Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_ss \qquad U_{spring}=\frac{1}{2}k_ss^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots}{m_1+m_2+\ldots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\ldots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

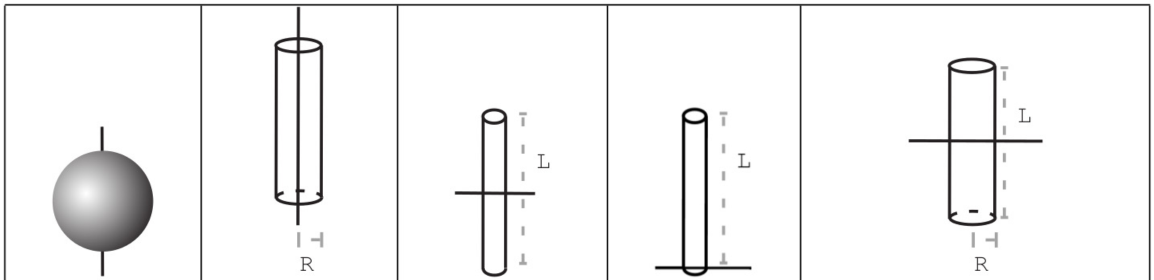
$$\text{prob}(E)\propto\Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\ldots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\ldots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$



$I = \dfrac{2}{5}MR^2$	$I = \dfrac{1}{2}MR^2$	$I = \dfrac{1}{12}ML^2$	$I = \dfrac{1}{3}ML^2$	$I = \dfrac{1}{12}ML^2 + \dfrac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\dfrac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \dfrac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 KMR - Test 2 - Spring 2022

Please clearly print your name & GTID in the lines below

Name: ______________________ GTID: ______________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

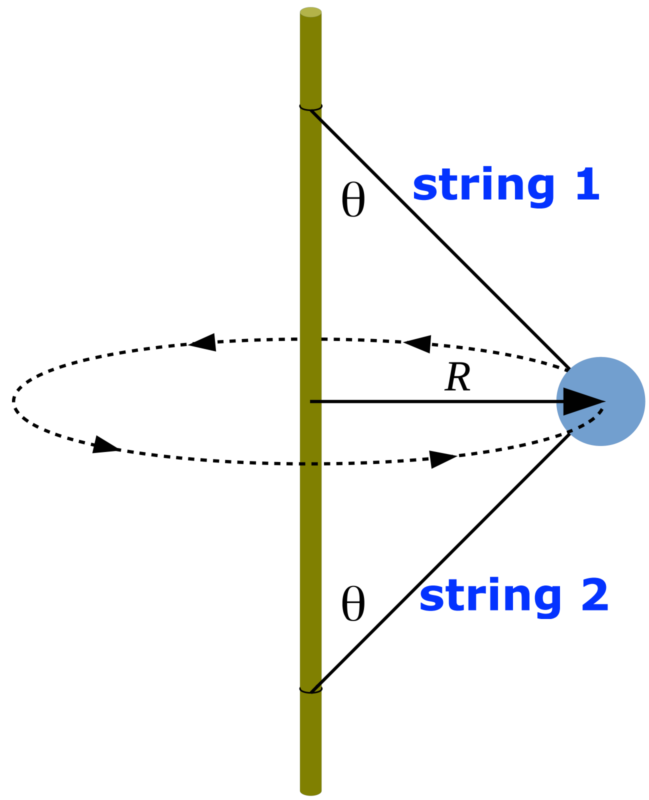
"In accordance with the Georgia Tech Honor Code, I have not given or received unauthorized aid on this test."

KEY

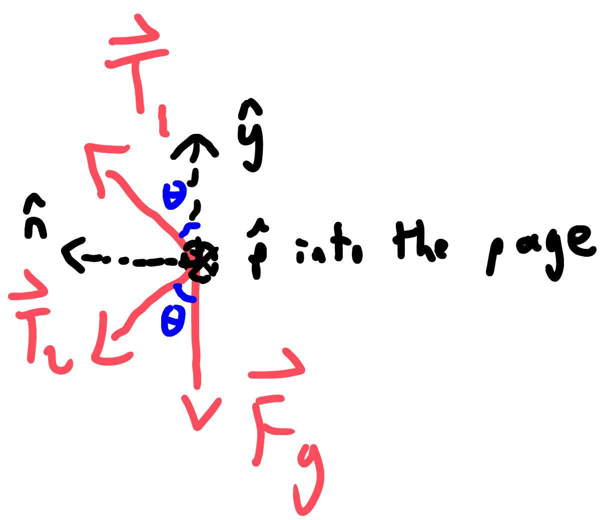
Sign your name on the line above

Spin Toy – Q2 in Gradescope [40 pts]

