# PHYS 2211 M - Final Exam - Summer 2022

**Please clearly print your name & GTID in the lines below**

Name: ________________________________ GTID: ________________________________

**Instructions**

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

**Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.**

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

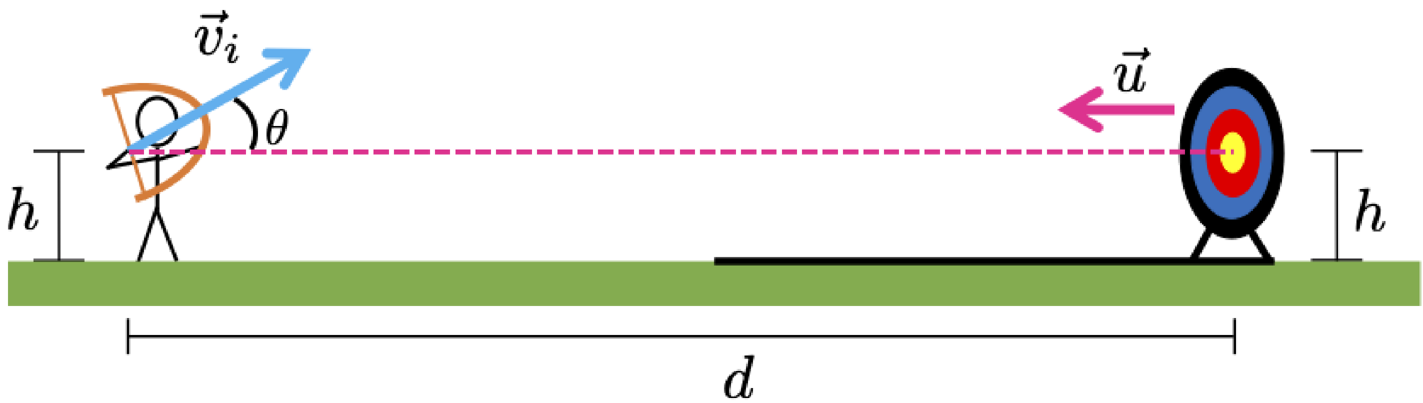
**"In accordance with the Georgia Tech Honor Code,**
**I have not given or received unauthorized aid on this test."**

______________________________________

Sign your name on the line above

**Archery – Q2 in Gradescope [15 pts]**

You are learning in archery class that to hit the bullseye (the center of the target) you need to aim a bit above it. The day of the test you are given a target that is a distance $d = 40$ m away, and it can move along a rail on the floor, coming towards you at a constant speed of $|\vec{u}| = 0.5$ m/s. You prepare your bow and arrow in such a way that the arrow will fly off from the same height above the ground as the bullseye ($h = 1.4$ m).




If you shoot the arrow with initial speed $|\vec{v}_i| = 30$ m/s, how high should you aim so you can hit the moving bullseye? In other words, **find the value (in degrees) for the angle $\theta$ in the diagram**. Assume there is no air resistance and that this is happening on Earth. The following trigonometric identity may be useful to you: $\sin(2\theta) = 2\sin\theta\cos\theta$.

$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$ , $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$

$\Rightarrow \cancel{y_f}^{0} = \cancel{y_i}^{0} + v_{i,y}\Delta t - \frac{1}{2}g(\Delta t)^2$

$\Rightarrow v_{i,y} = \frac{1}{2}g\Delta t = |v_i|\sin\theta$

$\Rightarrow \sin\theta = \frac{g\Delta t}{2|v_i|}$

Also, $v_x = |v_i|\cos\theta$ , and $\Delta x_{total} = d - |u|\Delta t$

$\Rightarrow |v_i|\cos\theta = \frac{d - |u|\Delta t}{\Delta t} = \frac{d}{\Delta t} - |u|$

