Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

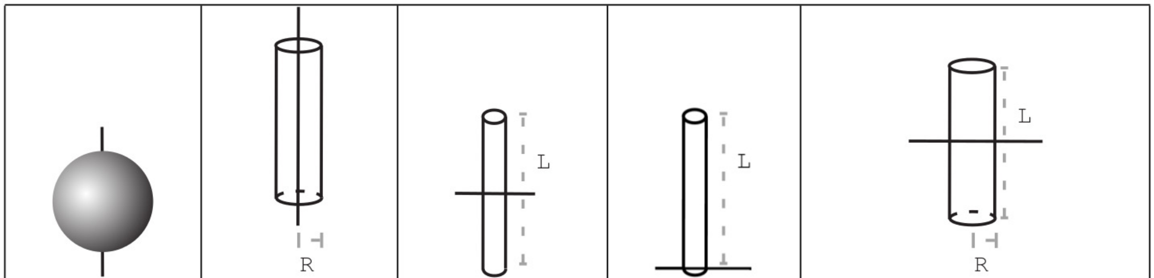
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 - Summer 2024 - Test 1

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You must join the appropriate proctoring meeting in MS Teams and keep your camera on and microphone muted throughout the exam period.
  - Other than MS Teams and Gradescope (and a PDF annotation app if applicable), you must not access any other app or website during the exam.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically (i.e., symbolically).
  - Numerical solutions should only be evaluated (i.e., plug in numbers) at the last step.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be marked as incorrect.**
  - Incorrect work or explanations mixed in with correct work will be marked as incorrect. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\dfrac{a \cdot b}{c \cdot d} = \dfrac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

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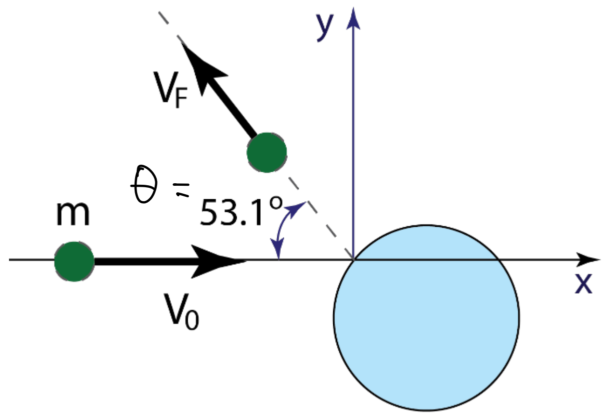
Sign your name on the line above

Collision with a surface [30 pts]

As a result of a glancing collision with the surface of the sphere (see Figure), the velocity of a small ball (mass $m = 139g$, shown in green) changes from $V_0 = 7.0\ m/s$ directed rightward to $V_F = 5.0\ m/s$ directed 53.1° above the horizontal. The ball was in contact with the sphere for a time $0.20s$.



1. [20 pts] What is the magnitude of the change in momentum of the ball?

$$|\Delta\vec{p}| = |\vec{p}_F - \vec{p}_0|.$$

$$\vec{p}_0 = \langle mV_0, 0\rangle \quad , \quad \vec{p}_F = \langle -mV_F\cos(\theta), mV_F\sin(\theta)\rangle$$

$$\Rightarrow \vec{p}_F - \vec{p}_0 = \langle -mV_F\cos(\theta), mV_F\sin(\theta)\rangle - \langle mV_0, 0\rangle$$

$$= \langle -mV_F\cos(\theta) - mV_0 , mV_F\sin(\theta)\rangle$$

$$\Rightarrow |\vec{p}_F - \vec{p}_0| = \sqrt{[-mV_F\cos(\theta) - mV_0]^2 + [mV_F\sin(\theta)]^2}$$

$$= m\sqrt{[V_F\cos(\theta) + V_0]^2 + [V_F\sin(\theta)]^2}$$

$$= (0.139\ kg)\sqrt{[(5\ m/s)\cos(53.1°) + (7\ \tfrac{m}{s})]^2 + [(7\ \tfrac{m}{s})\sin(53.1°)]^2}$$

$$= 1.497\ kg\ \frac{m}{s} = |\Delta\vec{p}|$$

2. [10 pts] Find the magnitude of the average force that acted on the ball during the collision.

