Week 8

Potential Energy

Topics for this week

1. Potential Energy
2. Gravitational Potential
3. Energy Graphs

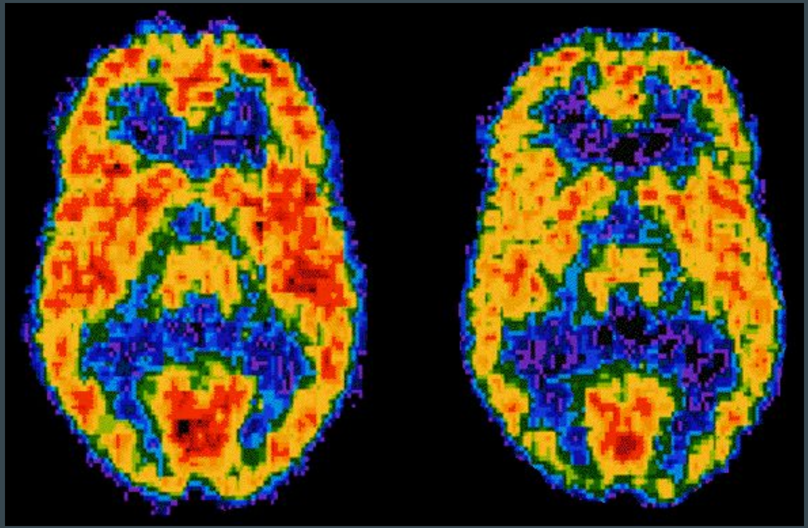
By the end of this week

1. Calculate potential energy changes
2. Compute gravitational potential
3. Graph energies in a system

Example: Positron Emission Tomography

Compounds containing unstable nuclei that emit positrons are introduced into the brain. When a positron is emitted, it goes only a short distance before coming nearly to rest and forming a bound state with an electron. These two particles annihilate, converting all of their rest mass energy into two photons (zero mass). These gamma rays are emitted at 180 degrees to each other.

What gamma ray energy should a detector be made sensitive to?



Example: PET Solution

- Look at the instant just before and just after the positron and electron combine

$$\Delta E = 0$$

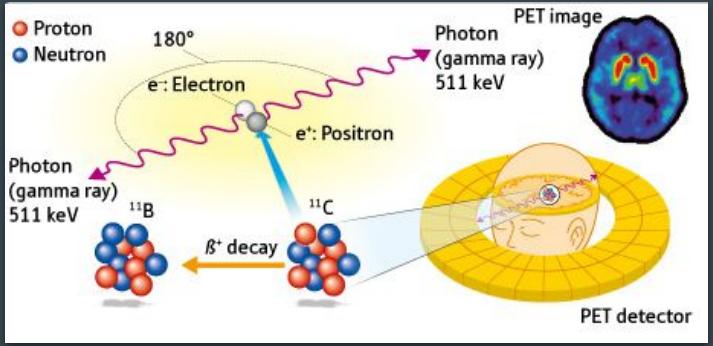
$$\Delta E_{rest,-} + \Delta E_{rest,+} + \Delta E_{\gamma} = 0$$

$$(0 - E_{rest,-}) + (0 - E_{rest,+}) + (2E_{\gamma} - 0) = 0$$

$$2E_{\gamma} = (m_{-} + m_{+})c^2$$

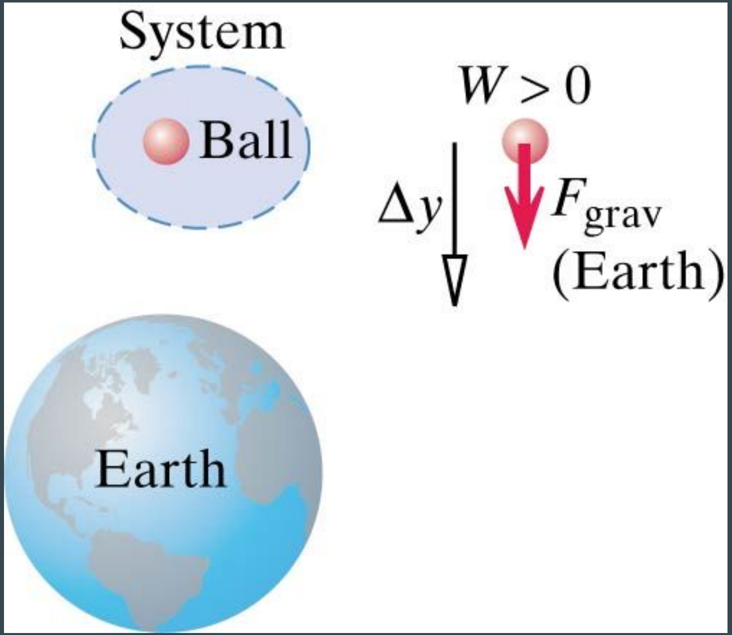
$$E_{\gamma} = \frac{1}{2}(2 * m_{electron})c^2$$

