Week 13

Angular Momentum Principle

Topics for this week

1. Total Angular Momentum
2. Changes in Angular Momentum
3. Torque

By the end of the week

1. Have two methods for determining angular momentum
2. Calculate torque
3. Never forget which way you need to turn a screw!

The total angular momentum

- How does the angular momentum principle generalize to multiparticle systems?
  - The same way we thought of total kinetic energy
- The total L is the superposition of the individual L

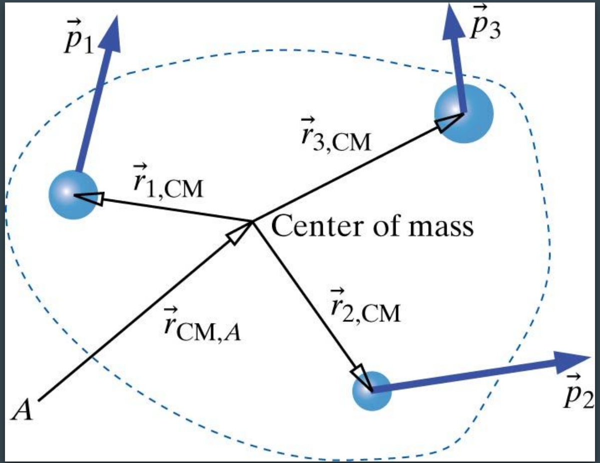
$$\vec{L}_{A,total} = \vec{L}_{A,1} + \vec{L}_{A,2} + \vec{L}_{A,3}$$

  - Define the position of each particle with respect to the center of mass

$$\vec{L}_{A,1} = (\vec{r}_{A,cm} + \vec{r}_{cm,1}) \times \vec{p}_1$$

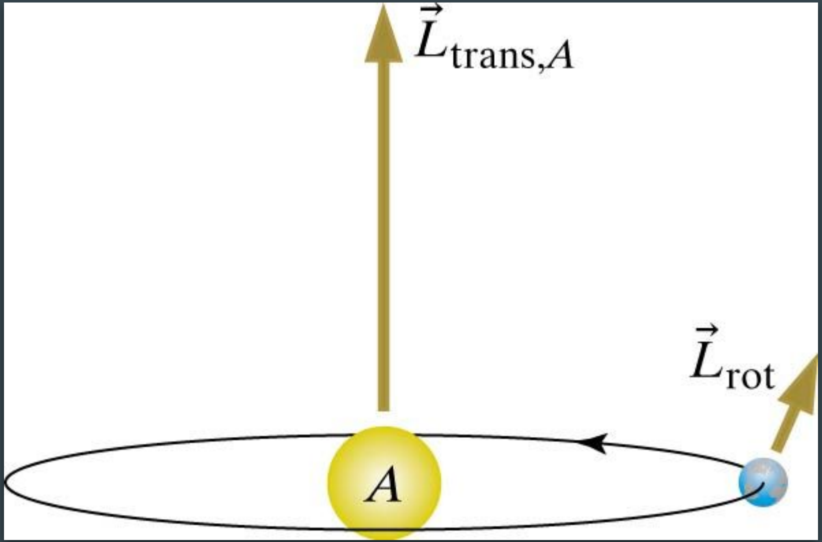
$$\vec{L}_{A,total} = (\vec{r}_{A,cm} \times \vec{p}_{total}) + \sum_{i=1}^{3} \vec{r}_{i,cm} \times \vec{p}_i$$

$$\vec{L}_{A,total} = \vec{L}_{A,trans} + \vec{L}_{cm,rot}$$



Decomposing Angular Momentum

- The translational angular momentum is associated with a rotation of the center of mass about some point A
  - Differs for different choices of the location for the point A
- The rotational angular momentum is associated with a rotation about the center of mass
  - Independent of the location of the point A and the motion of the CM
  - For solid body rotations about a single axis



$$\vec{L}_{rot} = I\vec{\omega} \quad \longrightarrow \quad K_{rot} = \frac{L_{rot}^2}{2I}$$

Changes in angular momentum

- What is the time rate of change of angular momentum for a point particle?
  - Take the derivative

