PHYS 2211 K

Week 10, Lecture 1

2022/03/15

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2 clicker questions today

On today's class…

1. Center of mass
2. Translational kinetic energy
3. Relative kinetic energy
4. Spinny stuff: angular speed, moment of inertia, rotational kinetic energy

CLICKER 1: Best Star Wars Trilogy?

A. Prequels

B. OT

C. Sequels

Energy Review

- Energy principle: $\Delta E = W + Q$
- Expanded with all the energies we know so far:

$$\Delta K + \Delta E_{\text{rest}} + \Delta U_g + \Delta U_e + \Delta U_s + \Delta E_{\text{th}} =$$

$$= \underbrace{\left(\vec{F}_{\text{net}} \cdot \Delta \vec{r}\right)}_{} + Q$$

This can only be done when the forces all act on the center of mass of the system; if they don't, then we need to do other stuff (this last bit will be on Thursday)

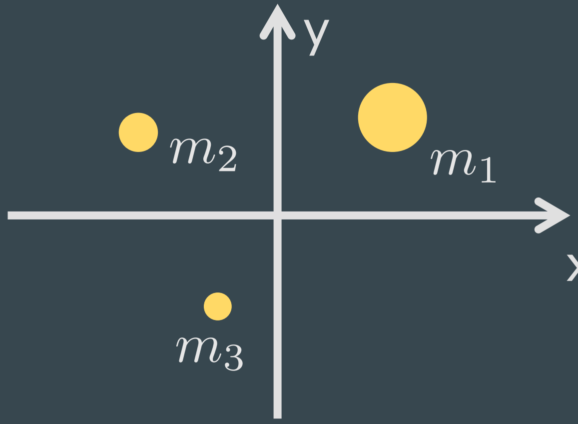
Problem here: this assumes we CAN add up all the forces into F_{net} and that they all act over the same displacement

Center of mass

- The thing we use when we represent an object or system as a point
- Located in the geometric center for regularly shaped objects that have uniform mass distribution
- For multi-particle systems, the position of the center of mass is given by:

$$\vec{r}_{\mathrm{cm}} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

Example: Find the position of the center of mass of this three-particle system, if m_1 = 8 kg, m_2 = 3 kg, and m_3 = 2 kg



$$\vec{r}_1 =< 3, 3, 0 > \text{m}$$

$$\vec{r}_2 =< -4, 2, 0 > \text{m}$$

$$\vec{r}_2 =< -2, -3, 0 > \text{m}$$

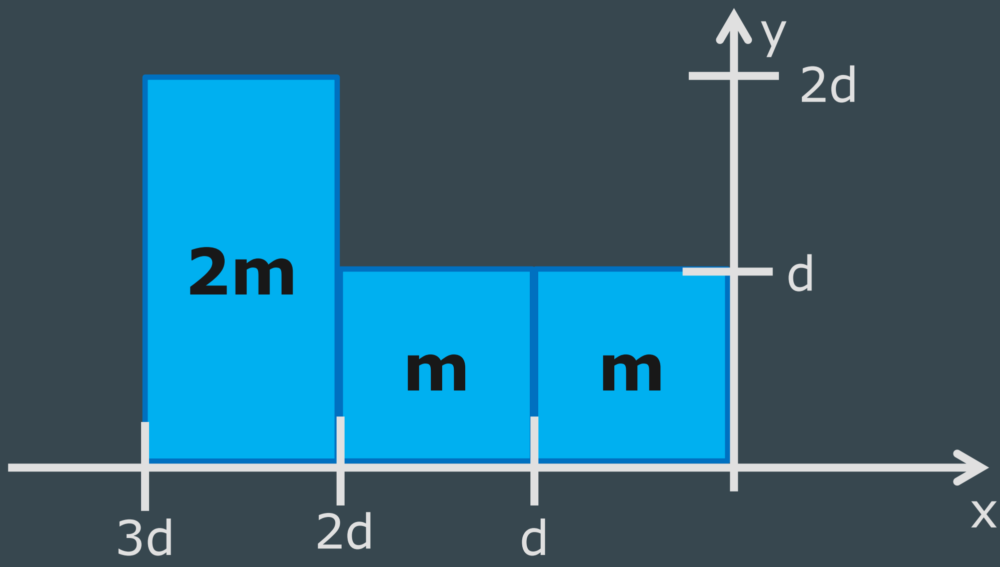
CLICKER 2: Find the position of the center of mass of this three-block system. All blocks have uniform density. Each square block has sides of length d, and the rectangle's sides are d and 2d.

