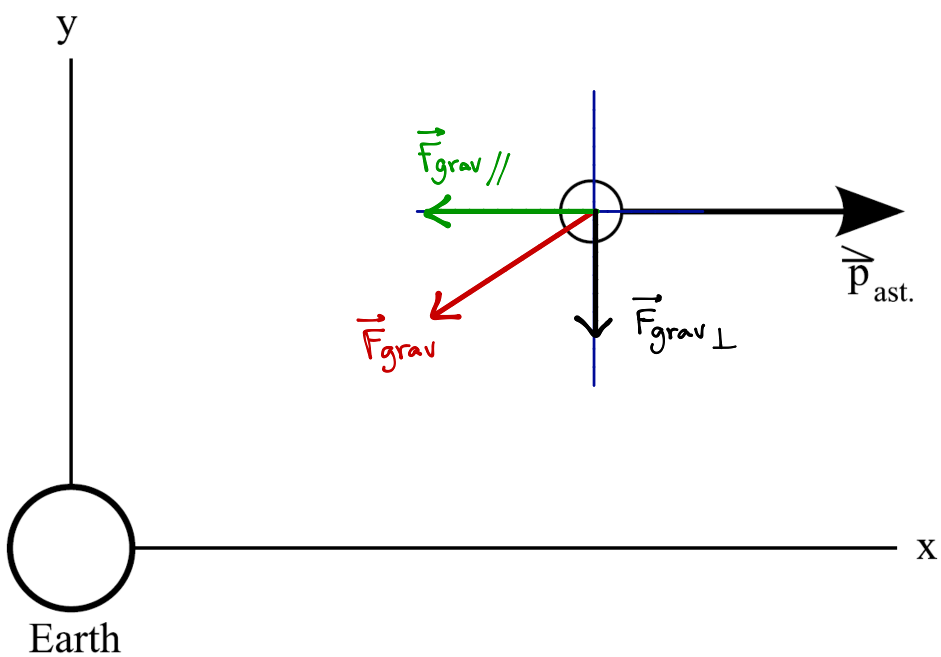
Physics 2211 GPS Week 6

Problem #1



The diagram above (not to scale) shows an asteroid passing near Earth. At the instant shown, the Earth is at position $< 0, 0, 0 >$ m, the asteroid is at position $< 1.7 \times 10^8, 1.1 \times 10^8, 0 >$m, and the asteroid has a momentum of $\vec{p}_{ast.} =< 2.4 \times 10^{16}, 0, 0 >$ kg m/s. The Earth's mass is 6×10^{24} kg and the asteroid's mass is 2×10^{13} kg.

(a) On the diagram, draw and label an arrow representing the gravitational force vector on the asteroid due to the Earth. (red)

(b) On the diagram, draw and label an arrow representing the component of the gravitational force on the asteroid that is parallel to its momentum. This arrow must be drawn to the same scale as the arrow you drew for part a. (green)

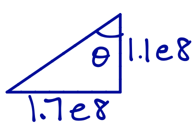
(c) On the diagram, draw and label an arrow representing the component of the gravitational force on the asteroid that is perpendicular to its momentum. This arrow must be drawn to the same scale as the arrow you drew for part a. (black)

(d) Which of the components of the force, parallel or perpendicular, is responsible for changing the direction of the asteroid's momentum? perpendicular

(e) At the instant shown, is the magnitude of the asteroids momentum increasing, <u>decreasing</u>, or not changing?

Decreasing b/c $\vec{F}_{net\,//}$ is antiparallel with $\vec{p}$

(f) Determine $(\frac{d\vec{p}}{dt})_{\|}$, $(\frac{d\vec{p}}{dt})_{\perp}$, and R_{kiss} the radius of the kissing circle for the asteroid. Briefly explain your answer.



$$\theta = \tan^{-1}(1.7e8/1.1e8) = \tan^{-1}(1.55) = 57^\circ$$

$$|\vec{F}_{grav}| = \frac{GMm}{r^2} = \frac{(6.7e{-}11)(6e24)(2e13)}{(1.7e8)^2 + (1.1e8)^2} = 1.96e11\ N$$

