Week 6

Moving Reference Frames

Topics for this week

1. Parallel and perpendicular force decomposition
2. Determining forces from motion
3. Curvature

By the end of the week

1. Be able to decompose forces into arbitrary components
2. Predict speeds from curvature

Non-equilibrium motion

- What does the direction of the net force have to do with motion?
- A bowling ball rolls in a straight line. To make it travel in a circle, in what direction did the student repeatedly apply a force to the ball with a rubber mallet?
  - In the direction of the ball's momentum
  - Perpendicular to the ball's momentum, toward the inside of the circle
  - Perpendicular to the ball's momentum, toward the outside of the circle

Force decomposition

- We have already seen how to decompose a force into the x, y, and z components
  - There was nothing special about that coordinate system
- What happens if we choose a coordinate system that is instantaneously synchronized to the motion of the system?

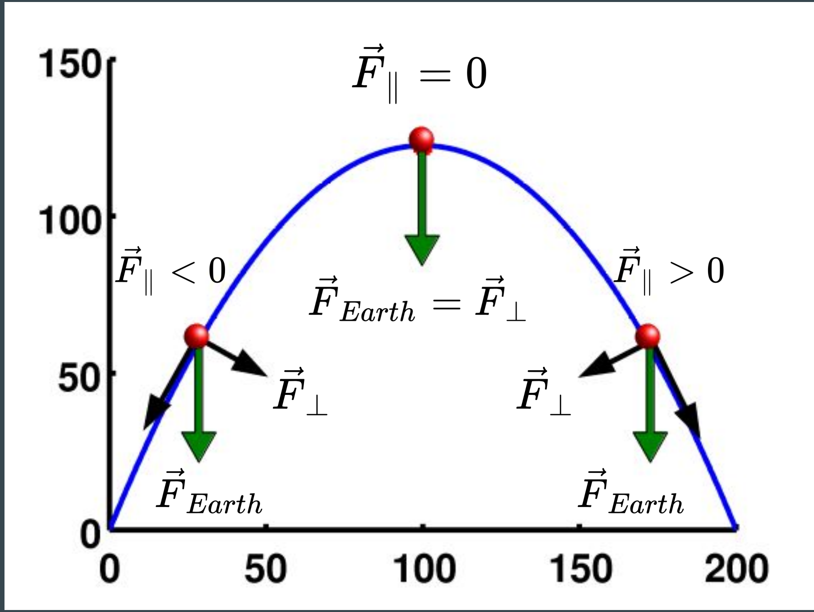
$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

  - $F_{\parallel}$ is the component of the net force acting parallel to the direction of motion
  - $F_{\perp}$ is the component of the net force acting perpendicular to the direction of motion
- This can be mathematically cumbersome for some problems but dramatically simplify the momentum principle for others!

Example: Projectile motion revisited

When we can neglect air resistance, the motion of an object near the surface of the Earth follows a parabolic trajectory. Instead of an xy coordinate system what happens when we choose a parallel and perpendicular coordinate system.

Is the net force still constant? Would this be a convenient coordinate system to use if we wanted to predict motion?

