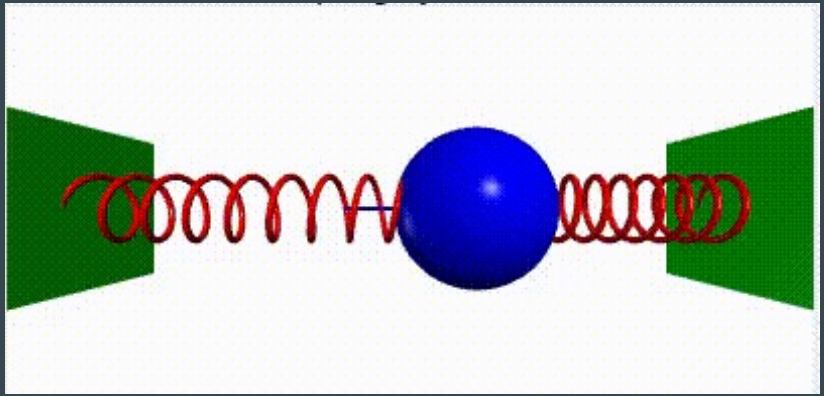

PHYS 2211 K

Week 3, Lecture 2
2022/01/27
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6 clicker questions today

On today’s class…

1. Spring force
2. Iteration with constant and non-constant forces
3. Universal gravitation

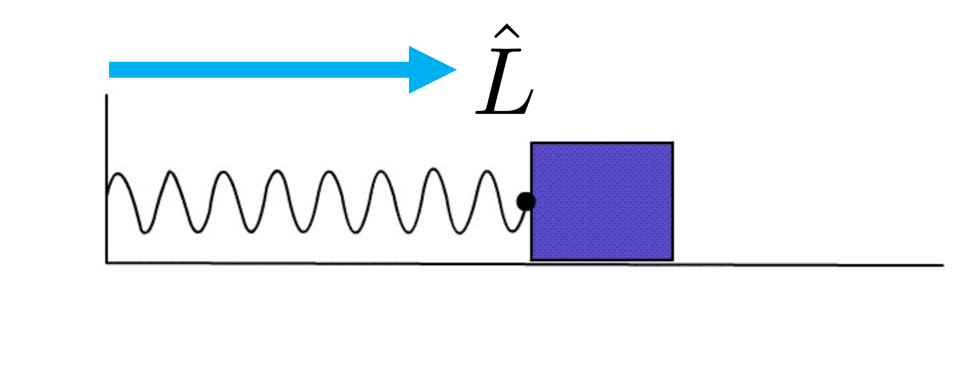
From Tuesday

- Spring force $\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$

- L vector points from fixed end to moving end of the spring (same as position vector of the mass, when the origin is located at the fixed end of the spring)

- $L > L_0$ = stretched spring (force pulls in)

- $L < L_0$ = compressed spring (force pushes out)



Also from Tuesday

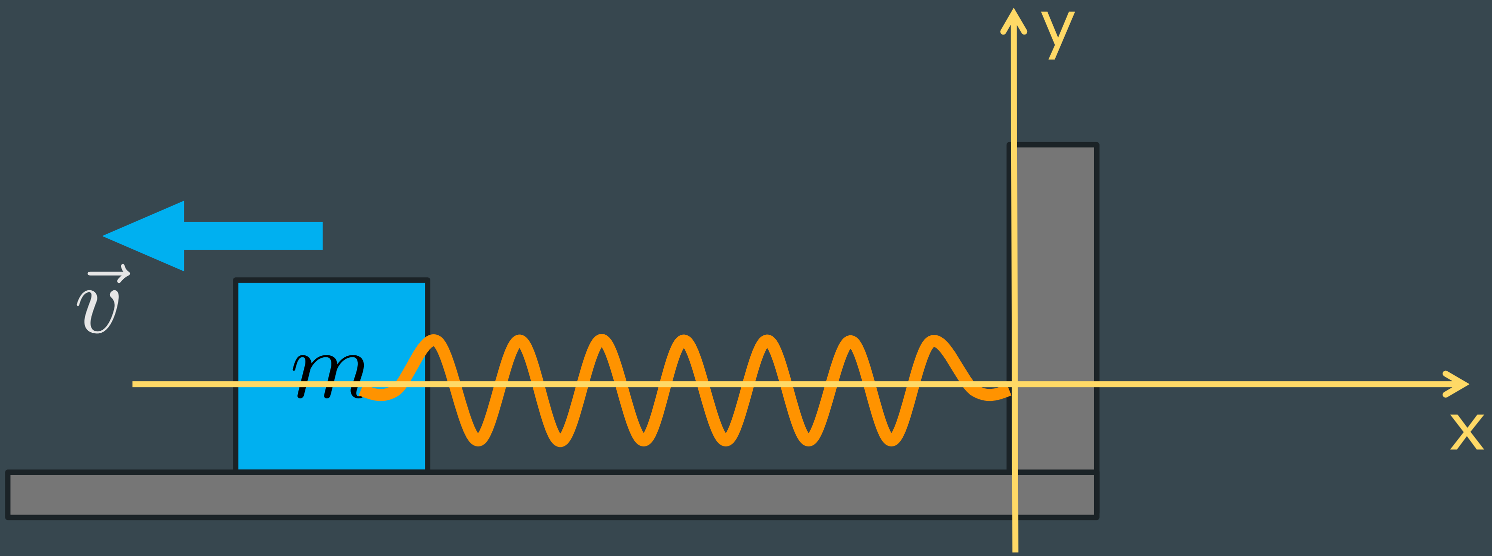
- Iteration means to predict the motion of an object in several very small consecutive time steps ← smaller deltat means more accurate prediction

