Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s \qquad U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M \qquad \Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots} \qquad I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots$$

$$K_{tot} = K_{trans} + K_{rel} \qquad K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I} \qquad K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot} \qquad \vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}} \qquad v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

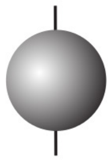
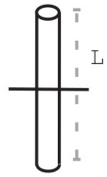
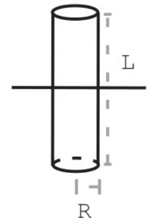
$$\text{prob}(E) \propto \Omega\left(E\right)e^{-\frac{E}{kT}} \qquad E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1,2,3\dots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0,1,2\dots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 Test 2 - Fall 2018

Please circle your lab section and then clearly print your name & GTID

Sections (M) Parker, (N) Yunker		
Day	12-3pm	3-6pm
Monday	M01 N01	M02 N02
Tuesday	M03 N03	M04 N04
Wednesday	M05 N05	M06 N06
Thursday	M07	N07

Name: Key

GTID:

Final Answers are in RED

Grading Decisions are in PURPLE

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code, I have not given or received unauthorized aid on this test."

Sign your name on the line above

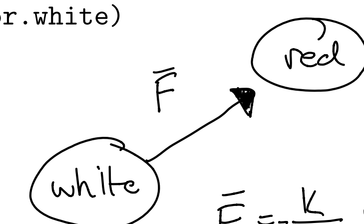
Problem 1 [25 pts]

Below is a programming shell to model the interaction of two particles. The force acting on the white particle is directed along the line connecting the two particles and is attractive. Its magnitude is given by $|\vec{F}| = \frac{k}{|\vec{r}|^3}$, where $|\vec{r}|$ is the separation between the particles and k is a positive constant. There are no other forces acting on the particles.

Write the necessary statements to compute the force on the white particle and update its momentum. The red particle will remain still.

```
GlowScript 2.7 VPython

redParticle = sphere(pos=vector(5,4,0), radius=0.25, color=color.red)
whiteParticle = sphere(pos=vector(-3,-2,0), radius=0.25, color=color.white)
k = 0.3 # Nm^3
redParticle.m = 1 # kg
whiteParticle.m = 5e-3 # kg
whiteParticle.p = whiteParticle.m*vector(800,800,0)
t = 0 # seconds
deltat = 5e-6 # seconds
while t < 1:
    ## A. [10pts] Compute the net force on the white particle
```



$\bar{F} = -\frac{k}{|r|^3}\hat{r}$

$\bar{r} = \bar{r}_{white} - \bar{r}_{red}$

+3 pt r = whiteParticle.pos - redParticle.pos

+2pt rhat = norm(r)

+2pt rmag = mag(r)

+3pt Fnet = -(k/rmag**3)*rhat

Note: They may define $r = r.pos - w.pos$ and then use $F_{net} = + \frac{k}{\|r\|^3}\hat{r}$

* If they combine it all into one line that's fine

```
    ## B. [10pts] Update the momentum of the white particle
```

ALL whiteParticle.p = whiteParticle.p + Fnet*deltat

```
    ## C. [5pts] Update the position of the white particle
```

ALL whiteParticle.pos = whiteParticle.pos + whiteParticle.p/whiteParticle.m*deltat

```
    ## Increase time
    t = t + deltat
```

Problem 2 [25 pts]

Two UFOs are floating through space, far from any planet. UFO_1 has mass 8500 kg, and initial velocity $< 15, 0, 0 >$ m/s. UFO_2 also has mass 8500 kg, and initial velocity $< 0, 0, 0 >$ m/s. The UFOs collide!

