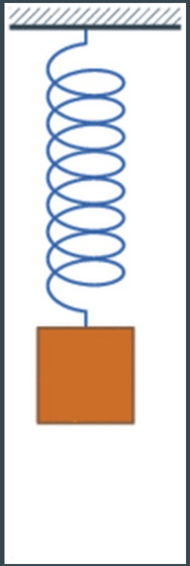

PHYS 2211 K

Week 3, Lecture 1

2022/01/25

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~~6~~ clicker questions today

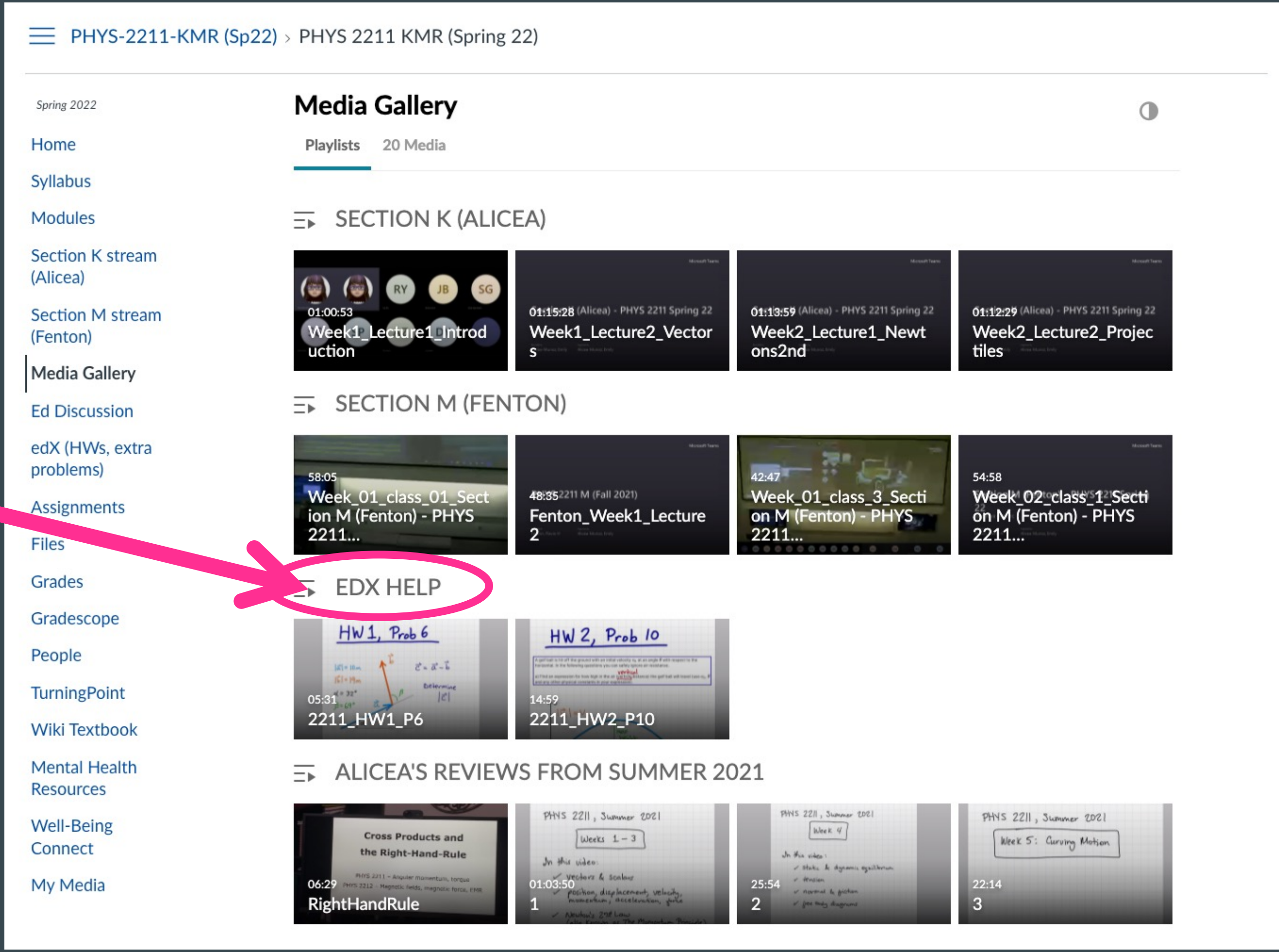
5

On today’s class…

1. Wrapping up projectile motion
2. Spring force
3. Iteration with constant and non-constant forces

Reminders!

