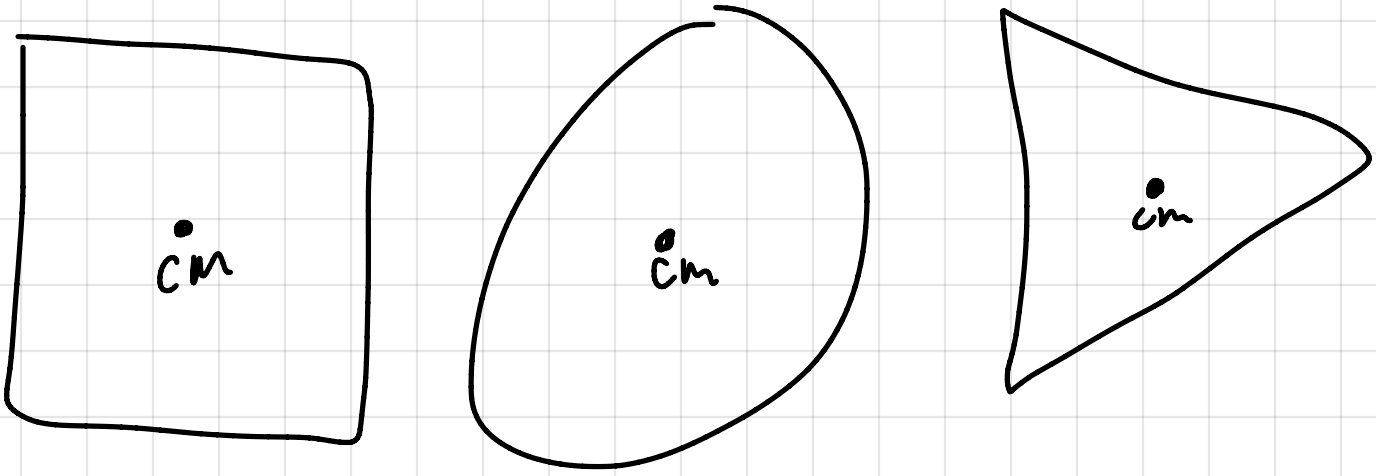
PHYS 2211, Summer 2021

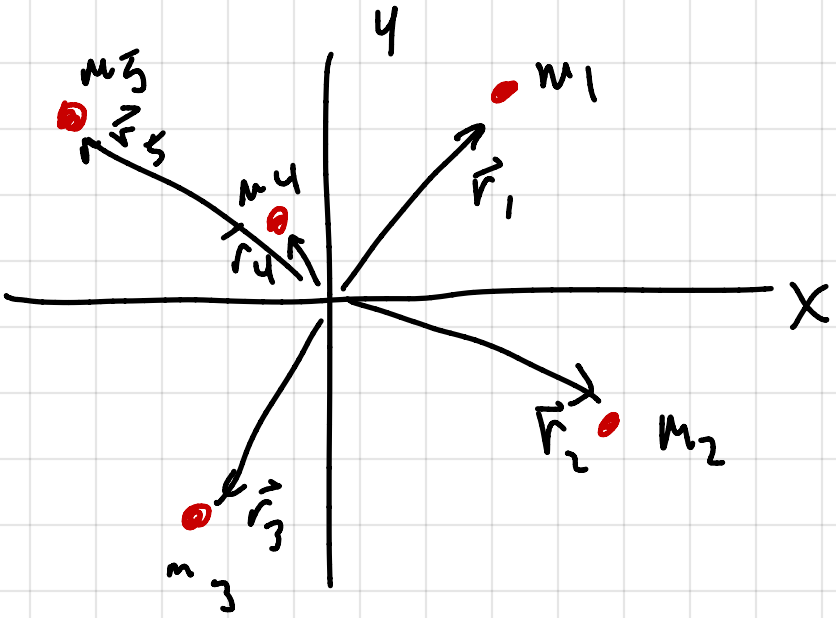
Week 8: Center of Mass & Multiparticle Systems

- ✓ center of mass
- ✓ translational kinetic energy (K_{trans})
- ✓ moment of inertia
- ✓ rotational kinetic energy (K_{rot})
- ✓ point particle vs real system
  - point particle → center of mass
  - real system → multi-particle

Center of Mass



uniform mass distribution ⇒ CM is geometric center



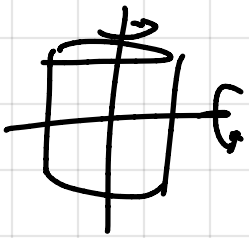
$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i} =$$

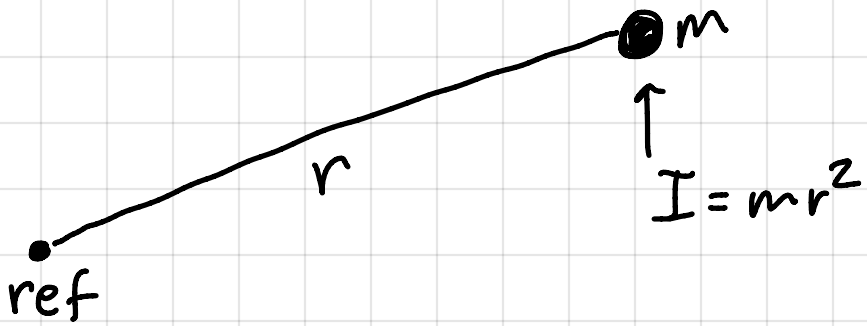
$$= \frac{m_1\vec{r}_1 + m_2\vec{r} + m_3\vec{r}_3 + m_4\vec{r}_4 + m_5\vec{r}_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

Moment of Inertia

$$I = m r^2$$

Solid $I = \frac{2}{5} m r^2$





Formula Sheet

Translational & Rotational Kinetic Energy

$$K_{trans} = \frac{1}{2} M_{total} V_{cm}^2 \leftarrow |\vec{V}_{cm}|^2 \rightarrow \text{linear motion}$$

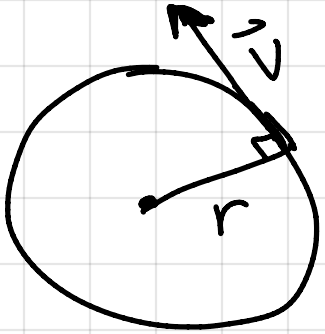
$$\vec{P}_{total} = \Sigma \vec{P}_i = M_{total} \vec{V}_{cm}$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

rotational motion

I: moment of inertia

ω: angular speed



$V = r\omega \Rightarrow \omega = \frac{V}{r}$

Point Particle vs Real System

center of mass → point particle

multiparticle → real system

$$\Delta E_{sys} = W_{surr}$$

$$\Delta K_{trans} = W_{cm}$$

$$\Delta K_{trans} = \vec{F}_{net} \cdot \Delta \vec{r}_{cm}$$

- * Net force
- * displacement of CM
- * translational kinetic energy

$$\Delta K_{trans} + \Delta K_{rot} + \Delta U_g + \Delta U_s + \Delta U_e + \Delta E_{th} + \dots = W_{real}$$

$$= \vec{F}_1 \cdot \Delta \vec{r}_1 + \vec{F}_2 \cdot \Delta \vec{r}_2 + \vec{F}_3 \cdot \Delta \vec{r}_3 + \dots$$

multiparticle

* real work is each force dotted with its own $\Delta \vec{r}$