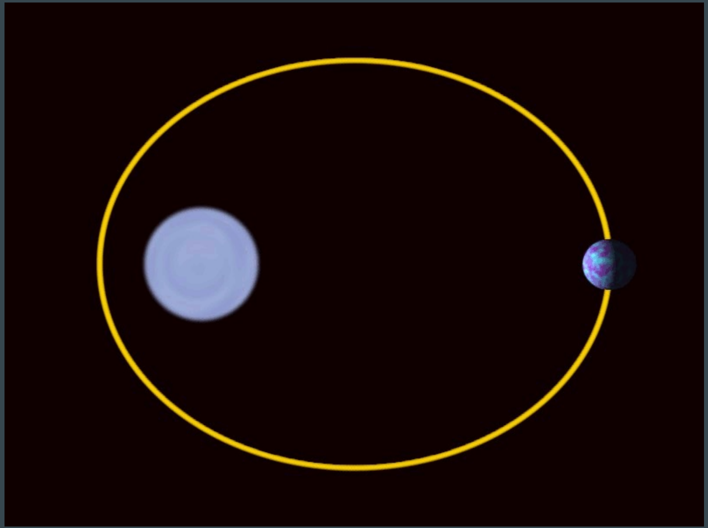


PHYS 2211 K

Week 4, Lecture 1

2022/02/01

Dr Alicea (ealicea@gatech.edu)

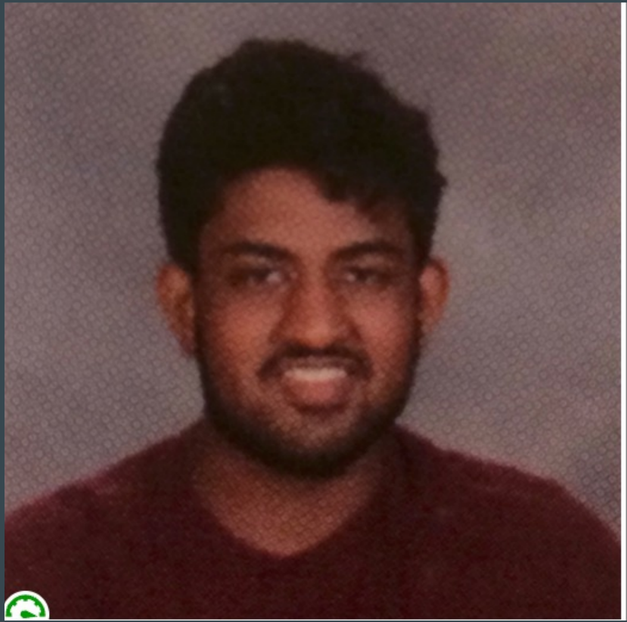
4 clicker questions today

On today's class...

1. Universal gravitation
2. Predicting motion with universal gravitation

Reminders!

- We now have a PLUS leader! Contact info and timing of PLUS sessions are in the canvas front page (scroll all the way down)



PLUS Leader: Vishnav Deenadayalan

- Contact: vishnavdeena@gatech.edu
- **PLUS Sessions**
  - Day: Mondays and Thursdays
  - Time: 5:00pm - 6:00pm
  - Location: CULC 280

Reminders!

- Test 1 is next Monday!

- Details are in edstem, post #128: https://edstem.org/us/courses/17074/discussion/1069706

QR code to the link (you still need to sign in though)

READ THE ENTIRE TEST DETAILS POST!!!

What we've learned so far...

- Newton's 2nd Law $$\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$$
- Position update $$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\Delta t$$
- Gravity near surface of Earth (constant force) $$\vec{F}_g =< 0, -mg, 0 >$$
- Spring force (non-constant) $$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

And also, iteration...

