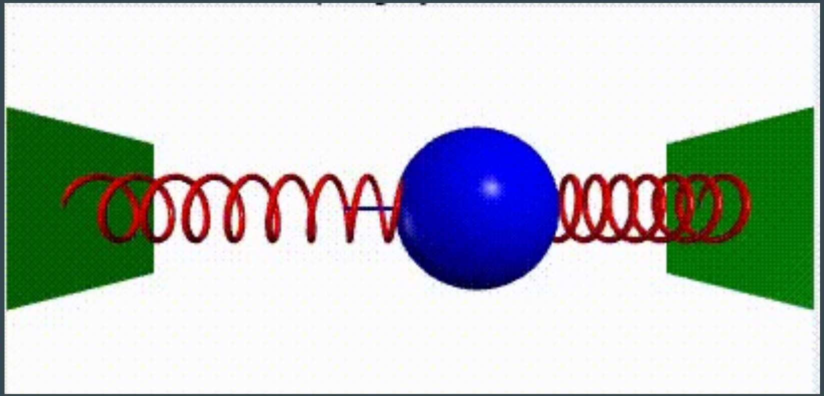
~~6~~ 3 clicker questions today



PHYS 2211 K

Week 3, Lecture 2
2022/01/27
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On today’s class…

1. Spring force
2. Iteration with constant and non-constant forces
3. ~~Universal gravitation~~

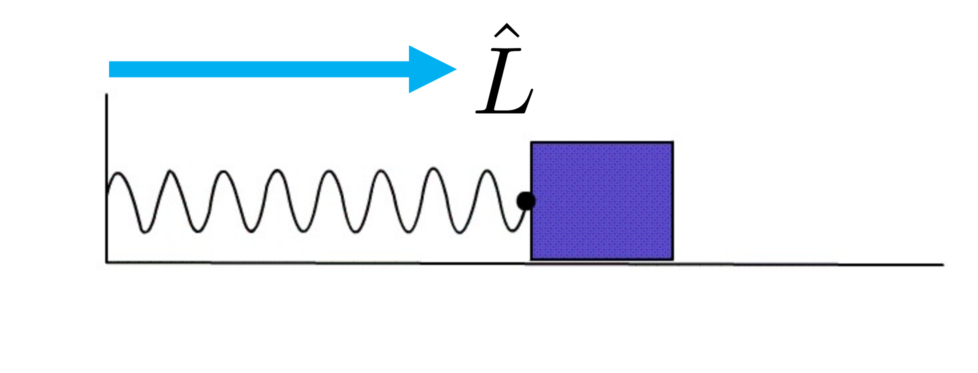
From Tuesday

- Spring force $$\vec{F}_s = -k(|\vec{L}| - L_0)\hat{L}$$

- L vector points from fixed end to moving end of the spring (same as position vector of the mass, when the origin is located at the fixed end of the spring)

- $L > L_0$ = stretched spring (force pulls in)

- $L < L_0$ = compressed spring (force pushes out)