- Procedure:
  - Find Fnet
  - Update velocity (v_final) with Newton's 2nd Law
  - Update position with position (r_final) update formula
    - For constant force: v_avg = arithmetic average of v_initial & v_final
    - For non-constant force: v_avg = v_final
  - Go to the next time step (increase t by an amount deltat)
  - Repeat: find new Fnet, find new v_final, find new r_final, etc

CLICKER 1: What is your favorite season?

Example

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.



CLICKER 2: What is the direction of the spring force?

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

A. To the left

B. To the right

C. Zero magnitude

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

What is the net force on the block at $t=0$?

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

What is the velocity of the block at $t=0.05$ s?

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

What is the position of the block at $t=0.05$ s?

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

What is the new net force acting on the block at $t=0.05$ s?

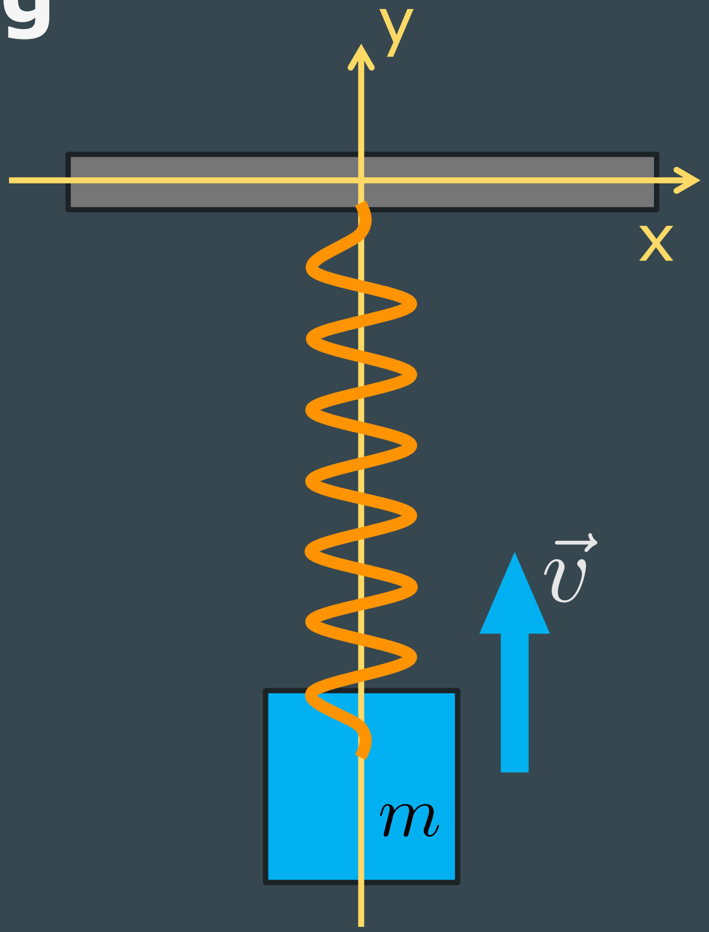
Summary of iterative procedure

- Physical properties of the system (m, k, L_0)
- Initial conditions: position ($\vec{r}_0$) and velocity ($\vec{v}_0$)
- First time step:
  - Force at t=0 ($\vec{F}_{net,0}$)
  - New velocity after Δt ($\vec{v}_1$)
  - New position after Δt ($\vec{r}_1$)
- If we wanted to go further, in the second time step we would do:
  - New net force after Δt ($\vec{F}_{net,1}$)
  - New new velocity after another Δt ($\vec{v}_2$)
  - New new position after another Δt ($\vec{r}_2$)
- And continue repeating for any additional time steps