- Find net force $\vec{F}_{\text{net}}$
- Update velocity using Newton's 2nd Law $\vec{v}_f = \vec{v}_i + (\vec{F}_{\text{net}}/m)\Delta t$
- Update position using position update formula
  - For constant force:

$$\vec{v}_{\text{avg}} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

  - For non-constant force:

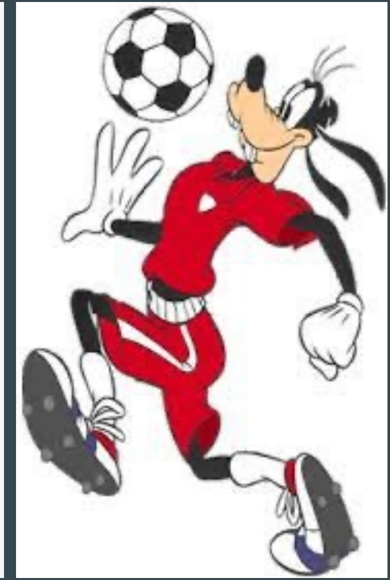
$$\vec{v}_{\text{avg}} \approx \vec{v}_f$$

CLICKER 1: What’s your favorite sportsball?

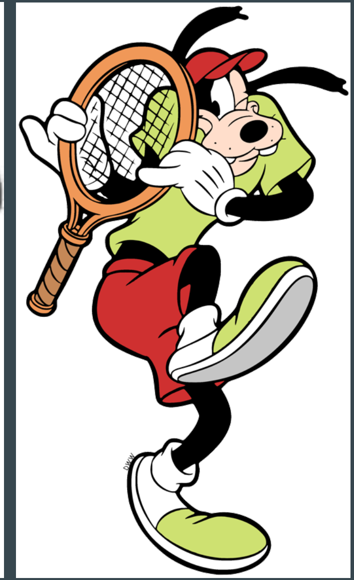
A. Basketball

B. Baseball

C. American Football



D. Soccer



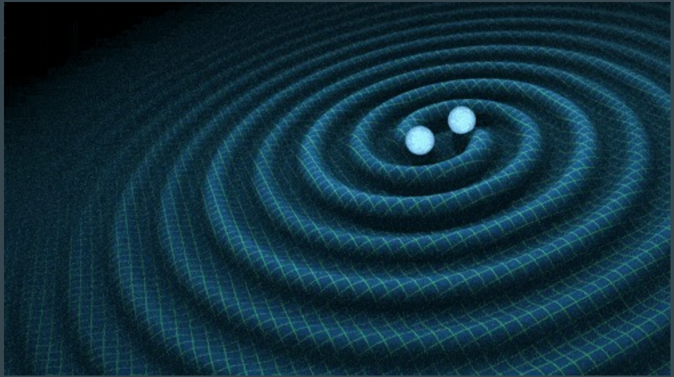
E. Tennis

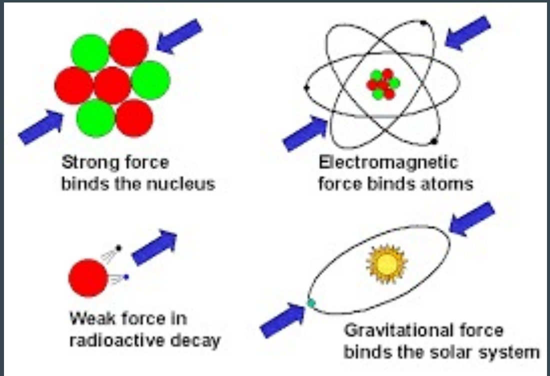
$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$$

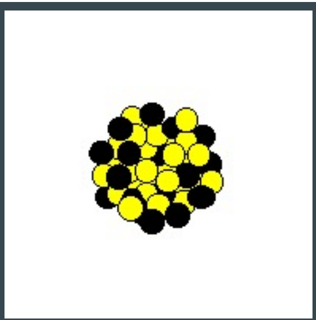
Universal gravitation

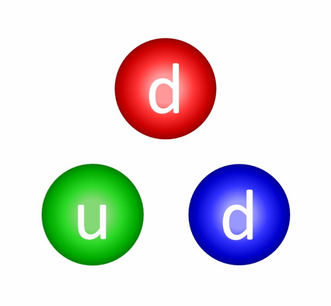
- Gravitation is one of the four fundamental interactions
  - Responsible for things falling to the ground, things orbiting other things, black holes, gravitational waves, the large-scale structure of the universe, etc

