location B $\langle 0, y_0, 0\rangle$. Having no way to measure the passage of time, the keeper decides to use her vast knowledge of the sea to estimate the speed of the sailboat. The keeper estimates that the net force acting on the sailboat is constant and given by $\langle a, b, 0\rangle$ where both a and b are positive constants. What would the lighthouse keeper predict for the speed of the sailboat at location B?

$$\vec{r}_A = \langle x_0, 0, 0 \rangle$$

$$|\vec{v}_A| = v_0$$

$$\vec{r}_B = \langle 0, y_0, 0 \rangle$$

$$\vec{F}_{net} = \langle a, b, 0 \rangle \quad \left\{ \begin{matrix} a > 0 \\ b > 0 \end{matrix} \right\}$$

$$\Delta E = W$$

$$\Delta K = \int \vec{F}_{net} \cdot d\vec{l}$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \vec{F}_{net} \cdot \Delta \vec{L} \qquad \text{where } \Delta \vec{L} = \vec{r}_f - \vec{r}_i$$

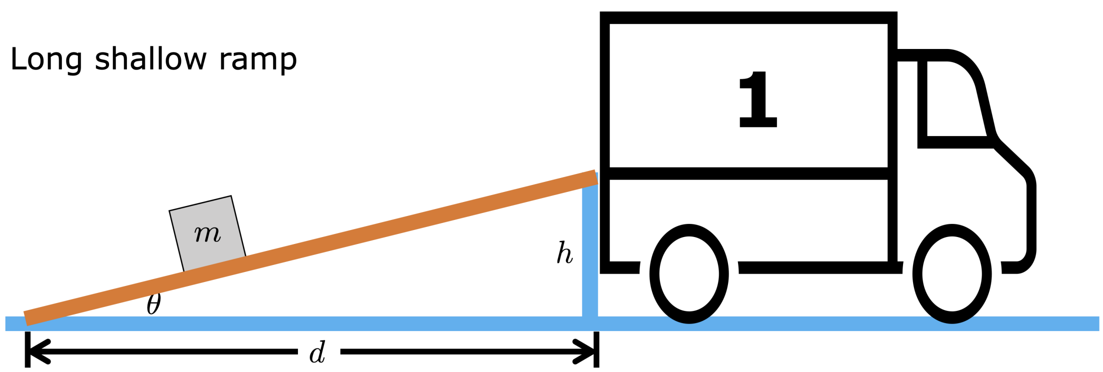
$$= \vec{F}_{net} \cdot \langle -x_0, +y_0, 0 \rangle \qquad = \langle -x_0, +y_0, 0 \rangle$$

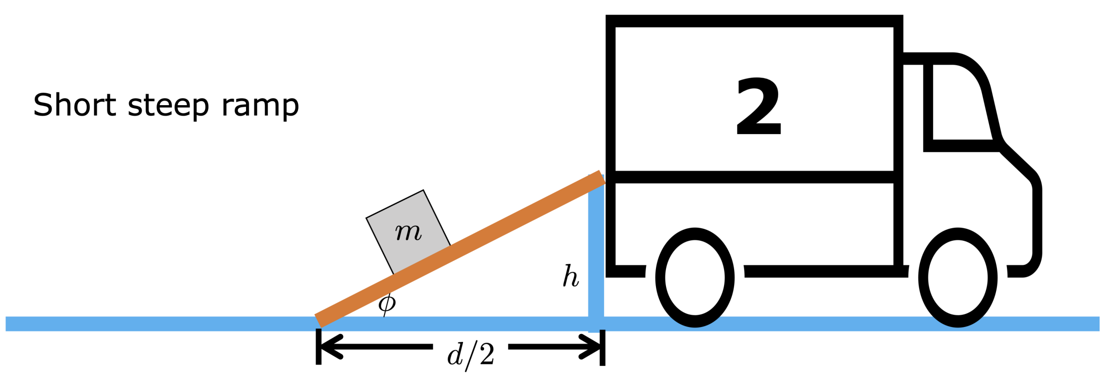
$$\frac{1}{2} m (v_f^2 - v_0^2) = -a x_0 + b y_0$$