  - $\vec{F}_{net,2}$, then $\vec{v}_3$, then $\vec{r}_3$, then $\vec{F}_{net,3}$, then $\vec{v}_4$, then $\vec{r}_4$, etc...

Example: A vertical spring

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

What is the net force acting on the block at this moment?



A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Initial position:

Initial velocity:

L vector and Lhat vector:

CLICKER 3: What is the net force acting on the block?

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

A. $\vec{F}_{\text{net}} =< 0, -k(L - L_0) + mg, 0 >$

B. $\vec{F}_{\text{net}} =< 0, k(L - L_0) - mg, 0 >$

C. $\vec{F}_{\text{net}} =< 0, -k(L - L_0) - mg, 0 >$

D. $\vec{F}_{\text{net}} =< 0, k(L - L_0) + mg, 0 >$

Solution: What is the net force acting on the block?

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Determine the position of the block at $t=T$ by iterating over two consecutive equal-sized time-steps.

Procedure: break the full time T into two smaller intervals: Δt_1 which goes from $t=0$ to $t=T/2$, and Δt_2 which goes from $t=T/2$ to $t=T$

- We already know Fnet at $t=0$
- Find velocity at the end of the interval Δt_1
- Find position at the end of the interval Δt_1
- Find new Fnet at the end of the interval Δt_1 (start of Δt_2)
- Find new velocity at the end of the interval Δt_2
- Find new position at the end of the interval Δt_2

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Velocity at the end of Δt_1

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Position at the end of Δt_1

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

New net force at the end of Δt_1, which is the start of Δt_2

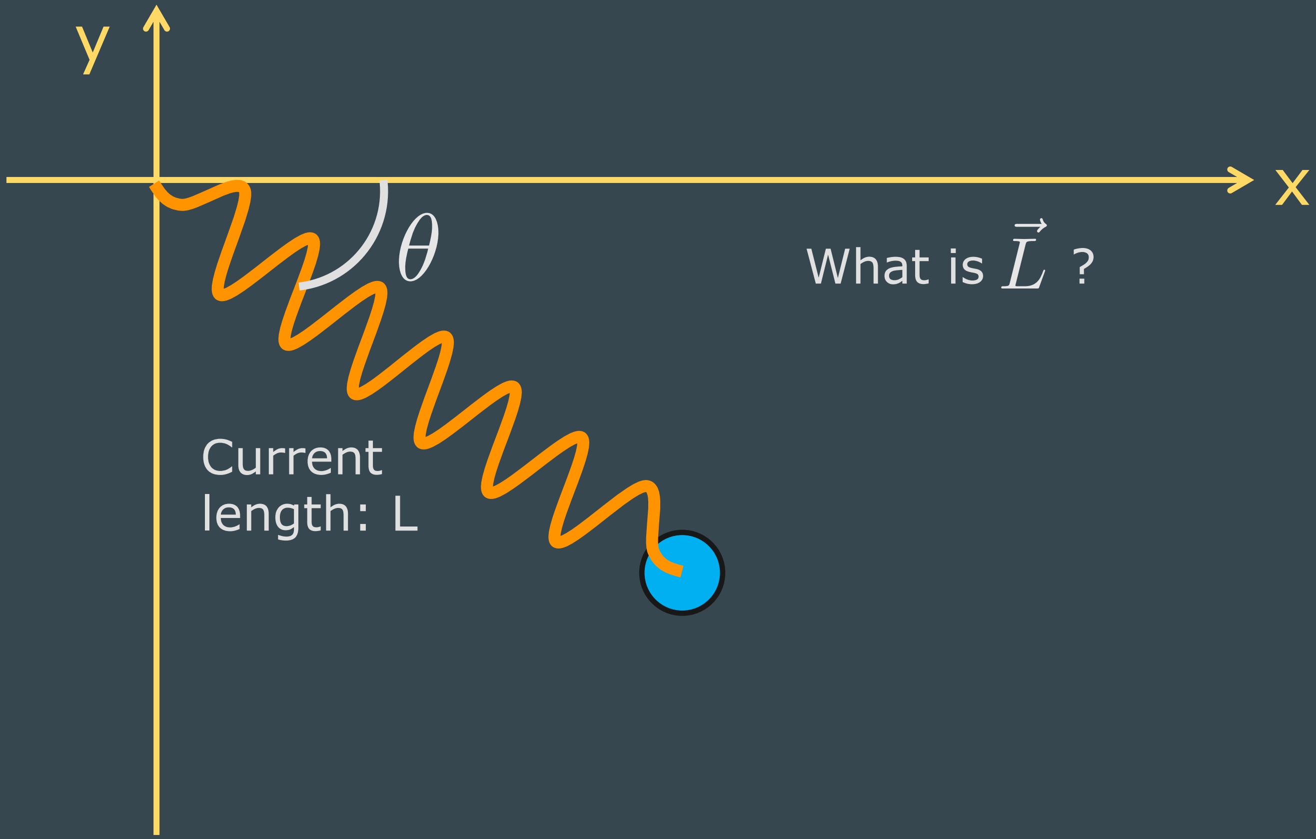
A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