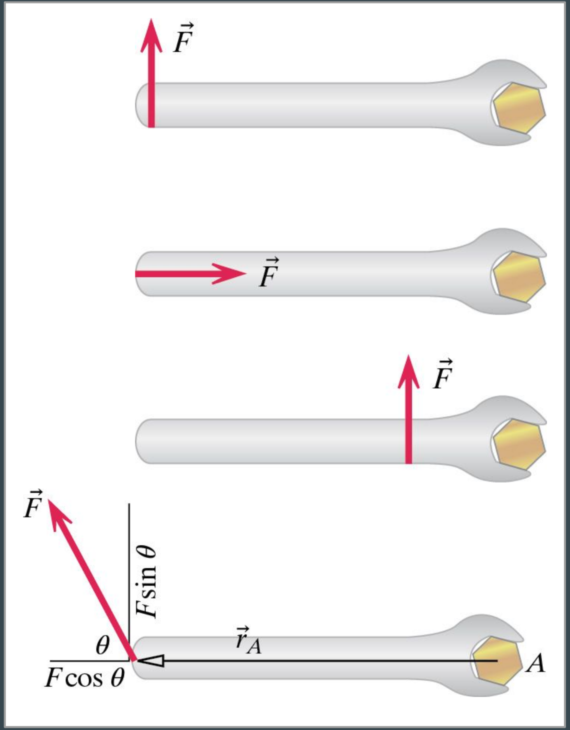
$$\frac{d}{dt}(\vec{r}_A \times \vec{p}) = \left(\frac{d\vec{r}_A}{dt} \times \vec{p}\right) + \left(\vec{r}_A \times \frac{d\vec{p}}{dt}\right)$$

  - The first term is zero because those vectors are parallel
  - The second term can be simplified by substitution of the momentum principle
- Tau stands for the torque or “twist” on the system from the surroundings

$$\frac{d\vec{L}_A}{dt} = \vec{r}_A \times \vec{F}_{net} = \vec{\tau}_{A,net}$$

Torque

- We can conceptualize torque by imagining that we are using a wrench to tighten a bolt.
  - Righty Tighty: Push up on the wrench and the torque on the wrench is LF and the bolt spins into the page
  - Push to the right and the bolt has zero spin
  - Righty Tighty: Push up on the wrench close to the bolt and the torque on the wrench is LF/4 and the bolt spins into the page more slowly than before
  - Righty Tighty: Push up and out on the wrench and the torque on the wrench is LFsinθ and the bolt spins into the page more slowly than before
- What happens if we push down on the wrench?
  - Visualize the direction of the torque and the bolt with the thumb of your right hand!



Decomposing Torque

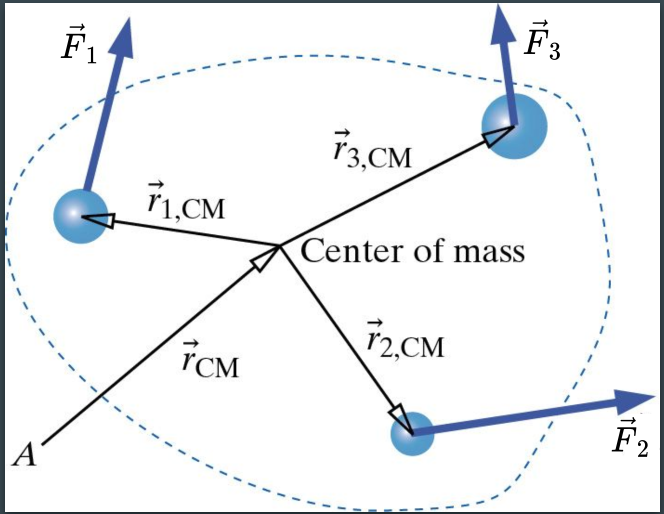
- A change in translational L results from the net force acting on the center of mass

$$\frac{d\vec{L}_{trans}}{dt} = \vec{r}_{A,cm} \times \vec{F}_{net}$$

- A change in rotational L results from forces acting relative to the center of mass

$$\frac{d\vec{L}_{rot}}{dt} = \sum \vec{r}_{cm,i} \times \vec{F}_i$$

- The location A can be chosen to eliminate part or all of the change in L



$$\frac{d\vec{L}_{A,tot}}{dt} = \frac{d\vec{L}_{trans}}{dt} + \frac{d\vec{L}_{rot}}{dt}$$

$$\frac{d\vec{L}_{A,tot}}{dt} = \vec{\tau}_{trans} + \vec{\tau}_{rot}$$

Conservation of angular momentum

- In some systems with forces present there can be a change in momentum but no change in angular momentum
  - More often we can pick the reference point "A" to eliminate torque.
  - Changes in the moment of inertia can lead to change in the rotation rate of the system

