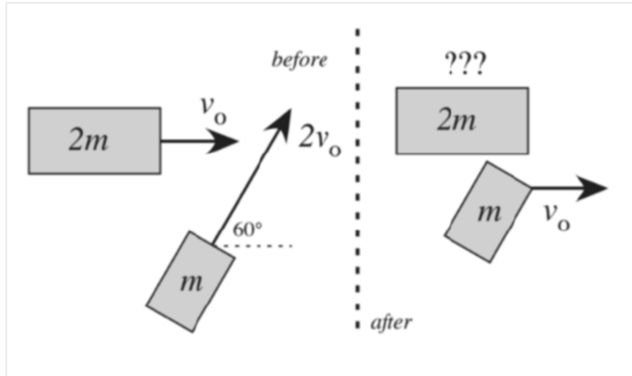
Physics 2211 GPS Week 12

Problem #1

A cruise ship is moving due east with some speed v_0 when it collides with a boat moving 60° north of east with a speed $2v_0$. The boat's mass is m and the cruise ship's mass is $2m$. Immediately after the collision, the boat is observed to be floating due east with a speed v_0.



(a) Determine the velocity of the cruise ship just after the collision. You need to express your answer in terms of the parameter v_0.

$\vec{V}_{ci} = \langle V_0, 0, 0 \rangle$ $\vec{V}_{bi} = 2V_0 \langle \cos 60°, \sin 60°, 0 \rangle$

$\vec{V}_{cf} = ?$ $\vec{V}_{bf} = \langle V_0, 0, 0 \rangle$

$m_c = 2m$ $m_b = m$

$$\vec{P}_{ci} + \vec{P}_{bi} = \vec{P}_{cf} + \vec{P}_{bf}$$

$$m_c \vec{V}_{ci} + m_b \vec{V}_{bi} = m_c \vec{V}_{cf} + m_b \vec{V}_{bf}$$

$$m_c \vec{V}_{ci} + m_b (\vec{V}_{bi} - \vec{V}_{bf}) = m_c \vec{V}_{cf}$$

$$\vec{V}_{cf} = \frac{\cancel{m_c}\vec{V}_{ci}}{\cancel{m_c}} + \frac{m_b}{m_c}(\vec{V}_{bi} - \vec{V}_{bf}) = \vec{V}_{ci} + \frac{\cancel{m}}{2\cancel{m}}(\vec{V}_{bi} - \vec{V}_{bf}) =$$

$$= \langle V_0, 0, 0 \rangle + \frac{1}{2}\left[2V_0 \langle \cos 60°, \sin 60°, 0 \rangle - \langle V_0, 0, 0 \rangle\right] =$$

$$= \langle V_0, 0, 0 \rangle + V_0 \langle \cos 60°, \sin 60°, 0 \rangle - \langle \tfrac{V_0}{2}, 0, 0 \rangle =$$

$$= \langle \tfrac{V_0}{2}, 0, 0 \rangle + V_0 \langle \cos 60°, \sin 60°, 0 \rangle =$$

$$= \langle \tfrac{V_0}{2} + V_0 \cos 60°, V_0 \sin 60°, 0 \rangle = \langle \tfrac{V_0}{2} + \tfrac{V_0}{2}, 0.866\, V_0, 0 \rangle =$$

$$= \langle V_0, 0.866\, V_0, 0 \rangle = \boxed{V_0 \langle 1, 0.866, 0 \rangle}$$

(b) What percentage of the original kinetic energy was lost in the collision?

$$\checkmark\ K_i = K_{ci} + K_{bi} = \frac{1}{2}m_c v_{ci}^2 + \frac{1}{2}m_b v_{bi}^2 = \frac{1}{\cancel{2}}(\cancel{2}m)\,v_0^2 + \frac{1}{2}m(2v_0)^2 =$$

$$= mv_0^2 + \frac{1}{\cancel{2}}m\overset{2}{\cancel{4}}v_0^2 = mv_0^2 + 2mv_0^2 = 3mv_0^2$$

$$\checkmark\ K_f = K_{cf} + K_{bf} = \frac{1}{2}m_c v_{cf}^2 + \frac{1}{2}m_b v_{bf}^2 = \frac{1}{\cancel{2}}(\cancel{2}m)\,\underbrace{v_{cf}^2} + \frac{1}{2}mv_0^2 =$$

$$v_{cf}^2 = v_0^2(1+0.866^2) = v_0^2(1+0.75) = 1.75v_0^2$$

$$= 1.75mv_0^2 + \frac{1}{2}mv_0^2 = (1.75+0.5)\,mv_0^2 = 2.25\,mv_0^2$$

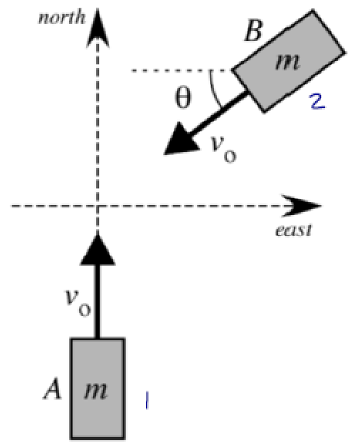
$$\checkmark\ |\Delta K| = |K_f - K_i| = |2.25mv_0^2 - 3mv_0^2| = |-0.75\,mv_0^2| = 0.75mv_0^2$$

$$\Rightarrow \frac{K_f}{K_i} = \frac{2.25\cancel{mv_0^2}}{3\cancel{mv_0^2}} = \frac{2.25}{3} = 0.75$$

$$\Rightarrow \%\ \text{lost} = (1-0.75)\times 100\% = \boxed{25\%}$$

Problem #2

Two cars of identical mass m are involved in a collision. Both cars are moving at the same speed v_0. One of the cars is initially moving due north, and the other is initially moving south of west at an angle θ (where $45° < \theta < 90°$). The collision between the two vehicles is maximally inelastic; in other words, the vehicles stick together after the collision.



(a) Determine the magnitude of the final velocity of each car after the collision, in terms of the quantities m, v_0, and θ.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_f \implies \vec{v}_f = \frac{m_1}{m_1+m_2}\vec{v}_1 + \frac{m_2}{m_1+m_2}\vec{v}_2$$

$$\vec{v}_f = \frac{\cancel{m}}{2\cancel{m}}\langle 0, v_0, 0\rangle + \frac{\cancel{m}}{2\cancel{m}}\langle -v_0\cos\theta, -v_0\sin\theta, 0\rangle = \frac{v_0}{2}\langle 0,1,0\rangle + \frac{v_0}{2}\langle -\cos\theta, -\sin\theta, 0\rangle =$$

$$= \frac{v_0}{2}\langle -\cos\theta, 1-\sin\theta, 0\rangle$$

$$|\vec{v}_f| = \sqrt{\left(\frac{v_0}{2}\right)^2}\sqrt{(-\cos\theta)^2 + (1-\sin\theta)^2} = \frac{v_0}{2}\sqrt{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta} =$$

$$= \frac{v_0}{2}\sqrt{\cos^2\theta + \sin^2\theta + 1 - 2\sin\theta} = \frac{v_0}{2}\sqrt{1+1-2\sin\theta} =$$

$$= \boxed{\frac{v_0}{2}\sqrt{2-2\sin\theta}}$$