PHYS 2211 MNR - Test 1 - Fall 2022

Please clearly print your name & GTID in the lines below

Name: ______________________________ GTID: ______________________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

KEY

Sign your name on the line above

Hockey Puck [30 pts]

An ice hockey puck of mass $m = 170$ g enters the goal with a momentum of $\vec{p}_i =< -4.6, 2.9, 0 >$ kg m/s, crossing the goal line at location $\vec{r}_g =< -27, 0, 0 >$ m relative to the origin which is located in the center of the rink. The puck had been hit by a player 0.4 seconds before reaching the goal.

1. [15 pts] What was the location of the puck $\vec{r}_i$ when it was hit by the player? You can assume negligible friction between the puck and the ice (that is, constant velocity).

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t \qquad \vec{v}_{avg} = \vec{v}_f = \frac{\vec{p}_i}{m}$$

$$\vec{r}_i = \vec{r}_f - \vec{v}_{avg}\Delta t$$

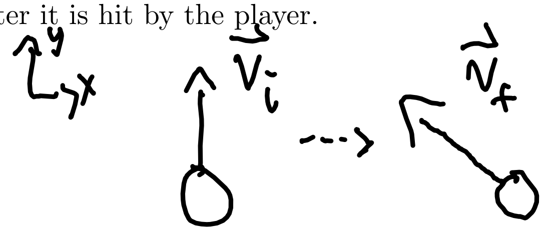
$$= \vec{r}_g - \frac{\vec{p}_i}{m}\Delta t$$

$$= <-27, 0, 0> \text{m} - \frac{0.4\text{ s}}{0.17\text{ kg}} <-4.6, 2.9, 0> \text{kg m/s}$$

$$\boxed{\vec{r}_i = <-16.18, -6.82, 0> \text{m}}$$

2. [15 pts] The player had hit the puck with a constant force for a very short time $\Delta t = 0.1$ s, which changed only the direction of motion of the puck, not its speed. Before it was hit, the velocity of the puck was along the $+\hat{y}$ axis. What is the force? Your answer must be a vector.

Hint: schematically draw the puck's momentum before and after it is hit by the player.



$$\vec{F} \approx \frac{\Delta \vec{p}}{\Delta t}$$

$$= \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$= \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

$$\vec{V}_f = \frac{\vec{p}_i}{m} = \frac{1}{0.17\,kg} \langle -4.6, 2.9, 0 \rangle\ kg\ m/s$$

$$\vec{V}_f = \langle -27.06, 17.06, 0 \rangle\ m/s$$

$$|\vec{V}_f| = 31.99\ m/s$$

$$|\vec{V}_i| = 31.99\ m/s \quad \text{(same speed)}$$

$$\Rightarrow \vec{V}_i = \langle 0, 31.99, 0 \rangle\ m/s$$

$$= \frac{0.17\,kg}{0.1\,s}\left[\langle -27.06, 17.06, 0 \rangle\ m/s - \langle 0, 31.99, 0 \rangle\ m/s \right]$$

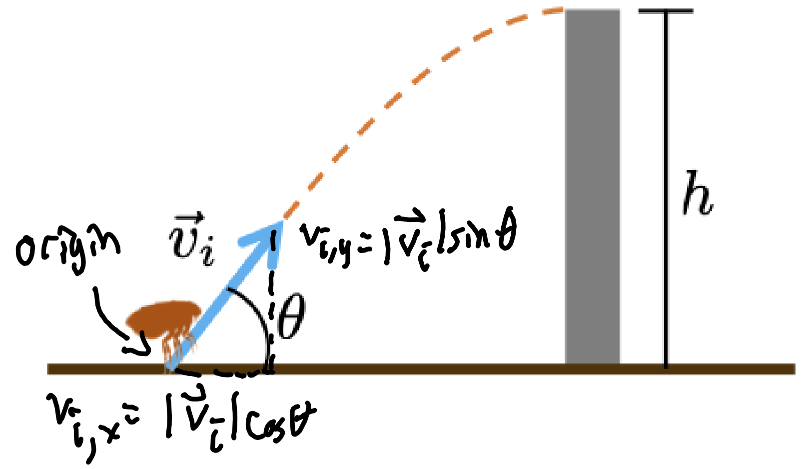
$$\boxed{\vec{F} = \langle -46.00, -25.38, 0 \rangle\ N}$$

