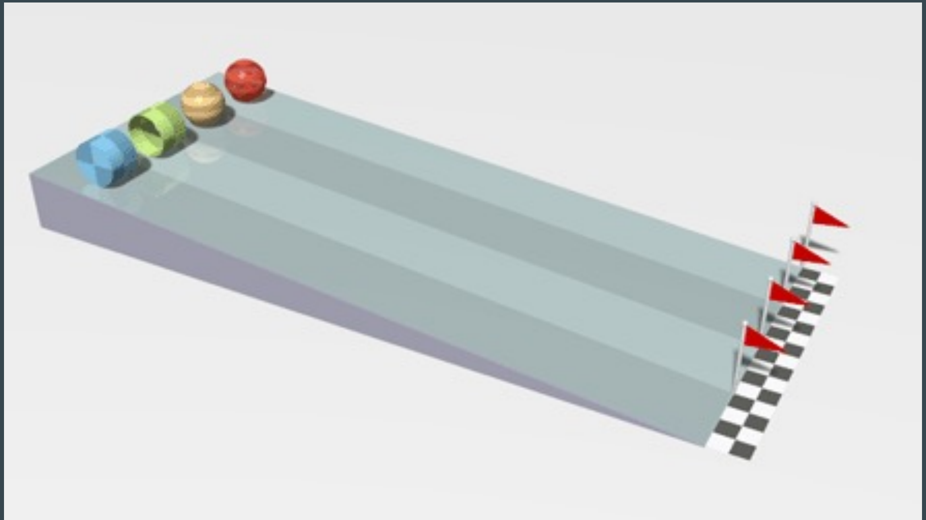

PHYS 2211 K

Week 10, Lecture 2

2022/03/17

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6 clicker questions today

On today's class…

1. Spinny stuff: angular speed, moment of inertia, rotational kinetic energy
2. "Point particle" (center-of-mass) vs "Real system" (extended, multiparticle)

Road map for the rest of the semester

- Week 12
  - Lecture topics: Collisions, scattering
  - Lab 5 submission at the end of the week (April 3)
- Week 13
  - Test 3 on April 4 (coverage: weeks 9, 10, 12)
  - Lectures topics: Cross product, Torque, Angular momentum
- Week 14
  - Lecture topics: Angular momentum principle, multiparticle angular momentum, angular momentum of rigid systems
- Week 15
  - Lecture topics: Wrapping up angular momentum; Quantum stuff
  - Hard deadline for everything (edx, etc) on April 24
- Week 16 – Final exam on Friday April 29

CLICKER 1: Free response!
What are your spring break plans?

From Tuesday

- Center of mass $\vec{r}_{\rm cm} = \dfrac{\sum m_i \vec{r}_i}{M_{\rm total}}$
- Translational kinetic energy $K_{\rm trans} = \dfrac{1}{2} M_{\rm total} v_{\rm cm}^2$
- Rotational kinetic energy $K_{\rm rot} = \dfrac{1}{2} I \omega^2$
- Parallel axis theorem $I_{\rm pa} = I_{\rm cm} + Md^2$

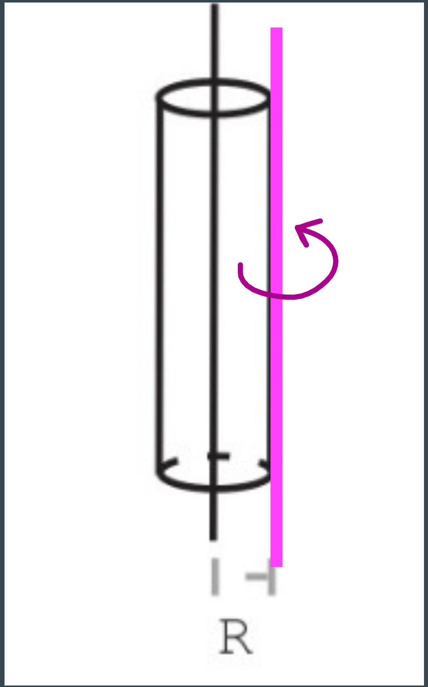
CLICKER 2: What is the moment of inertia of a solid cylinder of radius R spinning about the axis shown? Note that $I_{cm} = (1/2)MR^2$

A. $I_{pa} = (1/2)\ MR^2$

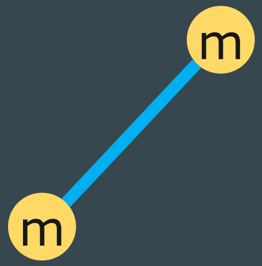
B. $I_{pa} = MR^2$

C. $I_{pa} = (3/2)\ MR^2$

D. $I_{pa} = 2\ MR^2$



Example: Find the total moment of inertia about an axis passing through the CM and into the page for a dumbbell that consists of two point masses m at the ends of a massless rod of length L.



