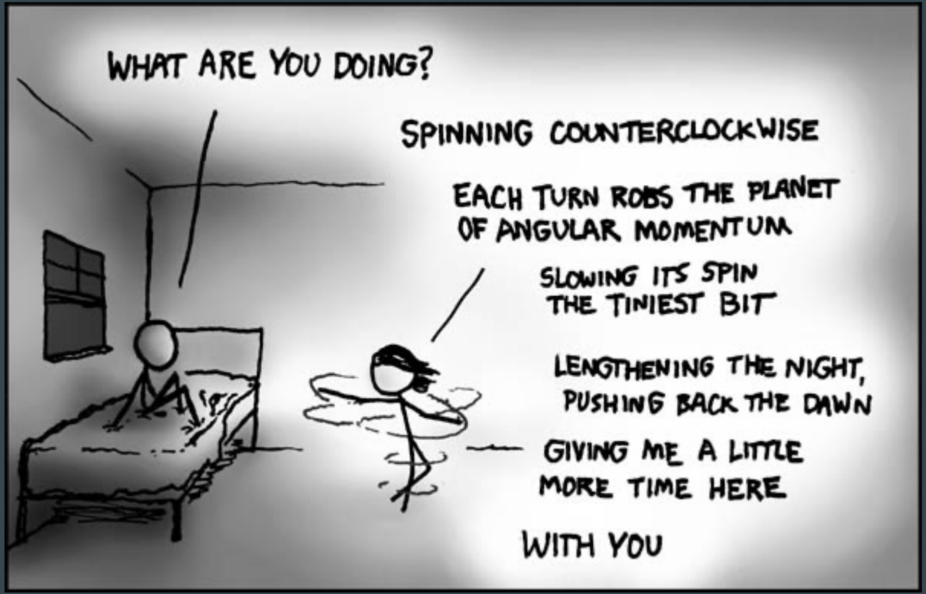


PHYS 2211 K

Week 14, Lecture 1

2022/04/12

Dr Alicea (ealicea@gatech.edu)

4 clicker questions today

On today's class…

1. Multiparticle angular momentum: L_{trans} and L_{rot}
2. Rotational angular momentum (multiparticle and rigid bodies)
3. The Angular Momentum Principle

Road map for the rest of the semester

- Week 14 ← you are here
  - Test 3 was yesterday!
  - Lecture topics: Angular momentum principle, multiparticle angular momentum, angular momentum of rigid systems
  - Lab 5 peer evals due at the end of the week (Sunday April 17)
- Week 15
  - Lecture topics: Wrapping up angular momentum; Quantum stuff
  - Hard deadline for **EVERYTHING** on Sunday April 24
- Week 16
  - (Optional) review session on Tuesday's class period (April 26)
  - Final exam on Friday April 29

CLICKER 1: How was the test?

A B C D E

Torque and angular momentum

- Torque: $\vec{\tau}_A = \vec{r}_A \times \vec{F}$
- Angular momentum for a single point mass: $\vec{L}_A = \vec{r}_A \times \vec{p}$ (translational)
- Total angular momentum for a multiparticle system: $\vec{L}_{\mathrm{total,A}} = \sum_i \vec{L}_{A,i}$

  and also → $\vec{L}_{\mathrm{total,A}} = \vec{L}_{\mathrm{trans,A}} + \vec{L}_{\mathrm{rot}}$
- Translational angular momentum for a multiparticle system: $\vec{L}_{\mathrm{trans,A}} = \vec{r}_{\mathrm{cm,A}} \times \vec{p}_{\mathrm{total}}$

Translational and Rotational

- The translational angular momentum of a multiparticle system with respect to some external reference point A is calculated as the cross product of the position vector of the CM with respect to point A and the total linear momentum of the system

$$\vec{L}_{\mathrm{trans,A}} = \vec{r}_{\mathrm{cm,A}} \times \vec{p}_{\mathrm{total}}$$