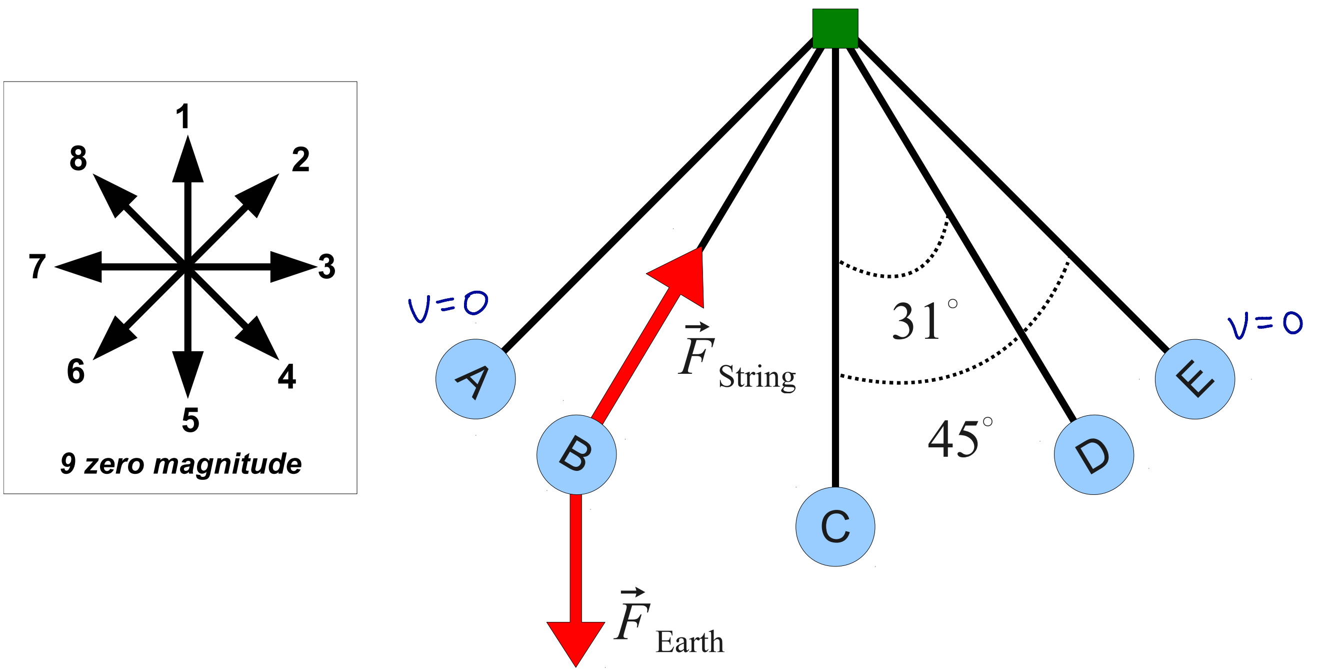
$$\Rightarrow \left(\frac{d\vec{p}}{dt}\right)_{//} = |\vec{F}_{grav}| \sin\theta = (1.96e11)\sin(57^\circ) = 1.64e11\ N\ (-\hat{x})$$

$$\Rightarrow \left(\frac{d\vec{p}}{dt}\right)_{\perp} = |\vec{F}_{grav}| \cos\theta = (1.96e11)\cos(57^\circ) = 1.07e11\ N\ (-\hat{y})$$

$$\Rightarrow R_{kiss} = \frac{|\vec{p}||\vec{v}|}{|\vec{F}_{\perp}|} = \frac{p}{F_\perp}\frac{p}{m} = \frac{p^2}{mF_\perp} = \frac{(2.4e16)^2}{(2e13)(1.07e11)} = \boxed{2.7e8\ m}$$

Problem #2

A 5 kg ball is attached to one end of a string that is 5 m in length; the other end of the string is attached to a fixed support. The ball swings freely in a circular arc from left to right. Figure 1 shows snapshots of the ball at different times. When the ball is at location C, the string is vertical. When the ball is at location A or E, the ball is momentarily at rest and the string makes an of 45 degree angle with the vertical. When the ball is at location B or D, the string make an angle of 31 degrees with the vertical and the ball's speed is 3.83 m/s (air resistance has been neglected).

