Physics 2211 GPS Week 10

Problem #1

During the spring semester at MIT, residents of the parallel buildings of the East Campus Dorms battle one another with large sling-shots made from surgical hose mounted to window frames. Water balloons (with a mass of about 0.5 kg) are placed in a pouch attached to the hose, which is then stretched nearly the width of the room (about 3.5 meters). If the hose obeys Hooke's Law, with a spring constant of 100 N/m, how fast is the balloon traveling when it leaves the dorm room window?

System: water ballon

Surroundings: hose

Initial: max stretch, balloon at rest

Final: no stretch, balloon released

Assumption: no vertical displacement between stretch and release (horizontal spring)

$$\Delta K_{sys} = W_{surr}$$

$$K_f - \cancel{K_i}^{0} = \int_{s_i}^{f} \vec{F} \cdot d\vec{r}$$

$$\frac{1}{2} m v_f^2 = \int_{s_i}^{s_f} -ks \, ds = -k \int_{s_i}^{0} s \, ds$$

$$\cancel{\frac{1}{2}} m v_f^2 = -k \left(\frac{s^2}{2}\right)_{s_i}^{0} = -k\left(0 - \frac{1}{2} s_i^2\right) = \cancel{\frac{1}{2}} k s_i^2$$

$$m v_f^2 = k s_i^2$$

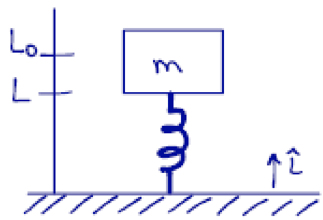
$$v_f^2 = \frac{k}{m} s_i^2$$

$$v_f = \sqrt{\frac{k}{m}}\, s_i = \sqrt{\frac{100}{0.5}}\,(3.5) = \boxed{49.5 \text{ m/s}}$$

Problem #2

A spring with stiffness k_s and relaxed length L_0 stands vertically on a table. You hold a mass M just barely touching the top of the spring.

(a) You very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. How much did the spring compress? Hint: Use Newton's 2nd law.



(Note: 1D motion)

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring} = 0$$

$$mg(-\hat{y}) + -k(L-L_0)\hat{\imath} = 0$$

$$-mg\,\hat{y} - k(L-L_0)\,\hat{y} = 0$$

$$(-mg - ks)\,\hat{y} = 0$$

$$-ks = mg$$

$$\boxed{s = \frac{-mg}{k}}$$ negative b/c compressed

∗ OK to say $s = mg/k$ only if student explicitly states that the spring is compressed.

(b) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block to be the system and use the energy principle to determine the work done by the Earth, the spring and your hand. Hint: the spring force is not constant.

System: block
Surroundings: Earth, spring, hand

Initial: block at rest, spring relaxed
Final: block at rest, spring compressed

✓ $W_{grav} = \vec{F}_{grav} \cdot \Delta\vec{r} = mg(-\hat{y}) \cdot \left(\frac{mg}{k}\right)(-\hat{y}) = \frac{m^2g^2}{k} \longrightarrow$ positive

✓ $W_{spring} = \int_i^f \vec{F}_{spring} \cdot d\vec{r} = \int_{s_i}^{s_f} -ks\,ds = -k\int_0^{-mg/k} s\,ds = -k\left(\frac{s^2}{2}\right)_0^{-mg/k} = \frac{-\cancel{k}}{2}\frac{m^2g^2}{k^{\cancel{2}}} = \frac{-1}{2}\frac{m^2g^2}{k}$

$$\Longrightarrow \Delta E = \Delta K = W_{total}$$

$$\cancel{K_f} - \cancel{K_i} = W_{grav} + W_{spring} + W_{hand}$$

$$0 = W_{grav} + W_{spring} + W_{hand}$$

$$\Rightarrow W_{hand} = -W_{grav} - W_{spring} = -\frac{m^2g^2}{k} - \frac{-1}{2}\frac{m^2g^2}{k} = \frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k}$$

$$\Rightarrow \boxed{W_{hand} = -\frac{1}{2}\frac{m^2g^2}{k}}$$

(c) In part (a) you very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. Choose the block+spring+Earth to be the system and use the energy principle to determine the work done by your hand.

