PHYS 2211 Exam 4 - Spring 2018

Please circle your lab section and fill in your contact info below.

Section (K Curtis) and (M Fenton)		
Day	12-3pm	3-6pm
Monday	K01 M01	K02 M02
Tuesday	K03 M03	K04 M04
Wednesday	K05 M05	K06 M06
Thursday	K07 M07	K08 M08

The final exam is Monday April 30 from 6-8:50 PM

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

Key

Sign your name on the line above

Instructions

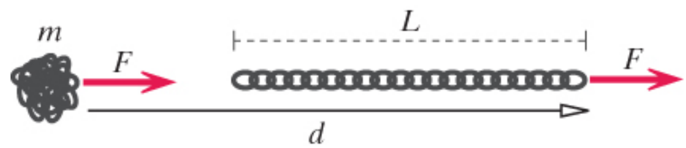
- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a\cdot b}{c\cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Problem 1 [25 pts] Grader & Score: Key

A chain of metal links with total mass m is coiled up in a tight ball on a low-friction table. Initially the chain is at rest before you pull on a link at one end of the chain with a constant force F. Eventually the chain straightens out to its full length L, and you keep pulling until you have pulled your end of the chain a total distance d as shown in the diagram.



A. [10 pts] Consider the point particle (i.e. center of mass) system: What is the speed of the chain when the end of the chain has reached a distance d?

Energy principle

$$\Delta E = W$$

$$\Delta K_{trans} = F\left(d - \frac{L}{2}\right)$$

$$\frac{1}{2} m v_f^2 = F\left(d - \frac{L}{2}\right)$$

$$v_f^2 = \frac{2F}{m}\left(d - \frac{L}{2}\right)$$

$$\boxed{v_f = \sqrt{\frac{2F}{m}\left(d - \frac{L}{2}\right)}}$$

-1 Clerical / Units
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-8 BTN

Common mistakes:
-4 Including internal energy
-2 Using the distance traveled by the end instead of center of mass
-2 For incorrect distance used for work

B. [5 pts] Consider the real (i.e. extended) system: What is the change in the total energy of the chain?

$$\Delta E = W$$

$$\boxed{\Delta E = Fd}$$

All or Nothing

C. [10 pts] In straightening out, the links of the chain bang against each other, and their temperature rises. Assume that the process is so fast that there is insufficient time for significant thermal transfer of energy from the chain to the table, and ignore the small amount of energy radiated away as sound produced in the collisions among the links. Calculate the increase in thermal energy of the chain.

$$\Delta E = \Delta K_{trans} + \Delta E_{thermal}$$

$$\underbrace{Fd}_{\text{From part b}} = \underbrace{F\left(d - \frac{L}{2}\right)}_{\text{From part a}} + \Delta E_{thermal}$$

