Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s \qquad U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M \qquad \Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} \qquad I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel} \qquad K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I} \qquad K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot} \qquad \vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}} \qquad v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

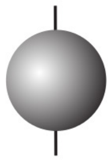
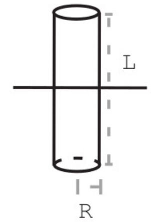
$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}} \qquad E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1,2,3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0,1,2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 Test 3 - Spring 2019

Please circle your lab section and then clearly print your name & GTID

Sections (M) 10AM, (K) 11AM		
Day	12-3pm	3-6pm
Monday	M01 K01	M02 K02
Tuesday	M03 K03	M04 K04
Wednesday	M05 K05	M06 K06
Thursday	M07 K07	M08 K08

Name: Key

GTID: ______________________

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

____

Sign your name on the line above


THE FINAL EXAM IS MONDAY, APRIL 29 6:00 PM - 8:50 PM
THE REGISTRAR SAYS IT'S FRIDAY
SCROLL DOWN!
IT SAYS WEDNESDAY!
LOOK UNDER COMMON EXAMS
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Problem 1 [25 pts]

Below is an incomplete program that calculates the motion of a ball attached to spring. The ball experience three forces: gravitational, spring, and drag. The spring has been damaged from overuse so that the magnitude of the spring force is proportional to the third power of the stretch $|\vec{F}_s| = k * s^3$. The drag force always points in the direction opposite to the velocity and has magnitude $|\vec{F}_{drag}| = b * |\vec{v}|^2$ where the constant b is a positive number. Fill in the missing lines of code below. **Please note that the program is spread over two pages.**

```
GlowScript 2.7 VPython
## constants
g = 9.81 ## acceleration from gravity (in N/kg)
mball = .1524   ## Mass of the ball (in kg)
L0 = 0.3     ## Relaxed length of the spring (in m)
k = 12   ## Spring constant you measured (in N/m^3)
b = 0.25 ## Ns^2/m^2

## objects
ceiling = box(pos=vector(0,0,0), size=vector(0.2, 0.01, 0.2))
ball = sphere(pos=vector(-0.1,-0.2,-0.3),radius=0.025, color=color.orange)
spring = helix(pos=ceiling.pos, color=color.cyan, thickness=.003, coils=40, radius=0.015)

## initial values
Fgrav = vector(0,-g*mball,0)
ball.vel = vector(0.024,0.084,-0.522)
ball.p = mball*ball.vel

deltat = 0.001  ## timestep (in s)
t = 0       ## start counting time at zero
while t < 10
    ## A. [14 pts] Add statements to calculate Fnet on the ball

    s = mag(ball.pos) - L0

(+6) Fs =  -k*s**3*norm(ball.pos)

(+6) Fdrag =  -b*mag2(ball.vel)*norm(ball.vel)

(+2) Fnet = Fgrav +  Fspring + Fdrag
```

GRADING

(-1) Clerical/Syntax
(-3) for incorrect vector direction
(-3) if Fs is proportional to s^2

Points assignments per line are given on the left

```
## B. [6 pts] Add statements to update the momentum, velocity and position of the ball
```

(+2) `ball.p =` ball.p + Fnet*deltat

(+2) `ball.vel =` ball.p/mball

(+2) `ball.pos =` ball.pos + ball.vel*deltat

GRADING

-1 Clerical/Syntax

Points assignments per line are given on the left

```
## C. [2pts] Add statements to calculate the kinetic energy (K) of the ball
```

`K =` 1/2*mball*mag(ball.vel)**2

GRADING

-1 Clerical/Syntax

```
## D. [3pts] Add statements to calculate the spring potential energy (U)
```

`U =` 1/4*k*s**4

For this irregular spring force:

$$U_s = \int k s^3 dx = \int k s^3 ds = \frac{1}{4} k s^4$$

GRADING

-1 Clerical/Syntax

```
t = t+deltat
```

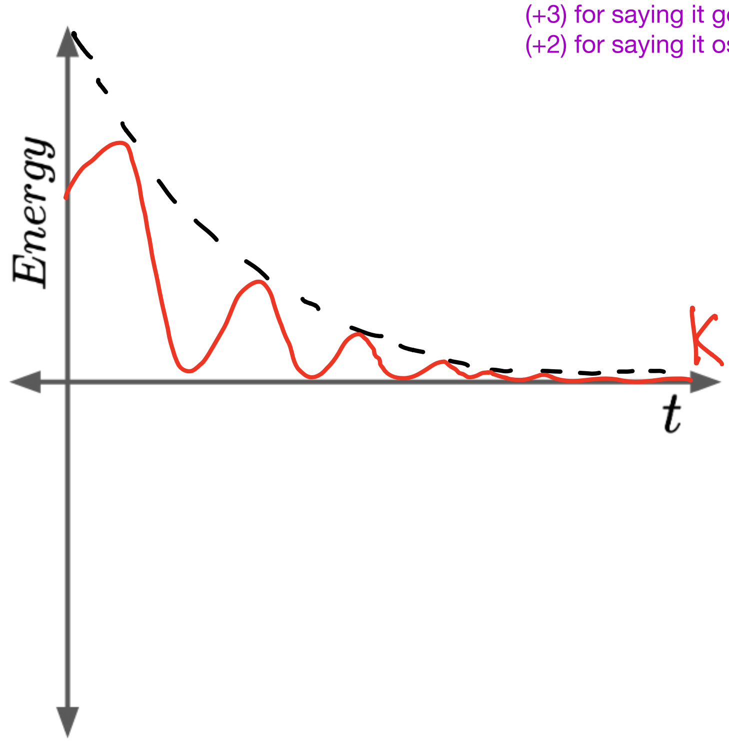
E. [5 pts] Extra Credit: Suppose your program ran for a very long time. Briefly describe the final velocity of the ball and sketch how the kinetic energy of the ball changed during the computation.

