Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

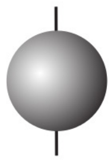
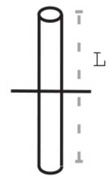
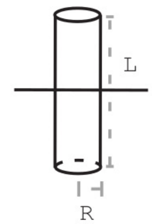
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/K/M/N/HP) - Fall 2023 - Test 3

Please clearly print your name & GTID in the lines below

Name: ______________________________ GTID: ______________________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format, and they must be **readable** in order to be graded. Unreadable files will earn a zero.
  - We recommend you upload a single PDF file for your entire work. You **must** indicate which page corresponds to each problem when you upload and submit.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

KEY

Sign your name on the line above

Problem 1: Thermal Energy [20 pts]

Georgia P. Burdell burns her mouth drinking a cup of 85°C hot coffee. She pours some of the coffee into the sink and fills the cup back up with 8°C cold milk. She then measures a final temperature of 60°C in the insulated cup of coffee and milk. **What is the ratio of milk to coffee in the cup?** The specific heat of coffee is $C_c = 4.2\,\mathrm{J/(g{\cdot}°C)}$, and the specific heat of milk is $C_m = 3.9\,\mathrm{J/(g{\cdot}°C)}$.

Thermal Energy

Want $\dfrac{m_{milk}}{m_{coffee}}$

$$\Delta E_{therm} = Q$$

$$m_{milk} C_{milk} \Delta T_{milk} + m_{coffee} C_{coffee} \Delta T_{coffee} = 0$$

$$\frac{m_{milk}}{m_{coffee}} C_{milk} \Delta T_{milk} + C_{coffee} \Delta T_{coffee} = 0$$

$$\frac{m_{milk}}{m_{coffee}} = -\frac{C_{coffee} \Delta T_{coffee}}{C_{milk} \Delta T_{milk}}$$

$$= -\frac{(4.2\ \mathrm{J/g\,°C})(60\,°C - 85\,°C)}{(3.9\ \mathrm{J/g\,°C})(60\,°C - 8\,°C)}$$

$$\approx \underline{0.518}$$

$$\boxed{\frac{m_{milk}}{m_{coffee}} \approx 0.518 \qquad m_{milk} \approx 0.518\, m_{coffee}}$$

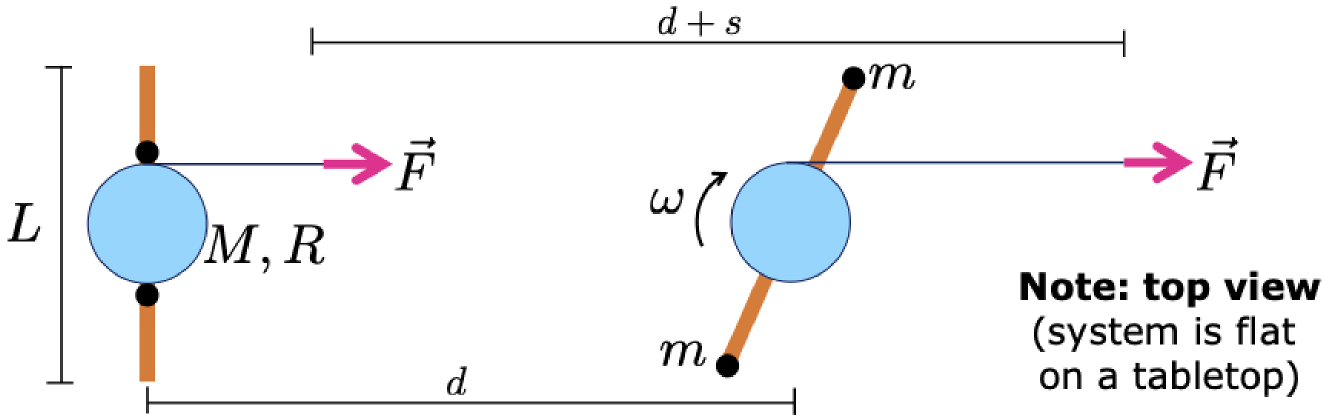
$$\boxed{\frac{m_{coffee}}{m_{milk}} \approx 1.931 \qquad m_{coffee} \approx 1.931\, m_{milk}}$$

4:31

Problem 2: Point Particle / Real System [30 pts]

A system consists of a **uniform solid disk** of mass M and radius R attached to the middle of a **massless rod** of length L. Two small **point masses** m rest on the rod next to the disk. The entire apparatus sits at rest on top of a table (see the left side of the figure).

A string is wound up around the disk, and you start pulling on the string with a constant force of magnitude F. The apparatus slides to the right along the frictionless tabletop. When the center of the disk has moved a distance d, a length of string s has come off the disk, the disk is spinning, and the two point masses have slid along the rod and got stuck at the ends of the rod (see the right side of the figure).



2.1 [10 pts] Use the **point-particle system** to find what is the speed v of the disk in the final state (right side of the figure).

