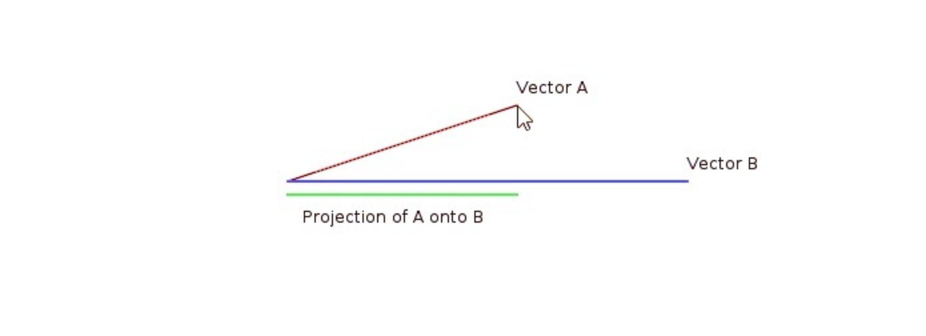

PHYS 2211 K

Week 7, Lecture 2

2022/02/24

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6 clicker questions today

On today's class…

1. Work and the Dot Product
2. Solving problems using the Energy Principle

CLICKER 1: Favorite Eeveelution

A. Vaporeon

B. Jolteon

C. Flareon

About the lab peer reviews...

- Should the peer graders be anonymous or not?

- 210 responses
  - Not anonymous: 57%
  - Anonymous: 43%

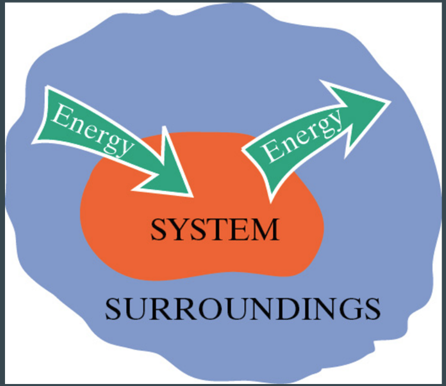
- Problem: the comments mentioned harassment, doxing, others being rude, etc, outside of the class (e.g., groupme) to the students who didn't give perfect 100s in peer reviews. THIS IS COMPLETELY UNACCEPTABLE!

- Solution: ??????????????????????

The Energy Principle

- This is the second fundamental principle that we'll learn about in this class (the first was Newton's 2^{nd} Law, the Momentum Principle)
- Also known as the First Law of Thermodynamics

$$\Delta E_{\text{sys}} = W_{\text{surr}} + Q$$



change in total energy of the system

work done on the system by the surroundings (macroscopic)

energy exchanged between system and surroundings due to a difference in temperature (microscopic work)

Energy is neither created nor destroyed, but it can be transformed or transferred.
The total energy in the universe (all systems plus all surroundings) is constant = conserved!

Energies we've seen so far

- Total energy for a (relativistic) particle

$$E_{\text{total}} = \gamma m c^2$$

- Rest mass energy $E_{\text{rest}} = mc^2$

- Kinetic energy $K = \frac{1}{2}mv^2$

Work

- The transfer of energy between system and surroundings due to the application of a force
- Only the component of the force that is parallel to the displacement of the system contributes to the work done by the surroundings
- When the force is constant,

$$W = \vec{F} \cdot \Delta\vec{r}$$

Work is a scalar quantity
(not a vector!)

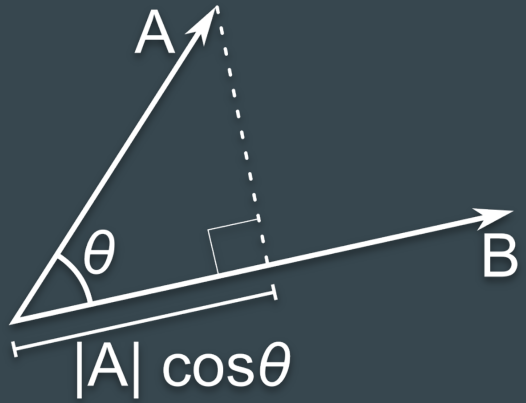
- When the force is not constant,

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

Multiplying vectors: the dot product

- The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$



- If you have the vectors in component form, such that $\vec{A}$ = <A_x, A_y, A_z> and $\vec{B}$ = <B_x, B_y, B_z>, then:

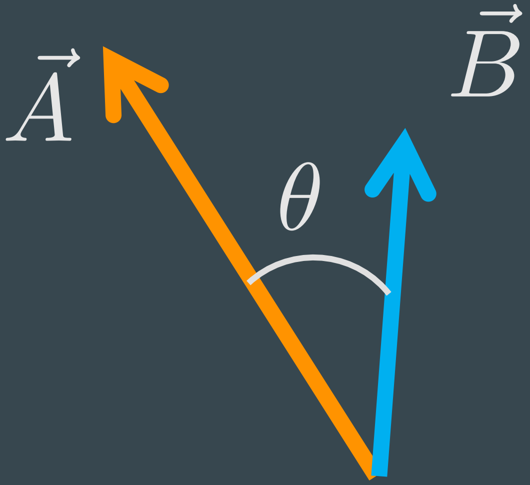
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Multiplying vectors: the dot product

If $\vec{A}$ = <1,2,3> and $\vec{B}$ = <6,5,4>, what is the dot product of $\vec{A}$ and $\vec{B}$?

Multiplying vectors: the dot product

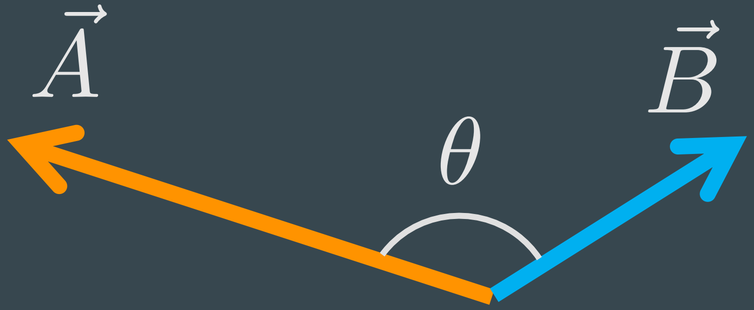
If $|\vec{A}| = 35$ and $|\vec{B}| = 12$, what is the dot product of $\vec{A}$ and $\vec{B}$?



$$\theta = 25^\circ$$

Multiplying vectors: the dot product

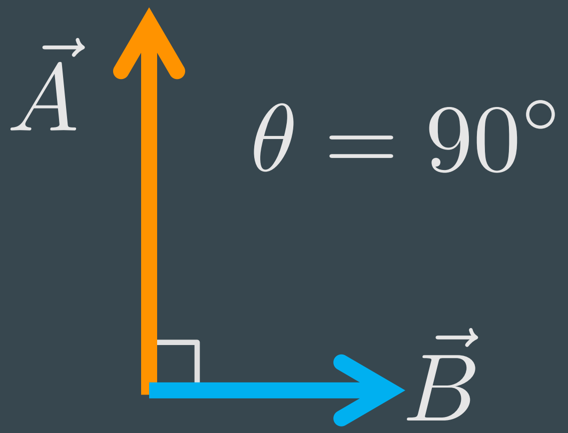
If $|\vec{A}|$ = 35 and $|\vec{B}|$ = 12, what is the dot product of $\vec{A}$ and $\vec{B}$?



$$\theta = 130^\circ$$

Multiplying vectors: the dot product

If $|\vec{A}|$ = 35 and $|\vec{B}|$ = 12, what is the dot product of $\vec{A}$ and $\vec{B}$?



Properties of the dot product

- The dot product takes as input two vectors and outputs a scalar
- The dot product commutes $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- The dot product is distributive

$$\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$$

- The dot product defines orthogonality

$$\vec{A} \perp \vec{B} \text{ if and only if } \vec{A} \cdot \vec{B} = 0$$

CLICKER 2: An object moves from position $\vec{r}_1 =< 2, 0, -2 > \mathrm{m}$ to position $\vec{r}_2 =< 0, -1, 4 > \mathrm{m}$ while being acted upon by a force $\vec{F} =< 10, 3, -2 > \mathrm{N}$. How much work did the force do?

A. W = 35 J

B. W = -35 J

C. W = 20 J

D. W = -20 J

Work and Energy

- When there's no temperature difference between system and surroundings, we can reduce the energy principle to:

$$\Delta E_{\text{sys}} = W_{\text{surr}}$$

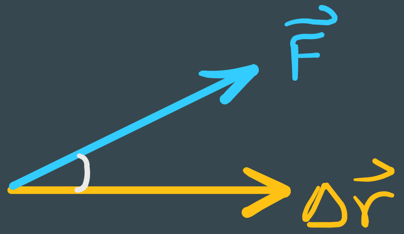
- If the system only has kinetic energy, then that means:

$$\Delta K = W$$

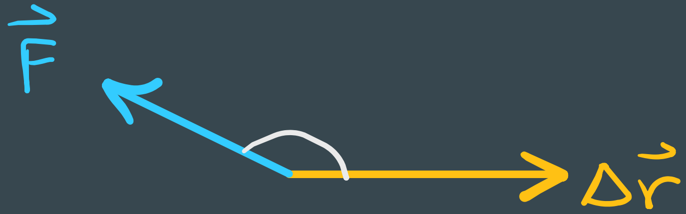
$$\Delta K = \vec{F} \cdot \Delta \vec{r}$$