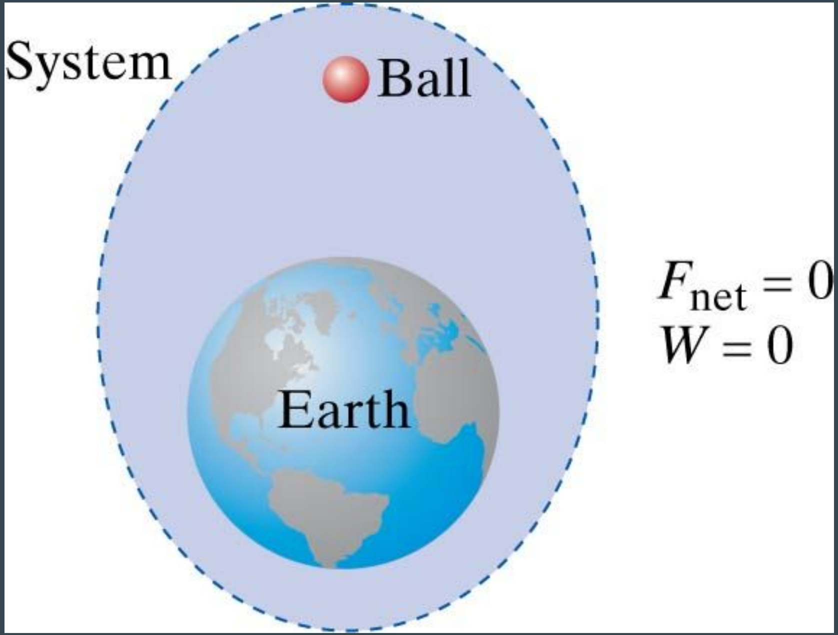
$$E_{\gamma} = 8.1 \times 10^{-14}\ J = 506\ keV$$