$$\Delta E_{thermal} = Fd - F\left(d - \frac{L}{2}\right)$$

$$= Fd - Fd + \frac{FL}{2}$$

$$= \frac{FL}{2}$$

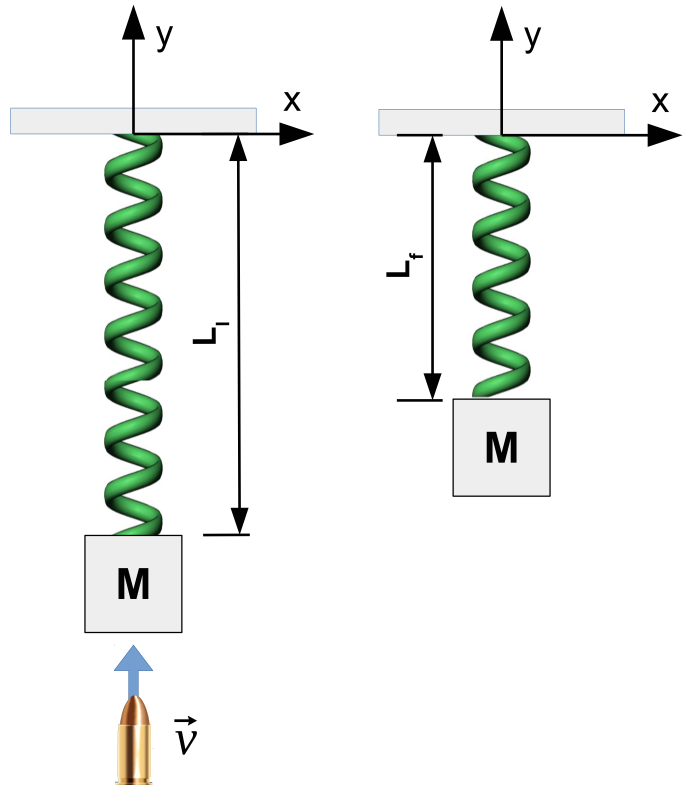
$$\boxed{\Delta E_{thermal} = \frac{FL}{2}}$$

-1 Clerical / Units
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-8 BTN

Watch for POE

Problem 2 [25 pts] Grader & Score: Key

A block of mass M hangs from a spring with stiffness k and rest length L_0. A bullet of mass m traveling at a speed of $|\vec{v}|$ straight upward buries itself in the block. The bullet+block then move upward, compressing the spring against gravity before momentarily coming to rest.



A. [10 pts] Calculate the speed of the bullet+block immediately after the bullet hits.

Momentum conservation

$$\Delta \vec{p} = 0$$

$$\vec{p}_f - \vec{p}_i = 0$$

$$(M+m)|\vec{v}_f|\hat{y} = m|\vec{v}|\hat{y}$$

$$\boxed{|\vec{v}_f| = \frac{m}{M+m}|\vec{v}|}$$

-1 Clerical / Units
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-8 BTN

Common mistakes:
-4 not including mass of bullet in final momentum

B. [15 pts] The length of the spring before the collision with the bullet was L_i. The length of the spring when the bullet+block momentarily come to rest is L_f. Write down an equation that could be used to determine L_f in terms of the known quantities: m, M, $|\vec{v}|$, g, k, L_0, and L_i. You do not need to solve this equation but please start from a fundamental principle and show all of your work.

Energy conservation
System: block+bullet, spring, Earth
Surroundings: none

$$\Delta E = 0$$

$$\Delta K + \Delta U_{spring} + \Delta U_{grav} = 0$$

$$K_f - K_i + U_{spring,f} - U_{spring,i} + U_{grav,f} - U_{grav,i} = 0$$

$$0 - \frac{1}{2}(M+m)|\vec{v}_f|^2 + \frac{1}{2}k(L_f - L_0)^2 - \frac{1}{2}k(L_i - L_0)^2 + (M+m)g(L_i - L_f) = 0$$

$$-\frac{1}{2}(M+m)\frac{m^2}{(M+m)^2}|v|^2 + \frac{1}{2}k(L_f - L_0)^2 - \frac{1}{2}k(L_i - L_0)^2 + (M+m)g(L_i - L_f) = 0$$

$$-\frac{1}{2}\frac{m^2}{M+m}|v|^2 + \frac{1}{2}k(L_f - L_0)^2 - \frac{1}{2}k(L_i - L_0)^2 + (M+m)g(L_i - L_f) = 0$$

They can stop at any point of simplification as long as they only have the answer in terms of the variables given

