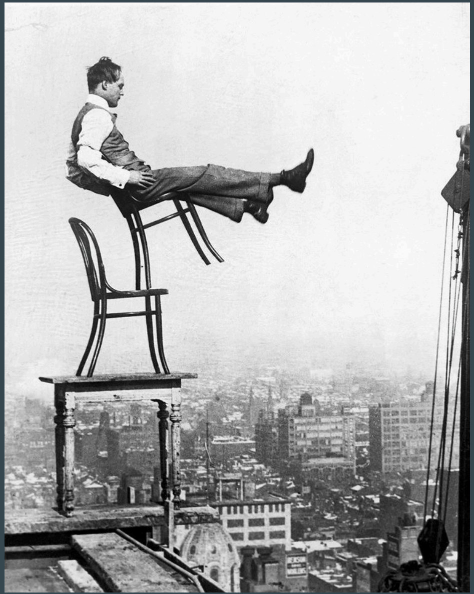


PHYS 2211 K

Week 5, Lecture 2

2022/02/10

Dr Alicea (ealicea@gatech.edu)

~~6~~ 4 clicker questions today

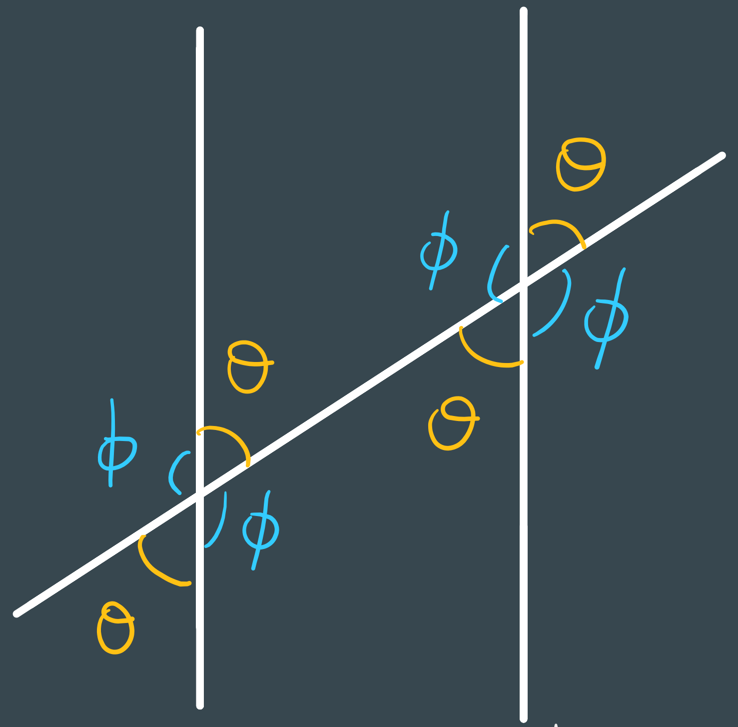
On today's class...