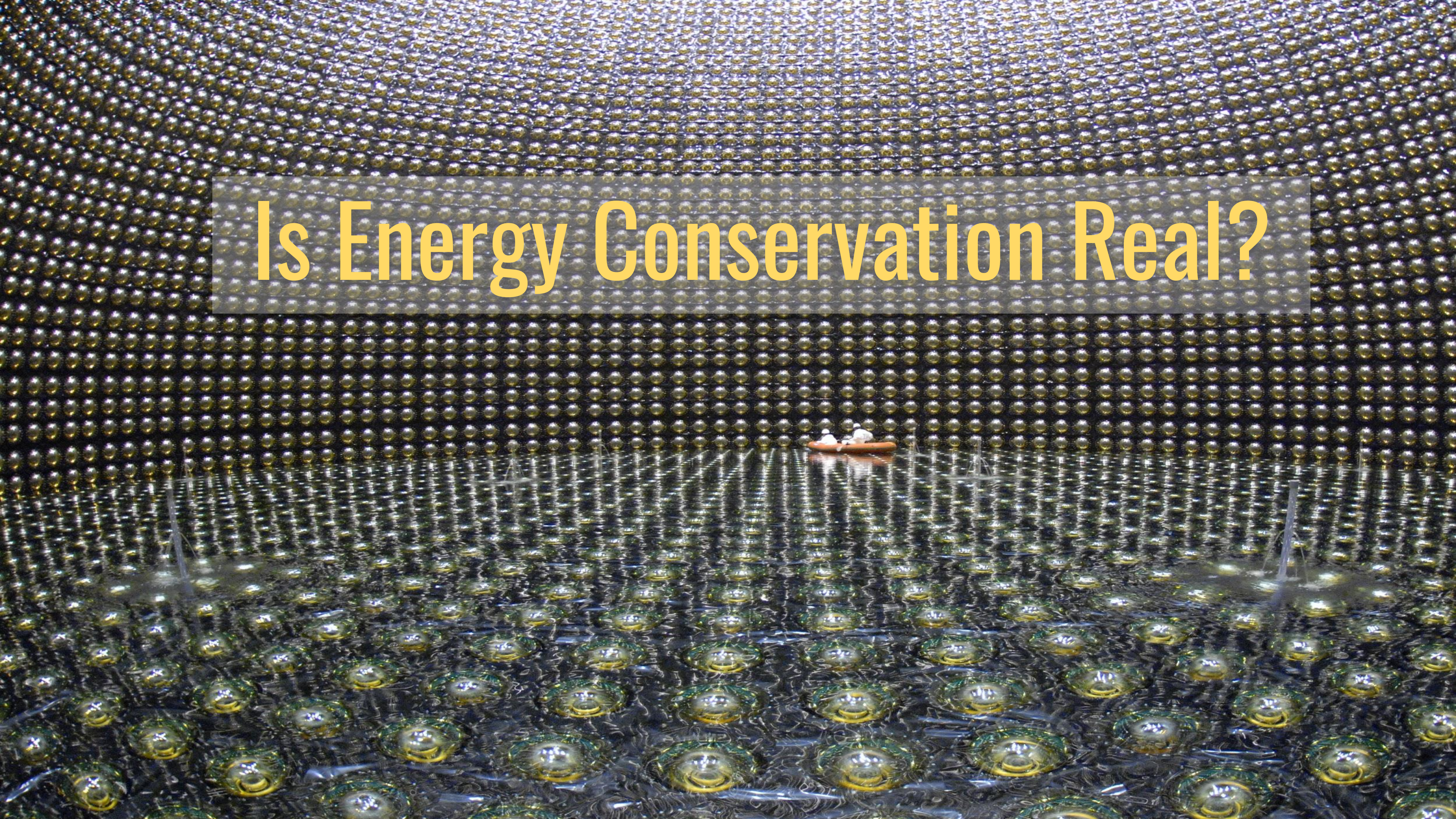


A puzzle

- Consider a ball initially at rest that begins to fall towards the Earth
  - In one system is the ball alone
  - In the other system is the ball and the Earth

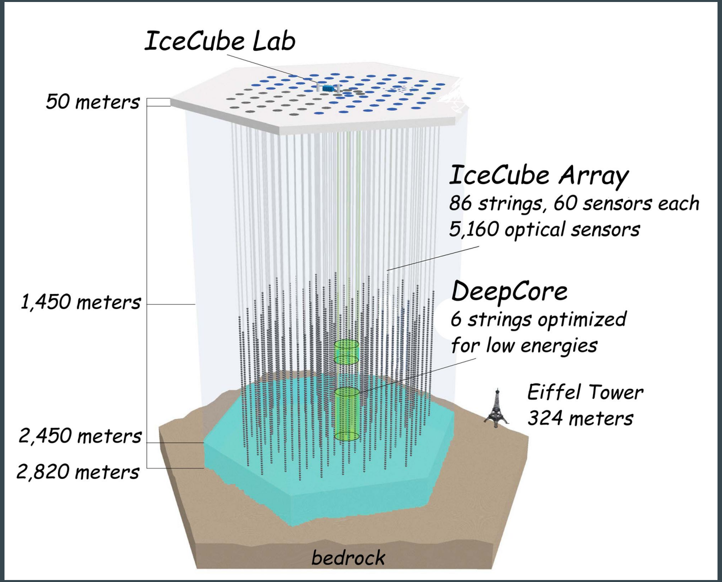





Is Energy Conservation Real?

