Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

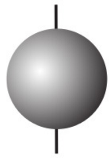
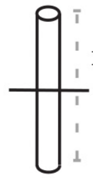
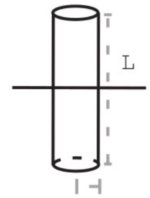
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/HP) - Spring 2024 - Test 1

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\frac{a \cdot b}{c \cdot d} = \frac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code, I have completed this test while adhering to these instructions."

KEY

Sign your name on the line above

Problem 1 - Constant Forces [25 pts]

A frog with mass $m = 0.5$ kg jumps from the surface of a pond to catch a perched dragonfly. The frog jumps out of the water with an initial velocity of $\vec{v}_i = \langle 1, 5, 0 \rangle$ m/s.

1.1 [10 pts] If the frog aimed to catch the dragonfly at the peak of its jump, how much time does the dragonfly have to escape after the frog has left the surface of the water?

Peak of jump $\Rightarrow V_{f,y} = 0$ m/s

$$\vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}}{m}\Delta t \Rightarrow V_{f,y} = V_{i,y} - \frac{mg}{m}\Delta t = V_{i,y} - g\Delta t$$

$$\Rightarrow \Delta t = \frac{V_{i,y} - V_{f,y}}{g}$$

$$= \frac{(5 \text{ m/s}) - (0 \text{ m/s})}{(9.8 \text{ m/s}^2)}$$

$$\approx \boxed{0.51 \text{ s}}$$

3:10

1.2 [15 pts] If it takes the dragonfly (which has a mass $m_d = 0.003$ kg) a time $\Delta t_2 = 0.2$ s to notice the frog after the frog leaves the surface of the water, and it needs to be at least at a height $h = 0.1$ m directly above its original perch to escape the frog's mouth, what constant force $\vec{F}$ must it produce onto its perch to get away? You can assume that the dragonfly takes off vertically from rest, and don't forget that gravity is also acting on the dragonfly!

$$\Delta t_{esc} = \Delta t - \Delta t_{react} = (0.51\text{ s}) - (0.2\text{ s}) = \underline{0.31\text{ s}}$$

$\Delta t = \Delta t$ from 1.1

$\Delta t_{react} = \Delta t_2$

$$\vec{r}_f = \vec{r}_i + \vec{V}_i \Delta t + \frac{1}{2}\frac{F_{net}}{m}(\Delta t)^2$$

$$\Rightarrow y_f = y_i + V_{i,y}\Delta t_{esc} + \frac{1}{2}\frac{F_{net}}{m_d}(\Delta t_{esc})^2$$

$$\Rightarrow F_{net,y} = \frac{2m_d}{(\Delta t_{esc})^2}\left[y_f - y_i - V_{i,y}\Delta t_{esc}\right] = \frac{2m_d}{(\Delta t_{esc})^2}\left[\Delta y - V_{i,y}\Delta t_{esc}\right]$$

$$= \frac{2(0.003\text{ kg})}{(0.31\text{ s})^2}\left[(0.1\text{ m}) - (0\text{ m/s})(0.31\text{ s})\right]$$

$$\approx \underline{6.24 \cdot 10^{-3}\text{ N}}$$

$$F_{net,y} = F - F_g$$

$$\Rightarrow F = F_{net,y} + F_g = F_{net,y} + m_d g$$

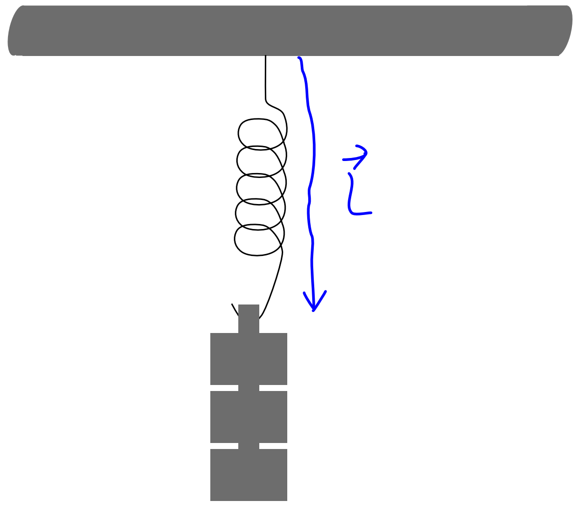
$$= (6.24 \cdot 10^{-3}\text{ N}) + (0.003\text{ kg})(9.8\text{ m/s}^2)$$

$$\approx \boxed{0.036\text{ N}}$$

10:00

Problem 2: Spring Forces [30 pts]

A stack of weights hangs on a spring from a rod as shown. The relaxed length of the spring is $L_0 = 0.5$ m and its stiffness is $k = 19.6$ N/m.



2.1 [10 pts] The weights have a total mass of $m = 0.4$ kg and are released from rest when the length of the spring is $L_1 = 0.60$ m. What is the **net force** $\vec{F}_{\text{net}}$ on the weights at this moment? Write your vector in a coordinate system with the $+\hat{y}$ axis pointing upwards.

