Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

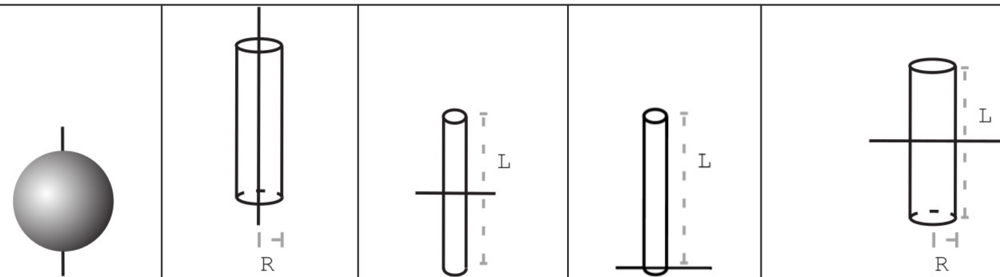
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/HP) - Spring 2024 - Test 3

Name: ______________________ GTID: ______________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\frac{a \cdot b}{c \cdot d} = \frac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

Sign your name on the line above

Problem 1: Thermal Energy [20 pts]

A container holds $m = 620$ g of water at an initial temperature of $T_i = 20°C$. An electric heater with power $P = 1200$ W is attached to the the container, and is turned on for exactly one minute. The container and heater are both inside an insulating enclosure. At the same time that the heater is on, you are also shaking the entire enclosure, doing 15000 J of work. What is the **final temperature of the water**, after the heater has been turned off and you've stopped shaking the container? The specific heat of water is $C = 4.2$ J/(g°C).

$$\Delta E = W + Q$$

$Q_{heater} \neq 0$ because heater is inside insulated cup

$$\Delta E_{therm} = W_{person} + Q_{heater}$$

$$mC(T_f - T_i) = W_{person} + P\Delta t$$

$$\Rightarrow T_f = \frac{W_{person} + P\Delta t}{mC} + T_i$$

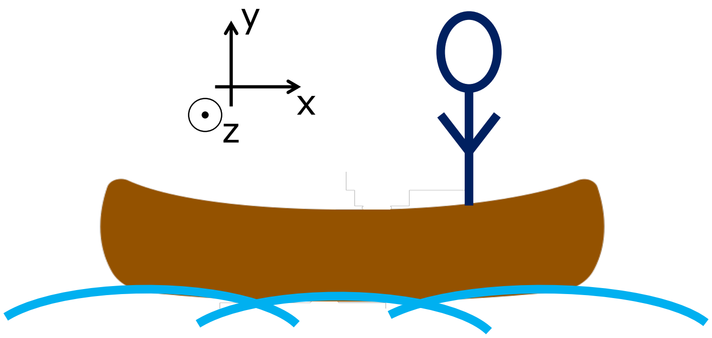
$$= \frac{(15000\ J) + (1200\ W)(60\ s)}{(620\ g)(4.2\ J/g\ °C)} + (20\ °C)$$

$$\approx \boxed{53.4\ °C}$$

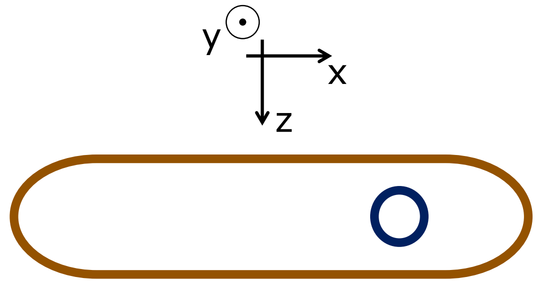
5:00

Problem 2: Center of Mass & Collisions [35 pts]

A professor is sitting in a canoe at a distance of 1.3 m to the right of the center of the canoe. The mass of the professor is $m_1 = 75$ kg. The canoe has a mass of $m_c = 30$ kg and its center is **initially** located at the origin. Gravity points in the $-\hat{y}$ direction. The canoe can glide perfectly so that all forces exerted by the water are in the $+\hat{y}$ direction.