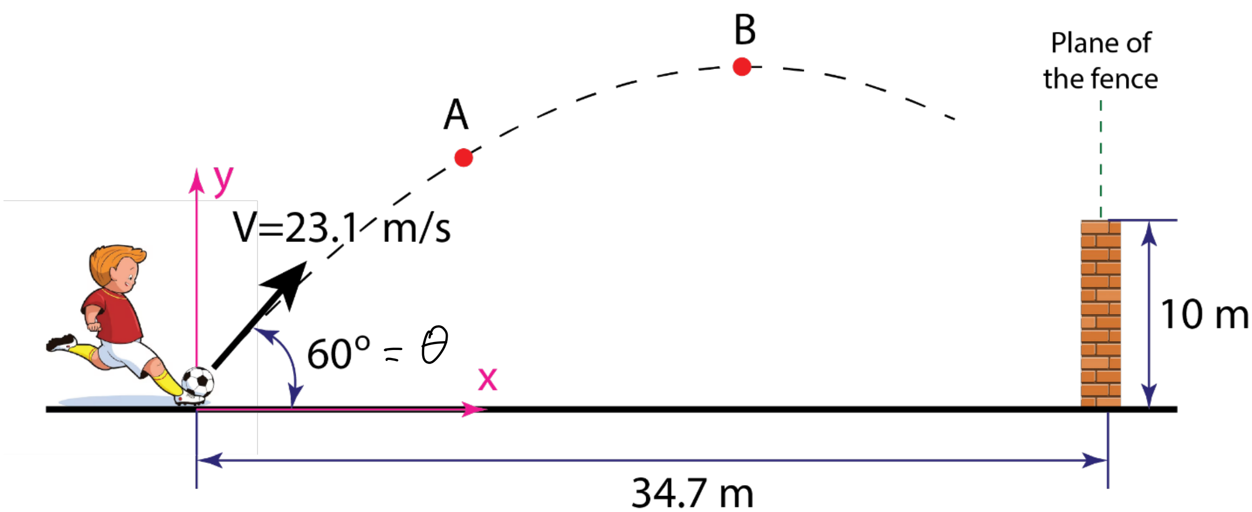
Newton's second Law: $\dfrac{\Delta \vec{p}}{\Delta t} = \vec{F}_{av}$

$\Rightarrow \dfrac{|\Delta \vec{p}|}{\Delta t} = |\vec{F}_{av}|$

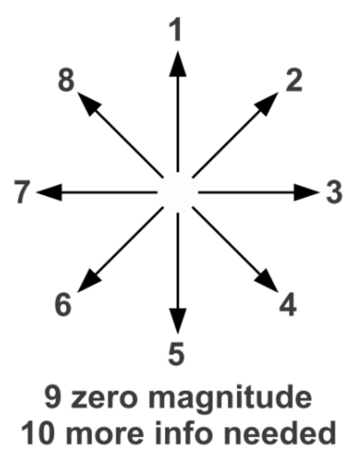
$\Rightarrow \dfrac{1.497 \text{ kg} \frac{m}{s}}{0.2 \text{ s}} = 7.49 \text{ kg} \frac{m}{s^2} = 7.49 \text{ N}$

Soccer Ball [40 pts]

A ball is kicked directly toward a fence from a point 34.7 meters away, as shown in the Figure. The ball's velocity as it leaves the kicker's foot is 23.1 m/s at an angle of 60° above the horizontal. The top of the fence is 10 meters high. The ball hits nothing while in flight, and air resistance is negligible. The Figure (not up to scale) also shows a part of the ball trajectory.



1. [10 pts] Using the numbered directions shown by the rosette, indicate (by number) which arrow best represents the direction of the quantities listed below. If a quantity has zero magnitude or cannot be determined, indicate that using the corresponding number.



a. [2 pts] __2__ The velocity of the ball at point A.

b. [2 pts] __5__ The acceleration of the ball at point A.

c. [2 pts] __3__ The velocity of the ball at point B.

d. [2 pts] ___5___ The acceleration of the ball at point B.

e. [2 pts] ___5___ The change in velocity of the ball between points A and B.