Solution videos to selected edX problems



GPS video solutions will be here too, in a separate playlist

Reminders!

Lab meetings begin THIS week!

GPS problem sets are in Files → GPS

GTA/UTA Contact Info: 2211_TA_Schedule.xlsx

- First tab: Lab schedule
- Second tab: GTA and UTA contact info (emails)
- Last updated: 2022/01/18

(this ⬆️ is on the canvas class front page, scroll down)

CLICKER 1: Avatar State!

A. H O N O R ! ! !

B. Yip yip!

C. *TEARBENDING*

D. I see by releasing a sonic wave from my mouth

The story so far…

- Newton's 2nd Law (in velocity update form) $$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$
- Position update formula $$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\Delta t$$
- Gravity near Earth $$\vec{F}_g =< 0, -mg, 0 >$$
- Kinematic equations in x and y (only valid for constant force)

$$x_f = x_i + v_{ix}\Delta t$$

$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

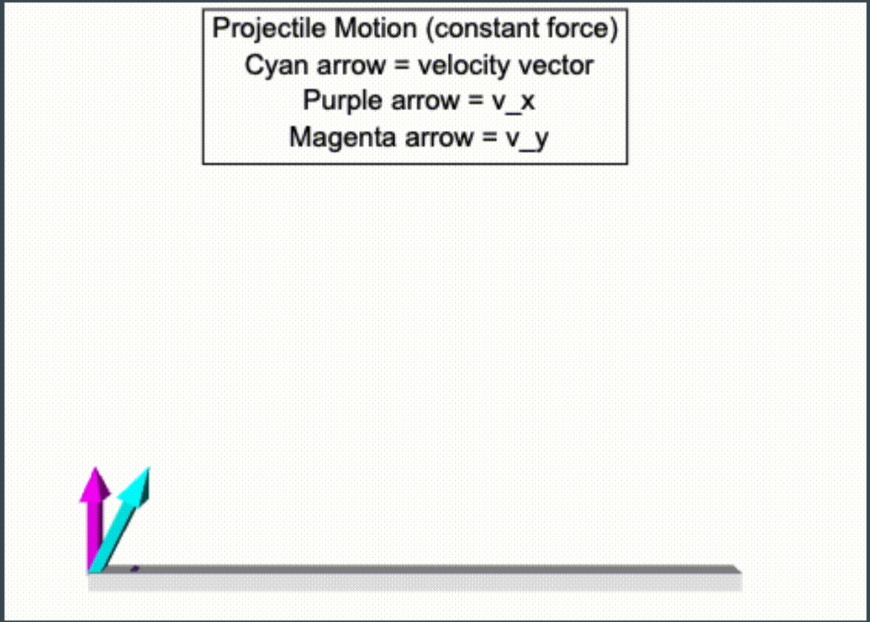
Projectile Motion

- Constant velocity motion in the x direction
- Constant force motion in the y direction