$$\rightarrow \quad v_f = \sqrt{\frac{2}{m}(b y_0 - a x_0) + {v_0}^2}$$

Problem #3

You are moving and want to use your knowledge from PHYS 2211 to help you decide which truck to rent out of two options. Truck number one has a **long ramp** at a shallow angle θ. Truck number two has a **short ramp** at a steep angle ϕ. You start with the simple problem of pushing a box of mass m up to the height h of the truck. You can assume both trucks have **frictionless** ramps.





Consider the **box** to be the system, the **initial state** to be when the box is motionless at the bottom of the ramp, and the **final state** to be when the box is at the top of the ramp.

(a) What is the **work done by gravity** as the system goes from its initial state to its final state?

$$W_{grav} = \int_i^f \vec{F}_g \cdot d\vec{r}$$

$$= \vec{F}_g \cdot \vec{d} \quad \text{(b/c } \vec{F}_g \text{ is constant)}$$

$$= mg(-\hat{y}) \cdot h(\hat{y})$$

$$\boxed{W_{grav} = -mgh}$$

(b) If you push the box with a force of magnitude F that is **parallel to the ramp**, how much work W_{long} do you do on the box if you use the **LONG** ramp?

$$W_{long} = \vec{F}_{long} \cdot \vec{d}_{long}$$

$$|\vec{d}_{long}| = \sqrt{d^2 + h^2}$$

$$\vec{d}_{long} = \langle d, h, 0 \rangle$$

$$\boxed{W_{long} = F\sqrt{d^2 + h^2}}$$

$$(\vec{F}_{long} \parallel \vec{d}_{long})$$

(c) If you push the box with a force of magnitude F that is **parallel to the ramp**, how much work W_{short} do you do on the box if you use the **SHORT** ramp?

$$W_{long} = \vec{F}_{short} \cdot \vec{d}_{short}$$

$$= F\sqrt{\frac{d^2}{4} + h^2}$$

$$|\vec{d}_{short}| = \sqrt{\left(\frac{d}{2}\right)^2 + h^2}$$

$$= \sqrt{\frac{d^2}{4} + h^2}$$

$$\vec{d}_{short} = \langle \frac{d}{2}, h, 0 \rangle$$

$$(\vec{F}_{short} \parallel \vec{d}_{short})$$

(d) Which ramp should you use if you want the box to move slower when it reaches the top of the ramp? Use the energy principle to determine the speed of the box at the top of each ramp.

$$\Delta E_{box} = W_{net\ on\ box}$$

Short

$$\Delta E_{box} = W_{short} + W_{grav}$$

$$= F\sqrt{\frac{d^2}{4}+h^2} - mgh$$

$$= \Delta KE_{box} = K_f - \cancel{K_i}^{0}$$

$$= \frac{1}{2}mv_f^2$$

$$\Rightarrow \boxed{v_{f,short} = \sqrt{\frac{2F}{m}\sqrt{\frac{d^2}{4}+h^2} - 2gh}}$$