GRADING

(+3) for saying it goes to zero
(+2) for saying it oscillates

Since the drag force is dissapitive, as $t \to \infty$, we will find $K \to 0$.

As it decays, the kinetic energy will oscillate.



Problem 2 [25 pts]

Trevor, Hunter, and Adele each purchase a cup of coffee that contains 450 grams of coffee at 96° C. The specific heat of coffee is 4.2 J/(g·C). The cups are well insulated so that no energy is transferred between the coffee and the cup.

A. [10 pts] Trevor prefers to drink his coffee at 82° C. Cream has a specific heat of 3.8 J/(g·C) and an initial temperature of 5° C. How many grams of cream should he add to bring his coffee to the desired temperature?

Conservation of energy: (Let 1 be coffee and 2 cream)

$$\Delta E_{th,1} + \Delta E_{th,2} = 0$$

$$m_1 c_1 (T_f - T_1) + m_2 c_2 (T_f - T_2) = 0$$

So,

$$m_2 = -\frac{m_1 c_1 (T_f - T_1)}{c_2 (T_f - T_2)}$$

$$= -\frac{(450\,g)(4.2\,J/gC)(82°C - 96°C)}{(3.8\,J/gC)(82°C - 5°C)}$$

$$\boxed{m_2 = 90.4\,g}$$

GRADING

(-2) Clerical
(-5) Minor
(-10) Major
(-20) BTN

B. [10 pts] Hunter likes to add sugar to his coffee. Sugar has a specific heat of 1.2 J/(g·C) and an initial temperature of 25° C. Hunter adds 30 grams of sugar to his coffee and stirs. By stirring the coffee, Hunter is doing 500 J of work on his coffee. What is the final temperature of the coffee after the liquid comes to rest?

Conservation of Energy:

$$\Delta E_{th,s} + \Delta E_{th,c} = W_{ext}$$

$$m_s C_s (T_f - T_s) + m_c C_c (T_f - T_c) = W_{ext}$$

So

$$T_f = \frac{W_{ext} + m_s C_s T_s + m_c C_c T_c}{m_s C_s + m_c C_c}$$

$$= \frac{(500\,J) + (30g)(1.2\,J/gC)(25°C) + (450g)(4.2\,J/gC)(96°C)}{(30g)(1.2\,J/gC) + (450g)(4.2\,J/gC)}$$

$$\boxed{T_f = 94.9\,°C}$$

GRADING

(-1) Clerical
(-2) Minor
(-4) Major
(-8) BTN

C. [5 pts] After purchasing her coffee, Adele decides to take a nap. While sleeping, Hunter stirs in 60 grams of sugar at a temperature of 25° C. By stirring, Hunter does 600 J of work on Adeles coffee. When Adele wakes up, she finds that her coffee has reached thermal equilibrium with the air and is at a temperature of 25° C. Taking the coffee and sugar as the system, determine the thermal transfer of energy Q between the system and the surroundings from just before Adele's nap until the moment she awoke.

$$\Delta E_{th} = Q + W_{ext}$$

$$Q = \Delta E_{th,s} + \Delta E_{th,c} - W_{ext}$$

$$= m_c C_c (T_f - T_c) + m_s C_s (T_f - T_s) - 600\,J$$

$$= (450g)(4.2\,J/gC)(25°C - 96°C) + (60g)(1.2\,J/gC)\underbrace{\cancel{(25°C - 25°C)}}_{0} - 600\,J$$

$$\boxed{\Delta Q = -134790\,J}$$

GRADING

(-1) clerical
(-2) minor error
(-2) if they forget W_ext

Problem 3 [25 pts]

Consider a system consisting of two particles:
$m_1 = 5$ kg, vector $\vec{v}_1 = \langle 7,\ -8,\ 14 \rangle$ m/s
$m_2 = 10$ kg, vector $\vec{v}_2 = \langle -15,\ 2,\ -6 \rangle$ m/s