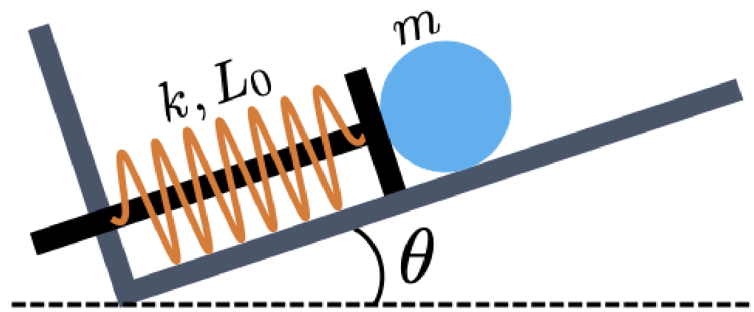
$\Rightarrow |v_i|\cos\theta + |u| = \frac{d}{\Delta t} \Rightarrow \Delta t = \frac{d}{|v_i|\cos\theta + |u|}$

$\Rightarrow \sin\theta = \frac{g}{2|v_i|}\frac{d}{|v_i|\cos\theta + |u|} = \frac{gd}{2|v_i|^2\cos\theta + 2|v_i||u|}$

$\Rightarrow \boxed{\sin\theta\,[2|v_i|^2\cos\theta + 2|v_i||u|] = gd}$

**Pinball – Q3 in Gradescope [15 pts]**

In a pinball machine, you launch a very small ball of mass $m$ by pulling on a handle that compresses a spring. The spring has relaxed length $L_0$ and stiffness $k$. The table on which the spring and ball rest is tilted at an angle $\theta$ as shown in the diagram.




Pulling the handle as far as it can go makes the spring compress to a length $L$. In this initial state, the ball is at rest. When you let go, the handle pushes the ball to launch it.

**How fast is the ball moving at the moment when it just loses contact with the spring?** You can assume there's no friction anywhere and this is happening on Earth.

System: ball + spring + Earth

$$\Delta E_{sys} = \cancel{W_{ext}}^{\,0} \Rightarrow \Delta E = \Delta K + \Delta U_g + \Delta U_s = 0$$

$$K_i = 0, \; K_f = \tfrac{1}{2} m v_f^2$$

$$U_{s,i} = \tfrac{1}{2} k s_i^2 = \tfrac{1}{2} k (L - L_0)^2, \; U_{s,f} = 0$$

$$\Delta U_g = mg(L_0 \sin\theta - L \sin\theta)$$

$$\Rightarrow \tfrac{1}{2} m v_f^2 - 0 + 0 - \tfrac{1}{2} k (L - L_0)^2 + mg(L_0 - L)\sin\theta = 0$$