- At maximum height, $v_y = 0$

- Always start from Newton's 2nd Law, use the kinematic equation as needed

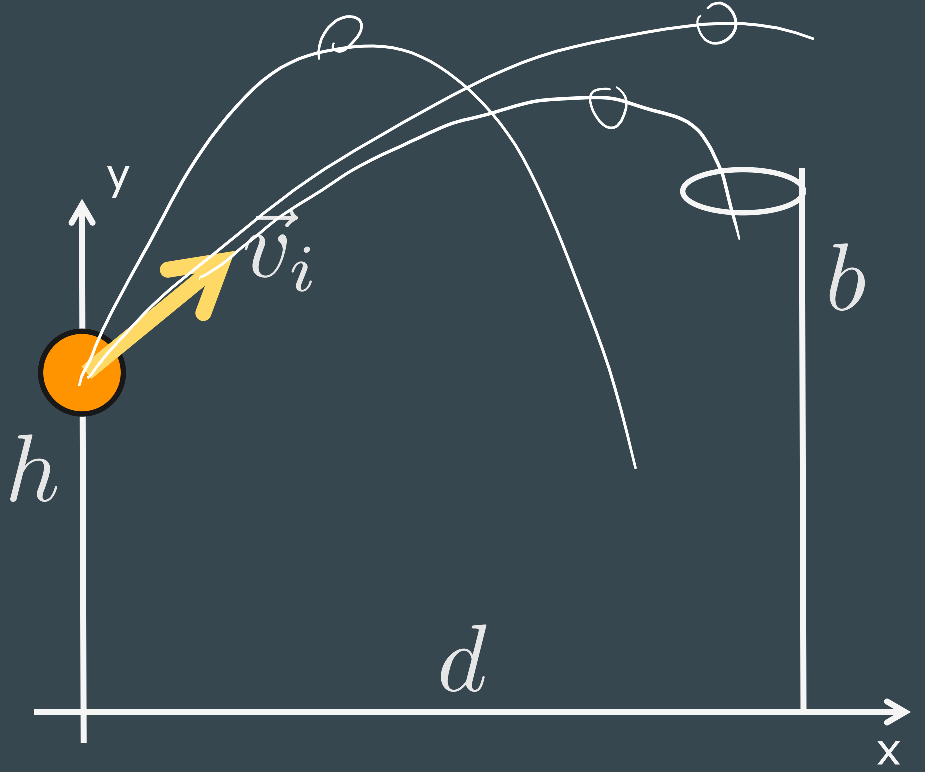
- Solve for intermediate unknowns to build towards your final answer



Example: Is it an airball?

A basketball player shoots a free-throw from a height h = 2 m above the ground. The free-throw line is d = 4.6 m away from the basket, and the basket is b = 3 m above the ground.

If the player releases the ball with an initial speed v = 6 m/s at an angle θ = 55 degrees from the horizontal, will he make the basket?



Spoiler alert: https://www.glowscript.org/#/user/ealicea/folder/Public/program/basketball

Knowns and unknowns

- Initial position of ball: $\vec{r}_i = < 0, h, 0 >$
- Initial velocity of ball: $\vec{v}_i = < v\cos\theta, v\sin\theta, 0 >$
- Position of basket: $\vec{r}_b = < d, b, 0 >$

- Numbers:
  - h = 2 m
  - v = 6 m/s
  - θ = 55 deg
  - d = 4.6 m
  - b = 3 m

Unknowns:

- Times! There's no time info at all!
- What does the trajectory look like? (is it steep? is it shallow?) = we don't know how high the ball gets

In what order do we calculate things?

- We want to divide the trajectory into two sections:
  (note that this is usually the best approach for projectile motion)
  1. From the moment we shoot to the maximum height
  2. From the maximum height to when the ball should go into the basket

- We don't know if the ball goes into the basket, so what we'll determine in the end is the y position of the ball at the x position of the basket
  - If $y_{ball} = y_{basket}$, then you made the shot
  - If $y_{ball} \neq y_{basket}$, then you missed

In what order do we calculate things?

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$
$$x_f = x_i + v_{ix}\Delta t$$
$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

- From shooting to maximum height

2 y_{max}, $\boxed{v_f \text{ @ } y_{max}} = 0$

1 Δt_{max}

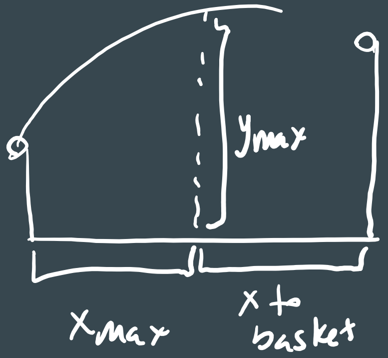
3 horizontal distance to $y_{max} \Rightarrow x_{max}$

- From maximum height to basket

✓ distance between x_{max} & basket $\Rightarrow$ "known"

1 time from (x_{max}, y_{max}) to basket.