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_g$$

$$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L} = -k(L_1 - L_0)(-\hat{y}) = k(L_1 - L_0)\hat{y}$$

$$\vec{F}_g = mg(-\hat{y}) = -mg\hat{y}$$

$$\vec{F}_{net} = k(L_1 - L_0)\hat{y} - mg\hat{y} = [k(L_1 - L_0) - mg]\hat{y}$$

$$= [(19.6\text{ N/m})(0.60\text{ m} - 0.5\text{ m}) - (0.4\text{ kg})(9.8\text{ m/s}^2)]\hat{y}$$

$$= \boxed{-1.96\text{ N}\,\hat{y}}$$

3:52

2.2 [10 pts] As the weights fall, what must be the length L_2 of the spring in order for the **net force** to be exactly opposite to the value it had when released?

$$\vec{F}_{net2} = -\vec{F}_{net} = \underline{1.96\ N\ \hat{y}}$$

$$\vec{F}_{net2} = \vec{F}_{s2} + \vec{F}_g \Rightarrow \vec{F}_{s2} = \vec{F}_{net2} - \vec{F}_g$$

$$\Rightarrow k(L_2 - L_0)\,\hat{y} = F_{net2}\,\hat{y} - [-mg\,\hat{y}]$$

$$\Rightarrow L_2 = \frac{F_{net2} + mg}{k} + L_0$$

$$= \frac{(1.96\ N) + (0.4\ kg)(9.8\ m/s^2)}{(19.6\ N/m)} + (0.5\ m)$$

$$= \boxed{0.8\ m}$$

9:09

2.3 [10 pts] What is the magnitude of the net force on the weights when they are halfway between these two positions? Remember to provide some calculations or justification for your answer.

$$L_{half} = \frac{L_1 + L_2}{2} = \frac{(0.60\ m) + (0.8\ m)}{2} = \underline{0.7\ m}$$

$$\vec{F}_s = k\,(L_{half} - L_0)\ \hat{y}$$

$$\vec{F}_{net} = [k(L_{half} - L_0) - mg]\ \hat{y}$$

$$= [(19.6\ N/m)(0.7\ m - 0.5\ m) - (0.4\ kg)(9.8\ m/s^2)]\ \hat{y}$$

$$= \boxed{0\ N\ \hat{y}}$$

Gravity is constant, spring force depends linearly on position

$\Rightarrow$ net force depends linearly on position

$\Rightarrow$ net force at halfway point is average of endpoints

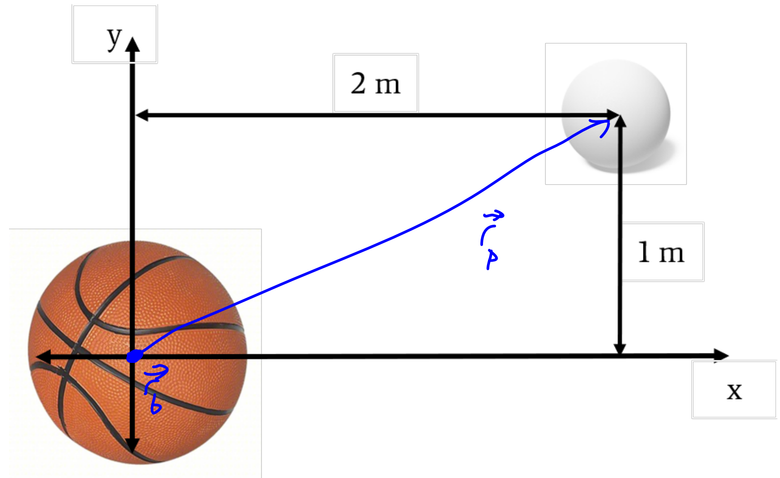
Net forces at endpoints are equal and opposite

$\Rightarrow$ net force at halfway point is zero

15:09

Problem 3: Gravitational Forces [25 pts]

In deep space, far from any stars or planets, a ping-pong ball sits near a basketball. The ping-pong ball has mass $m_p = 0.003$ kg, and the basketball has mass $m_b = 0.3$ kg. At $t = 0$ s, the position of the basketball is $\vec{r}_b = \langle 0, 0, 0\rangle$ m and the position of the ping-pong ball is $\vec{r}_p = \langle 2, 1, 0\rangle$ m. Both objects are at rest at $t = 0$ s.



3.1 [10 pts] What is the gravitational force $\vec{F}_g$ exerted by the basketball on the ping-pong ball?

$$\vec{r} = \vec{r}_p - \vec{r}_b = (\langle 2,1,0\rangle\ m) - (\langle 0,0,0\rangle\ m) = \underline{\langle 2,1,0\rangle\ m = \vec{r}_p}$$

$$|\vec{r}| = |\vec{r}_p| = \sqrt{(2\ m)^2 + (1\ m)^2 + (0\ m)^2} = \underline{\sqrt{5}\ m}$$

$$\hat{r} = \hat{r}_p = \frac{\vec{r}_p}{|\vec{r}_p|} = \frac{\langle 2,1,0\rangle\ m}{\sqrt{5}\ m} = \underline{\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle}$$

$$\vec{F}_g = -\frac{G m_b m_p}{|\vec{r}|^2}\hat{r}$$

$$= -\frac{(6.7\cdot 10^{-11}\ N m^2/kg^2)(0.3\ kg)(0.003\ kg)}{(\sqrt{5}\ m)^2}\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle$$

$$\approx \boxed{\langle -1.08\cdot 10^{-14}, -5.39\cdot 10^{-15}, 0\rangle\ N}$$