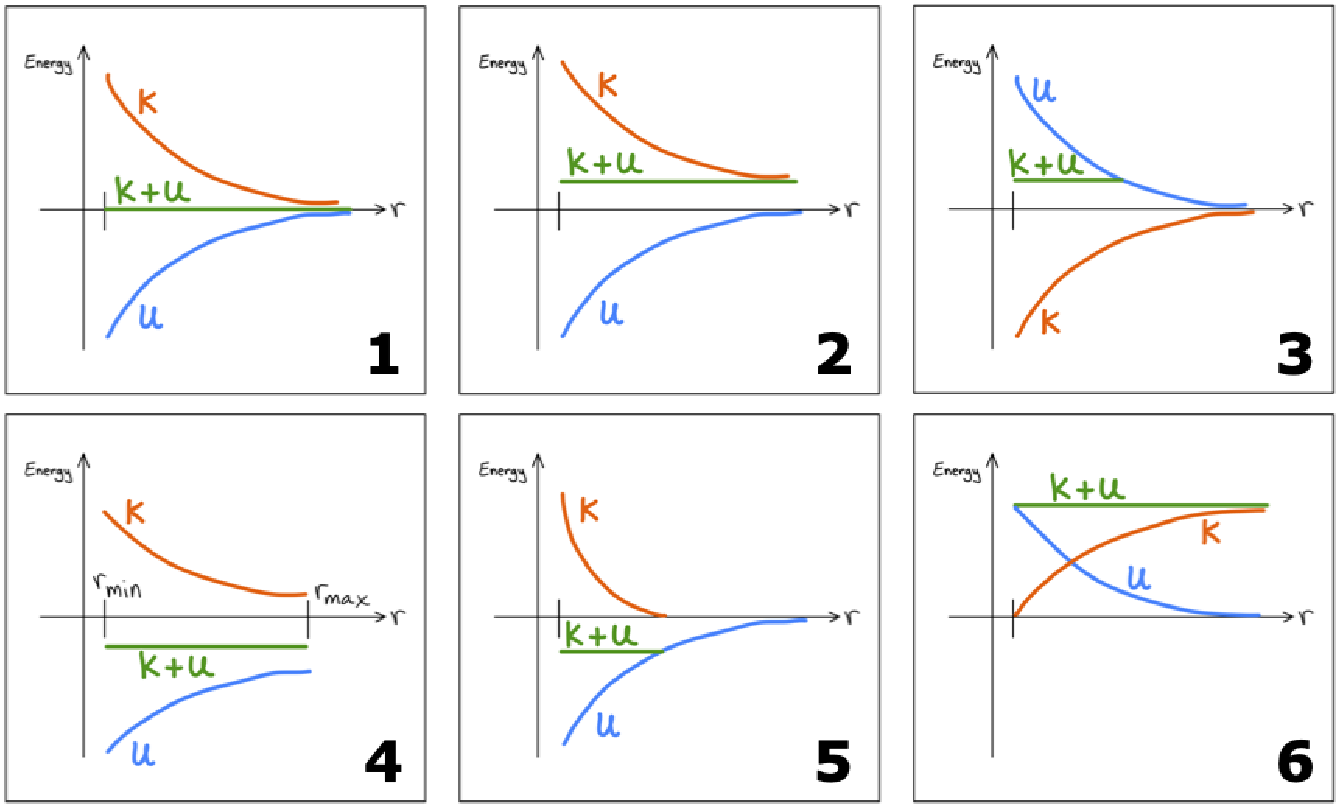
$$\Rightarrow v_f^2 = \frac{2}{m}\left[\tfrac{1}{2} k (L - L_0)^2 - mg(L_0 - L)\sin\theta\right]$$

$$\Rightarrow \boxed{v_f = \sqrt{\frac{2}{m}\left[\tfrac{1}{2} k (L - L_0)^2 - mg(L_0 - L)\sin\theta\right]}}$$

Note: full credit if submission gets this far, the math gets messy after this. Do not take off points for algebra errors after this point

**Graphs – Q4 in Gradescope [20 pts]**

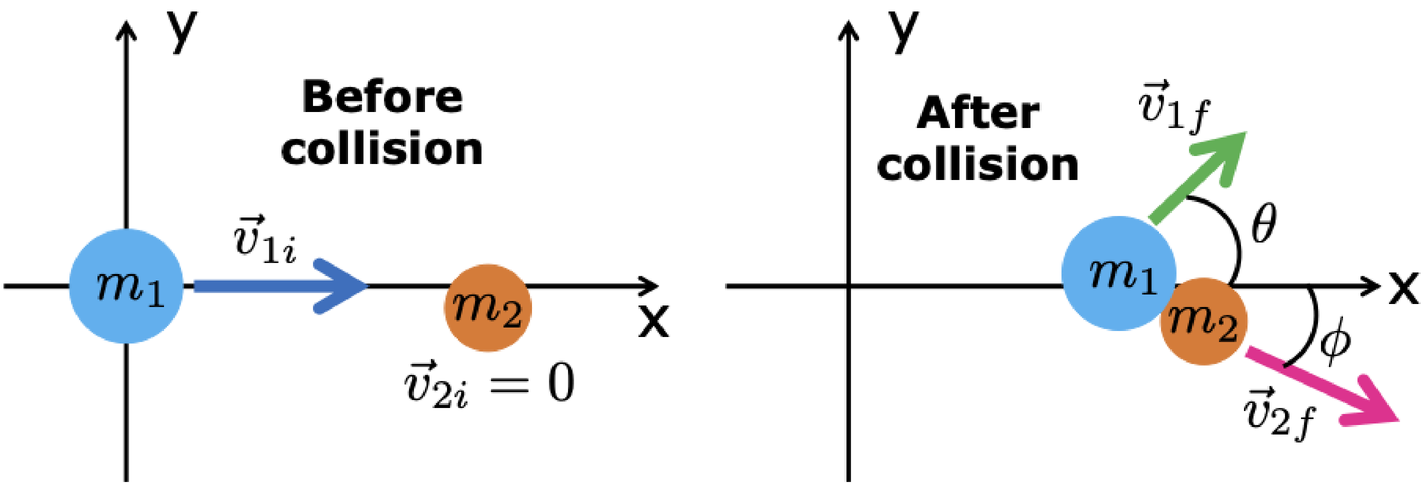
The following image shows six graphs of Energy versus distance ($r$). In each graph, the orange curve labeled $K$ represents kinetic energy, the blue curve labeled $U$ represents the potential energy, and the green line labeled $K+U$ represents the total energy. **Indicate which graph (by number) best represents each of the scenarios described below.**




