Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}}$$

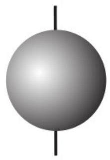
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

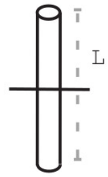
$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

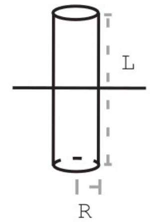
Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$







$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 Test 3 - Fall 2018

Please circle your lab section and then clearly print your name & GTID

Sections (M) Parker, (N) Yunker		
Day	12-3pm	3-6pm
Monday	M01 N01	M02 N02
Tuesday	M03 N03	M04 N04
Wednesday	M05 N05	M06 N06
Thursday	M07	N07

Name: Test Key v1

GTID: ____________________

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

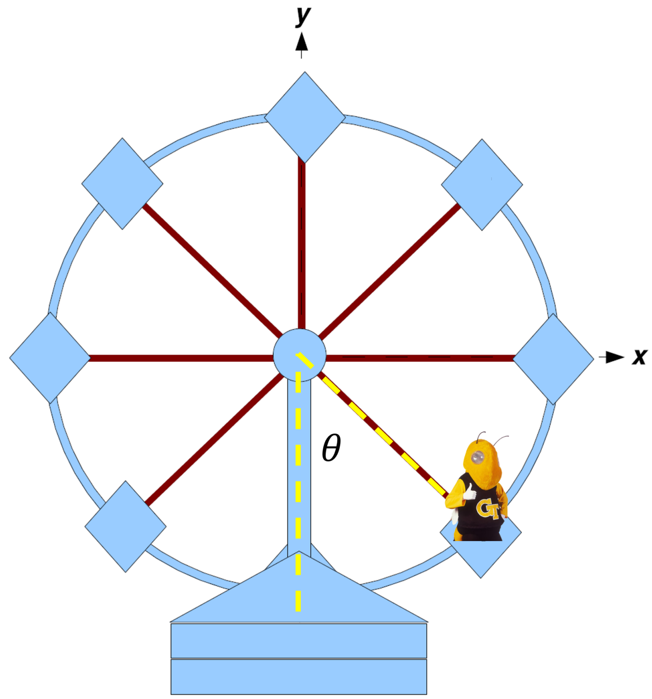
"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

____

Sign your name on the line above

Problem 1 [25 pts]

Buzz is riding the SkyView Ferris wheel in Centennial Park. The Ferris wheel is rotating clockwise so that Buzz has a constant speed, v_0. The Ferris wheel has radius R, and Buzz has mass m. Buzz's is currently at an angle θ, as defined in the figure.



All

A. [2 pts] What is the net force on Buzz in the direction parallel to the motion?

Constant speed $\rightarrow |\vec{F}_{\parallel}| = 0$

So $\boxed{\vec{F}_{net_{\parallel}} = 0 \qquad \hat{p}}$

All

B. [2 pts] What is the net force on Buzz in the direction perpendicular to the motion?

• Full credit if subscript on V_0 is dropped

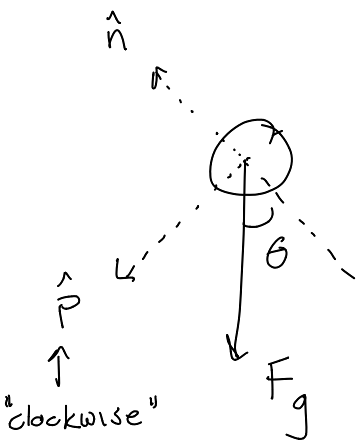
Circular motion $\rightarrow |\vec{F}_{\perp}| = \frac{mv^2}{r}$