A. $\vec{r}_{cm}$ = < -d, 0.75d, 0 >

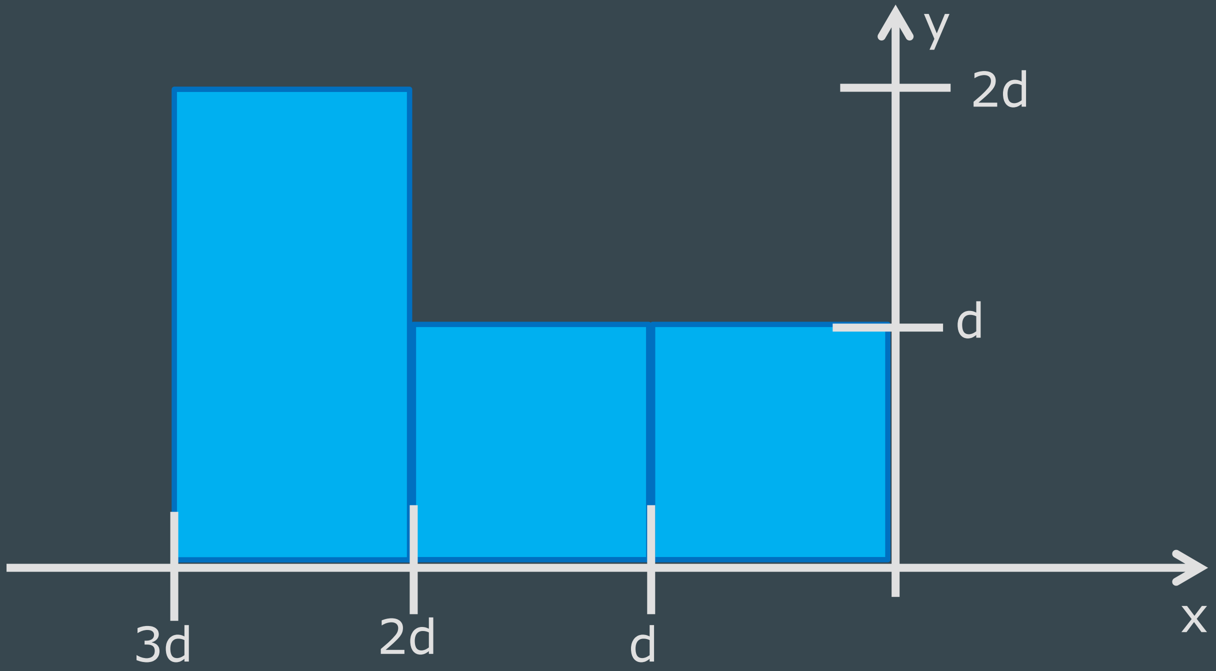
B. $\vec{r}_{cm}$ = < -1.5d, d, 0 >

C. $\vec{r}_{cm}$ = < -1.75d, 0.75d, 0 >

D. $\vec{r}_{cm}$ = < -2d, d, 0 >



Solution: Find the position of the center of mass of this three-block system. All blocks have uniform density. Each square block has sides of length d, and the rectangle's longer side has length 2d.