(side view)



(top view)

2.1 [10 pts] If the system consists of the canoe and the professor, determine what is $\vec{r}_{cm}$, the **position of the center of mass** of the system.

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_c \vec{r}_c}{m_1 + m_c}$$

$$= \frac{(75\text{ kg})(\langle 1.3, 0, 0\rangle\text{ m}) + (30\text{ kg})(\langle 0, 0, 0\rangle\text{ m})}{(75\text{ kg}) + (30\text{ kg})}$$

$$\approx \boxed{\langle 0.93, 0, 0\rangle\text{ m}}$$

2:29

2.2 [10 pts] The professor carefully walks to the other side of the canoe so that he is now a distance 1.3 m to the left of the center of the canoe. Determine what is the **new position vector** of the professor. Hint: Think about what is the net external force acting on the system.

$$\vec{F}_{net} = \vec{0} \Rightarrow \vec{v}_{cm} \text{ constant}, \quad \vec{v}_{cm} = \vec{0} \Rightarrow \underline{\vec{r}_{cm} \text{ constant}}$$

$$x_{c,new} = x_{1,new} + d \qquad d = 1.3 \text{ m}$$

$$x_{cm} = \frac{m_1 x_{1,new} + m_c x_{c,new}}{m_1 + m_c}$$

$$= \frac{m_1 x_{1,new} + m_c (x_{1,new} + d)}{m_1 + m_c}$$

$$\Rightarrow x_{1,new} = \frac{(m_1 + m_c) x_{cm} - m_c d}{m_1 + m_c}$$

$$= \frac{[(75 \text{ kg}) + (30 \text{ kg})](0.93 \text{ m}) - (30 \text{ kg})(1.3 \text{ m})}{(75 \text{ kg}) + (30 \text{ kg})}$$

$$\approx \underline{0.56 \text{ m}}$$

$$\Rightarrow \boxed{\vec{r}_{1,new} = \langle 0.56, 0, 0 \rangle \text{ m}}$$

Alternative: symmetry argument

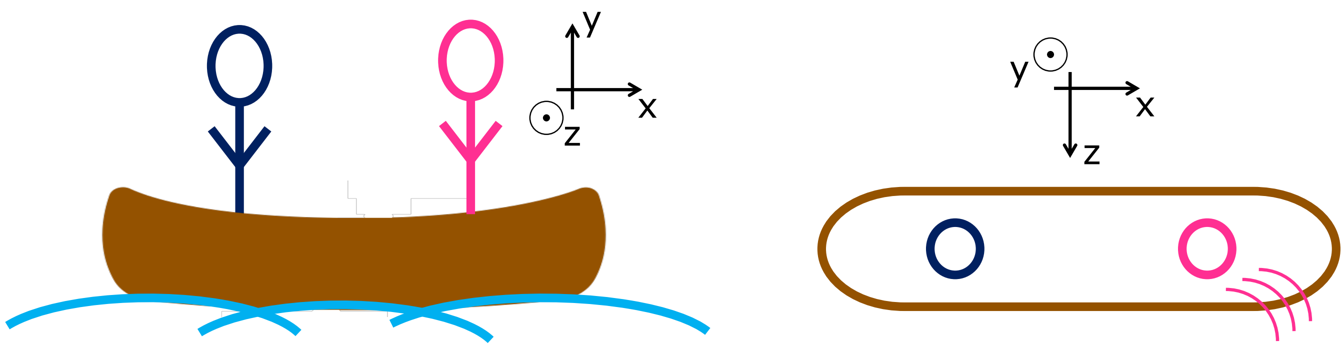
Professor is distance $\Delta d = d_1 - x_{cm}$ to the right of the center of mass at one end of the canoe, so he is Δd to the left of the center of mass at the other end.

$$x_{1,new} = x_{cm} - \Delta d = x_{cm} - (d_1 - x_{cm}) = 2 x_{cm} - d_1$$

$$= 2(0.93 \text{ m}) - (1.3 \text{ m}) \approx \boxed{0.56 \text{ m}}$$

7:32

2.3 [5 pts] A second professor with the same mass as the first ($m_2 = 75$ kg) leaps into the canoe and lands in the same place where the first professor was initially located inside the canoe (i.e., 1.3 m to the right of the center of the canoe). The velocity of the leap was $\vec{v} = < -2, 0, -2 >$ m/s. Which one of the following statements best describes the canoe **immediately after the second professor lands**?