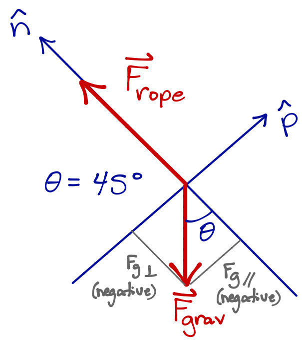


(a) Using the rosette of arrows shown above, indicate the number of the arrow that best represents the direction of the following quantities at locations A through E:

	A	B	C	D	E
$(\frac{d\vec{p}}{dt})_{\parallel}$	4	4	9	6	6
$(\frac{d\vec{p}}{dt})_{\perp}$	9	2	1	8	9
$\vec{F}_{net}$	4	3	1	7	6

(b) Draw a force diagram showing all of the forces acting in the ball at location $\boxed{\text{E.}}$ What is the magnitude of the net force acting on the ball at this location? Please show all of your work in this calculation and put off substituting numerical values into you work until you have a final expression to evaluate.

Swinging forward (A to E)



Note that $\vec{F}_{rope}$ is entirely perpendicular

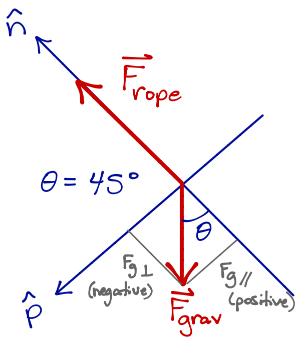
Parallel: $\vec{F}_{net//} = \vec{F}_{grav//} = -mg\sin\theta = 34.6 \text{ N } (-\hat{p})$

Perpendicular: at turning point, so $v = 0 \Rightarrow \vec{F}_{net\perp} = \frac{mv^2}{R}\hat{n} = 0$

Net force: $\vec{F}_{net} = \vec{F}_{//} + \cancel{\vec{F}_{\perp}}^{\,0} = 34.6 \text{ N } (-\hat{p})$

$$\Longrightarrow \boxed{|\vec{F}_{net}| = 34.6 \text{ N}}$$

Alternative: swinging back (E to A) $\Rightarrow$ $\hat{p}$ axis is reversed



Parallel: $\vec{F}_{net//} = \vec{F}_{grav//} = mg\sin\theta = 34.6 \text{ N } (\hat{p})$

Perpendicular: $\vec{F}_{net\perp} = 0$ (same reason as before)

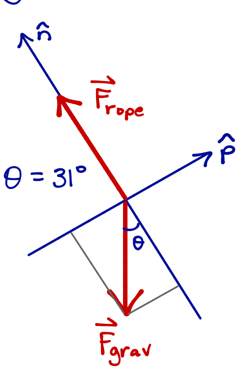
Net force: $\vec{F}_{net} = \vec{F}_{//} + \cancel{\vec{F}_{\perp}}^{\,0} = 34.6 \text{ N } (\hat{p})$

$$\Longrightarrow \boxed{|\vec{F}_{net}| = 34.6 \text{ N}}$$

SAME ANSWER REGARDLESS OF METHOD

(c) Draw a force diagram showing all of the forces acting in the ball at location $\boxed{\text{D.}}$ What is the magnitude of the tension force acting on the ball at this location? Please show all of your work in this calculation and put off substituting numerical values into you work until you have a final expression to evaluate.

Swinging forward (A to E)



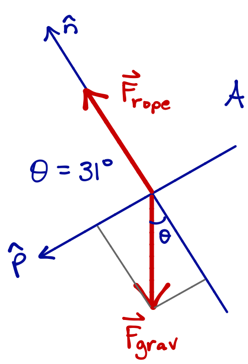
$$\vec{F}_{net\perp} = \vec{F}_{grav\perp} + \vec{F}_{rope\perp} = \frac{mv^2}{R}\hat{n}$$

$$-mg\cos\theta\,\hat{n} + T\hat{n} = \frac{mv^2}{R}\hat{n}$$

$$T\hat{n} = \left(\frac{mv^2}{R} + mg\cos\theta\right)\hat{n}$$

$$\vec{T} = \frac{\cancel{(5)}(3.83)^2}{\cancel{(5)}} + (5)(9.8)\cos(31°) = 56.7 \text{ N } (\hat{n})$$