New velocity at the end of Δt_2

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

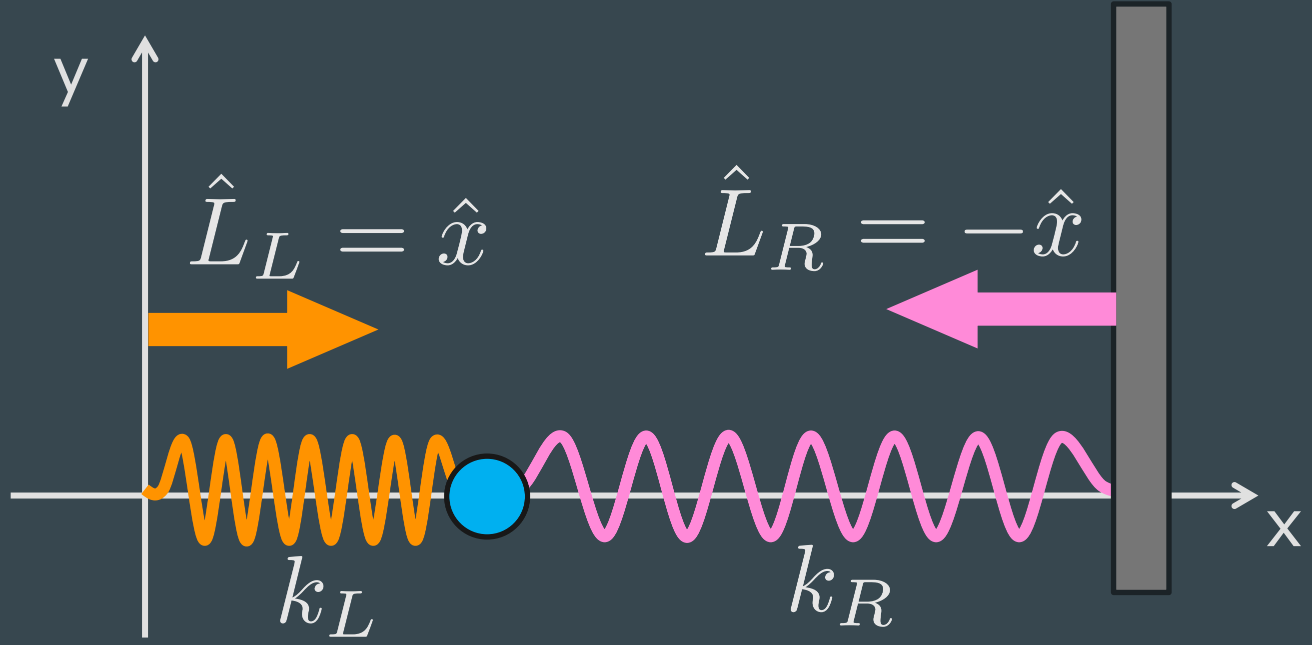
New position at the end of Δt_2

Springs can also be diagonal...



What is $\vec{L}$?

And there can be more than one spring...