ALL

A. [3 pts] What is the total momentum before they collide?

$$\bar{P}_{tot,i} = \bar{P}_{1,i} + \bar{P}_{2,i}$$

$$= m(\bar{v}_{1,i} + \bar{v}_{2,i})$$

$$= (8500\text{ kg})\left(\langle 15,0,0\rangle \tfrac{m}{s} + \langle 0,0,0\rangle \tfrac{m}{s}\right)$$

$$= \boxed{\langle 127500, 0, 0\rangle \text{ kg}\tfrac{m}{s}}$$

$\vec{p} = <$ 127500 , 0 , 0 $>$ kg$\frac{\text{m}}{\text{s}}$

ALL

B. [3 pts] What is the total momentum after they collide?

$\bar{F}_{ext} = \bar{0} \rightarrow$ conservation of momentum

$$\bar{P}_{tot,f} = \bar{P}_{tot,i} = \boxed{\langle 127500, 0, 0\rangle \text{ kg}\tfrac{m}{s}}$$

POE from part A

$\vec{p} = <$ 127500 , 0 , 0 $>$ kg$\frac{\text{m}}{\text{s}}$

ALL

C. [4 pts] If the velocity of UFO_1 after the collision is $< 0, 0, 0 >$ m/s, what is the momentum of UFO_2?

$$\bar{P}_{tot,f} = m(\cancel{v_{1,f}}^{\,0} + v_{2,f}) = \bar{P}_{2,f}$$

$$\bar{P}_{2,f} = \bar{P}_{tot,f} = \boxed{\langle 127500, 0, 0\rangle \text{ kg}\tfrac{m}{s}}$$

POE from part B

$\vec{p} = <$ 127500 , 0 , 0 $>$ kg$\frac{\text{m}}{\text{s}}$

D. [3 pts] What is the velocity of UFO_2 after the collision?

POE from part C

ALL

$$\vec{v}_{2,f} = \frac{\vec{p}_{2,f}}{m} = \frac{\langle 127500,0,0\rangle \ \mathrm{kg\,\frac{m}{s}}}{8500\ \mathrm{kg}} = \boxed{\langle 15,0,0\rangle \ \frac{m}{s}}$$

$\vec{v} = <$ 15 , 0 , 0 $> \frac{\mathrm{m}}{\mathrm{s}}$

E. [5 pts] If the collision took 0.2 seconds, what was the average net force on UFO_1?

ALL

$$\vec{F}_{avg} = \frac{\Delta \vec{p}_1}{\Delta t} = \frac{m}{\Delta t}\left(v_{1,f} - v_{1,i}\right) = \frac{8500\ \mathrm{kg}}{0.2\ \mathrm{s}}\left(\langle 0,0,0\rangle \frac{m}{s} - \langle 15,0,0\rangle \frac{m}{s}\right)$$

$$\vec{F}_{avg} = \boxed{\langle -637500,0,0\rangle\ N}$$

$\vec{F} = <$ -637500, 0 , 0 $>$ N

F. [3 pts] Two new UFOs collide, far from any planet. UFO_3 has mass 8500 kg, and initial velocity $< 15, 0, 0 >$ m/s. UFO_4 also has mass 8500 kg, and initial velocity $< 0, 0, 0 >$ m/s. After their collision, the UFOs stick together. What is the momentum of the UFOs after the collision?

ALL

$$\vec{p}_{tot,f} = \vec{p}_{tot,i}$$
$$= \vec{p}_{3,i} + \vec{p}_{4,i}$$
$$= m\left(\vec{v}_{3,i} + \vec{v}_{4,i}\right)$$
$$= (8500\ \mathrm{kg})\left(\langle 15,0,0\rangle \frac{m}{s} + \langle 0,0,0\rangle \frac{m}{s}\right)$$
$$= \boxed{\langle 127500,0,0\rangle\ \mathrm{kg\,\frac{m}{s}}}$$

$\vec{p} = <$ 127500 , 0 , 0 $>$ kg$\frac{\mathrm{m}}{\mathrm{s}}$

G. [4 pts] What is the speed of UFO_3 after the collision?

* -2 if they give a correct velocity instead of speed

Stuck together, so $\vec{v}_{3,f} = \vec{v}_{4,f} = \vec{v}_f$

ALL

$$\vec{p}_{tot,f} = m\left(\vec{v}_{3,f} + \vec{v}_{4,f}\right)$$
$$= 2m v_f$$

POE from Part F

So,

$$v_f = \frac{\vec{p}_{tot,f}}{2m} = \frac{\vec{p}_{tot,i}}{2m} = \frac{m v_{3,i}}{2m} = \frac{v_{3,i}}{2}$$

$$v_f = \left\langle \frac{15}{2}, 0, 0 \right\rangle \frac{m}{s} \rightarrow \boxed{|v_f| = \frac{15}{2}\ \frac{m}{s}}$$

