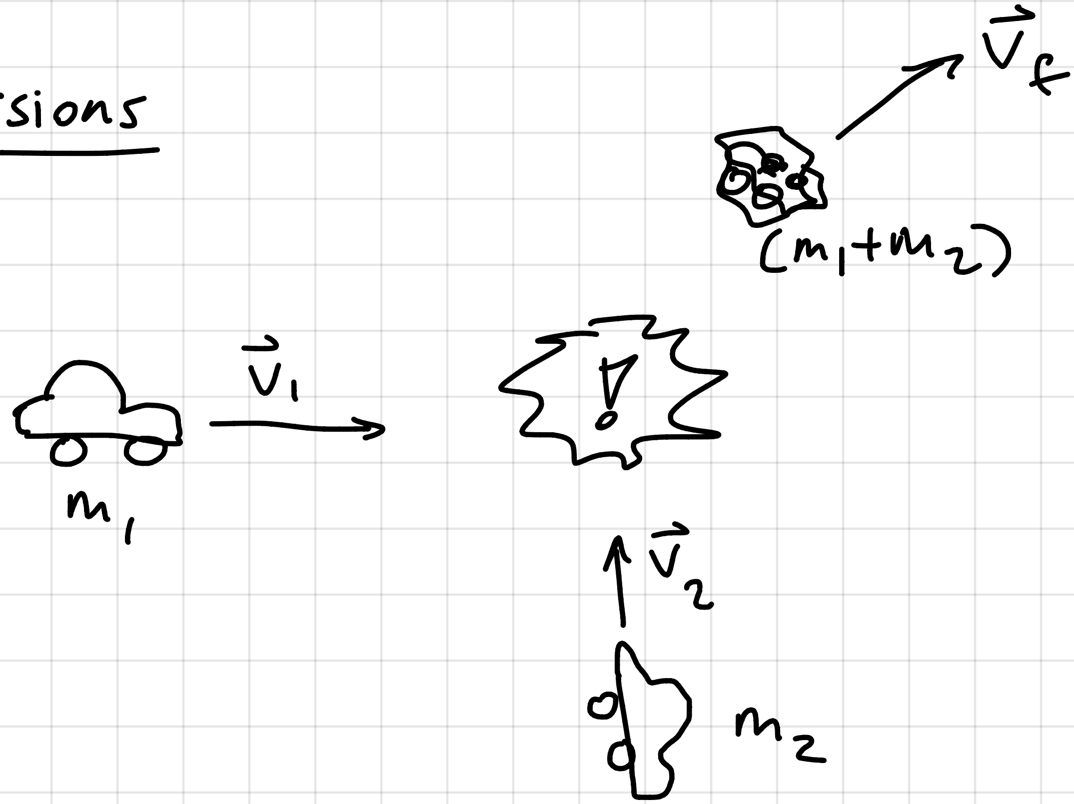
PHYS 2211, Summer 2021

Weeks 9 & 10: Collisions & Angular Momentum

- ✓ collisions: conservation of momentum
- ✓ energy in collisions (elastic, inelastic)
- ✓ the vector cross product
- ✓ torque
- ✓ angular momentum
- ✓ translational/rotational angular momentum
- ✓ the angular momentum principle

Collisions



In a collision: $\boxed{\Delta \vec{p}_{system} = 0}$

$$\vec{P}_{total,i} = \vec{P}_{total,f}$$

momentum is a **vector**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \underbrace{(m_1+m_2)}_{\text{Sticking together after collision}} \vec{v}_f$$