So $\boxed{\vec{F}_{net_{\perp}} = \frac{mV_0^2}{R} \qquad \hat{n}}$

All

C. [3 pts] What is the magnitude of the perpendicular component of the force of gravity?

along $\hat{n}$



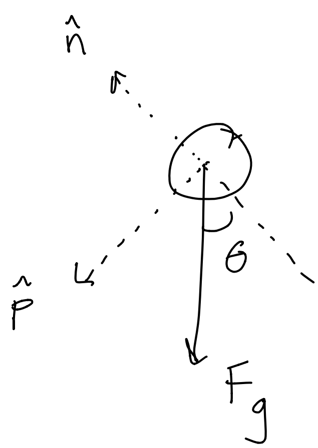
• full credit if $|\cdot|$ is missing

$|\vec{F}_{g_\perp}| = mg\,|\cos\theta|$

All

D. [3 pts] What is the magnitude of the parallel component of the force of gravity?

along $\hat{p}$



• full credit if $|\cdot|$ is missing

$|\vec{F}_{g_\parallel}| = mg\,|\sin\theta|$

E. [15 pts] What is the magnitude of the force of the Ferris wheel on Buzz?

$$\bar{F}_g + \bar{N} = \bar{F}_{net} = +\frac{mv_0^2}{R}\hat{n} \Rightarrow \left(mg\sin\theta\,\hat{p} - mg\cos\theta\,\hat{n}\right) + \bar{N} = \frac{mv_0^2}{R}\hat{n}$$

So,

$$\bar{N} = \left(\frac{mv_0^2}{R} + mg\cos\theta\right)\hat{n} - (mg\sin\theta)\hat{p}$$

Therefore,

$$\boxed{|\bar{N}| = \sqrt{\left(\frac{mv_0^2}{R} + mg\cos\theta\right)^2 + (mg\sin\theta)^2}}$$

Watch out for POE from parts A-D

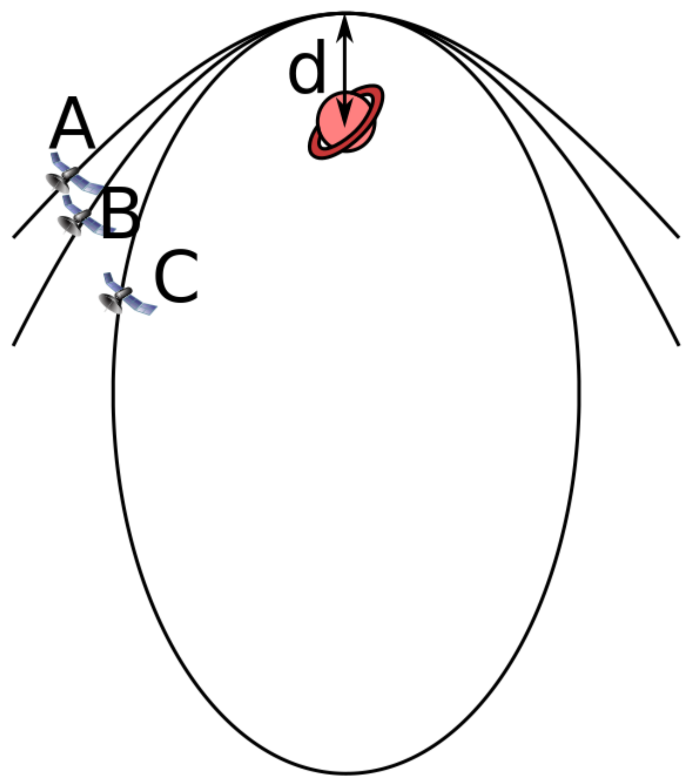
GRADING

-1 Clerical
-3 Minor
-6 Major
-12 BTN

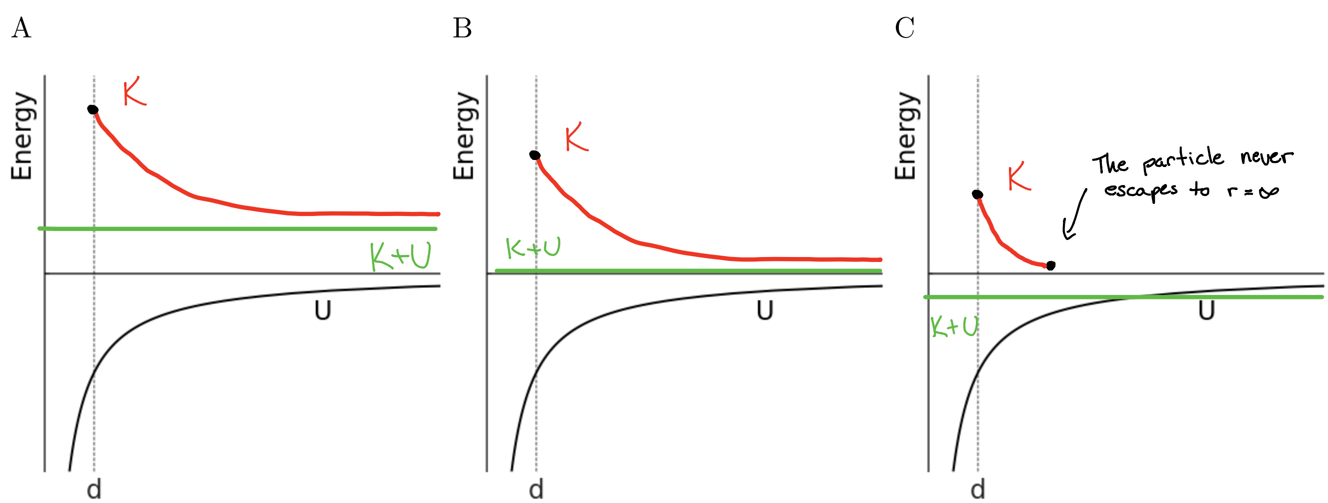
• full credit if they assume counterclockwise rotation

