PHYS 2211 K

Week 12, Lecture 2
2022/03/31

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4 clicker questions today

On today's class…

1. Collisions and conservation of linear momentum
2. Elastic, inelastic, maximally inelastic collisions
3. Ballistic pendulum, Rutherford scattering

Road map for the rest of the semester

- Week 12 ← you are here
  - Lecture topics: Wrapping up pp vs real; Collisions, Scattering
- Week 13
  - Lectures topics: Cross product, Torque, Angular momentum
  - New due date: Lab 5 submission due at end of week 13 (April 10)
- Week 14
  - New test date: Test 3 on April 11 (coverage: weeks 9, 10, 12)
  - Lecture topics: Angular momentum principle, multiparticle angular momentum, angular momentum of rigid systems
- Week 15
  - Lecture topics: Wrapping up angular momentum; Quantum stuff
  - Hard deadline for everything (edx, etc) on April 24
- Week 16 – Final exam on Friday April 29

CLICKER 1: Did you miss me? ☺

A. Yes!

B. Kinda?

C. Maybe

D. Nah

E. No comment

Thank you for all your kind messages ♥

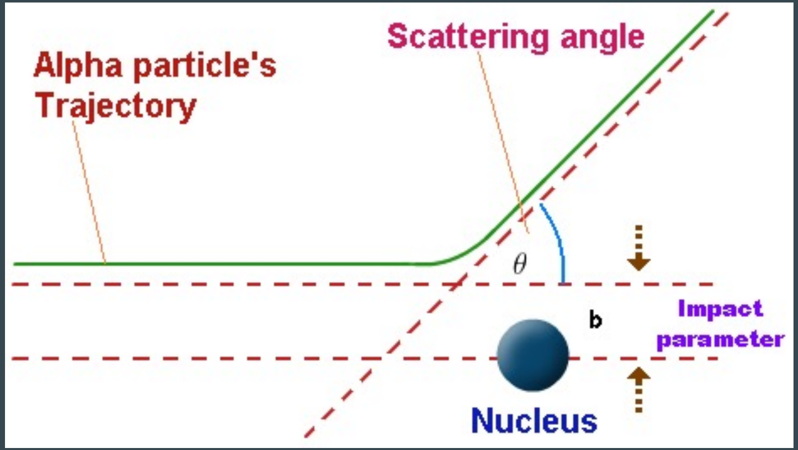
Please note that Tuesday’s clickers won’t count towards the class participation grade because I’m not sure how to sync clickers for this section from Prof Fenton’s turningpoint account.

Collisions

- A collision is an interaction between two objects that takes place in a very short amount of time

- The interaction between the two objects is much larger than the interactions between the objects and their surroundings

- The objects do not have to make physical contact for there to be a collision!



Conservation of Linear Momentum

- The total linear momentum of a system during a collision is conserved

$$\Delta\vec{p}_{\text{total}} = 0$$

- This means that the total linear momentum (the momentum of all the objects involved in the collision) is the same before and after the collision, regardless of what happens during the collision

$$\underbrace{(\vec{p}_1 + \vec{p}_2)_i}_{\text{just before the collision}} = \underbrace{(\vec{p}_1 + \vec{p}_2)_f}_{\text{just after the collision}}$$

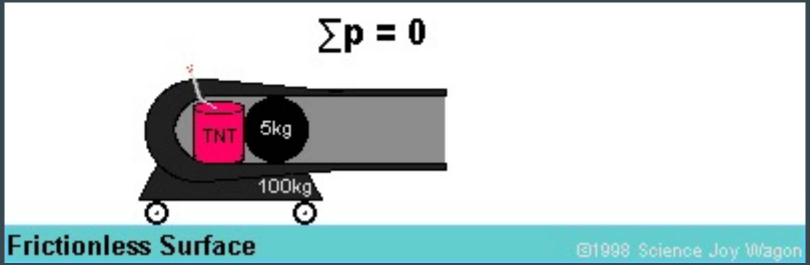
- Remember that momentum is a vector, and therefore you have to consider the x,y,z components independently from each other