2 find height of ball @ distance to basket



In what order do we calculate things?

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$
$$x_f = x_i + v_{ix}\Delta t$$
$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

- From shooting to maximum height
  1. time to max height, Δt_{max}
  2. max height, y_{max}
  3. horizontal distance at max height, x_{max}

- From maximum height to basket
  1. time from max height to horizontal distance of basket, Δt_d
  2. height of ball at horizontal distance of basket, y_d

CLICKER 2: What is the **maximum height** of the ball, y_{max}?

A. $y_{max} = 0.5$ m

B. $y_{max} = 3.23$ m

C. $y_{max} = 1.23$ m

D. $y_{max} = 2.6$ m

Solution: What is the maximum height of the ball, y_{max}?

① $\Delta t_{max} \Rightarrow \vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}}{m}\Delta t$

$$\underset{0}{V_{\cancel{max}\,y}} = V_{iy} + \frac{-\cancel{m}g}{\cancel{m}}\Delta t_{max}$$

$$0 = V\sin\theta - g\Delta t_{max}$$

$$V\sin\theta = +g\Delta t_{max}$$

$$\Delta t_{max} = \frac{V\sin\theta}{g}$$

$$= \frac{(6\,\cancel{m}/\cancel{s})\sin 55}{9.8\,\cancel{m}/s^{\cancel{2}}}$$

$$= 0.5\text{ sec}$$

② $y_{max} \Rightarrow$ kinematics

$$y_f = y_i + V_{iy}\Delta t - \tfrac{1}{2}g\Delta t^2$$

$$y_{max} = h + V\sin\theta\,\Delta t_{max} - \tfrac{1}{2}g\,\Delta t_{max}^2$$

$$= 2m + (6)(\sin 55)\,m/\cancel{s}\,(0.5\,\cancel{sec})$$

$$- \tfrac{1}{2}(9.8\,m/\cancel{s^2})(0.5\,\cancel{sec})^2 =$$

$$= \left[2 + 6(\sin 55)(0.5) - \tfrac{1}{2}(9.8)(0.5)^2\right]m =$$

$$= \boxed{3.23\text{ m}}\ y_{max}$$

In what order do we calculate things?

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$

$$x_f = x_i + v_{ix}\Delta t$$

$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

- From shooting to maximum height
  1. time to max height, Δt_{max}
  2. max height, y_{max}
  3. horizontal distance at max height, x_{max}
- From maximum height to basket
  1. time from max height to horizontal distance of basket, Δt_d
  2. height of ball at horizontal distance of basket, y_d

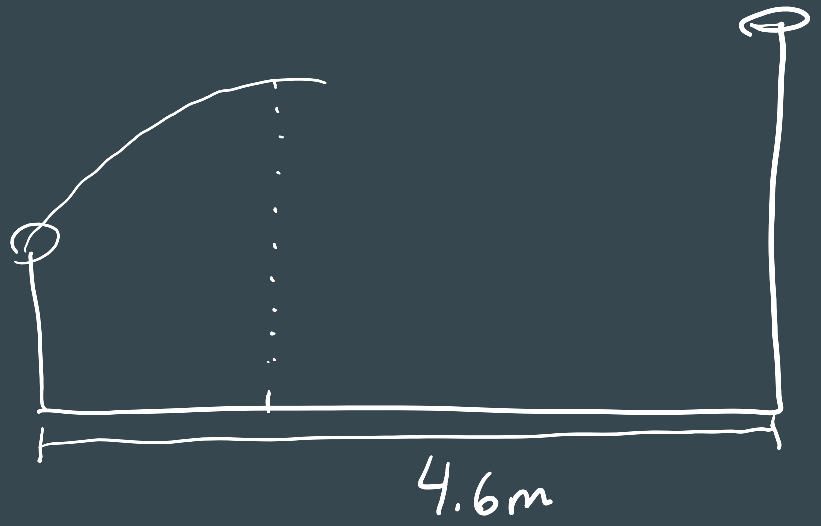
What is the horizontal distance of the ball when it reaches the max height?