CLICKER 3: Find the total moment of inertia about an axis passing through the CM and into the page for a dumbbell that consists of two solid spheres, each of mass m and radius R, whose centers are attached to the ends of a thin rod of length L and mass M. Don't simplify the final answer.

A. $I = (1/12)ML^2 + (2/5)mR^2$

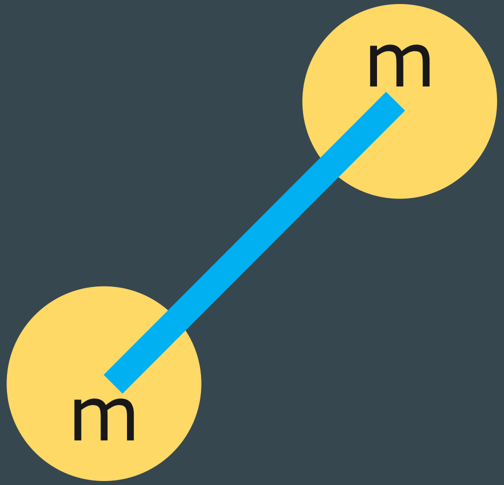
B. $I = (1/12)ML^2 + (4/5)mR^2$

C. $I = (1/12)ML^2 + (2/5)mR^2 + (1/2)mL^2$

D. $I = (1/12)ML^2 + (2/5)mR^2 + (1/4)mL^2$

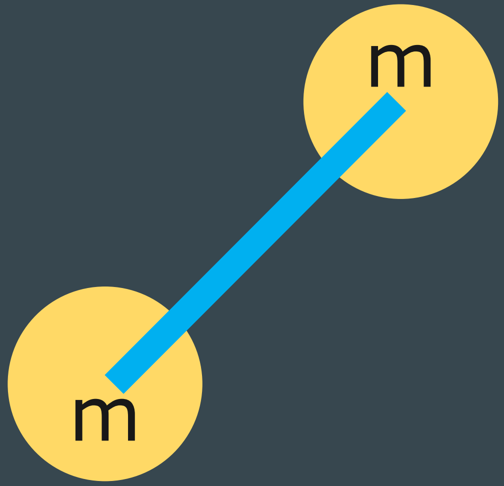
E. $I = (1/12)ML^2 + (4/5)mR^2 + (1/2)mL^2$

F. $I = (1/12)ML^2 + (4/5)mR^2 + (1/4)mL^2$



(Note: at each CM, $I_{sphere} = (2/5)mR^2$ and $I_{rod} = (1/12)ML^2$)

Solution: Find the total moment of inertia about an axis passing through the CM and into the page for a dumbbell that consists of two solid spheres, each of mass m and radius R, whose centers are attached to the ends of a thin rod of length L and mass M. Don't simplify the final answer.



Downhill racing

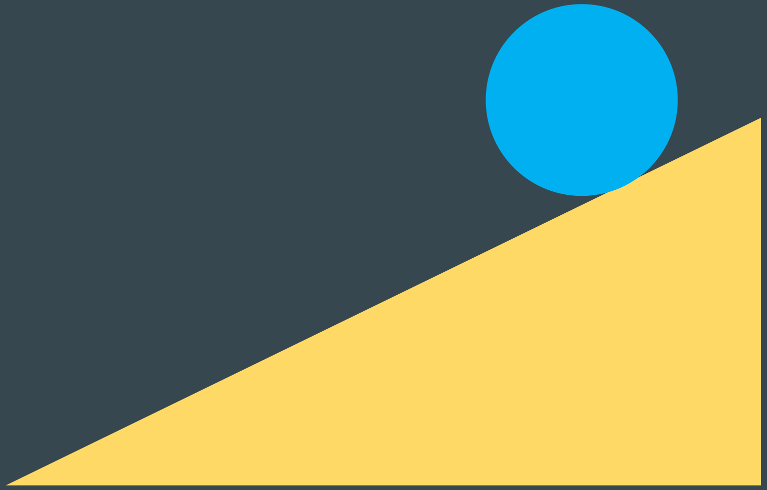
- If you have two spheres (or disks, or cylinders), one solid and one hollow, how can you tell them apart if they both have the same mass M and the same radius R? Roll them down a ramp!
  - Moment of inertia is a “rotational mass”: higher I means it’s more difficult to make things turn