$$\Longrightarrow \boxed{|\vec{T}| = 56.7 \text{ N}}$$



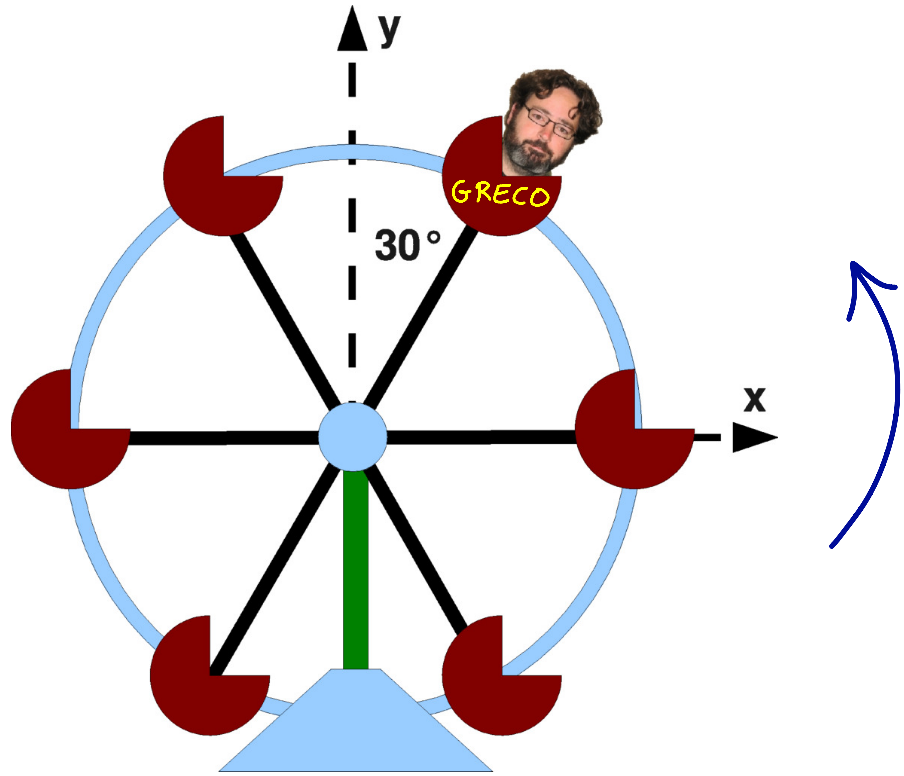
Alternative: swinging back (E to A)

$\hat{p}$ axis is reversed;

calculation is NOT affected

Problem #3

The Singapore Flyer (the world's largest Ferris wheel) has a diameter of 165 m. The wheel rotates counter-clockwise at a *constant rate*, completing a full rotation in 4 minutes. Dr. Greco, a 100 kg passenger, is riding in the gondola at an angle of 30 degrees from the vertical.



(a) Choosing Dr. Greco as your system, which objects in the surroundings exert a force on him.

Earth, seat

(b) What is the parallel component of the rate of change of Dr. Greco's momentum? Explain how you know this.

$\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = 0$ b/c constant angular speed

(c) What is the perpendicular component of the rate of change of Dr. Greco's momentum?

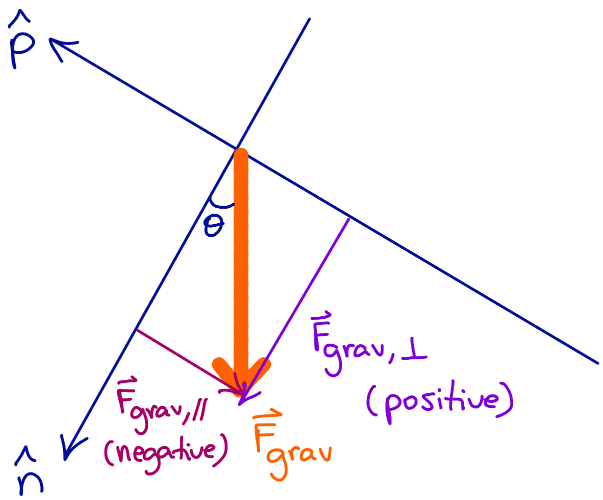
$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^{\cancel{2}} m}{\cancel{R}\, T^2} = \frac{4\pi^2 R m}{T^2} =$$

$$= \frac{(4\pi^2)(165/2)(100)}{[(4)(60)]^2} = \boxed{5.65\ \text{N}}$$

direction: $\hat{n}$

(perpendicular to $\vec{p}$,
towards center of turning circle)