1. Static and Dynamic Equilibrium
2. Lots and lots of problems with static and dynamic equilibrium

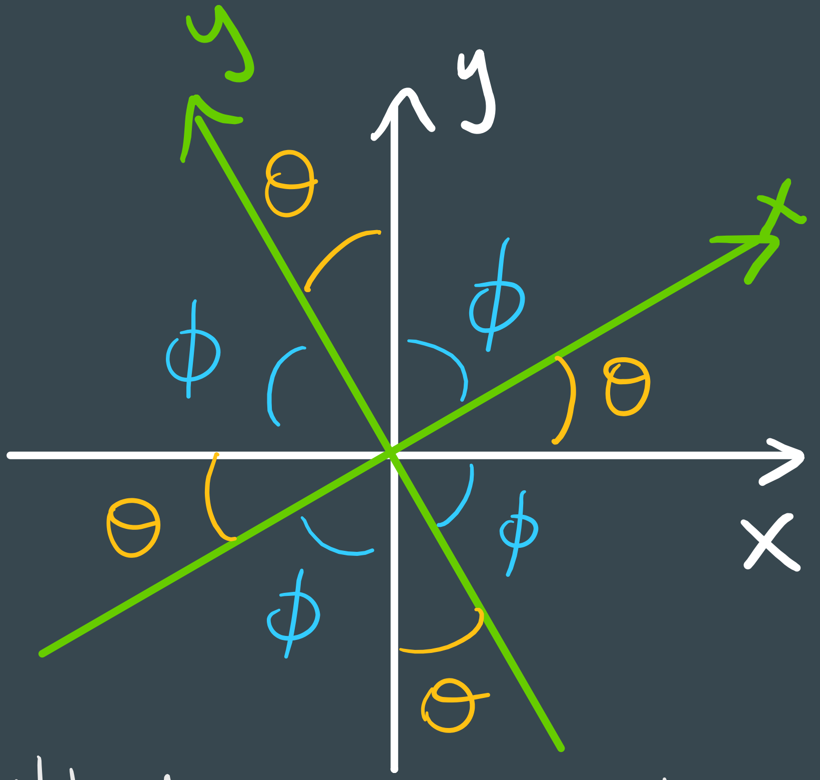
From Tuesday

- Contact forces need to be in contact with the system
- Tension force pulls the system along a rope/string
- Surface contact forces have a component perpendicular to the surface (called normal) and a component parallel to the surface (called friction)
- Magnitude of friction is proportional to magnitude of normal $$|\vec{f}| = \mu|\vec{N}|$$
- Free body diagrams show the forces acting on a system

Geometry Refresher...



alternate angles



tilted coordinates

CLICKER 1: How do you pronounce "gif"?

A. With a **hard** "g" like in "get"

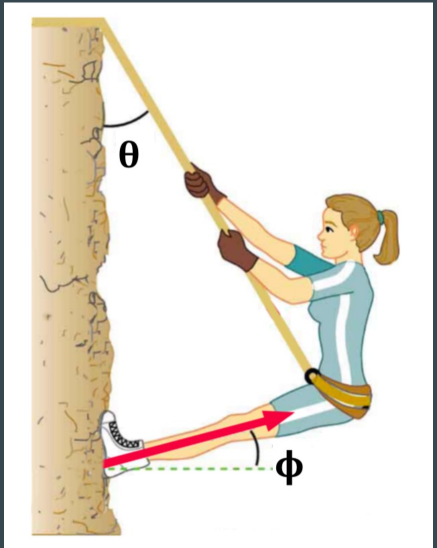
B. With a **soft** "g' like in "giraffe"

The results were 50/50 ?!?!

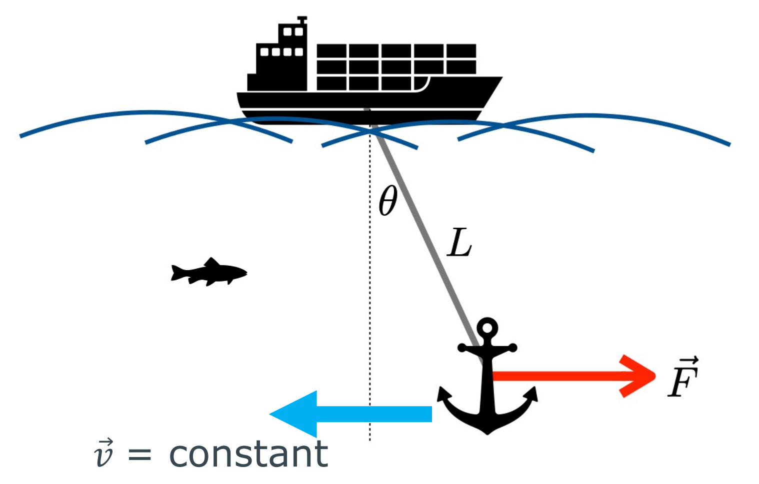
Static & Dynamic Equilibrium

A system is in static equilibrium when Fnet = 0 and v = 0

Engineering students may take a class that's all about this, called "Statics"



A system is in dynamic equilibrium when Fnet = 0 but v = constant ≠ 0



If Fnet ≠ 0, then the system is **NOT** in equilibrium

Equilibrium problems

- When $\vec{F}_{\text{net}} = 0$ this means all the forces acting on the system are balanced

- All the x components of all the forces add up to zero
- All the y components of all the forces add up to zero
- All the z components of all the forces add up to zero

$$\vec{F}_{\text{net,x}} = 0 \qquad \vec{F}_{\text{net,y}} = 0 \qquad \vec{F}_{\text{net,z}} = 0$$

Equilibrium problems

- Apply $\vec{F}_{\text{net}} = \sum_{i=1}^{n} \vec{F}_i = 0$ separately for x, y, z

  sum of all forces along a particular direction (e.g., "x")

- Will need FBD and trigonometry for finding x, y, z components

- Solve for unknowns (sometimes will end up with two equations and two unknowns)