- System: the thing that’s rolling down + Earth

- Surroundings: ramp where the things roll without slipping
  - “rolling without slipping” means static friction acting only at the point of contact

$$\Delta K_{\text{trans}} + \Delta K_{\text{rot}} + \Delta U_g = 0$$

Initial state: $v_i=0$, $y_i=h$, $\omega_i=0$
Final state: $v_f=?$, $y_f=0$, $\omega_f=v_f/R$



Downhill racing

- The object that moves fastest wins the race

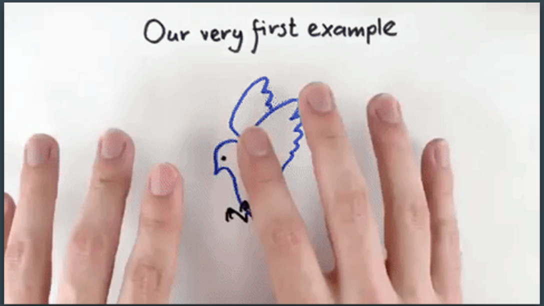
$$v_f = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

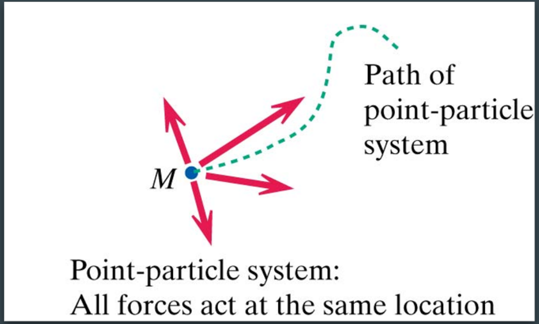
- Higher I means lower v_f, which means objects that have lower I move faster down the ramp

Point Particle vs Real System

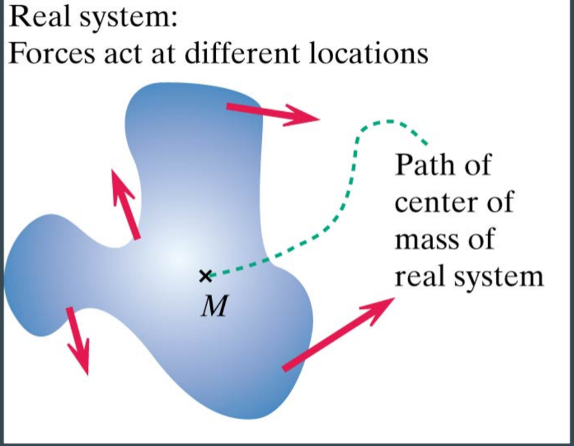
- Imagine an entire object/system crushed down to a point at the center of mass
- We call this the "point particle system" (also known as "center of mass system")
  - The system now can only have mass and speed/velocity, because it's a point (a point has no structure or dimensions)
  - All external forces act on the center of mass
  - Work is calculated with the net external force acting across the displacement of the CM
  - Energy principle looks like this: $\Delta E_{sys} = W_{cm}$

$$\Delta K_{\text{trans}} = \vec{F}_{\text{net}} \cdot \Delta \vec{r}_{cm}$$





Point Particle vs Real System



- If the object/system is not a point, then you need to consider the shape of the system AND its motion relative to the CM

- We call this the "real system" (also known as "extended system" or "multiparticle system")
  - System is not a point = there are more energies
  - Works need to be calculated for each separate external force acting across their own separate displacements
  - Energy principle looks like this: $\Delta E_{sys} = Q + W_{real}$

$$\begin{aligned}\Delta K_{\text{trans}} + \Delta K_{\text{rot}} + \Delta K_{\text{vib}} + \Delta U_g + \Delta U_e \\ + \Delta U_s + \Delta E_{\text{th}} + \Delta E_{\text{chem}} + \Delta E_{\text{int}} + \ldots\end{aligned} = \begin{aligned}Q + (\vec{F}_1 \cdot \Delta\vec{r}_1) + (\vec{F}_2 \cdot \Delta\vec{r}_2) \\ + (\vec{F}_3 \cdot \Delta\vec{r}_3) + \ldots\end{aligned}$$

General Strategy for PP vs Real

- Draw a diagram that identifies initial and final states, and locate the CM

- Analyze the point particle system
  - Find the net force, $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$
  - Find the displacement of the CM, $\Delta\vec{r}_{cm} = \vec{r}_f - \vec{r}_i$
  - Determine W_{cm} (scalar!) and ΔK_{trans} (scalar!)