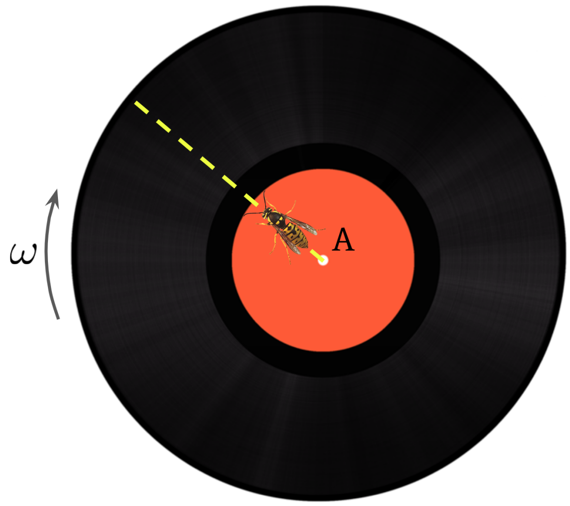
Watch for POE (for the velocity of the block+bullet)

-1 Clerical / Units
-3 Math Error / Minor Physics Error
-6 Major Physics Error
-12 BTN

Common mistakes:
-6 Leaving it in terms of variables not given in the problem statement

Problem 3 [25 pts] Grader & Score: Key

A vinyl record has mass M and radius R and rotates about the center of the record with a constant angular velocity $\vec{\omega} = -\omega_0 \hat{z}$ as indicated in the diagram. You notice a confused yellow jacket with mass m is sitting on the record at the origin. At a time $t = 0$, the yellow jacket starts walking outward from the center of the record. From the perspective of the yellow jacket, she follows a straight line as indicated by the dashed yellow line in the diagram.



A. [5 pts] Calculate the total vector angular momentum of the record about a point A located at the origin at time $t = 0$.

$$\vec{L} = I\vec{\omega}$$
$$= \left(\tfrac{1}{2}MR^2\right)\left(-\omega_0 \hat{z}\right)$$
$$\boxed{\vec{L} = -\tfrac{1}{2}MR^2\omega_0 \hat{z}}$$

+3 for angular momentum equation
+2 for correct moment of inertia

B. [5 pts] Calculate the total vector angular momentum of the yellow jacket about a point A located at the origin at time $t = 0$ (i.e. when the yellow jacket is at the center of the record). The yellow jacket is so small it can be considered a point mass.

None because it is located at the origin

$$\vec{L} = \langle 0, 0, 0 \rangle$$

All or nothing

C. [10 pts] Consider the system of the record and yellow jacket to determine the angular speed of the record when the yellow jacket has walked to the edge (i.e. a distance R from the origin). You can assume that the torque on the record about a point A located at the origin is zero and that the yellow jacket can be treated as a point mass.

Conservation of angular momentum

$$\Delta \vec{L} = 0$$

$$\vec{L}_f - \vec{L}_i = 0$$

$$I_f \vec{\omega}_f + \frac{1}{2} M R^2 \omega_0 \hat{z} = 0$$

$$|\vec{\omega}_f| = \frac{\frac{1}{2} M R^2 \omega_0}{I_f}$$

$$I_f = \frac{1}{2} M R^2 + m R^2$$
$$= \left(\frac{1}{2} M + m\right) R^2$$

$$|\vec{\omega}_f| = \frac{\frac{1}{2} M R^2 \omega_0}{\left(\frac{1}{2} M + m\right) R^2}$$

$$\boxed{|\vec{\omega}_f| = \frac{M}{(M + 2m)} \omega_0}$$

-1 Clerical / Units
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-8 BTN

Common mistakes:
-2 Incorrect moment of inertia

Watch for POE

D. [5 pts] When the yellow jacket reaches the edge of the record she is thrown from the record. A few moments later you observe her to be at position $\vec{r} =< 2R, R, 0 >$ relative to the origin. At this instant the yellow jacket is moving with velocity $\vec{v} =< a, 0, 0 >$ where a is a positive constant. Calculate the yellow jacket's total vector angular momentum relative to a point B located at $\vec{r}_B =< R, 0, 0 >$. The yellow jacket is so small it can be considered a point mass.