Also from Tuesday

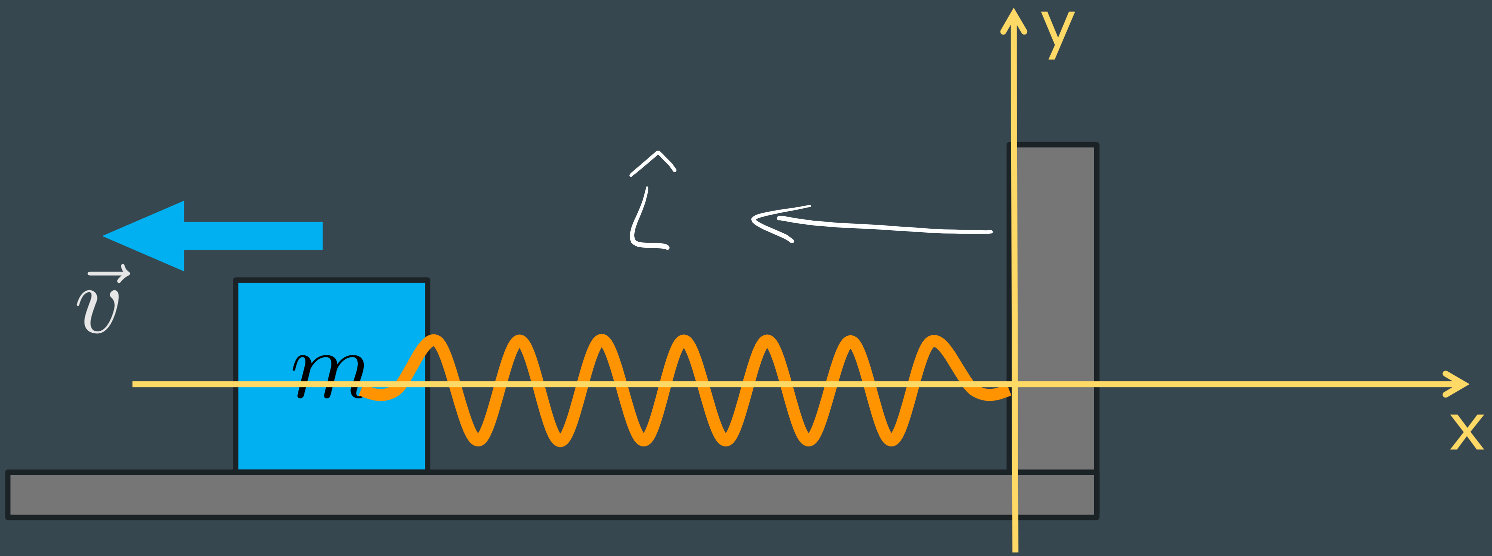
- Iteration means to predict the motion of an object in several very small consecutive time steps ← smaller deltat means more accurate prediction

- Procedure:
  - Find Fnet
  - Update velocity (v_final) with Newton's 2nd Law
  - Update position with position (r_final) update formula
    - For constant force: v_avg = arithmetic average of v_initial & v_final
    - For non-constant force: v_avg = v_final
  - Go to the next time step (increase t by an amount deltat)
  - Repeat: find new Fnet, find new v_final, find new r_final, etc

CLICKER 1: What is your favorite season?

Example

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.



CLICKER 2: What is the **direction** of the spring force?

A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

A. To the left

B. To the right

C. Zero magnitude

$$\vec{F}_s = -k(L - L_0)\hat{\imath}$$

$$= -k(L - L_0)(-\hat{x})$$

$$\Rightarrow (-)(+)(-)$$

$$\Rightarrow +\hat{x}$$

A block of mass m = 2.5 kg is attached to a spring with stiffness k = 12 N/m and relaxed length L_0 = 25 cm. The block moves horizontally and there is no friction between the block and the table. At t=0, the spring has length L = 30 cm and moves at a speed of 1 m/s to the left.

What is the net force on the block at t=0?

$$\vec{F}_{net} = -k(L-L_0)\hat{L} = -k(L-L_0)(-\hat{x}) =$$

$$= (-12\,\tfrac{N}{\cancel{m}})(0.3\cancel{m} - 0.25\cancel{m})(-\hat{x}) =$$

$$= (-12)(0.3-0.25)(-\hat{x})\,N =$$

$$= (-0.6)(-\hat{x})\,N = \boxed{0.6\,N\,\hat{x}}$$

$$= \boxed{\langle 0.6, 0, 0\rangle\,N}$$

A block of mass m = 2.5 kg is attached to a spring with stiffness k = 12 N/m and relaxed length L_0 = 25 cm. The block moves horizontally and there is no friction between the block and the table. At t=0, the spring has length L = 30 cm and moves at a speed of 1 m/s to the left.

What is the velocity of the block at t=0.05 s?

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = \langle -1,0,0\rangle \text{ m/s} + \frac{\langle 0.6,0,0\rangle \text{ N}}{2.5 \text{ kg}}(0.05 \text{ sec})$$

$$= \left[\langle -1,0,0\rangle + \frac{\langle (0.05)(0.6),\ 0,\ 0\rangle}{2.5}\right] \text{ m/s} =$$

$$\frac{N}{kg}s = \frac{\cancel{kg}\,m}{s^2}\,\frac{s}{\cancel{kg}} = \frac{m}{s}$$

$$= \boxed{\langle -0.988,\ 0,\ 0\rangle \text{ m/s}}$$

$$= \boxed{0.988 \text{ m/s } (-\hat{x})}$$

A block of mass m = 2.5 kg is attached to a spring with stiffness k = 12 N/m and relaxed length L_0 = 25 cm. The block moves horizontally and there is no friction between the block and the table. At t=0, the spring has length L = 30 cm and moves at a speed of 1 m/s to the left.