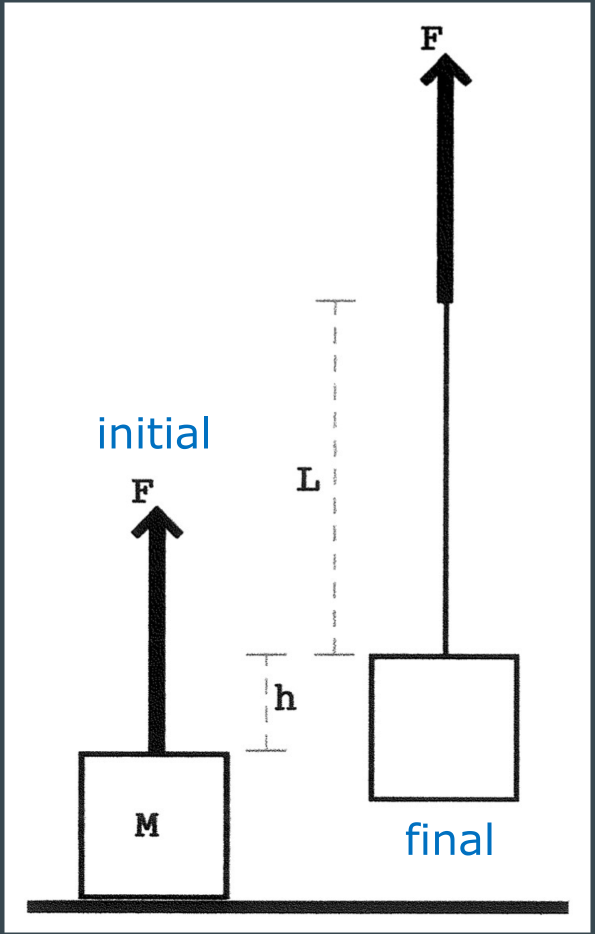
- Analyze the extended system
  - Find the displacement associated with each external force
  - Calculate separately the work done by each force
  - Determine W_{real} by adding up all the works done by all the forces
  - Apply the energy principle to find the unknowns
    - Remember that you already calculated ΔK_{trans} as being equal to the work done by the net force on the center of mass

Example: A box contains machinery that can rotate. The total mass of the box and machinery is M. A string wound around the machinery comes out through a small hole at the top of the box.

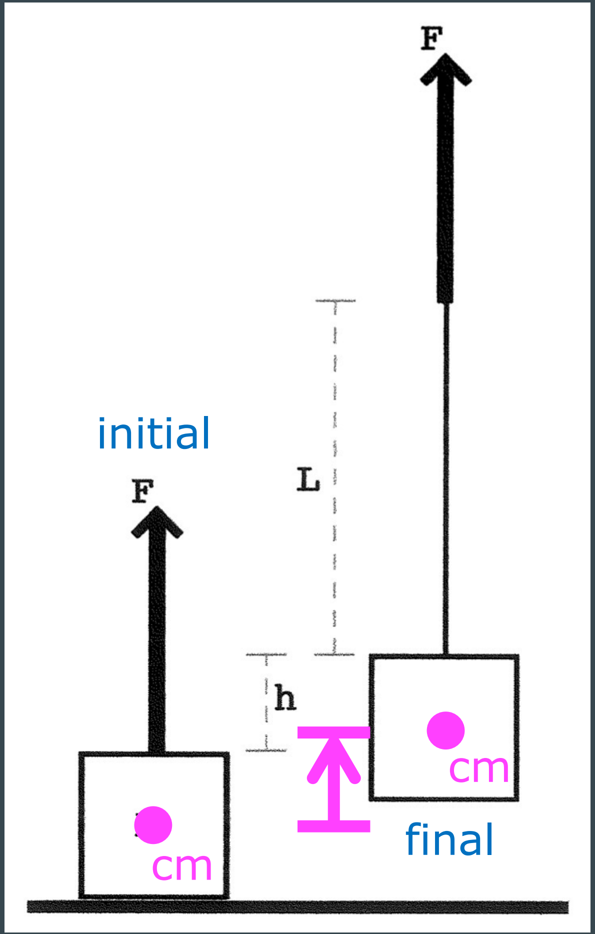
Initially the box sits on the ground and the machinery is not rotating. Then you pull upwards on the string with a constant force of magnitude F. At an instant where you have pulled a length of string L, the box has risen to a height h.

