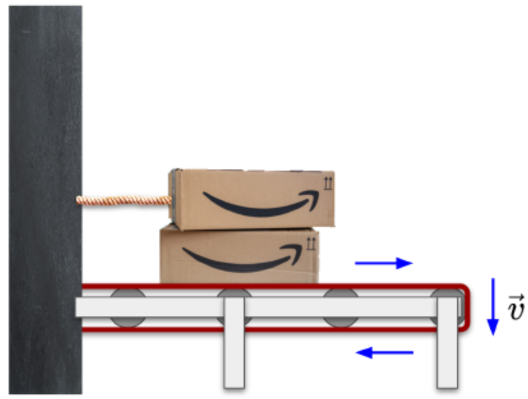
Physics 2211 GPS Week 6

Problem #1

A conveyor belt rotates clockwise with speed v as indicated in the diagram. Two identical boxes, each with mass m, are stacked on top of each other placed on the conveyor belt. The top box is attached to a rope that is connected to the wall. As a result, neither box is moving and tension in the rope is measured to be T. Your answers to the questions below may contain this measured value.



A. Determine the coefficient of kinetic friction μ_k between the bottom box and the conveyor belt. Hint: treat the two boxes as the system.

The normal force is $\vec{F}_n = 2mg\hat{y} \Rightarrow \vec{F}_{friction} = \mu_k|\vec{F}_n|\hat{x}$

$= 2\mu_k mg\,\hat{x}$

Since $\Delta\vec{p} = 0$, $\vec{F}_{net} = 0 \Rightarrow |T| = |\vec{F}_{friction}|$

$$\Rightarrow \boxed{\mu_k = \frac{T}{2mg}}$$

B. The top box is just on the verge of sliding. Determine the coefficient of static friction μ_s between the top and bottom box.

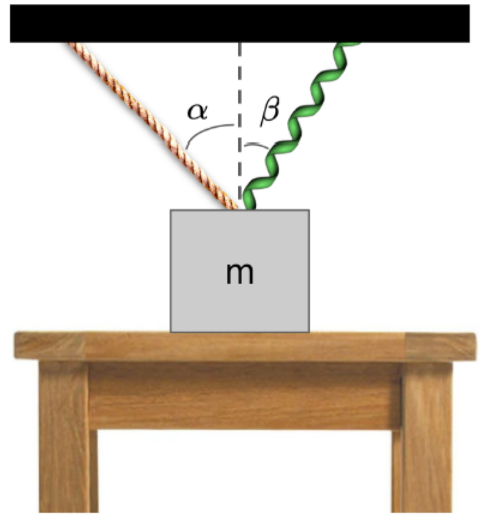
$\vec{F}_n = -mg\hat{y} \Rightarrow \vec{F}_{friction} = \mu_2|\vec{F}_n|\hat{x}$

Since $\Delta\vec{p} = 0$, $|T| = |\vec{F}_{friction}|$

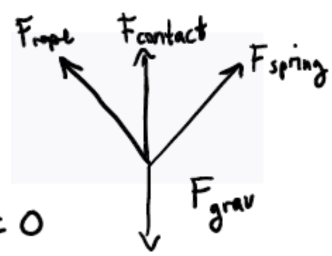
$$\Rightarrow \boxed{\mu_2 = \frac{T}{mg}}$$

Problem #2

A box of mass m hangs from a rope and a spring. The spring, which has spring stiffness k_s and relaxed length L_0, is stretched to a length L. The rope and spring make angles α and β respectively with the vertical. A frictionless table located beneath the suspended box is in contact with the box. Given the information about the spring, the rope and the angles they make, you will ultimately answer the question: does the table help support the box?