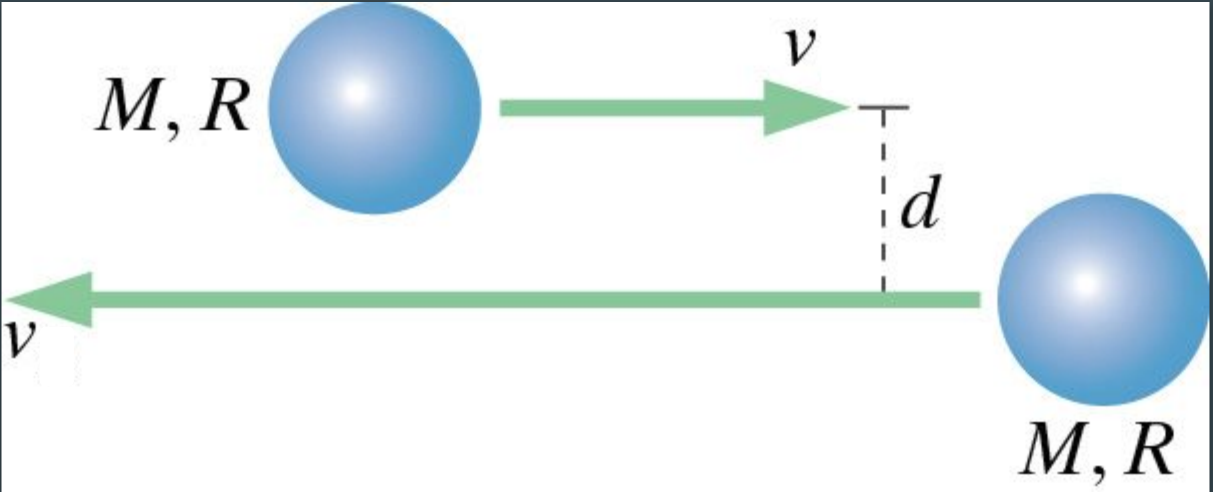
$$\Delta \vec{L}_{rot} = 0$$

$$I_i \vec{\omega}_i = I_f \vec{\omega}_f$$

Spin level: Over 9000!!!

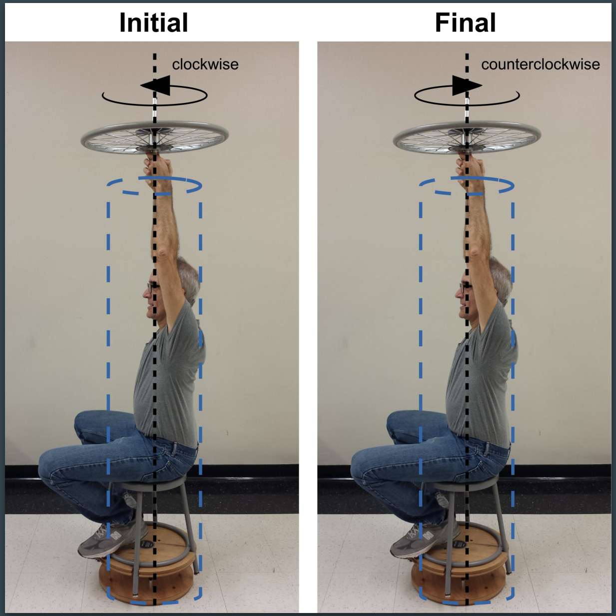
Example: The colliding balls

Two identical asteroids of mass M and radius R collide and stick together. Before the collision, on asteroid was moving to the right with speed v and the other was moving to the left with speed v and neither asteroid was rotating. After the collision, determine the velocity of the center of mass of the stuck together asteroids and it's angular velocity.



Example: The spinning prof.

- Prof. Schatz is sitting in a swivel chair such that the wheel, chair, and professor Schatz all share the same rotational axis as indicated in the figure. Initially only the wheel is spinning
  - The wheel is spinning clockwise with constant angular speed ω when viewed from above. The moment of inertia for the wheel is mR^2 and $(½)Mr_s^2$ for Prof. Schatz and the chair.
  - Professor Schatz turns the wheel over while maintaining it directly overhead so that the angular speed of the wheel is unchanged but it is now rotating counterclockwise when viewed from above with speed ω_0. Determine the magnitude of angular velocity of Prof. Schatz.