System: block, spring, Earth
Surroundings: hand

Initial: block at rest, spring relaxed, height $= L_0$
Final: block at rest, spring compressed, height $= L$

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = W_{hand}$$

$$\underbrace{\cancel{\tfrac{1}{2}m(v_f^2 - v_i^2)}}_{0} + mg(h_f - h_i) + \tfrac{1}{2}k(s_f^2 - \underbrace{\cancel{s_i^2}}_{0}) = W_{hand}$$

$$mg(L - L_0) + \tfrac{1}{2}ks_f^2 = W_{hand}$$

$$mg\left(\frac{-mg}{k}\right) + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = W_{hand}$$

$$\frac{-m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2\cancel{k}}{k^{\cancel{2}}} = W_{hand}$$

$$\Rightarrow W_{hand} = \frac{1}{2}\frac{m^2g^2}{k} - \frac{m^2g^2}{k} = \boxed{-\frac{1}{2}\frac{m^2g^2}{k}}$$

Same as in part (a)

(d) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block
Surroundings: Earth, spring

Initial: block at rest, spring relaxed
Final: block moving, spring compressed

✓ W_{grav} = same as in part (b) b/c moves same distance $= \frac{m^2g^2}{k}$

✓ W_{spring} = same as in part (b) b/c same compression $= \frac{-1}{2}\frac{m^2g^2}{k}$

$$\Rightarrow \Delta E = \Delta K = W_{total}$$

$$\tfrac{1}{2}m(v_f^2 - \underbrace{\cancel{v_i^2}}_{0}) = W_{grav} + W_{spring} = \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k}$$

$$\cancel{\tfrac{1}{2}}\cancel{m}v_f^2 = \cancel{\tfrac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Longrightarrow \boxed{v_f = g\sqrt{m/k}}$$

(e) Now you again hold the mass just barely touching the top of the spring, and then let go. Choose the block+spring+Earth to be the system and use the energy principle to calculate the speed of the block when the spring has the same compression you found in part (a).

System: block, spring, Earth
Surroundings: nothing

Initial: block at rest, spring relaxed, height $= L_0$
Final: block moving, spring compressed, height $= L$

$$\Delta E = \Delta K + \Delta U_{grav} + \Delta U_{spring} = 0$$

$$\frac{1}{2}m(v_f^2 - \cancel{v_i^2}) + mg(h_f - h_i) + \frac{1}{2}k(s_f^2 - \cancel{s_i^2}) = 0$$

$$\frac{1}{2}mv_f^2 + mg(L - L_0) + \frac{1}{2}ks_f^2 = 0$$

$$\frac{1}{2}mv_f^2 + mg\left(\frac{-mg}{k}\right) + \frac{1}{2}\cancel{k}\left(\frac{m^2g^2}{k^{\cancel{2}}}\right) = 0$$

$$\frac{1}{2}mv_f^2 - \frac{m^2g^2}{k} + \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\cancel{\frac{1}{2}}\cancel{m}v_f^2 = \cancel{\frac{1}{2}}\frac{m^{\cancel{2}}g^2}{k}$$

$$v_f^2 = \frac{mg^2}{k} \Rightarrow \boxed{v_f = g\sqrt{m/k}}$$ same as part (d)

(f) Compare your answers in parts (b) and (c), and the answers in parts (d) and (e). What does that tell you about your choices when you have an energy principle problem?

(b) and (c) are the same

(d) and (e) are the same

You can choose what to put in your system and what to count as part of the surroundings when solving an energy principle problem, so pick whatever makes the problem easier.

Problem #3

After watching "The Big Lebowski" for the first time this summer, you and a friend get into an argument about how much ice to add when making the perfect white russian cocktail. You both agree that, for optimum taste, the cocktail should be enjoyed at 10 degrees Celsius. The two ingredients for the cocktail, cream and a "vodka & kahlua" mix, both leave the fridge at 15 degrees Celsius. Ice from a standard freezer is at a temperature of -10 degrees Celsius. If typical white russian calls for 0.06 L of cream and 0.14 L of the "vodka & kahlua" mix, how much ice is needed to bring the drink down to its optimum temperature?