What is the position of the block at t=0.05 s?

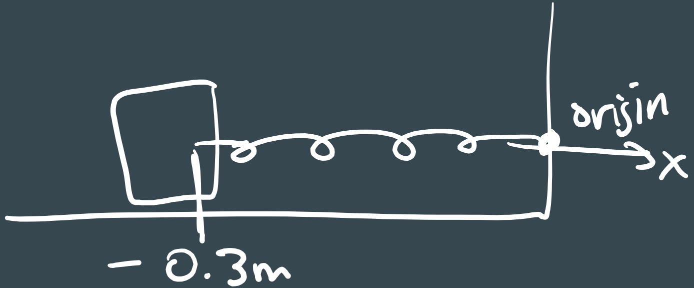
$\vec{r}_i = \langle -0.3, 0, 0 \rangle \text{ m}$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{Avg}\Delta t = \vec{r}_i + \vec{v}_f \Delta t =$$

$$= \langle -0.3, 0, 0 \rangle \text{m} + (\langle -0.988, 0, 0 \rangle \text{m}/\cancel{\text{s}})(0.05\ \cancel{\text{s}}) =$$

$$= \langle -0.3494, 0, 0 \rangle \text{ m}$$

$$= 0.3494 \text{ m } (-\hat{x})$$



A block of mass $m = 2.5$ kg is attached to a spring with stiffness $k = 12$ N/m and relaxed length $L_0 = 25$ cm. The block moves horizontally and there is no friction between the block and the table. At $t=0$, the spring has length $L = 30$ cm and moves at a speed of 1 m/s to the left.

What is the new net force acting on the block at t=0.05 s?

