Physics 2211 GPS Week 5

Problem #1

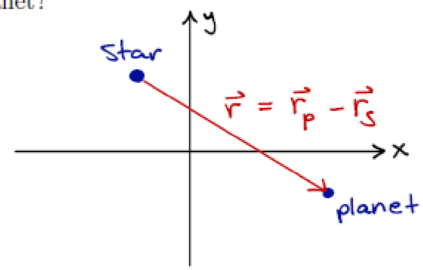
A planet of mass 4×10^{24} kg is at location $< 5e11, -2e11, 0 >$ m. A star of mass 5×10^{30} kg is at location $< -2e11, 3e11, 0 >$ m. It will be useful to draw a diagram of the situation.

(a) What is the relative position vector $\vec{r}$ pointing from the star (initial) to the planet (final)?

$$\vec{r} = \vec{r}_p - \vec{r}_s = \langle 5e11, -2e11, 0\rangle - \langle -2e11, 3e11, 0\rangle =$$

$$= \langle 5e11 - -2e11, -2e11 - 3e11, 0\rangle =$$

$$= \boxed{\langle 7e11, -5e11, 0\rangle \text{ m}}$$



(b) What is the (vector) force exerted on the planet by the star?

$$|\vec{r}| = \sqrt{(7e11)^2 + (-5e11)^2} = \sqrt{7.4e23} = 8.6e11 \text{m}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle 7e11, -5e11, 0\rangle}{8.6e11} = \langle 0.814, -0.581, 0\rangle$$

$$|\vec{F}| = \frac{G m_p m_s}{|\vec{r}|^2} = \frac{(6.7e-11)(4e24)(5e30)}{7.4e23} = 1.81e21 \text{ N}$$

$$\Rightarrow \vec{F}_{\substack{\text{on planet}\\ \text{by star}}} = -|\vec{F}|\hat{r} = -(1.81e21)\langle 0.814, -0.581, 0\rangle =$$

$$= \boxed{\langle -1.47e21, 1.05e21, 0\rangle \text{ N}}$$

(c) What is the (vector) force exerted on the star by the planet?

$$\vec{F}_{\substack{\text{on star}\\ \text{by planet}}} = -\vec{F}_{\substack{\text{on planet}\\ \text{by star}}} = \boxed{\langle 1.47e21, -1.05e21, 0\rangle \text{ N}}$$

(b/c Newton's 3rd law)

(d) The velocity of the planet at this instant is $< 0.5 \times 10^4, 1.5 \times 10^4, 0 >$ m/s. Determine the velocity of the planet 2×10^7 seconds later.

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{net}/m_p)\Delta t =$$

$$= \langle 0.5e4, 1.5e4, 0\rangle + \frac{2e7}{4e24}\langle -1.47e21, 1.05e21, 0\rangle =$$

$$= \langle 0.5e4, 1.5e4, 0\rangle + \langle -7350, 5250, 0\rangle =$$

$$= \boxed{\langle -2350, 20250, 0\rangle \text{ m/s}}$$

(e) Calculate the new position of the planet 2×10^7 seconds later.

Approximate $\vec{v}_{Avg} \approx \vec{v}_f$ b/c non-constant force

$$\vec{r}_{p,f} = \vec{r}_{p,i} + \vec{v}_f \Delta t = \langle 5e11, -2e11, 0\rangle + (2e7)\langle -2350, 20250, 0\rangle =$$

$$= \langle 5e11, -2e11, 0\rangle + \langle -4.7e10, 4.05e11, 0\rangle =$$

$$= \boxed{\langle 4.53e11, 2.05e11, 0\rangle \text{ m}}$$

(f) The planet has a new position and a new velocity. Starting from these new conditions, briefly explain how you would determine the new vector gravitational force exerted by the star on the planet and update the position of the planet. What approximations or simplifying assumptions did you make?