- The other three are:
  - Electromagnetic force
  - Weak nuclear force
  - Strong nuclear force









Universal gravitation

- We’ve already seen gravity near the surface of Earth:

$$\vec{F}_g =< 0, -mg, 0 >$$

- Now we’ll talk about Newton’s Law of Universal Gravitation
    - Attractive force between any two objects with mass
    - Proportional to product of masses
    - Inversely proportional to square of separation distance
    - Gravity near the surface of Earth is a good approximation as long as the height at which the object is located is much smaller than the radius of Earth

Gravitation

$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$$

- Relative position vector

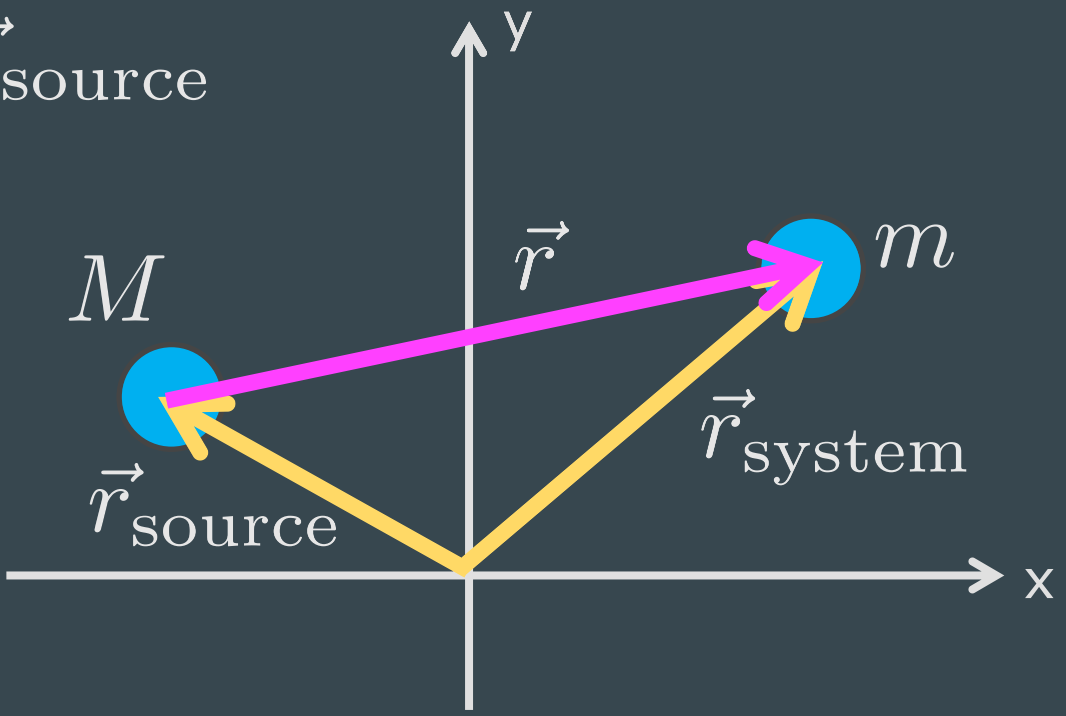
$$\vec{r} = \vec{r}_{\mathrm{system}} - \vec{r}_{\mathrm{source}}$$

- Direction of gravity: towards the source

$$-\hat{r}$$

- Gravitational constant:

$$G = 6.7 \times 10^{-11}\ \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$