A. Calculate the tension in the rope. Give your answer in vector form.



The box is motionless, so $\vec{F}_{net} = 0$

$$= \sum F_x = 0 = |\vec{F}_{rope}|(-\sin\alpha) + |\vec{F}_{spring}|\sin\beta$$

$$|\vec{F}_{spring}| = (L - L_0)k_s \Rightarrow |\vec{F}_{rope}| = \frac{(L-L_0)k_s \sin\beta}{\sin\alpha}$$

$$\Rightarrow \vec{F}_{rope} = \frac{(L-L_0)k_s\sin\beta}{\sin\alpha}\langle -\sin\alpha, \cos\alpha, 0\rangle$$

$$= \boxed{(L-L_0)k_s\sin\beta\left\langle -1, \frac{1}{\tan\alpha}, 0\right\rangle}$$

B. Determine the magnitude of the normal component of the contact force on the box due to the table.

Because $\vec{F}_{net} = 0$,

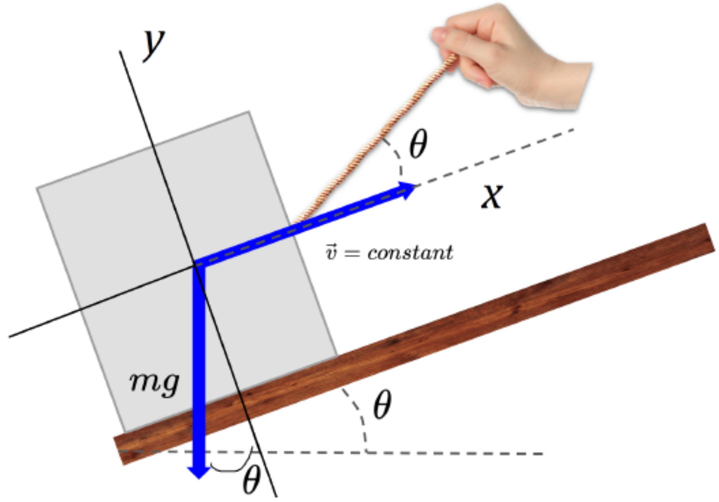
$$\sum \vec{F}_y = 0 = \vec{F}_{grav} + \vec{F}_{contact} + \vec{F}_{spring,y} + \vec{F}_{rope,y} .$$

$$\vec{F}_{spring,y} = |\vec{F}_{spring}| \cos\beta = k_s(L - L_0)\cos\beta$$

$$\Rightarrow \sum \vec{F}_y = -mg + |\vec{F}_{contact}| + k_s(L-L_0)\cos\beta + k_s(L-L_0)\frac{\sin\beta}{\tan\alpha} .$$

$$\Rightarrow \boxed{|\vec{F}_{contact}| = mg - k_s(L-L_0)\left(\cos\beta + \frac{\sin\beta}{\tan\alpha}\right)}$$

3. Using your hand you pull a block of mass m up a wooden board at a constant speed. As indicated in the diagram, your hand is pulling a string at an angle θ with the wooden board which makes an angle θ with the ground. The coeffecient of sliding friction between the block and the board is μ. Determine the tension in the string.



The block is moving at a constant speed, meaning the net force on the block is 0. We can use Newton's second law, broken down into the components of forces along the direction up the ramp (x) and the direction perpendicular to the ramp (y) to solve for the tension T in the string since we know that in each direction the net force will be 0. We can break the contact force into a friction force and a normal force, where $F_f = \mu F_n$.

$$F_x = 0 = T\cos\theta - mg\sin\theta - F_f$$
$$F_y = 0 = T\sin\theta + F_n - mg\cos\theta$$

From the second line we can write $F_n = mg\cos\theta - T\sin\theta$. Plugging this multiplied by μ into the first equation for F_f,

$$0 = T\cos\theta - mg\sin\theta - \mu(mg\cos\theta - T\sin\theta)$$

Rearranging and solving for T,

$$\boxed{T = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\theta + \mu\sin\theta}} \quad \text{or} \quad \boxed{T = mg\frac{\tan\theta + \mu}{1 + \mu\tan\theta}}$$