Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

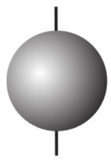
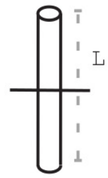
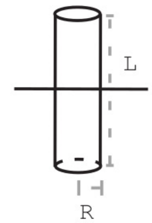
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/K/M/N/HP) - Fall 2023 - Test 1

Please clearly print your name & GTID in the lines below

Name: ______________________________ GTID: ______________________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format, and they must be **readable** in order to be graded. Unreadable files will earn a zero.
  - We recommend you upload a single PDF file for your entire work. You **must** indicate which page corresponds to each problem when you upload and submit.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

KEY

Sign your name on the line above

Problem 1: Coding [10 pts]

A crow is hopping along the top of a construction crane at a height of 80 m above the ground. The crow sees a small pebble (mass $m = 45$ g), and deciding to have a bit of fun, kicks it off the crane. The pebble flies off with velocity $\vec{v}_i = < 0.1, 0.05, 0 >$ m/s. The crow watches the pebble for a bit, then gets bored and flies away in the opposite direction. As the pebble falls, it experiences an air resistance (i.e., drag) force that is proportional to the square of its speed and points in the opposite direction of the pebble's motion, $\vec{F}_d = -bv^2\hat{v}$, where $b = 0.001$ is a proportionality constant. You decide to write a GlowScript code to model the motion of the pebble.

1. [2 pts] Which of the following sets of statements correctly displays the **initial conditions** of the system?

- ```
  b = 0.001
  g = 9.8
  t = 0
  deltat = 0.0001
  pebble.m = 45
  pebble.pos = 80
  pebble.vel = vec(0.1, 0.05, 0)
  ```

- ```
  b = 0.001
  g = 9.8
  t = 0
  deltat = 0.0001
  pebble.m = 0.045
  pebble.pos = vec(0, 80, 0)
  pebble.vel = vec(0.1, 0.05, 0)
  ```

- ```
  b = 0.001
  g = 9.8
  t = 0
  deltat = 0.0001
  pebble.m = vec(45, 0, 0)
  pebble.pos = vec(80, 0, 0)
  pebble.vel = vec(0.1, 0.05, 0)
  ```

- ```
  b = 0.001
  g = 9.8
  pebble.m = 0.045
  pebble.pos = 80
  pebble.vel = vec(0.1, 0.05, 0)
  ```

2. [2 pts] Which of the following statements correctly computes the **force of gravity** acting on the pebble?

- `Fgrav = pebble.m * g`
- `Fgrav = -pebble.m * g`
- `Fgrav = -pebble.m * g * vec(0, -1, 0)`
- `Fgrav = vec(0, -pebble.m * g, 0)`

0:55

3. [2 pts] Which of the following statements correctly computes the **air resistance force** acting on the pebble?

- `Fd = -b * mag(pebble.vel)**2 * norm(pebble.vel)`
- `Fd = -b * pebble.vel**2 * norm(pebble.vel)`
- `Fd = -b * mag(pebble.vel)**2 * vec(0, -1, 0)`
- `Fd = -b * pebble.vel**2 * vec(0, -1, 0)`

4. [2 pts] Which of the following statements correctly computes the **net force** acting on the pebble?

- `Fnet = Fgrav - Fd`
- `Fnet = Fd - Fgrav`
- `Fnet = Fgrav + Fd`
- `Fnet = vec(0, Fd, 0) - vec(0, Fgrav, 0)`

5. [2 pts] Which of the following sets of statements **correctly applies Newton's second law** to predict the motion of the pebble?

- ```
  pebble.pos.final = pebble.pos.initial + pebble.vel*deltat
  pebble.vel.final = pebble.vel.initial + (Fnet/pebble.m)*deltat
  ```
- ```
  pebble.vel = pebble.vel + (Fnet/pebble.m)*deltat
  pebble.pos = pebble.pos + pebble.vel*deltat
  ```
- ```
  pebble.pos = pebble.pos + pebble.vel*deltat
  pebble.vel = pebble.vel + (Fnet/pebble.m)*deltat
  ```
- ```
  pebble.vel.final = pebble.vel.initial + (Fnet/pebble.m)*deltat
  pebble.pos.final = pebble.pos.initial + pebble.vel*deltat
  ```