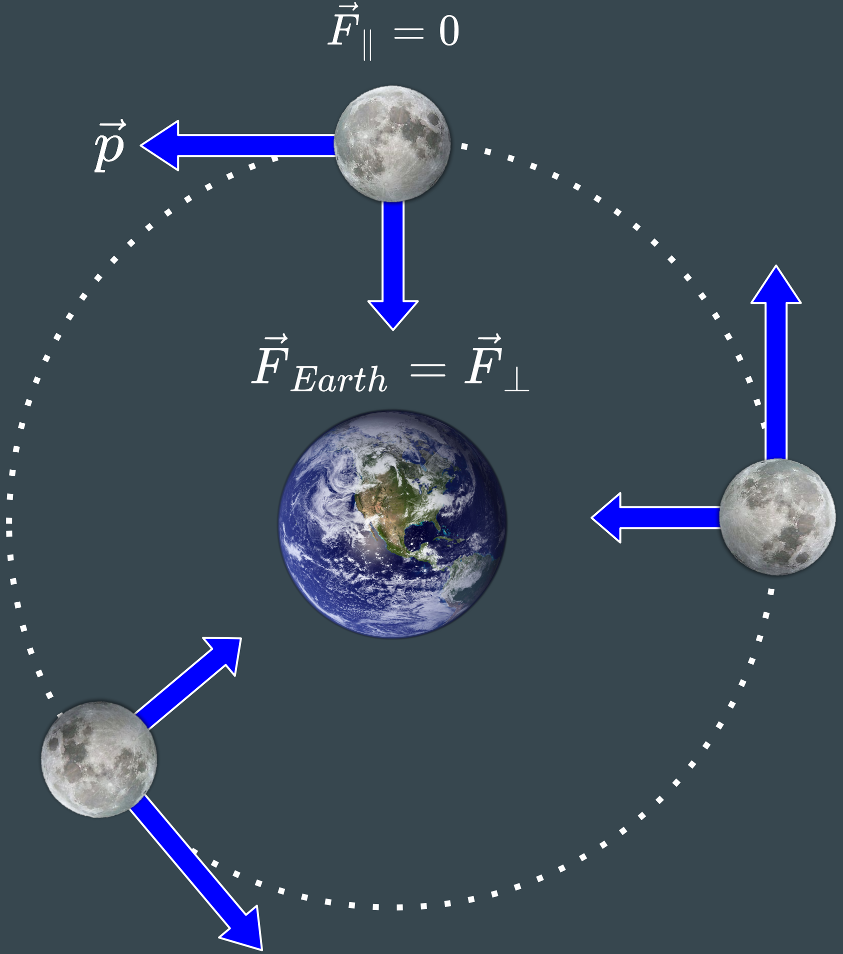


Example: Orbital motion

Consider the Moon moving around the Earth in a circular trajectory at a constant speed.

Instead of an xy coordinate system what happens when we choose a parallel and perpendicular coordinate system.

Is the net force still constant? Would this be a convenient coordinate system to use if we wanted to predict motion?



Decomposing the momentum principle

- If we decompose the net force into parallel and perpendicular components then we should do the same to the change in moment
    - This will allow us to match up the changes and forces

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(|\vec{p}|\hat{p})$$

- Use the product rule

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

Matching up changes in momentum to force

- Each half of the total change in momentum corresponds to different aspects of the motion and the net force
- The first half points in the direction of momentum with a magnitude equal to the change in speed

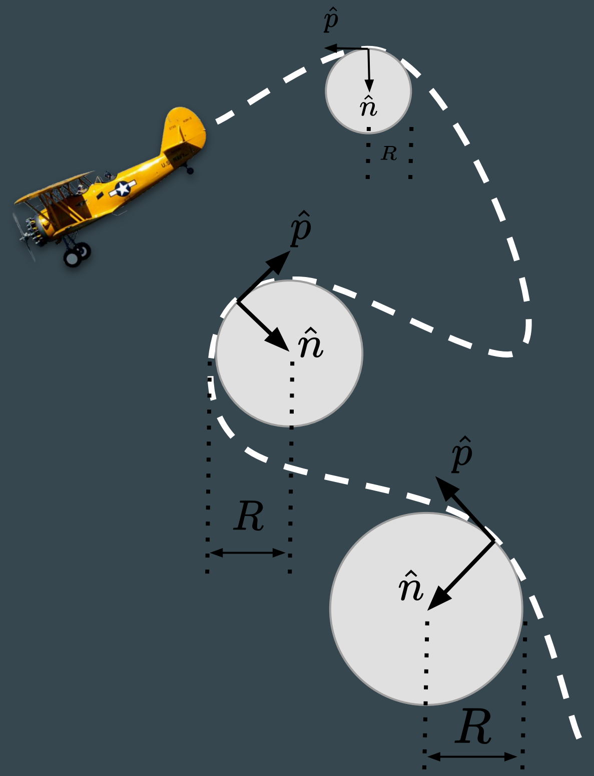
$$\frac{d|\vec{p}|}{dt}\hat{p} = \left(\vec{F}_{net}\right)_{\parallel}$$

- The second half is the derivative of a unit vector
  - No longer a unit vector and will point in a direction perpendicular to the original vector

$$|\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \left(\vec{F}_{net}\right)_{\perp}$$