(side view) (top view)

(A) The center of the canoe moves in the x direction only and the canoe rotates clockwise (when viewed from above)

(B) The center of the canoe moves in the x direction only and the canoe rotates counterclockwise (when viewed from above)

(C) The center of the canoe is moving along both x and z axes and the canoe rotates clockwise (when viewed from above)

(D) The center of the canoe is moving along both x and z axes and the canoe rotates counterclockwise (when viewed from above)

2.4 [10 pts] Intending to stop the motion of the canoe's center, the first professor uses a paddle to apply a constant force to the canoe for one second. What is the (vector) value of this force? $\vec{P}_f = \vec{0}$

Momentum conserved during collision

$$\vec{P}_{total} = \vec{P}_{i2} \Rightarrow (m_c + m_1 + m_2)\vec{V}_{total} = m_2 \vec{V}_2$$

$$\Delta\vec{p} = \vec{F}_{net}\,\Delta t \Rightarrow \vec{P}_f - \vec{P}_i = \vec{F}\,\Delta t$$

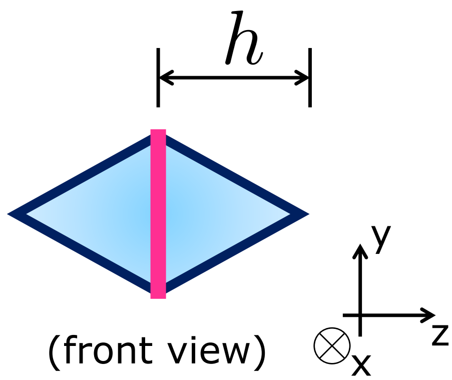
$$\Rightarrow \vec{F} = \frac{-\vec{P}_{i2}}{\Delta t} = -\frac{m_2 \vec{V}_2}{\Delta t} = -\frac{(75\ kg)(<-2, 0, -2> m/s)}{(1\ s)}$$

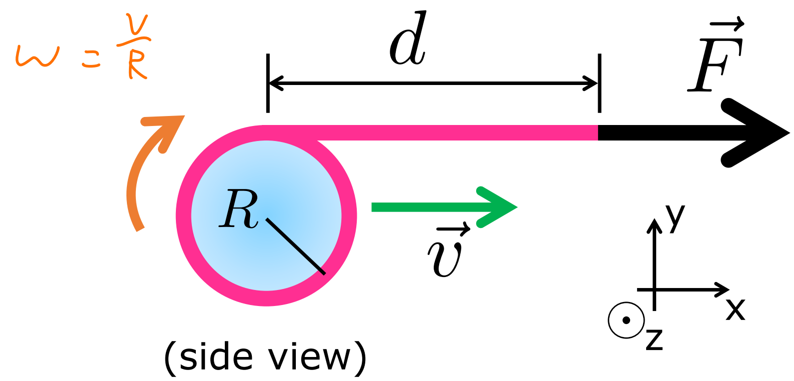
$$= \boxed{<150, 0, 150>\ N}$$

18:24

Problem 3: Point Particle & Real System [20 pts]

A toy is shaped like two cones stuck together along their bases, with a pull cord wrapped around its widest point as shown. The mass of the toy is $M = 0.1$ kg, its radius at the widest point is $R = 5$ cm, and the length from the middle to the point of one side-cone is $h = 10$ cm.





The toy is originally at rest, and you use the pull cord to make it roll without slipping along the surface of a table. After pulling the cord a distance of $d = 50$ cm towards you with a constant but unknown force $\vec{F}$, the top is spinning and moving towards you at speed $v = 0.5$ m/s. What is the **magnitude of the force** applied to the pull cord? The moment of inertia of this toy is $I = (3/10)MR^2$.