Ice: density = 0.91 kg/L, C = 4.18 J/(Cg)
Mix: density = 0.8 kg/L, C = 2.44 J/(Cg)
Cream: density = 1 kg/L, C = 3.77 J/(Cg)

We assume that the ingredients are sufficiently isolated from their surroundings when they are mixed, so that a negligible amount of heat transfers to the surroundings. This means that the system is isolated and the energy principle predicts that

$$\Delta E_{sys} = 0.$$

Taking the initial and final state to be before the mixing process and after the combination has reached equilibrium, respectively, there are then no appreciable changes in kinetic or potential energy, and so

$$\Delta E_{therm,sys} = 0.$$

Each ingredient contributes to the total change in thermal energy:

$$\Delta E_{therm,sys} = \Delta E_{therm,mix} + \Delta E_{therm,cream} + \Delta E_{therm,\ ice}$$

$$= m_{mix} C_{mix} \Delta T_{mix} + m_{cream} C_{cream} \Delta T_{cream} + m_{ice} C_{ice} \Delta T_{ice} = 0.$$

Solving for the mass of the ice we find that

$$m_{ice} = -\frac{m_{mix} C_{mix} \Delta T_{mix} + m_{cream} C_{cream} \Delta T_{cream}}{C_{ice} \Delta T_{ice}}$$

$$= -\frac{\left(\left(\left(0.8\frac{kg}{L}\right)(0.14L)\left(2.44\frac{J}{gC^\circ}\right)\right)(10°C - 15°C) + \left(\left(1.0\frac{kg}{L}\right)(0.06L)\left(3.77\frac{J}{gC^\circ}\right)\right)(10°C - 15°C)\right)}{\left(4.18\,\frac{J}{g\,C^\circ}\right)(10°C - (-10°C))}$$

$$\approx 0.030\ kg = 30\ g$$

Note: The heat capacities and the densities do not use the same mass units, so in principle we could convert first. However, in this case there's no need because the same conversion factor appears in the numerator and the denominator so they cancel out.

Problem #4 - If time permits

During 3 hours one winter afternoon, when the outside temperature was 11° C, a house heated by electricity was kept at 25° C with the expenditure of 58 kwh (kilowatt·hours) of electric energy.

(a) What was the average energy leakage in joules per second (watts) through the walls of the house to the environment (the outside air and ground)?

$$\checkmark\ \Delta t = (3\,\text{hrs})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 10\,800\ \text{sec}$$

$$\checkmark\ \Delta E = Q = (58\ \text{kWh})(1000\,\text{W}/1\,\text{kW})(60\,\text{min}/1\,\text{hr})(60\,\text{sec}/1\,\text{min}) = 2.088e8\ \text{W}\cdot\text{s}$$

(same as Joules)

$$\Rightarrow \text{energy leakage} = \frac{\Delta E}{\Delta t} = \frac{2.088e8\ \text{W}\cdot\text{s}}{10800\ \text{sec}} = \boxed{1.93e4\ \text{Watts}}$$

$$\text{Watt} = \frac{\text{Joule}}{\text{second}}$$

Alternative: $\dfrac{\Delta E}{\Delta t} = \dfrac{58\ \text{kWh}}{3\,\text{h}} = \dfrac{19.3\ \text{kW}}{} \Big| \dfrac{1000\ \text{W}}{1\ \text{kW}} = \boxed{1.93e4\ \text{Watts}}$

(b) The rate at which energy is transferred between two systems due to a temperature difference is often proportional to their temperature difference. Assuming this to hold in this case, if the house temperature had been kept at 28° C (82.4° F), how many kwh of electricity would have been consumed?

$$\text{energy transfer} \propto \text{temperature difference} \Longrightarrow \frac{Q_1}{\Delta T_1} = \frac{Q_2}{\Delta T_2}$$

$$\Longrightarrow \frac{58\ \text{kWh}}{(25-11)^\circ\text{C}} = \frac{Q_2}{(28-11)^\circ\text{C}}$$

$$\frac{58}{14} = \frac{Q_2}{17}$$

$$14\,Q_2 = (58)(17)$$

$$Q_2 = \frac{(58)(17)}{(14)} = \boxed{70.4\ \text{kWh}}$$