$$\vec{F}_s = -k(L - L_0)\hat{i} = -k(|\vec{r}_f| - L_0)(-\hat{x}) =$$

$$= (-12)(+0.3494 - 0.25)(-\hat{x}) =$$

$$= 1.1928\,\text{N}\,(+\hat{x})$$

$$= \langle 1.1928, 0, 0 \rangle\,\text{N}$$

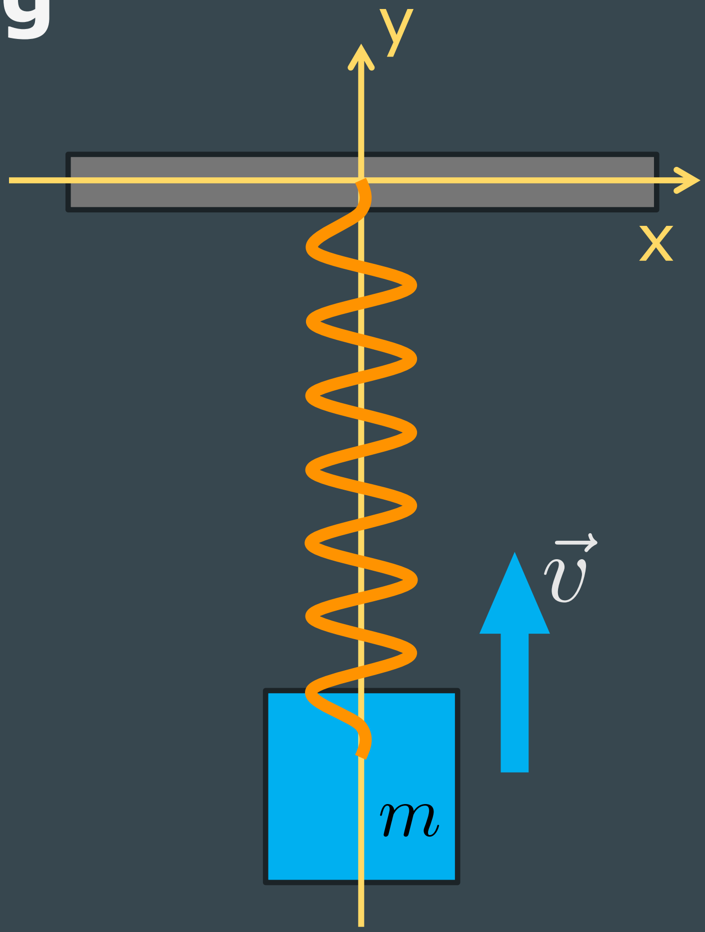
Summary of iterative procedure

- Physical properties of the system (m, k, L_0)
- Initial conditions: position ($\vec{r}_0$) and velocity ($\vec{v}_0$)
- First time step:
  - Force at t=0 ($\vec{F}_{net,0}$)
  - New velocity after Δt ($\vec{v}_1$)
  - New position after Δt ($\vec{r}_1$)
- If we wanted to go further, in the second time step we would do:
  - New net force after Δt ($\vec{F}_{net,1}$)
  - New new velocity after another Δt ($\vec{v}_2$)
  - New new position after another Δt ($\vec{r}_2$)
- And continue repeating for any additional time steps
  - $\vec{F}_{net,2}$, then $\vec{v}_3$, then $\vec{r}_3$, then $\vec{F}_{net,3}$, then $\vec{v}_4$, then $\vec{r}_4$, etc…

Example: A vertical spring

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

What is the net force acting on the block at this moment?



A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

Initial position: $\vec{r}_i = \langle 0, -L, 0 \rangle$

Initial velocity: $\vec{v}_i = \langle 0, v, 0 \rangle$

L vector and Lhat vector:

$$\vec{L} = \vec{r}_i = \langle 0, -L, 0 \rangle$$

$$\hat{L} = -\hat{y} = \langle 0, -1, 0 \rangle$$

CLICKER 3: What is the **net force** acting on the block?

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

A. $\vec{F}_{\text{net}} =< 0, -k(L - L_0) + mg, 0 >$

B. $\vec{F}_{\text{net}} =< 0, k(L - L_0) - mg, 0 >$

C. $\vec{F}_{\text{net}} =< 0, -k(L - L_0) - mg, 0 >$

D. $\vec{F}_{\text{net}} =< 0, k(L - L_0) + mg, 0 >$

Solution: What is the **net force** acting on the block?

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

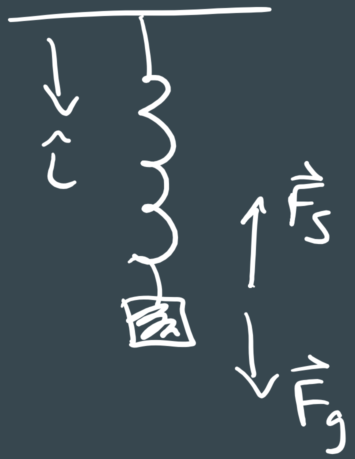
$$\vec{F}_g = \langle 0, -mg, 0 \rangle$$

$$\vec{F}_s = -k(L-L_0)\hat{L} = -k(L-L_0)(-\hat{y})$$

$$= \underline{k(L-L_0)\hat{y}}$$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_s = mg(-\hat{y}) + k(L-L_0)(+\hat{y}) =$$

$$= \left[k(L-L_0) - mg\right](+\hat{y})$$



A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Determine the position of the block at $t=T$ by iterating over two consecutive equal-sized time-steps.

Procedure: break the full time T into two smaller intervals: Δt_1 which goes from $t=0$ to $t=T/2$, and Δt_2 which goes from $t=T/2$ to $t=T$

- We already know Fnet at $t=0$
- Find velocity at the end of the interval Δt_1
- Find position at the end of the interval Δt_1
- Find new Fnet at the end of the interval Δt_1 (start of Δt_2)
- Find new velocity at the end of the interval Δt_2
- Find new position at the end of the interval Δt_2

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at $t=0$. At this moment, the spring is stretched to length L.