Long

$$\Delta E_{box} = W_{long} + W_{grav}$$

$$= F\sqrt{d^2+h^2} - mgh$$

$$= \Delta KE_{box} = K_f - \cancel{K_i}^{0}$$

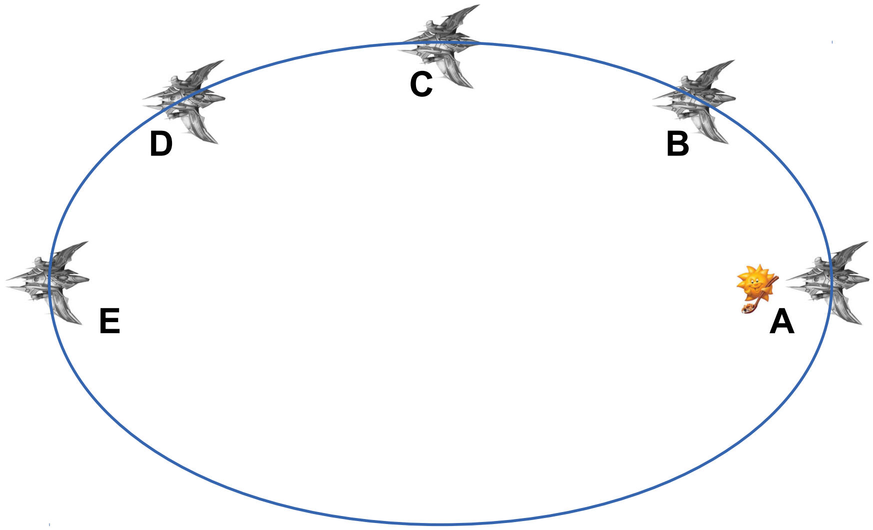
$$= \frac{1}{2}mv_f^2$$

$$\Rightarrow \boxed{v_{f,long} = \sqrt{\frac{2F}{m}\sqrt{d^2+h^2} - 2gh}}$$

Use the short ramp. Less work is done to move it to the top due to F being constant, so the box has less KE, and hence, a lower speed at the top.

Problem #4

The diagram shows the path of a spacecraft orbiting a star. You will be asked to rank order various quantities in terms of their values at the locations marked on the path, with the largest first. You can use the symbols “<” and “=”. For example, if you were asked to rank order the locations in terms of their distance from the star: “A <B <C <D <E”



(a) Rank order the locations on the path in terms of the spacecraft’s **kinetic energy** at each location, starting with the location where the kinetic energy is the largest.

A > B > C > D > E

(b) Consider the system of the spacecraft plus the star. **Which of the following statements are correct?**

A. As the kinetic energy of the system increases, the gravitational potential energy of the system decreases.

B. As the spacecraft slows down, the kinetic energy of the system decreases.

C. As the spacecraft slows down, energy is lost from the system.

D. External work must be done on the system to speed up the spacecraft.

E. As the spacecraft’s kinetic energy increases, the gravitational potential energy of the system also increases.

F. Along this path the gravitational potential energy of the system is never zero.

G. The sum of the kinetic energy of the system plus the gravitational potential energy of the system is a positive number.

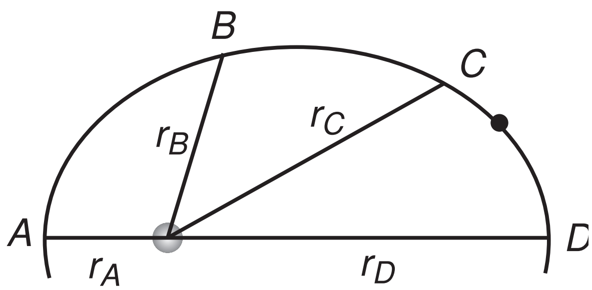
H. The gravitational potential energy of the system is inversely proportional to the square of the distance between the spacecraft and star.

I. The sum of the kinetic energy of the system plus the gravitational potential energy of the system is the same at every location along this path.