Real system:

$$\Delta E = W$$

$$\Delta K_{trans} + \Delta K_{rot} = W_f$$

$$\frac{1}{2}M(v_f^2 - v_i^2) + \frac{1}{2}I(\omega_f^2 - \omega_i^2) = \vec{F} \cdot \Delta\vec{r}$$

$v_i = 0, \quad \omega_i = 0$

$$\frac{1}{2}M v_f^2 + \frac{1}{2}\left(\frac{3}{10}MR^2\right)\omega_f^2 = F d$$

$\omega = \frac{v}{R}$

$$\frac{1}{2}M v_f^2 + \frac{3}{20}MR^2\left(\frac{v_f}{R}\right)^2 = F d$$

$$\frac{1}{2}M v_f^2 + \frac{3}{20}M v_f^2 = F d$$

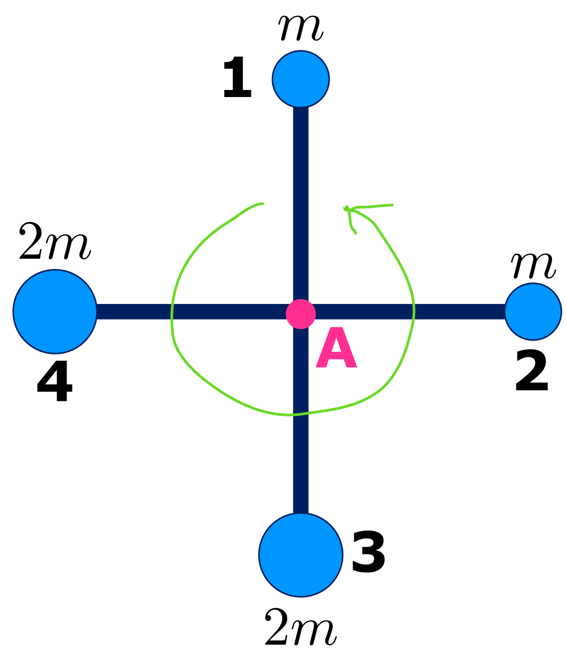
$$F = \frac{13}{20}\frac{M v_f^2}{d}$$

$$= \frac{13}{20}\frac{(0.1\text{ kg})(0.5\text{ m/s})^2}{(0.5\text{ m})}$$

$$= \boxed{0.0325\text{ N}}$$

6:48

Problem 4: Angular Momentum Principle [25 pts]



Four **point particles** (labeled 1, 2, 3, 4) are attached to the ends of two **massless** rods, each of which has length $2d$, as shown in the figure to the right. The two rods are connected to each other at their centers (the location labeled "A"). The full system, consisting of the two rods and the four particles, is free to spin about location A.

The masses and initial velocities of the particles are:

$M_1 = m, \quad \vec{v}_1 = < -v, 0, 0 >$ $\qquad \vec{r}_1 = \langle 0, d, 0 \rangle$

$M_2 = m, \quad \vec{v}_2 = < 0, v, 0 >$ $\qquad \vec{r}_2 = \langle d, 0, 0 \rangle$

$M_3 = 2m, \quad \vec{v}_3 = < v, 0, 0 >$ $\qquad \vec{r}_3 = \langle 0, -d, 0 \rangle$

$M_4 = 2m, \quad \vec{v}_4 = < 0, -v, 0 >$ $\qquad \vec{r}_4 = \langle -d, 0, 0 \rangle$

4.1 [10 pts] Determine the **total angular momentum** ($\vec{L}$) of the full system with respect to point A.