$$x_f = x_i + v_{ix} \Delta t$$

$$x_{max} = \cancel{x_i}^{\,0} + v\cos\theta\,\Delta t_{max}$$

$$= (6\,\text{m/s})(\cos 55)(0.5\,\text{s})$$

$$= \boxed{1.72\,\text{m}}$$



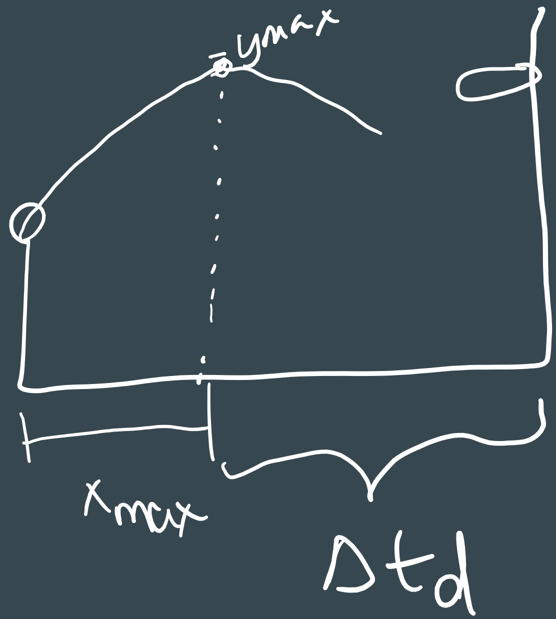
CLICKER 3: How much time does it take for the ball to go from its maximum height to the basket?

A. $\Delta t_d = 0.75$ s

B. $\Delta t_d = 1.3$ s

C. $\Delta t_d = 0.50$ s

D. $\Delta t_d = 0.84$ s



Solution: How much time does it take for the ball to go from its maximum height to the basket?

$$x_f = x_i + v_{ix}\Delta t$$

initial: @ max height

find: @ distance of basket

$$d = x_{max} + v\cos\theta\,\Delta t_d$$

$$d - x_{max} = v\cos\theta\,\Delta t_d$$

$$\Delta t_d = \frac{d - x_{max}}{v\cos\theta} = \frac{4.6\text{ m} - 1.72\text{ m}}{(6\text{ m/s})(\cos 55)} = \boxed{0.84\text{ sec}}$$

In what order do we calculate things?

- From shooting to maximum height
  1. time to max height, Δt_{max}
  2. max height, y_{max}
  3. horizontal distance at max height, x_{max}

$$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$

$$x_f = x_i + v_{ix}\Delta t$$

$$y_f = y_i + v_{iy}\Delta t - (1/2)g(\Delta t)^2$$

- From maximum height to basket
  1. time from max height to horizontal distance of basket, Δt_d
  2. height of ball at horizontal distance of basket, y_d

CLICKER 4: Did the ball go into the basket?

A. Yes

B. No

Solution: Did the ball go into the basket?

