Week 10

Internal Energy Change

Topics for this week

1. The moment of inertia
2. Point Particle (center of mass)
3. Extended Systems
4. Work done by contact forces

By the end of the week

1. Know how to include rotations in energy calculations
   a. Pass a job interview

Multiparticle Kinetic Energy

- The net force on an object changes the momentum of the center of mass
- When a force does not act at the center of mass
  - We still have the same change in the total momentum
  - We also see motion relative to the center of mass
- We can model the total macroscopic kinetic energy as a sum
  - Energy due to motion of the center of mass
  - Energy due to motion about the center of mass (rotation & vibration)

$$K_{total} = K_{translation} + K_{relative}$$

$$K_{translation} = \frac{1}{2} M_{sys} v_{cm}^2 = \frac{P_{total}^2}{2M_{sys}}$$

$$K_{relative} = K_{rotation} + K_{vibration}$$

Rotational Kinetic Energy

- We can model the rotational kinetic energy when all of the objects in the system are rotating about the center of mass with the same rotation rate
  - For rigid body rotations consider an object that is stationary but rotating at a constant rate omega

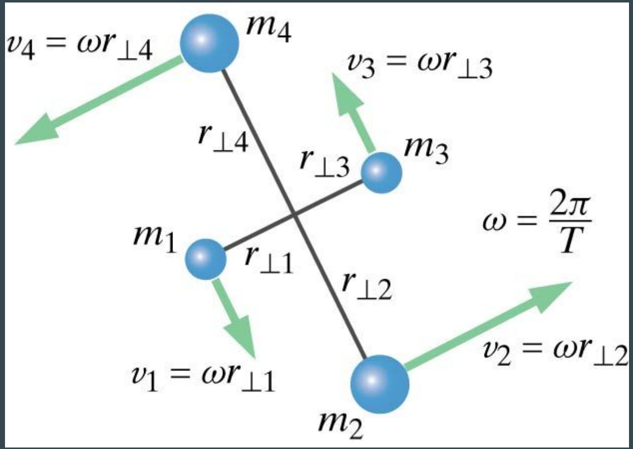
$$v = \frac{2\pi r}{T} = \omega r$$

$$K_{total} = K_{rotation}$$

$$K_{total} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2 + m_3 v_3^2 + m_4 v_4^2)$$

$$K_{total} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2)\omega^2$$

$$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$$



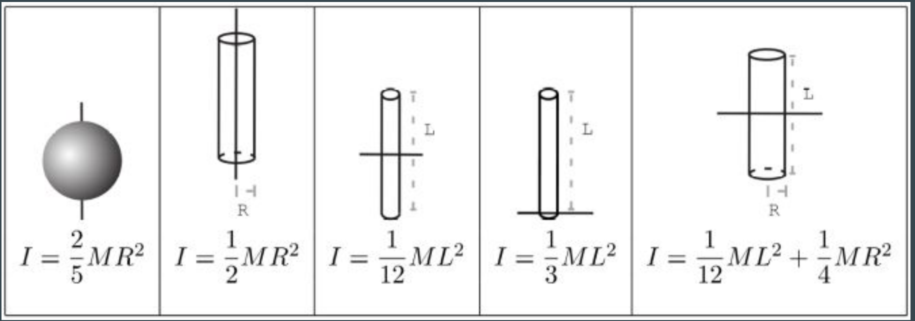
The moment of inertia

- The moment of inertia is a measure of how hard it is to change an object's rotation rate
  - A rotational mass that is a property of the geometry and density of an object
  - Specific to the prescribed axis of rotation
- Modeled as a sum of squares
  - For a collection of particles we take the perpendicular distance from the axis of rotation

