Physics 2211 GPS Week 11

Problem #1

After watching "The Big Lebowski" for the first time this summer, you and a friend get into an argument about how much ice to add when making the perfect white russian cocktail. You both agree that, for optimum taste, the cocktail should be enjoyed at 10 degrees Celsius. The two ingredients for the cocktail, cream and a "vodka & kahlua" mix, both leave the fridge at 15 degrees Celsius. Ice from a standard freezer is at a temperature of -10 degrees Celsius. If typical white russian calls for 0.06 L of cream and 0.14 L of the "vodka & kahlua" mix, how much ice is needed to bring the drink down to its optimum temperature?

Ice: density = 0.91 kg/L, C = 4.18 J/(Cg)
Mix: density = 0.8 kg/L, C = 2.44 J/(Cg)
Cream: density = 1 kg/L, C = 3.77 J/(Cg)

We assume that the ingredients are sufficiently isolated from their surroundings when they are mixed, so that a negligible amount of heat transfers to the surroundings. This means that the system is isolated and the energy principle predicts that

$$\Delta E_{sys} = 0.$$

Taking the initial and final state to be before the mixing process and after the combination has reached equilibrium, respectively, there are then no appreciable changes in kinetic or potential energy, and so

$$\Delta E_{therm,sys} = 0.$$

Each ingredient contributes to the total change in thermal energy:

$$\Delta E_{therm,sys} = \Delta E_{therm,mix} + \Delta E_{therm,cream} + \Delta E_{therm,\ ice}$$

$$= m_{mix}C_{mix}\Delta T_{mix} + m_{cream}C_{cream}\Delta T_{cream} + m_{ice}C_{ice}\Delta T_{ice} = 0.$$

Solving for the mass of the ice we find that

$$m_{ice} = -\frac{m_{mix}C_{mix}\Delta T_{mix} + m_{cream}C_{cream}\Delta T_{cream}}{C_{ice}\Delta T_{ice}}$$

$$= -\frac{\left(\left(\left(0.8\frac{kg}{L}\right)(0.14L)\left(2.44\frac{J}{gC^\circ}\right)\right)(10°C - 15°C) + \left(\left(1.0\frac{kg}{L}\right)(0.06L)\left(3.77\frac{J}{gC^\circ}\right)\right)(10°C - 15°C)\right)}{\left(4.18\frac{J}{g\,C^\circ}\right)(10°C - (-10°C))}$$

$$\approx 0.030\ kg = 30\ g$$

Note: The heat capacities and the densities do not use the same mass units, so in principle we could convert first. However, in this case there's no need because the same conversion factor appears in the numerator and the denominator so they cancel out.

Problem #2

During 3 hours one winter afternoon, when the outside temperature was 11° C, a house heated by electricity was kept at 25° C with the expenditure of 58 kwh (kilowatt·hours) of electric energy.

(a) What was the average energy leakage in joules per second (watts) through the walls of the house to the environment (the outside air and ground)?

✓ $\Delta t = (3\,hrs)(60\,min/1\,hr)(60\,sec/1\,min) = 10\,800\ sec$

✓ $\Delta E = Q = (58\,kWh)(1000\,W/1\,kW)(60\,min/1\,hr)(60\,sec/1\,min) = 2.088e8\ W\cdot s$

(same as Joules)

$\Rightarrow$ energy leakage $= \dfrac{\Delta E}{\Delta t} = \dfrac{2.088e8\ W\cdot s}{10800\ sec} = \boxed{1.93e4\ Watts}$

$$\text{Watt} = \frac{\text{Joule}}{\text{second}}$$

Alternative: $\dfrac{\Delta E}{\Delta t} = \dfrac{58\ kWh}{3\,h} = \dfrac{19.3\ kW \mid 1000\ W}{\mid 1\ kW} = \boxed{1.93e4\ Watts}$

(b) The rate at which energy is transferred between two systems due to a temperature difference is often proportional to their temperature difference. Assuming this to hold in this case, if the house temperature had been kept at 28° C (82.4° F), how many kwh of electricity would have been consumed?

energy transfer $\propto$ temperature difference $\Longrightarrow \dfrac{Q_1}{\Delta T_1} = \dfrac{Q_2}{\Delta T_2}$

$$\Longrightarrow \frac{58\ kWh}{(25-11)^\circ C} = \frac{Q_2}{(28-11)^\circ C}$$

$$\frac{58}{14} = \frac{Q_2}{17}$$

$$14 Q_2 = (58)(17)$$

$$Q_2 = \frac{(58)(17)}{(14)} = \boxed{70.4\ kW\cdot h}$$

Problem #3

Consider a system consisting of two particles connected by a spring of negligible mass:
$m_1 = 5$ kg, vector $\vec{v}_1 = \langle 5,\ -10,\ 15\rangle$ m/s
$m_2 = 10$ kg, vector $\vec{v}_2 = \langle -10,\ 0,\ -5\rangle$ m/s