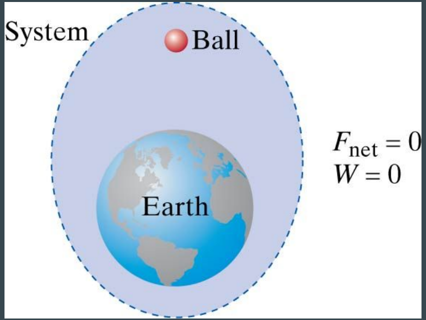
Yes, energy conservation is real!

- Not only are neutrinos real but we have a team here at Tech that have built a telescope to observe them at the south pole



Potential Energy

- Energy that is associated with pairs of interacting objects inside a system
- Common types of potential energy
    - Compressed or stretched springs
    - Galaxies of stars interacting gravitationally
    - Atoms in which the protons and electrons interact electrically
- Changes in interaction energy are associated with changes in shape of a multiparticle system
- A change in potential energy is negative internal work



$$\Delta E = \Delta K + \Delta U = 0$$

$$\Delta U \equiv -W_{int}$$

Energy of a multiparticle system

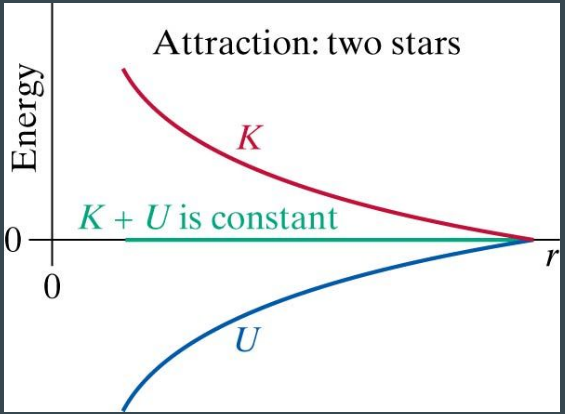
- The sum of the rest energies and the kinetic energies of each particle in the system, plus the sum of the potential energies due to the interactions of all pairs of particles in the system

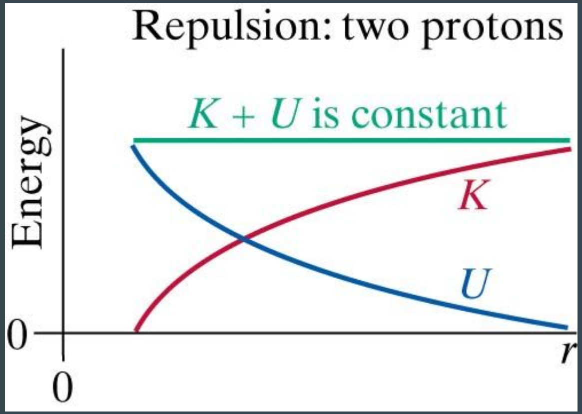
$$E_{sys} = (m_1 c^2 + m_2 c^2 + \ldots) + (K_1 + K_2 + \ldots) + (U_{12} + \ldots)$$

- Choose your system and be consistent!
  - Be careful not to double count
  - Forces internal to the system do no externals work
    - Internal forces have potential energy
- Potential energy depends on the separation between pairs of particles, not on their individual positions
  - Any change of potential energy is associated with a change of shape for the system

Properties of Potential Energy

- Potential energy must approach zero as the separation between pairs of particles becomes very large





    - For an attractive interaction, potential energy becomes negative as the separation distance decreases
    - For a repulsive interaction, potential energy becomes positive as the separation distance decreases

Gravitational Potential Energy

- The work done on mass m_1 by mass m_2

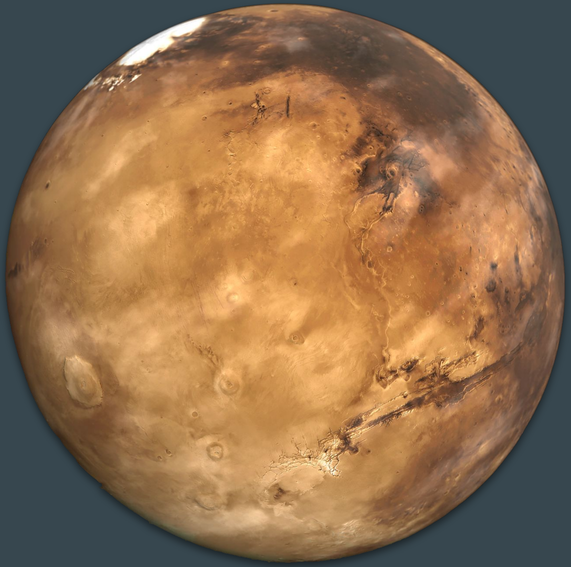
$$U_{grav} = -\int \vec{F}_{grav} \cdot d\vec{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{r}$$