initial: @ max height

time: Δt_d

$$y_f = y_i + v_{iy}\Delta t - \frac{1}{2}g\Delta t^2$$

$$= y_{max} + \cancel{v_{max_y}\Delta t_d} - \frac{1}{2}g\Delta t_d^2$$

$$= y_{max} - \frac{1}{2}g\Delta t_d^2 =$$

$$= 3.23\,m - \frac{1}{2}(9.8\,m/\cancel{s^2})(0.84\,\cancel{sec})^2$$

$$= 3.23 - \frac{1}{2}(9.8)(0.84^2) = -0.22\,m$$

Basket is @ $b = 3m$

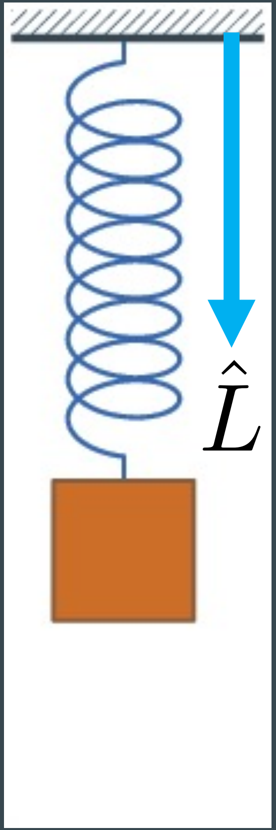
Not the same

Simulation: https://www.glowscript.org/#/user/ealicea/folder/Public/program/basketball

The Spring Force

$$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

- k is the spring stiffness (property of material; units: N/m)
- L_0 is the relaxed length of the spring (units: m)
- $\vec{L}$ is a vector that points from the fixed end of the spring to the moving end of the spring ($\hat{L}$ is its unit vector, Lhat)
- $|\vec{L}|$ (also written as L) is the stretched ($L>L_0$) or compressed ($L<L_0$) length of the spring
- The spring force is a non-constant force: it depends on the position of the object that is attached to the spring



The Spring Force

$$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

- The thing in parenthesis can be represented as “s”

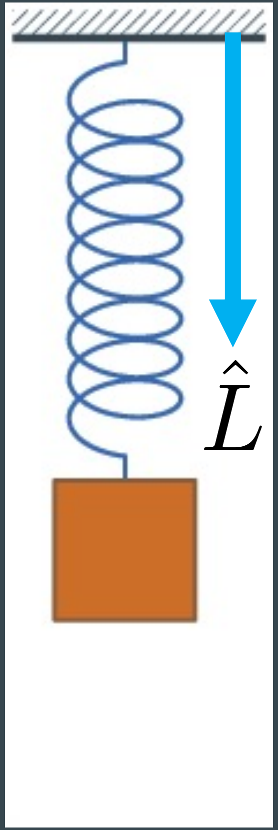
$$s = |\vec{L}| - L_0$$

(s stands for “stretch” but it can also mean compression)

- The spring force therefore can also be written as:

$$\vec{F}_s = -ks\hat{L}$$

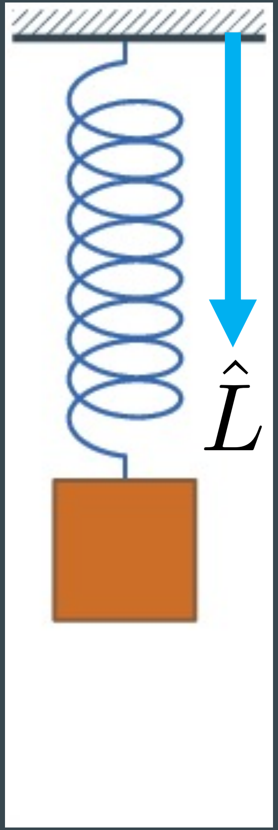
(Hooke’s Law)



The Spring Force is restorative

$$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

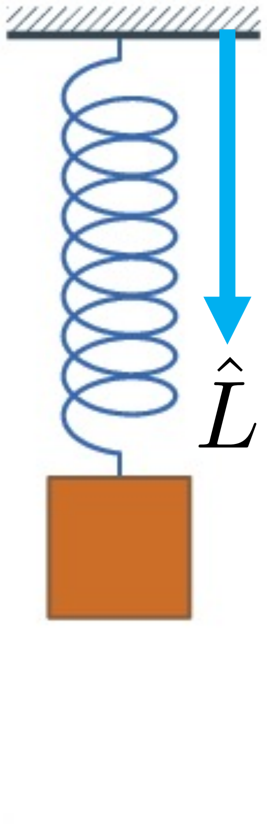
- If the spring is stretched ($L>L_0$), then the thing in parentheses is positive and the force points in the direction of negative Lhat = towards the fixed end
    - A stretched spring wants to compress (pulls)



The Spring Force is restorative

$$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

- If the spring is compressed ($L<L_0$), then the thing in parentheses is negative and the force points in the direction of positive Lhat = towards the moving end
    - A compressed spring wants to stretch (pushes)



Example: https://www.glowscript.org/#/user/ealicea/folder/Public/program/springexample

CLICKER 5: A spring stands vertically with its fixed end attached to a table as shown. What is Lhat for this spring?