Non-Equilibrium problems

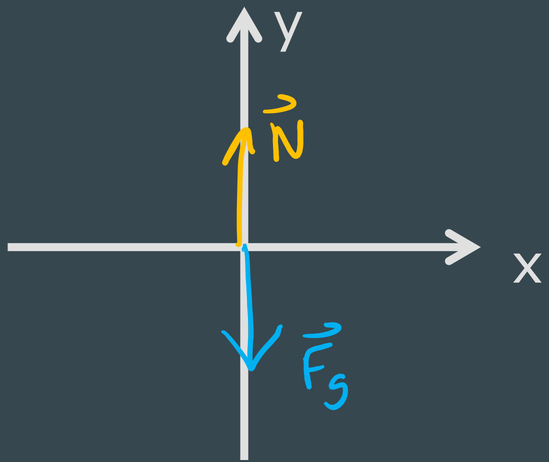
- When $\vec{F}_{\text{net}} \neq 0$ this means the forces acting on the system are unbalanced, and therefore the system is accelerating

- In this case,

$$\vec{F}_{\text{net,x}} = m\vec{a}_x \quad \vec{F}_{\text{net,y}} = m\vec{a}_y \quad \vec{F}_{\text{net,z}} = m\vec{a}_z$$

- But we'll worry more about that next week ☺

Example: A block of mass m sits motionless on a table. Find the magnitude of the normal force exerted by the table on the block.



$$F_{net_y} = 0$$

$$\vec{N} + \vec{F}_g = 0$$

$$N(\hat{y}) + mg(-\hat{y}) = 0$$

$$(N - mg)\hat{y} = 0$$

$$N - mg = 0$$

$$\boxed{N = mg}$$

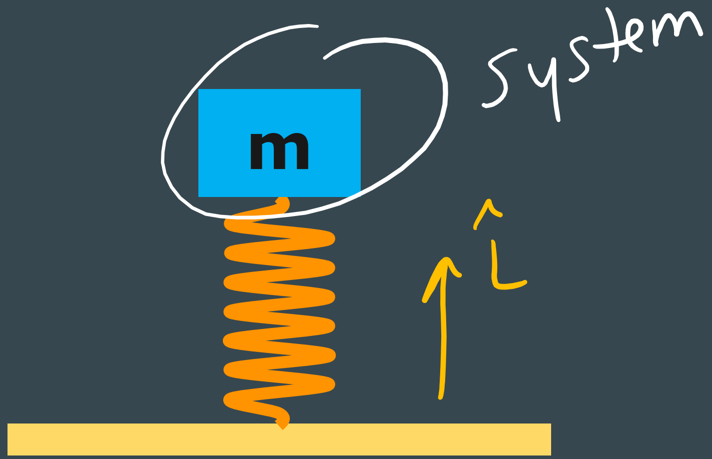
CLICKER 2: A block of mass m = 3 kg sits motionless on a spring that is compressed by an amount s = 12 cm. What is the stiffness of the spring, k?

~~A.~~ k = -2.45 N/m

B. k = 245 N/m

~~C.~~ k = -245 N/m

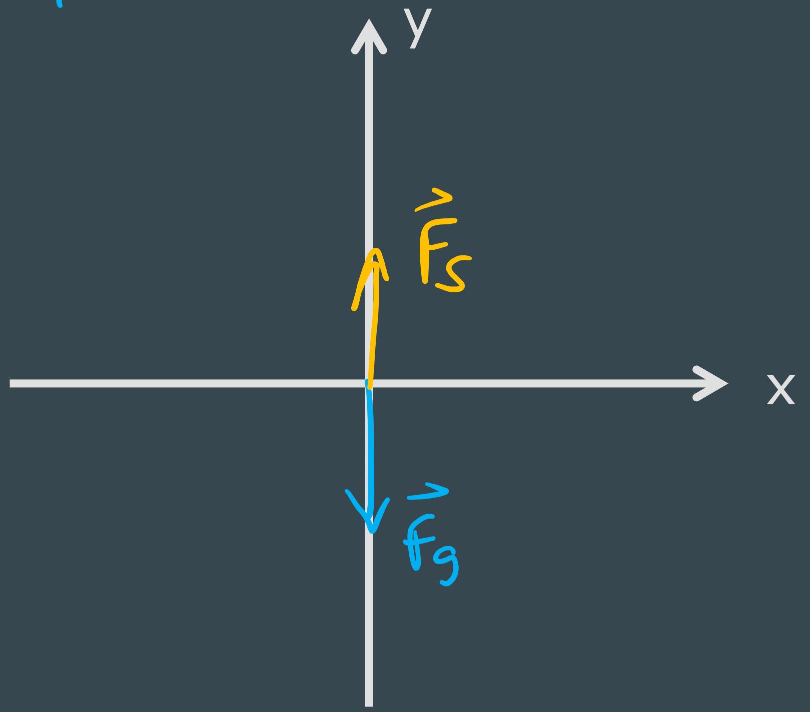
D. k = +2.45 N/m



Solution: A block of mass m = 3 kg sits motionless on a spring that is compressed by an amount s = 12 cm. What is the stiffness of the spring, k?