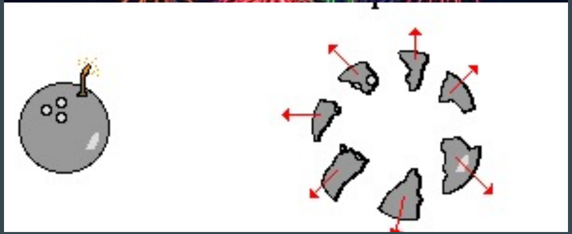
Conservation of Linear Momentum

- $\Delta\vec{p} = 0$ comes from Newton’s 2^{nd} law: if Fnet=0, then the total momentum of the system doesn’t change

- This means that the total linear momentum of an isolated system is also conserved

- Recoil and the shape of fireworks are a result of the conservation of total linear momentum





CLICKER 2: (1D collision) Princess Azula (m = 45 kg) was moving with a velocity of <4,0,0> m/s when she collided with Uncle Iroh (M = 100 kg) who was moving with a velocity of <-1,0,0> m/s. After the collision, Azula's velocity is <-2,0,0> m/s. What is Iroh's velocity after the collision?

A. <-0.1, 0, 0> m/s

B. <0.1, 0, 0> m/s

C. <-1.7, 0, 0> m/s

D. <1.7, 0, 0> m/s

E. <-3.7, 0, 0> m/s

F. <3.7, 0, 0> m/s

Solution: (1D collision) Princess Azula (m = 45 kg) was moving with a velocity of <4,0,0> m/s when she collided with Uncle Iroh (M = 100 kg) who was moving with a velocity of <-1,0,0> m/s. After the collision, Azula's velocity is <-2,0,0> m/s. What is Iroh's velocity after the collision?

$$\Delta \vec{p} = 0 \Rightarrow \vec{p}_i = \vec{p}_f \Rightarrow \vec{p}_{iA} + \vec{p}_{iI} = \vec{p}_{fA} + \vec{p}_{fI}$$

$$(45)(4)\hat{x} + (100)(1)(-\hat{x}) = (45)(2)(-\hat{x}) + \vec{p}_{fI}$$

$$(180 - 100)\hat{x} = -90\hat{x} + \vec{p}_{fI}$$

$$(80 + 90)\hat{x} = \vec{p}_{fI}$$

$$170\hat{x} = \vec{p}_{fI}$$

$$\vec{p}_{fI} = m_I \vec{v}_{fI} \Rightarrow \vec{v}_{fI} = \frac{\vec{p}_{fI}}{m_I} = \frac{170}{100} = \boxed{1.7 \text{ m/s}}\ \hat{x}$$

$$\Rightarrow \boxed{<1.7, 0, 0> \text{ m/s}}$$

Types of Collisions

- Collisions are classified according to what happens to the kinetic energy of the system before and after the collision

- There are three types of collisions:
  - Elastic Collision: $\Delta K = 0$
    - the system doesn’t change shape, temperature, etc
    - this never actually happens in real life for macroscopic objects, but can be a good approximation for solid heavy things colliding
  - Inelastic Collision: $\Delta K \neq 0$
    - energy is lost to the surroundings in the form of sound, heat, deformation, etc
    - most real-world collisions are inelastic
  - Maximally Inelastic Collision: an inelastic collision that results in the two objects sticking together after colliding

CLICKER 3: What kind of collision did Azula and Iroh have?