- If two objects are attracted to each other due to a gravitational force the potential energy of the objects decreases
    - This can be exploited in many ways!

Example: Escape from Mars

The radius of Mars is 3400 km and its mass is 6e23 kg. After nearly dying, Matt Damon launches an escape pod straight up from the surface of mars. What initial speed of the pod is needed so that when the probe is far from Mars it has a final speed is 2000 m/s?



Example: Mars solution

- Use the energy principle
  - Mars and the pod in our system and ignore everything else in the surroundings
  - Initial state is at the surface of Mars with initial unknown speed
  - Final state is far from Mars with given final speed.

$$\Delta E_{rest} + \Delta K + \Delta U = 0$$

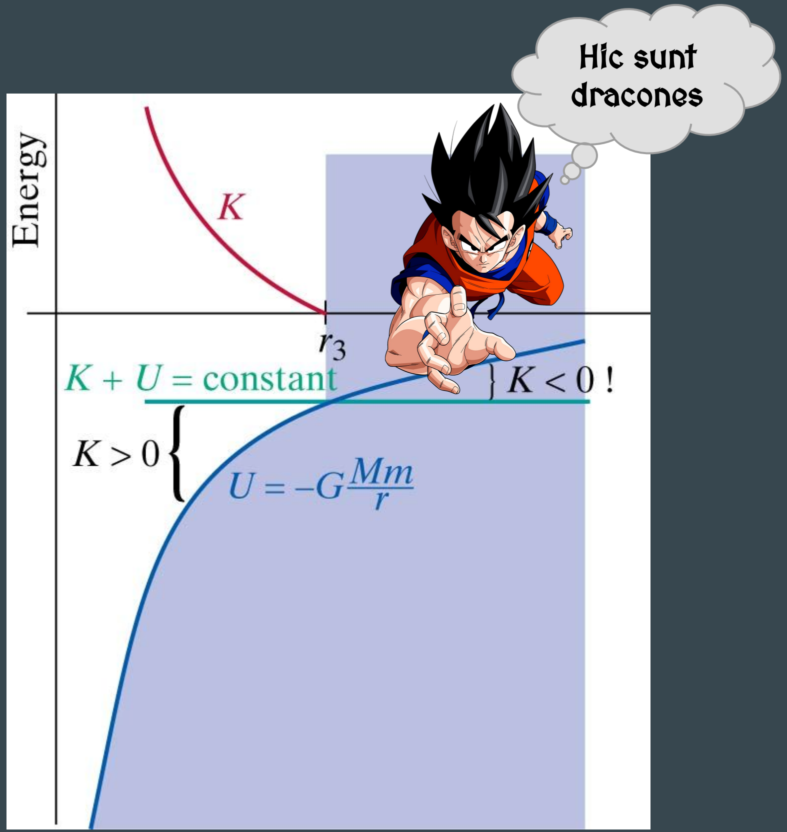
$$\Delta K_{Mars} + \Delta K_{pod} + \Delta U_{grav} = 0$$