Velocity at the end of Δt_1

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = v + \left[k(L-L_0)-mg\right]\frac{T/2}{m} =$$

$$= \left\{ v + \frac{T}{2m}\left[k(L-L_0)-mg\right] \right\} (+\hat{y})$$

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

Position at the end of Δt_1

$$\vec{r}_f = \vec{r}_i + \vec{v}_f \Delta t = \left\{ -L + \frac{I}{2}\left[v + \frac{I}{2m}\left(k(L-L_0) - mg\right)\right]\right\}\hat{y}$$

First time step

$\vec{F}_{net_0}$

$\vec{v}_1$

$\vec{r}_1$

Second time step

$\vec{F}_{net_1}$

$\vec{v}_2$

$\vec{r}_2$

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

New net force at the end of Δt_1, which is the start of Δt_2

$$\vec{F}_{net_1} = \vec{F}_g + \vec{F}_{s_{new}} =$$

$$= mg(-\hat{y}) + -k(L_{new} - L_0)(-\hat{y}) =$$

$$L_{new} \rightarrow |\vec{r}_1|$$

$$= \underbrace{\left[k(|\vec{r}_1| - L_0) - mg\right](\hat{y})}\ = \vec{F}_{net_1}$$

A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

New velocity at the end of Δt_2

$$\vec{v}_2 = \vec{v}_1 + \frac{\vec{F}_{net,1}}{m} \Delta t =$$

$$= \vec{v}_1 + \frac{T}{2m} \vec{F}_{net,1}$$

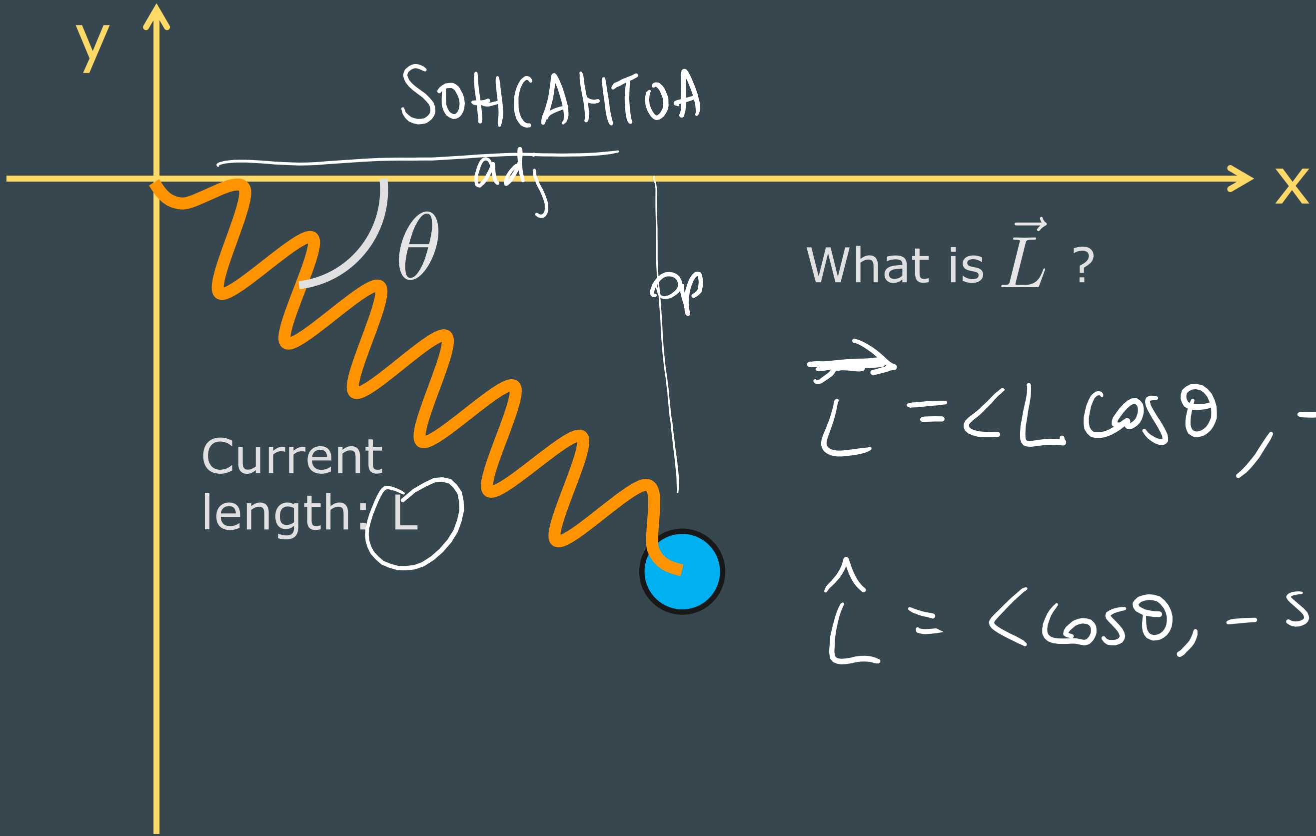
A spring with stiffness k and relaxed length L_0 hangs vertically from the ceiling. A block of mass m is attached to the free end of the spring and moving upwards with speed v at t=0. At this moment, the spring is stretched to length L.