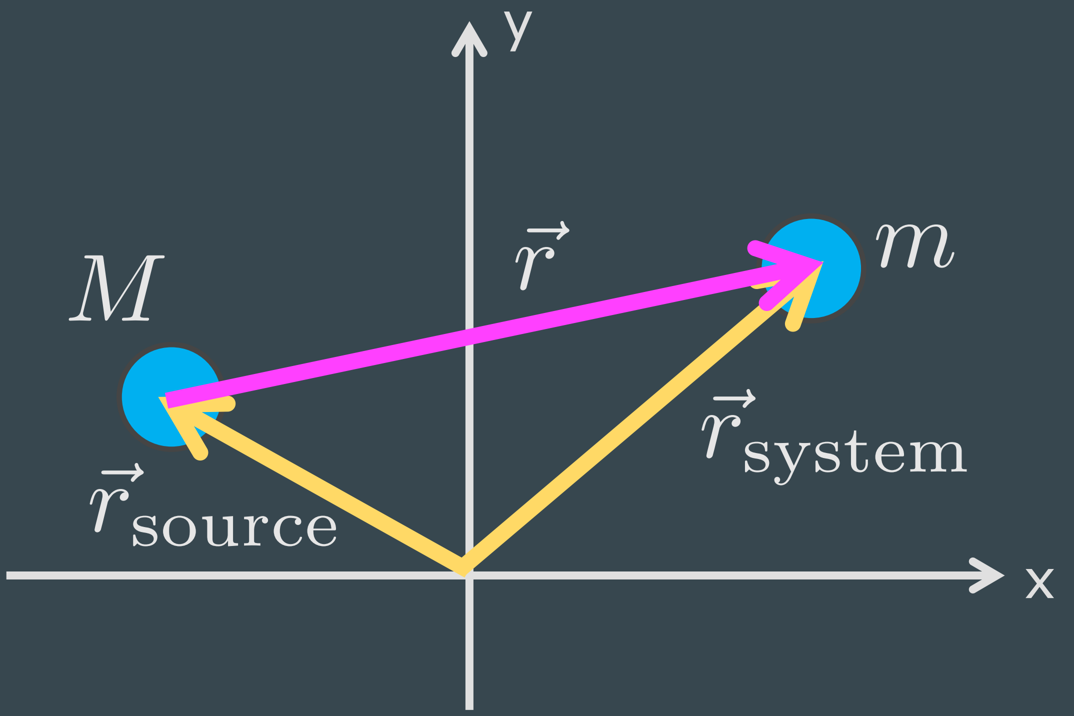


Gravitation

$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$$

- This is the force felt by the system due to the mass of the object which is the source

- If the source is M and the system is m, then we call this "F on m by M"



Gravitation

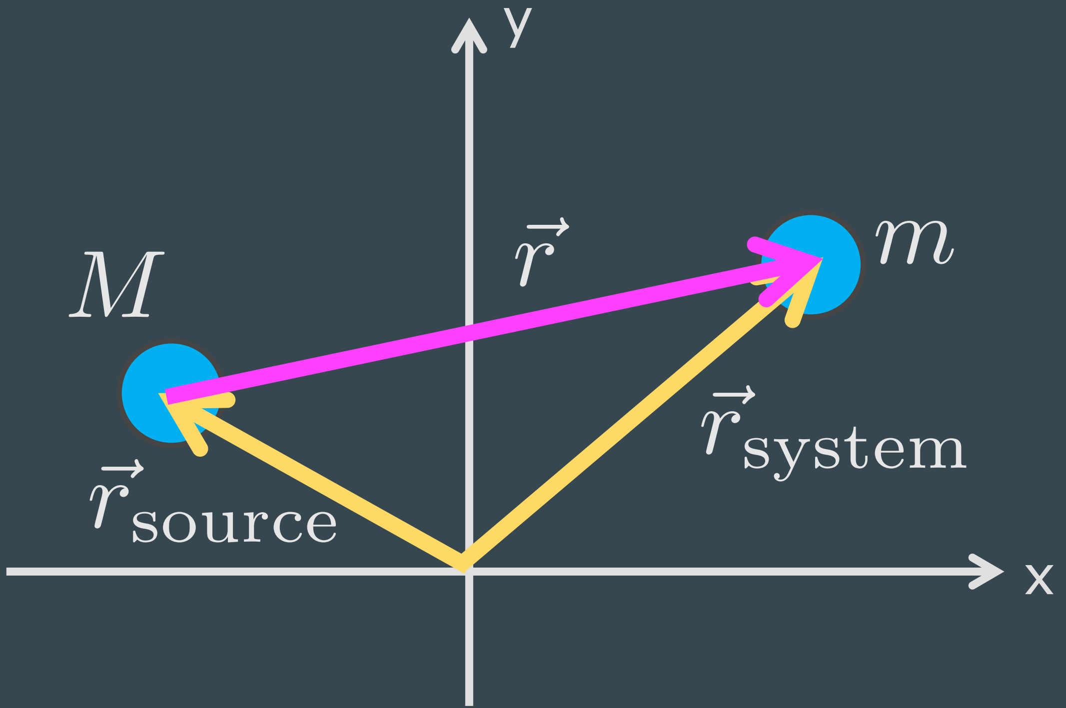
$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$$