1:41

Problem 2: Projectile Motion [35 pts]

An American bullfrog named Jeremiah spots a stationary insect perched at a height 0.5 m above the ground. The bullfrog decides to leap into the air to catch the insect. The bullfrog jumps with an initial speed $v_i = 8$ m/s at an angle of $\theta = 30°$ above the horizontal ground.

1. [10 pts] How much **time** does it take for the bullfrog to reach the maximum height of its trajectory?

Velocity update:

$$V_{f,y} = V_{i,y} - g\Delta t$$

$$\Rightarrow 0 = V_{i,y} - g\Delta t$$

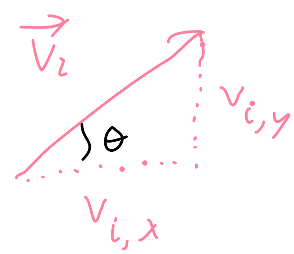
$$\Rightarrow \Delta t = \frac{-V_{i,y}}{-g}$$

$$= \frac{V_i \sin(\theta)}{g}$$

$$= \frac{(8\ m/s)\sin(30°)}{(9.8\ m/s^2)} \approx \boxed{0.408\ s}$$

Top of trajectory:

$V_y = V_{f,y} = 0$



$$V_{i,y} = V_i \sin(\theta)$$

S:21

2. [15 pts] What is the **maximum height** above the ground reached by the bullfrog?

Position update (kinematics):

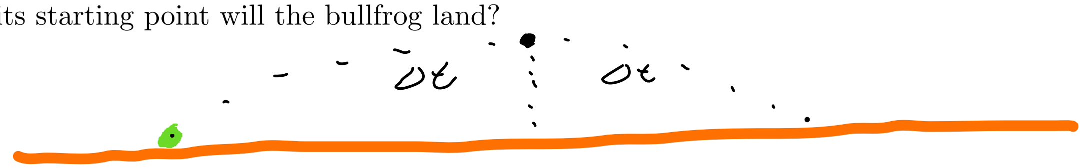
$$y_f = y_i + v_{i,y} \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$= v_i \sin(\theta) \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$= (8 \text{ m/s}) \sin(30^\circ)(0.408 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(0.408 \text{ s})^2$$

$$\approx \boxed{0.816 \text{ m}}$$

3. [10 pts] The insect suddenly flies away just as the bullfrog is about to reach it. Horizontally, **how far away** from its starting point will the bullfrog land?



By symmetry, Δt back to ground $= 2\Delta t$

Position update:

$$x_f = x_i + v_{i,x} \Delta t = v_i \cos(\theta)(2\Delta t) = 2 v_i \cos(\theta) \Delta t$$

$$= 2(8 \text{ m/s}) \cos(30^\circ)(0.408 \text{ s}) \approx \boxed{5.66 \text{ m}}$$

13:17

2. [15 pts] What is the **maximum height** above the ground reached by the bullfrog?

Position update (kinematics):

$$y_f = y_i + v_{i,y}\,\Delta t - \frac{1}{2} g (\Delta t)^2$$

$$= v_i \sin(\theta)\,\Delta t - \frac{1}{2} g (\Delta t)^2$$

$$= (8\ m/s)\sin(30°)(0.408\ s) - \frac{1}{2}(9.8\ m/s^2)(0.408\ s)^2$$

$$\approx \boxed{0.816\ m}$$

3. [10 pts] The insect suddenly flies away just as the bullfrog is about to reach it. Horizontally, **how far away** from its starting point will the bullfrog land?

Assume bullfrog lands on leaf.

