Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

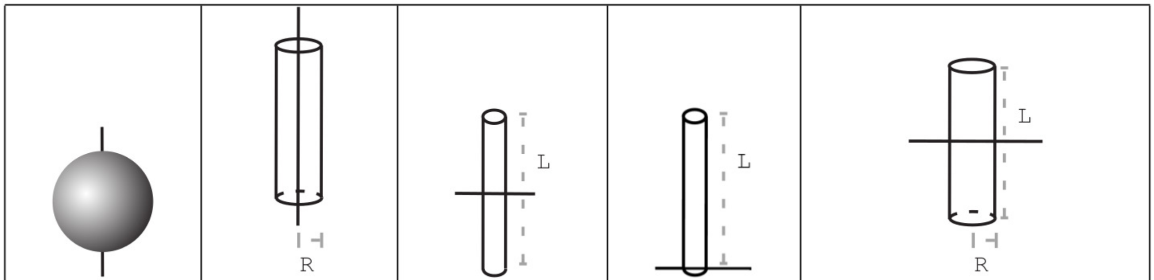
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\dots$$

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

Moment of inertia for rotation about indicated axis



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Summer 2024 - Final Exam

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You must join the appropriate proctoring meeting in MS Teams and keep your camera on and microphone muted throughout the exam period.
  - Other than MS Teams and Gradescope (and a PDF annotation app if applicable), you must not access any other app or website during the exam.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically (i.e., symbolically).
  - Numerical solutions should only be evaluated (i.e., plug in numbers) at the last step.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be marked as incorrect.**
  - Incorrect work or explanations mixed in with correct work will be marked as incorrect. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\dfrac{a \cdot b}{c \cdot d} = \dfrac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

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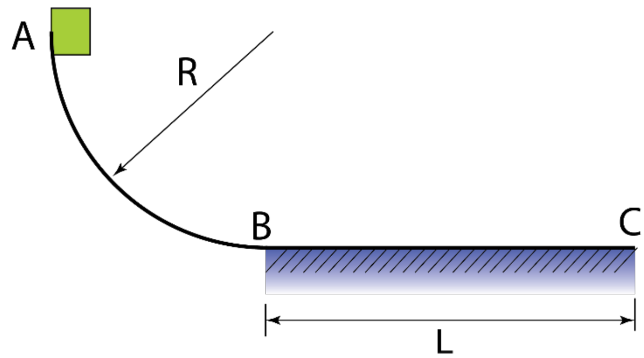
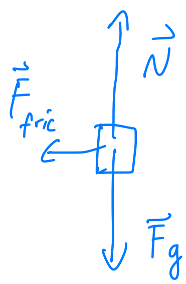
Sign your name on the line above

A block on the incline [40 pts]

A block of mass m is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R (point A in the Figure). The block then slides onto a horizontal plane (point B) where it finally comes to rest distance L from the beginning of the plane (point C). The curved incline is frictionless, but there is friction on the block while it slides horizontally between points B and C.



1. [6 pts] Draw a free body diagram for the block as it moves along horizontal plane. Show all forces.



2. [15 pts] Express the work done by the force of friction on the block as it moves along the horizontal plane in terms of m, L, and the coefficient of kinetic friction μ. Determine the coefficient of kinetic friction μ between the block and the horizontal plane. Express your answer for μ in terms of the given quantities R, L.

$$W_{Fric} = \mu m g L \cos(180°) = -\mu m g L$$

Point A: initial , Point C: final

$$\Rightarrow \quad \Delta E = \Delta K = 0 = W_g + W_{Fric} = mgR - \mu m g L$$

$$\Rightarrow \quad \mu = \frac{R}{L}$$

3. [6 pts] Determine the speed of the block at point B. Express your answer in terms of the known quantities (R and g).