Point-particle system: $\Delta\vec{r} = \Delta\vec{r}_{cm}$

$\Delta E = W_{net}$

$\Delta K_{trans} = \vec{F}_{net} \cdot \Delta\vec{r}_{cm}$

$\frac{1}{2} M_{tot} (v_f^2 - v_i^2) = (F\hat{x}) \cdot (d\,\hat{x})$ $\qquad v_i = 0$

$\frac{1}{2} M_{tot} v_f^2 = Fd$

$v_f = \sqrt{\frac{2Fd}{M_{tot}}} = \boxed{\sqrt{\frac{2Fd}{M+2m}}}$

3:30

2.2 [20 pts] Use the **real system** to find what is the angular speed ω of the disk in the final state (right side of the figure).

Real system: $\Delta\vec{r} = \Delta\vec{r}_{pc}$

$w_i = 0 \Rightarrow K_{rot,i} = 0 \Rightarrow \Delta K_{rot} = K_{rot,f} = \frac{1}{2} I_f w_f^2$

$I_f = I_{disk} + 2 I_{p.m.} = I_{disk,cm} + 2 I_{p.m.,\parallel}$

$= (\frac{1}{2} M R^2) + 2[m(\frac{L}{2})^2] = \underline{\frac{1}{2} M R^2 + \frac{1}{2} m L^2}$

$\Delta E = W_F$

$\Delta K_{trans} + \Delta K_{rot} = \vec{F} \cdot \Delta\vec{r}_{pc}$ From 2.1: $\Delta K_{trans} = F d$

$F d + \frac{1}{2} I_f w_f^2 = (F\hat{x}) \cdot ((d+s)\hat{x})$

$F d + \frac{1}{2} I_f w_f^2 = F(d+s)$

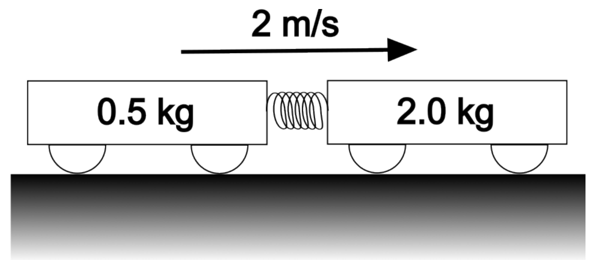
$\frac{1}{2} I_f w_f^2 = F s$

$w_f = \sqrt{\frac{2Fs}{I_f}} = \boxed{\sqrt{\frac{2Fs}{\frac{1}{2}MR^2 + \frac{1}{2}mL^2}} = \sqrt{\frac{4Fs}{MR^2 + mL^2}}}$

9:37

Problem 3: Collisions and Explosions [20 pts]

A small cart with mass $m_1 = 0.5$ kg (on the left) and another larger cart with mass $m_2 = 2.0$ kg (on the right) are attached together and moving to the right with a speed of $v_i = 2$ m/s. Suddenly a spring-loaded plunger pops out and blows the two carts apart from each other. The smaller cart shoots to the left at speed $v_{f1} = 2$ m/s.



3.1 [10 pts] What is the **final velocity** (speed and direction) of the larger cart, $\vec{v}_{f2}$?

Conservation of Momentum

$$\vec{p}_i = \vec{p}_f$$

$$(m_1 + m_2)\vec{v}_i = m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2}$$

$$\vec{v}_{f2} = \frac{1}{m_2}\left[(m_1+m_2)\vec{v}_i - m_1 \vec{v}_{f1}\right]$$

$$= \frac{1}{(2.0\text{ kg})}\left[(0.5\text{ kg} + 2.0\text{ kg})\langle 2\text{ m/s}, 0, 0\rangle - (0.5\text{ kg})\langle -2\text{ m/s}, 0, 0\rangle\right]$$

$$= \boxed{\langle 3, 0, 0\rangle\text{ m/s}}$$

3:48

3.2 [10 pts] If the stiffness of the spring in the plunger is $k = 25,000$ N/m, what was the **initial compression** of the spring before the explosion? Hint: use the Energy Principle and put everything (both carts and spring) in the system.

Energy Principle

System: cart 1, cart 2, spring

Surroundings: nothing

$$\Delta E = W$$

$$\Delta K_{trans} + \Delta U_s = 0$$

$$\frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2 - \frac{1}{2}(m_1+m_2)v_i^2 + \frac{1}{2}k(s_f^2 - s_i^2) = 0 \qquad s_f = 0$$

$$\frac{1}{2}k s_i^2 = \frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2 - \frac{1}{2}(m_1+m_2)v_i^2$$

$$s_i = \sqrt{\frac{1}{k}\left[m_1 v_{f1}^2 + m_2 v_{f2}^2 - (m_1+m_2)v_i^2\right]}$$

$$= \sqrt{\frac{1}{(25000\ N/m)}\left[(0.5\,kg)(2\,m/s)^2 + (2.0\,kg)(3\,m/s)^2 - (0.5\,kg + 2.0\,kg)(2\,m/s)^2\right]}$$

