Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

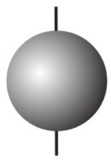
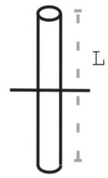
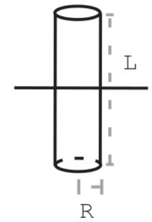
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Planck's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/F/HP) - Fall 2024 - Test 2

Name: ______________________ GTID: ______________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\dfrac{a \cdot b}{c \cdot d} = \dfrac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY
____
Sign your name on the line above

Problem 1 - Concepts [10 points]

1.1 [1 pt] The **work** done on a system can be...

(a) only positive
(b) only negative
(c) only zero
(d) positive, negative, or zero

1.2 [1 pt] The **kinetic** energy of a system can be...

(a) zero or positive
(b) zero or negative
(c) only zero
(d) positive, negative, or zero

1.3 [1 pt] The **potential** energy of a system consisting of a mass m and a spring with stiffness k can be...

(a) zero or positive
(b) zero or negative
(c) only zero
(d) positive, negative, or zero

1.4 [1 pt] The **rest** energy of a system of mass m can be...

(a) only positive
(b) only negative
(c) only zero
(d) positive, negative, or zero

1.5 [1 pt] If we define the total energy of a system as the sum of its potential and kinetic energies (E = K + U), then the **total energy** of a system can be...

(a) only positive
(b) only negative
(c) only zero
(d) positive, negative, or zero

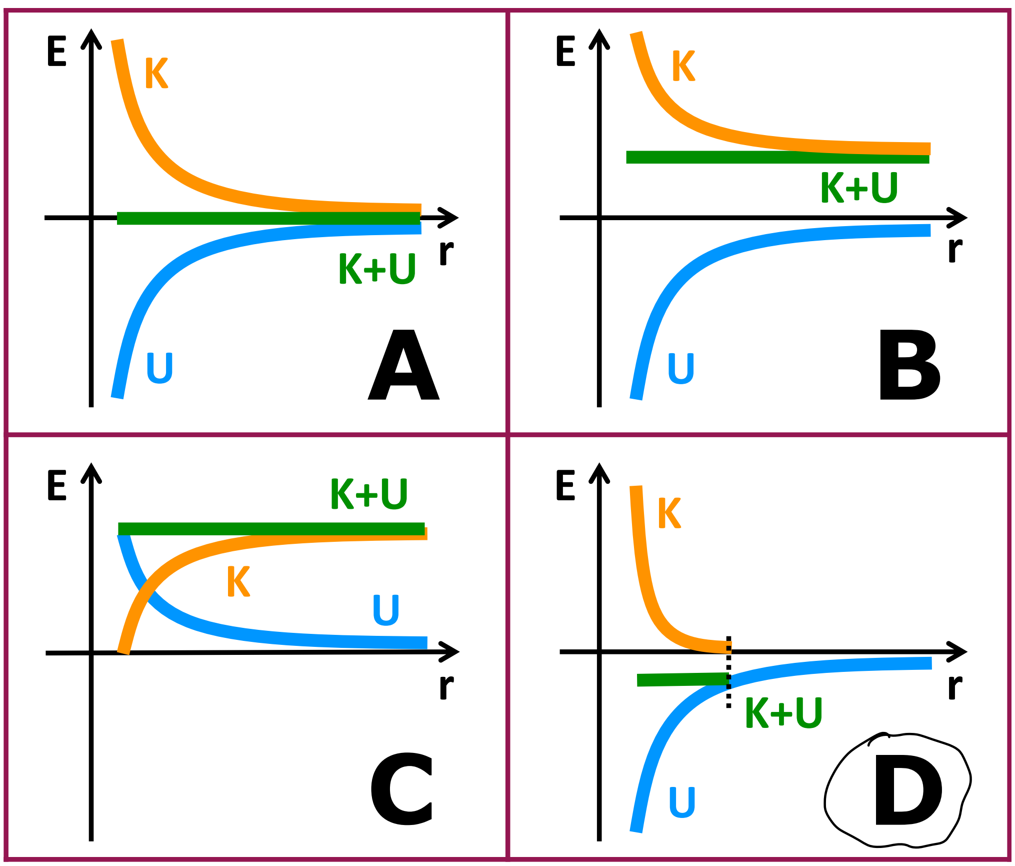
1.6 [1 pt] If we define the total energy of a system as the sum of its potential and kinetic energies (E = K + U), then the total energy of a **bound** system can be...