Position update (kinematics):

$$v_{i,y} = v_{apex,y} = 0$$

$$y_f = y_i + v_{i,y}\,\Delta t - \frac{1}{2} g (\Delta t)^2$$

$$= y_{apex} - \frac{1}{2} g (\Delta t)^2$$

$$\Rightarrow \Delta t_{fall} = \sqrt{\frac{2(y_{apex} - y_f)}{g}} = \sqrt{\frac{2[(0.816\ m) - (0.5\ m)]}{(9.8\ m/s^2)}} \approx \underline{0.254\ s}$$

$$\Delta t_{total} = \Delta t_{rise} + \Delta t_{fall} = (0.408\ s) + (0.254\ s) \approx \underline{0.662\ s}$$

Position update

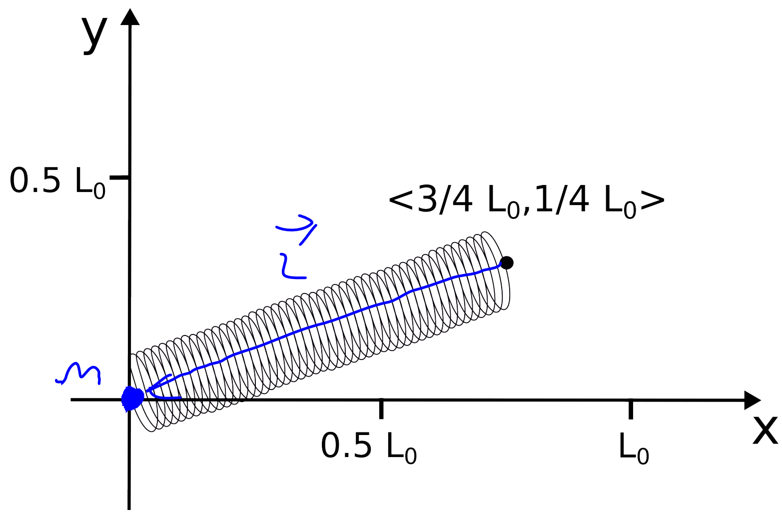
$$x_f = x_i + v_{i,x}\,\Delta t = v_i \cos(\theta)\,\Delta t_{total}$$

$$= (8\ m/s)\cos(30°)(0.662\ s) \approx \boxed{4.585\ m}$$

13:17

Problem 3: Spring Force [20 pts]

A spring with stiffness k and relaxed length L_0 has one end fixed at position $< \frac{3}{4}L_0, \frac{1}{4}L_0, 0 >$ as shown in the figure. A small mass m (not pictured) is attached to the other end of the spring, located at the origin of the coordinate system.



1. [10 pts] What is the vector force $\vec{F}_1$ that the spring would exert on the mass that is **at the origin**? Your answer must be a symbolic expression.

$$\vec{L} = \langle -\tfrac{3}{4}L_0, -\tfrac{1}{4}L_0, 0 \rangle$$

$$|\vec{L}| = \sqrt{L_x^2 + L_y^2 + L_z^2} = \sqrt{(-\tfrac{3}{4}L_0)^2 + (-\tfrac{1}{4}L_0)^2} = \sqrt{\tfrac{9}{16}L_0^2 + \tfrac{1}{16}L_0^2}$$

$$= \sqrt{\tfrac{10}{16}L_0^2} = \frac{\sqrt{10}}{4}L_0 \approx 0.791 L_0$$

$$\hat{L} = \frac{\vec{L}}{|\vec{L}|} = \frac{\langle -\frac{3}{4}L_0, -\frac{1}{4}L_0, 0 \rangle}{\frac{\sqrt{10}}{4}L_0} = \langle -\tfrac{3}{\sqrt{10}}, -\tfrac{1}{\sqrt{10}}, 0 \rangle$$

$$\vec{F}_1 = -k(|\vec{L}| - L_0)\hat{L} = -k\left(\tfrac{\sqrt{10}}{4}L_0 - L_0\right)\langle -\tfrac{3}{\sqrt{10}}, -\tfrac{1}{\sqrt{10}}, 0 \rangle$$

$$\boxed{\begin{aligned} &= \left(\tfrac{\sqrt{10}}{4} - 1\right) k L_0 \langle \tfrac{3}{\sqrt{10}}, \tfrac{1}{\sqrt{10}}, 0 \rangle \\ &\approx -0.209\, k L_0 \langle \tfrac{3}{\sqrt{10}}, \tfrac{1}{\sqrt{10}}, 0 \rangle \end{aligned}}$$