(a) Point particle system: What is the speed of the box when it is at a height h above the ground?

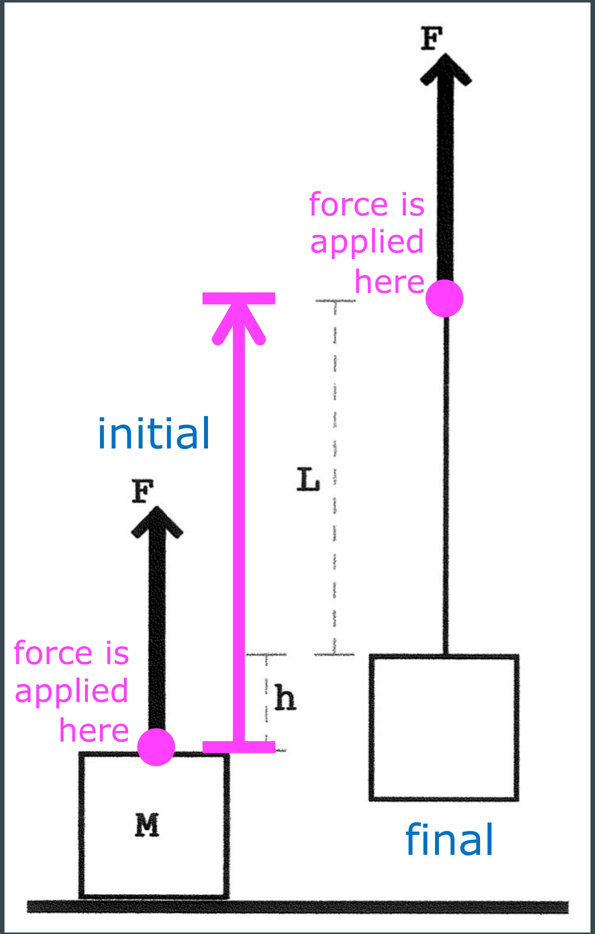
(b) Real system: What is the rotational kinetic energy of the machinery inside the box at this moment?



(a) Point particle system: What is the speed of the box when it is at a height h above the ground?



(b) Real system: What is the rotational kinetic energy of the machinery inside the box at this moment?



CLICKER 4: A chain of **mass M** is coiled up into a tight ball (**at rest**) on a **frictionless** table. You then pull on a link at the end of the chain with a constant force **F**. Eventually the chain straightens out to its full **length L**, but you keep pulling until you have pulled your end of the chain a total **distance d**. What is the work done on the **point particle system**?

A. F(d - L)

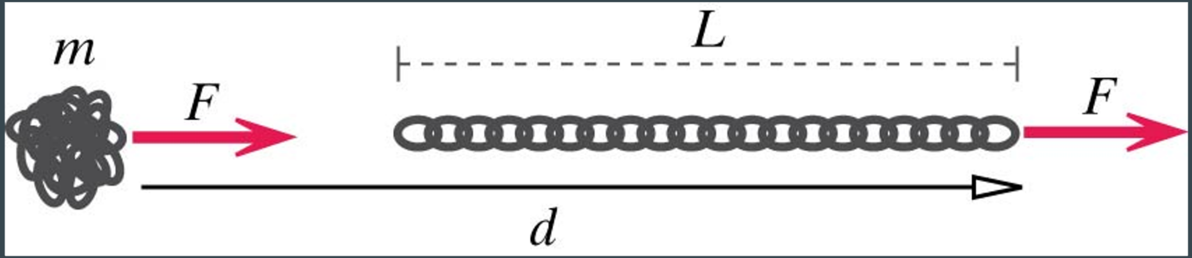
B. Fd

C. F(d - L/2)

D. FL/2

E. FL

F. 0



CLICKER 5: A chain of mass M is coiled up into a tight ball (at rest) on a frictionless table. You then pull on a link at the end of the chain with a constant force F. Eventually the chain straightens out to its full length L, but you keep pulling until you have pulled your end of the chain a total distance d. What is the work done on the extended system?