Example: The spinning prof. continued

- Prof. Schatz lowers the wheel so that it is directly in front of him a distance d from his axis of rotation. The wheel is still rotating counterclockwise when viewed from above. During this process there is zero net torque on the system. You can assume that there is negligible friction in both the axles of the wheel and chair.
  - Determine the new magnitude of the angular velocity of Prof. Schatz after having moved the wheel in front of him.

Example: How fast can a car navigate a turn

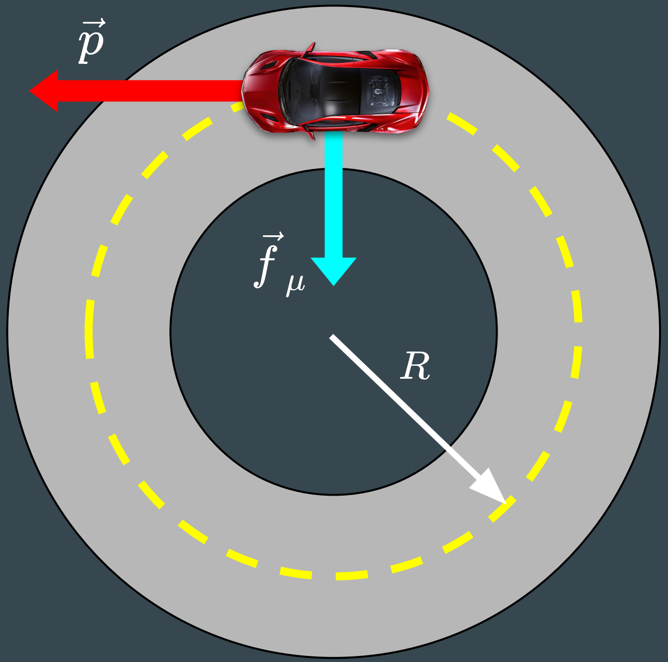
A car is travelling through a circular turn of radius R at a constant speed v. The car has dimensions of length L, height h, width w, and a total mass M. How fast can the car travel through the turn before it begins to tip over?

Example: Cont.

- When viewed from above, the force of gravity and the normal component of the contact force cancel
  - The net frictional force is the only force responsible for moving the car along a circular path
  - Use the momentum principle to determine the magnitude of this force

$$\vec{F}_{net} = \vec{F}_{\perp} = \vec{f}_{\mu}$$

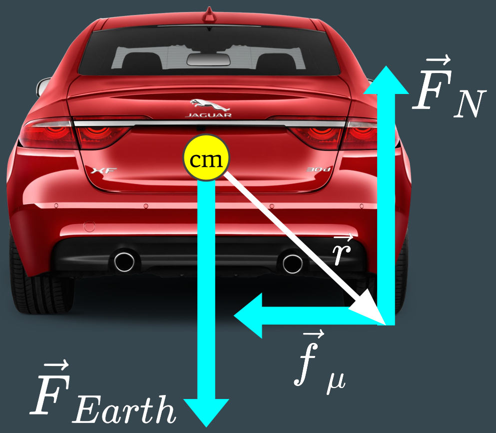
$$\vec{f}_{\mu} = \frac{m|\vec{v}|^2}{R}$$



Example: Cont.

- When viewed from behind, the car will begin to rotate as soon as the net torque on the car is non-zero
  - At this instant the inside tires lose contact
  - Calculate the net torque about the center of mass

Car has momentum into the page and is turning left



$$\vec{\tau}_{cm,net} = \vec{r}_f \times \vec{f}_\mu + \vec{r}_{F_N} \times \vec{F}_N$$

$$\vec{\tau} = \langle \frac{w}{2}, -\frac{h}{2}, 0 \rangle \times \langle -\frac{mv^2}{R}, mg, 0 \rangle$$

$$|\vec{\tau}| = \frac{wmg}{2} - \frac{hmv^2}{2R}$$

Example: Cont.

- The net torque is zero when the velocity has the value

$$v^2 = \left(\frac{w}{h}\right) gR$$

- This ratio of the width to the height is called the static stability factor
  - Formula 1 car approx 1.89
  - Ford Bronco II approx 0.99
    - 1 in 500 Ford Bronco II’s were involved in a rollover