- Magnitude (always positive):

$$|\vec{F}_g| = \frac{GMm}{r^2}$$

- Full vector combines magnitude and direction:

$$\vec{F}_g = |\vec{F}_g|(-\hat{r})$$



CLICKER 2: The gravitational force exerted by a planet on one of its moons is 3e23 N when the moon is at a particular location. If the mass of the moon were three times as large, what would the force on the planet be due to the moon?

A. 1e23 N
B. 3e23 N
C. 6e23 N
D. 9e23 N
E. We need more info

CLICKER 3: The gravitational force exerted by a planet on one of its moons is 3e23 N when the moon is at a particular location. If the distance between the moon and the planet was cut in half, what would the force on the moon be?

A. 1.2e24 N
B. 6e23 N
C. 3e23 N
D. 1.5e23 N
E. 0.33e23 N

Gravity on Earth

$$|\vec{F}_g| = \frac{GMm}{r^2}$$

M = mass of Earth = 6e24 kg
m = mass of whatever object near the surface of Earth
r = radius of Earth + distance between Earth's surface and object

radius of Earth: 6.4e6 m (six million meters)

What is the magnitude of the force of gravity felt by a rock of mass 3kg sitting at the top of Mt Everest? (elevation: 8848 m)

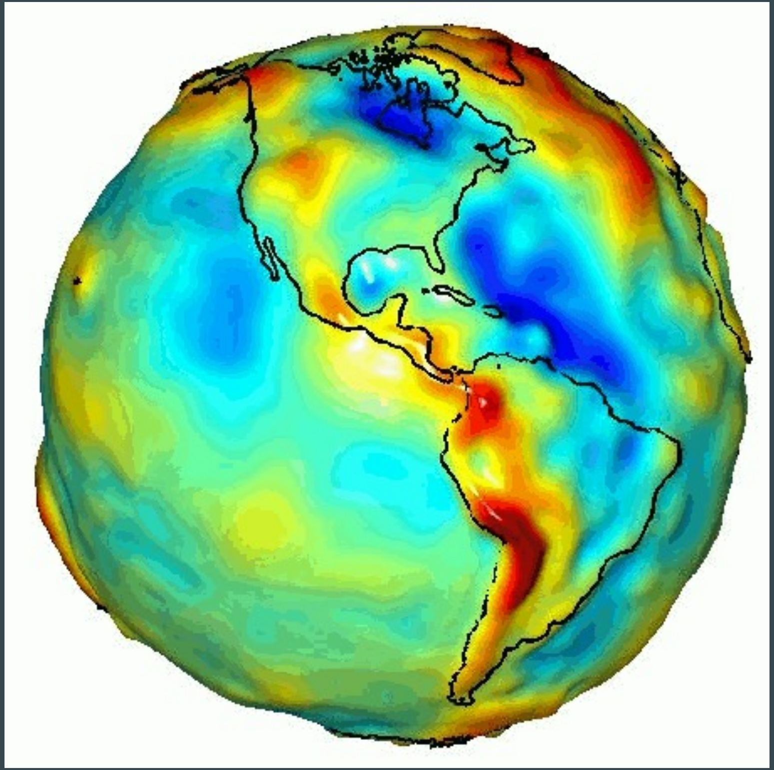
r = 6.4e6 m + 8848 m = essentially 6.4e6 m → height doesn't matter, it'll always be small enough compared to the radius of Earth!