$$\vec{L}_1 = \vec{r}_1 \times \vec{p}_1 = M_1(\vec{r}_1 \times \vec{v}_1) = m(\langle 0, d, 0\rangle \times \langle -v, 0, 0\rangle) = \langle 0, 0, mvd \rangle$$

$$\vec{L}_2 = \vec{r}_2 \times \vec{p}_2 = M_2(\vec{r}_2 \times \vec{v}_2) = m(\langle d, 0, 0\rangle \times \langle 0, v, 0\rangle) = \langle 0, 0, mvd \rangle$$

$$\vec{L}_3 = \vec{r}_3 \times \vec{p}_3 = M_3(\vec{r}_3 \times \vec{v}_3) = 2m(\langle 0, -d, 0\rangle \times \langle v, 0, 0\rangle) = \langle 0, 0, 2mvd \rangle$$

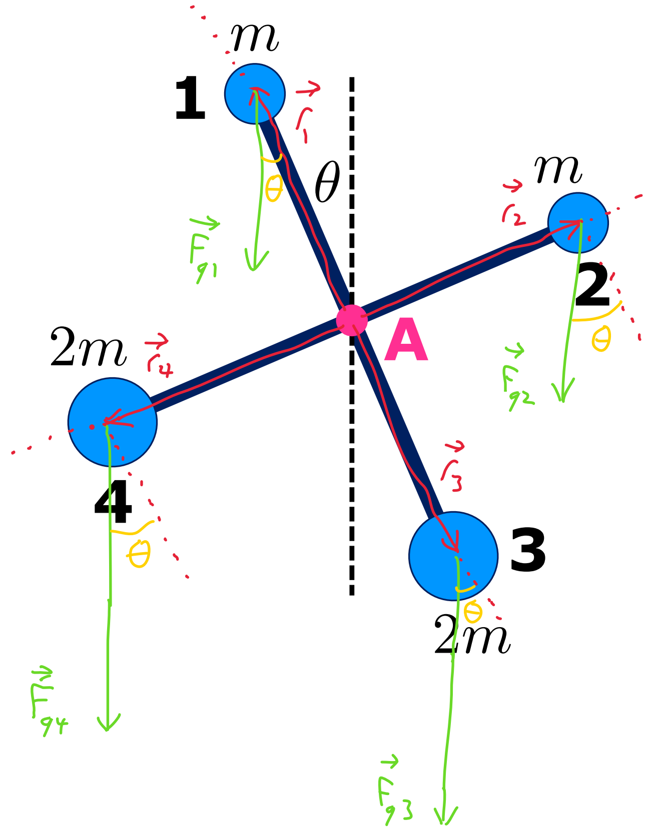
$$\vec{L}_4 = \vec{r}_4 \times \vec{p}_4 = M_4(\vec{r}_4 \times \vec{v}_4) = 2m(\langle -d, 0, 0\rangle \times \langle 0, -v, 0\rangle) = \langle 0, 0, 2mvd \rangle$$

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \vec{L}_4$$

$$= \boxed{\langle 0, 0, 6mvd \rangle}$$

5 : 10

4.2 [15 pts] At some moment the system has spun so it is tilted at an angle θ with respect to the vertical, as shown in the diagram on the right. Someone stops the system and holds it **motionless** in this position. After keeping it motionless for a while, the person **lets go** and the system is allowed to spin again. What is the **final angular momentum** ($\vec{L}_f$) of the system with respect to point A, a short time T after being let go? Remember that gravity exists and points downwards.



$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_{g,1} = mgd \sin(\theta)\,\hat{z}$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_{g,2} = mgd \cos(\theta)\,(-\hat{z})$$

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_{g3} = 2mgd \sin(\theta)\,(-\hat{z})$$

$$\vec{\tau}_4 = \vec{r}_4 \times \vec{F}_{g,4} = 2mgd \cos(\theta)\,\hat{z}$$

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 = \underline{\left[mgd\,(\cos(\theta) - \sin(\theta))\right]\,\hat{z}}$$

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t} = \frac{\vec{L}_f}{T}$$

$$\Rightarrow \vec{L}_f = T\vec{\tau} = \boxed{\left[Tmgd\,(\cos(\theta) - \sin(\theta))\right]\,\hat{z}}$$

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