Problem 2 [25 pts]

Three satellites of equal mass m (labeled A, B, and C in the figure) are on trajectories that pass through the same point of closest approach a distance d from a heavy planet (which has mass M and can be assumed stationary). One satellite has negative, one positive, and one zero total energy.



A. [12 pts] Complete the following three energy diagrams by adding plots of the kinetic energy and the total energy for the corresponding orbits (the potential energy is shown already). Pay attention to the labels!



- +2 pts for each correct curve (6 curves drawn in total)
- −2pts for switching graphs (i.e. A and B graphs are switched)

B. [7 pts] What is the speed of the satellite at the point of closest approach for the satellite with zero total energy?

Zero total energy means that, at $r = d$,

$$K + U = \frac{1}{2}mv^2 - G\frac{mM}{d} = 0$$

So,

$$\boxed{v = \sqrt{\frac{2GM}{d}}}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-6 BTN

C. [6 pts] What is the radius of the kissing circle at the point of closest approach for the trajectory with zero total energy? [Hint: you do NOT need to know advanced geometry for this]

Equation for kissing circle: $|F_\perp| = \frac{mv^2}{R}$

So for this scenario,

POE

$$|F_g| = \frac{m}{R}\left(\sqrt{\frac{2GM}{d}}\right)^2$$

$$G\frac{mM}{d^2} = \frac{m}{R}\left(\frac{2GM}{d}\right)$$

$$\boxed{R = 2d}$$

GRADING

-1 Clerical
-2 Minor
-3 Major
-5 BTN

Problem 3 [25 pts]

Consider a ^{7}Be atom (mass 1.165189×10^{-26} kg) undergoing a reaction to become a ^{7}Li (mass 1.165035×10^{-26} kg) atom and a fast moving neutrino. (You may either assume that the neutrino mass is something small such as 10^{-36} kg or use a formula for massless particles as you prefer).

A. [4 pts] If the ^{7}Be atom begins at rest, what is the total kinetic energy of the system after the reaction?

By conservation of Energy, $E_f = E_i$

So

$$\cancel{K_i}^{\,0} + E_{rest,i} = K_f + E_{rest,f}$$

$$K_f = E_{Rest,i} - E_{rest,f}$$

$$= c^2 (M_{o,i} - M_{o,f})$$

$$= \left(3 \times 10^8 \tfrac{m}{s}\right)^2 \left(1.165189 \times 10^{-26} kg - (1.165035 \times 10^{-26} + 10^{-36}) kg\right)$$

$$= \boxed{1.386 \times 10^{-13} J}$$

GRADING

-1 Clerical
-2 Minor
-3 Major

B. [10 pts] If all of this kinetic energy were given to the neutrino, what would the magnitude of the neutrino's momentum be?

$$E^2 = (pc)^2 + (m_o c^2)^2$$

for $m_o = 10^{-36}$,

$$E^2 \approx (pc)^2$$

So

$$p = \frac{E}{c} = \frac{K_f}{c}$$

POE

$$p = \frac{(1.386 \times 10^{-13} J)}{3 \times 10^8 \frac{m}{s}} = \boxed{4.62 \times 10^{-22} \; kg \frac{m}{s}}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

C. [8 pts] What would be the kinetic energy of a ^{7}Li atom with the same momentum? (Hint: the previous answer should be around 4×10^{-22} kg · m/s. If you didn't get something close to that, please use the above number instead.)