A. [5 pts] What is the total momentum $\vec{p}_{total}$ of this system? ***Show all steps in your work.***

$$\bar{P}_{tot} = m_1\bar{v}_1 + m_2\bar{v}_2$$

$$= (5\text{ kg})\langle 7,-8,14\rangle \tfrac{m}{s} + (10\text{ kg})\langle -15,2,-6\rangle \tfrac{m}{s}$$

$$\boxed{P_{tot} = \langle -115,-20,10\rangle \text{ kg}\tfrac{m}{s}}$$

GRADING

All or Nothing

B. [5 pts] What is $\vec{V}_{CM}$, the velocity of the center of mass of this system? ***Show all steps in your work.***

POE

$$\bar{V}_{cm} = \frac{\bar{P}_{tot}}{M_{tot}} = \frac{1}{(15\text{ kg})}\langle -115,-20,10\rangle$$

$$\boxed{\bar{V}_{cm} = \langle -7.66,-1.33,0.66\rangle \ m/s}$$

GRADING

All or Nothing

C. [5 pts] What is $K_{translational}$, the translational kinetic energy of this system? ***Show all steps in your work.***

$$K_{trans} = \frac{1}{2}M_{tot}\bar{V}_{cm}^2 = \frac{(15\text{ kg})}{2}\left[(-7.66)^2+(-1.33)^2+(0.66)^2\right]\frac{m^2}{s^2}$$

POE

$$\boxed{K_{trans} = 457.5 \text{ J}}$$

GRADING

All or Nothing

D. [5 pts] What is K_{total}, the total kinetic energy of this system? ***Show all steps in your work.***

$$K_{tot} = \frac{1}{2}m_1\bar{v}_1^2 + \frac{1}{2}m_2\bar{v}_2^2$$

$$= \frac{(5\text{ kg})}{2}\left[(7)^2+(-8)^2+(14)^2\right]\frac{m^2}{s^2} + \frac{(10\text{ kg})}{2}\left[(-15)^2+(2)^2+(-6)^2\right]\frac{m^2}{s^2}$$

$$\boxed{K_{tot} = 2097.5 \text{ J}}$$

GRADING

All or Nothing

E. [5 pts] What is K_{rel}, the kinetic energy of this system relative to the center of mass? ***Show all steps in your work.***

POE POE

$$K_{tot} = K_{trans} + K_{Rel}, \text{ So}$$

$$K_{Rel} = 2097.5 \text{ J} - 457.5 \text{ J}$$

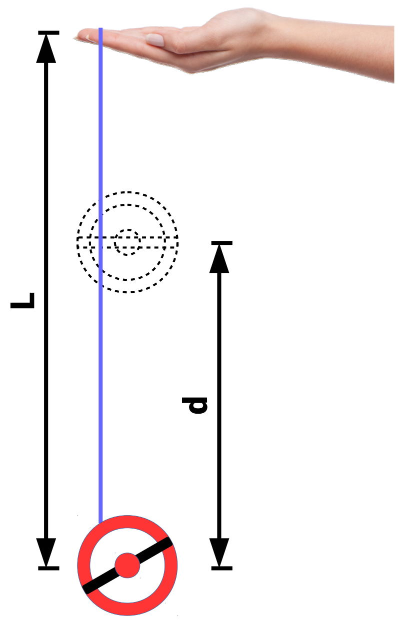
$$\boxed{K_{Rel} = 1640 \text{ J}}$$

GRADING

All or Nothing

Problem 4 [25 pts]

A physics student is playing with a yo-yo. The yo-yo is initially held motionless in midair when the student releases the yo-yo and pulls up on the string with a constant force F. Their hand moves up as a length of string L unravels and the yo-yo falls a distance d under the force of gravity. The yo-yo has mass m, and the mass of the string can be ignored.



A. [10 pts] Determine the speed of the center of mass for the yo-yo after it has fallen a distance d?

Conservation of energy: (point particle)

$$\Delta K = W_{ext}$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) = F_{net} \cdot \Delta x_{cm}$$

$$\tfrac{1}{2}mv^2 = (F - mg)(-d)$$

So

$$\boxed{V_{cm} = \sqrt{\frac{2}{m}(mgd - Fd)}}$$

GRADING

(-1) Clerical
(-2) Minor
(-4) Major
(-8) BTN

B. [15 pts] Calculate the rotational speed ω for the yo-yo after it has fallen a distance d? You can model the yo-yo as a cylinder of radius R.

Conservation of energy

$$\Delta K_{trans} + \Delta K_{Rot} = W_{ext}$$

$$(mg - F)d + \left(\tfrac{1}{2} I\omega^2 - 0\right) = F\,\Delta X_{poc,F} + F_g\,\Delta X_{poc,g}$$

$$(mg - F)d + \tfrac{1}{2} I\omega^2 = (F)(L-d) + (-mg)(-d)$$

$$\tfrac{1}{2} I\omega^2 = FL$$

$$\omega = \left(\frac{2FL}{I}\right)^{1/2}$$

for a cylinder, $I = \frac{1}{2} m R^2$, so

$$\boxed{\omega = \left(\frac{4FL}{mR^2}\right)^{1/2}}$$

GRADING

(-1) Clerical
(-3) Minor
(-6) Major
(-12) BTN

This page is for extra work, if needed.