Gravity on Earth

G = 6.7e-11 Nm^2/kg^2
M_E = 6e24 kg
R_E = 6.4e6 m

$$|\vec{F}_g| = \frac{GMm}{r^2}$$ but near the Earth's surface, $$|\vec{F}_g| = mg$$

Gravity on Earth



Deviations from

g=9.81 m/s^2 are

about ± 0.02 m/s^2

(therefore constant, as far as we’re concerned)

https://grace.jpl.nasa.gov/resources/6/grace-global-gravity-animation/

Example: The mass of the moon is 7.35e22 kg. The diameter of the moon is 3.47e6 m. What is the magnitude of the acceleration due to gravity at the surface of the moon, g_m?

Procedure for finding $\vec{F}_g$

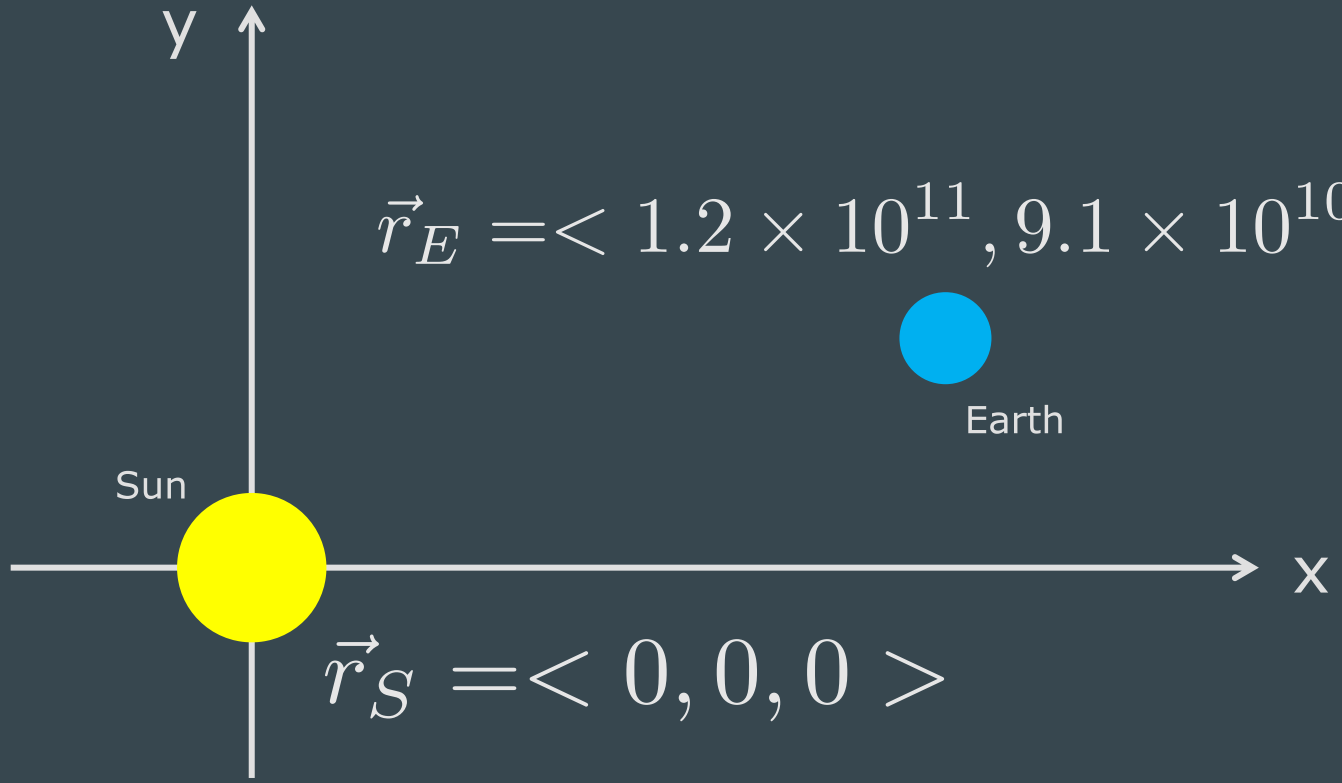
1. Draw a picture that includes position vectors for each object
2. Calculate the relative position vector (points to the object feeling the force)
3. Calculate the distance between the objects (magnitude of the relative position vector)
4. Calculate the direction of the relative position vector (r-hat)
5. Calculate the magnitude of the force
6. Combine the magnitude and direction (remember the minus sign!)
7. Check against your picture