- The total angular momentum, which we already said is the sum of all the individual angular momentums, is also equal to the sum of the translational and rotational angular momentums:

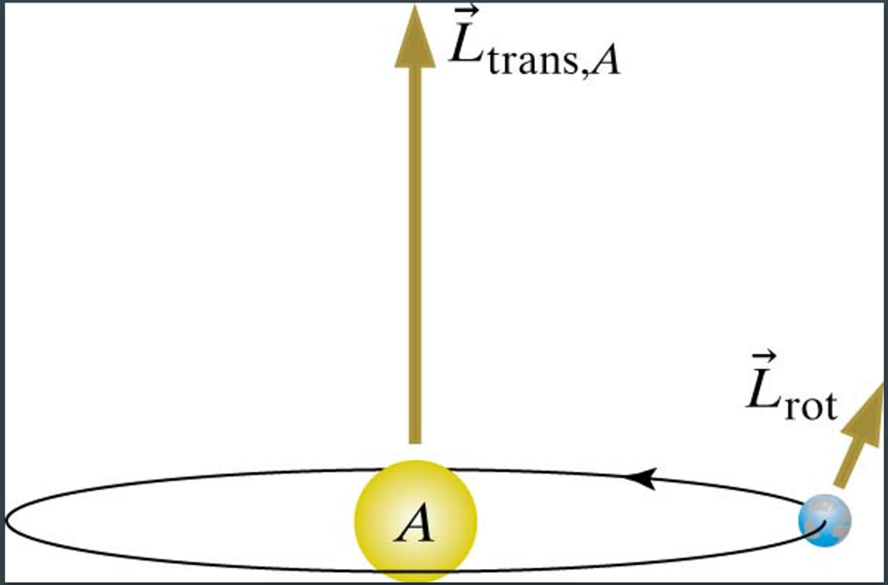
$$\vec{L}_{\mathrm{total,A}} = \vec{L}_{\mathrm{trans,A}} + \vec{L}_{\mathrm{rot}}$$

Translational and Rotational

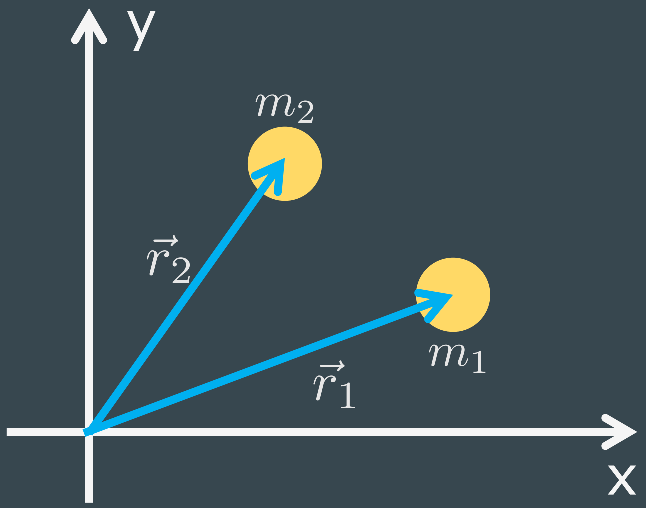
- Example: the Earth and the Sun, with the position of the Sun as the reference point

- L_{trans} for Earth is from its motion around the Sun (which causes the year) – orbit

- L_{rot} for Earth is from its motion about its own rotational axis (which causes the day/night cycle) – spin



Example: Find the rotational angular momentum of this two-mass system with respect to the origin.



$$m_1 = m_2 = 5 \text{ kg}$$
$$\vec{r}_1 = < 5, 2, 0 > \text{ m}$$
$$\vec{r}_2 = < 3, 4, 0 > \text{ m}$$
$$\vec{v}_1 = < 0, 2, 0 > \text{ m/s}$$
$$\vec{v}_2 = < 0, -1, 0 > \text{ m/s}$$

Procedure:

1. Find L1 and L2
2. Add them to find Ltotal
3. Find the CM
4. Determine ptotal
5. Find Ltrans for the system
6. Use Ltotal = Ltrans + Lrot to find Lrot

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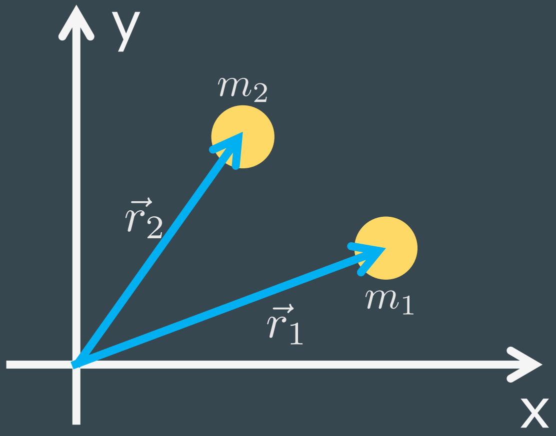

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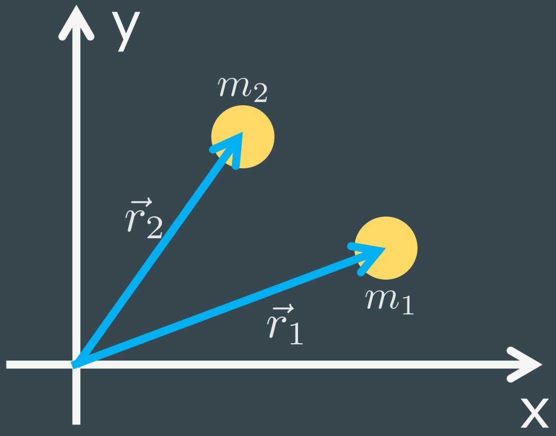

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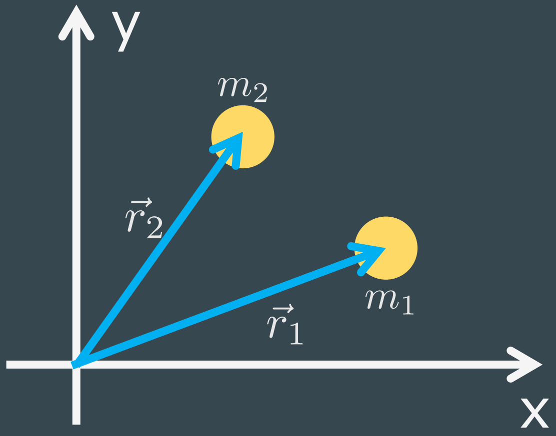