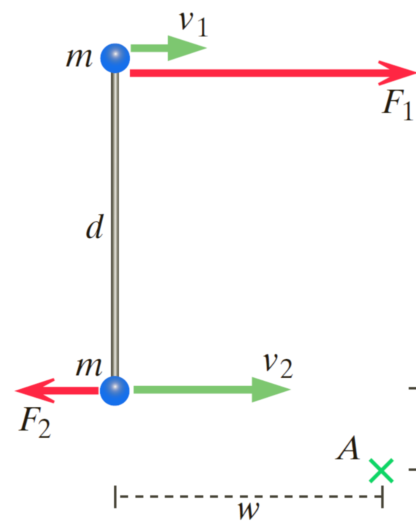
$$= \boxed{0.02\ m = 2\ cm}$$

9:44

Problem 4: Angular Momentum and Torque [30 pts]

Two identical point masses $m_1 = m_2 = 0.3$ kg are connected by a massless rod of length $d = 1$ m. At a particular instant, the two point masses have speeds $v_1 = 30$ m/s and $v_2 = 60$ m/s and are subjected to external forces $F_1 = 40$ N and $F_2 = 15$ N (see the diagram for the directions of the velocities and forces).

An external reference point A is located at a distance $h = 0.3$ m and $w = 0.7$ m below and to the right of the lower of the point masses. The entire system is moving out in space, far away from any other objects. You can assume a standard coordinate system with $+x$ pointing to the right, $+y$ upwards, and $+z$ out of the page.



4.1 [15 pts] Calculate $\vec{L}_A$, the **total angular momentum** of the system with respect to the external reference point A.

$$\vec{L}_A = \vec{L}_{1A} + \vec{L}_{2A} = (\vec{r}_{1A} \times \vec{p}_1) + (\vec{r}_{2A} \times \vec{p}_2)$$

$$\vec{r}_{1A} = \langle -w,\ h+d,\ 0\rangle \qquad \vec{p}_1 = \langle mv_1,\ 0,\ 0\rangle$$

$$\vec{r}_{2A} = \langle -w,\ h,\ 0\rangle \qquad \vec{p}_2 = \langle mv_2,\ 0,\ 0\rangle$$

$$\vec{r}_{1A} \times \vec{p}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -w & h+d & 0 \\ mv_1 & 0 & 0 \end{vmatrix} = \langle 0,\ 0,\ -(h+d)(mv_1)\rangle$$

$$\vec{r}_{2A} \times \vec{p}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -w & h & 0 \\ mv_2 & 0 & 0 \end{vmatrix} = \langle 0,\ 0,\ -(h)(mv_2)\rangle$$

$$\vec{L}_A = (\vec{r}_{1A} \times \vec{p}_1) + (\vec{r}_{2A} \times \vec{p}_2) = \langle 0, 0, -mv_1(h+d)\rangle + \langle 0, 0, -mv_2 h\rangle$$

$$= -[mv_1(h+d) + mv_2 h]\ \hat{z}$$

$$= -[(0.3\text{ kg})(30\text{ m/s})(0.3\text{ m} + 1\text{ m}) + (0.3\text{ kg})(60\text{ m/s})(0.3\text{ m})]\ \hat{z}$$

$$= \boxed{-17.1\text{ kg m}^2/\text{s}\ \hat{z}}$$

7:14

4.2 [15 pts] Calculate $\vec{\tau}_{\text{cm}}$, the **net torque** on the system with respect to the system's center of mass.

$$\vec{\tau}_{cm} = \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{r}_1 = \langle 0, \tfrac{d}{2}, 0 \rangle \qquad \vec{F}_1 = \langle F_1, 0, 0 \rangle$$

$$\vec{r}_2 = \langle 0, -\tfrac{d}{2}, 0 \rangle \qquad \vec{F}_2 = \langle -F_2, 0, 0 \rangle$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \frac{d}{2} & 0 \\ F_1 & 0 & 0 \end{vmatrix} = \langle 0, 0, -(\tfrac{d}{2})(F_1) \rangle$$

$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -\frac{d}{2} & 0 \\ -F_2 & 0 & 0 \end{vmatrix} = \langle 0, 0, -(-\tfrac{d}{2})(-F_2) \rangle$$

$$\vec{\tau}_{cm} = (\vec{r}_1 \times \vec{F}_1) + (\vec{r}_2 \times \vec{F}_2) = \langle 0, 0, -\tfrac{F_1 d}{2} \rangle + \langle 0, 0, -\tfrac{F_2 d}{2} \rangle$$

$$= -\left[\tfrac{d}{2}(F_1 + F_2)\right] \hat{z}$$

$$= -\left[\tfrac{(1\,m)}{2}(40\,N + 15\,N)\right] \hat{z}$$

$$= \boxed{-27.5\ Nm\ \hat{z}}$$

12:52

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