(a) only positive
(b) only negative
(c) only zero
(d) positive, negative, or zero

1.7 [1 pt] If we define the total energy of a system as the sum of its potential and kinetic energies (E = K + U), then the total energy of an **unbound** system can be...

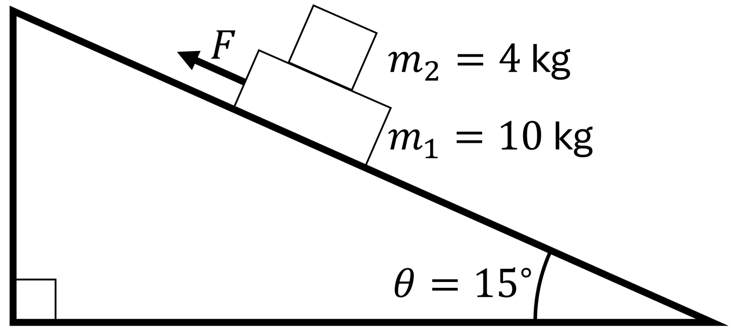
(a) zero or positive
(b) zero or negative
(c) only zero
(d) positive, negative, or zero

1.8 [3 pts] An electron and a proton are held at rest a small distance away from each other, then they are let go. Which of the following best represents the **energy graph** (kinetic energy K, potential energy U, total energy K+U) for the system consisting of the proton and the electron?

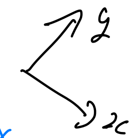


Problem 2 - Equilibrium [30 points]

Two boxes of masses $m_1 = 10$ kg (the bigger box on the bottom) and $m_2 = 4$ kg (the smaller box on the top) are stacked on each other and placed on a frictionless ramp at an angle $\theta = 15°$. There is friction between the two boxes but we don't know the coefficient of friction. You pull m_1 with a force of unknown constant magnitude F parallel to the ramp such that m_1 moves up the ramp at constant speed. While you do this, m_2 slides downwards on m_1, also at constant speed.

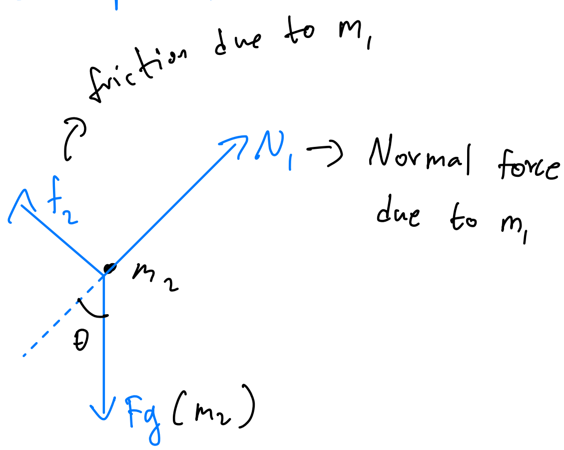


2.1 [10 pts] Draw two free-body diagrams: (A) a diagram that includes all the forces acting on m_2 (the smaller box on top), and (B) another diagram that includes all the forces acting on m_1 (the bigger box on bottom). Your diagram should include angles where needed.

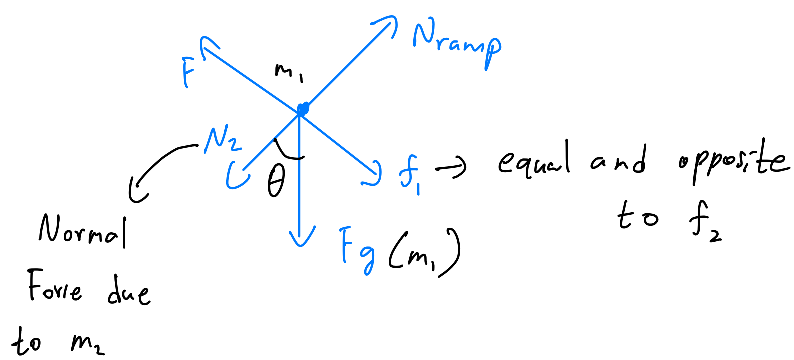


Coordinate system

① Top box



② Bottom box



2.2 [10 pts] Find the coefficient of kinetic friction μ between the two boxes.

