# Physics 2211 GPS Week 12

## Problem #1

Consider a system consisting of two particles connected by a spring of negligible mass:
$m_1 = 5$ kg, vector $\vec{v}_1 = \langle 5, \ -10, \ 15 \rangle$ m/s
$m_2 = 10$ kg, vector $\vec{v}_2 = \langle -10, \ 0, \ -5 \rangle$ m/s

(a) What is the total momentum $\vec{p}_{total}$ of this system?

$$\vec{p}_1 = m_1 \vec{v}_1 = (5) <5, -10, 15> = <25, -50, 75> \text{ kg m/s}$$

$$\vec{p}_2 = m_2 \vec{v}_2 = (10) <-10, 0, -5> = <-100, 0, -50> \text{ kg m/s}$$

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = <25, -50, 75> + <-100, 0, -50> =$$

$$= \boxed{<-75, -50, 25> \text{ kg m/s}}$$

(b) What is $\vec{V}_{CM}$, the velocity of the center of mass of this system?

$$\vec{p}_{total} = M_{total} \vec{V}_{cm} \implies \vec{V}_{cm} = \frac{\vec{p}_{total}}{M_{total}}$$

$$\vec{V}_{cm} = \frac{<-75, -50, 25>}{5+10} = \boxed{<-5, -3.3, 1.7> \text{ m/s}}$$

(c) What is $K_{trans}$, the translational kinetic energy of this system?

$$K_{trans} = \frac{1}{2} M_{total} V_{cm}^2$$

$$\checkmark \; V_{cm}^2 = (-5)^2 + (-3.3)^2 + (1.7)^2 = 38.68$$

$$\Longrightarrow K_{trans} = \frac{1}{2}(15)(38.68) = \boxed{290 \text{ J}}$$

(d) What is $K_{total}$, the total kinetic energy of this system?

$$K_{total} = K_1 + K_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\checkmark \; v_1^2 = (5)^2 + (-10)^2 + (15)^2 = 350$$

$$\checkmark \; v_2^2 = (-10)^2 + (-5)^2 = 125$$

$$\Longrightarrow K_{total} = \frac{1}{2}(5)(350) + \frac{1}{2}(10)(125) = 875 + 625 = \boxed{1500 \text{ J}}$$

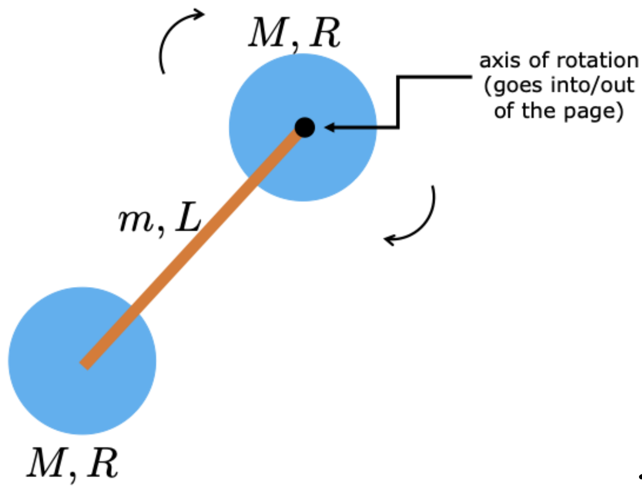
(e) What is $K_{rel}$, the kinetic energy of this system relative to the center of mass?

$$K_{total} = K_{trans} + K_{rel} \Longrightarrow K_{rel} = K_{total} - K_{trans}$$

$$K_{rel} = 1500 - 290 = \boxed{1210 \text{ J}}$$

**Problem #2**

A barbell is made up of two solid spheres of mass $M$ and radius $R$ whose centers are attached to the ends of a thin rod that has mass $m$ and length $L$. The entire thing rotates about an axis that goes through the center of sphere 1. Determine the total moment of inertia of the barbell about this axis of rotation. Hint: remember the parallel axis theorem.