A. Elastic

B. Inelastic

Azula (m = 45 kg)
v_initial = 4 m/s
v_final = -2 m/s

Iroh (M = 100 kg)
v_initial = -1 m/s
v_final = 1.7 m/s

Initial

$$K_{Ai} = \frac{1}{2} m_A v_{iA}^2 = \frac{1}{2}(45)(4^2) = 360$$

$$K_{Ii} = \frac{1}{2} M_I v_{iI}^2 = \frac{1}{2}(100)(-1)^2 = 50$$

$$K_i = 360 + 50 = \underline{410 J}$$

Final

$$K_{Af} = \frac{1}{2} m_A v_{fA}^2 = \frac{1}{2}(45)(-2^2) = 90$$

$$K_{If} = \frac{1}{2} M_I v_{fI}^2 = \frac{1}{2}(100)(1.7)^2 = 144.5$$

$$K_f = 90 + 144.5 = \underline{234.5 J}$$

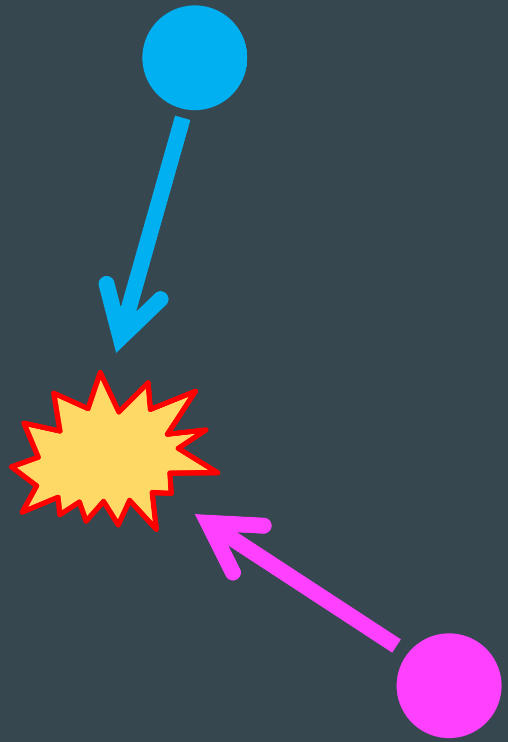
$$K_i \neq K_f$$

Collisions in 2D and 3D

- Exactly the same as collisions in 1D, but you need to make sure to remember the vector nature of momentum and keep track of both x and y components

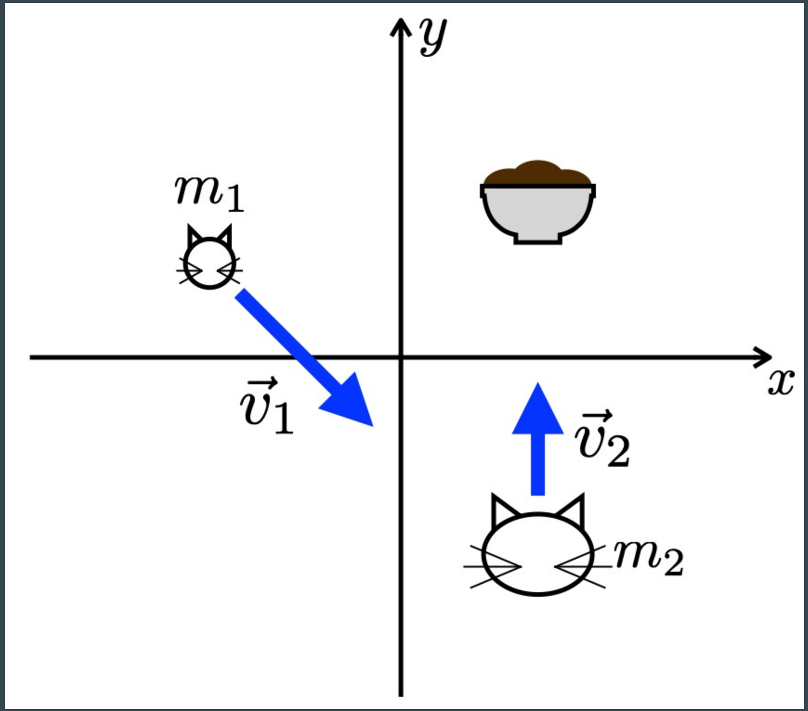
- The physics is the same, but the math can get a bit ugly

- Same for 3D collisions, only that you then have to also keep track of the z components



CLICKER 4: (2D collision) Little Kitty (m_1 = 2 kg) runs with velocity <4.1, -8.7, 0> m/s and collides with Big Kitty (m_2 = 9 kg) who was moving at 0.9 m/s towards the food bowl. What's the final velocity of the single giant ball of claws and fur resulting from this collision?