$$\Delta K = \frac{1}{2} m v_B^2 - \frac{1}{2} m \cancel{v_A^2}^{\,0} = mgR$$

$$\Rightarrow \quad v_B = \sqrt{2gR}$$

4. [7 pts] Determine the magnitude of the acceleration of the block while it slides along the horizontal plane. Use your result in part 2 to express your answer in terms of the known quantities (R, L and g).

$$|\vec{F}_{fric}| = \mu m g = \frac{R}{L} m g .$$

$$|\vec{a}| = \frac{|\vec{F}_{fric}|}{m} = \frac{R}{L} g$$

5. [6 pts] How much time elapses while the block is sliding horizontally between points B and C? Express your answer in terms of the known quantities (R, L and g). Hint: use your results for the speed at point B (part 3) and the acceleration between points B and C (part 4).

Constant force between points B and C, so

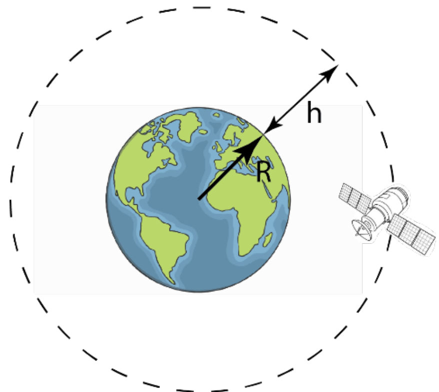
$$v_C = 0 = v_B - |\vec{a}| t \quad \Rightarrow \quad \sqrt{2gR} = \frac{R}{L} g\, t$$

$$\Rightarrow \quad t = \frac{L}{Rg} \sqrt{2gR} = L \sqrt{\frac{2}{gR}}$$

A satellite in geostationary orbit [30 pts]

A satellite is said to be in geostationary orbit when it moves in uniform circular motion directly above the Earth's equator, with an orbital period that matches the Earth's rotation rate.

Consider a communications satellite in geostationary orbit. The satellite has mass $m = 2000\ kg$ and has orbital period T equal to the rotation rate of the Earth ($T = 24\ hrs$). What is the height h of this satellite above the surface of the Earth (see the Figure)?



Note that the mass of the Earth is $M = 6 \times 10^{24}\ kg$, the radius of the Earth is $R = 6.4\ \times\ 10^{6} m$, and remember that for universal gravitation you should always use the center-to-center distance. You can assume the satellite is a point mass.

$$V = \frac{2\pi(R+h)}{T}.$$

Uniform circular motion

$$\Rightarrow \vec{F}_{net} = \frac{mv^2}{r}\hat{n}, \quad r = R+h$$

$$\Rightarrow \frac{\cancel{m}v^2}{R+h}\cancel{\hat{n}} = \frac{GM\cancel{m}}{(R+h)^2}\cancel{\hat{n}}$$

$$\Rightarrow \frac{4\pi^2(R+h)^2}{T^2(R+h)} = \frac{GM}{(R+h)^2}$$

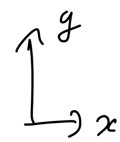
$$\Rightarrow (R+h)^3 = \frac{T^2 GM}{4\pi^2} \Rightarrow h = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3} - R$$

$$= 35.96 \times 10^6\ m$$

$$= 35.96 \times 10^3\ km$$

Bullet and the block [50 pts]

A rubber bullet of mass m is moving horizontally with speed v_0 when it hits a wooden block of mass $M = 6m$ that is hanging at rest at the end of a thin rod of negligible mass (see Figure 1). After the collision, the bullet recoils back with the speed $v_f = v_0/2$ and the wooden block acquires velocity v_b (see Figure 2).



