

Calculus BC Equations

LIMITS

$$\text{If } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}, \text{ or } \frac{\infty}{\infty}$$
$$\text{then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

APPLICATIONS OF DIFFERENTIATION

$$V = \frac{4}{3}\pi r^3 \quad V = \frac{1}{3}h\pi r^2$$

INTEGRATION

$$\int u \, dv = uv - \int v \, du \quad \frac{1}{b-a} \int_a^b f(x) \, dx$$
$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$
$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

APPLICATIONS OF INTEGRATION

$$\pi \int_a^b (R_2)^2 - (R_1)^2 \, dx$$

INFINITE SERIES

Monotonic Sequence A sequence $\{a_n\}$ that is nondecreasing (i.e. $\{1, 1, 2, 3\}$) where

$$a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n \leq \dots$$

or if terms are nonincreasing like

$$a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq \dots$$

Bounded Monotonic Sequence A bounded monotonic sequence converges. A sequence is bounded if it is bounded above by M and below by N such that $N < a_n < M, \forall n \geq 0$.

Infinite Series Infinite series are defined as

$$S = \sum_{n=1}^{\infty} a_n$$

where S_n denotes the n^{th} partial sum

Convergence: For an infinite series $S = \sum a_n$, where S_n denotes the n^{th} partial sum, if the sequence $\{S_n\}$ converges to S then the series $S = \sum a_n$ converges. The limit

S is called the sum of the series.

Integral Test: For an infinite series $S = \sum f(x)$ if the improper integral $\int f(x) = L$ converges then the series converges and if the improper integral $\int f(x)$ does not exist or is infinity, it diverges. It does not give any information about the actual sum of the series.

P series:

$$N = \sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$$

$P = 1$ diverges $P > 1$ converges $P < 1$ diverges $0 > P > 1$ diverges

Taylor Polynomials If f has n derivatives at c , then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is defined as the **n^{th} degree Taylor polynomial**.

Taylor Series If f is infinitely differentiable, then f is represented exactly by the series, centered at $x = c$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

PARAMETRIC EQUATIONS

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Distance Formula:

$$\Delta s = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$x(t) = x_0 + v_{x0} + \frac{1}{2}at^2$$

$$y(t) = y_0 + v_{y0} + \frac{1}{2}at^2$$

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Projectile Motion

Maximum Height:

$$H = \frac{v_o^2 \sin^2 \theta}{2g}$$

Horizontal Range:

$$R = \frac{v_o^2 \sin 2\theta}{g}$$

Flight Time:

$$t = \frac{2v_{y0} \sin \theta}{g}$$

Example

Eliminating the Parameter:

Finding a rectangular equation that represents the graph of a set of parametric equations is called *eliminating the parameter*.

1. Parametric Equations $x = t^2 - 4$ $y = \frac{t}{2}$

2. Solve for t in one equation.

$$x = (2y)^2 - 4$$

3. Rectangular Equation

$$x = 4y^2 - 4$$

POLAR EQUATIONS

$$\int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

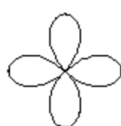
$$\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

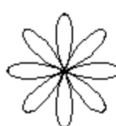
$$\tan \theta = \frac{y}{x}$$



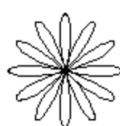
$$r = a \cos 2\theta$$



$$r = a \cos 3\theta$$



$$r = a \cos 4\theta$$



$$r = a \cos 6\theta$$

VECTORS

Angle Between Two Vectors

If θ is the angle between two nonzero vectors \vec{u} and \vec{v} , then

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

Alternatively,

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

This form can be used to calculate the dot product without knowing the component form of the vectors.

VECTOR-VALUED FUNCTIONS

A function of the form:

$$\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

Can also be written as:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

Projectile Motion equations (without air resistance) can be written as Vector Valued Functions:

$$\vec{s}(t) = \langle x_0 + v_{x0}t, y_0 + v_{y0}t - \frac{1}{2}gt^2 \rangle$$

$$\vec{v}(t) = \langle v_{x0}, v_{y0} - gt \rangle$$

$$\vec{a}(t) = \langle 0, -g \rangle$$

DIFFERENTIAL EQUATIONS

• Separable Differentiable Equations

1. Separate the variables into standard form:

$$F(y) dy = G(x) dx$$

• First Order Differentiable

1. Rearrange equation into standard form:

$$y' + py = q$$

2. Integrating factor:

$$u(x) = e^{\int p dx}$$

3. Multiply both sides:

$$uy' + upy = uq$$

$$(uy)' = uq$$

4. Integrate:

$$uy = \int (uq) dx$$

• Euler's Method Formula

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - \cdots + (-1)^n (x-1)^n + \cdots$$

$$1 - x + x^2 - x^3 + x^4 - x^5 + \cdots + (-1)^n x^n + \cdots$$

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots + \frac{(-1)^{n-1} (x-1)^n}{n} + \cdots$$

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$$

$$x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots + \frac{(2n)!x^{2n+1}}{(2^n n!)^2 (2n+1)} + \cdots$$

$$1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \frac{k(k-1)(k-2)(k-3)x^4}{4!} + \cdots$$

LIMITS

- Limits at infinity
 - Three possibilities for horizontal asymptotes
- Removeable vs. non-removeable discontinuity
 - One-sided limits
- L'Hôpital's Rule
 - Conditions for use

DIFFERENTIATION

- Definition of derivative at a point
- Derivatives of polynomials, trig, and exponential functions
- Differentiation rules
- Equation of a tangent line to a curve
- Interpreting the signs of the first and second derivative
 - Find the min or max of a function
- Sketching the first and second derivative from a graph
- Implicit differentiation

APPLICATIONS OF DIFFERENTIATION

- Optimization
 - Distance, area, volume
- Newton's Method
- Related Rates
 - Distance, area, volume, depth, ladder, and shadows

INTEGRATION

- Reimann Sums
- Difference between area and definite integral
- Integration by substitution
- Integration by parts
- Partial fractions
 - Distinct linear, repeated linear, quadratic and repeated quadratic factors
- Improper integrals

APPLICATIONS OF INTEGRATION

- Volumes of revolution
- Work done by a variable force
- Average value of a function
- Arc length of a curve
- Area between two curves

INFINITE SERIES

- P-series
- Geometric Series
- Convergence or divergence
 - Integral test, ratio test, comparison test and root test
- Taylor polynomial approximation with desired accuracy
- Taylor and Maclaurin series for elementary functions
 - Radius of convergence
 - Interval of convergence

PARAMETRIC EQUATIONS

- Converting to/from rectangular functions
- Difference between $\frac{dy}{dx}$, $\frac{dy}{dt}$ and $\frac{dx}{dt}$
- Second derivative of a parametric equation
- Arc length
- Projectile Motion: Range, hangtime, and max height

POLAR EQUATIONS

- Converting to/from rectangular functions
- Area and arc length of polar functions

VECTORS

- Dot product of two vectors
- Differentiation and integration of vector-valued functions (Initial value problems)
- Tangential acceleration and centripetal acceleration

DIFFERENTIAL EQUATIONS

- Logistic differential equations and population growth
- Standard form of first order linear differential equations
- Solve by integrating factor