S: 54

3.2 [15 pts] What is the new position of the ping-pong ball two days later? Consider only one iterative step (i.e., $\Delta t = 2$ days).

$$\Delta t = 2 \text{ days} \cdot \frac{86400 \text{ s}}{1 \text{ day}} = \underline{172800 \text{ s}}$$

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m} \Delta t$$

$$\Rightarrow \vec{v}_{f,p} = \vec{v}_{i,p} + \frac{\vec{F}_g}{m_p} \Delta t$$

$$= (\langle 0, 0, 0 \rangle \text{ m/s}) + \frac{(\langle -1.08 \cdot 10^{-14}, -5.39 \cdot 10^{-15}, 0 \rangle \text{ N})}{(0.003 \text{ kg})} (172800 \text{ s})$$

$$\approx \underline{\langle -6.21 \cdot 10^{-7}, -3.11 \cdot 10^{-7}, 0 \rangle \text{ m/s}}$$

Non-constant force $\Rightarrow \vec{v}_{avg} \approx \vec{v}_f$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\Rightarrow \vec{r}_{f,p} = \vec{r}_{i,p} + \vec{v}_{f,p} \Delta t$$

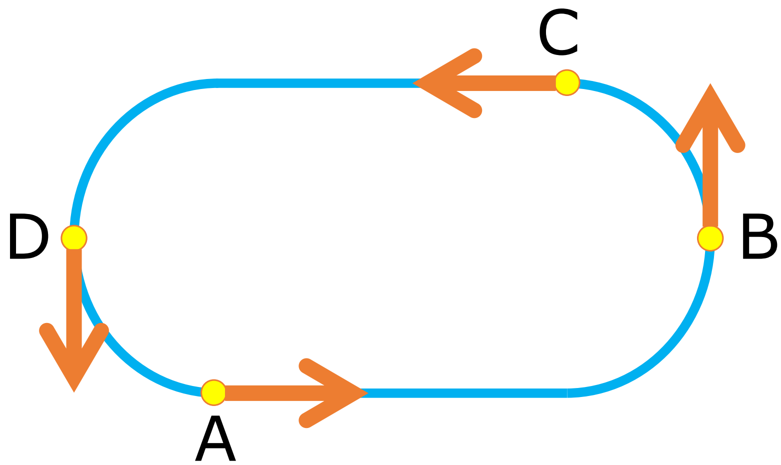
$$= (\langle 2, 1, 0 \rangle \text{ m}) + (\langle -6.21 \cdot 10^{-7}, -3.11 \cdot 10^{-7}, 0 \rangle \text{ m/s})(172800 \text{ s})$$

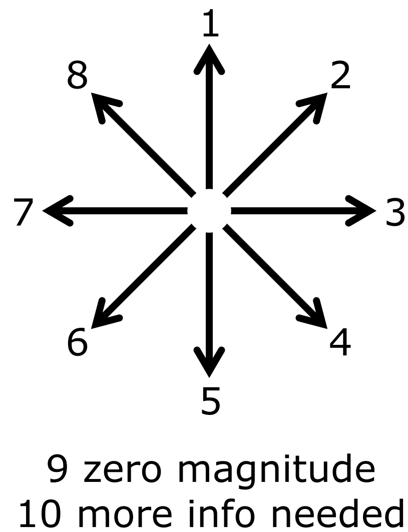
$$\approx \boxed{\langle 1.89, 0.95, 0 \rangle \text{ m}}$$

12:09

Problem 4: Vectors [20 pts]

An athlete runs counterclockwise on a track as shown. The athlete begins at location A, then runs along the track towards location B, then location C, then location D, and then keeps going to do another lap. At each location, an orange arrow indicates the **instantaneous** velocity of the athlete. All the velocities have the same magnitude.





Using the numbered directions shown by the rosette, indicate (by number) **which arrow best represents the direction** of the quantities listed below. If a quantity has zero magnitude or cannot be determined, indicate that using the corresponding number.

4.1 [2 pts] 10 The initial position vector at location A.

4.2 [2 pts] 1 The (instantaneous) momentum at location B.

4.3 [2 pts] 4 The change in position (displacement) from location D to location A.

4.4 [2 pts] 8 The average velocity from location B to location C.

4.5 [2 pts] 6 The change in velocity from location B to location C.

4.6 [2 pts] 5 The change in velocity from location B to location D.

4.7 [2 pts] 8 The change in momentum from location A to location B.

4.8 [2 pts] 4 The change in momentum from location C to location D.

4.9 [2 pts] 7 The average acceleration from location A to location C.

4.10 [2 pts] 2 The net force from location D to location A.

2:01

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