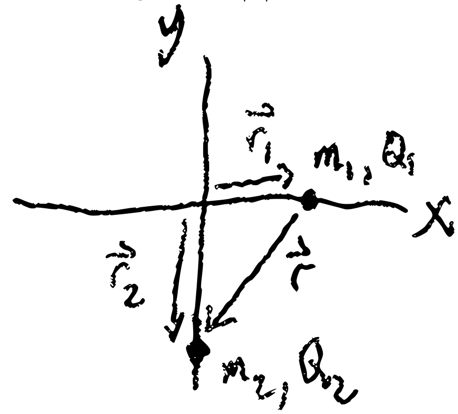
Assume star doesn't move. Calculate new $\vec{r} = \vec{r}_p - \vec{r}_s$ ($\vec{r}_p$ changed but $\vec{r}_s$ stayed the same), then use $\vec{F} = \left(\frac{G m_p m_s}{|\vec{r}|^2}\right)(-\hat{r})$ to find new vector gravitational force. Use the new $\vec{F}$ to find the new $\vec{v}$, and then use new $\vec{v}$ to find new $\vec{r}_p$

Problem #2

Two objects are located in the xy plane. They both have the same mass ($m_1 = m_2 = 1000$ kg) and the same positive electric charge ($Q_1 = Q_2 = 1 \times 10^{-8}$ C). Object 1 is located at $\vec{r}_1 =< 3, 0, 0 >$ m and Object 2 is located at $\vec{r}_2 =< 0, -4, 0 >$ m.

A. [10 pts] Find the position vector $\vec{r}$ that points from Object 1 to Object 2, its magnitude $|\vec{r}|$, and its unit vector, $\hat{r}$.

$\vec{r} = \vec{r}_2 - \vec{r}_1$ (see picture)



$= <0, -4, 0> m - <3, 0, 0> m$

$$\boxed{\begin{gathered}\vec{r} = <-3, -4, 0> m \\ |\vec{r}| = \sqrt{(-3m)^2 + (-4m)^2} = 5\ m \\ \hat{r} = \frac{\vec{r}}{|\vec{r}|} = <-\frac{3}{5}, -\frac{4}{5}, 0>\end{gathered}}$$

B. [10 pts] What is the **vector gravitational force on Object 2** due to Object 1?

$$\vec{F}_{grav,1 \text{ on } 2} = -\frac{G m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$= -\frac{(6.7 \times 10^{-11}\ N\ m^2/kg^2)(1000\ kg)(1000\ kg)}{(5m)^2} <-\frac{3}{5}, -\frac{4}{5}, 0>$$

$$\boxed{\vec{F}_{grav,1 \text{ on } 2} = < 1.61 \times 10^{-6}, 2.14 \times 10^{-6}, 0 > N}$$

C. [10 pts] What is the **vector electric force on Object 2** due to Object 1?

$$\vec{F}_{electric,1on2} = \frac{kQ_1Q_2}{|\vec{r}|^2}\hat{r}$$

$$= \frac{(9\times10^9\ Nm^2/C^2)(1\times10^{-8}\ C)(1\times10^{-8}C)}{(5m)^2}\left\langle -\frac{3}{5}, -\frac{4}{5}, 0\right\rangle$$

$$\boxed{\vec{F}_{electric,1on2} = \langle -2.16\times10^{-8}, -2.88\times10^{-8}, 0\rangle\ N}$$