1. 6 [2 pts] Two electrons are a significant distance away from each other and move with nonzero initial speeds heading straight towards each other. When they get very close, they momentarily stop and then start moving away from each other.
2. 2 [2 pts] A proton and an electron are moving away from each other with high initial speeds. When they are infinitely far away from each other, they are still moving.
3. 1 [2 pts] A comet swings by close to the Sun and then moves away. The comet's speed is zero when it is very, very far away from the Sun.
4. 1 [2 pts] A rock is ejected from a volcano on Io (one of Jupiter's moons) with a speed that is exactly equal to Io's escape speed.
5. 2 [2 pts] Voyager II taking off from Earth with an initial speed that is just a little bit higher than Earth's escape speed.
6. 5 [2 pts] An electron and a proton are held at rest at some distance away from each other and then are let go.
7. 4 [2 pts] Pluto and Charon, its biggest moon, in bound orbits around each other.
8. 6 [2 pts] Two protons are held near each other at rest and then are let go.
9. 4 [2 pts] A planet in an elliptical orbit around a star.
10. 3 [2 pts] An impossible set of energy graphs.

**Collision – Q5 in Gradescope [20 pts]**

A ball of mass $m_1$ moves atop the frictionless surface of a table in the +x direction with initial velocity $\vec{v}_{1i} = < v_{1i}, 0, 0 >$. It collides **elastically** off-center with a smaller ball of mass $m_2$ which was originally at rest.




After the collision, ball $m_1$ moves with final speed $v_{1f}$ at an unknown angle $\theta$ above the x axis, and ball $m_2$ moves with final speed $v_{2f}$ at an unknown angle $\phi$ below the x axis. **Find the values of $\theta$ and $\phi$** in terms of the known quantities in the problem (the masses $m_1$ and $m_2$, and the speeds $v_{1i}$, $v_{2i}$, $v_{1f}$, $v_{2f}$).

Conservation of angular momentum: $\vec{p}_f = \vec{p}_i$.

(Also, $K_i = K_f$ because collision is elastic, but not needed for solution.)

$\vec{p}_i = m_1 \langle v_{1i}, 0, 0 \rangle$, $\vec{p}_f = m_1 v_{1f} \langle \cos\theta, \sin\theta, 0 \rangle + m_2 v_{2f} \langle \cos\phi, -\sin\phi, 0 \rangle$

$p_x$: $m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$

$p_y$: $0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$

$$\Rightarrow \boxed{\sin\phi = \frac{m_1 v_{1f}}{m_2 v_{2f}} \sin\theta}$$

$$\Rightarrow \boxed{\cos\phi = \frac{m_1 v_{1i} - m_1 v_{1f} \cos\theta}{m_2 v_{2f}}}$$

