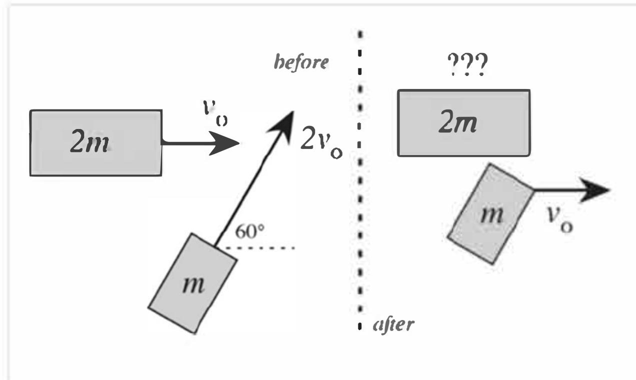
Physics 2211 GPS Week 13

Problem #1

A cruise ship is moving due east with some speed v_0 when it collides with a boat moving 60° north of east with a speed $2v_0$. The boat's mass is m and the cruise ship's mass is $2m$. Immediately after the collision, the boat is observed to be floating due east with a speed v_0.



(a) Determine the velocity of the cruise ship just after the collision. You need to express your answer in terms of the parameter v_0.

$\vec{v}_{ci} = \langle v_0, 0, 0\rangle$ $\quad\quad$ $\vec{v}_{bi} = 2v_0\langle \cos 60°, \sin 60°, 0\rangle$

$\vec{v}_{cf} = ?$ $\quad\quad$ $\vec{v}_{bf} = \langle v_0, 0, 0\rangle$

$m_c = 2m$ $\quad\quad$ $m_b = m$

$$\vec{P}_{ci} + \vec{P}_{bi} = \vec{P}_{cf} + \vec{P}_{bf}$$

$$m_c\vec{v}_{ci} + m_b\vec{v}_{bi} = m_c\vec{v}_{cf} + m_b\vec{v}_{bf}$$

$$m_c\vec{v}_{ci} + m_b(\vec{v}_{bi} - \vec{v}_{bf}) = m_c\vec{v}_{cf}$$

$$\vec{v}_{cf} = \frac{\cancel{m_c}\vec{v}_{ci}}{\cancel{m_c}} + \frac{m_b}{m_c}(\vec{v}_{bi} - \vec{v}_{bf}) = \vec{v}_{ci} + \frac{\cancel{m}}{2\cancel{m}}(\vec{v}_{bi} - \vec{v}_{bf}) =$$

$$= \langle v_0, 0, 0\rangle + \frac{1}{2}\left[2v_0\langle \cos 60°, \sin 60°, 0\rangle - \langle v_0, 0, 0\rangle\right] =$$

$$= \langle v_0, 0, 0\rangle + v_0\langle \cos 60°, \sin 60°, 0\rangle - \langle \tfrac{v_0}{2}, 0, 0\rangle =$$

$$= \langle \tfrac{v_0}{2}, 0, 0\rangle + v_0\langle \cos 60°, \sin 60°, 0\rangle =$$

$$= \langle \tfrac{v_0}{2} + v_0\cos 60°, v_0 \sin 60°, 0\rangle = \langle \tfrac{v_0}{2} + \tfrac{v_0}{2}, 0.866\, v_0, 0\rangle =$$

$$= \langle v_0, 0.866\, v_0, 0\rangle = \boxed{v_0\langle 1, 0.866, 0\rangle}$$

(b) What percentage of the original kinetic energy was lost in the collision?

$$\checkmark\ K_i = K_{ci} + K_{bi} = \tfrac{1}{2} m_c v_{ci}^2 + \tfrac{1}{2} m_b v_{bi}^2 = \tfrac{1}{\cancel{2}}(\cancel{2}m)\, v_0^2 + \tfrac{1}{2} m (2v_0)^2 =$$

$$= m v_0^2 + \tfrac{1}{\cancel{2}} m \overset{2}{\cancel{4}} v_0^2 = m v_0^2 + 2 m v_0^2 = 3 m v_0^2$$

$$\checkmark\ K_f = K_{cf} + K_{bf} = \tfrac{1}{2} m_c v_{cf}^2 + \tfrac{1}{2} m_b v_{bf}^2 = \tfrac{1}{\cancel{2}}(\cancel{2}m)\, \underbrace{v_{cf}^2} + \tfrac{1}{2} m v_0^2 =$$

$$v_{cf}^2 = v_0^2 (1 + 0.866^2) = v_0^2 (1 + 0.75) = 1.75 v_0^2$$

$$= 1.75 m v_0^2 + \tfrac{1}{2} m v_0^2 = (1.75 + 0.5)\, m v_0^2 = 2.25 m v_0^2$$