Using sum of forces from the x ang y directions of diagram 1:

$F_{net,x} = 0$

$\Rightarrow \quad m_2 g \sin\theta - f_2 = 0 \quad \Rightarrow \quad \cancel{m_2}\cancel{g}\sin\theta = \mu \cancel{m_2}\cancel{g}\cos\theta$

$f_2 = \mu N_1$

$F_{net,y} = 0$

$\Rightarrow \quad \boxed{\mu = \dfrac{\sin\theta}{\cos\theta} = \tan\theta}$

$\Rightarrow \quad N_1 - m_2 g \cos\theta = 0$

$\Rightarrow \quad N_1 = m_2 g \cos\theta$

Numeric:

$\mu = \tan(15°) = \boxed{0.268}$

2.3 [10 pts] Find the magnitude of the force F.

Sum of forces in the x and y directions using diagram 2:

$F_{net,x} = 0$

$\Rightarrow f_1 + m_1 g \sin\theta - F = 0$

$|f_1| = |f_2|$

$\Rightarrow \underbrace{\mu m_2 g \cos\theta}_{\mu \cos\theta \,=\, \tan\theta \cos\theta \,=\, \sin\theta} + m_1 g \sin\theta = F$

$\Rightarrow m_2 g \sin\theta + m_1 g \sin\theta = F$

$\Rightarrow \boxed{F = (m_1 + m_2) g \sin\theta}$

$F_{net,y} = 0$

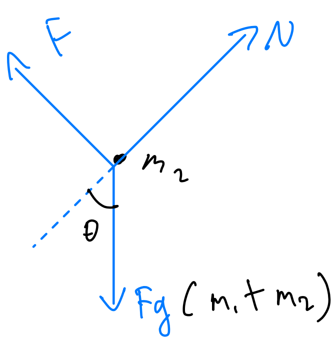
$\Rightarrow N_{ramp} - N_2 - m_1 g \cos\theta = 0$

We don't need this since ramp is frictionless.

Numeric:

$F = (10\ \text{kg} + 4\ \text{kg})(9.8\ \frac{\text{m}}{\text{s}^2}) \sin(15^\circ) = \boxed{35.51\ \text{N}}$