2. [5 pts] Determine the time it takes for the ball to reach the plane of the fence.

$$\vec{V} = \langle V\cos(\theta),\ V\sin(\theta)\rangle = \langle V_x,\ V_y\rangle.$$

The object needs to travel $d = 34.7\ m$ in the x-direction. Thus,

$$V\cos(\theta)\,t = d \Rightarrow t = \frac{d}{V\cos(\theta)}$$

$$= \frac{34.7\ m}{(23.1\ \frac{m}{s})\cos(60°)}$$

$$= 3.00\ s$$

3. [15 pts] Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

Position update formula for constant forces in y-direction:

$$y_F = y_i + v\sin(\theta)\,t + \frac{F_y}{2m}t^2$$

$$\vec{F} = \langle 0, -mg, 0\rangle \Rightarrow F_y = -mg \Rightarrow \frac{F_y}{2m} = \frac{-\cancel{m}g}{2\cancel{m}} = -4.9\ m/s^2$$

$$v\sin(\theta) = (23.1)\sin(60^\circ) = 20.0\ m/s$$

$y_i = 0$. t is the same as question 2. So,

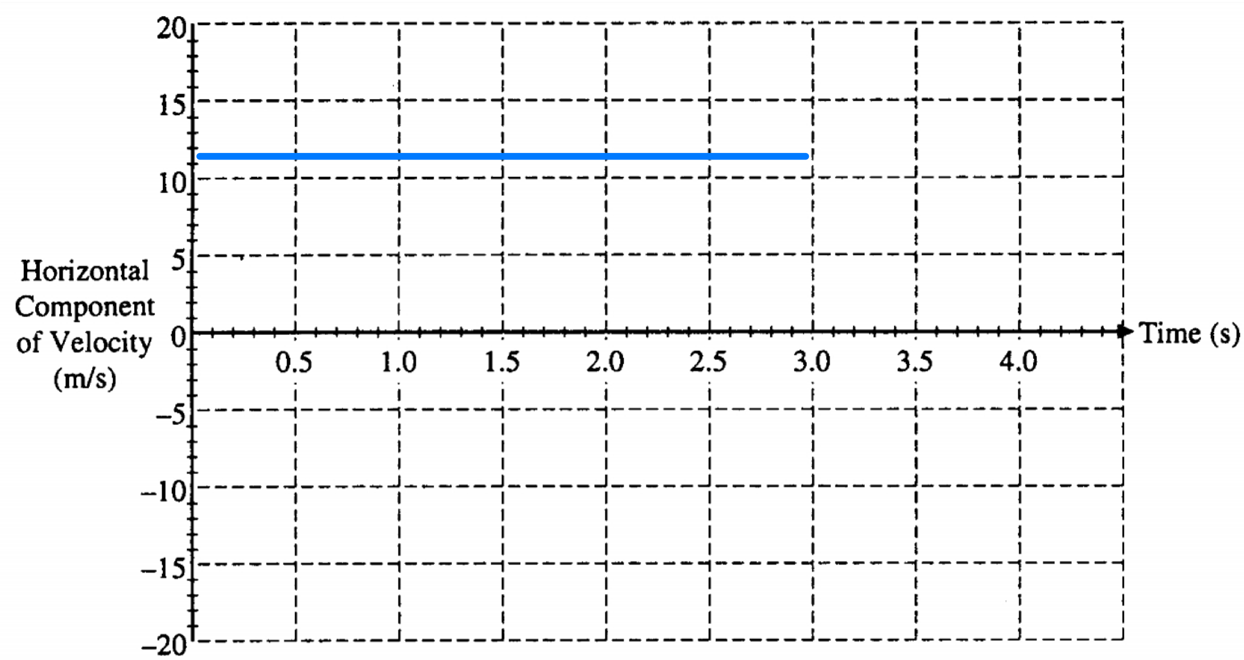
$$y_F = (20.0\ \tfrac{m}{s})(3\ s) - (4.9\ m/s^2)(3\ s)^2$$