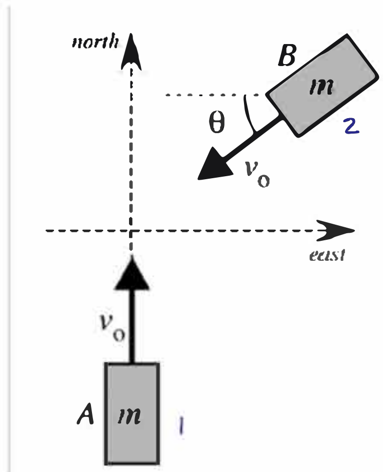
$$\checkmark\ |\Delta K| = |K_f - K_i| = |2.25 m v_0^2 - 3 m v_0^2| = |-0.75 m v_0^2| = 0.75 m v_0^2$$

$$\Rightarrow \frac{K_f}{K_i} = \frac{2.25 \cancel{m v_0^2}}{3 \cancel{m v_0^2}} = \frac{2.25}{3} = 0.75$$

$$\Rightarrow \%\ \text{lost} = (1 - 0.75) \times 100\% = \boxed{25\%}$$

Problem #2

Two cars of identical mass m are involved in a collision. Both cars are moving at the same speed v_0. One of the cars is initially moving due north, and the other is initially moving south of west at an angle θ (where $45° < \theta < 90°$). The collision between the two vehicles is maximally inelastic; in other words, the vehicles stick together after the collision.



(a) Determine the magnitude of the final velocity of each car after the collision, in terms of the quantities m, v_0, and θ.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_f \Rightarrow \vec{v}_f = \frac{m_1}{m_1+m_2}\vec{v}_1 + \frac{m_2}{m_1+m_2}\vec{v}_2$$

$$\vec{v}_f = \frac{\cancel{m}}{2\cancel{m}}\langle 0, v_0, 0\rangle + \frac{\cancel{m}}{2\cancel{m}}\langle -v_0\cos\theta, -v_0\sin\theta, 0\rangle = \frac{v_0}{2}\langle 0,1,0\rangle + \frac{v_0}{2}\langle -\cos\theta, -\sin\theta, 0\rangle =$$

$$= \frac{v_0}{2}\langle -\cos\theta, 1-\sin\theta, 0\rangle$$

$$|\vec{v}_f| = \sqrt{\left(\frac{v_0}{2}\right)^2}\sqrt{(-\cos\theta)^2 + (1-\sin\theta)^2} = \frac{v_0}{2}\sqrt{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta} =$$

$$= \frac{v_0}{2}\sqrt{\cos^2\theta + \sin^2\theta + 1 - 2\sin\theta} = \frac{v_0}{2}\sqrt{1+1-2\sin\theta} =$$

$$= \boxed{\frac{v_0}{2}\sqrt{2-2\sin\theta}}$$

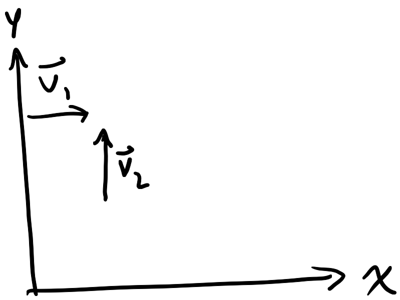
Problem #3

A person of mass m_1 is walking at constant velocity $\vec{v_1} =< v_1, 0, 0 >$ along the sidewalk towards an intersection, looking down at their phone. At the same time, another person riding a scooter (total mass m_2) approaches the intersection with unknown velocity $\vec{v}_2 =< 0, v_2, 0 >$. Neither the person walking nor the person on the scooter notice each other, and so they crash.

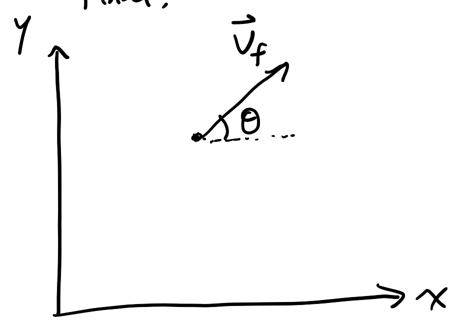
The two people and the scooter stick together after the crash, moving together with an **unknown final speed** at an angle θ above the x-axis.

1. [15 pts] How fast was the scooter moving before the crash, i.e., what was the initial speed v_2? **Your answer must only use variables that represent known quantities** (m_1, m_2, v_1, θ).