ALTERNATIVE APPROACH: Take two blocks as the system



$\Rightarrow$

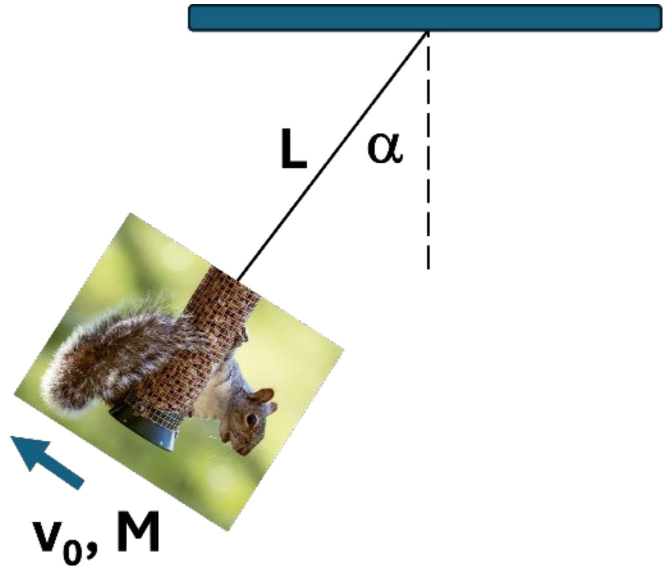
$F_{net,x} = 0$

$(m_1 + m_2) g \sin\theta - F = 0$

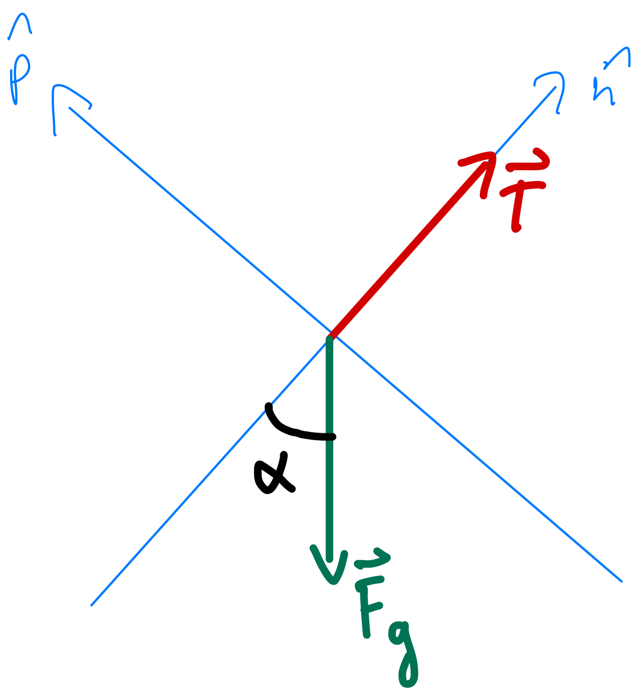
$\Rightarrow F = (m_1 + m_2) g \sin\theta$

Problem 3 - Curving Motion [30 points]

A squirrel jumps on a bird feeder hanging on Dr. Curtis' porch. The squirrel and bird feeder swing upward with a speed v_0 at the moment shown in the schematic. Dr. Curtis is worried that the fraying rope holding the bird feeder might break. The rope has a length L. The total mass of the bird feeder and squirrel is M.



3.1 [5 pts] Draw a free body diagram of the forces acting on the system consisting of the bird feeder and the squirrel for the moment shown in the schematic. Your diagram should clearly include the $\hat{p}$ and $\hat{n}$ axes, all the forces, and any necessary angles.



For the instant in time shown in the schematic, select the correct answer.

3.2 [1 pt] The direction of $\vec{F}_{\text{net}\parallel}$ is...

(a) $+\hat{p}$ (b) $-\hat{p}$ (c) $+\hat{n}$ (d) $-\hat{n}$ (e) zero

3.3 [1 pt] The direction of $\vec{F}_{\text{net}\perp}$ is...

(a) $+\hat{p}$ (b) $-\hat{p}$ (c) $+\hat{n}$ (d) $-\hat{n}$ (e) zero

3.4 [1 pt] The direction of $\left(\frac{d\vec{p}}{dt}\right)_{\parallel}$ is...

(a) $+\hat{p}$ (b) $-\hat{p}$ (c) $+\hat{n}$ (d) $-\hat{n}$ (e) zero

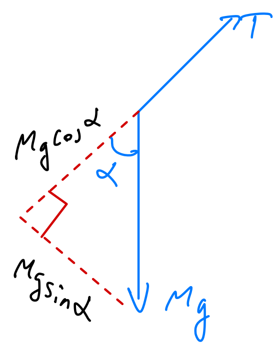
3.5 [1 pt] The direction of $\left(\frac{d\vec{p}}{dt}\right)_{\perp}$ is...

(a) $+\hat{p}$ (b) $-\hat{p}$ (c) $+\hat{n}$ (d) $-\hat{n}$ (e) zero

3.6 [1 pt] The speed of the bird feeder is...

(a) increasing (b) decreasing (c) constant (d) zero

3.7 [10 pts] Find the tension in the rope T, for the moment shown in the schematic. Your answer should be in terms of some combination of the variables: v_0, M, L, g, α.



Net force in $\perp$ direction: $\frac{Mv^2}{L}\hat{n}$

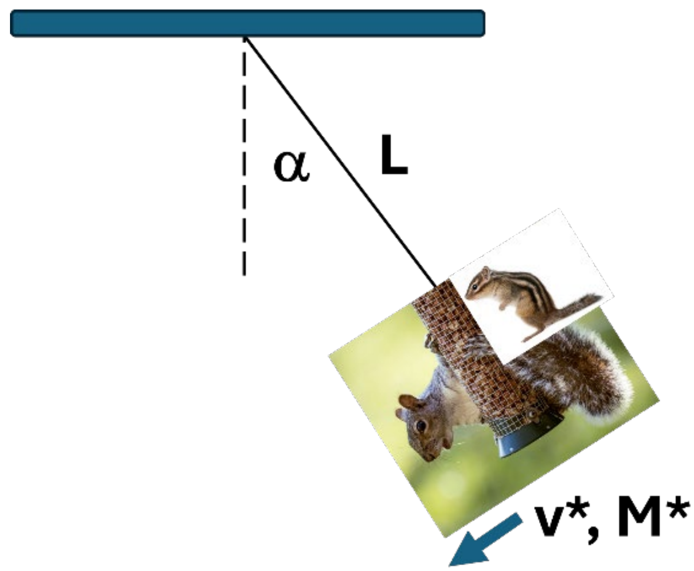
$$T\hat{n} - Mg\cos(\alpha)\hat{n} = \frac{Mv^2}{L}\hat{n}$$