Flea [30 pts] SOLUTION W/OUT KINEMATICS

(look after this solution for sol'n w/ kinematics)

Fleas are some of the best jumpers in the Animal Kingdom, relative to body size. A flea with mass $m = 0.001$ g is seen jumping with unknown initial speed $|\vec{v}_i|$ at an angle $\theta = 60°$ above the horizontal. At the maximum height of its trajectory, the flea lands on an obstacle that is $h = 14$ cm tall.

Throughout this problem you should keep 2 decimal places in all calculations. You can assume there is no air resistance.



1. [10 pts] What is the initial speed $|\vec{v}_i|$ of the flea?

$$\vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

y-direction:

$$V_{f,y} = V_{i,y} + \frac{F_{net,y}}{m}\Delta t$$

$$0 = |\vec{v}_i|\sin\theta + \frac{-\cancel{m}g}{\cancel{m}}\Delta t$$

$$|\vec{v}_i| = \frac{g\Delta t}{\sin\theta}$$

$$= \frac{g}{\sin\theta}\left[\frac{2h}{|\vec{v}_i|\sin\theta}\right]$$

$$\Rightarrow \boxed{|\vec{v}_i| = \sqrt{\frac{2gh}{\sin^2\theta}} = 1.91\ m/s}$$

$$\vec{r}_f = \vec{r}_i + \vec{V}_{avg}\Delta t$$

y-direction:

$$r_{f,y} = r_{i,y} + V_{avg,y}\Delta t$$

const. force

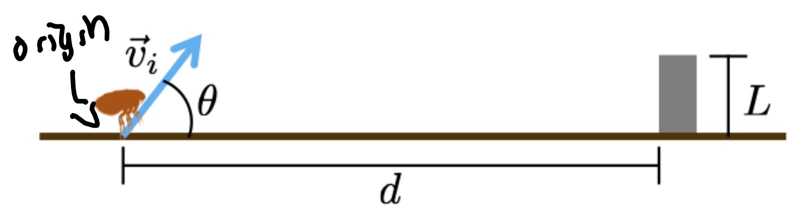
$$V_{avg,y} = \frac{1}{2}(v_{i,y} + \cancel{v_{f,y}}^{\,0})$$

$$V_{avg,y} = \frac{1}{2}|\vec{v}_i|\sin\theta$$

$$h = 0 + \frac{1}{2}|\vec{v}_i|\sin\theta\,\Delta t$$

$$\Rightarrow \Delta t = \frac{2h}{|\vec{v}_i|\sin\theta}$$

2. [20 pts] Our little flea once again jumps in exactly the same way that it did before (i.e., same initial velocity). This time, however, there's a low wall $L = 3$ cm tall at a distance $d = 31$ cm away from the flea. Can the flea fly above this obstacle?



Hint: Find the x and y coordinates of the flea at the position of the obstacle.

At horizontal distance d away from where the flea jumps, what height (y-pos) is the flea?