New position at the end of Δt_2

$$\vec{r}_f = \vec{r}_i + \vec{v}_f \Delta t$$

$$\boxed{\vec{r}_2 = \vec{r}_1 + \vec{v}_2\left(\frac{T}{2}\right)}$$

Springs can also be diagonal...

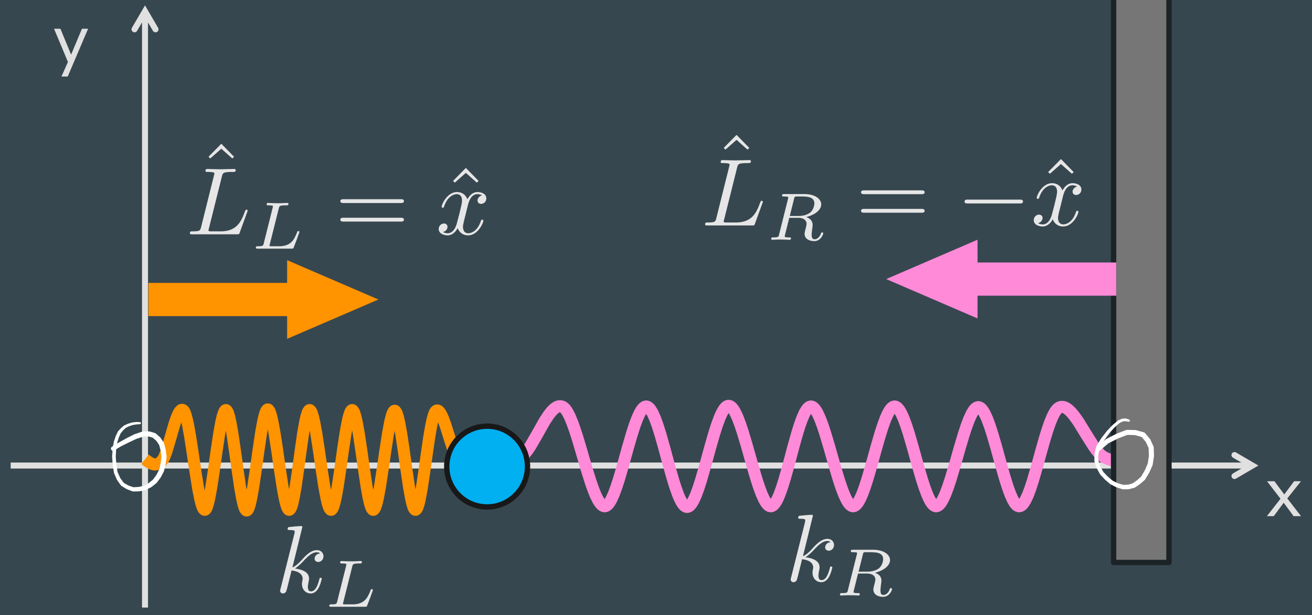


What is $\vec{L}$?

$$\vec{L} = \langle L\cos\theta, -L\sin\theta, 0\rangle$$

$$\hat{L} = \langle \cos\theta, -\sin\theta, 0\rangle$$

And there can be more than one spring...



Net force on the mass is the vector sum of the two spring forces

$$\vec{F}_{\text{net}} = \vec{F}_{sL} + \vec{F}_{sR}$$