$$\frac{1}{2} m_p \left(v_f^2 - v_i^2 \right) + G \frac{m_p m_M}{r_i} = 0$$

$$v_i = \sqrt{v_f + 2G \frac{m_M}{R_M}} = 5258\ m/s$$

Plotting energy

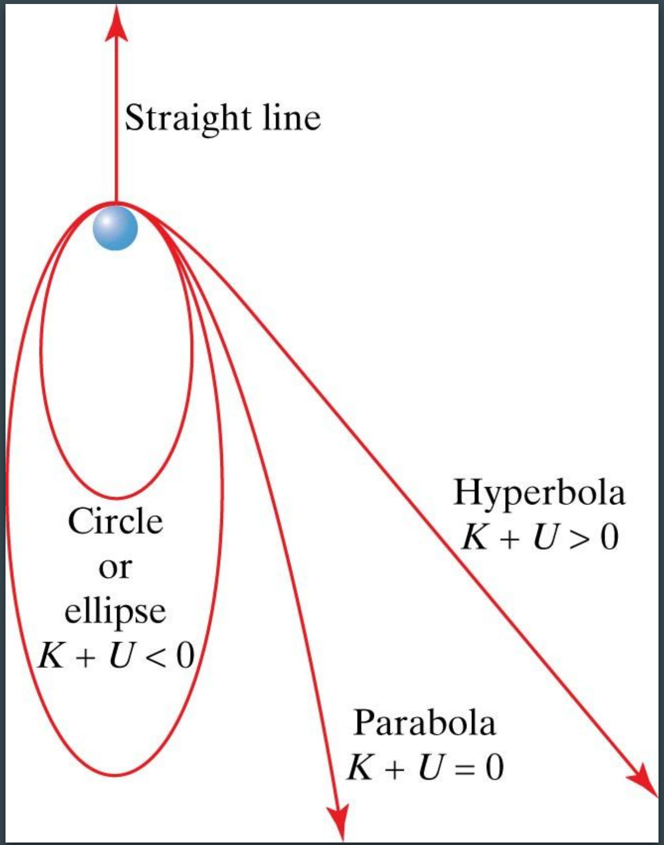
- Plotting energy versus separation distance can be helpful in understanding a broad range of dynamics
  - Can indicate limits on allowable motion
  - If a system is bounded or unbounded
- Draw U(r) vs r for a particular interaction
- Plot the total energy K+U at some point (a straight line)
- Plot K for all separation distances
  - K can never be negative



Energy Graphs

- Energy diagrams can provide us with information about the limits and types of motion a system will exhibit

1. When the total energy is less than zero the system is bounded
2. When the total energy is greater than or equal to zero the system is unbounded



Gravitational Potential near Earth

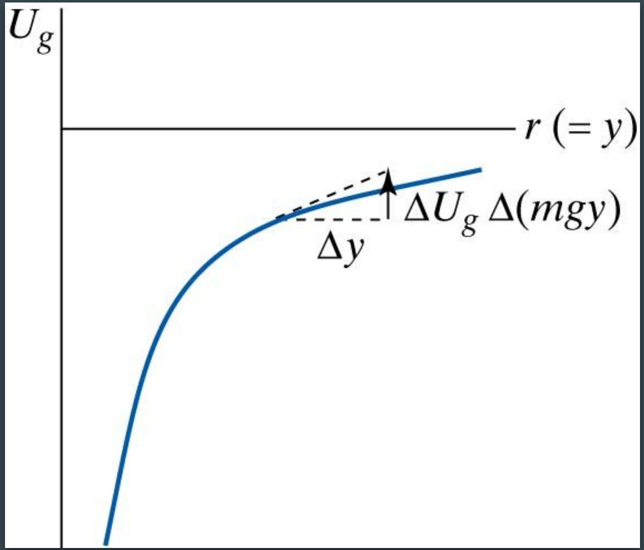
- Consider an object of mass "m" starting at the surface of the Earth and ends at height "h"
  - The change in gravitational potential is given by

$$\Delta U = -GmM_{Earth}\left(\frac{1}{R+h} - \frac{1}{R}\right)$$

$$\Delta U = \frac{GmM_{Earth}h}{R}\left(\frac{1}{(R+h)}\right) \approx mgh$$

- Moving away from the Earth corresponds to an increase in potential energy

$$\Delta U = mg\Delta y$$



The Electric Potential

- The electric force law for two charged particles has the same r^{-2} dependence as the gravitational force law

$$U_{grav} = -G\frac{m_1 m_2}{r} \quad \Longrightarrow \quad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- Why doesn't the electric potential have a minus sign?
  - Mass is always positive but charges can be positive or negative
- The ionization energy for a neutral atom is the minimum energy required to for an electron to become unbounded from the atom (about 14 eV)