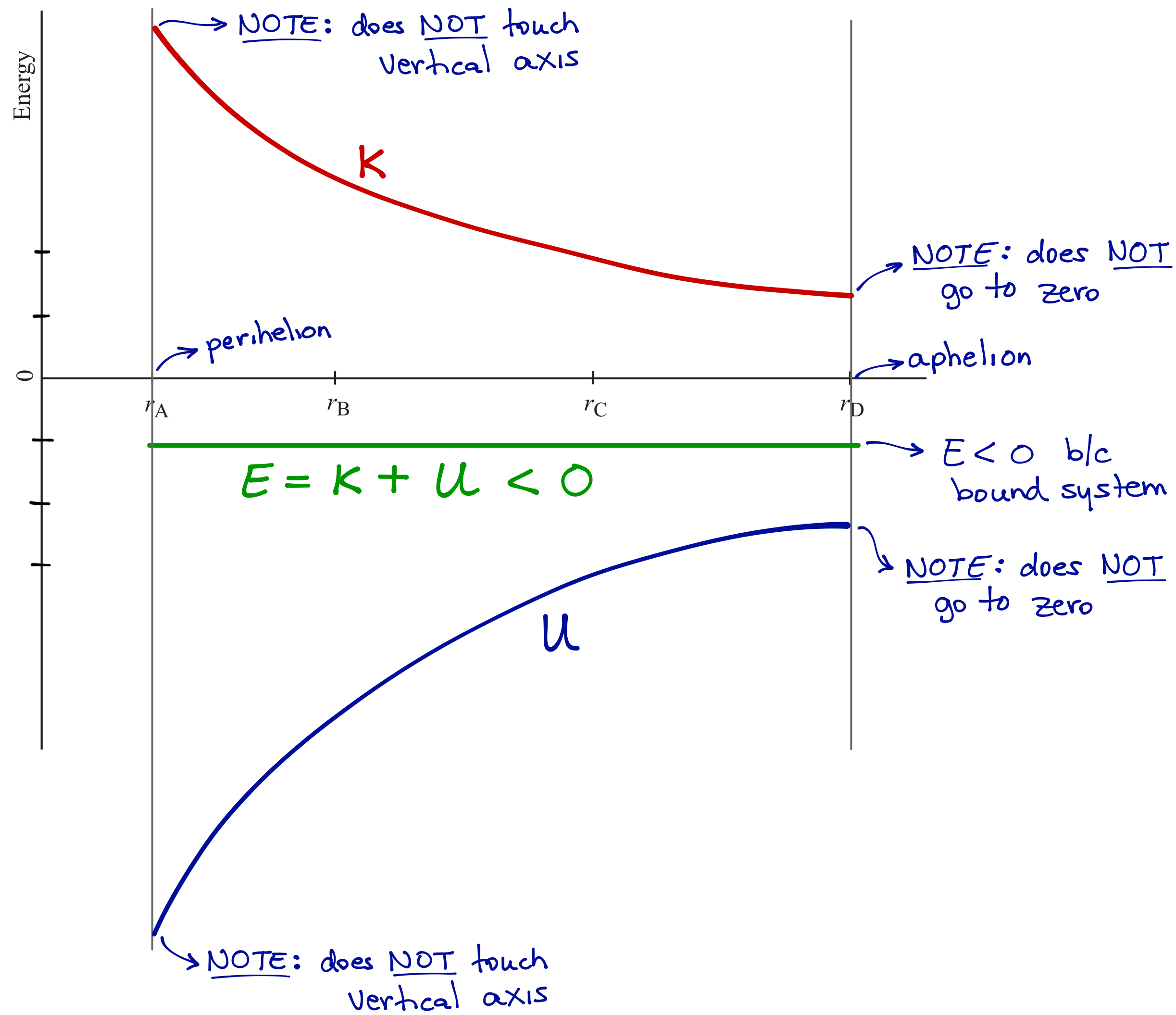
J. At every location along the spacecraft’s path the gravitational potential energy of the system is negative.