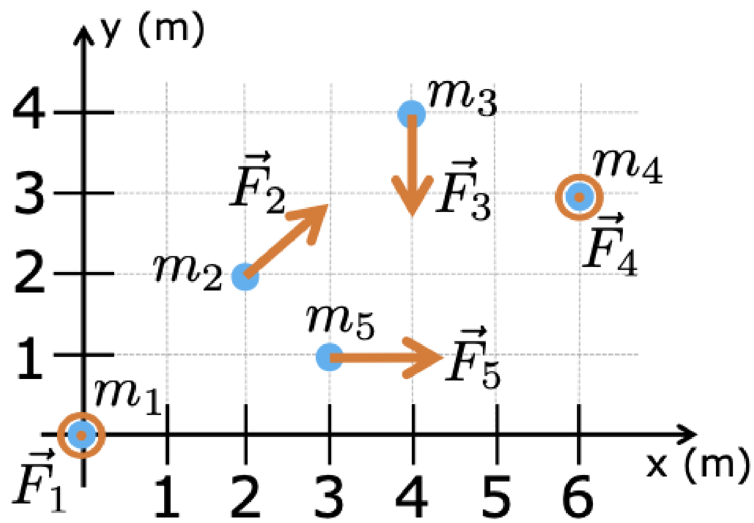
Note: full credit if submission gets this far, the math gets messy after this. Do not take off points for algebra errors after this point

**Torque – Q6 in Gradescope [15 pts]**

Five identical point masses with mass $m = 2$ kg are located in the xy plane as shown in the diagram.

Five external forces of identical magnitude $|\vec{F}| = 4$ N but pointing in different directions are applied individually to each of the particles as shown. Note that Forces 1 and 4 point out of the page, and Force 2 points 45 degrees above the horizontal.

Find the **net torque** $\vec{\tau}$ being exerted on this system of four particles, with respect to the origin. Your answer should be a vector with appropriate units.




$\vec{\tau}_1 = 0 \quad (\vec{r}_1 = 0)$

$\vec{\tau}_2 = 0 \quad (\vec{F}_2 \parallel \vec{r}_2)$

$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3 = (4m)(4N)(-\hat{z}) = -16 \, N \cdot m \, \hat{z}$

$$\vec{\tau}_4 = \vec{r}_4 \times \vec{F}_4 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 6 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = (3)(4)\hat{x} - (6)(4)\hat{y} + 0\hat{z}$$

$$= 12\hat{x} - 24\hat{y}$$

$\vec{\tau}_5 = \vec{r}_5 \times \vec{F}_5 = (1m)(4N)(-\hat{z}) = -4 \, N \cdot m \, \hat{z}$

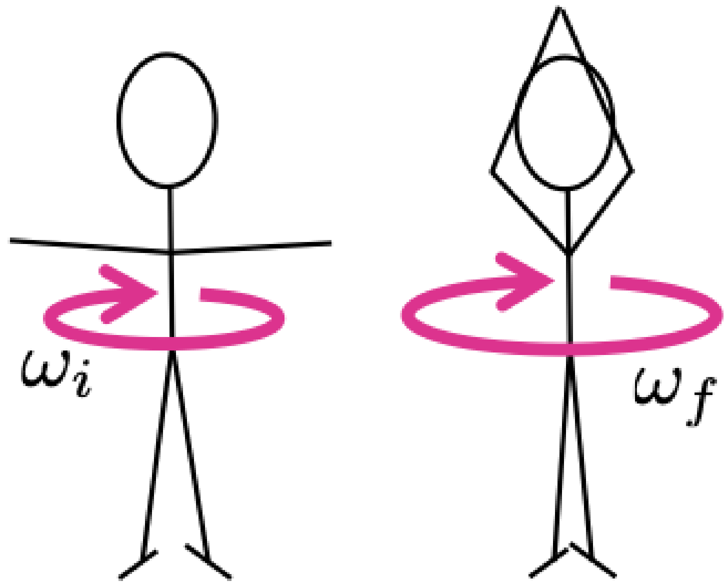
$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 + \vec{\tau}_5 = \boxed{\langle 12, -24, -20 \rangle \, N \cdot m}$$

**Figure Skater – Q7 in Gradescope [15 pts]**

A spinning figure skater who brings their arms close in to their body will then spin faster.