Work can be positive, negative, or zero

- Positive work
  force parallel to displacement
  (increases the system's energy)



- Negative work
  force antiparallel to displacement
  (decreases the system's energy)



- Zero work
  force perpendicular to displacement
  (system's energy remains unchanged)

CLICKER 3: A box of mass **m = 3 kg** is pushed off the roof of a building from an initial height of **h = 10 m** above the ground. The initial velocity of the box is $\vec{v}_i$ = <5, 0, 0> m/s. Some time later the box hits the ground, **d = 7.14 m** away from the building. **How much work did gravity do** on the box as it fell?

A. W = 362 J

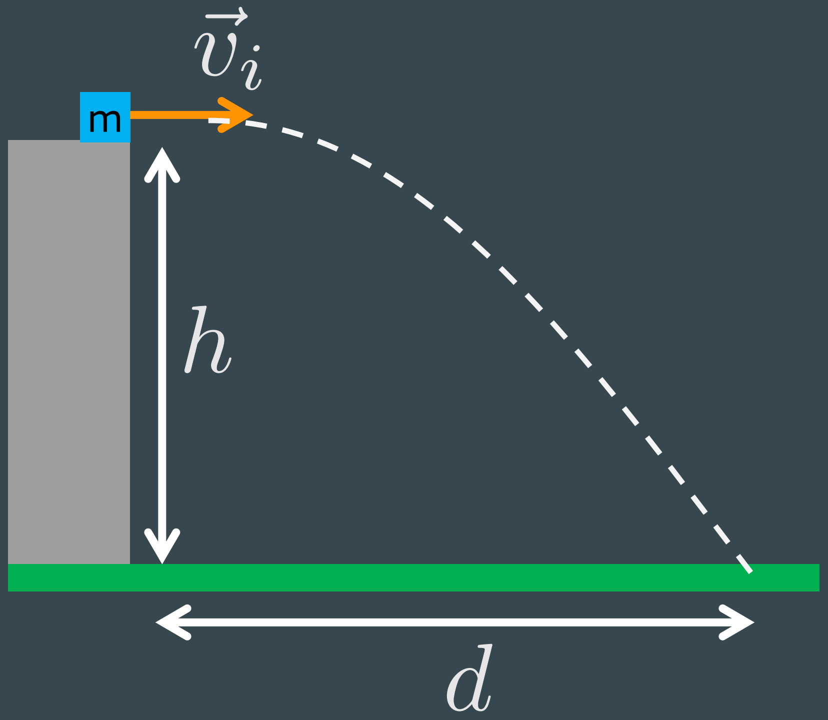
B. W = -362 J

C. W = 294 J

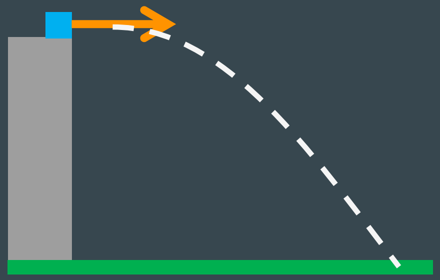
D. W = -294 J

E. W = 210 J

F. W = -210 J



Solution: A box of mass m = 3 kg is pushed off the roof of a building from an initial height of h = 10 m above the ground. The initial velocity of the box is $\vec{v}_i$ = <5, 0, 0> m/s. Some time later the box hits the ground, d = 7.14 m away from the building. How much work did gravity do on the box as it fell?



Applying the Energy Principle

- Clearly identify what is in the system and what is in the surroundings
- Identify the initial state and the final state of the system (position, speed…)
- Determine the types of energies that the system has, and what are the initial and final energies for the system
- Identify the forces that the surroundings are exerting on the system, and the displacements over which those forces are acting
- Calculate the total work done by the surroundings on the system, making sure to not double-count (important when dealing with potential energy)
- Apply the energy principle, $\Delta E = W + Q$, and solve for the unknowns

CLICKER 4: Out in space, an astronaut pushed an 8 kg box that was initially floating at rest at location <5,0,10> m to location <1,0,14> m, by applying a constant net force <1,0,5> N. What is the speed of the box when it gets to the final position?