Solution: Find the position of the center of mass of this three-block system. All blocks have uniform density. Each square block has sides of length d, and the rectangle's longer side has length 2d.

$$\vec{r}_1 =$$

$$\vec{r}_2 =$$

$$\vec{r}_3 =$$

$$\vec{r}_{\mathrm{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} =$$

Motion of the center of mass

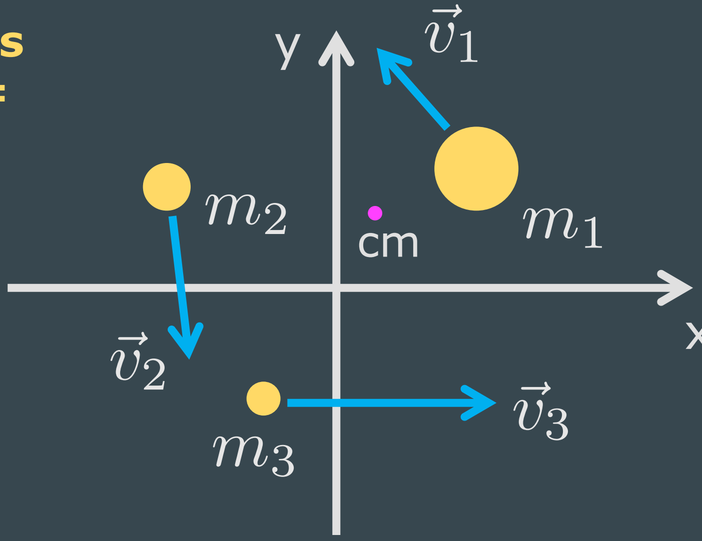
- The energy associated with the motion of the center of mass is the translational kinetic energy of the system (K_{trans})

$$K_{\text{trans}} = \frac{1}{2} M_{\text{total}} v_{\text{cm}}^2$$

- This is what we've been calling "kinetic energy" so far – but now the distinction is important because we'll learn about other types of kinetic energy soon
- Similarly, the total momentum of a multiparticle system is the momentum of the center of mass, and it also equals the sum of all the momentums of all the particles in the system

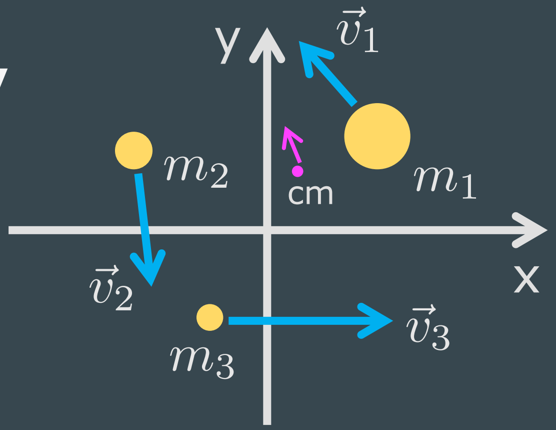
$$\vec{p}_{\text{cm}} = M_{\text{total}} \vec{v}_{\text{cm}}$$

Example: Find the velocity of the center of mass of this three-particle system, if m_1 = 8 kg, m_2 = 3 kg, and m_3 = 2 kg



$$\vec{v}_1 =< -2, 2, 0 > \mathrm{m/s}$$
$$\vec{v}_2 =< 1, -3, 0 > \mathrm{m/s}$$
$$\vec{v}_3 =< 4, 0, 0 > \mathrm{m/s}$$

Translational Kinetic Energy



- For this system we now know:

$$M_{\text{total}} = 13\ \text{kg}$$

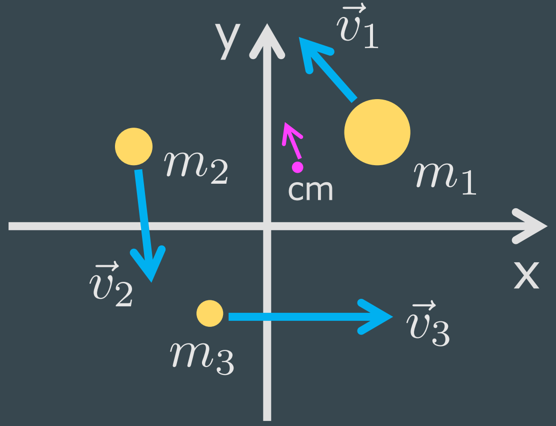
$$\vec{r}_{\text{cm}} =< 0.62, 1.85, 0 > \ \text{m}$$

$$\vec{v}_{\text{cm}} =< -0.38, 0.54, 0 > \ \text{m/s}$$

- The translational kinetic energy of the system is then:

Total Kinetic Energy

- To get the total kinetic energy of the system, we need to add up the kinetic energy of each particle that makes up the system



m_1 = 8 kg $\quad \vec{v}_1 =< -2, 2, 0 > \mathrm{m/s}$

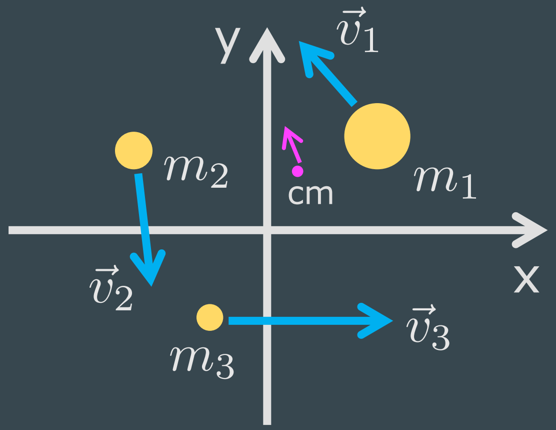
m_2 = 3 kg $\quad \vec{v}_2 =< 1, -3, 0 > \mathrm{m/s}$

m_3 = 2 kg $\quad \vec{v}_3 =< 4, 0, 0 > \mathrm{m/s}$

Wait a second...

$$K_{\text{trans}} = 2.83 \text{ J}$$

$$K_{\text{total}} = 63 \text{ J}$$



Why are these different? Because the system is in motion AND there is motion within the system!

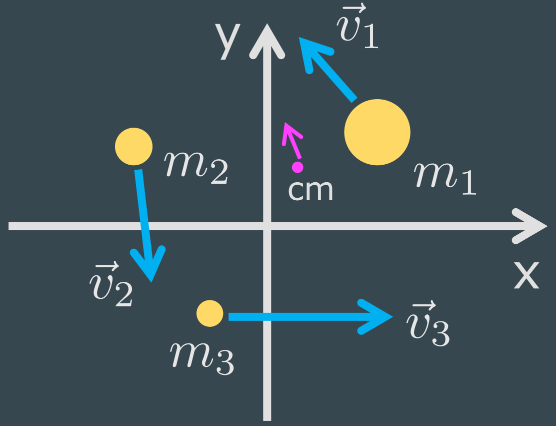
- The motion of the center of mass leads us to K_{trans}
- The difference between K_{total} and K_{trans} is the relative kinetic energy

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rel}}$$

K_{rel} for this system

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rel}}$$

$$\rightarrow K_{\text{rel}} = K_{\text{total}} - K_{\text{trans}}$$



This idea of breaking down the motion of a complex system into what the CM is doing (translation) and what's happening around the CM (e.g., rotation), is something that we'll be seeing a lot from now on

What can go into K_{rel}?

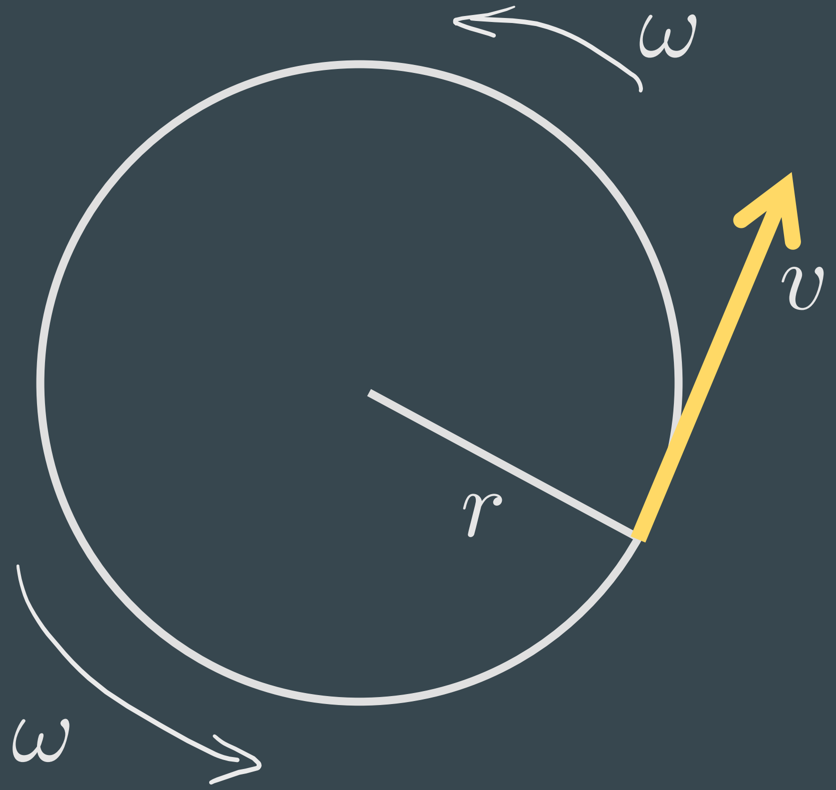
- Any kind of motion that is relative to the center of mass of the system, namely:
  - Vibrations
  - Rotations