Net force on the mass is the vector sum of the two spring forces

$$\vec{F}_{\text{net}} = \vec{F}_{sL} + \vec{F}_{sR}$$


$$\vec{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}$$

Universal gravitation

- We've already seen gravity near the surface of Earth:

$$\vec{F}_{\text{grav}} =< 0, -mg, 0 >$$

- Now we'll talk about Fgrav in general: Newton's Law of Universal Gravitation
  - Attractive force between any two objects with mass
  - Proportional to product of masses
  - Inversely proportional to square of separation distance

Gravitation

$$\vec{F}_{\mathrm{grav}} = -\frac{GMm}{r^2}\hat{r}$$

- Relative position vector

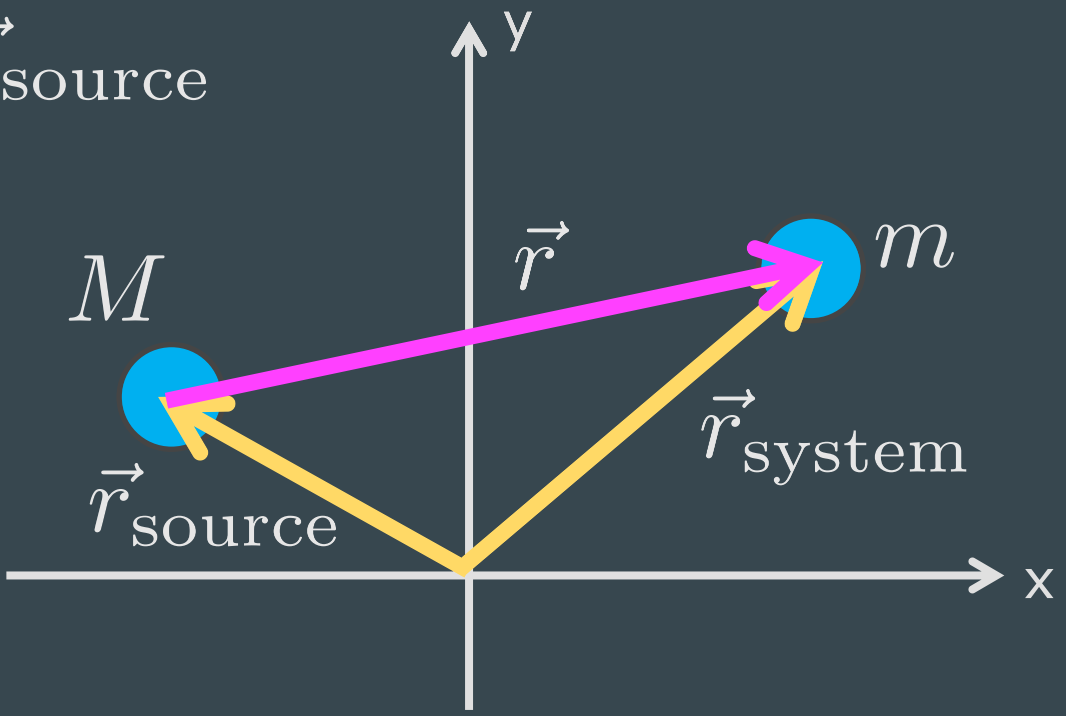
$$\vec{r} = \vec{r}_{\mathrm{system}} - \vec{r}_{\mathrm{source}}$$

- Direction of gravity: towards the source

$$-\hat{r}$$

- Gravitational constant:

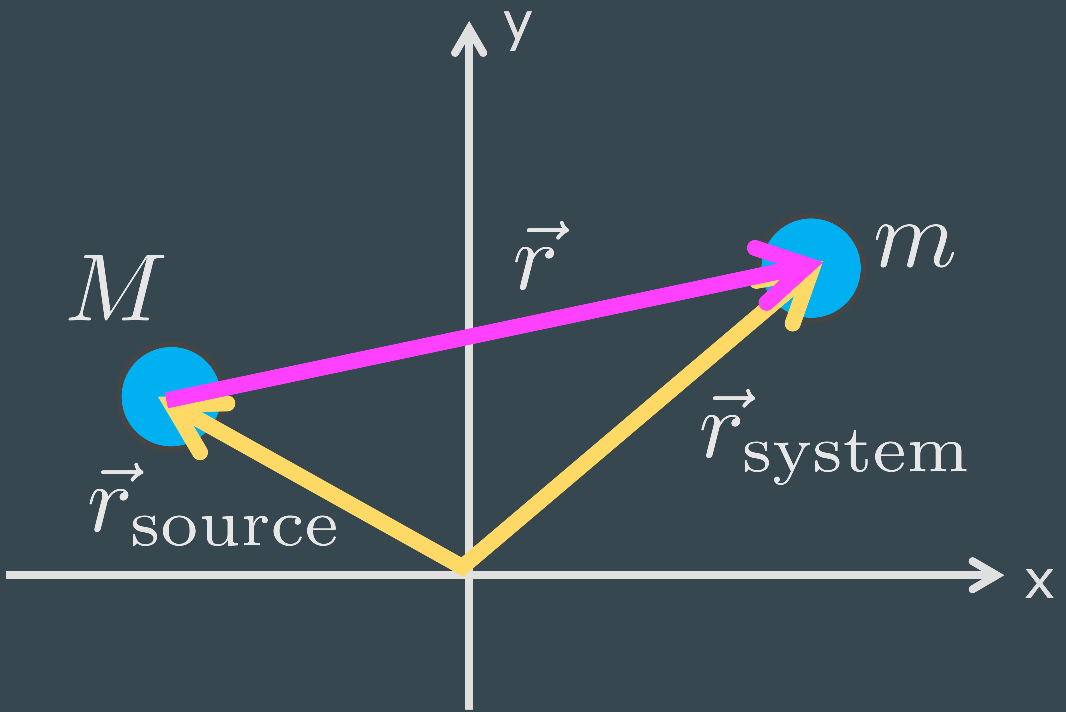
$$G = 6.7 \times 10^{-11}\ \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$



Gravitation

$$\vec{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}$$

- This is the force felt by the system (m) due to the mass of the object at the source (M)
- If the source is 1 and the system is 2, then we call this "F on 2 by 1"



Gravitation

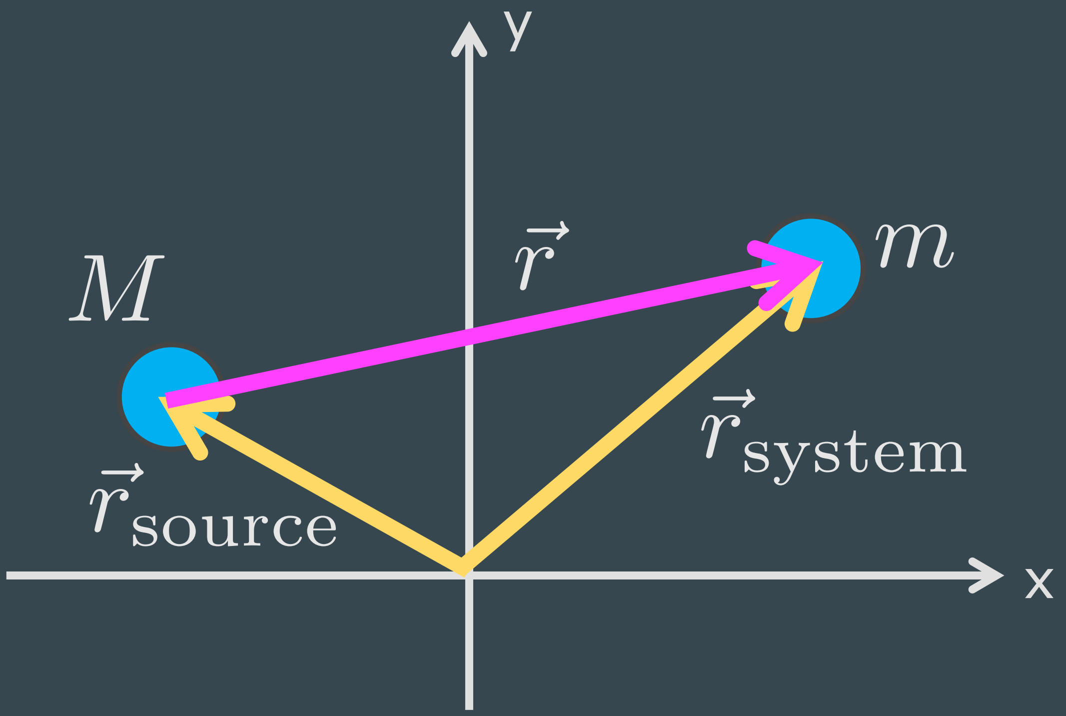
$$\vec{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}$$

- Magnitude:

$$|\vec{F}_{\text{grav}}| = \frac{GMm}{r^2}$$

- Full vector combines magnitude and direction:

$$\vec{F}_{\text{grav}} = |\vec{F}_{\text{grav}}|(-\hat{r})$$



CLICKER 4: The gravitational force exerted by a planet on one of its moons is 3e23 N when the moon is at a particular location. If the mass of the moon were three times as large, what would the force on the planet be due to the moon?