(d) What is the magnitude of the parallel component of the contact force exerted by the Ferris wheel on Dr. Greco?



$$\left(\frac{d\vec{p}}{dt}\right)_{//} = \vec{F}_{grav,//} + \vec{F}_{c,//} = 0$$

$$\Rightarrow \vec{F}_{c,//} = -\vec{F}_{grav,//} = -(-mg\sin\theta) =$$

$$= mg\sin\theta = (100)(9.8)\sin(30^\circ) =$$

$$= \boxed{490\ \text{N}} \quad \text{direction: } \hat{p}$$

(e) What is the magnitude of the perpendicular component of the contact force exerted by the Ferris wheel on Dr. Greco?

$$\vec{F}_{net\perp} = \vec{F}_{grav\perp} + \vec{F}_{c\perp} = \frac{mv^2}{R}\hat{n}$$

$$\Rightarrow \vec{F}_{c\perp} = \frac{mv^2}{R}\hat{n} - \vec{F}_{grav\perp} = \frac{mv^2}{R}\hat{n} - mg\cos\theta\,\hat{n} =$$

$$= \left(\frac{mv^2}{R} - mg\cos\theta\right)\hat{n} = 5.65 - (100)(98)\cos(30^\circ) =$$

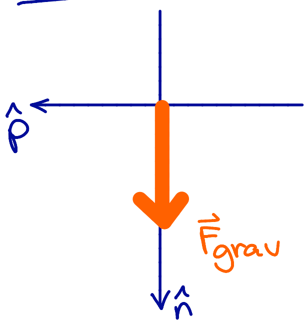
$$= 5.65 - 848.7 = -843.05\ \text{N}$$

$$\Rightarrow \text{magnitude} = \boxed{843.05\ \text{N}}$$

direction: $-\hat{n}$ (radially outwards)

(f) Now consider the instant where Dr. Greco is momentarily at the top of the Ferris wheel. With what speed must Dr. Greco be traveling so that he feels weightless?

At top:



Weightlessness means no contact force, $\vec{F}_c = 0$

$$\vec{F}_{net\perp} = \frac{mv^2}{R}\hat{n} = \cancel{\vec{F}_c} + \vec{F}_{grav} = mg\,\hat{n}$$

$$\Rightarrow \frac{\cancel{m}v^2}{R} = \cancel{m}g$$

$$\frac{v^2}{R} = g$$

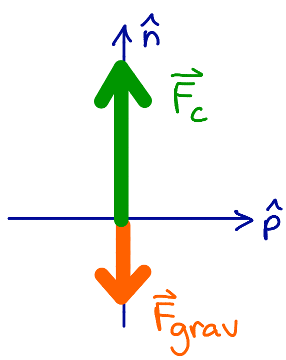
$$v^2 = gR$$

$$\Rightarrow v = \sqrt{gR} = \sqrt{(9.8)(165/2)} = \boxed{28.4 \text{ m/s}}$$

(64 mph; one rotation every 18 sec)

(g) Now consider the instant where Dr. Greco is momentarily at the bottom of the Ferris wheel. With what speed must Dr. Greco be traveling so that he feels three times as heavy as his usual weight?

At bottom:



Feeling 3x as heavy means $|\vec{F}_c| = 3|\vec{F}_{grav}|$

$$\vec{F}_{net\perp} = \frac{mv^2}{R}\hat{n} = \vec{F}_c + \vec{F}_{grav} = 3|\vec{F}_{grav}|\hat{n} + |\vec{F}_{grav}|(-\hat{n})$$

$$\frac{mv^2}{R}\hat{n} = 3mg\,\hat{n} - mg\,\hat{n} = 2mg\,\hat{n}$$

$$\frac{\cancel{m}v^2}{R} = 2\cancel{m}g$$

$$\frac{v^2}{R} = 2g \Rightarrow v^2 = 2gR$$

$$\Rightarrow v = \sqrt{2gR} = \sqrt{(2)(9.8)(165/2)} = \boxed{40.2 \text{ m/s}}$$

(90 mph; one rotation every 12 sec)