GRADING

-1 Clerical
-2 Minor
-4 Major
-6 BTN

Relatwistic:

$$K = mc^2 - m_o c^2$$

$$= \sqrt{(pc)^2 + (m_o c^2)^2} - m_o c^2$$

$$= \left[\left((4.62 \times 10^{-22}\, kg \tfrac{m}{s})(3\times10^8 \tfrac{m}{s})\right)^2 + \left((1.165035 \times 10^{-26}\, kg)(3\times10^8 \tfrac{m}{s})^2\right)^2\right]^{1/2} - \left[(1.165035 \times 10^{-26}\, kg)(3\times10^8 \tfrac{m}{s})^2\right]$$

$$= \boxed{9.1604 \times 10^{-18}\ J}$$

Non-Relativistic:

$$K = \frac{p^2}{2m}$$

$$= \frac{(4.62 \times 10^{-22}\ kg \tfrac{m}{s})^2}{2(1.165035 \times 10^{-26}\ kg)}$$

$$= \boxed{9.1604 \times 10^{-18}\ J}$$

note: if you actually solve for it, $v \approx 4 \times 10^4 \ \frac{m}{s}$

So non relativistic limit is fine

All

D. [3 pts] In a more accurate calculation, the kinetic energy is shared between the ^{7}Li atom and the neutrino in order to conserve momentum. What fraction of the kinetic energy goes to the neutrino?

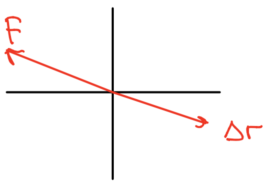
(a) All
(b) Almost all
(c) Close to half
(d) Very little
(e) None

Problem 4 [25 pts]

Prof. Parker and Prof. Yunker are maneuvering a boat with a combined mass of 3000 kg. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.3 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Prof. Parker exerts a force $\langle -400, 0, 200\rangle$ N and Prof. Yunker exerts a force $\langle 150, 0, 300\rangle$ N.

A. [5 pts] How much work does Prof. Parker do?

Diagram



Constant Force, so $W = \bar{F}\cdot\Delta\bar{r}$

$\Delta r = \langle 4,0,2\rangle - \langle 2,0,3\rangle = \langle 2,0,-1\rangle$

So

$W = \langle -400, 0, 200\rangle \cdot \langle 2,0,-1\rangle$

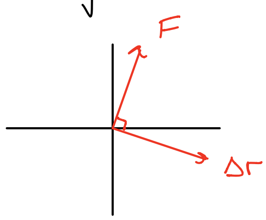
$= -800 - 200 = \boxed{-1000\ \text{J}}$

GRADING

-1 Clerical
-2 Minor
-5 Major

B. [5 pts] How much work does Prof. Yunker do?

Diagram



$W = \bar{F}\cdot\Delta\bar{r}$

$= \langle 150, 0, 300\rangle \cdot \langle 2,0,-1\rangle$

$= 300 - 300$

$= \boxed{0\ \text{J}}$

GRADING

-1 Clerical
-2 Minor
-5 Major

C. [5 pts] What is the angle between the (vector) force that Prof. Yunker exerts and the average vector velocity of the boat.

Since, $\bar{F}\cdot\Delta\bar{r} = 0$, we know that $\Delta\bar{r} \perp \bar{F}$.

Since $\bar{V}_{avg} = \dfrac{\Delta\bar{r}}{\Delta t}$ (i.e. pallalel to $\Delta\bar{r}$), it follows that

$\bar{F} \perp \bar{V}_{avg} \rightarrow \boxed{\theta = 90^\circ}$

GRADING

-1 Clerical
-2 Minor
-5 Major

D. [10 pts] The boat comes to rest (zero velocity) just as it reaches its final position. Determine the work done by the water on the boat during this maneuver.

By the conservation of energy

$E_f = 0 = E_i + W_{ext} = \frac{1}{2}mv_i^2 + W_{parker} + W_{water}$

So, $W_{water} = -W_{parker} - \frac{1}{2}mv_i^2$

$= 1000\ \text{J} - \frac{1}{2}(3000\ \text{kg})(1.3\ \tfrac{\text{m}}{\text{s}})^2$

$\boxed{= -1535\ \text{J}}$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

This page is for extra work, if needed.

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