A. <8.2, -9.3, 0> m/s
B. <0.75, -0.85, 0> m/s
C. <0.37, -0.71, 0> m/s
D. <4.1, -0.3, 0> m/s
E. 0 m/s



Solution: (2D collision) Little Kitty (m_1 = 2 kg) runs with velocity <4.1, -8.7, 0> m/s and collides with Big Kitty (m_2 = 9 kg) who was moving at 0.9 m/s towards the food bowl. What's the final velocity of the ball of claws and fur resulting from this collision?

$$\Delta\vec{p} = 0 \Rightarrow \vec{p}_i = \vec{p}_f \Rightarrow p_{ix} = p_{fx} \; \& \; p_{iy} = p_{fy}$$

$$\vec{P}_{Li} + \vec{P}_{Bi} = \vec{P}_{f(L+B)}$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{(2)\langle 4.1, -8.7, 0\rangle + (9)\langle 0, 0.9, 0\rangle}{2+9} =$$

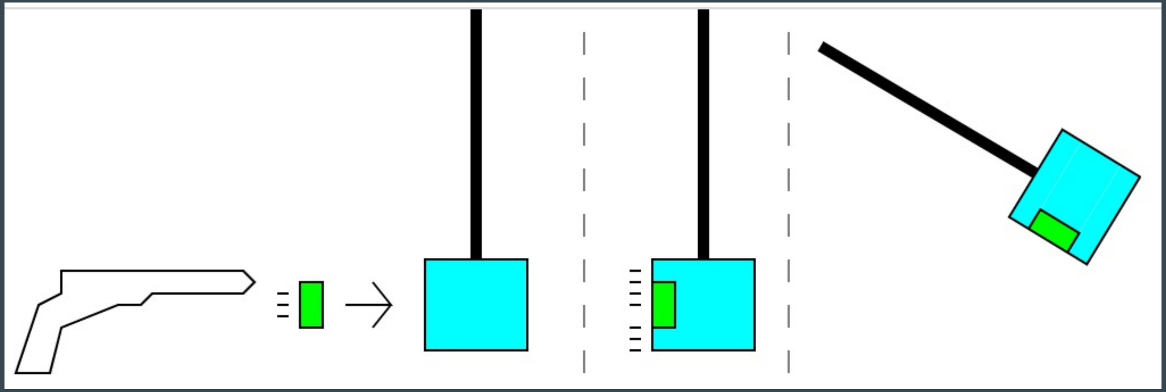
$$= \frac{\langle 8.2, -17.4, 0\rangle + \langle 0, 8.1, 0\rangle}{11} =$$

$$= \frac{\langle 8.2, -9.3, 0\rangle}{11} = \boxed{\langle 0.745, -0.845, 0\rangle \text{ m/s}}$$

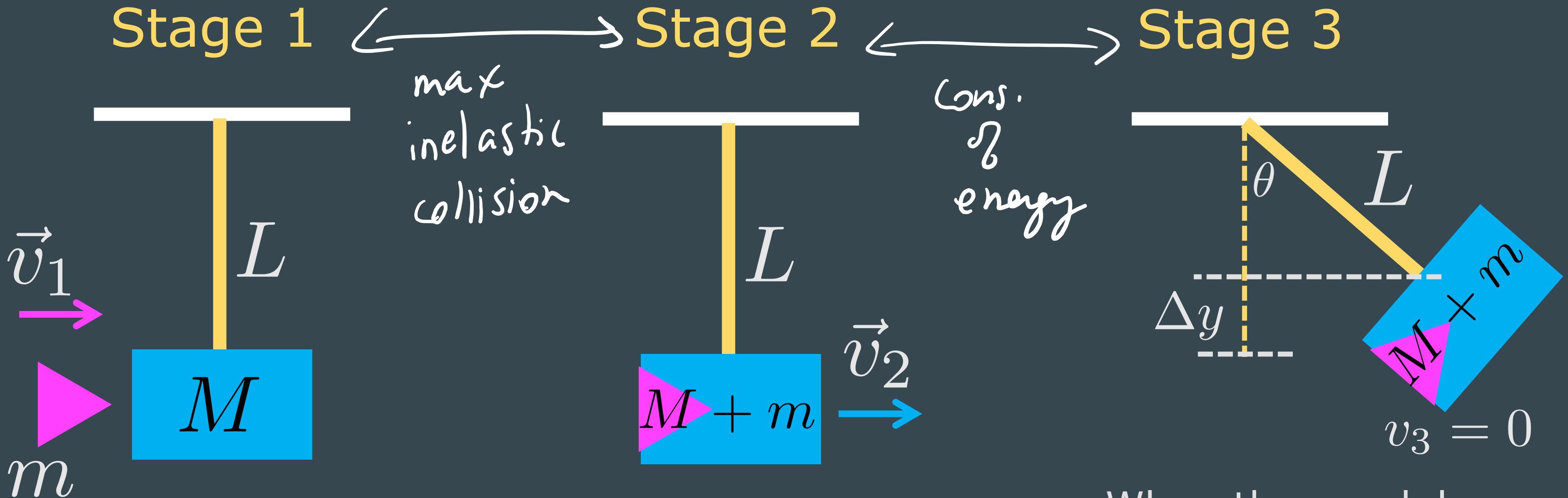
Example of collisions and conservation of energy: The Ballistic Pendulum