A. 1e23 N
B. 3e23 N
C. 6e23 N
D. 9e23 N
E. We need more info

CLICKER 5: The gravitational force exerted by a planet on one of its moons is 3e23 N when the moon is at a particular location. If the distance between the moon and the planet was cut in half, what would the force on the moon be?

A. 1.2e24 N
B. 6e23 N
C. 3e23 N
D. 1.5e23 N
E. 0.75e23 N
F. 0.33e23 N

Procedure for solving gravitation problems

1. Draw a picture that includes position vectors for each object
2. Calculate the relative position vector (points to the object feeling the force)
3. Calculate the distance between the objects (magnitude of the relative position vector)
4. Calculate the magnitude of the force
5. Calculate the direction of the force (remember the negative sign on rhat!)
6. Combine the magnitude and direction
7. Check against your picture

Gravity on Earth

$$|\vec{F}_{\text{grav}}| = \frac{GMm}{r^2}$$

M = mass of Earth = 6e24 kg

m = mass of whatever object near the surface of Earth

r = radius of Earth + distance between Earth's surface and object

(radius of Earth: 6.4e6 m (six million meters)

What is the magnitude of the force of gravity felt by a rock of mass 3kg sitting at the top of Mt Everest? (elevation: 8848 m)

r = 6.4e6 m + 8848 m = essentially 6.4e6 m → height doesn't matter, it'll always be small enough compared to the radius of Earth!

Gravity on Earth

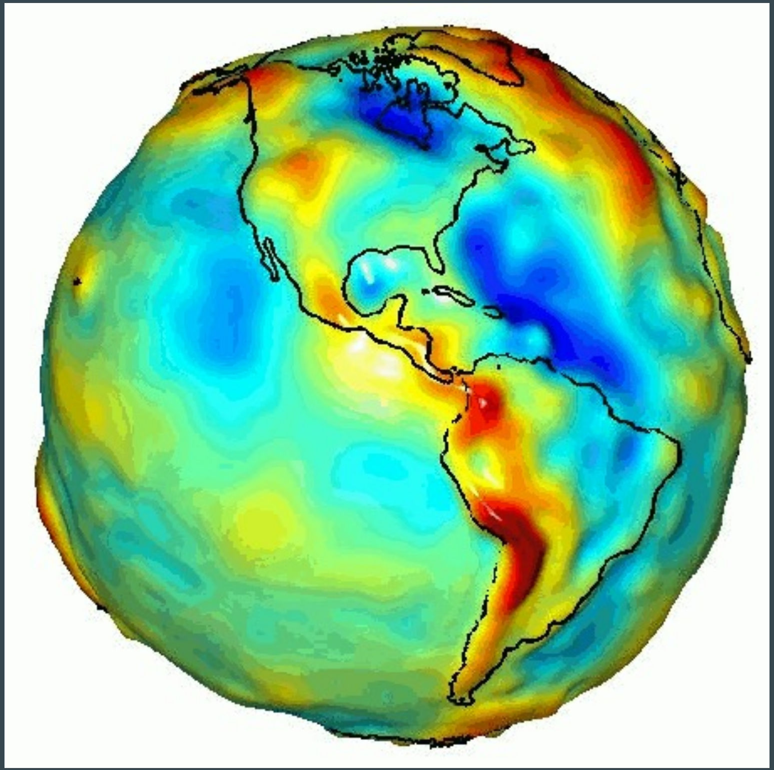
$$|\vec{F}_g| = \frac{GMm}{r^2}$$ but near the Earth's surface, $$|\vec{F}_g| = mg$$

$G = 6.7\text{e-}11\ Nm^2/kg^2$

$M_E = 6\text{e}24$ kg

$R_E = 6.4\text{e}6$ m

Gravity on Earth



Deviations from

$g=9.81\ m/s^2$ are

about $\pm\ 0.02\ m/s^2$

(therefore constant, as far as we're concerned)

https://grace.jpl.nasa.gov/resources/6/grace-global-gravity-animation/

CLICKER 6: The mass of the moon is 7.35e22 kg. The diameter of the moon is 3.47e6 m. What is the magnitude of the acceleration due to gravity at the surface of the moon, g_m?