$L - L_0 = \text{“}s\text{”}$

System: block



$$F_{net_y} = 0$$

$$\vec{F}_s + \vec{F}_g = 0$$

$$-k(L-L_0)\hat{\imath} + mg(-\hat{y}) = 0$$

$$-k(L-L_0)\hat{y} + mg(-\hat{y}) = 0$$

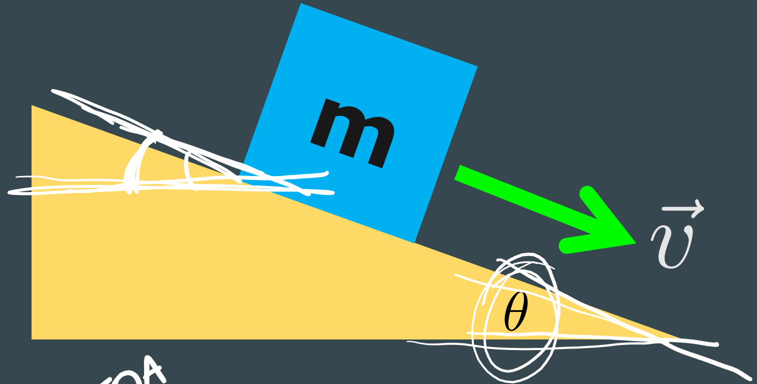
$$-k(-s)\hat{y} + mg(-\hat{y}) = 0$$

$$(ks - mg)\hat{y} = 0$$

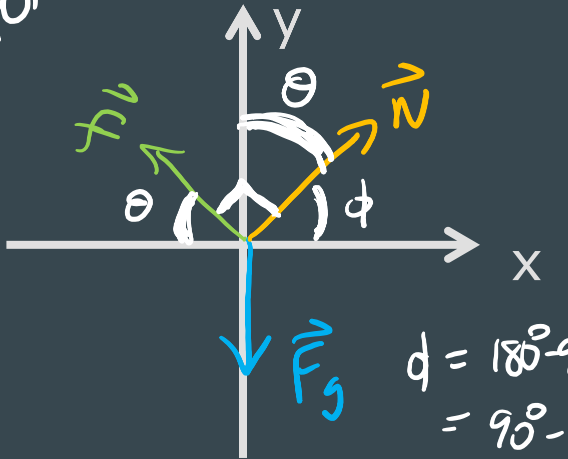
$$ks = mg$$

$$k = \frac{mg}{s} = \frac{(3)(9.8)}{0.12} = \boxed{245\ N/m}$$

Example: A block of mass m slides down a ramp inclined at an angle theta at constant velocity. What is the magnitude of the friction force acting on the block?



SOHCAHTOA



$\phi = 180° - 90° - \theta$
$= 90° - \theta$

Method 1: xy coordinates as usual

$F_{net_x} = 0$

$-f\cos\theta + N\sin\theta = 0$

$f\cos\theta = N\sin\theta$

$N = f\dfrac{\cos\theta}{\sin\theta}$

$F_{net_y} = 0$

$f\sin\theta + N\cos\theta - mg = 0$

$f\sin\theta + N\cos\theta = mg$

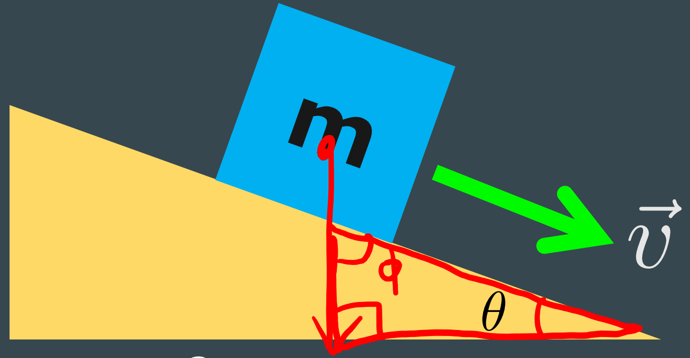
$f\sin\theta + \left(f\dfrac{\cos\theta}{\sin\theta}\right)\cos\theta = mg$

$f\left(\sin\theta + \dfrac{\cos^2\theta}{\sin\theta}\right) = mg$

$f\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}\right) = mg$

$\dfrac{f}{\sin\theta} = mg \Rightarrow \boxed{f = mg\sin\theta}$

Example: A block of mass m slides down a ramp inclined at an angle theta at constant velocity. What is the magnitude of the friction force acting on the block?