$$= 15.9\ m.$$

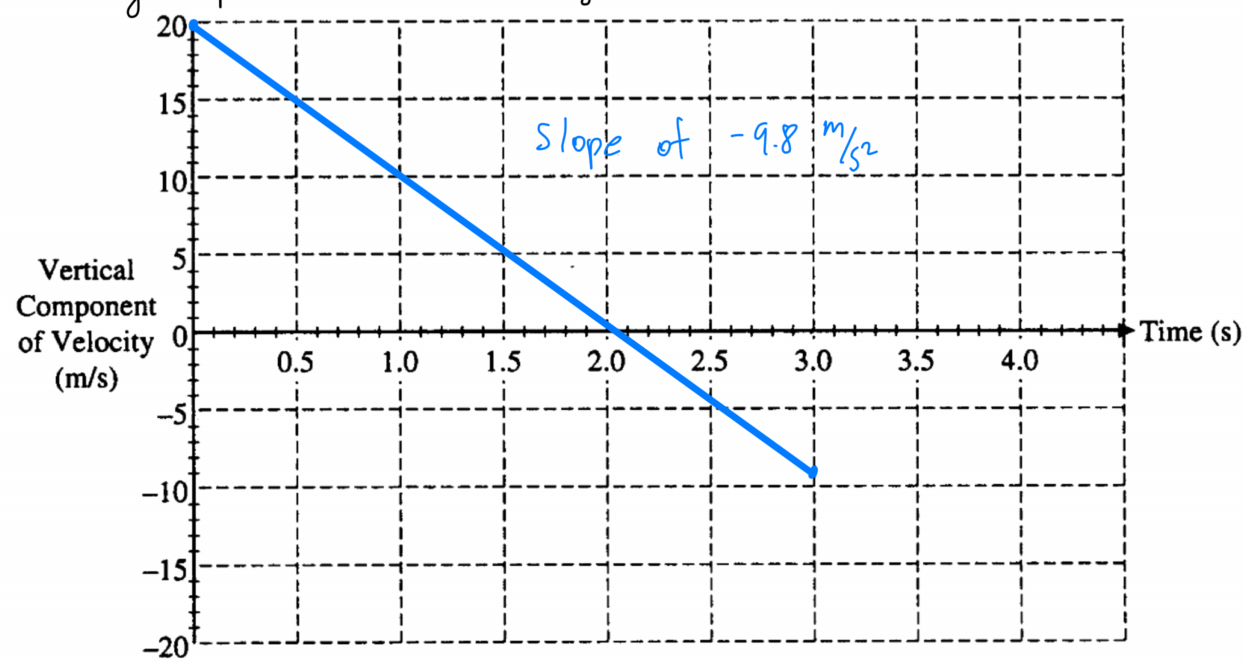
The ball will pass $15.9 - 10 = 5.9$ m above the fence.

4. [10 pts] On the axes below, sketch the horizontal and vertical components of the ball's velocity as functions of time until the ball reaches the plane of the fence. If you are not using this template sketch these graphs on clear paper.

$V_x = V\cos(60°) = 11.55 \; m/s$

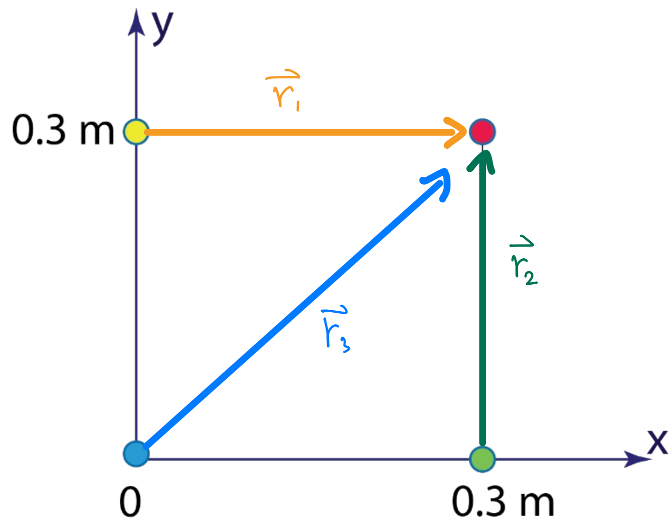


Velocity update formula $V_y = V\sin(\theta) - gt = 20 \; m/s - (9.8 \; \frac{m}{s^2})t$