S:48

2. [10 pts] The original spring is replaced with another spring. This second spring has twice the stiffness ($k_2 = 2k$) and half the relaxed length ($L_{02} = L_0/2$) of the first spring. This new spring is also attached to a mass at the origin on one side and fixed at position $< \frac{3}{4}L_0, \frac{1}{4}L_0, 0 >$ on the other side. What is the **magnitude** of the force $|\vec{F}_2|$ that this second spring exerts on the mass at the origin? Express your answer as a multiple of $|\vec{F}_1|$.

$$|\vec{F}_{spring}| = k \left| |\vec{L}| - L_0 \right| \qquad \underline{|\vec{F}_1| = \left| \frac{\sqrt{10}}{4} - 1 \right| k L_0}$$

$$|\vec{F}_2| = k_2 \left| |\vec{L}| - L_{02} \right| = 2k \left| \frac{\sqrt{10}}{4} L_0 - \frac{L_0}{2} \right|$$

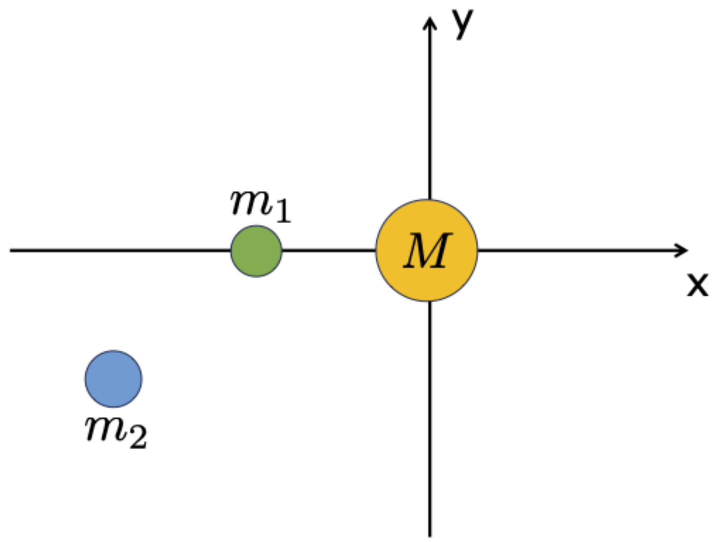
$$= \underline{\left| \frac{\sqrt{10}}{2} - 1 \right| k L_0}$$

$$\frac{|\vec{F}_2|}{|\vec{F}_1|} = \left| \frac{\left(\frac{\sqrt{10}}{2} - 1\right) k L_0}{\left(\frac{\sqrt{10}}{4} - 1\right) k L_0} \right| = \left| \frac{\frac{\sqrt{10}}{2} - 1}{\frac{\sqrt{10}}{4} - 1} \right| \Rightarrow \boxed{|\vec{F}_2| = \left| \frac{\frac{\sqrt{10}}{2} - 1}{\frac{\sqrt{10}}{4} - 1} \right| |\vec{F}_1| \approx 2.77 |\vec{F}_1|}$$

10:54

Problem 4: Gravitation [35 pts]

Two planets are revolving around a star. The star has mass M and is located at the origin. The planets have masses m_1 and m_2, and are located at positions $\vec{r}_1 =< -A, 0, 0 >$ and $\vec{r}_2 =< -2A, -\sqrt{3}A, 0 >$ respectively, where A is a positive scalar with appropriate dimensions.