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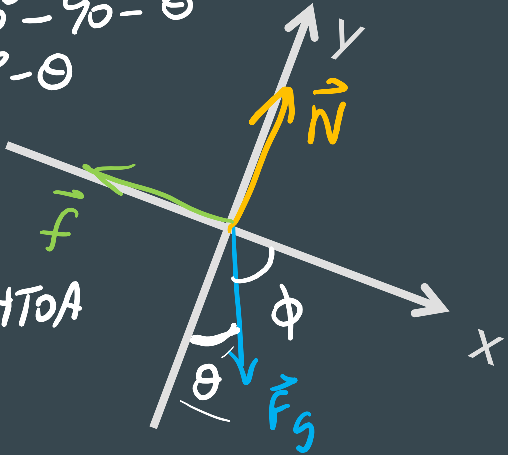


SOHCAHTOA

Method 2: Tilted xy coordinates

$F_{net_x} = 0$

$F_{net_y} = 0$

$-f + mg\sin\theta = 0$

$$\boxed{f = mg\sin\theta}$$

CLICKER 3: A block of mass m = 3 kg sits motionless (but just on the verge of sliding) on a ramp that is inclined by an angle θ = 35°. What is the coefficient of friction between the ramp and the block?

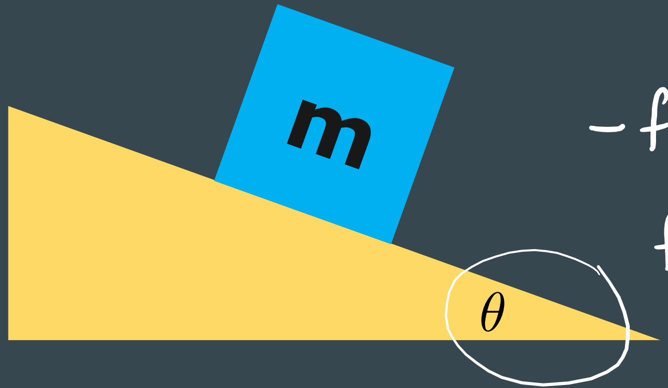
A. $\mu_s = 0.70$

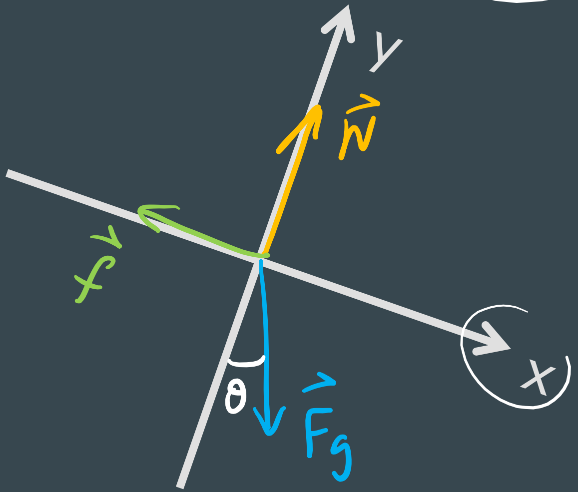
B. $\mu_s = 1.43$

C. $\mu_s = 0.57$

D. $\mu_s = 0.82$

Solution: A block of mass m = 3 kg sits motionless (but just on the verge of sliding) on a ramp that is inclined by an angle θ = 35°. What is the coefficient of friction between the ramp and the block?