$|\vec{v}| = \frac{15}{2}$ $\frac{\mathrm{m}}{\mathrm{s}}$

Problem 3 [25 pts]

Prof. Yunker has a pile of scrambled eggs. When you look closer, you realize he isn't eating them, but measuring their physical properties. The eggs have been shaped into a disc with a height of 0.2 m and a radius of 0.1 m. Prof. Yunker compresses the eggs with a force of 0.5 N, and measures the new height of the eggs to be 0.17 m. During this experiment the effect of gravity on the eggs can be ignored.

A. [7 pts] Determine the spring constant of the egg disc.

$$s = |L| - L_0 = .17 - .2 = -.03$$

Equilibrium $\rightarrow$ $\bar{F}_{net} = 0 = \bar{F} + \bar{F}_s$. So $|F| = |F_s|$.

$$|F| = |ks| \rightarrow k = \left|\frac{F}{s}\right| = \frac{.5\,N}{.03\,m}$$

$$\boxed{k = \frac{50}{3}\frac{N}{m} = 16.67\ \frac{N}{m}}$$

GRADING

-1 Clerical
-2 Minor
-3 Major
-6 BTN

- (-2) Negative spring constant
- (-3) incorrect stretch

B. [8 pts] Calculate the Young's modulus of the egg disc.

GRADING

-1 Clerical
-2 Minor
-3 Major
-6 BTN

- (-2) for negative Y

F is <u>into</u> material $\rightarrow$ stess is negative.

$$Y = \frac{(-F/A)}{(\Delta L/L)} = \frac{(-.5\,N)/(\pi(.1\,m)^2)}{(-.03\,m)/(.2\,m)}$$

$$\boxed{Y = 106.1\ \frac{N}{m^2}}$$

C. [2 pts] Prof. Yunker cuts the egg disc in half, so it now has a height of 0.1 m. What is the Young's modulus for the new egg disc?

ALL

Young's Modulus is a material property—it would not change.

GRADING

- (-2) for thinking ΔL stays the same

D. [8 pts] The eggs are made from proteins that are 4 nm in diameter. What is the stiffness of these proteins?

$$d = 4 \times 10^{-9} \text{ m}$$

$$K_{protein} = k_s \frac{(\#\text{ proteins in series})}{(\#\text{ proteins in parallel})}$$

$$= k_s \frac{(L/d)}{(A/d^2)}$$

$$= k_s \left(\frac{Ld}{A}\right)$$

Assumes proteins take up a cubic volume. Using a spherical volume:

$$K_p^{(sphere)} = \frac{k_s\,(L/d)}{(A/(\pi(\frac{d}{2})^2))} = \frac{\pi}{4} K_p^{(cube)}$$

$$= 3.33 \times 10^{-7} \frac{N}{m}$$

$$= \underbrace{16.67 \frac{N}{m}}_{\text{POE from part A}} \left(\frac{(.2\text{ m})(4 \times 10^{-9}\text{ m})}{\pi\,(.1\text{ m})^2}\right) = \boxed{4.24 \times 10^{-7} \frac{N}{m}}$$