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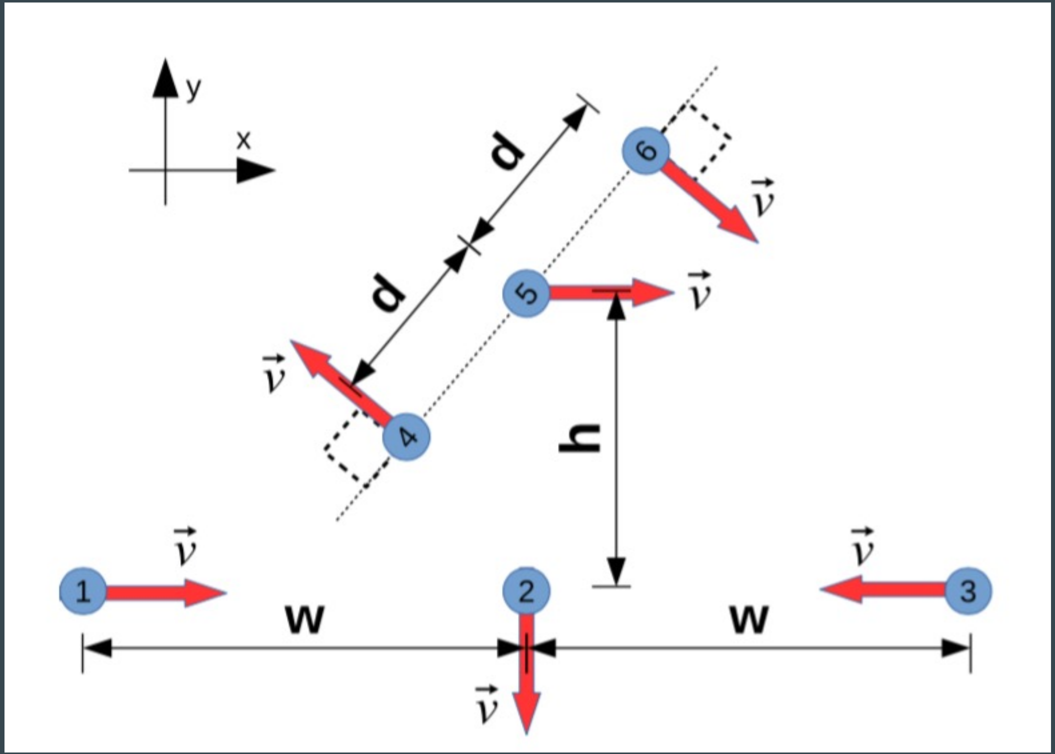
CLICKER 2: What is the translational angular momentum of this multiparticle system with respect to the location of particle 5? All masses are equal (m), and all speeds are equal (v).



A. (1/2) hmv (+zhat)

B. (1/2) hmv (-zhat)

C. hmv (+zhat)

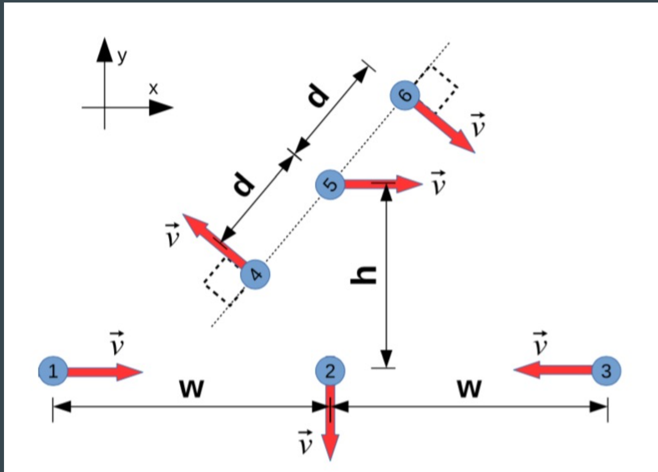
D. hmv (-zhat)

E. 2hmv (+zhat)

F. 2hmv (-zhat)

Reminder! $\vec{L}_{\text{trans,A}} = \vec{r}_{\text{cm,A}} \times \vec{p}_{\text{total}}$

Solution: What is the translational angular momentum of this multiparticle system with respect to the location of particle 5? All masses are equal (m), and all speeds are equal (v).

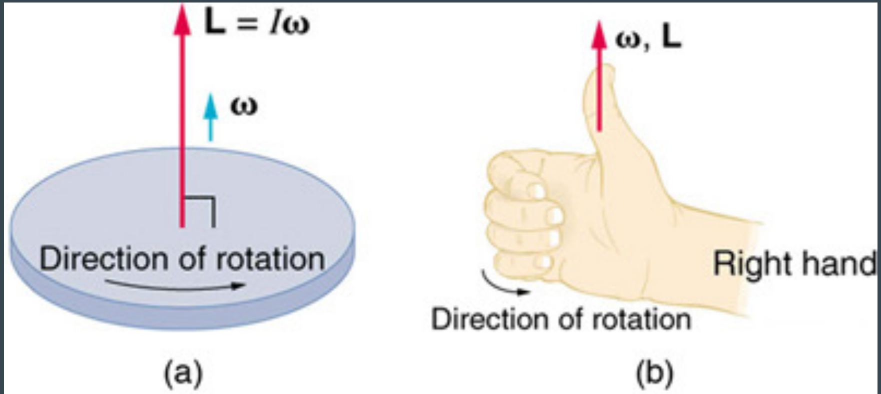


Rotational Angular Momentum

- For a rigid body: $\vec{L}_{\text{rot}} = I\vec{\omega}$ ← This can be used for systems of many point particles, and it is also equal to:
(i.e., a solid thing)