How long does it take the flea to travel a horizontal distance d? Write velocity and position update eq'ns in the x:

$$V_{f,x} = V_{i,x} + \frac{\cancel{F_{net,x}}^{\to 0}}{m} \Delta t_{\text{reach wall}}$$

$$\Rightarrow V_{f,x} = V_{i,x} = V_{avg,x}$$

$$r_{f,x} = r_{i,x} + V_{avg,x} \Delta t_{\text{reach wall}}$$

$$d = 0 + v_{i,x} \Delta t_{\text{reach wall}}$$

$$\Rightarrow \Delta t_{\text{reach wall}} = \frac{d}{V_{i,x}} = \frac{d}{|\vec{v}_i| \cos\theta}$$

What is the Flea's height (y-pos) at this time?

$$r_{f,y} = r_{i,y} + v_{avg,y}\,\Delta t_{\text{reach wall}}$$

$$v_{avg,y} = \frac{1}{2}(v_{i,y} + v_{f,y})$$

$$v_{f,y} = v_{i,y} + \frac{F_{net,y}}{m}\Delta t_{\text{reach wall}}$$

$$= |\vec{v}_i|\sin\theta + \frac{-\cancel{m}g}{\cancel{m}}\Delta t_{\text{reach wall}}$$

$$v_{avg,y} = \frac{1}{2}\left[2|\vec{v}_i|\sin\theta - g\Delta t_{\text{reach wall}}\right]$$

$$r_{f,y} = 0 + \left[|\vec{v}_i|\sin\theta - \frac{g\,\Delta t_{\text{reach wall}}}{2}\right]\Delta t_{\text{reach wall}}$$

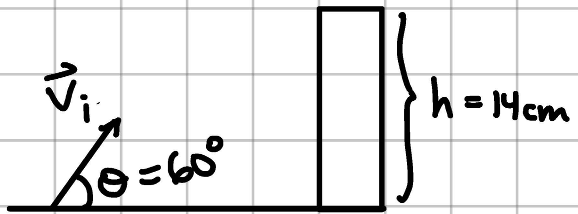
Note: can start here w/ kinematic eq'n from formula sheet

$$= |\vec{v}_i|\sin\theta\,\Delta t_{\text{reach wall}} - \frac{g\,\Delta t^2_{\text{reach wall}}}{2}$$

$$= \cancel{|\vec{v}_i|}\sin\theta\,\frac{d}{\cancel{|\vec{v}_i|}\cos\theta} - \frac{g}{2}\left(\frac{d}{|\vec{v}_i|\cos\theta}\right)^2$$

$$= 0.02\,\text{m} < L = 0.03\,\text{m}$$

$\boxed{\text{No}}$, the flea will not fly over the wall.



$\cancel{v_{fy}}^{0} = v_{iy} - g\Delta t$

$v_{iy} = g\Delta t \Rightarrow \Delta t = \frac{v_{iy}}{g} = \frac{v_i \sin\theta}{g}$

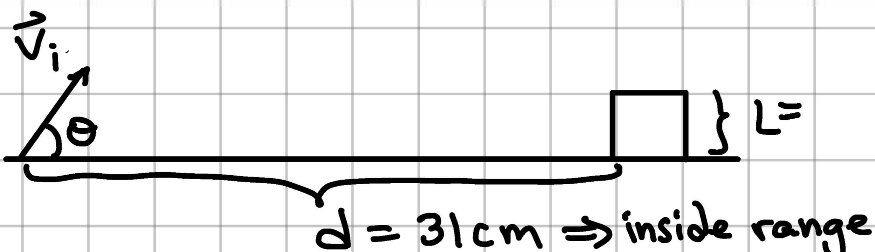
$= 0.17 \text{ sec}$

$y_f = \cancel{y_i}^{0} + v_{iy}\Delta t - \frac{1}{2} g \Delta t^2$

SOLUTION W/ KINEMATICS
(see previous solution for sol'n w/out kinematics

$h = v_i \sin\theta \frac{v_i \sin\theta}{g} - \frac{1}{2} g \left(\frac{v_i \sin\theta}{g}\right)^2 = \frac{(v_i \sin\theta)^2}{2g}$

$v_i = \sqrt{\frac{2gh}{(\sin\theta)^2}} = \sqrt{\frac{(2)(9.8)(0.14)}{(\sin 60°)^2}} = \boxed{1.91 \text{ m/s}}$



Range:

$x_f = \cancel{x_i}^{0} + v_{ix}\Delta t =$

$= v_i \cos\theta \left(\frac{2v_i \sin\theta}{g}\right) =$

$= \frac{2v_i^2 \sin\theta \cos\theta}{g} =$

$= \boxed{0.32 \text{ m}}$

Horizontal distance at max height

$x_{y_{max}} = v_{ix}\Delta t = v_i \cos\theta \frac{v_i \sin\theta}{g} = \boxed{0.16 \text{ m}}$

Distance from max height to wall

$w = d - x_{y_{max}} = 0.31 - 0.16 = \boxed{0.15 \text{ m}}$

Time to cover that horizontal distance (max height to wall)

$w = v_{ix}\Delta t' \Rightarrow \Delta t' = \frac{w}{v_i \cos\theta} = \boxed{0.16 \text{ sec}}$

y position of flea at horizontal distance of wall (from ymax)

$y_f = y_{max} + \cancel{v_{y_{max}}}^{0}\Delta t' - \frac{1}{2} g (\Delta t')^2 = y_{max} - \frac{1}{2} g (\Delta t')^2 = \boxed{0.015 \text{ m}}$

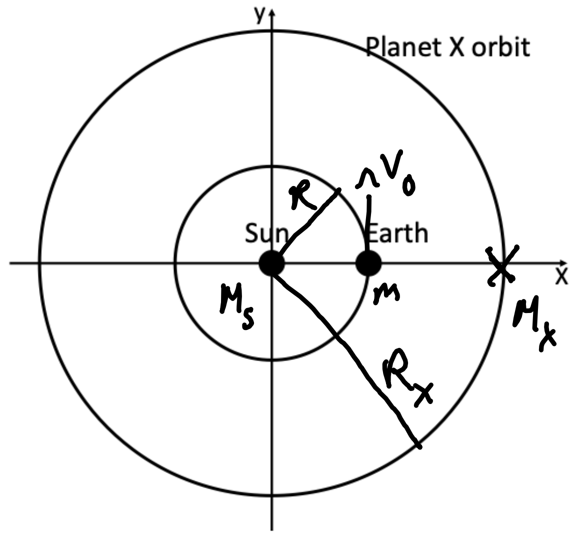
Wall is 3cm tall. Flea is 1.5 cm above ground. $\boxed{\text{SPLAT !}}$

Planet X [40 pts]

We investigate what would be the (tiny) gravitational influence of an elusive 9th planet (Planet X) on the circular motion of the Earth around the Sun. Planet X is believed to make one full orbit around the Sun in 400 Earth years.

The mass of the Sun is M_S, the mass of Planet X is M_X, and the mass of Earth is m. The radius of Earth's orbit is R, and the radius of Planet X's orbit is R_X.

The diagram on the right shows the positions of the Sun and the Earth at time $t = 0$.



1. [5 pts] At $t = 0$, where does Planet X have to be in its own orbit such that its gravitational influence on Earth is at its strongest value? Mark this position in the diagram with an **X**.

2. [20 pts] Starting from the positions of the Sun, Earth, and Planet X at $t = 0$, determine the new position of the Earth a short time Δt later. The Earth was already moving counterclockwise with speed v_0. You can assume Planet X has not moved.

$$\vec{F}_{net,\,on\ Earth}(t=0) = \vec{F}_{grav,\,Sun\ on\ Earth}(t=0) + \vec{F}_{grav,\,X\ on\ Earth}(t=0)$$

$$\boxed{\begin{array}{l} \vec{r}_{Earth}(t=0) = R\hat{x} \\ \vec{r}_{planet\text{-}X}(t=0) = R_x\hat{x} \\ \vec{r}_{Sun}(t=0) = \vec{0} \end{array}}$$