Problem #3

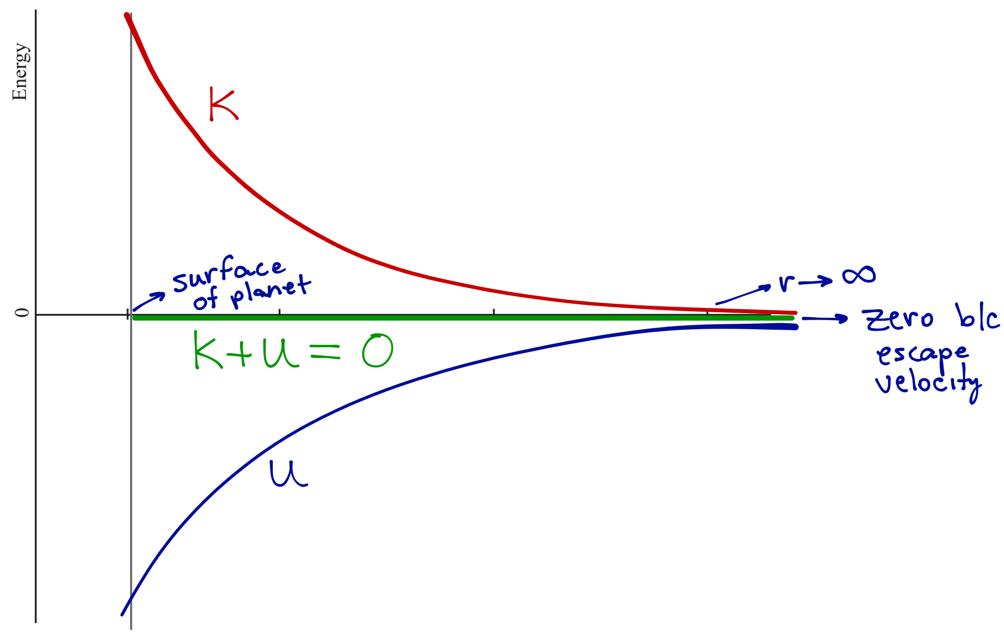
Here is a portion of the orbit of an asteroid around the Sun in an elliptical orbit, moving from A to B to C to D.



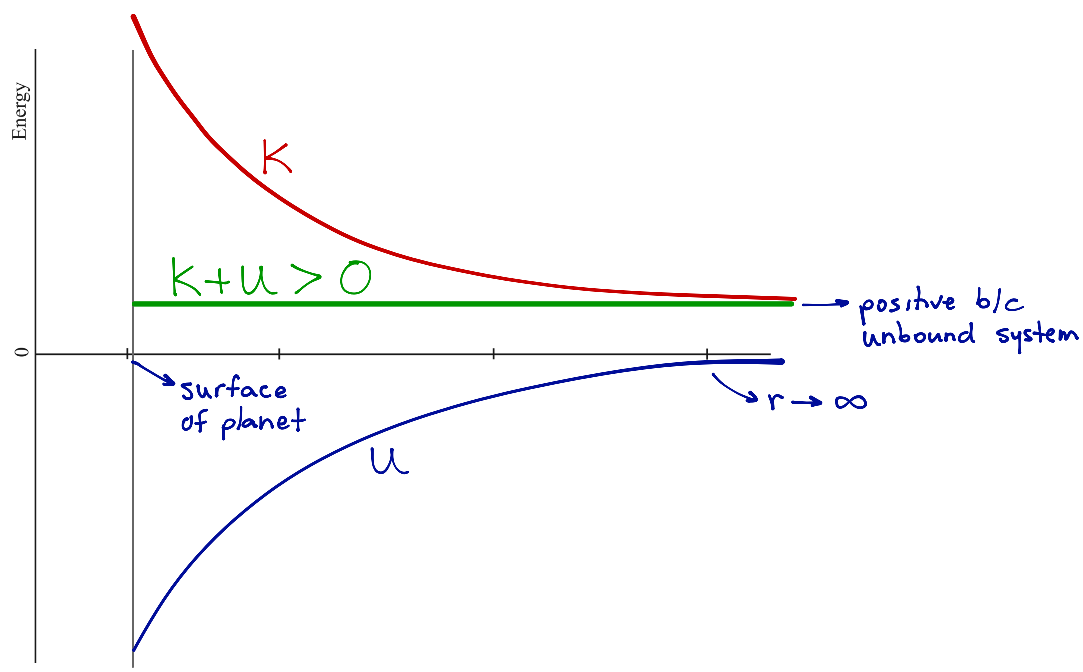
(a) For the system consisting of the Sun plus the asteroid, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between Sun and asteroid. ***Label each curve.*** Along the r axis are shown the various distances between Sun and asteroid.