$$\vec{L}_{\text{rot}} = \sum(\vec{r}_{i,cm} \times \vec{p}_i)$$

- About that omega:
  - ω (not a vector) is angular speed
  - $\vec{\omega}$ (vector) is angular velocity
- The direction of the angular velocity is given by the right-hand-rule (same as with angular momentum)

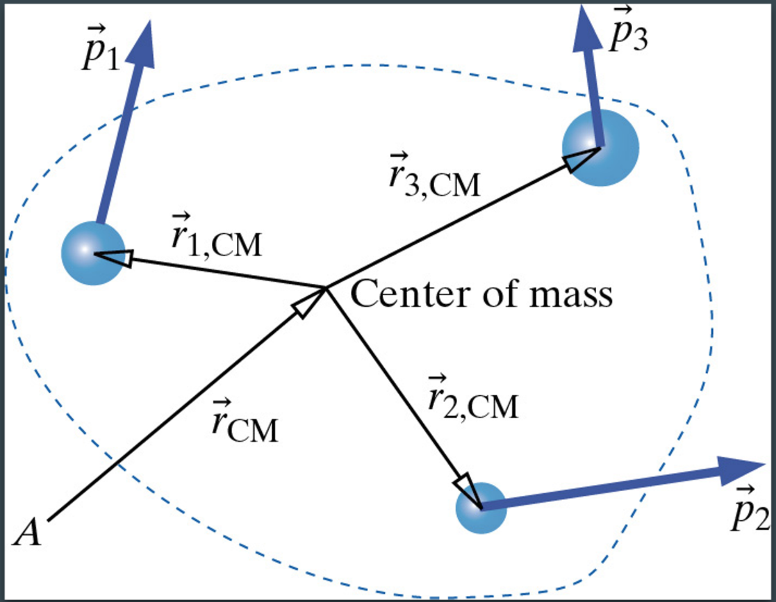


CLICKER 3: What is the magnitude of the rotational angular momentum of the Earth? R_E = 6.4e6 m, T = 24 hr, M_E = 6e24 kg, assume the Earth is a solid sphere so its moment of inertia is $I = (2/5)\ MR^2$.

A. 7.15e33 kg m^2/s
B. 1.14e33 kg m^2/s
C. 1.12e27 kg m^2/s
D. 2.57e37 kg m^2/s
E. None of the above

Where does $\vec{L}_{\text{rot}} = I\vec{\omega}$ come from?

Warning: long derivation ahead (but it's a fun one!)



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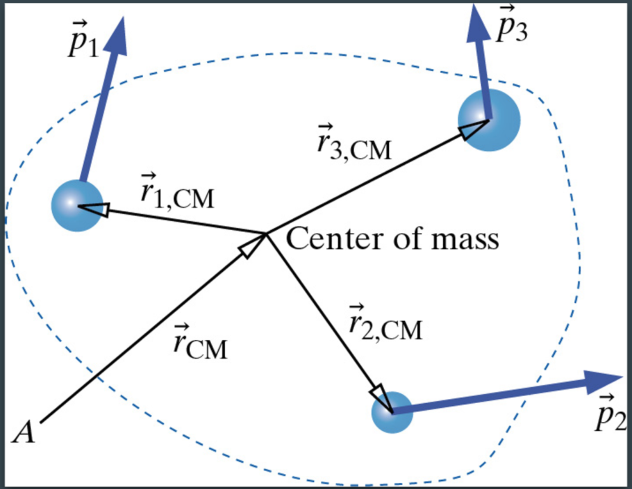