A. F(d - L)

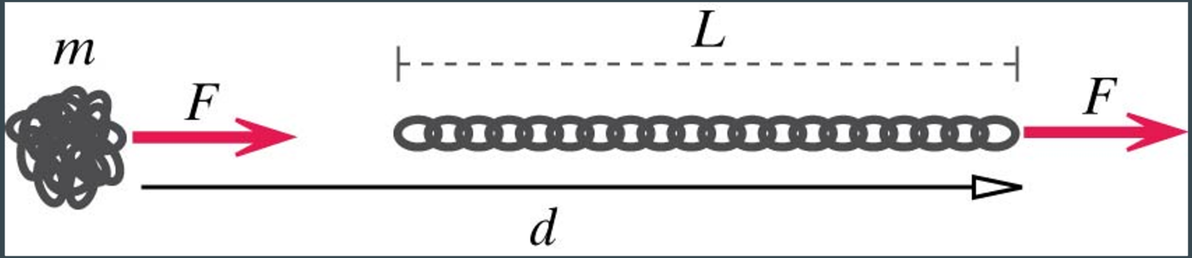
B. Fd

C. F(d - L/2)

D. FL/2

E. FL

F. 0



CLICKER 6: A string is wrapped around a disk of mass M and radius R. Starting from rest, you pull the string with a constant force F along a frictionless surface. At the instant when the CM has moved a distance d, a length L of the string has unwound off the disk. At the final state, what is the rotational kinetic energy of the disk?