$$= -\frac{GM_s m}{R^2}\hat{x} - \frac{GM_x m}{(R_x-R)^2}(-\hat{x})$$

$$= Gm\left[-\frac{M_s}{R^2} + \frac{M_x}{(R_x-R)^2}\right]\hat{x}$$

Update Earth's velocity:

$$\boxed{\vec{v}_{i,Earth}(t=0) = v_0\hat{y}}$$

$$\vec{v}_{f,Earth}(t=\Delta t) \approx \vec{v}_{i,Earth}(t=0) + \frac{\vec{F}_{net}(t=0)}{m}\Delta t$$

$$\vec{v}_{f,Earth} \approx \left\langle G\Delta t\left[-\frac{M_s}{R^2} + \frac{M_x}{(R_x-R)^2}\right],\ v_0,\ 0\right\rangle$$

Update Earth's position:

$$\vec{r}_{f,\text{Earth}}(t=\Delta t) = \underset{\underset{R\hat{x}}{\parallel}}{\vec{r}_{i,\text{Earth}}}(t=0) + \vec{v}_{avg}\,\Delta t$$

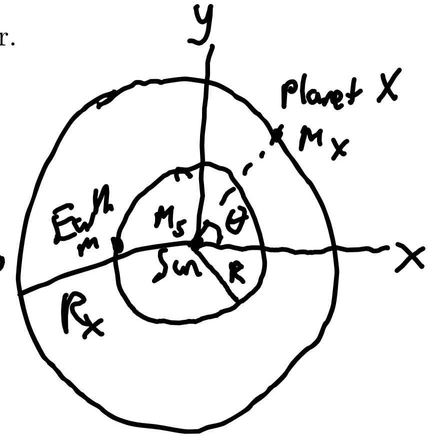
$$\vec{v}_{avg} \approx \vec{v}_f(t=\Delta t)$$

non-const. force

$$\boxed{\vec{r}_{f,\text{Earth}}(t=\Delta t) = \left\langle R + G(\Delta t)^2\left[-\frac{M_S}{R^2} + \frac{M_X}{(R_X-R)^2}\right],\ v_0\,\Delta t,\ 0\right\rangle}$$

3. [15 pts] Half an Earth year after $t = 0$, Planet X has moved an angle θ counterclockwise on its own circular orbit. Calculate the new net gravitational force on Earth. Your answer must be a vector. Note that $\theta < 90°$.

Hint: Think about where the Earth would be located half a year later.



(Assume the Sun hasn't moved.)

$$\vec{r}_{Earth} = R(-\hat{x}), \quad \vec{r}_{planet-x} = \langle R_X\cos\theta, R_X\sin\theta, 0\rangle$$

$$\vec{r}_{X\ to\ Earth} = \vec{r}_{Earth} - \vec{r}_{planet-x}$$

$$= \langle -R - R_X\cos\theta, -R_X\sin\theta, 0\rangle$$

$$|\vec{r}_{X\ to\ Earth}| = \sqrt{(R+R_X\cos\theta)^2 + (R_X\sin\theta)^2}$$

$$= \sqrt{R^2 + 2RR_X\cos\theta + R_X^2}$$

(using trig identity $\cos^2\theta + \sin^2\theta = 1$)

$$\vec{F}_{net,\ on\ Earth} = \vec{F}_{grav,\ Sun\ on\ Earth} + \vec{F}_{grav,\ X\ on\ Earth}$$

$$= -\frac{GM_S m}{R^2}(-\hat{x}) - \frac{GM_X m}{|\vec{r}_{X\ to\ Earth}|^3}\vec{r}_{X\ to\ Earth}$$

$$\boxed{\vec{F}_{net,\ on\ Earth} = Gm\left\langle \frac{M_S}{R^2} + \frac{M_X(R+R_X\cos\theta)}{[R^2+2RR_X\cos\theta+R_X^2]^{3/2}}, \frac{M_X R_X\sin\theta}{[R^2+2RR_X\cos\theta+R_X^2]^{3/2}}, 0\right\rangle}$$