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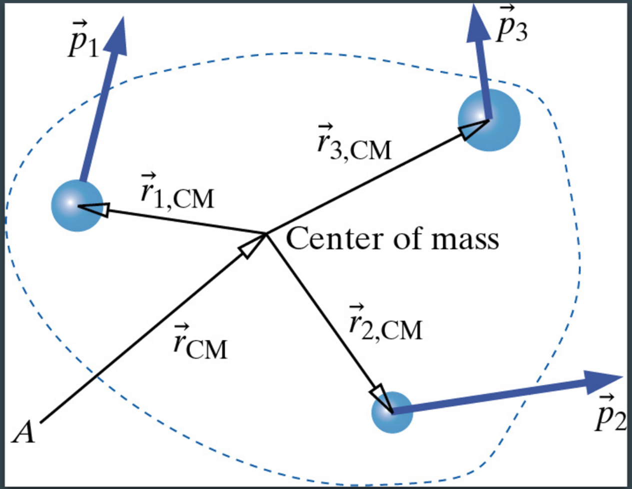


The Relationship between Torque and Changes in Angular Momentum

Start here: $\vec{L} = \vec{r} \times \vec{p}$ then differentiate with respect to time (remembering the product rule)

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

The Angular Momentum Principle

$$\frac{d\vec{L}_{\text{total}}}{dt} = \vec{\tau}_{\text{net}}$$

$$\vec{L}_f = \vec{L}_i + \vec{\tau}_{\text{net}}\Delta t$$

- The rate of change of the TOTAL angular momentum of the system equals the NET torque acting on the system
- This is our third fundamental principle (along with the Momentum Principle and the Energy Principle)

CLICKER 4: A barbell has two identical point masses m at the ends of a massless rod of length 2d. The whole thing spins counterclockwise about its CM with angular speed ω at t=0. Two forces act on the barbell as shown. What will be the angular momentum of the system about its CM a short time T later?

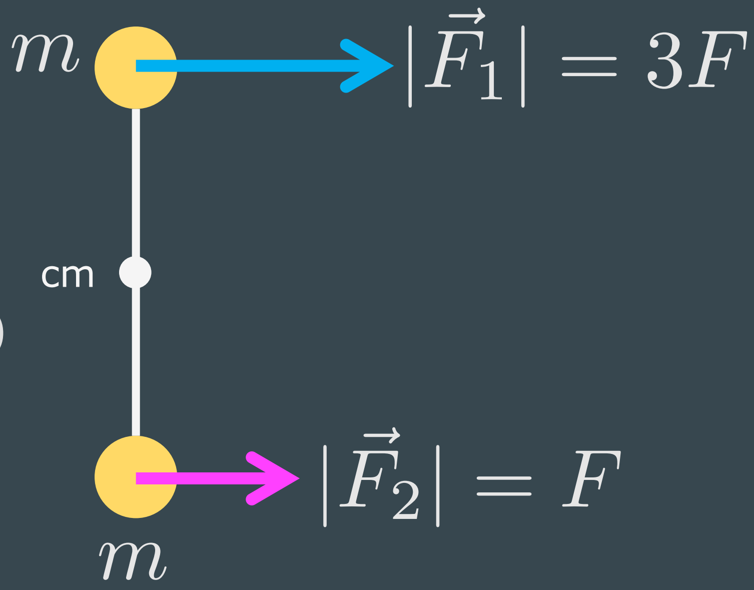
A. ($md^2\omega$ + dFT) (+zhat)