$$\Rightarrow \boxed{\vec{T} = M\left(\frac{v^2}{L} + g\cos(\alpha)\right)\hat{n}}$$

3.8 [5 pts] The bird feeder continues to swing with the greedy squirrel. At what value of the angle α is the tension the largest? (e.g. 32 degrees, 90 degrees, etc).

At $\boxed{\alpha = 0}$, $\cos(\alpha) = 1$. Thus, Tension is largest there.

3.9 [5 pts] To Dr. Curtis' dismay, the squirrel has a chipmunk friend who jumps on to join the feast, increasing the total mass to $M*$. The bird feeder is moving at a higher speed $v*$ when at angle α, as shown in the new schematic. If the tension to break the rope is T_B, how fast must $v*$ be to break the rope at α? Your answer should be in terms of some combination of the variables: T_B, $M*$, L, g, α.



The equation for tension from 3.7 still applies here:

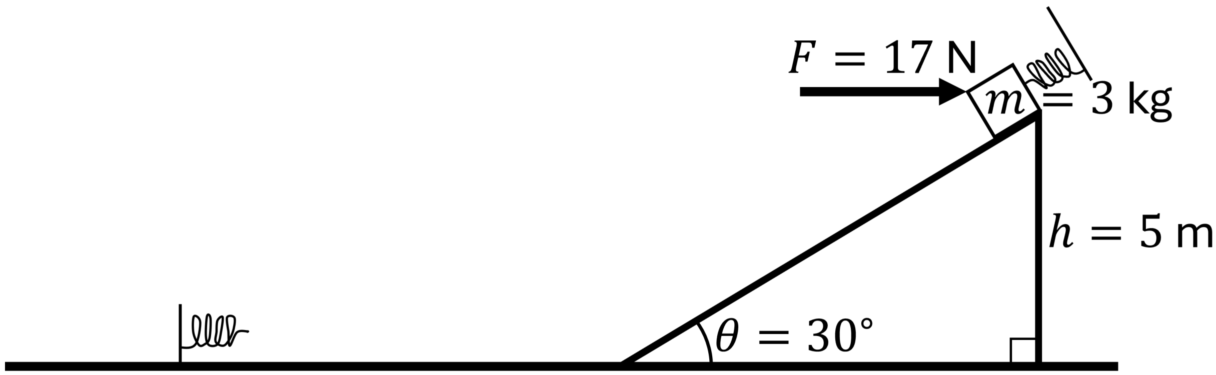
$$T_B = M_* \left(\frac{v_*^2}{L} + g\cos(\alpha) \right)$$

$$\Rightarrow \quad T_B - M_* g \cos(\alpha) = \frac{M_* V_*^2}{L}$$

$$\Rightarrow \quad \boxed{V_* = \sqrt{\frac{L}{M_*} \left[T_B - M_* g \cos(\alpha) \right]}}$$

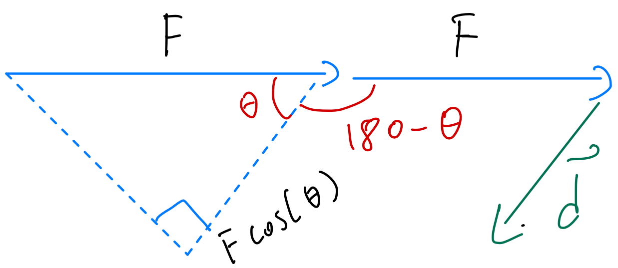
Problem 4 - Energy Principle [30 points]

You hold a box of mass $m = 3$ kg motionless at the top of a frictionless ramp. The ramp is inclined at an angle $\theta = 30°$ and has a height $h = 5$ m. A spring of stiffness $k = 15$ N/m is held against the box, compressed by an amount $s_1 = 40$ cm. When you release the box, the spring pushes it down the ramp.