A. Fd

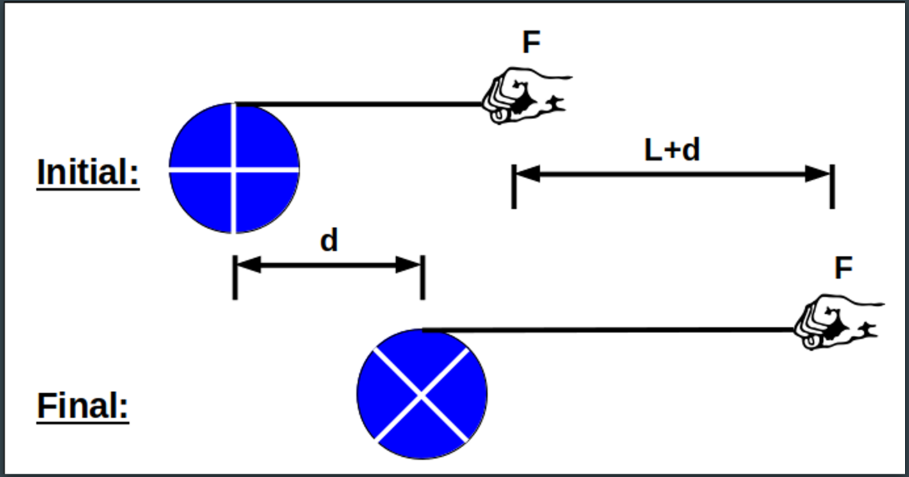
B. FL

C. F(L + d)

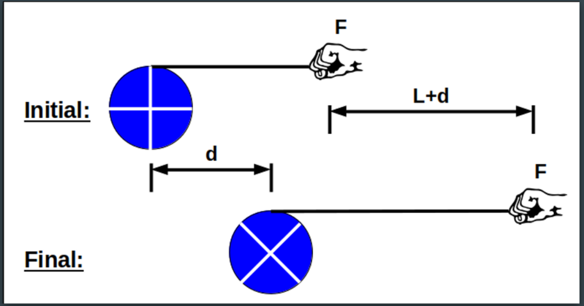
D. 0

E. F(L - d)

F. FLd



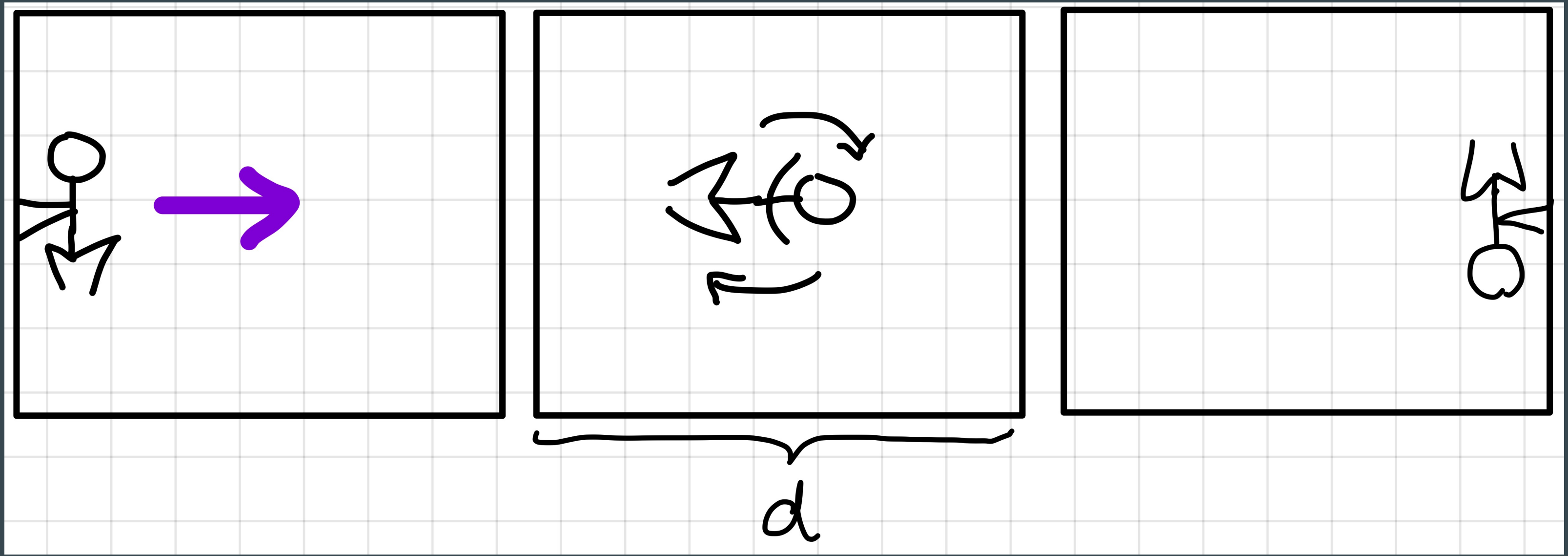
Solution: A string is wrapped around a disk of mass M and radius R. Starting from rest, you pull the string with a constant force F along a frictionless surface. At the instant when the CM has moved a distance d, a length L of the string has unwound off the disk. At the final state, what is the rotational kinetic energy of the disk?



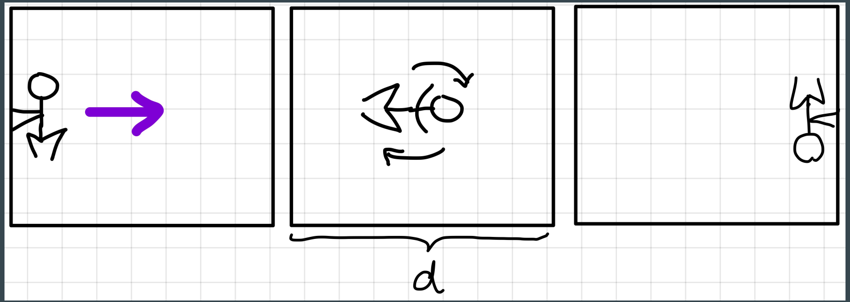
CM system to find ΔK_{trans}

Extended system to find ΔK_{rot}

Example: An astronaut (**mass m**, **moment of inertia I**) in the space station is bored. To keep herself entertained, she grabs hold of one wall, getting as **close to it** as she can. Then she extends her arms (**length L**) to push away from the wall with a **constant force F**. As she moves away she spins clockwise, so she's upside down when she's just about to hit the opposite wall a distance **d** away. What was the **change in the astronaut's internal energy**, from pushing off the left wall until just before she hits the right wall?



Solution: An astronaut (mass m, moment of inertia I) in the space station is bored. To keep herself entertained, she grabs hold of one wall, getting as close to it as she can. Then she extends her arms (length L) to push away from the wall with a constant force F. As she moves away she spins clockwise, so she's upside down when she's just about to hit the opposite wall a distance d away. What was the change in the astronaut's internal energy, from pushing off the left wall until just before she hits the right wall?



CM system to find her final speed

Reminder: full spin is $\omega = 2\pi/T$, so half spin is π/T

Real system to find ΔE_{int}