(b) A spacecraft leaves the surface of a planet at exactly the escape speed. For the system consisting of a planet and a spacecraft, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between planet and the spacecraft. ***Label each curve.***

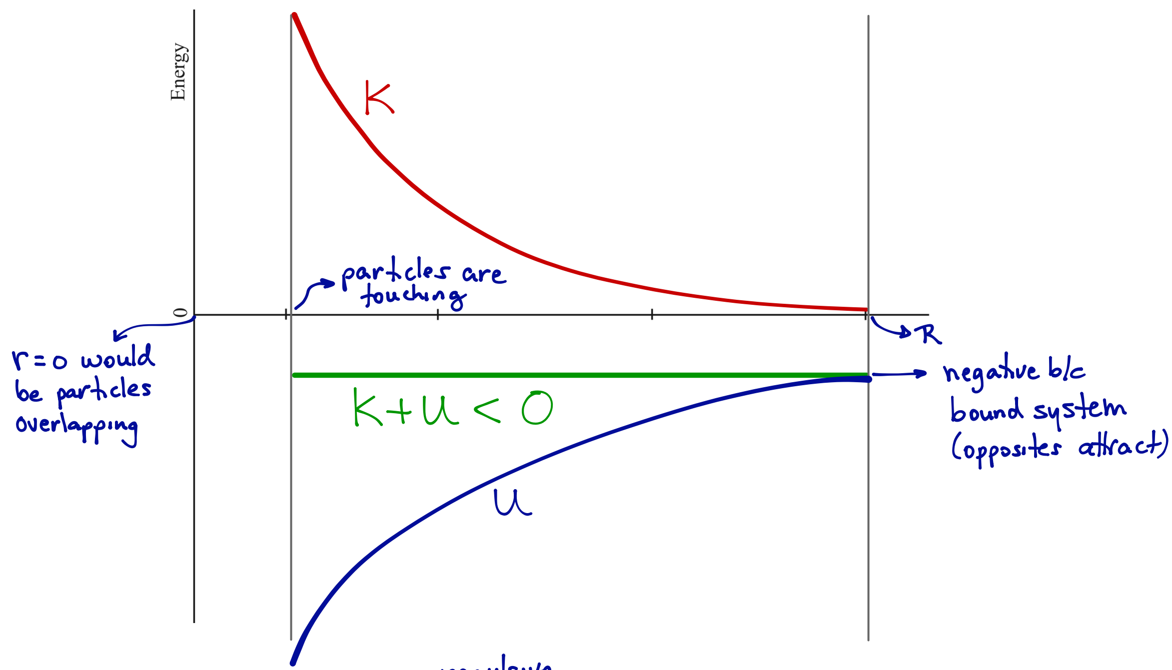


(c) A spacecraft leaves the surface of a planet with a velocity that is twice the escape speed. For the system consisting of a planet and a spacecraft, graph the gravitational potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between planet and the spacecraft. ***Label each curve.***



