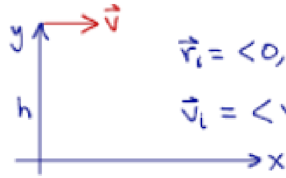
Physics 2211 GPS Week 4

Problem #1

A man standing at the top of a building kicks a small rock of mass m. The height of the building is h and initial velocity of the rock after the kick is $\vec{v} =< v, 0, 0 >$. Assume no air resistance and let g be the magnitude of the acceleration due to gravity.



$\vec{r}_i = <0, h, 0>$

$\vec{v}_i = <v, 0, 0>$

(a) Find the time taken by the rock to hit the ground.

Motion in x-direction is constant velocity ($F_{net,x} = 0$)

Motion in y-direction is constant acceleration ($F_{net,y} = -mg$)

Momentum principle, y-component:

$$F_{net,y} = \frac{m \Delta v_y}{\Delta t}$$

$$-\cancel{m}g = \frac{\cancel{m}(v_{y,f} - \cancel{v_{y,i}})}{\Delta t}$$

$$-g = \frac{v_{y,f}}{\Delta t} \implies v_{y,f} = -g\Delta t$$

Finding average velocity (y-component):

$$v_{y,Avg} = \frac{v_{yi} + v_{yf}}{2} = \frac{0 - g\Delta t}{2} = -\frac{1}{2} g \Delta t$$

Using the position-update formula (y-component):

$$r_{yf} = r_{yi} + v_{y,Avg} \Delta t$$

$$0 = h + \left(-\frac{1}{2} g \Delta t\right) \Delta t$$

$$0 = h - \frac{1}{2} g \Delta t^2$$

$$\frac{1}{2} g \Delta t^2 = h$$

$$\Delta t^2 = \frac{2h}{g} \implies \boxed{\Delta t = \sqrt{\frac{2h}{g}}}$$

(b) How far away from the building did the rock travel before hitting the ground?

Motion in x-direction is constant velocity ($F_{net,x} = 0$)

$$r_{x,f} = r_{x,i} + v_{x,i}\,\Delta t$$

$$r_{x,f} = 0 + v\,\Delta t$$

$$r_{x,f} = v\,\Delta t = \boxed{v\sqrt{\frac{2h}{g}}}$$

(c) Determine the magnitude of velocity of the rock before hitting the ground.

x-component : $v_{xf} = v_{xi} = v$ (b/c $F_{net,x} = 0$)

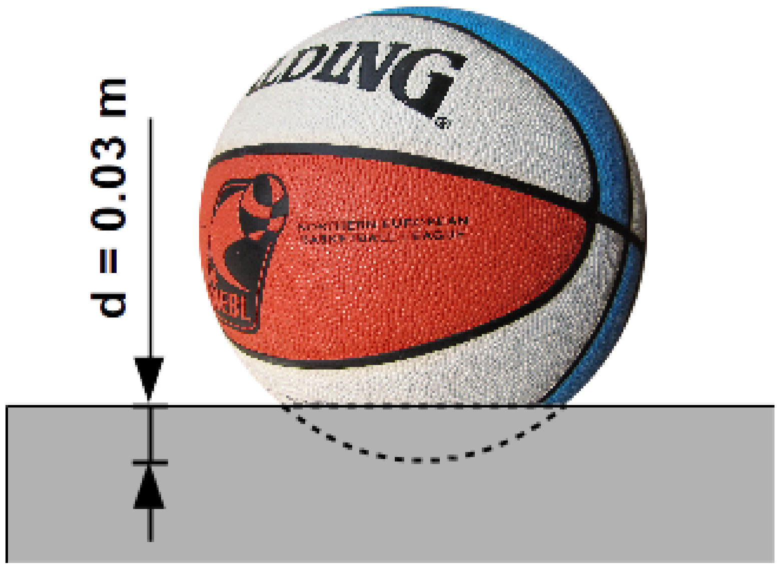
y-component : $v_{yf} = \underbrace{-g\,\Delta t}_{\text{from part (a)}} = -g\sqrt{\frac{2h}{g}}$

$$\Rightarrow \vec{v}_f = \langle v, -g\sqrt{\frac{2h}{g}}, 0 \rangle$$

$$\Rightarrow |\vec{v}_f| = \sqrt{v^2 + \left(-g\sqrt{\frac{2h}{g}}\right)^2} = \sqrt{v^2 + g^2\frac{2h}{g}} = \boxed{\sqrt{v^2 + 2gh}}$$

Problem #2

A basketball has a mass of 0.625 kg is resting on the rim of the hoop, and falls to the ground a distance 3.048 m. The ball moves down, hits the floor and bounces straight back up with almost the same speed. As indicated in the diagram, high-speed photography shows that the ball is compressed an amount $d = 3$ cm at the instant when its speed is momentarily zero, before rebounding.