Consider the toy represented in the image to the right. This toy is made by attaching a ball of mass m to a wooden stick by means of two identical strings, each of which makes an angle θ with the stick. When you rotate the stick between the palms of your hands, the ball travels in a circle of radius R at constant speed v. In the diagram gravity points down, and the circular path of the ball is flat and in a plane that is parallel with the ground.



1. [10 pts] Draw a free-body diagram where the system is the ball at the location shown in the diagram. Your diagram should include all the forces acting on the ball in the correct directions and with correct angles, where applicable. Hint: think of what are the $\hat{p}$, $\hat{n}$, and $\hat{y}$ axes in this problem.



2. [30 pts] Determine the magnitude of the tension T_2 due to string 2.

Because the ball stays in a flat, horizontal plane, it isn't accelerating in the y-direction, and hence

sum of forces in the y-dir $= \sum F_y = 0.$ (1)

The ball undergoes a centripetal acceleration in the plane, and hence

sum of forces in the n-dir $= \sum F_n = \frac{mv^2}{R}\hat{n}$ (2)

(There are no forces in the p-direction. It travels with constant speed in a circle.)

Using Equation (1), we can relate T_1 and T_2:

$$\sum F_y = 0$$

$$\Rightarrow T_1 \cos\theta - T_2 \cos\theta - mg = 0$$

$$T_1 = T_2 + \frac{mg}{\cos\theta}$$

Using Equation (2), we can get another relation and solve for T_2:

$$\sum F_n = \frac{mv^2}{R}\hat{n}$$

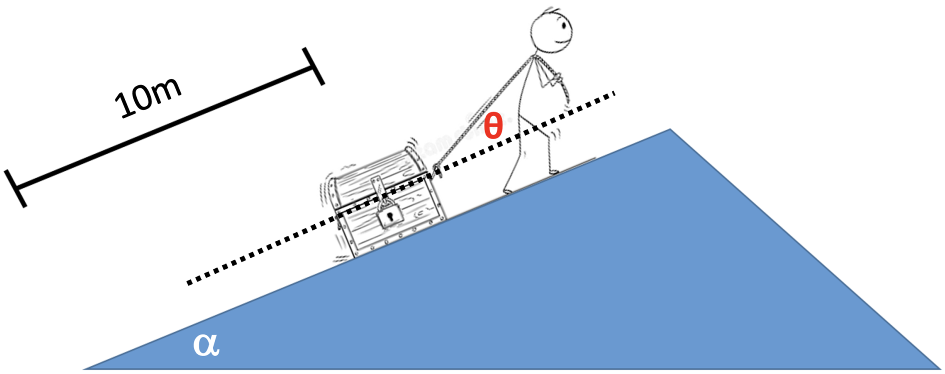
$$\Rightarrow T_1 \sin\theta + T_2 \sin\theta = \frac{mv^2}{R}$$

$$\Rightarrow \left[T_2 + \frac{mg}{\cos\theta}\right]\sin\theta + T_2 \sin\theta = \frac{mv^2}{R}$$

$$\boxed{T_2 = \frac{mv^2}{2R\sin\theta} - \frac{mg}{2\cos\theta}}$$

Treasure – Q3 in Gradescope [30 pts]

Stick Man is pulling his treasure chest up a hill that has an angle $\alpha = 30°$. He pulls with a force $|\vec{F}| = 900$ N at an angle $\theta = 25°$ with respect to the surface of the hill. The mass of the chest is $m = 200$ kg, and there's a coefficient of kinetic friction $\mu_k = 0.35$ between the chest and the hill.

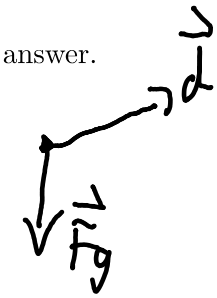


Consider the **treasure chest alone** to be the system, and that it's moving up along the hill.