A. 9.8 m/s^2

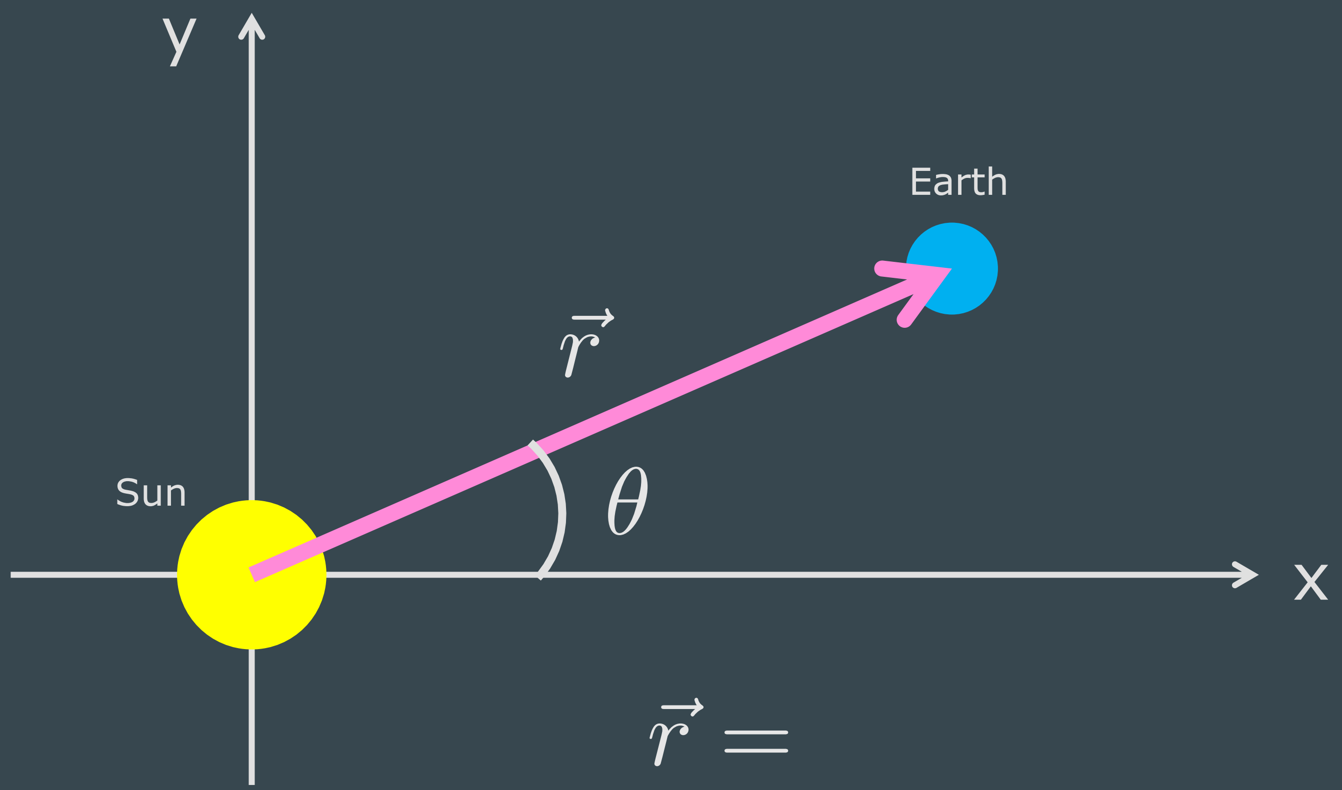
B. 1.63 m/s^2

C. 3.7 m/s^2

D. 0.41 m/s^2

Example: $\vec{F}_g$ on Earth due to Sun

$$M_S = 2 \times 10^{30}\text{ kg}$$
$$M_E = 6 \times 10^{24}\text{ kg}$$
$$|\vec{r}| = 1.5 \times 10^{11}\text{ m}$$
$$\theta = 45^\circ$$



$$\vec{r} =$$

Example: $\vec{F}_g$ on Earth due to Sun

$$M_S = 2 \times 10^{30}\ \text{kg}$$
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$$|\vec{r}| = 1.5 \times 10^{11}\ \text{m}$$
$$\theta = 45^\circ$$

Find r vector, and magnitude, and rhat

Find magnitude of Fgrav

Combine magnitude and direction