1. [15 pts] What is the gravitational force on Planet 1 **due to the star**, $\vec{F}_{\text{on 1 by S}}$?

$$\vec{r}_{s\,to\,1} = \vec{r}_1 - \vec{r}_s = \vec{r}_1 = \langle -A, 0, 0 \rangle$$

$$|\vec{r}_{s\,to\,1}| = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{(-A)^2} = \sqrt{A^2} = A$$

$$\vec{F}_g = -\frac{G M_1 M_2}{|\vec{r}|^2}\hat{r} = -\frac{G M_1 M_2}{|\vec{r}|^2}\left(\frac{\vec{r}}{|\vec{r}|}\right) = -\frac{G M_1 M_2}{|\vec{r}|^3}\vec{r}$$

$$\vec{F}_{on\,1\,by\,s} = -\frac{G M_s m_1}{|\vec{r}_{s\,to\,1}|^3}\vec{r}_{s\,to\,1} = -\frac{G M m_1}{(A)^3}\langle -A, 0, 0 \rangle$$

$$= \boxed{\left\langle \frac{G M m_1}{A^2}, 0, 0 \right\rangle}$$

5:01

2. [15 pts] What is the gravitational force on Planet 1 **due to Planet 2**, $\vec{F}_{\text{on 1 by 2}}$?

$$\vec{r}_{2\text{ to }1} = \vec{r}_1 - \vec{r}_2 = \langle -A, 0, 0\rangle - \langle -2A, -\sqrt{3}A, 0\rangle$$

$$= \langle A, \sqrt{3}A, 0\rangle$$

$$|\vec{r}_{2\text{ to }1}| = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{(A)^2 + (\sqrt{3}A)^2} = \sqrt{A^2 + 3A^2} = 2A$$

$$\vec{F}_{\text{on 1 by 2}} = -\frac{Gm_1m_2}{|\vec{r}_{2\text{ to }1}|^3}\vec{r}_{2\text{ to }1} = -\frac{Gm_1m_2}{(2A)^3}\langle A, \sqrt{3}A, 0\rangle$$

$$= -\frac{Gm_1m_2}{8A^3}\langle A, \sqrt{3}A, 0\rangle$$

$$= \boxed{\left\langle -\frac{Gm_1m_2}{8A^2}, -\frac{Gm_1m_2\sqrt{3}}{8A^2}, 0\right\rangle}$$

*Algebraically-equivalent answers also acceptable

9:56

3. [5 pts] What is the **net gravitational force** on Planet 1, $\vec{F}_{\text{net}}$?

$$\vec{F}_{net} = \vec{F}_{on\,1\,by\,3} + \vec{F}_{on\,1\,by\,2}$$

$$= \left\langle \frac{GMm_1}{A^2}, 0, 0 \right\rangle + \left\langle -\frac{Gm_1m_2}{8A^2}, -\frac{Gm_1m_2\sqrt{3}}{8A^2}, 0 \right\rangle$$

$$= \boxed{\left\langle \frac{GMm_1}{A^2} - \frac{Gm_1m_2}{8A^2}, -\frac{Gm_1m_2\sqrt{3}}{8A^2}, 0 \right\rangle}$$

* Same as 4.3

12:17

4. **[EXTRA CREDIT: 5 pts]** You are standing on the surface of Planet 1. This planet has a large total electric charge $Q = 1000$ C. If the radius of the planet is $R = 1 \times 10^5$ m, what is the **force per unit charge** due to the electric force at the surface of the planet?

Hint: This is analogous to the gravitational acceleration on the surface of a planet ($g = 9.8$ m/s^2 on Earth), but it is due to the electric force instead of being due to the gravitational force.

$$|\vec{F}_g| = \frac{GMm}{R^2} \rightarrow mg \Rightarrow \underline{g = \frac{GM}{R^2}}$$

$$|\vec{F}_e| = \frac{kQq}{R^2} \rightarrow q a_e \Rightarrow a_e = \frac{|\vec{F}_e|}{q} = \frac{kQ}{R^2}$$

$$= \frac{(9 \cdot 10^9\ Nm^2/C^2)(1000\ C)}{(10^5\ m)^2}$$

$$= \boxed{900\ N/C}$$

4:40

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