B. ($md^2\omega$ - dFT) (+zhat)

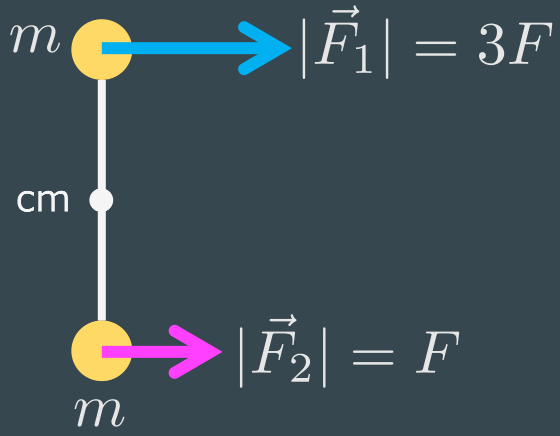
C. ($2md^2\omega$ – 2dFT) (+zhat)

D. ($2md^2\omega$ + 2dFT) (+zhat)

E. None of the above



Solution: A barbell has two identical point masses m at the ends of a massless rod of length 2d. The whole thing spins counterclockwise about its CM with angular speed ω_i at t=0. Two forces act on the barbell as shown. What will be the angular momentum of the system about its CM a short time T later?



The Angular Momentum Principle

- Can be decomposed into translational and rotational:

$$\frac{d\vec{L}_{\text{total}}}{dt} = \frac{d\vec{L}_{\text{trans}}}{dt} + \frac{d\vec{L}_{\text{rot}}}{dt}$$

$$\frac{d\vec{L}_{\text{total}}}{dt} = \vec{\tau}_{\text{trans}} + \vec{\tau}_{\text{rot}}$$

Translational and Rotational (torques)

Angular momentums

Torques

$$\vec{L}_{\text{trans,A}} = \vec{r}_{\text{cm,A}} \times \vec{p}_{\text{total}} \longleftrightarrow \vec{\tau}_{\text{trans}} = \vec{r}_{\text{cm,A}} \times \vec{F}_{\text{net}}$$

$$\vec{L}_{\text{rot}} = \sum(\vec{r}_{i,cm} \times \vec{p}_i) \longleftrightarrow \vec{\tau}_{\text{rot}} = \sum(\vec{r}_{i,cm} \times \vec{F}_i)$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

What do $\vec{\tau}_{\text{trans}}$ and $\vec{\tau}_{\text{rot}}$ mean?

- Translational torque is caused by the net external force acting on the center of mass of the system (with respect to some reference point A), and it causes a change in the $\vec{L}_{\text{trans}}$ of the system

$$\vec{\tau}_{\text{trans}} = \vec{r}_{\text{cm,A}} \times \vec{F}_{\text{net}}$$

- Rotational torque is caused by individual forces acting on the individual particles at their positions relative to the center of mass of the system, and it causes a change in the $\vec{L}_{\text{rot}}$ of the system

$$\vec{\tau}_{\text{rot}} = \sum(\vec{r}_{i,cm} \times \vec{F}_i)$$

Translational and Rotational (torques)

- If there's a torque, there will be a change in angular momentum (just like if there's a force, there's a change in velocity)
- If there's a rotational torque, the rotational angular momentum of the system will change
- If there's a translational torque, the translational angular momentum of the system will change
- Sometimes, we can select the reference point such that some or even ALL of the torques end up vanishing, leaving us with a much simpler problem to solve where $\vec{\tau} = 0$ (we'll see this on Thursday)

Linear Motion

Rotational Motion

$\vec{v} = \frac{d\vec{r}}{dt}$ velocity | angular velocity $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ acceleration | angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$

$\vec{p} = m\vec{v}$ momentum | angular momentum $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$\vec{F} = m\vec{a}$ force | torque $\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha}$

$\vec{F} = \frac{d\vec{p}}{dt}$ momentum principle | angular momentum principle $\vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$ momentum update | angular momentum update $\vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

$\vec{v}_f = \vec{v}_i + (\vec{F}/m)\Delta t$ velocity update | angular velocity update $\vec{\omega}_f = \vec{\omega}_i + (\vec{\tau}/I)\Delta t$