$$\vec{L} = \vec{r}\,' \times \vec{p}$$

$$\vec{r}\,' = \vec{r} - \vec{r}_B = \langle R, R, 0 \rangle$$

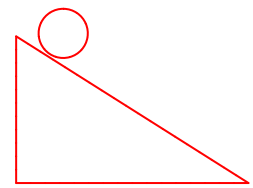
$$\vec{p} = m\vec{v} = \langle ma, 0, 0 \rangle$$

$$\boxed{\vec{L} = -maR\hat{z}}$$

+1 for having L=r x p
+2 for correct r
+2 for correctly calculating cross product

Problem 4 [25 pts] Grader & Score: Key

A. [8 pts] A solid sphere of steel and a smaller solid sphere with the same mass but (1/4) the radius roll down an incline plane. Both spheres start from rest and at the same time from the same height. Which one will arrive at the bottom first? Please show how you determined this.

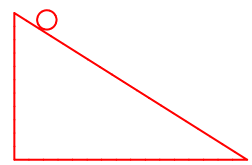


Energy conservation
System: sphere and Earth

$\Delta E = 0$

$\Delta K_{trans} + \Delta K_{rot} + \Delta U_{grav} = 0$

$\frac{1}{2}m|\vec{v}_f|^2 + \frac{1}{2}I|\vec{\omega}_f|^2 - mg\,\Delta h = 0$



$|\omega_f| = \frac{2|\vec{v}_f|}{R}$

Error here. v = r * omega. The 2 shouldn't be there.

$\frac{1}{2}m|\vec{v}_f|^2 + \cancel{X}\frac{I\,|\vec{v}_f|^2}{R^2} - mg\,\Delta h = 0$

$\left(\frac{1}{2}m + \cancel{X}\frac{I}{R^2}\right)|\vec{v}_f|^2 = mg\,\Delta h$

$|\vec{v}_f|^2 = mg\,\Delta h \frac{1}{\frac{1}{2}m + \cancel{X}\frac{I}{R^2}}$

sphere
$I = \frac{2}{5}mR^2$

$|\vec{v}_f|^2 = mg\,\Delta h \frac{1}{\frac{1}{2}m + \frac{2}{5}m}$

Doesn't depend on R

$\boxed{\text{They both reach the bottom at the same time}}$

-1 Clerical
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-6 BTN

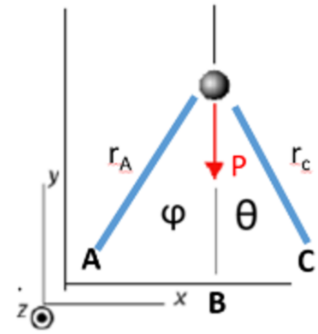
ALTERNATIVELY
Full points for simply stating that they both reach the bottom at the same time. No work necessary.

B. [3 pts] A large massive sphere of steel (like the one above) and another sphere of the same size and same mass but hollow in the center (that is, all the mass is at the surface as we used a more dense material) roll down from rest and at the same time from the same height. Which one will arrive at the bottom first?

The solid one reaches the bottom faster because the fraction in the moment of inertia is smaller

All or nothing

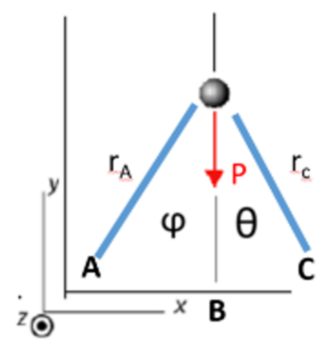
C. [3 pts] A ball is moving in the y direction with a momentum $\vec{p}$ (as shown in the figure). A,B and C are possible origins with $\vec{r}_A$, $\vec{r}_B$ and $\vec{r}_C$ the vector positions from those origins to the ball (B origin is directly below the ball). Where ϕ and θ are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use B as your origin.



$\boxed{0}$

All or nothing

D. [3 pts] A ball is moving in the y direction with a momentum $\vec{p}$ (as shown in the figure). A,B and C are possible origins with $\vec{r}_A$, $\vec{r}_B$ and $\vec{r}_C$ the vector positions from those origins to the ball (B origin is directly below the ball). Where ϕ and θ are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use C as your origin.