In 1742, the ballistic pendulum was invented to determine the muzzle velocity of a musket (the speed at which the bullets get shot). A bullet (mass m) is shot at a block (mass M) attached to a pendulum. The bullet and the block undergo a maximally inelastic collision. The pendulum (length L) swings as a result of the impact, when it reaches its maximum angle θ, a latch secures it in place so it doesn't move anymore.

You can only measure the masses (m and M), the length of the pendulum (L) and the maximum angle (θ). Can you determine the muzzle velocity of the musket?



Ballistic Pendulum



Bullet is fired, collision hasn't happened yet

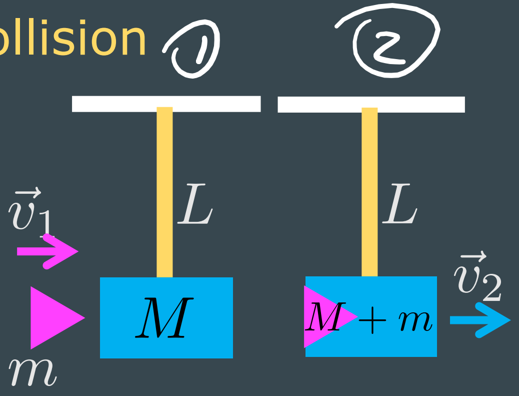
Immediately after maximally inelastic collision, block and bullet move together

When the pendulum stops at the block's maximum height

Ballistic Pendulum

From stage 1 to stage 2 there's a maximally inelastic collision
= conservation of linear momentum

We'll end up with one equation that has two unknowns



$$\Delta\vec{p} = 0 \Rightarrow \vec{p}_i = \vec{p}_f$$

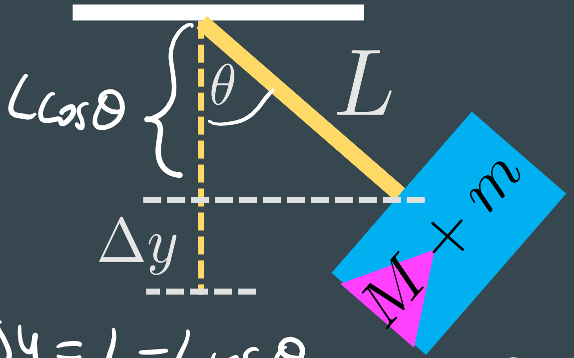
$$m\vec{v}_1 = (m+M)\vec{v}_2$$

$$\boxed{v_1 = \left(\frac{m+M}{m}\right) v_2}$$

what we want

?

Ballistic Pendulum



From stage 2 to stage 3 it's a conservation of energy problem, which we'll solve to find v_2

$$\Delta E = \cancel{W}^{\,0} \qquad \text{system: everything}$$

$$\Delta y = L - L\cos\theta$$

$$v_3 = 0$$

$$\Delta K + \Delta U_g = 0$$

$$\frac{1}{2}(m+M)(\cancel{v_3^2}^{\,0} - v_2^2) + (m+M)g\,(L - L\cos\theta) = 0$$

