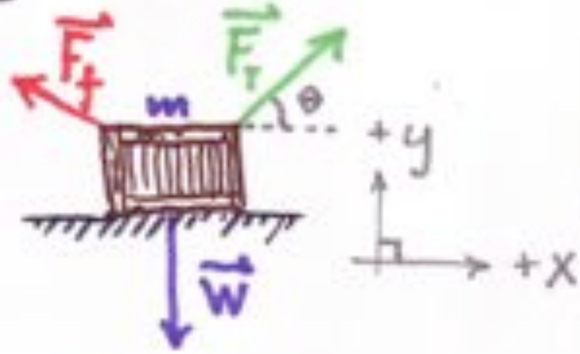
Here's our system:



$\vec{v} = \vec{0}$
CONSTANT

In this coordinate system, we have

$$\vec{W} = \langle 0, -mg, 0 \rangle$$

(as usual)

$$\vec{F}_T = \langle F_{T_x}, F_{T_y}, 0 \rangle$$

(we don't know what <u>these</u> are yet, but we do know they're <u>positive</u>, since $\vec{F}_T$ points in the $+x, +y$ direction)

$$\vec{F}_f = \langle f, N, 0 \rangle$$

(likewise, we don't know the values of f or N yet, but $\vec{F}_f$ points $-x, +y$, so $f < 0$ and $N > 0$)

Since $\vec{v}$ is constant, we know

$$\vec{F}_{net} = \vec{F}_f + \vec{F}_T + \vec{W} = \vec{0}$$

Moreover, each component of $\vec{F}_{net}$ is zero, so we can set up 3 equations (one for each coordinate):

$$x: \; f + F_{T_x} + 0 = 0$$

$$y: \; N + F_{T_y} + (-mg) = 0$$

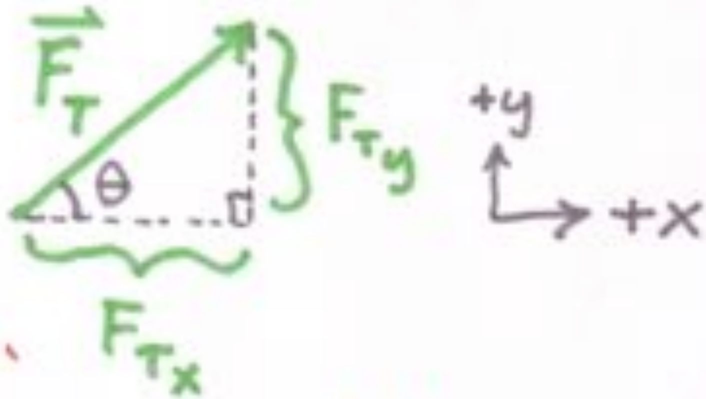
$$z: \; 0 + 0 + 0 = 0$$

we don't learn anything from the z equation...

... so we have 2 equations and 4 unknowns (f, N, F_{T_x}, F_{T_y}).

WE CAN DO BETTER!

Geometrically, we see



$$F_{T_x} = |\vec{F}_T| \cos\theta$$

$$F_{T_y} = |\vec{F}_T| \sin\theta$$

so now we have 2 equations and 3 unknowns (f, N, $|\vec{F}_T|$). We know θ, so we also know $\sin\theta$ & $\cos\theta$.

we just need to get rid of one more unknown!

WE HAVE A MODEL FOR

STATIC FRICTION

and it tells us

$$|f| = \mu_s |N|$$

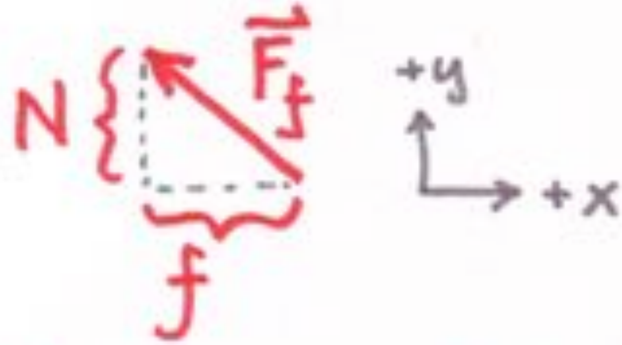
when the box is on the verge of sliding.

⇧

WE'LL MAKE THIS ASSUMPTION

Now, our static friction model only tells us about how the MAGNITUDES $|f|$ & $|N|$ are related — f and N could each be positive or negative.

We can figure that out geometrically:



f is negative

N is positive

so we have

$$f = -\mu_s N$$

and we're good to go!

we finally have 2 equations:

$$x: \; -\mu_s N + |\vec{F}_T|\cos\theta = 0$$

$$y: \; N + |\vec{F}_T|\sin\theta - mg = 0$$

and 2 unknowns: $(N, |\vec{F}_T|)$

FIRST, SOLVE FOR N:

$$x: \; -\mu_s N = -|\vec{F}_T|\cos\theta$$

$$N = \frac{|\vec{F}_T|\cos\theta}{\mu_s}$$

NEXT, SOLVE FOR $|\vec{F}_T|$:

$$y: \; N + |\vec{F}_T|\sin\theta - mg = 0$$

$$\frac{|\vec{F}_T|\cos\theta}{\mu_s} + |\vec{F}_T|\sin\theta - mg = 0$$

$$|\vec{F}_T|\left(\frac{\cos\theta}{\mu_s} + \sin\theta\right) = mg$$

$$|\vec{F}_T| = mg\left(\frac{1}{\frac{\cos\theta}{\mu_s} + \sin\theta}\right)$$

$$|\vec{F}_T| = \frac{mg}{\frac{\cos\theta}{\mu_s} + \sin\theta}\left(\frac{\mu_s}{\mu_s}\right)$$

this is equal to one

$$|\vec{F}_T| = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Now, if we want the numerical value of $|\vec{F}_T|$, we can plug in our given quantities

$$m = 40\,kg$$
$$g = 9.8\,m/s^2$$
$$\theta = 35^\circ$$
$$\mu_s = 0.25$$

and get

$$|\vec{F}_T| = \frac{(0.25)(40\,kg)(9.8\,m/s^2)}{(0.25)(0.574) + (0.819)} = 102\,N$$

and if we want the numerical values of each component of $\vec{F}_T$, we can use

$$F_{T_x} = |\vec{F}_T| \cos\theta$$

$$F_{T_y} = |\vec{F}_T| \sin\theta$$

to get

$$\vec{F}_T = \langle 83, 59, 0 \rangle \text{ N}$$

To go further, since we now know the numerical values of F_{T_x} & F_{T_y}, we can use

$$f + F_{T_x} = 0 \quad \text{and} \quad N + F_{T_y} - mg = 0$$

to get

$$\vec{F}_f = \langle -83, 333, 0 \rangle \text{ N}$$