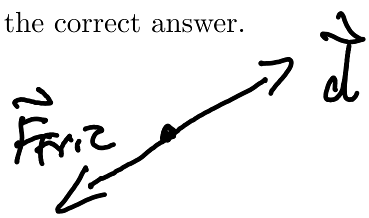
1. [3 pts] Is the work done by **gravity** positive, negative, or zero? Circle the correct answer.
   - positive
   - negative
   - zero

$W_F = \int \vec{F}\cdot d\vec{r} = \vec{F}\cdot\vec{d}$

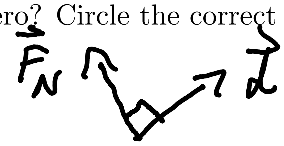
constant force



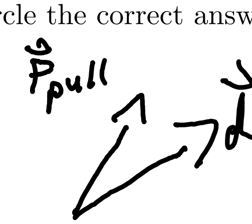
2. [3 pts] Is the work done by **friction** positive, negative, or zero? Circle the correct answer.
   - positive
   - negative
   - zero



3. [2 pts] Is the work done by **the normal force** positive, negative, or zero? Circle the correct answer.
   - positive
   - negative
   - zero

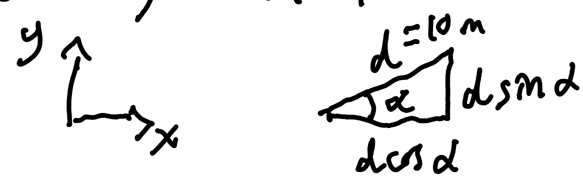


4. [2 pts] Is the work done by **the pulling force** positive, negative, or zero? Circle the correct answer.
   - positive
   - negative
   - zero



5. [20 pts] Calculate how much total work has been done on the system as it moved a distance $d = 10$ m up along the hill.

$W_{total} = \sum_i W_{\vec{F}_i}$ = sum of work done by each force



$\vec{d} = \langle d\cos\alpha, d\sin\alpha \rangle$

$W_{grav} = \vec{F}_{grav} \cdot \vec{d}$ $\qquad \vec{F}_g = \langle 0, -mg \rangle$

$\boxed{W_{grav} = -mg\,d\sin\alpha = -9800\text{ J}}$

$W_{fric} = \vec{F}_{fric} \cdot \vec{d}$ $\qquad$ dir.

$= \mu_K |\vec{F}_N|$ (down ramp) $\cdot\ d$ (up ramp)

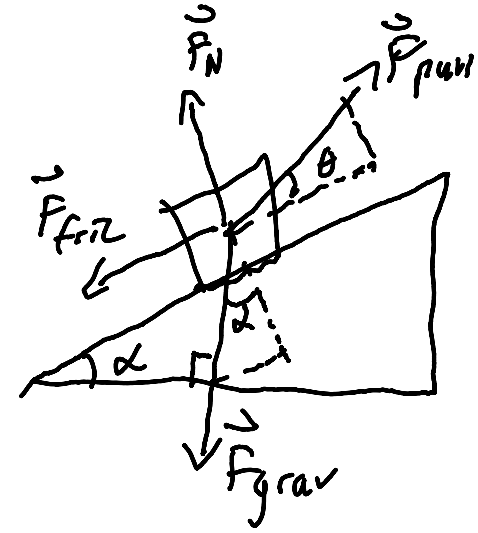
?

Balance the forces normal to the ramp to find $|\vec{F}_N|$:

$\sum$(forces ⊥ to ramp) $= 0$

$|\vec{F}_N| + |\vec{F}|\sin\theta - mg\cos\alpha = 0$

$\Rightarrow |\vec{F}_N| = mg\cos\alpha - |\vec{F}|\sin\theta$



$|\vec{F}_N| = 1317$ N

So

$\boxed{W_{fric} = -\mu_K |\vec{F}_N| d = -4610\text{ J}}$

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$W_N = |\vec{F}_N|$ ($\perp$ to ramp) $\cdot$ d (along ramp)

$$\boxed{W_N = 0 \text{ J}}$$

Using coordinates along the ramp and $\perp$ to the ramp:

$\vec{F}_{pull} = \langle |\vec{F}|\cos\theta, |\vec{F}|\sin\theta \rangle$

up ramp $\qquad$ $\perp$ to ramp

$\vec{d} = \langle 10, 0 \rangle$

$W_{pull} = \vec{F}_{pull} \cdot \vec{d}$

$$\boxed{W_{pull} = |\vec{F}|d\cos\theta = 8157 \text{ J}}$$

So

$$\boxed{W_{total} = \text{sum of works} = -6253 \text{ J}}$$

Energies – Q4 in Gradescope [30 pts]

Americium-241 is a radioactive isotope that is commonly found in smoke detectors. A nucleus of americium-241 at rest decays into a nucleus of neptunium-237 by emitting a fast-moving alpha particle and an x-ray photon:

$$\text{Am}^{241} \rightarrow \text{Np}^{237} + \alpha \text{ particle} + \text{photon}$$

Quantity	Variable
mass of americium	m_A
mass of neptunium	m_N
mass of alpha particle	m_α
charge of alpha particle	q (positive)
charge of neptunium	Q (positive
kinetic energy of neptunium	K_n
energy of emitted photon	E_x

1. [10 pts] Write the energy principle equation that you would need to solve to determine the speed of the alpha particle v when it is located at a distance d from the neptunium nucleus. You can treat all speeds as non-relativistic. Your answer should use the variables listed in the table above and any necessary fundamental physical constants. You can neglect all gravitational interactions.

$$E_{initial} = E_{final}$$

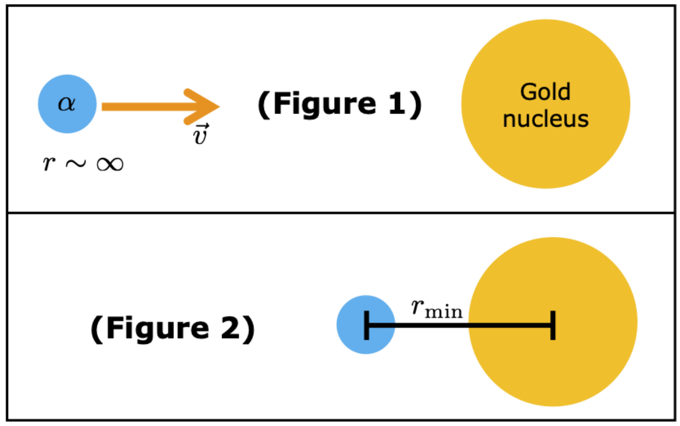
(non-relativistic KE)

$$E_{rest, Am^{241}} = E_{rest, Np^{237}} + K_n + E_{rest, \alpha} + K_\alpha + E_x + U_{electric, \alpha - Np^{237}}$$

$$\boxed{m_A c^2 = m_N c^2 + K_n + m_\alpha c^2 + \frac{1}{2} m_\alpha v_\alpha^2 + E_x + \frac{kQq}{d}}$$

any algebraically equivalent equation is valid

The alpha particle that got emitted in Part 1 travels a very long distance through free space for a very long time, moving with a speed of $v = 2.19 \times 10^7$ m/s. Eventually, it turns out to be in a direct collision course with the nucleus of a gold atom (Figure 1). However, due to the electric repulsion between like charges, the alpha particle never actually touches the gold nucleus. Instead, it gets to a center-to-center distance of r_{min}, where it momentarily stops (Figure 2) before it starts moving straight back the way it came.



Alpha	Gold
$m = 6.645 \times 10^{-27}$ kg	$M = 3.271 \times 10^{-25}$ kg
$q = +3.200 \times 10^{-19}$ C	$Q = +1.264 \times 10^{-17}$ C

2. [20 pts] Use the energy principle to determine what is the distance r_{min}. You can assume that the gold nucleus never moves, that the alpha particle's speed is non-relativistic, and that the gravitational force between the particles is negligible. All your calculations should keep 3 decimal places.

$$E_i = E_f$$

$$\underset{r \sim \infty}{\uparrow} \qquad \underset{r = r_{min}}{\uparrow}$$

$$K_i + \underbrace{\cancel{U_i}}_{0 \text{ (at } r \sim \infty)} = \underbrace{\cancel{K_f}}_{0 \text{ (stops momentarily)}} + U_f$$

$$\frac{1}{2} m v^2 = \frac{kQq}{r_{min}}$$

$$\Rightarrow \boxed{r_{min} = \frac{2kQq}{mv^2} = 2.284 \times 10^{-14} \text{ m}}$$