Maximally inelastic collision

Types of Collisions

Elastic: $\Delta K = 0$

Kinetic energy is **conserved**

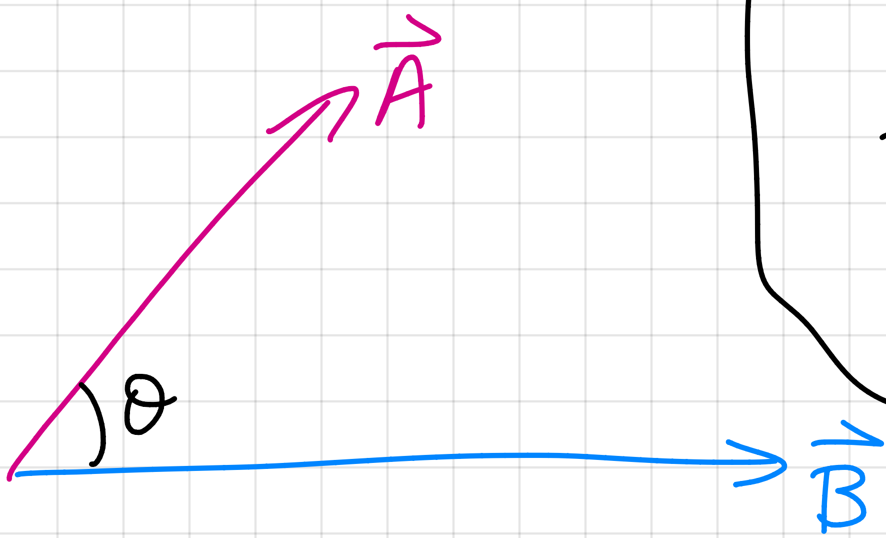
Inelastic: $\Delta K \neq 0$

Kinetic energy is **NOT** conserved

↓

maximally inelastic (stick together)

Cross Product



Recall
dot product

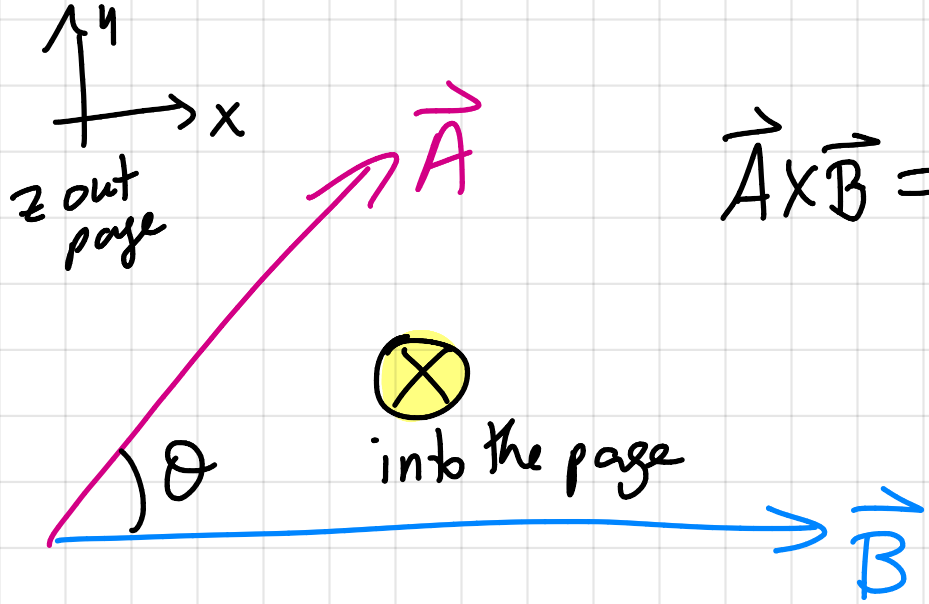
$\vec{A}\cdot\vec{B} = \underbrace{AB\cos\theta}_{\text{scalar}}$

$\vec{A}\times\vec{B} = AB\sin\theta$ in direction RHR

Right hand Rule

Point your fingers in the direction of the first vector, then curl them towards the second vector, and your thumb will point in the direction of the cross product.

SEE RIGHT HAND RULE video!



$$\vec{A} \times \vec{B} = AB\sin\theta \; (-\hat{z})$$

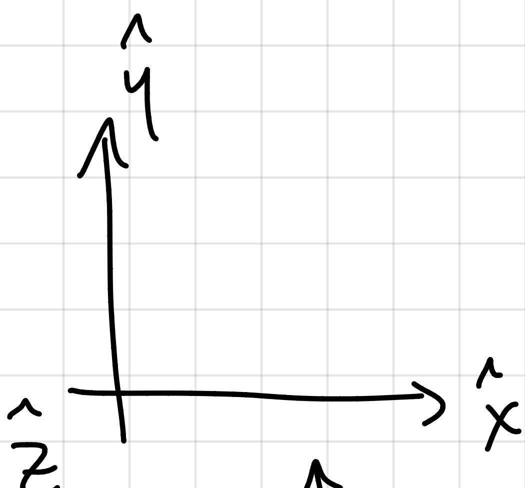
(into the page)

The cross product does NOT commute

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{B} \times \vec{A} = AB\sin\theta \; \odot \; (+\hat{z})$$

out of page



$$\hat{x} \times \hat{y} = \hat{z}$$

right handed coordinate system

$$\vec{A} = \langle A_x,\ A_y,\ A_z \rangle$$

$$\vec{B} = \langle B_x,\ B_y,\ B_z \rangle$$

determinant method

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\hat{x}$: $A_y B_z - A_z B_y$