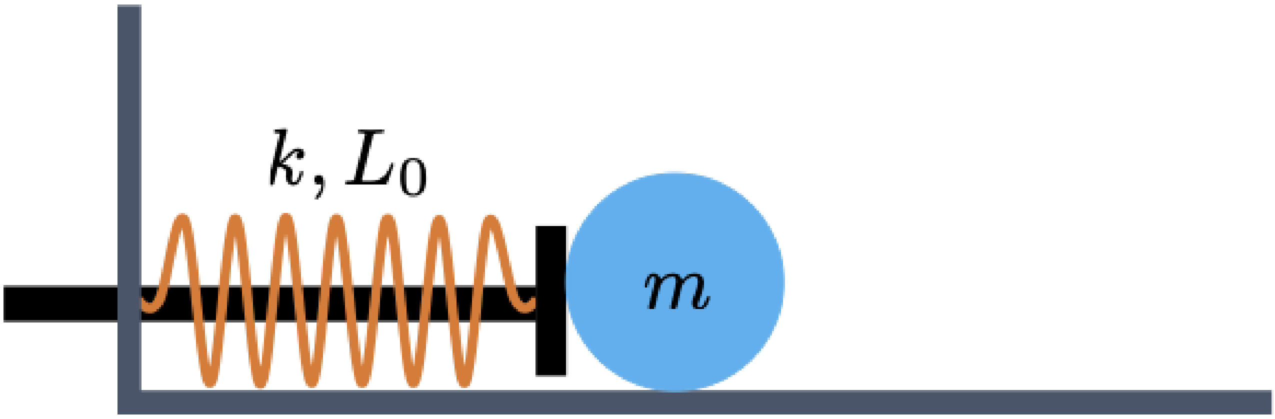
D. [5 pts] If nothing else is interacting with the two objects, will they move towards each other or away from each other? **Explain your reasoning**. You do not have to do any new calculations.

They will move towards each other, because the two forces acting on each of the particles, gravity and electric force due to the other particle, act in opposite directions (gravity is attractive and like charges repel). But since gravity is stronger (ie has a greater magnitude), the net force on each particle points toward the other. (The reason we can say the same thing is happening to object 1 is because it is identical to object 2 OR Newton's 3rd Law.)

Problem #3

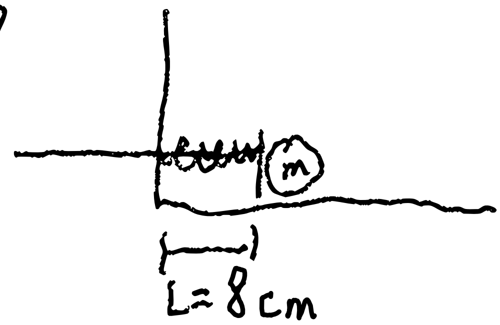
In a pinball machine, you launch a ball of mass $m = 150$ g by pulling on a handle that compresses a horizontal spring. The spring has relaxed length $L_0 = 12$ cm and stiffness $k = 350$ N/m. At $t = 0$ the handle is pulled as far out as it can go, and the spring is maximally compressed to a length of $L = 8$ cm.

In this problem you can assume that the origin of the coordinate system is located at the fixed end of the spring (on the left side of the image), the $+x$ axis goes to the right, and the $+y$ axis goes upwards. There is no friction or air resistance anywhere in this problem.



A. [10 pts] Determine the **vector net force** on the ball at $t = 0$.

$t=0$



What forces are acting on the ball? (at moment of launch)

cancel out $\begin{cases} \bullet \text{ Earth's gravity} \\ \bullet \text{ Normal force of floor} \end{cases}$

• Spring force $\Rightarrow \vec{F}_{net} = \vec{F}_{spring}$

$$\vec{F}_{spring} = -k_s(|\vec{L}| - L_0)\hat{L} \quad , \quad \vec{L} = L\hat{x}$$

$$= -k_s(L - L_0)\hat{x} \qquad |\vec{L}| = L = 0.08\text{m}$$

$$\hat{L} = \frac{\vec{L}}{||\vec{L}||} = \hat{x}$$

$$= -(350 \text{ N/m})(0.08\text{m} - 0.12\text{m})\hat{x}$$

$$= 14 \text{ N } \hat{x}$$

$$\Rightarrow \boxed{\vec{F}_{net}(t=0) = \langle 14, 0, 0 \rangle \text{ N}}$$

$\rightarrow \Delta t = 0.1\,s$

B. [15 pts] Determine the **position** of the ball at $t = 0.1$ s after launching it.

Update the velocity:

$$\vec{v}(t=0.1\,s) = \cancelto{0}{\vec{v}(t=0\,s)} + \frac{\vec{F}_{net}(t=0\,s)}{m}\Delta t$$

$$= \frac{0.1\,s}{0.15\,kg}\langle 14, 0, 0\rangle\,N$$

$$= \langle 9.33, 0, 0\rangle\,m/s$$

Update the position:

$$\vec{r}(t=0.1\,s) = \vec{r}(t=0\,s) + \vec{V}_{avg}\,\Delta t$$

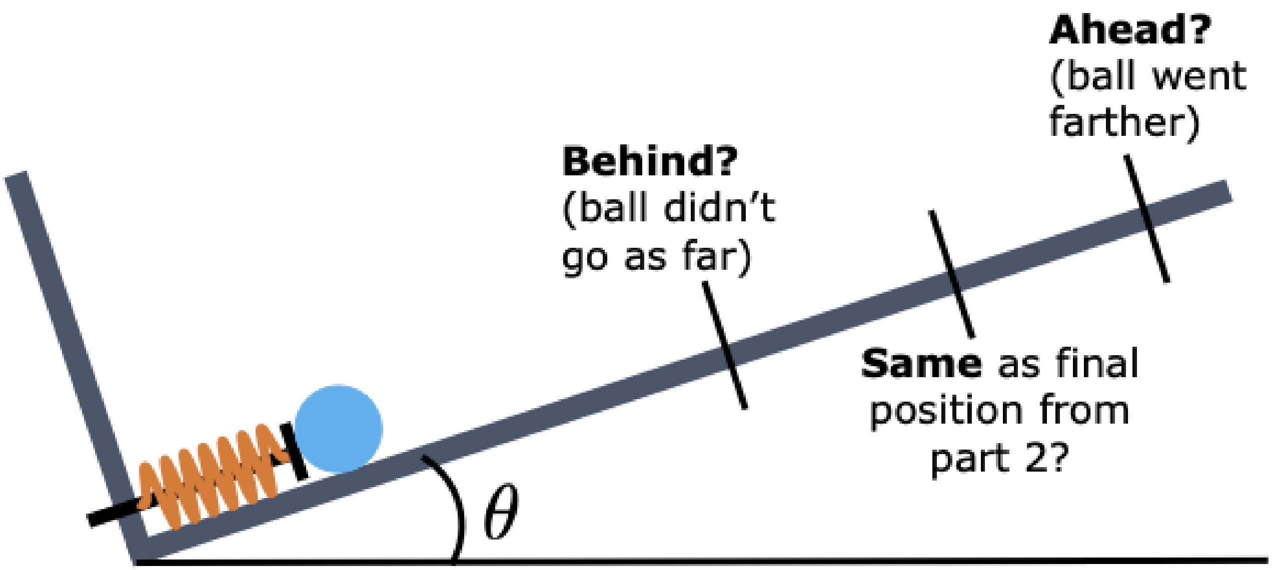
non-constant force

$$\simeq \vec{r}(t=0\,s) + \vec{v}(t=0.1\,s)\,\Delta t$$

$$= \langle 0.08, 0, 0\rangle\,m + (0.1\,s)\langle 9.33, 0, 0\rangle\,m/s$$

$$\boxed{\vec{r}(t=0.1\,s) = \langle 1.013, 0, 0\rangle\,m}$$

C. [5 pts] In a real pinball machine, the "floor" in which the ball rests is actually tilted upwards (the ball is higher than the handle) by an angle of $\theta = 7°$. If you were to repeat your calculations with everything the same except but this time taking into account the elevated floor, would the ball's position be ahead, behind, or the same as the position you calculated in the previous part? **Explain your reasoning.** You do not have to redo the calculations.



Behind, When doing iterations to find the new position of the ball, the net force along the direction of motion, i.e. along the tilted floor, is reduced by part of Earth's gravity which no longer cancels with the normal force.