The stick person figure skater we're looking at has a mass of $M = 50$ kg and arms of length $R_i = 0.8$ m. The skater originally spins with angular speed $\omega_i = 2$ radians/second, and has their arms fully extended. At this initial state, the figure skater can be modeled as a hollow cylinder with moment of inertia $I = MR^2$.

When the skater brings their arms in towards their body, they can be modeled as a solid cylinder with radius $R_f = 0.2$ m.




Determine the **final angular speed** $\omega_f$ of the skater when their arms are tucked in.

$$\vec{L}_f = \vec{L}_i \, .$$

$$\vec{L}_i = I\vec{\omega}_i = MR_i^2\vec{\omega}_i \, , \quad \vec{L}_f = \frac{1}{2}MR_f^2\vec{\omega}_f$$

$$\Rightarrow |\vec{\omega}_f| = \left| \frac{MR_i^2}{\frac{1}{2}MR_f^2}\vec{\omega}_i \right| = \frac{2R_i^2}{R_f^2}|\vec{\omega}_i|$$

$$= \frac{2(0.8\,\text{m})^2}{(0.2\,\text{m})^2}\left(2\,\frac{\text{rad}}{\text{sec}}\right) = \boxed{64\,\frac{\text{rad}}{\text{sec}}}$$

**EXTRA CREDIT – Q8 in Gradescope [5 pts]**

1. [1 pts] Which of the following equations is the correct formula for the Momentum Principle?
   - $\vec{p} = m\vec{v}$
   - $\vec{v}_f = \vec{v}_i + (\vec{F}/m)\Delta t$
   - $\vec{p}_f = \vec{p}_i$
   - $\vec{v}_f = \vec{v}_i + \vec{F}\Delta t$
2. [1 pts] Which of the following equations is the correct formula for the Energy Principle?
   - $\Delta E_{sys} = W + Q$
   - $\Delta E_{sys} = W - Q$
   - $\Delta K + \Delta U = 0$
   - $\Delta K + \Delta U = W$
3. [1 pts] Which of the following equations is the correct formula for the Angular Momentum Principle?
   - $\vec{L} = \vec{r} \times \vec{p}$
   - $\vec{L}_f = \vec{L}_i + (\vec{\tau}/m)\Delta t$
   - $\vec{L}_f = \vec{L}_i$
   - $\vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$
4. [2 pts] You're writing a GlowScript code to model the motion of the Moon in its orbit around the Earth. Which of the following code snippets is the correct way to apply Newton's 2nd Law inside the `while` loop for this simulation?
   - ```
     r = Earth.pos - Moon.pos
     rhat = r/mag(r)
     Fgrav = (G * Moon.m * Earth.m / mag(r)**2) * (-rhat)
     Moon.vel = Moon.vel + (Fgrav/Moon.m)*deltat
     Moon.pos = Moon.pos + Moon.vel*deltat
     ```
   - ```
     r = Moon.pos - Earth.pos
     rhat = r/mag(r)
     Fgrav = (G * Moon.m * Earth.m / mag(r)**2) * (-rhat)
     Moon.pos = Moon.pos + Moon.vel*deltat
     Moon.vel = Moon.vel + (Fgrav/Moon.m)*deltat
     ```
   - ```
     r = Moon.pos - Earth.pos
     rhat = r/mag(r)
     Fgrav = (G * Moon.m * Earth.m / mag(r)**2) * (-rhat)
     Moon.vel = Moon.vel + (Fgrav/Moon.m)*deltat
     Moon.pos = Moon.pos + Moon.vel*deltat
     ```
   - ```
     r = Moon.pos - Earth.pos
     rhat = r/mag(r)
     Fgrav = (G * Moon.m * Earth.m / mag(r)**2) * rhat
     Moon.vel = Moon.vel + (Fgrav/Moon.m)*deltat
     Moon.pos = Moon.pos + Moon.vel*deltat
     ```

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