At $t = 3$ s, $V_y = 20(m/s) - (9.8 \frac{m}{s^2})(3 \; s) = -9.4 \; \frac{m}{s}$

Force of Gravity [30 pts]



Four identical 23.2 kg masses are located at the corners of the square in the *x*-*y* plane, as shown in the figure.

$$G = 6.674 \times 10^{-11} Nm^2/kg^2$$

1. [5 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the upper left corner (in yellow). Your answer must be a vector with appropriate units.

$$\vec{F}_{yellow} = -G \frac{m_1 m_2}{r_1^2} \hat{r}_1 .$$

$m_1 = m_2 = 23.2\ kg$, $r_1 = 0.3\,m$,

$$\hat{r}_1 = \frac{\langle 0.3\ m, 0 \rangle}{0.3\ m} = \langle 1, 0 \rangle$$

$$\Rightarrow \vec{F}_{yellow} = -\left(6.674 \times 10^{-11} \frac{Nm^2}{kg^2}\right) \frac{(23.2\ kg)^2}{(0.3\ m)^2} \langle 1, 0 \rangle = \langle -3.99 \times 10^{-7} N, 0 \rangle$$

2. [5 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the lower right corner (in green). Your answer must be a vector with appropriate units.

$$\vec{F}_{green} = -G \frac{m_1 m_2}{r_2^2} \hat{r}_2 .$$

$m_1 = m_2 = 23.2\ kg$, $r_2 = 0.3\,m$,

$$\hat{r}_2 = \frac{\langle 0, 0.3\,m \rangle}{0.3\ m} = \langle 0, 1 \rangle$$

$$\Rightarrow \vec{F}_{green} = -\left(6.674 \times 10^{-11} \frac{Nm^2}{kg^2}\right) \frac{(23.2\ kg)^2}{(0.3\ m)^2} \langle 0, 1 \rangle = \langle 0, -3.99 \times 10^{-7} N \rangle$$

3. [15 pts] Find the force on the mass at the upper right corner (in red) due to the mass at the lower left corner (in blue). Your answer must be a vector with appropriate units.

$$\vec{F}_{blue} = -G \frac{m_1 m_2}{r_3^2} \hat{r}_3 . \qquad m_1 = m_2 = 23.2 \text{ kg},$$

$$r_3 = \sqrt{(0.3 \text{ m})^2 + (0.3 \text{ m})^2} = 0.3\sqrt{2} \text{ m}$$

$$\hat{r}_2 = \frac{\langle 0.3 \text{ m}, 0.3 \text{ m} \rangle}{0.3\sqrt{2} \text{ m}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{F}_{blue} = -\left(6.674 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}\right) \frac{(23.2 \text{ kg})^2}{(0.3\sqrt{2} \text{ m})^2} \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \langle -1.41 \times 10^{-7} \text{ N}, -1.41 \times 10^{-7} \text{ N} \rangle$$

4. [5 pts] Find the net force on the mass at the upper right corner (in red) caused by the other three masses. Your answer must be a vector with appropriate units.

Add all the answers from questions 1 to 3:

$$\vec{F}_{red} = \vec{F}_{yellow} + \vec{F}_{green} + \vec{F}_{blue}$$

$$= \langle -3.99 \times 10^{-7} \text{ N}, 0 \rangle + \langle 0, -3.99 \times 10^{-7} \text{ N} \rangle$$

$$+ \langle -1.41 \times 10^{-7} \text{ N}, -1.41 \times 10^{-7} \text{ N} \rangle$$

$$= \langle -5.4 \times 10^{-7} \text{ N}, -5.4 \times 10^{-7} \text{ N} \rangle = (5.4 \times 10^{-7} \text{ N}) \langle -1, -1 \rangle$$