$\hat{y}$: $A_z B_x - A_x B_z$

$\hat{z}$: $A_x B_y - A_y B_x$

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

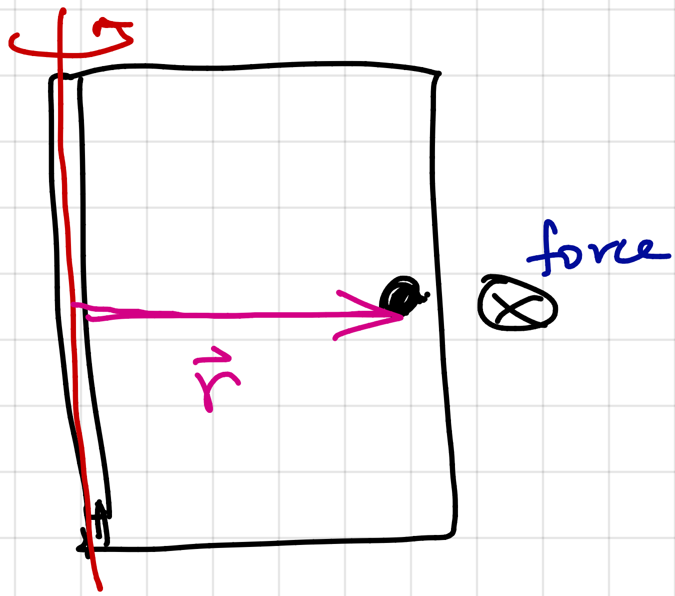
Torque & Angular Momentum

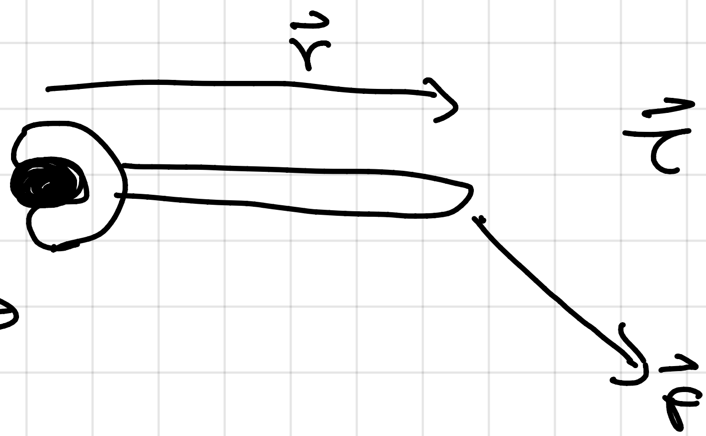
$$\vec{\tau} = \vec{r} \times \vec{F}$$

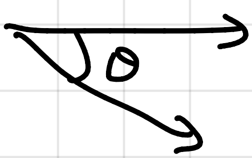
"tau" (torque)

from pivot to where you apply force

applied force



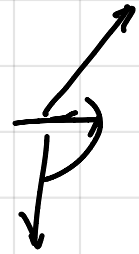


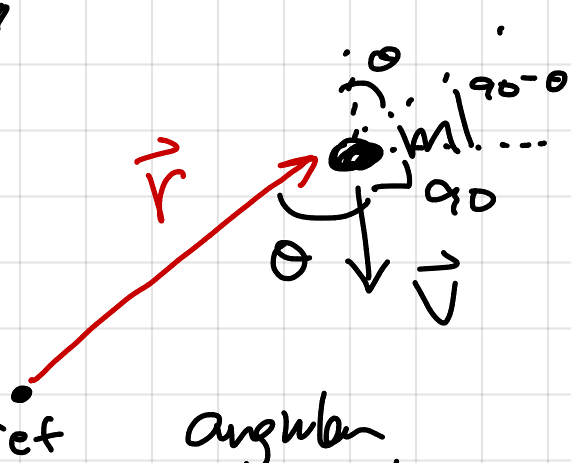


$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= rF\sin\theta \; (-\hat{z})$$