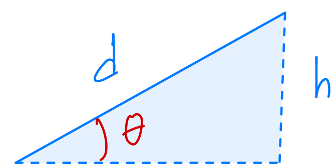


4.1 [10 pts] As the box slides down the ramp, a horizontal force of magnitude $F = 17$ N is applied to the box as shown in the diagram. Determine the work done by the horizontal force as the box moves from the top of the ramp to the bottom of the ramp.

First, find component of force F along the ramp:



Also,



$h = d\sin\theta \Rightarrow d = \dfrac{h}{\sin\theta}$

Since the block is moving to the left, and F is to the right, work is negative.

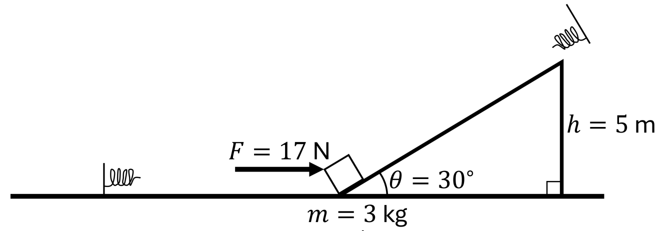
Thus, $W_F = \vec{F}\cdot\vec{d} = Fd\cos(180-\theta) = -Fd\cos\theta = -F\dfrac{h}{\sin\theta}\cos\theta$

$$\Rightarrow \boxed{W_F = -\frac{Fh}{\tan\theta} = -Fh\cot\theta}$$

<u>Numeric!</u>

$$W_F = -(17\,N)(5\,m)\cot(30°) = \boxed{-147.22\ J}$$

4.2 [10 pts] Find the kinetic energy K of the box when it reaches the bottom of the ramp.



By taking the block, spring and Earth as the system, we get

$$\Delta K + \Delta U_s + \Delta U_g = W_F$$

$$\Rightarrow K_f - \cancel{K_i}^{\,0} + \cancel{U_{s,f}}^{\,0} - U_{s,i} + \cancel{U_{g,f}}^{\,0} - U_{g,i} = W_F$$

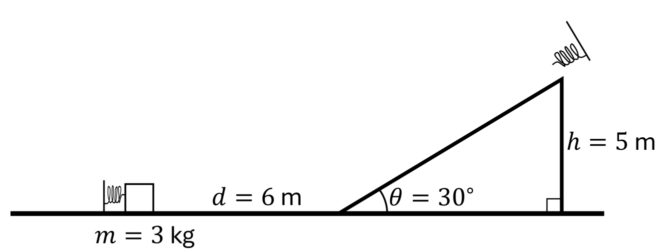
$$\Rightarrow K_f = W_F + U_{g,i} + U_{s,i}$$

$$\Rightarrow \boxed{K_f = W_F + mgh + \frac{1}{2} k s_1^2}$$

<u>Numeric</u> :

$$K_f = -147.22 \text{ J} + (3 \text{ kg})(9.8 \text{ m/s}^2)(5 \text{ m}) + \frac{1}{2}(15 \tfrac{\text{N}}{\text{m}})(0.4 \text{ m})^2 = \boxed{0.98 \text{ J}}$$

4.3 [10 pts] Once the box reaches the bottom of the ramp, it continues sliding along a flat horizontal frictionless surface for a distance $d = 6$ m. Then, it collides with another identical spring of stiffness $k = 15$ N/m. By what amount s_2 is this second spring compressed when the box comes to a stop?



Note, this K_i is the K_f from previous part (4.2).

By taking spring and block as system:

$$\Delta U_s + \Delta K = 0 \quad \Rightarrow \quad U_{f,s} - \cancel{U_{i,s}}^{\,0} + \cancel{K_f}^{\,0} - K_i = 0$$

$$\Rightarrow \frac{1}{2} k s_2^2 = K_i \qquad \Rightarrow \boxed{s_2 = \sqrt{\frac{2 K_i}{k}}}$$

<u>Numeric</u> :

$$s_2 = \sqrt{\frac{2(0.98 \text{ J})}{15 \text{ N/m}}} = \boxed{0.36 \text{ m or } 36 \text{ cm}}$$

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