

AUTHOR ONE AND AUTHOR TWO

ABSTRACT. This paper is a sample prepared to illustrate the use of the American Mathematical Society's L^AT_EX document class `amsart` and publication-specific variants of that class for AMS-L^AT_EX version 2.

Theorem. Ratio Test (1) $\left(\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1 \right) \Rightarrow (\sum_{n=0}^{\infty} a_n \text{ absolutely converges})$ Proof. Using the direct proof, $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1 \Leftrightarrow \forall \varepsilon > 0, \exists N \in \mathbb{N} \text{ such that } \left| \left| \frac{a_{n+1}}{a_n} \right| - L \right| < \varepsilon, \text{ for every } n > N$ That is, for every $\varepsilon > 0$, there exists a large number, $N \in \mathbb{N}$, such that $\left| \left| \frac{a_{n+1}}{a_n} \right| - L \right| < \varepsilon$ for $n = N+1, N+2, \dots \Leftrightarrow L - \varepsilon < \left| \frac{a_{n+1}}{a_n} \right| < L + \varepsilon$ for $n = N+1, N+2, \dots$ Since this equation is always true for every ε , choose its value $\varepsilon = \frac{1-L}{2} > 0$. Now we have a true statement (hypothesis), for $n = N+1, N+2, \dots L - \frac{1-L}{2} < \left| \frac{a_{n+1}}{a_n} \right| < L + \frac{1-L}{2} \Leftrightarrow \frac{3L-1}{2} < \left| \frac{a_{n+1}}{a_n} \right| < \frac{L+1}{2}$ $\left(\left| \frac{a_{n+1}}{a_n} \right| > 0 \text{ is true} \right) \wedge \left(\frac{L+1}{2} < 1 \text{ is true} \right) \Rightarrow \forall n > N, \max \left\{ 0, \frac{3L-1}{2} \right\} < \left| \frac{a_{n+1}}{a_n} \right| < \frac{L+1}{2} < 10 < \frac{L+1}{2} < 1$ $n = N+1, \left| \frac{a_{N+2}}{a_{N+1}} \right| < \frac{L+1}{2} \Leftrightarrow |a_{N+2}| < \frac{L+1}{2} |a_{N+1}|$ $n = N+2, \left| \frac{a_{N+3}}{a_{N+2}} \right| < \frac{L+1}{2} \Leftrightarrow |a_{N+3}| < \frac{L+1}{2} |a_{N+2}| < \left(\frac{L+1}{2} \right)^2 |a_{N+1}|$ $n = N+3, \left| \frac{a_{N+4}}{a_{N+3}} \right| < \frac{L+1}{2} \Leftrightarrow |a_{N+4}| < \frac{L+1}{2} |a_{N+3}| < \left(\frac{L+1}{2} \right)^2 |a_{N+2}| < \left(\frac{L+1}{2} \right)^3 |a_{N+1}|$ Therefore, we can construct the following inequality, $\sum_{n=0}^{\infty} |a_n| = (|a_0| + \dots + |a_N|) + (|a_{N+1}| + |a_{N+2}| + |a_{N+3}| + |a_{N+4}| + \dots)$ $(|a_1| + \dots + |a_N|) + \left\{ |a_{N+1}| + \frac{L+1}{2} |a_{N+1}| + \left(\frac{L+1}{2} \right)^2 |a_{N+1}| + \left(\frac{L+1}{2} \right)^3 |a_{N+1}| + \dots \right\} = (|a_1| + \dots + |a_N|) + |a_{N+1}| \left\{ 1 + \frac{L+1}{2} + \left(\frac{L+1}{2} \right)^2 + \left(\frac{L+1}{2} \right)^3 + \dots \right\} = (|a_1| + \dots + |a_N|) + |a_{N+1}| \left\{ \frac{1}{1 - \left(\frac{L+1}{2} \right)} \right\} = \left(\sum_{n=0}^N |a_n| \right) + |a_{N+1}| \left(\frac{2}{1-L} \right) \sum_{n=0}^{\infty} |a_n| < \sum_{n=0}^N |a_n| + |a_{N+1}| \left(\frac{2}{1-L} \right)$ Since the infinite sum is bounded and monotonic, it converges by the monotone convergence theorem. $(\sum_{n=0}^{\infty} a_n \text{ absolutely converges})$ \square Also, we have to consider the following two different hypotheses. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ and $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L = 1$

Theorem. Ratio Test (2) $\left(\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1\right) \Rightarrow (\sum_{n=0}^{\infty} a_n \text{ diverges})$ *We can prove the Ratio Test (2) same as we did in*

Theorem. Ratio Test (3) $\left(\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1\right) \Rightarrow (\sum_{n=0}^{\infty} a_n \text{ is inconclusive})$ Try to prove it. (Choose any positive value for

Exercise. Test the series by using the Ratio Test.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{n^4}{3^{n+2}}$
2. $\sum_{n=1}^{\infty} \frac{1}{n!}$

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$$\begin{aligned} 3. \quad & \sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!} \\ 5. \quad & \sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2+1} \end{aligned}$$

$$\begin{aligned} 4. \quad & \sum_{n=1}^{\infty} \frac{1}{n} \\ 6. \quad & \sum_{n=1}^{\infty} \frac{n^n}{n!} \end{aligned}$$

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THIS IS A SPECIAL SECTION HEAD

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1. THIS IS A NUMBERED FIRST-LEVEL SECTION HEAD

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1.1. This is a numbered second-level section head. This is an example of a numbered second-level heading.