Graphical representations

- Consider the path of an object through space
  - At any instant in time we can determine directions parallel and perpendicular to the motion
- "R" is the radius of the kissing circle at a particular point along a trajectory
  - 1/R is a measure of the curvature at that point
- "n" is the unit vector that is perpendicular to the momentum
  - This vector points toward the center of the kissing circle
- These directions and quantities are different at each point along the trajectory!
  - The coordinate system is local to the system



Example: The speed of the moon

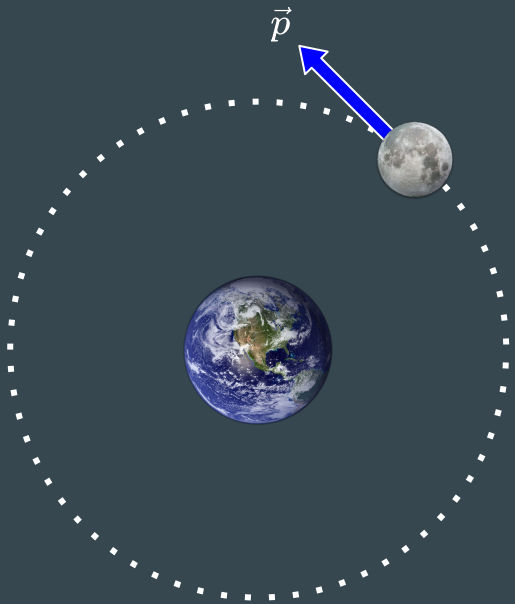
The moon orbits the Earth in a nearly circular orbit of radius R at a constant speed. Calculate the speed of the moon.

$$\frac{d\vec{p}}{dt} = \vec{F}_{grav}$$

$$\frac{d|\vec{p}|}{dt}\hat{p} = \left(\vec{F}_{grav}\right)_{\parallel} = 0$$

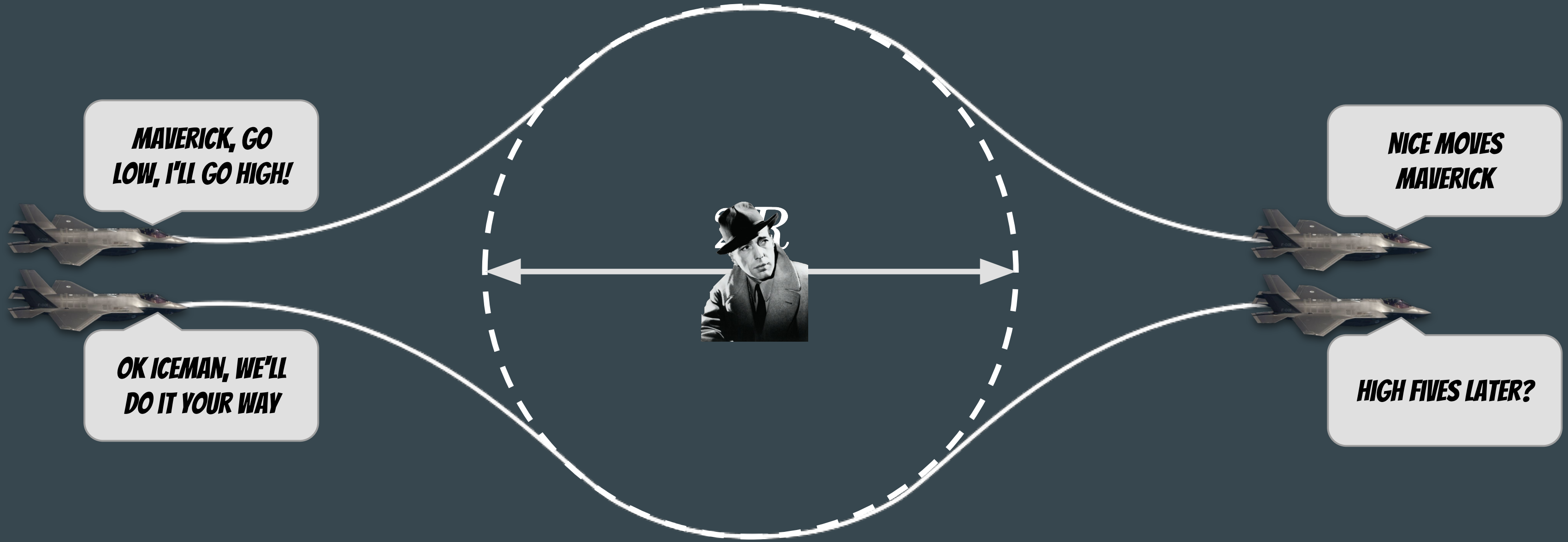
$$|\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \left(\vec{F}_{grav}\right)_{\perp} = G\frac{m_{moon}m_{Earth}}{R^2}\hat{n}$$