A. <1, 0, 0>

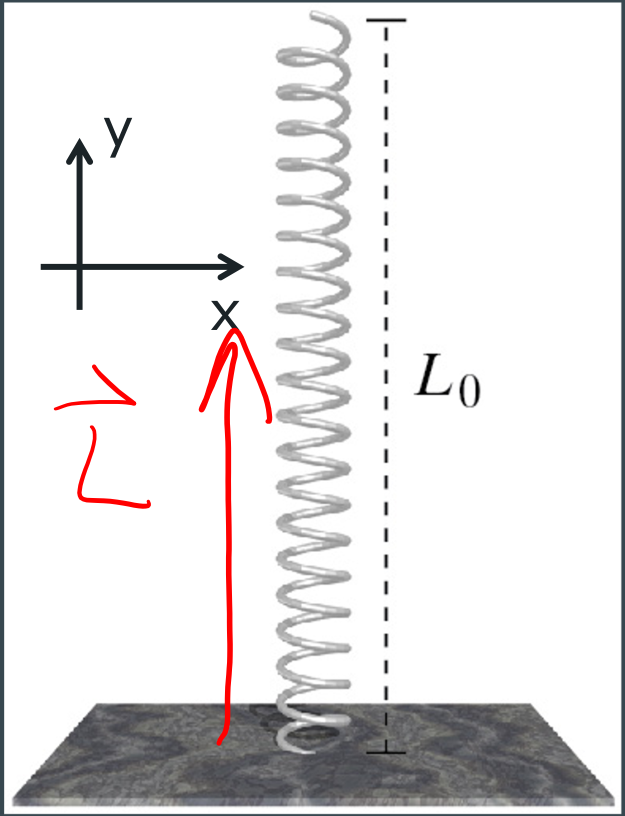
B. <-1, 0, 0>

C. <0, 1, 0>

D. <0, -1, 0>

E. <0, 0, 1>

F. <0, 0, -1>



Iteration

- This means to predict the motion of an object in several very small consecutive time steps
- When coding, this is done in the while loop
- When doing it by hand, you need to be aware of accumulating rounding errors

- Procedure:
  - Find Fnet
  - Update velocity (v_final) with Newton's 2nd Law
  - Update position with position (r_final) update formula
    - For constant force: v_avg = arithmetic average of v_initial & v_final
    - For non-constant force: v_avg = v_final
  - Go to the next time step (increase t by an amount deltat)
  - Repeat: find new Fnet, find new v_final, find new r_final, etc

What $\vec{v}_{\text{avg}}$ to use for position update?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}} \Delta t$$

- When $\vec{F}_{\text{net}}$ is constant, we can approximate $\vec{v}_{\text{avg}}$ as:

$$\vec{v}_{\text{avg}} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

This is exact for constant forces

- When $\vec{F}_{\text{net}}$ is not constant, we approximate $\vec{v}_{\text{avg}}$ as:

$$\vec{v}_{\text{avg}} \approx \vec{v}_f$$

This gives more accurate results when iterating non-constant forces

What $\vec{v}_{\mathrm{avg}}$ to use for position update?

- Example: horizontal springs (spring force = not constant)
  https://www.glowscript.org/#/user/ealicea/folder/Public/program/comparison1

- Example: an orbit (gravitational force = not constant)
  https://www.glowscript.org/#/user/ealicea/folder/Public/program/comparison2

- Example: projectile motion (gravity near Earth = constant)
  https://www.glowscript.org/#/user/ealicea/folder/Public/program/comparison3