Initial:



Final:



Conservation of momentum: $\vec{p}_f = \vec{p}_i$

$$\Rightarrow \vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = \langle m_1v_1, m_2v_2, 0\rangle$$

$$\vec{p}_f = (m_1+m_2)\,\vec{v}_f = (m_1+m_2)\langle v_f\cos\theta, v_f\sin\theta, 0\rangle$$

Comparing $p_{i,x}$ and $p_{f,x}$,

$$m_1v_1 = (m_1+m_2)\,v_f\cos\theta \Rightarrow v_f = \frac{m_1v_1}{(m_1+m_2)\cos\theta}$$

Comparing $p_{i,y}$ and $p_{f,y}$,

$$m_2v_2 = (m_1+m_2)\,v_f\sin\theta = \frac{(m_1+m_2)\,m_1v_1\sin\theta}{(m_1+m_2)\cos\theta}$$

$$= m_1v_1\tan\theta$$

$$\Rightarrow \boxed{v_2 = \frac{m_1v_1\tan\theta}{m_2}}$$

2. [5 pts] What was the change in internal energy of the system during the crash? Again, your answer must only use variables that represent known quantities in the problem.

Energy principle:

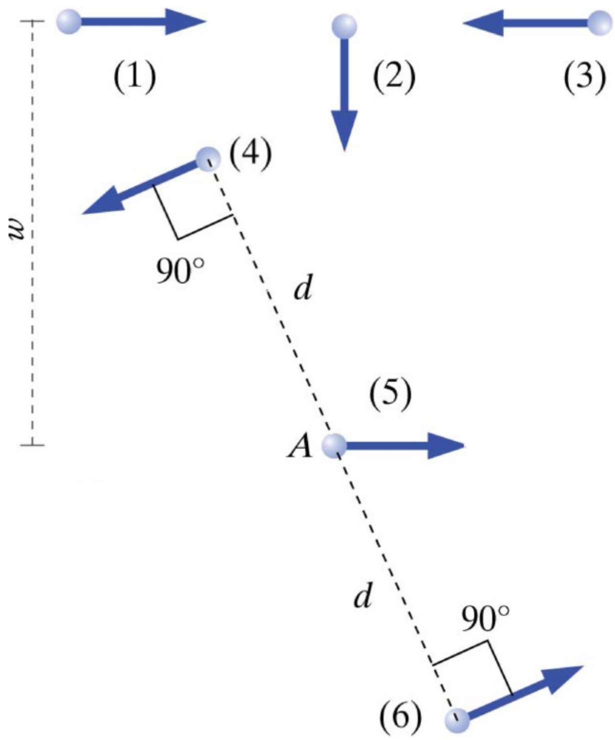
$$\Delta K + \Delta E_{int} = \cancel{W_{ext}}^{\,0}$$

$$\Rightarrow \Delta E_{int} = -\Delta K = K_i - K_f$$

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1+m_2) v_f^2$$

$$= \boxed{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1 v_1 \tan\theta}{m_2}\right)^2 - \frac{1}{2}(m_1+m_2)\left(\frac{m_1 v_1}{(m_1+m_2)\cos\theta}\right)^2}$$

Physics 2211 GPS Week 14



Problem #1

In the diagram on the right, six identical particles of mass m and speed v are moving relative to a point A, the current location of particle (5). The distance of these particles from point A is indicated in the diagram. The arrows indicate the directions of the particle's velocities.

As usual, +x is to the right, +y is up and +z is out of the page, towards you.

In the following calculations, remember that angular momen-tum is a vector.

(a) Calculate the angular momentum of particle 1 with respect to A.

$$\vec{L}_{1A} = \vec{r}_{1A} \times \vec{p}_1 = wmv\,(-\hat{z})$$

(b) Calculate the angular momentum of particle 2 with respect to A.

$$\vec{L}_{2A} = \vec{r}_{2A} \times \vec{p}_2 = 0$$ because $\vec{r}_{2A}$ and p_2 are antiparallel.

(c) Calculate the angular momentum of particle 3 with respect to A.

$$\vec{L}_{3A} = \vec{r}_{3A} \times \vec{p}_3 = wmv\,(+\hat{z})$$

(d) Calculate the angular momentum of particle 4 with respect to A.

$$\vec{L}_{4A} = \vec{r}_{4A} \times \vec{p}_4 = dmv\,(+\hat{z})$$

(e) Calculate the angular momentum of particle 5 with respect to A.

$$\vec{L}_{5A} = \vec{r}_{5A} \times \vec{p}_5 = 0$$ because $\vec{r}_{5A} = 0$.

(f) Calculate the angular momentum of particle 6 with respect to A.

$$\vec{L}_{6A} = \vec{r}_{6A} \times \vec{p}_6 = dmv\,(+\hat{z})$$

(g) Calculate the total angular momentum of the system of particles with respect to A.

$$\vec{L}_A = \sum_i \vec{L}_{iA} = (wmv - wmv + dmv + dmv)\,\hat{z} = 2dmv\,(\hat{z})$$