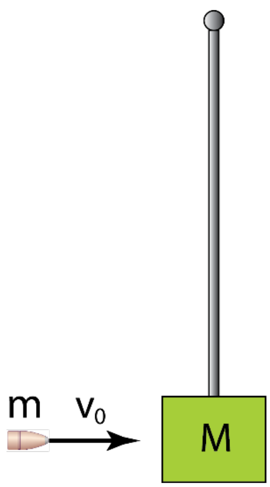


Figure 1 Initial state before the collision

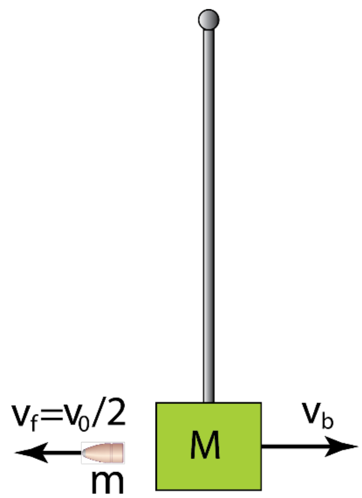


Figure 1 The state after the collision

1. [18 pts] Find the speed of the wooden block v_b immediately after the bullet recoils back (immediately after the collision). Express your answer in terms of the given quantity v_0.

$$\vec{p}_i = \vec{p}_f \Rightarrow m V_0 \cancel{\hat{x}} = -\frac{m v_0}{2} \cancel{\hat{x}} + M v_b \cancel{\hat{x}}$$

$$\Rightarrow \frac{3}{2} \frac{m}{M} v_0 = v_b = \frac{v_0}{4}$$

2. [16 pts] After the collision the block swings upward (see Figure 3). What is the maximum height h (relative to the original position) reached by the block? Express your answer in terms of the given quantities m, v_0.

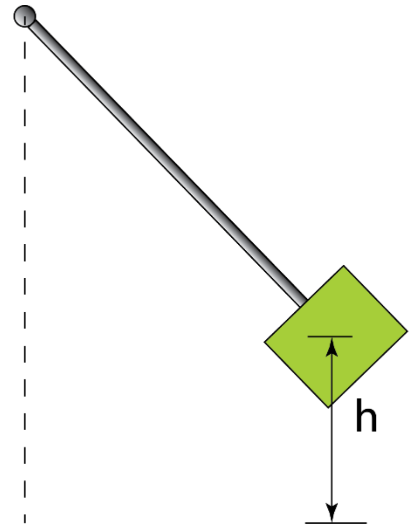


Figure 3 The final state

$$\Delta K = W_g$$

$$\frac{1}{2} M \cancelto{0}{v_f}^{2} - \frac{1}{2} \cancel{M} v_b^2 = \cancel{M} g h \cos(180°)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{v_0^2}{16}\right) = gh$$

$$h = \frac{1}{32} \frac{v_0^2}{g}$$

3. [16 points] How much of the initial kinetic energy of the bullet is converted to thermal energy during the collision? Express your answer in terms of the given quantities m, v_0.

$$K_f - K_i = \frac{1}{2} M v_b^2 + \frac{1}{2} m \frac{v_0^2}{4} - \frac{1}{2} m v_0^2$$

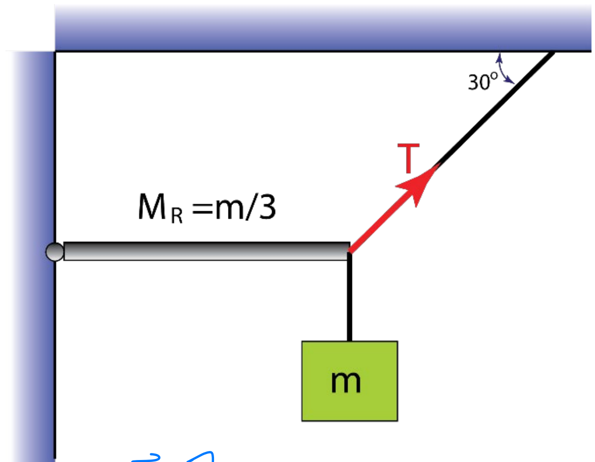
$$= \frac{3}{16} m v_0^2 + \frac{2}{16} m v_0^2 - \frac{8}{16} m v_0^2$$