(a) Determine the velocity of the ball at the moment when it hits the ground.

Rim to floor: need to find impact speed (easy to do with conservation of energy, but we haven't learned that yet!)

$\vec{v}_{rim} = 0$

$$\left.\begin{aligned} \vec{v}_{Avg} &= \tfrac{1}{2}(\cancel{\vec{v}_{rim}}^{\,0} + \vec{v}_{floor}) = \tfrac{1}{2}\vec{v}_{floor} \\ \vec{v}_{Avg} &= \frac{\Delta\vec{r}}{\Delta t} = \frac{\cancel{\vec{r}_{floor}}^{\,0} - \vec{r}_{rim}}{\Delta t} = \frac{-\vec{r}_{rim}}{\Delta t} \end{aligned}\right\} \quad \tfrac{1}{2}\vec{v}_{floor} = \frac{-\vec{r}_{rim}}{\Delta t}$$

$$\tfrac{1}{2}v_{floor}(\cancel{-}\hat{y}) = \frac{\cancel{-}r_{rim}\hat{y}}{\Delta t}$$

$$v_{floor} = \frac{2r_{rim}}{\Delta t} \Rightarrow \Delta t = \frac{2r_{rim}}{v_{floor}}$$

$$\Rightarrow \vec{v}_{floor} = \cancel{\vec{v}_{rim}}^{\,0} + (\vec{F}_{net}/m)\Delta t = \frac{\cancel{m}g(-\hat{y})}{\cancel{m}}\Delta t$$

$$v_{floor}(\cancel{-\hat{y}}) = g\Delta t(\cancel{-\hat{y}})$$

$$v_{floor} = g\Delta t = g\frac{2r_{rim}}{v_{floor}} \Longrightarrow v_{floor}^2 = 2gr_{rim}$$

$$v_{floor} = \sqrt{2gr_{rim}} = \sqrt{(2)(9.8)(3.048)} = 7.73 \text{ m/s}$$

$$\Longrightarrow \boxed{\vec{v}_{floor} = -7.73 \text{ m/s } \hat{y}}$$

(b) Determine the approximate average speed of the ball during the period from first contact with the floor to the moment the ball's speed is momentarily zero. You can assume that the force of contact is constant.

✓ Impact to stopping:

$v_i = v_{floor} = 7.73$ m/s

$v_f = 0$

$$\Rightarrow v_{Avg} = \tfrac{1}{2}(v_i + \cancel{v_f}) = \tfrac{1}{2}(7.73) = \boxed{3.865 \text{ m/s}}$$

(b) How much time elapses between first contact with the floor, and the ball coming to a stop?

$$V_{Avg} = \frac{\Delta r}{\Delta t} \Rightarrow \Delta t = \frac{\Delta r}{V_{Avg}} = \frac{d}{V_{Avg}} = \frac{3e\text{-}2}{3.865} = \boxed{0.00776 \text{ sec} = 7.76e-3 \text{ sec}}$$

(c) What is the magnitude of the force exerted by the floor on the ball during contact?

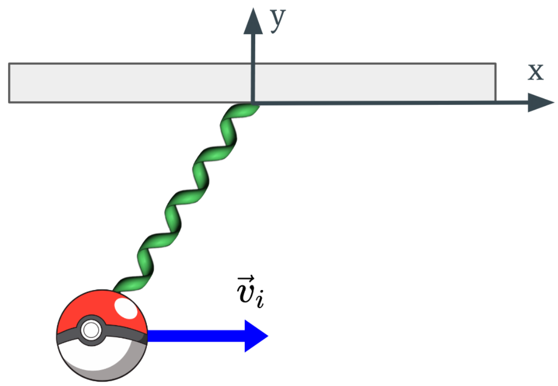
$$\vec{F} = \vec{F}_{ground} + \vec{F}_{grav} = \frac{\Delta \vec{p}}{\Delta t}$$

$$|\vec{F}| = \frac{|\Delta \vec{p}|}{\Delta t} = \frac{m|\Delta \vec{v}|}{\Delta t} = \frac{m|\cancel{v_f}^{0} - v_i|}{\Delta t} = \frac{mv_i}{\Delta t} = \frac{(0.625)(7.73)}{7.76e\text{-}3} = 622.58\ N$$

$$|\vec{F}_{ground}| = |\vec{F} - \vec{F}_{grav}| = 622.58\ N - (-6.13\ N) = \mathbf{628.71\ N}$$