Alternative Solution

POE from part B/C

$$K_p = Y d$$

$$= (106.1 \tfrac{N}{m^2})(4 \times 10^{-9}\text{ m})$$

$$= 4.24 \times 10^{-7} \frac{N}{m}$$

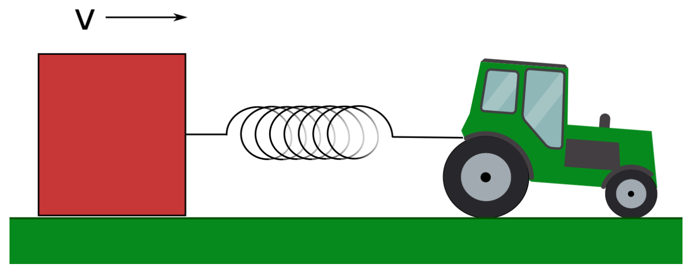
GRADING

-1 Clerical
-2 Minor
-3 Major
-6 BTN

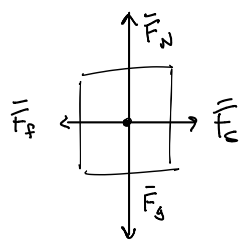
- (-2) for using spherical volume
- (-2) incorrect unit conversion

Problem 4 [25 pts]

A block of mass $m = 1000$ kg is being pulled across a surface, connected to a tractor by a spring of stiffness $k = 4900$ N/m and relaxed length $L_0 = 4$ m. The whole assemblage moves with constant speed $v = 3$ m/s. The coefficient of sliding friction between the block and the ground is $\mu_k = 0.5$, while the coefficient of static friction is $\mu_s = 0.75$.



A. [8 pts] Draw a force body diagram and label all of the forces acting on the block. The relative length of your arrows should scale with the forces so that if one force is twice as large as another it's arrow will be twice as long.



GRADING

- (+3) for drawing y-vectors
- (+3) for drawing x-vectors
- (+2) for y being twice as long
- If only x components are present, (+2) if vectors are the same length

Equilibrium: $|\bar{F}_N| = |\bar{F}_g|$ → lengths of this pair should be equal

$|\bar{F}_s| = |\bar{F}_f| = \mu_k |\bar{F}_g|$
$= \frac{1}{2} |\bar{F}_g|$ → lengths should be equal and half as long as x-direction

B. [10 pts] Calculate the length, L, of the spring as the block slides along the ground.

Equilibrium: $|\bar{F}_f| = |\bar{F}_s|$

$$\mu_k mg = k|L - L_0|$$

$$L = \frac{\mu_k mg}{k} + L_0$$

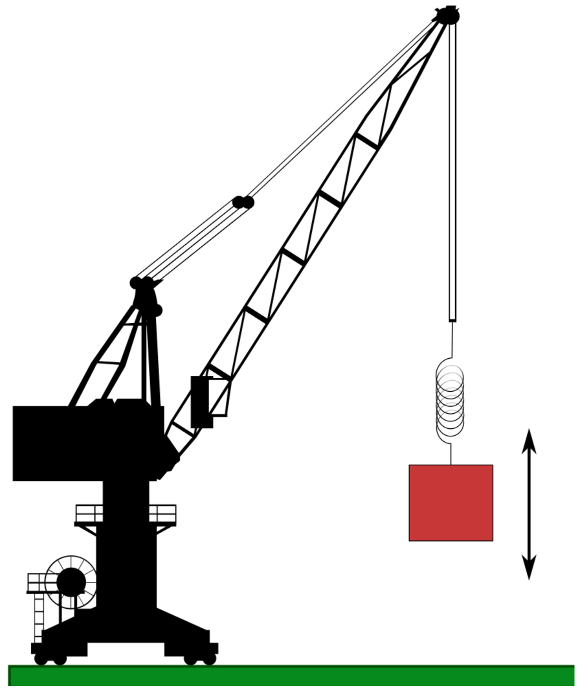
$$= \frac{(.5)(1000\,kg)(9.81\,\frac{m}{s^2})}{4900\,N/m} + 4m = \boxed{5.00\ m}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

- (-4) For not taking absolute value of Fs
- (-4) for forgetting L0
- (-4) using static friction coefficient

C. [7 pts] Suppose the block is detached from the tractor and lifted by a crane *using the same spring*. As the block bobs up and down, determine the period of oscillation.



$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{1000\,\text{kg}}{4900\,\frac{N}{m}}}$$

$$\boxed{T = 2.83\,s}$$

GRADING

-1 Clerical
-2 Minor
-3 Major
-6 BTN

This page is for extra work, if needed.

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