$$= -\frac{3}{16} m v_0^2$$

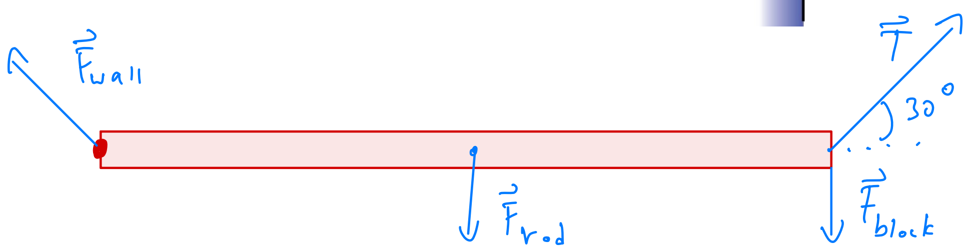
$\frac{3 m v_0^2}{16}$ was converted to thermal energy.

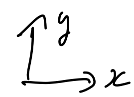
A rod in equilibrium [40 pts]

A horizontal uniform rod of mass $M_R = m/3$ and length L is attached on a pivot point to a wall. A block of mass m is hanging at the end of the rod. To prevent the rod from tilting a rope is also attached to the end of the rod with the other end of the rope tied to the ceiling. This rope makes an angle 30° with the ceiling (see the Figure).



1. [4 pts] Show all the forces that act on the rod. Sketch appropriate vectors at points where the forces are applied. You may use the provided figure or sketch the rod separately. Note: the wall exerts a force on the rod at the pivot point. This force is directed up and left.





z, outside page.

2. [6 pts] Determine the torque of the force by the wall about the pivot point.

Set pivot point as origin. Then $\vec{\tau}_{wall} = \underbrace{\vec{r}}_{0} \times \vec{F}_{wall} = 0$

3. [10 pts] Determine the torque of the weight of the block about the pivot point. Determine the torque of the weight of the rod about the pivot point. What are the directions of these torques?

$$\vec{\tau}_{block} = L\hat{x} \times mg(-\hat{y}) = mgL(-\hat{z})$$

$$\vec{\tau}_{rod} = \underbrace{\frac{L}{2}}_{\text{rod CM}}\hat{x} \times M_R g(-\hat{y}) = \frac{m}{6}gL(-\hat{z})$$

Both point into the page, $-\hat{z}$ direction.

4. [10 pts] Express the torque of the tension force T about the pivot point in terms of given quantities and (still unknown) magnitude of T. What is the direction of this torque?

$$\vec{\tau}_T = L\hat{x} \times \left[T\cos(30^\circ)\hat{x} + T\sin(30^\circ)\hat{y} \right]$$

$$= \underbrace{L\hat{x} \times T\cos(30^\circ)\hat{x}}_{\hat{x} \times \hat{x} = 0} + L\hat{x} \times T\sin(30^\circ)\hat{y}$$

$$= \frac{TL}{2}\hat{z},$$ points out the page.

5. [10 pts] Find the magnitude of the tension force T. Express your answer in terms of the given quantity m and g.

$$\sum_i \vec{\tau}_i = \vec{\tau}_T + \vec{\tau}_{block} + \vec{\tau}_{rod} = 0$$

$$\Rightarrow \frac{TL}{2}\cancel{\hat{z}} + mgL(-\cancel{\hat{z}}) + \frac{m}{6}gL(-\cancel{\hat{z}}) = 0$$

$$\Rightarrow \frac{TL}{2} = \frac{7}{6}mgL \Rightarrow T = \frac{7}{3}mg$$

Monkey pulls the rope [40 pts]

An object with mass m, on the end of a string, moves in a circle on a horizontal frictionless table as shown in Figure 1. At the initial state the speed of an object is v_i and the radius of the circle is R. A monkey pulls the string very slowly through a small hole in the table (point O).