- When there is rotation about the center of mass, then the system has rotational kinetic energy, K_{rot}

- K_{rot} can be calculated directly if you know the system's angular speed and its moment of inertia

$$K_{\mathrm{rot}} = \frac{1}{2} I \omega^2$$

Angular Speed



Linear (tangential) speed of a point in the circumference of a circle of radius r

$$v = \frac{2\pi r}{T} = \omega r$$

period of rotation

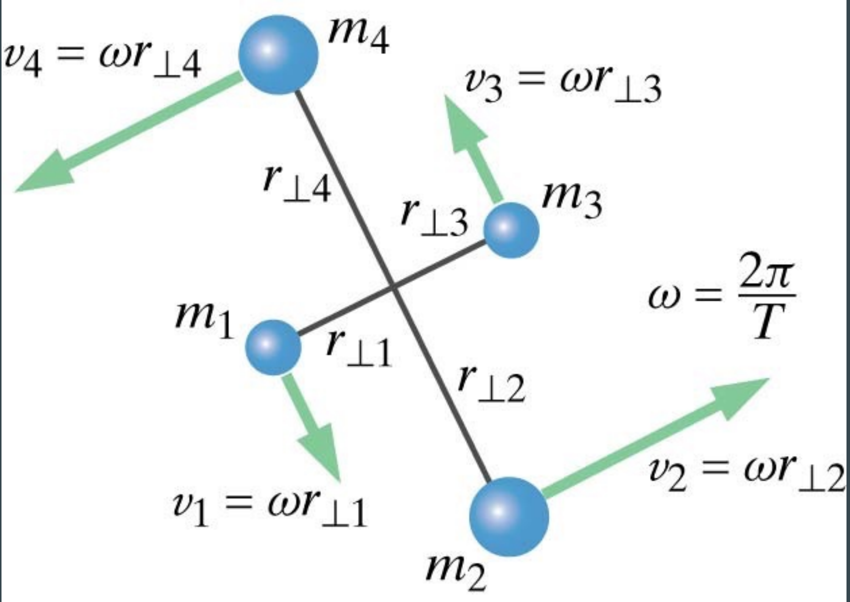
Therefore:

$$\omega = \frac{2\pi}{T} = v/r$$

(units: rads/sec)