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Lemma 1.1. *Let $f, g \in A(X)$ and let E, F be cozero sets in X .*

- (1) *If f is E -regular and $F \subseteq E$, then f is F -regular.*
- (2) *If f is E -regular and F -regular, then f is $E \cup F$ -regular.*
- (3) *If $f(x) \geq c > 0$ for all $x \in E$, then f is E -regular.*

The following is an example of a proof.

Proof. Set $j(\nu) = \max(I \setminus a(\nu)) - 1$. Then we have

$$\sum_{i \notin a(\nu)} t_i \sim t_{j(\nu)+1} = \prod_{j=0}^{j(\nu)} (t_{j+1}/t_j).$$

Hence we have

$$\begin{aligned} (1.1) \quad & \prod_{\nu} \left(\sum_{i \notin a(\nu)} t_i \right)^{|a(\nu-1)| - |a(\nu)|} \sim \prod_{\nu} \prod_{j=0}^{j(\nu)} (t_{j+1}/t_j)^{|a(\nu-1)| - |a(\nu)|} \\ & = \prod_{j \geq 0} (t_{j+1}/t_j)^{\sum_{j(\nu) \geq j} (|a(\nu-1)| - |a(\nu)|)}. \end{aligned}$$

By definition, we have $a(\nu(j)) \supset c(j)$. Hence, $|c(j)| = n - j$ implies (5.4). If $c(j) \notin a$, $a(\nu(j))c(j)$ and hence we have (5.5). \square

¹Here is an example of a footnote. Notice that this footnote text is running on so that it can stand as an example of how a footnote with separate paragraphs should be written.

And here is the beginning of the second paragraph.

This is an example of an ‘extract’. The magnetization M_0 of the Ising model is related to the local state probability $P(a) : M_0 = P(1) - P(-1)$. The equivalences are shown in Table ??.

TABLE 1.

	$-\infty$	$+\infty$
$f_+(x, k)$	$e^{\sqrt{-1}kx} + s_{12}(k)e^{-\sqrt{-1}kx}$	$s_{11}(k)e^{\sqrt{-1}kx}$
$f_-(x, k)$	$s_{22}(k)e^{-\sqrt{-1}kx}$	$e^{-\sqrt{-1}kx} + s_{21}(k)e^{\sqrt{-1}kx}$

Definition 1.2. This is an example of a ‘definition’ element. For $f \in A(X)$, we define

$$(1.2) \quad \mathcal{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}.$$

Remark 1.3. This is an example of a ‘remark’ element. For $f \in A(X)$, we define

$$(1.3) \quad \mathcal{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}.$$

Example 1.4. This is an example of an ‘example’ element. For $f \in A(X)$, we define

$$(1.4) \quad \mathcal{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}.$$

Exercise 1.5. This is an example of the `xca` environment. This environment is used for exercises which occur within a section.

The following is an example of a numbered list.

- (1) First item. In the case where in G there is a sequence of subgroups

$$G = G_0, G_1, G_2, \dots, G_k = e$$

such that each is an invariant subgroup of G_i .

- (2) Second item. Its action on an arbitrary element $X = \lambda^\alpha X_\alpha$ has the form

$$(1.5) \quad [e^\alpha X_\alpha, X] = e^\alpha \lambda^\beta [X_\alpha X_\beta] = e^\alpha c_{\alpha\beta}^\gamma \lambda^\beta X_\gamma,$$

- (a) First subitem.

$$-2\psi_2(e) = c_{\alpha\gamma}^\delta c_{\beta\delta}^\gamma e^\alpha e^\beta.$$

- (b) Second subitem.

- (i) First subsubitem. In the case where in G there is a sequence of subgroups

$$G = G_0, G_1, G_2, \dots, G_k = e$$

such that each subgroup G_{i+1} is an invariant subgroup of G_i and each quotient group G_{i+1}/G_i is abelian, the group G is called *solvable*.

- (ii) Second subsubitem.

- (c) Third subitem.

- (3) Third item.

Here is an example of a cite. See [A].

Theorem 1.6. *This is an example of a theorem.*

Theorem 1.7 (Marcus Theorem). *This is an example of a theorem with a parenthetical note in the heading.*



FIGURE 1. This is an example of a figure caption with text.



FIGURE 2.

2. SOME MORE LIST TYPES

This is an example of a bulleted list.

- \mathcal{J}_g of dimension $3g - 3$;
- $\mathcal{E}_g^2 = \{\text{Pryms of double covers of } C = \square \text{ with normalization of } C \text{ hyperelliptic of genus } g - 1\}$ of dimension $2g$;
- $\mathcal{E}_{1,g-1}^2 = \{\text{Pryms of double covers of } C = \square_{P^1}^H \text{ with } H \text{ hyperelliptic of genus } g - 2\}$ of dimension $2g - 1$;
- $\mathcal{P}_{t,g-t}^2$ for $2 \leq t \leq g/2 = \{\text{Pryms of double covers of } C = \square_{C''}^{C'} \text{ with } g(C') = t - 1 \text{ and } g(C'') = g - t - 1\}$ of dimension $3g - 4$.

This is an example of a ‘description’ list.

Zero case: $\rho(\Phi) = \{0\}$.

Rational case: $\rho(\Phi) \neq \{0\}$ and $\rho(\Phi)$ is contained in a line through 0 with rational slope.

Irrational case: $\rho(\Phi) \neq \{0\}$ and $\rho(\Phi)$ is contained in a line through 0 with irrational slope.

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