$$F_{net_x} = 0$$

$$-f + mg\sin\theta = 0$$

$$f = mg\sin\theta$$

$$F_{net_y} = 0$$

$$N - mg\cos\theta = 0$$

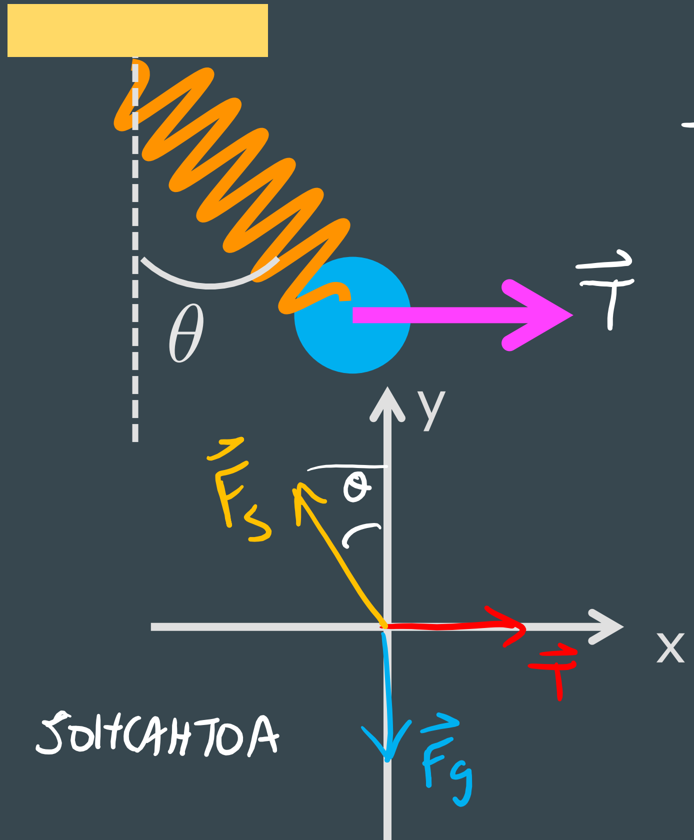
$$N = mg\cos\theta$$

$$f = \mu_s N$$

$$\mu_s = \frac{f}{N} = \frac{\cancel{mg}\sin\theta}{\cancel{mg}\cos\theta} = \tan\theta =$$

$$= \tan 35° = \boxed{0.70}$$

Example: A ball of mass m is attached to stretched spring. You pull the ball with a rope exerting on it a force of unknown magnitude. This causes the ball to hang motionless at an angle theta from the vertical. What is the magnitude of the unknown force?



$$F_{net\,x} = 0$$

$$-F_s \sin\theta + T = 0$$

$$F_s \sin\theta = T$$

$$F_{net\,y} = 0$$

$$F_s \cos\theta - mg = 0$$

$$F_s \cos\theta = mg$$

$$F_s = \frac{mg}{\cos\theta}$$

$$T = \left(\frac{mg}{\cos\theta}\right)\sin\theta = \boxed{mg\tan\theta}$$

CLICKER 4: A helicopter (m = 5500 kg) moves to the left at constant velocity. The lift force generated by the blades makes an angle of 30 degrees with respect to the vertical. What is the magnitude of the air resistance that opposes the helicopter's motion?

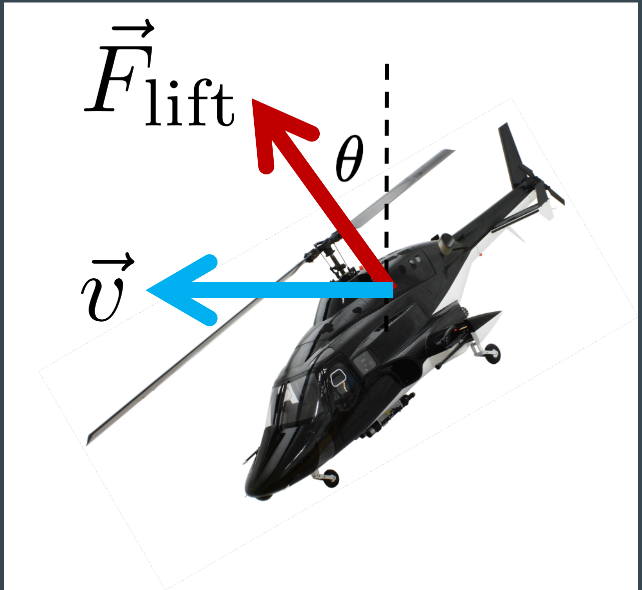
A. F_{air} = 3175 N

B. F_{air} = 53900 N

C. F_{air} = 93358 N

D. F_{air} = 31119 N

E. F_{air} = 46679 N



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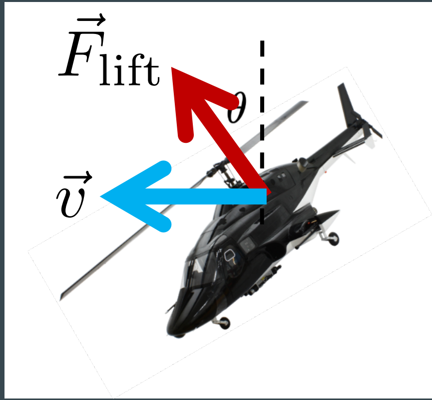