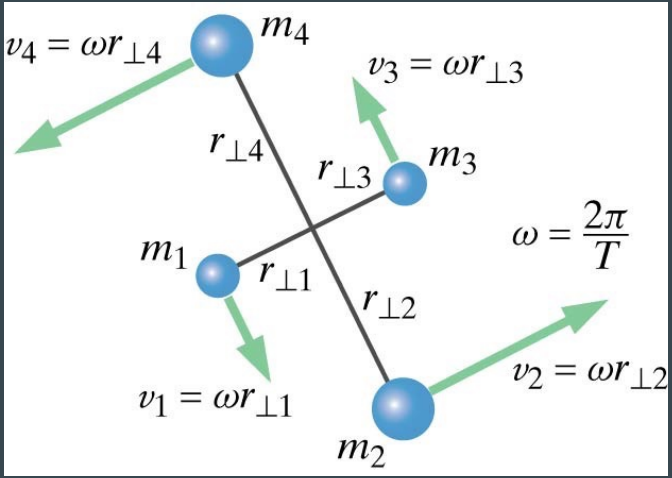
Moment of Inertia

- Let’s assume that we have a multiparticle system that:
  - has four particles with masses m_1 , m_2 , m_3 , m_4
  - each mass is at some distance from the CM of the system
  - all the masses spin about the CM with the same angular speed ω
  - has $K_{trans} = 0$ (center of mass doesn’t move)
  - the only thing the system is doing is spinning about an axis that goes through its center of mass (only rotation, no vibration)



Moment of Inertia: derivation

$$K_{\text{total}} = K_{\text{rot}}$$



$$v = \omega r$$

Moment of Inertia

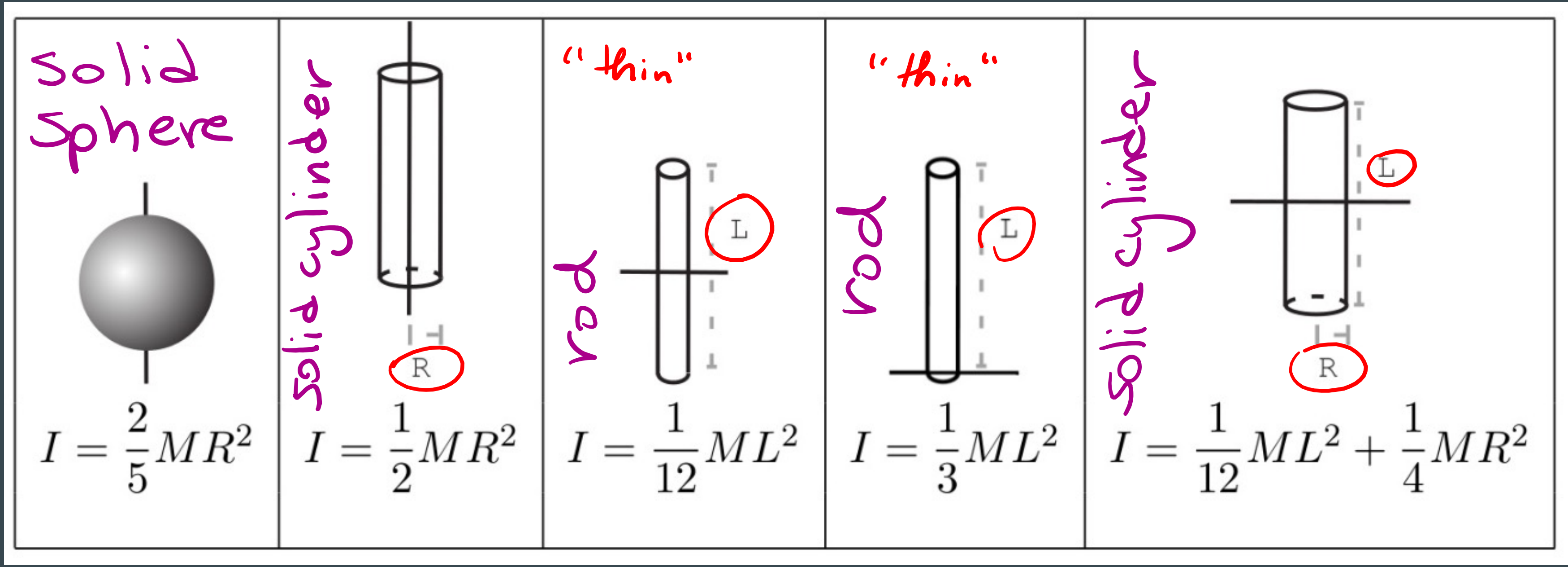
- The moment of inertia (I) is a measure of how difficult it is to change the rate of rotation of an object
- Can be thought of as a "rotational mass"
- It is a property associated with the geometry/shape/mass distribution of an object or system
- For a multiparticle system:

$$I \equiv \sum_{i=1}^{N} m_i \underbrace{r_{i\perp}^2}$$

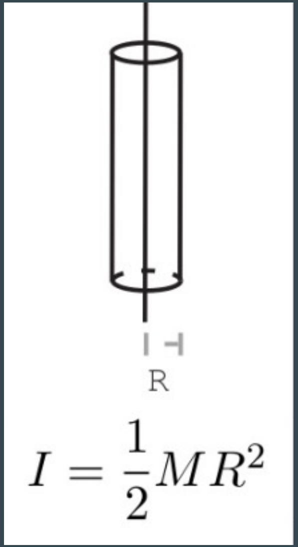
perpendicular distance to axis of rotation

Moment of Inertia

- For extended objects, you'd need to do an integral; the exact procedure is beyond the scope of this class, which is why the formula sheet gives you some useful moments of inertia

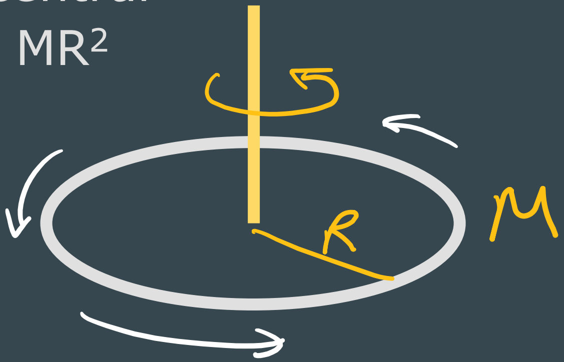


Moment of Inertia



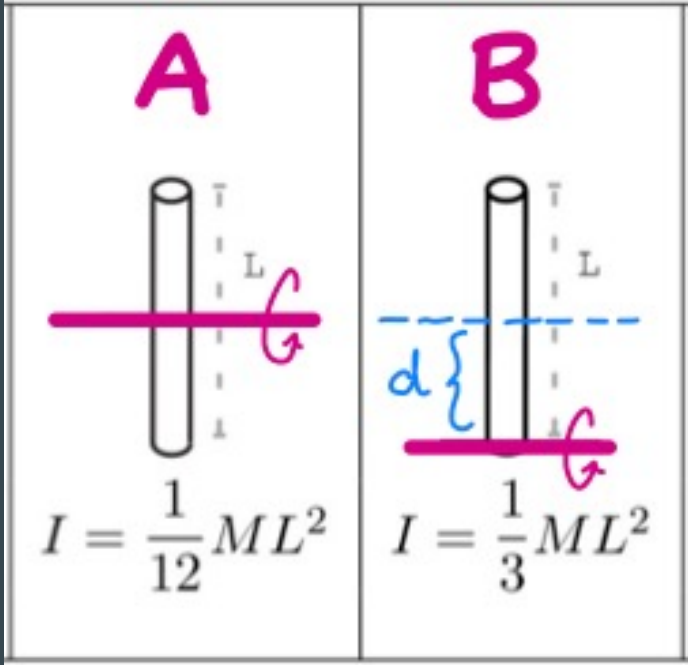
- For a solid cylinder rotating about its central axis, the moment of inertia only depends on the radius of the cylinder, not how long the cylinder is
- A solid disk is just a flattened-out cylinder, which means $I = (1/2)\ MR^2$ for a solid disk as well

- For a circular ring of mass M rotating about its central axis, all its mass is at the circumference, so $I = MR^2$
- Similarly, a hollow cylinder rotating about its central axis will have $I = MR^2$, because a ring is just a flattened hollow cylinder



Parallel Axis Theorem

- If you know the moment of inertia about an axis that goes through the center of mass (I_{cm}, like in figure A), and the distance d between that axis and another axis that is parallel to it (figure B), then the moment of inertia about this parallel axis, I_{pa}, is:



$$I_{\text{pa}} = I_{\text{cm}} + Md^2$$

Parallel Axis Theorem

- Example: deriving the moment of inertia in Figure B