(a) What is the total momentum $\vec{p}_{total}$ of this system?

$$\vec{P}_1 = m_1\vec{v}_1 = (5)<5,-10,15> = <25,-50,75> \text{ kg m/s}$$

$$\vec{P}_2 = m_2\vec{v}_2 = (10)<-10,0,-5> = <-100,0,-50> \text{ kg m/s}$$

$$\vec{P}_{total} = \vec{P}_1 + \vec{P}_2 = <25,-50,75> + <-100,0,-50> =$$

$$= \boxed{<-75,-50,25> \text{ kg m/s}}$$

(b) What is $\vec{V}_{CM}$, the velocity of the center of mass of this system?

$$\vec{P}_{total} = M_{total}\vec{V}_{cm} \Longrightarrow \vec{V}_{cm} = \frac{\vec{P}_{total}}{M_{total}}$$

$$\vec{V}_{cm} = \frac{<-75,-50,25>}{5+10} = \boxed{<-5,-3.3,1.7> \text{ m/s}}$$

(c) What is K_{trans}, the translational kinetic energy of this system?

$$K_{trans} = \frac{1}{2} M_{total} V_{cm}^2$$

$$\checkmark\ V_{cm}^2 = (-5)^2 + (-3.3)^2 + (1.7)^2 = 38.68$$

$$\Longrightarrow K_{trans} = \frac{1}{2}(15)(38.68) = \boxed{290\ \text{J}}$$

(d) What is K_{total}, the total kinetic energy of this system?

$$K_{total} = K_1 + K_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\checkmark\ v_1^2 = (5)^2 + (-10)^2 + (15)^2 = 350$$

$$\checkmark\ v_2^2 = (-10)^2 + (-5)^2 = 125$$

$$\Longrightarrow K_{total} = \frac{1}{2}(5)(350) + \frac{1}{2}(10)(125) = 875 + 625 = \boxed{1500\ \text{J}}$$

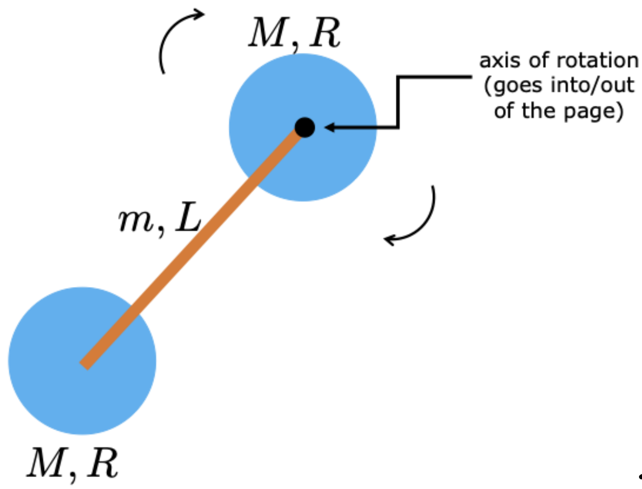
(e) What is K_{rel}, the kinetic energy of this system relative to the center of mass?

$$K_{total} = K_{trans} + K_{rel} \Longrightarrow K_{rel} = K_{total} - K_{trans}$$

$$K_{rel} = 1500 - 290 = \boxed{1210\ \text{J}}$$

Problem #4

A barbell is made up of two solid spheres of mass M and radius R whose centers are attached to the ends of a thin rod that has mass m and length L. The entire thing rotates about an axis that goes through the center of sphere 1. Determine the total moment of inertia of the barbell about this axis of rotation. Hint: remember the parallel axis theorem.



|| - axis theorem: $I_{\parallel\text{-axis}} = I_{CM} + Md^2$

(d = dist. b/w axis through CM and the || -axis)

$$I_{CM,\text{ solid sphere}} = \frac{2}{5}MR^2$$

$$I_{CM,\text{ thin rod}} = \frac{1}{12}mL^2$$

$$I_{\text{system}} = I_{\text{axis, sphere 1}} + I_{\text{axis, rod}} + I_{\text{axis, sphere 2}}$$