$$F_{net\,x} = 0$$

$$-F_\ell \sin\theta + F_{air} = 0$$

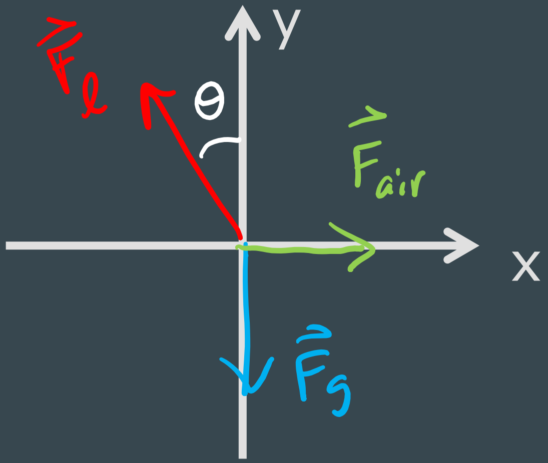
$$F_{air} = F_\ell \sin\theta$$

$$F_{net\,y} = 0$$

$$F_\ell \cos\theta - mg = 0$$

$$F_\ell \cos\theta = mg$$

$$F_\ell = \frac{mg}{\cos\theta}$$

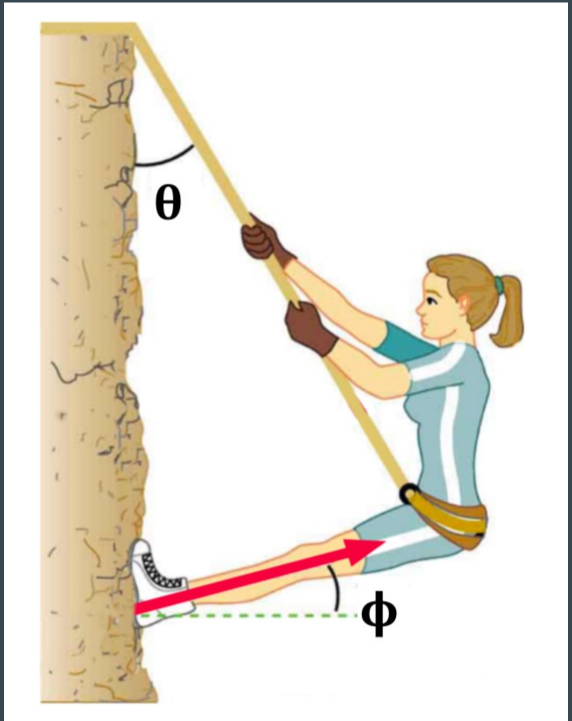


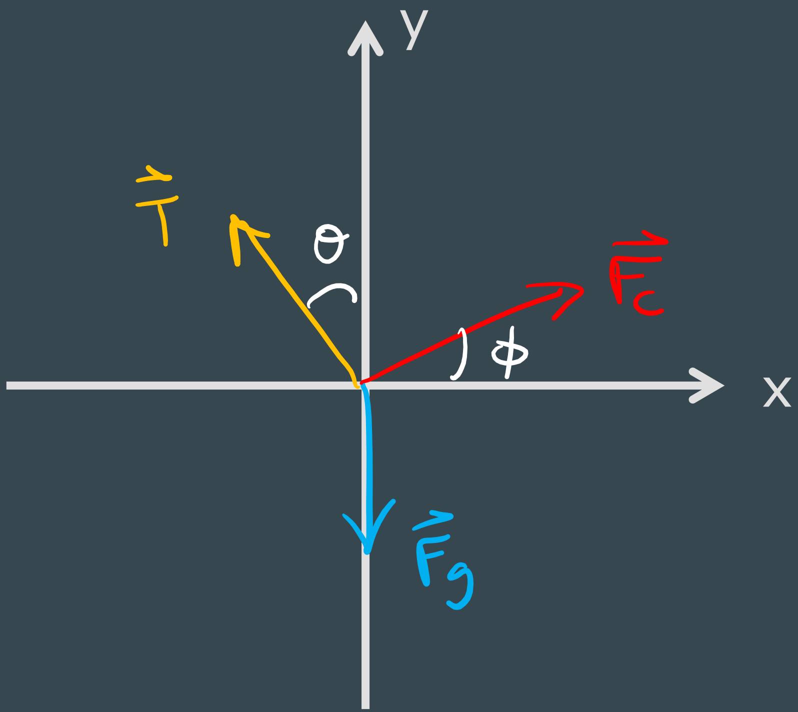
$$F_{air} = \frac{mg}{\cos\theta}\sin\theta =$$

$$= mg\tan\theta =$$

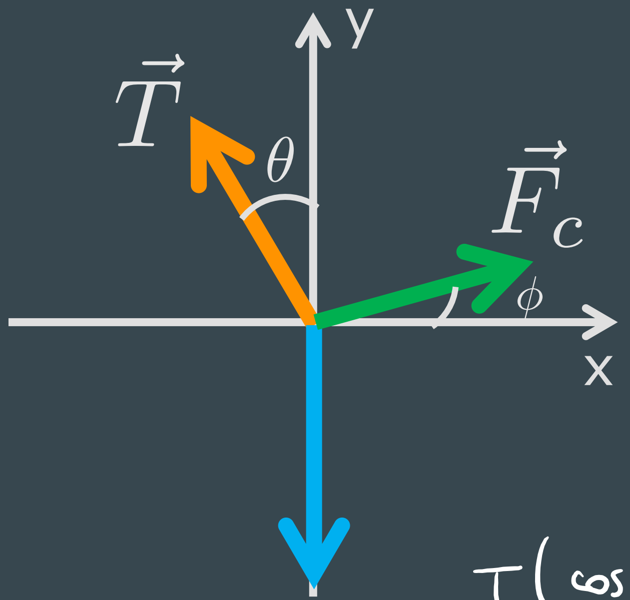
$$= (5500)(9.8)\tan 30^\circ = \boxed{31119\ \text{N}}$$