$$I = \sum_{i=1}^{N} m_i r_{i,\perp}^2$$

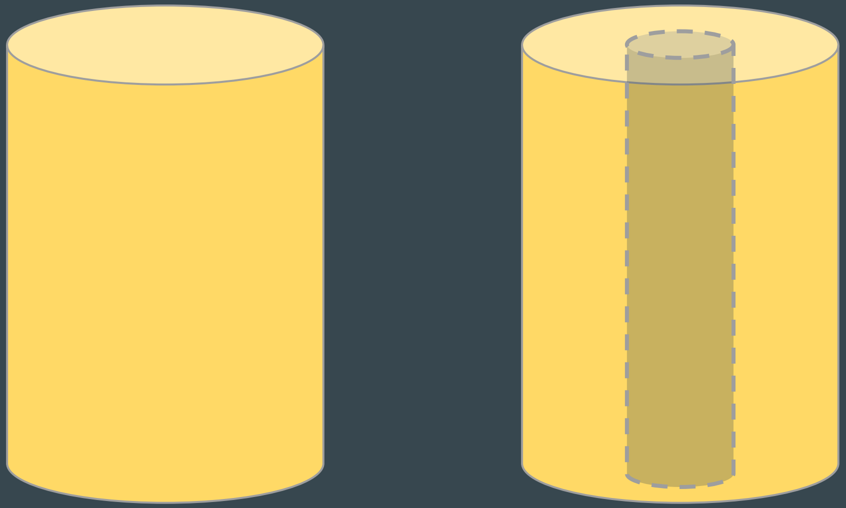
  - For a solid object we need to integrate

$$I = \int_i^f r_\perp^2 \, dm$$



Example: The Job Interview

During a job interview with GE you are asked to determine which of two solid cylinders has an interior void. The cylinders both have the same mass, radius and length. How can you tell without damaging the cylinders? (Group Question: Ideas?)



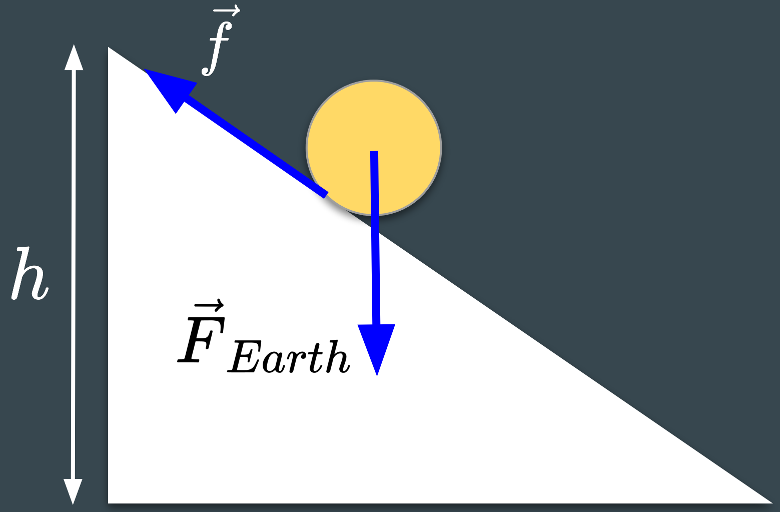
Example: Solution

- Use rotational kinetic energy to analyze each cylinder rolling, without slipping, down a hill of height h from rest
  - System: Cylinder, Surroundings: Ramp + Earth
  - Initial State: Motionless at top, Final State: Moving at bottom

$$\Delta K_{trans} + \Delta K_{rot} = W$$

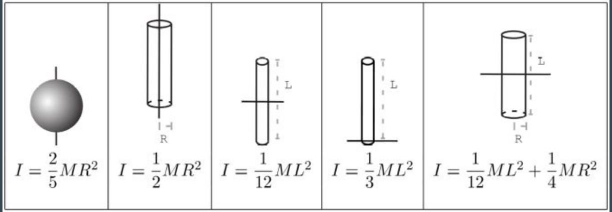
$$\frac{1}{2}mv_f^2 + \frac{1}{2}Iw_f^2 = mgh$$

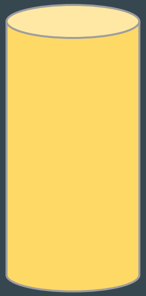
$$v_f^2 = \frac{2gh}{\left(1 + \frac{I}{mR^2}\right)}$$



Example: Solution Cont.