$$\vec{L} = \vec{r} \times \vec{p}$$

angular momentum





angular momentum of m with respect to the reference point

$$= |\vec{r}|\,|\vec{p}| \sin\theta$$

$$= r m v \sin\theta$$

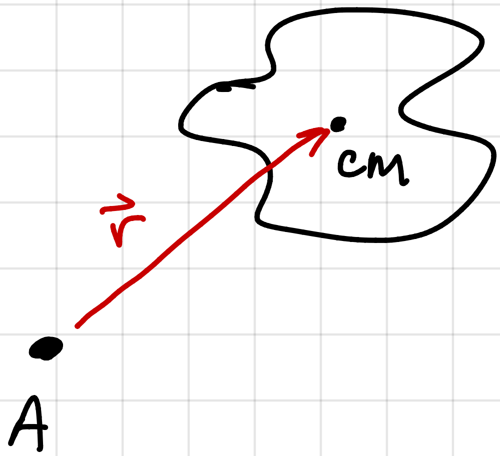
Translational Angular Momentum

Point particle

$$\vec{L}_{trans,A} = \vec{r}_A \times \vec{p}$$

$\vec{r}_A$: vector from the reference point to the system

extended system

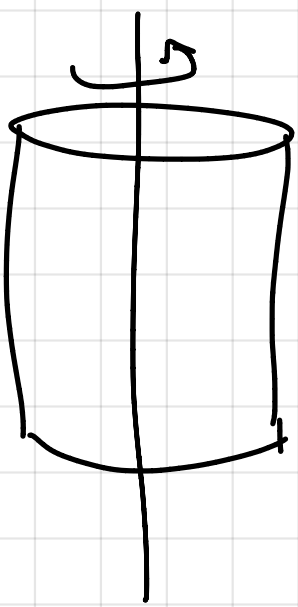


$$\vec{L}_{trans,A} = \vec{r}_A \times \vec{p}_{cm}$$

from A to cm

Rotational Angular Momentum

Solid cylinder

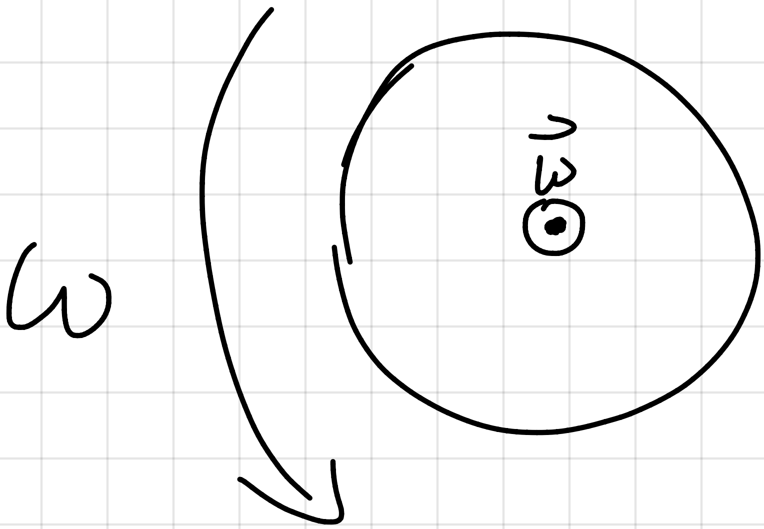


$$\vec{L}_{rot} = I\vec{\omega}$$

moment of inertia

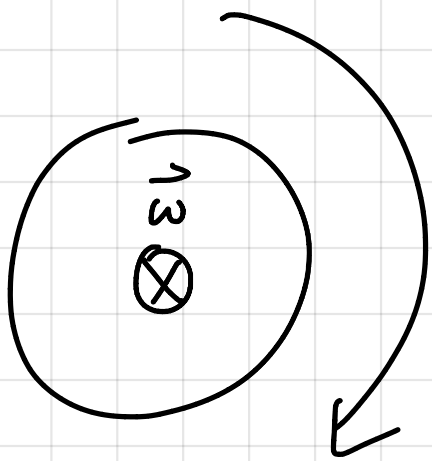
angular velocity

always with respect to an axis that passes through the cm



Spinning counterclockwise

$\vec{\omega}$ points out of the page



Spinning clockwise

$\vec{\omega}$ = into the page

"leftie loosy, righty tighty"

The Angular Momentum Principle

$$\vec{\tau}_{net} = \frac{d\vec{L}_{total}}{dt}$$

net torque

change in angular momentum

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

$$\Delta \vec{L} = \vec{\tau}\,\Delta t$$

$$\vec{L}_f = \vec{L}_i + \vec{\tau}\,\Delta t$$

angular momentum update

Linear	Angular
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$
$\vec{F} = \frac{d\vec{p}}{dt}$	$\vec{\tau} = \frac{d\vec{L}}{dt}$
$\vec{v} = \frac{d\vec{r}}{dt}$	$\omega = \frac{d\theta}{dt}$
$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$	$\vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$
	(divide by I)
$\vec{v}_f = \vec{v}_i + \underbrace{\frac{\vec{F}}{m}}_{\vec{a}}\Delta t$	$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\Delta t$
$\vec{F} = \underbrace{m}_{\text{mass}}\underbrace{\vec{a}}_{\text{linear acceleration}}$	$\vec{\tau} = \underbrace{I}_{\text{moment of inertia}}\underbrace{\vec{\alpha}}_{\text{angular acceleration}}$