Example: A rock climber (**m = 60 kg**) is leaning back from a rock wall, **motionless**, as shown in the diagram. The rope has some **unknown tension T** and makes an angle θ = 31° with the vertical. In this position, the wall exerts a contact force on the climber of unknown magnitude at an angle ϕ = 15° above the horizontal. **What is the magnitude of the tension force?**





Solution: A rock climber (m = 60 kg) is leaning back from a rock wall, motionless, as shown in the diagram. The rope has some unknown tension T and makes an angle θ = 31° with the vertical. In this position, the wall exerts a contact force on the climber of unknown magnitude at an angle ϕ = 15° above the horizontal. What is the magnitude of the tension force?



$$F_{net\,x} = 0$$

$$-T\sin\theta + F_c\cos\phi = 0$$

$$T\sin\theta = F_c\cos\phi$$

$$F_c = T\frac{\sin\theta}{\cos\phi}$$

$$F_{net\,y} = 0$$

$$T\cos\theta + F_c\sin\phi - mg = 0$$

$$T\cos\theta + F_c\sin\phi = mg$$

$$T\cos\theta + T\frac{\sin\theta}{\cos\phi}\sin\phi = mg$$

$$T\left(\cos\theta + \frac{\sin\theta\sin\phi}{\cos\phi}\right) = mg$$

$$T(\cos\theta + \sin\theta\tan\phi) = mg$$

$$T = \frac{mg}{\cos\theta + \sin\theta\tan\phi} = \frac{(60)(9.8)}{\cos 31 + \sin 31\tan 15} = \boxed{591\text{ N}}$$

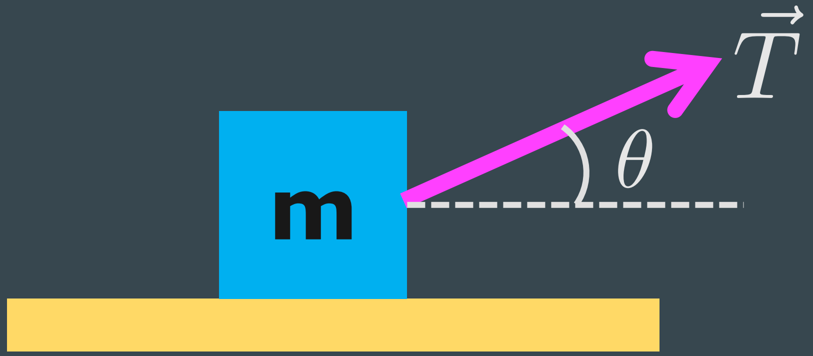
CLICKER 5: A block of mass m is dragged along a table by a string so it moves to the right at constant speed v. The string makes an angle θ with the table, and the coefficient of kinetic friction between table and block is μ. What is the magnitude of the tension force?

A. $T = \dfrac{mg}{\mu\cos\theta + \sin\theta}$

B. $T = \dfrac{\mu N}{\cos\theta}$

C. $T = N - mg\sin\theta$

D. $T = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$

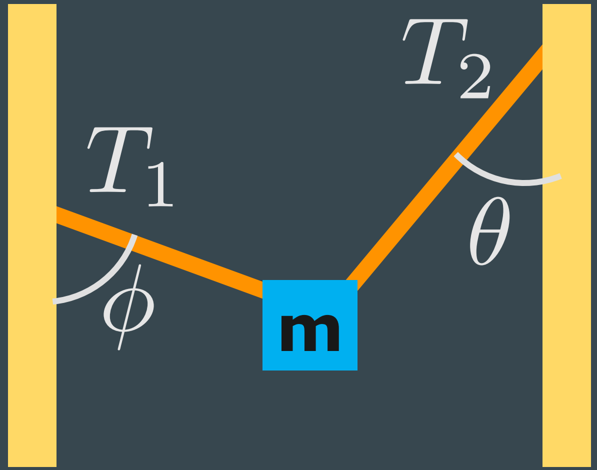


For Tuesday

CLICKER 6: A block of mass m hangs motionless tied to two ropes which make different angles on two parallel walls. What is the magnitude of Tension 2?

A. $$T_2 = \frac{mg\sin\phi}{\sin\theta\cos\phi + \cos\theta\sin\phi}$$

B. $$T_2 = \frac{mg\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}$$



For Tuesday