Example: $\vec{F}_g$ on Earth due to Sun

$M_S = 2 \times 10^{30}$ kg

$M_E = 6 \times 10^{24}$ kg

$G = 6.7 \times 10^{-11}$ Nm^2/kg^2



$$\vec{r}_E = < 1.2 \times 10^{11}, 9.1 \times 10^{10}, 0 > \text{m}$$

$$\vec{r}_S = < 0, 0, 0 >$$

Example: $\vec{F}_g$ on Earth due to Sun

$M_S = 2 \times 10^{30}$ kg

$M_E = 6 \times 10^{24}$ kg

$G = 6.7 \times 10^{-11}$ Nm^2/kg^2

Find r vector, and magnitude, and r-hat

Find magnitude of Fgrav

Combine magnitude and direction

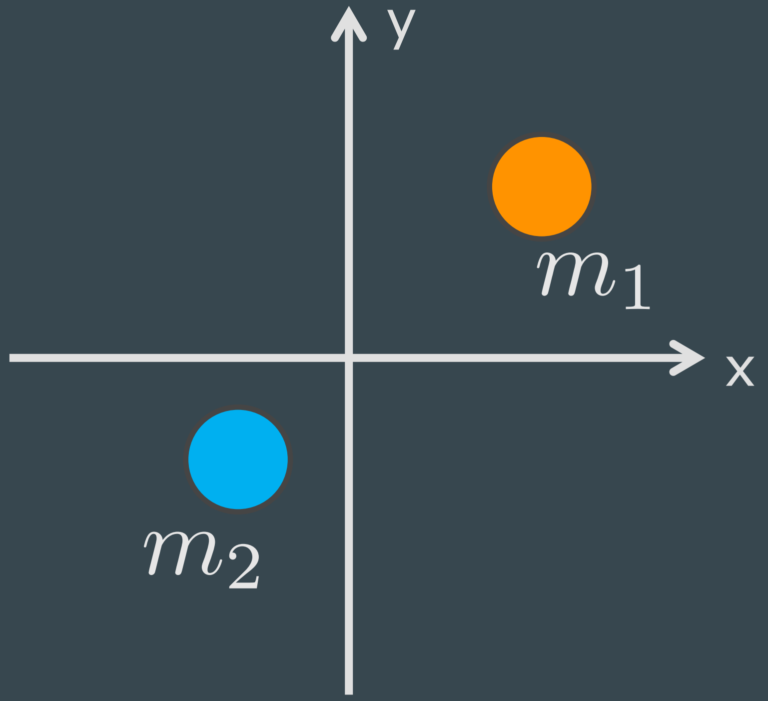
CLICKER 4: Two objects have the same mass (m_1=m_2=10 kg). One of them is located at $\vec{r}_1 =< 4, 3, 0 >$ m and the other is located at $\vec{r}_2 =< -2, -1, 0 >$ m. Determine the gravitational force exerted on m_2 by m_1.

A. <7.72e-10, 5.12e-10, 0> N

B. <1.07e-10, 7.1e-11, 0> N

C. <-7.72e-10, -5.12e-10, 0> N

D. <-1.07e-10, -7.1e-11, 0> N



Solution: Two objects have the same mass ($m_1=m_2=10$ kg). One of them is located at $\vec{r}_1 =< 4, 3, 0 >$ m and the other is located at $\vec{r}_2 =< -2, -1, 0 >$ m. Determine the gravitational force exerted on m_2 by m_1.