$\parallel$-axis theorem: $I_{\parallel\text{-axis}} = I_{CM} + Md^2$

($d$ = dist. b/w axis through CM and the $\parallel$-axis)

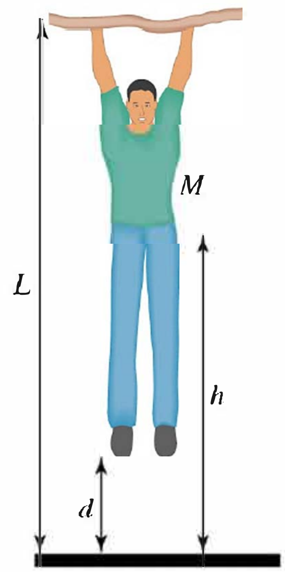
$$I_{CM,\text{solid sphere}} = \frac{2}{5}MR^2$$
$$I_{CM,\text{thin rod}} = \frac{1}{12}mL^2$$

$$I_{system} = I_{axis,\text{sphere }1} + I_{axis,\text{rod}} + I_{axis,\text{sphere }2}$$
$$= \left[I_{CM,\text{sphere }1} + I_{\parallel\text{-axis},\text{sphere }1}\right] + \left[I_{CM,\text{rod}} + I_{\parallel\text{-axis},\text{sphere }2}\right]$$
$$+ \left[I_{CM,\text{sphere }2} + I_{\parallel\text{-axis},\text{sphere }2}\right]$$
$$= \left[\frac{2}{5}MR^2 + M(0)^2\right] + \left[\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right]$$
$$+ \left[\frac{2}{5}MR^2 + M(L)^2\right]$$

$$\boxed{I_{system} = \frac{4}{5}MR^2 + \left(\frac{1}{3}m + M\right)L^2}$$

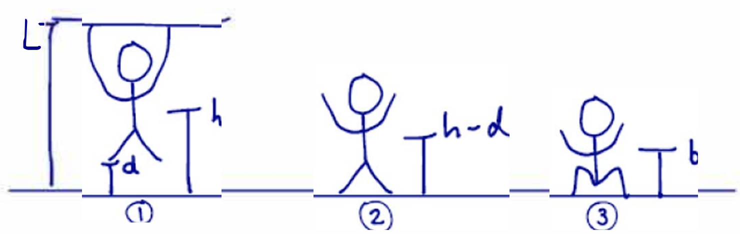
Problem #3

You hang by your hands from a tree limb that is a height $L = 6$ m above the ground, with your center of mass a height $h = 5$ m above the ground and your feet a height $d = 4$ m above the ground, as shown in the figure (not to scale). You then let yourself fall. You absorb the shock by bending your knees, ending up momentarily at rest in a crouched position with your center of mass a height $b = 0.25$ m above the ground. Your mass is $M = 110$ kg.




(a) Starting from the energy principle, find your speed just before your feet touch the ground.

System : point particle
Initial: (1)
Final: (2)




$$\Delta E = \Delta K = W_{grav}$$

$$\frac{1}{2} m (v_f^2 - \cancel{v_i^2}) = \vec{F}_{grav} \cdot \Delta\vec{r}_{cm} = mg(-\hat{y}) \cdot d(-\hat{y})$$

$$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g d$$

$$v_f^2 = 2gd \Rightarrow v_f = \sqrt{2gd} = \sqrt{(2)(9.8)(4)} = \boxed{8.85 \text{ m/s}}$$

(b) Starting from the energy principle (point particle model) and assuming that the contact force of the ground on your feet is constant, find the magnitude of the contact force during your landing.