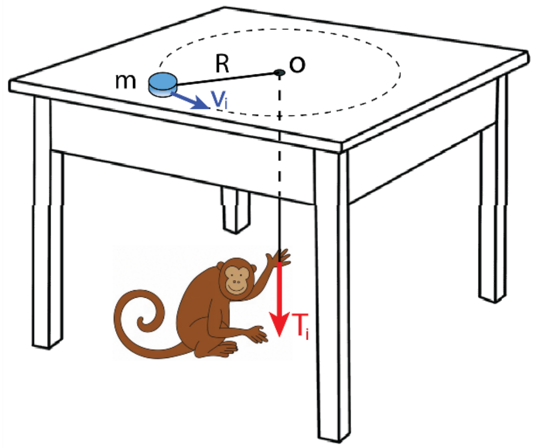


Figure 2 Initial State

1. [6 pts] What is the tension in the string T_i in the initial state? Express your answer in terms of the given quantities m, v_i, R.

$$T_i = F_{net} = \frac{m v_i^2}{R}$$

2. [6 pts] What is the torque of the tension force in the horizontal string about point O? Note: as the object moves the tension force in the string is always directed along the string towards point O.

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \vec{T} = \vec{r} \times (-T)\hat{r} = 0$$

3. [8 pts] Which of the following is correct as the monkey pulls the rope and the radius of the circle decreases (circle all the true statements):

(A) The angular momentum of an object about point O increases.

(B) The angular momentum of an object about point O decreases.

(C) The angular momentum of the object about point O remains constant.

(D) The kinetic energy of an object decreases.

(E) The kinetic energy of an object remains constant.

(F) The kinetic energy of the object increases.

4. [10 pts]. The monkey has pulled the string such that the radius of the circle is a third of its initial value: $R_f = R/3$ (see Figure 2). What is the speed of an object v_f at this final state? Express your answer in terms of the given quantity v_i.

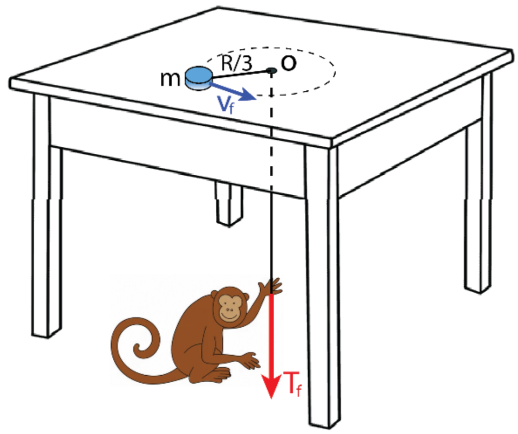


Figure 3 Final State

$$\vec{L}_f = \vec{L}_i$$

$$\Rightarrow \quad I_i w_i = I_f w_f$$

$$\Rightarrow \quad m R^2 \frac{v_i}{R} = m R_f^2 \frac{v_f}{R_f}$$

$$\Rightarrow \quad v_f = \frac{R}{R_f} v_i = 3 v_i$$

5. [2 pts] Is the tension in the string T_f in the final state larger or smaller than that in the initial state?

$$T_f = \frac{m v_f^2}{R_f} = \frac{9 m v_i^2}{R/3} = 27 T_i$$, larger

6. [8 pts] What is the work done by the monkey as it pulls the string (from the initial to the final state). Express your answer in terms of the given quantities m, v_i, R.

$$\Delta E = \Delta K_{rot} = W_{monkey}$$

$$\Rightarrow \quad \frac{1}{2} I_f w_f^2 - \frac{1}{2} I_i w_i^2 = \frac{1}{2} m R_f^2 \frac{v_f^2}{R_f^2} - \frac{1}{2} m R_i^2 \frac{v_i^2}{R_i^2}$$

$$= \frac{1}{2} m \left(9 v_i^2 - v_i^2 \right) = 4 m v_i^2 = W_{monkey}.$$