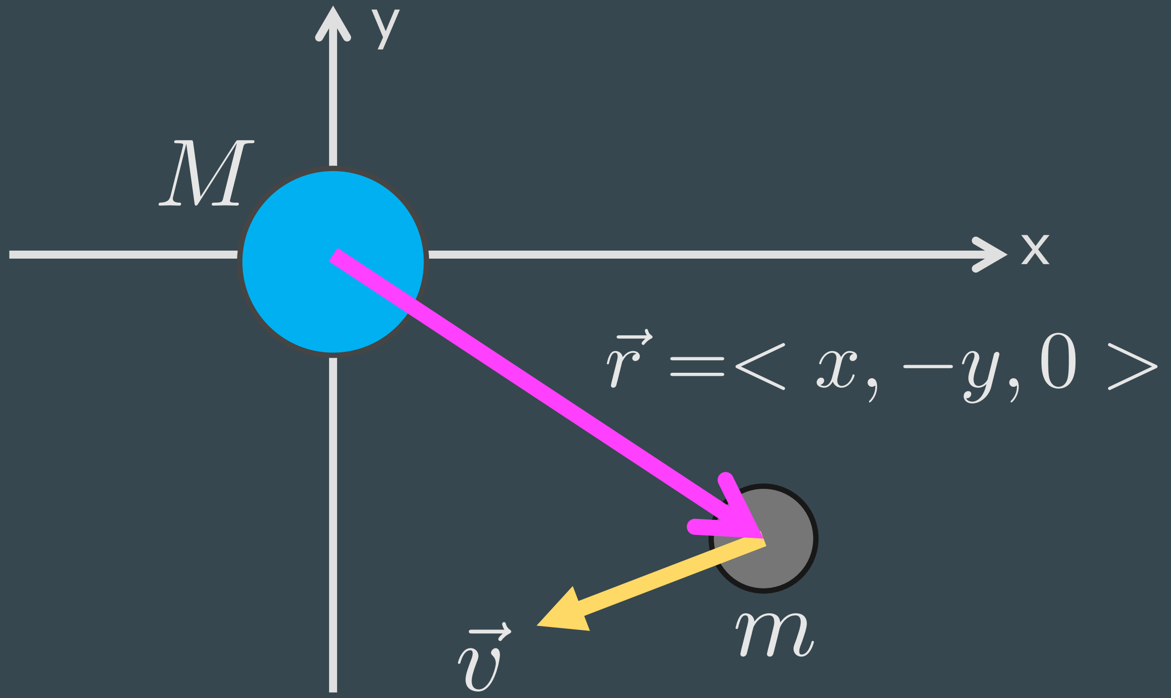
Predicting motion with $\vec{F}_{\text{net}} = \vec{F}_g$

1. Find the gravitational force on the system using the procedure we just discussed

2. Use Newton's 2nd Law to update the system's velocity

3. Use the position update formula for non-constant forces to update the position of the system

Example: A planet of mass M is located at the origin. The planet has a moon of mass m. At t=0, the moon is located at $<x,-y,0>$ and moves with velocity $<-v_x, -v_y, 0>$. Determine the position of the moon at time t=T using one iteration step.



Steps:

1. find F_g at t=0
2. update v at t=T
3. update r at t=T

Solution: A planet of mass M is located at the origin. The planet has a moon of mass m. At t=0, the moon is located at <x,-y,0> and moves with velocity <$-v_x$, $-v_y$, 0>. Determine the position of the moon at time t=T using one iteration step.

1. find F_g at t=0

2. update v at t=T

$$\vec{v}_f = \vec{v}_i + (\vec{F}_g/m)\Delta t$$

3. update r at t=T

$$\vec{r}_f = \vec{r}_i + \vec{v}_f \Delta t$$

Reminder about units!

- Symbolic answers have their dimensions built-in on the variables themselves (e.g., "m" is mass so that means kg, "r" is distance so that means meters, etc), so you don't need to add units at the end

- Numerical answers have no information about dimensions (e.g., "7" can be seven anything – meters, Newtons, oranges, cats, bottles of nail polish, etc), so it's necessary to include units with them

- If you put units in a symbolic answer, that's a clerical error
- If you don't put units in a numeric answer, that's a clerical error
- **Clerical errors mean -1pt in test problems**