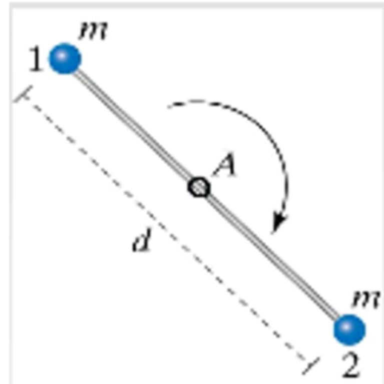


$$|\vec{L}| = |\vec{r} \times \vec{p}|$$
$$= r_c |\vec{p}| \sin\theta$$

$$\boxed{|\vec{L}| = r_c \, |\vec{p}| \, \sin\theta}$$

All or nothing

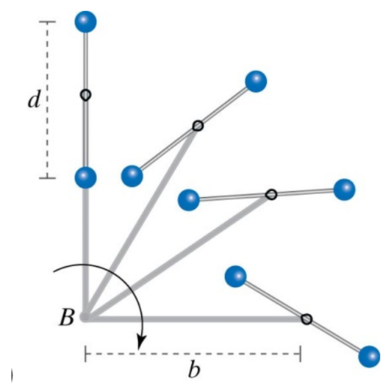
E. [3 pts] Calculate the rotational angular momentum of the two balls of mass m each, connected by a rod of length d, that rotates clockwise around the center of mass denoted by A with an angular velocity ω_1. You can assume the mass of the rod is zero.



$$\vec{L} = I\vec{\omega}$$
$$= -2m\left(\frac{d}{2}\right)^2 \omega_1 \hat{z}$$
$$\boxed{\vec{L} = -\frac{md^2}{2}\omega_1 \hat{z}}$$

All or nothing

F. [5 pts] Calculate the total angular momentum of the rotating barbell from part "E" if the center of mass is now connected to a bar of length b, that rotates clockwise around an axes labeled B with an angular velocity ω_2. The barbell is still rotating clockwise about the barbells center of mass. You can assume the mass of the rods are zero.



$$\vec{L} = \vec{L}_{rot} + \vec{L}_{trans}$$
$$= -\frac{md^2}{2}\omega_1 \hat{z} - 2mb^2\omega_2 \hat{z}$$
$$\boxed{\vec{L} = -\left(\frac{m}{2}d^2\omega_1 + 2mb^2\omega_2\right)\hat{z}}$$

+2 for recognizing sum of angular momentums
+3 for correct moment of inertia

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s \qquad U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M \qquad \Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} \qquad I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel} \qquad K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I} \qquad K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot} \qquad \vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}} \qquad v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

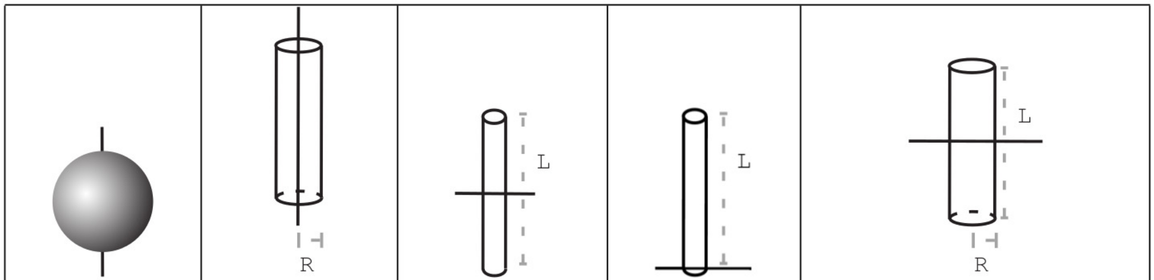
$$\text{prob}(E) \propto \Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1,2,3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0,1,2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}