Initial ②, final ③

$$\Delta E = \Delta K = W_{total} = \vec{F}_{net} \cdot \Delta \vec{r}_{cm}$$

$$\frac{1}{2}m(\cancel{v_f^2} - v_i^2) = (F_c - mg)[b - (h-d)]$$

$$-\frac{1}{\cancel{2}}m(\cancel{2}gd) = (F_c - mg)(b-h+d)$$

$$-mgd = (F_c - mg)(b-h+d)$$

$$F_c - mg = \frac{-mgd}{b-h+d}$$

$$F_c = mg - \frac{mgd}{b-h+d} = (110)(9.8) - \frac{(110)(9.8)(4)}{0.25-5+4} = \boxed{6827\ \text{N}}$$

(c) What is the (real) work done by the contact force?

$W_c = \vec{F}_c \cdot \Delta \vec{r} = 0$ b/c the floor doesn't move

(d) Starting from the energy principle (real model), find the change in your internal energy during landing.

Method #1
Initial ①, final ③

$$\Delta E = \cancel{\Delta K}^{0} + \Delta E_{int} = W_{grav} + \cancel{W_c}^{0}$$

$$\Delta E_{int} = -mg(b-h) = -(110)(9.8)(0.25-5) = \boxed{5121\ \text{J}}$$

Method #2
Initial ②, final ③

$$\Delta E = \Delta K + \Delta E_{int} = W_{grav} + \cancel{W_c}^{0}$$

$$\frac{1}{2}m(\cancel{v_f^2} - v_i^2) + \Delta E_{int} = -mg[b-(h-d)] = -mg(b-h+d)$$

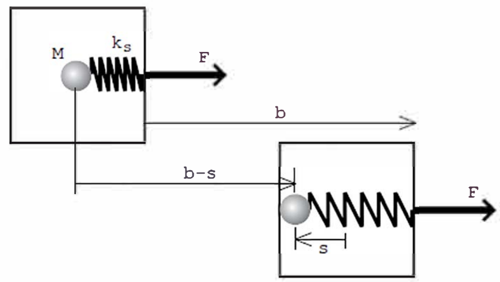
$$-\frac{1}{\cancel{2}}m(\cancel{2}gd) + \Delta E_{int} = -mg(b-h) - mgd$$

$$\cancel{-mgd} + \Delta E_{int} = -mg(b-h) - \cancel{mgd}$$

$$\Delta E_{int} = -mg(b-h) = \boxed{5121\ \text{J}}$$

Problem #4

A thin box in outer space contains a large ball of clay of mass $M$, connected to an initially relaxed spring of stiffness $k_s$. The mass of the box is negligible compared to $M$. The apparatus is initally at rest. Then a force of constant magnitude $F$ is applied to the box. When the box has moved a distance $b$, the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount $s$. See the diagram for distances.




(a) Immediately after the clay sticks to the box, how fast is the box moving?

System : the whole thing is a point

$$\Delta E = \Delta K_{trans} = W = \vec{F} \cdot \Delta \vec{r}_{cm}$$

$$\frac{1}{2} m(v_f^2 - \cancel{v_i^2}) = F(b-s)$$

$$\frac{1}{2} m v_f^2 = F(b-s)$$

$$m v_f^2 = 2F(b-s)$$

$$v_f^2 = \frac{2F}{m}(b-s)$$

$$\boxed{v_f = \sqrt{\frac{2F}{m}(b-s)}}$$

(b) What is the increase in thermal energy of the clay?

System : real (clay + spring)

$$\Delta E = \Delta K_{trans} + \Delta U_s + \Delta E_{int} = W = \vec{F} \cdot \Delta \vec{r}$$

$$F(b-s) + \frac{1}{2} k (s_f^2 - \cancel{s_i^2}) + \Delta E_{int} = Fb$$

$$\cancel{Fb} - Fs + \frac{1}{2} ks^2 + \Delta E_{int} = \cancel{Fb}$$

$$-Fs + \frac{1}{2} ks^2 + \Delta E_{int} = 0$$

$$\boxed{\Delta E_{int} = Fs - \frac{1}{2} ks^2}$$