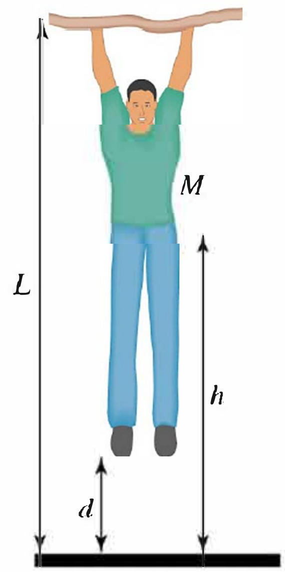
$$= \left[I_{CM,\text{ sphere 1}} + I_{\parallel\text{-axis, sphere 1}}\right] + \left[I_{CM,\text{ rod}} + I_{\parallel\text{-axis, sphere 2}}\right] + \left[I_{CM,\text{ sphere 2}} + I_{\parallel\text{-axis, sphere 2}}\right]$$

$$= \left[\frac{2}{5}MR^2 + M(0)^2\right] + \left[\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right] + \left[\frac{2}{5}MR^2 + M(L)^2\right]$$

$$\boxed{I_{\text{system}} = \frac{4}{5}MR^2 + \left(\frac{1}{3}m + M\right)L^2}$$

Problem #5

You hang by your hands from a tree limb that is a height $L = 6$ m above the ground, with your center of mass a height $h = 5$ m above the ground and your feet a height $d = 4$ m above the ground, as shown in the figure (not to scale). You then let yourself fall. You absorb the shock by bending your knees, ending up momentarily at rest in a crouched position with your center of mass a height $b = 0.25$ m above the ground. Your mass is $M = 110$ kg.

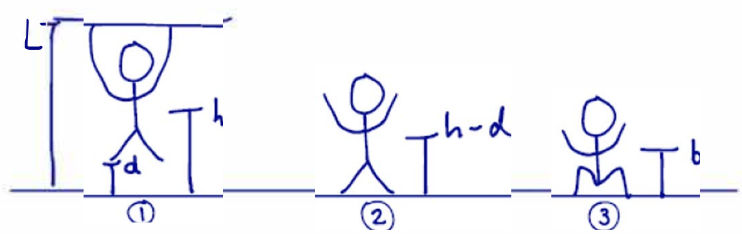


(a) Starting from the energy principle, find your speed just before your feet touch the ground.

System : point particle

Initial: (1)

Final: (2)



$\Delta E = \Delta K = W_{grav}$

$\frac{1}{2} m (v_f^2 - \cancel{v_i^2}) = \vec{F}_{grav} \cdot \Delta \vec{r}_{cm} = mg(-\hat{y}) \cdot d(-\hat{y})$

$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g d$

$v_f^2 = 2gd \Rightarrow v_f = \sqrt{2gd} = \sqrt{(2)(9.8)(4)} = \boxed{8.85 \text{ m/s}}$

(b) Starting from the energy principle (point particle model) and assuming that the contact force of the ground on your feet is constant, find the magnitude of the contact force during your landing.

Initial ②, final ③

$$\Delta E = \Delta K = W_{total} = \vec{F}_{net} \cdot \Delta \vec{r}_{cm}$$

$$\frac{1}{2} m (\cancel{v_f^2}^{\,0} - v_i^2) = (F_c - mg)[b - (h-d)]$$

$$-\frac{1}{\cancel{2}} m (\cancel{2} g d) = (F_c - mg)(b - h + d)$$

$$-mgd = (F_c - mg)(b - h + d)$$

$$F_c - mg = \frac{-mgd}{b-h+d}$$

$$F_c = mg - \frac{mgd}{b-h+d} = (110)(9.8) - \frac{(110)(9.8)(4)}{0.25 - 5 + 4} = \boxed{6827 \text{ N}}$$

(c) What is the (real) work done by the contact force?

$W_c = \vec{F}_c \cdot \Delta \vec{r} = 0$ b/c the floor doesn't move

(d) Starting from the energy principle (real model), find the change in your internal energy during landing.

Method #1
Initial ①, final ③

$$\Delta E = \cancel{\Delta K}^{\,0} + \Delta E_{int} = W_{grav} + \cancel{W_c}^{\,0}$$

$$\Delta E_{int} = -mg(b-h) = -(110)(9.8)(0.25 - 5) = \boxed{5121 \text{ J}}$$

Method #2
Initial ②, final ③

$$\Delta E = \Delta K + \Delta E_{int} = W_{grav} + \cancel{W_c}^{\,0}$$

$$\frac{1}{2} m (\cancel{v_f^2}^{\,0} - v_i^2) + \Delta E_{int} = -mg[b - (h-d)] = -mg(b - h + d)$$

$$-\frac{1}{\cancel{2}} m (\cancel{2} g d) + \Delta E_{int} = -mg(b-h) - mgd$$

$$\cancel{-mgd} + \Delta E_{int} = -mg(b-h) - \cancel{mgd}$$

$$\Delta E_{int} = -mg(b-h) = \boxed{5121 \text{ J}}$$