Problem #3

One end of a spring is attached to the ceiling at the origin and the other is attached to a ball of mass m which is free to move around. The spring has a rest length L_0 and stiffness k. At time $t = 0$ the ball is located at position $\vec{r}_i\langle -2L_0, -4L_0, 0\rangle$ and has a velocity $\vec{v}_i = \langle v, 0, 0\rangle$ where v is a positive constant. The gravitational force acting on the ball due to the Earth points in the negative y-direction.



A. Calculate the **net force** acting on the ball at time $t = 0$. Hint: since the spring's fixed end is at the origin, the position vector of the ball is the same as the $\vec{L}$ for the spring.

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring}$$

$$\vec{F}_{grav} = \langle 0, -mg, 0\rangle$$

$$\begin{aligned}\vec{F}_{spring} &= -k_s(|L| - L_0)\hat{L} \\ &= -k_s(\sqrt{(-2L_0)^2 + (-4L_0)^2 + 0^2} - L_0)\frac{\langle -2L_0, -4L_0, 0\rangle}{\sqrt{(-2L_0)^2 + (-4L_0)^2 + 0^2}} \\ &= -(2\sqrt{5} - 1)k_s L_0\langle -\tfrac{1}{\sqrt{5}}, -\tfrac{2}{\sqrt{5}}, 0\rangle\end{aligned}$$

$$\vec{F}_{net} = \left\langle \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}}, \frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}} - mg, 0 \right\rangle$$

B. Calculate the **new velocity** of the ball a short time later, when $t = \Delta t$.

Update the velocity:

$$\vec{v}(\Delta t) = \vec{v}(0) + \frac{\vec{F}_{net}}{m}\Delta t$$

$$\vec{a}_{net} = \left\langle \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}, \frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m} - g, 0 \right\rangle$$

$$\begin{aligned}\vec{v}(\Delta t) &= \langle v_0, 0, 0\rangle + \left\langle \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}, \frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m} - g, 0 \right\rangle \Delta t \\ &= \left\langle v_0 + \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}\Delta t, \frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}\Delta t - g\Delta t, 0 \right\rangle\end{aligned}$$

C. Calculate the **new position** of the ball at time $t = \Delta t$.

Update the position:

$$\vec{r}(\Delta t) = \vec{r}(0) + \vec{v}_{avg}\Delta t$$

$$\vec{v}_{avg} \approx \vec{v}(\Delta t)$$

$$\begin{aligned}\vec{r}(\Delta t) &= \langle -2L_0, -4L_0, 0\rangle + \left\langle v_0 + \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}\Delta t, \frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}\Delta t - g\Delta t, 0 \right\rangle \Delta t \\ &= \left\langle -2L_0 + v_0\Delta t + \frac{(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m}\Delta t^2, -4L_0 + \left(\frac{2(2\sqrt{5}-1)k_s L_0}{\sqrt{5}m} - g\right)\Delta t^2, 0 \right\rangle\end{aligned}$$

D. Determine the **new net force** acting on the ball at time $t = \Delta t$. You may use the variable $\vec{r}_f$ to indicate the current position of the ball (from part C).

(Note that $\vec{r}(\Delta t)$ should be replaced with $\vec{r}_f$.)

Now at $t = \Delta t$

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring}$$

$$\vec{F}_{grav} = \langle 0, -mg, 0 \rangle$$

$$\begin{aligned} \vec{F}_{spring} &= -k_s(|L| - L_0)\hat{L} \\ &= -k_s(|r(\vec{\Delta} t)| - L_0)\frac{\vec{r}(\Delta t)}{|\vec{r}(\Delta t)|} \end{aligned}$$

$$\vec{F}_{net} = \vec{F}_{grav} - k_s(|r(\vec{\Delta} t)| - L_0)\frac{\vec{r}(\Delta t)}{|\vec{r}(\Delta t)|}$$

E. How would you calculate the new position of the ball at time $t = 2\Delta t$? List the steps you'd need to take, and what quantities you'd need to determine.

Since you have the net force at time $t = 2\Delta t$, you need to use it to find the **velocity at t $= 2\Delta$t** with the velocity update formula and then use that velocity to find the **position at t $= 2\Delta$t** with the position update formula, just like for $t = \Delta t$.