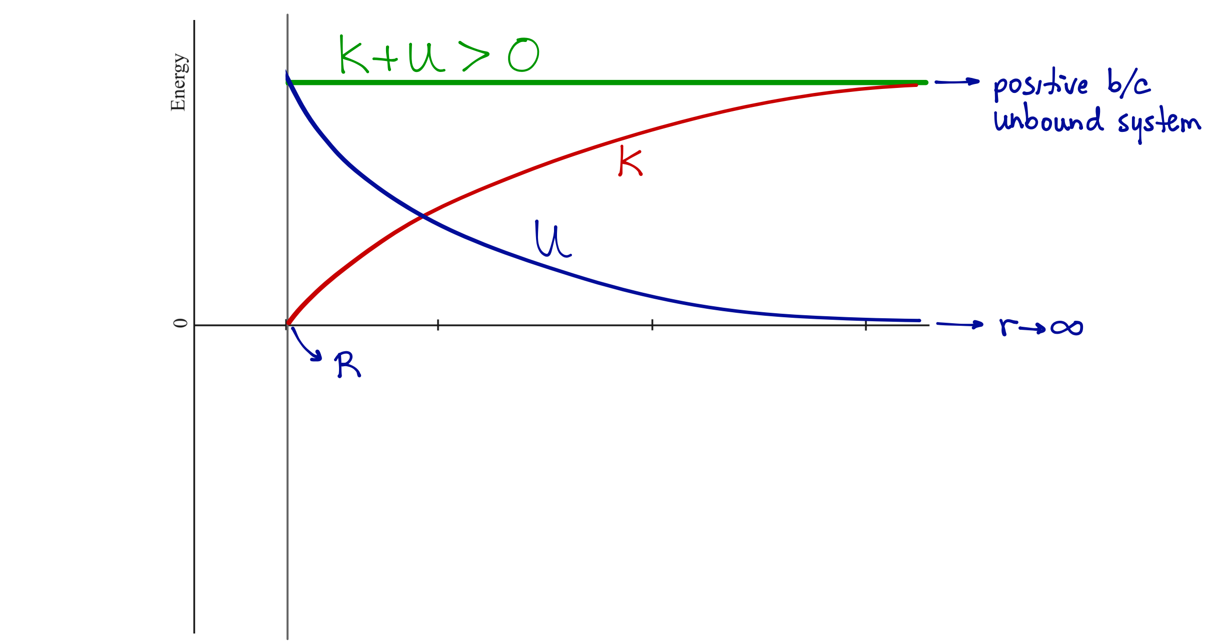
attractive

(d) Two charged particle with opposite charge and identical mass are released from rest a distance R from each other. For the system consisting of the two charges, graph the electric potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between the two charges. ***Label each curve.***



repulsive

(e) Two charged particle with identical charge and mass are released from rest a distance R from each other. For the system consisting of the two charges, graph the electric potential energy U, the kinetic energy K, and the sum $K + U$, as a function of the separation distance between the two charges. ***Label each curve.***



Problem #4

During the spring semester at MIT, residents of the parallel buildings of the East Campus Dorms battle one another with large sling-shots made from surgical hose mounted to window frames. Water balloons (with a mass of about 0.5 kg) are placed in a pouch attached to the hose, which is then stretched nearly the width of the room (about 3.5 meters). If the hose obeys Hooke's Law, with a spring constant of 100 N/m, how fast is the balloon traveling when it leaves the dorm room window?

System: water ball on

Surroundings: hose

Initial: max stretch, balloon at rest

Final: no stretch, balloon released

Assumption: no vertical displacement between stretch and release. (horizontal spring)

$$\Delta K_{sys} = W_{surr}$$

$$K_f - \cancel{K_i}^{0} = \int_{s_i}^{f} \vec{F} \cdot d\vec{r}$$

$$\frac{1}{2} m v_f^2 = \int_{s_i}^{s_f} -ks\, ds = -k \int_{s_i}^{0} s\, ds$$

$$\cancel{\tfrac{1}{2}} m v_f^2 = -k \left(\frac{s^2}{2}\right)\Big|_{s_i}^{0} = -k\left(0 - \frac{1}{2} s_i^2\right) = \cancel{\tfrac{1}{2}} k s_i^2$$

$$m v_f^2 = k s_i^2$$

$$v_f^2 = \frac{k}{m} s_i^2$$

$$v_f = \sqrt{\frac{k}{m}}\, s_i = \sqrt{\frac{100}{0.5}}\,(3.5) = \boxed{49.5 \text{ m/s}}$$