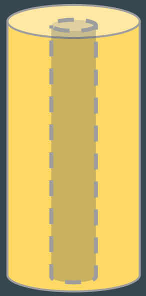
- The final speed, and how long it takes to traverse the hill, will depend on the moment of inertia of the rotating object
    - The cylinder with the larger moment of inertia will move more slowly





$$I = \frac{1}{2}mR^2$$

$$v_f^2 = \frac{4gh}{3}$$

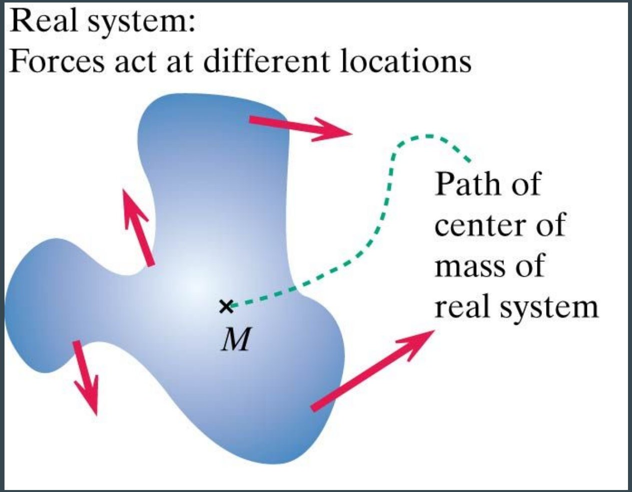


$$I = \frac{1}{2}m(r_{void}^2 + R^2)$$

$$v_f^2 = \frac{4gh}{3 + \left(\frac{r_{void}}{R}\right)^2}$$

The extended system

- Modelling a system as an extended object
  - Takes into account the shape, configuration and motion relative to the center of mass
  - Requires the work to be calculated for each individual force
    - By how much is each force displaced?
  - Include internal energy changes



$$\Delta E_{sys} = Q + W_1 + W_2 + \ldots$$

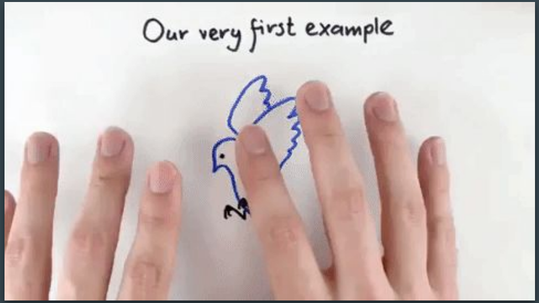
$$\Delta E_{sys} = \Delta K_{trans} + \Delta K_{rot} + \Delta K_{vib} + \Delta U_{grav} + \Delta U_{elec} + \Delta U_{spring} + \Delta E_{therm} + \Delta E_{chem} + \ldots$$

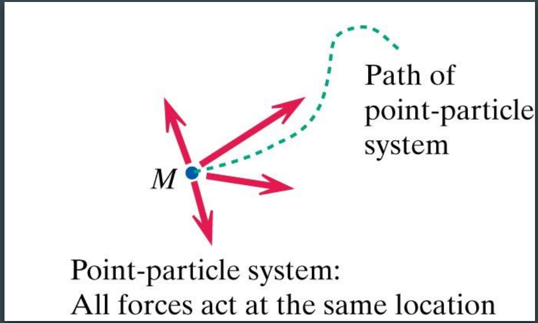
The center of mass system (point particle model)

- Modelling a system as a point located at the center of mass
    - Imagine you crush the whole system down to a point located at the center of mass
        - All external forces now act at the center of mass
        - Work is calculated as the displacement of the center of mass by the net force
    - A point has no size or dimension
        - No relative motion, no internal energy
        - The energy principle simplifies to:

$$\Delta K_{trans} = W_{cm}$$

$$\frac{1}{2}M_{sys}(v_{cm,f}^2 - v_{cm,i}^2) = \int_{\vec{r}_{cm,i}}^{\vec{r}_{cm,f}} \vec{F}_{net} \cdot \vec{d}l$$





Making predictions for complex systems

1. Draw a diagram that identifies the initial and final state of the system
2. Locate the center of mass on your diagram
3. Find the change in translational kinetic energy
    a. Determine the displacement of the center of mass

$$\Delta K_{trans} = W_{cm}$$

4. Calculate the work done by the displacement of each force and add them together
    a. Each force moves the same distance as its point of application
5. Write the energy equation for the real system and make a prediction!
    a. Substitute the change in translational kinetic energy

$$\Delta E_{sys} = \Delta K_{trans} + \Delta K_{rot} + \Delta K_{vib} + \Delta U_{grav}$$
$$+\Delta U_{elec} + \Delta U_{spring} + \Delta E_{therm} + \Delta E_{chem} + \ldots$$