$$-\frac{1}{2}\cancel{(m+M)}\,v_2^2 + \cancel{(m+M)}\,g\,L\,(1-\cos\theta) = 0$$

$$\boxed{v_2 = \sqrt{2gL(1-\cos\theta)}}$$

$$L - L\cos\theta$$

$$L(1-\cos\theta)$$

Ballistic Pendulum

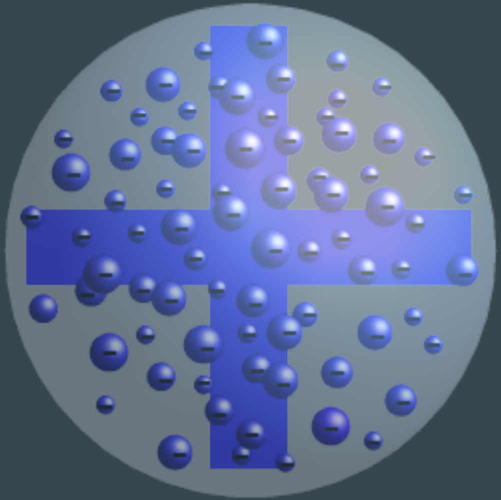
Now we plug in what we found for v_2 to determine v_1, which is the objective of this entire exercise

$$v_1 = \left(\frac{m+M}{m}\right) v_2$$

$$v_2 = \sqrt{2gL(1-\cos\theta)}$$

$$\Rightarrow \boxed{v_1 = \left(\frac{m+M}{m}\right)\sqrt{2gL(1-\cos\theta)}}$$

Example of collision with no contact: Rutherford Scattering



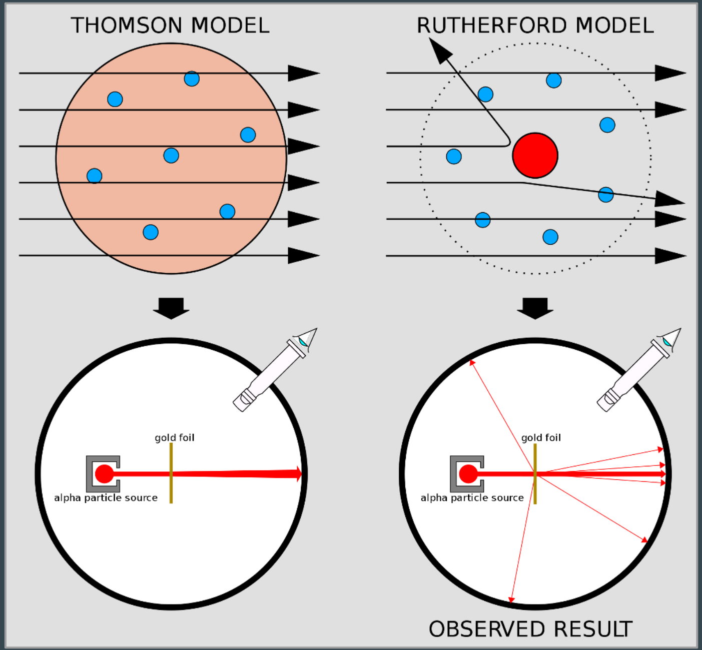
- Thomson (1877) proposed the "plum pudding" model of the atom
  - Positively charged "cake" of uniform density with tiny negative "raisins" distributed throughout (electrons)
- Rutherford (1911) set out to determine the distribution of the electrons in the plum pudding model
- The experiment consisted of shooting high-energy alpha particles at a thin sheet of gold foil – with wildly unexpected results!

Rutherford Scattering

- Imagine shooting a gun at a sheet of single-ply toilet paper, and having some of the bullets bounce back at you

- The deflection of the alpha particles (which are positively charged) indicated the presence of a positively charged nucleus in the center of the atom

- Deflection happens because the electric force between the nucleus and the alpha particle is repulsive, and because of conservation of momentum in collisions



Rutherford Scattering

- Glowscript code:
  - https://www.glowscript.org/#/user/ealicea/folder/Public/program/rutherford
- PhET simulation:
  - https://phet.colorado.edu/sims/html/rutherford-scattering/latest/rutherford-scattering_en.html
- Hypherphysics resource:
  - http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html

Using conservation of energy we can determine the minimum separation distance between alpha and gold nucleus in the case of a head-on collision (impact parameter b=0), which is what we did in Test 2

The scattering angle can be found from the initial and final momentums