$$|\vec{v}|^2 = G\frac{m_{Earth}}{R}$$



Example: Top Gun

Iceman and Maverick are on a night patrol when they encounter a bogey. They split and fly around the the bogey in a circular orbit. If they both travel at the same constant speed, who feels a larger contact force from their seat halfway around the bogey?



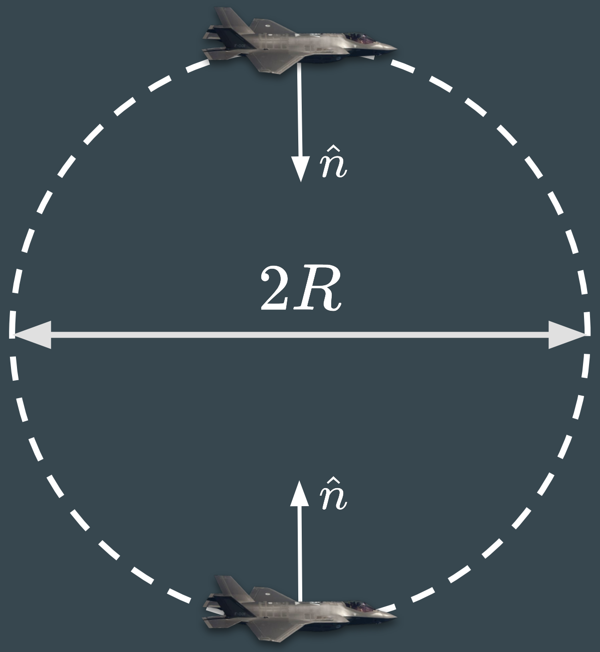
Example: Top Gun Solution

- Each pilot only feels two forces acting on their body
  - (1) Earth's gravity, (2) Contact force with seat
  - They do, however, have different reference frames

$$|\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_{seat,top} + (mg)\hat{n}$$

$$|\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_{seat,bottom} - (mg)\hat{n}$$

- The pilot on the bottom half of the path feels a larger contact force on their body

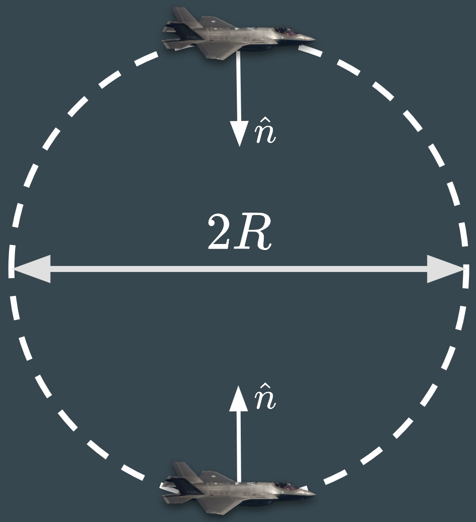


Kinesthetic sensations

- Our bodies perceive contact forces
  - It is possible to follow a trajectory through space that results in zero contact forces

$$\vec{F}_{seat,top} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} - (mg)\hat{n}$$

$$v^2 = Rg$$



- Our bodies place limits on maximum contact forces
  - Bones break according to Young's modulus and your heart is only able to accelerate your blood at the rate of about 9g

$$\frac{|\vec{v}|^2}{R} = 9g \quad \Longrightarrow \quad v^2 = 9Rg$$

Video solutions from chapter 5

- Practice problems that I solved on video over the years
  - You don’t gain much by just watching them you also need to try and work them out
- An object moving in a circle along an angled surface
  - https://vimeo.com/208202487
- Riding a sleigh over a hill
  - https://vimeo.com/158651602
- A bug slides off a sphere
  - https://vimeo.com/158393644
- A student riding a ferris wheel
  - https://vimeo.com/30277476
- The three body problem
  - https://vimeo.com/30277550