A. $v_f = 2.4$ m/s

B. $v_f = -2.4$ m/s

C. $v_f = 0$ m/s

D. $v_f = -2$ m/s

E. $v_f = 2$ m/s

F. v_f can't be calculated

Solution: Out in space, an astronaut pushed an 8 kg box that was initially floating at rest at location <5,0,10> m to location <1,0,14> m, by applying a constant net force <1,0,5> N. What is the speed of the box when it gets to the final position?

1. System:

2. Surroundings:

3. Initial state:

4. Final state:

5. Work:

6. Energy Principle:

CLICKER 5: Assume the Earth (mass m) moves around the Sun (mass M) at constant speed v in a circular orbit of radius R. How much work does the Sun do on the Earth after one half orbit?

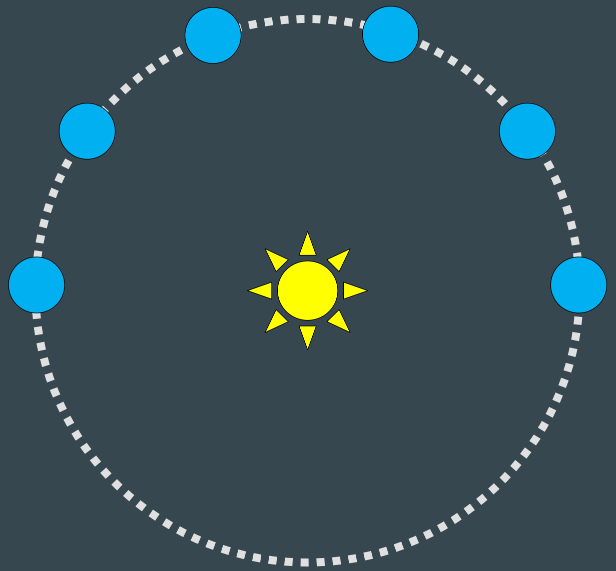
A. $\left(\frac{GMm}{R^2}\right)(2\pi R)$

B. $-\left(\frac{GMm}{R^2}\right)(2\pi R)$

C. $-\left(\frac{GMm}{R^2}\right)(\pi R)$

D. $\left(\frac{GMm}{R^2}\right)(\pi R)$

E. 0



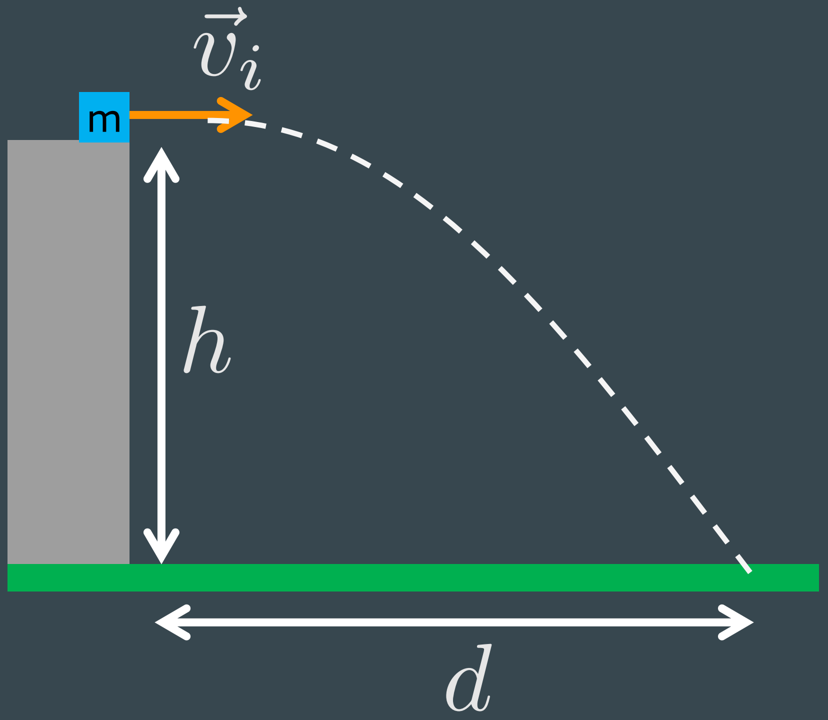
CLICKER 6: A box of mass m = 3 kg is pushed off the roof of a building from an initial height of h = 10 m above the ground. The initial velocity of the box is $\vec{v}_i$ = <5, 0, 0> m/s. Some time later the box hits the ground, d = 7.14 m away from the building. How fast was the box moving when it hit the ground?

A. v_f = 14 m/s

B. v_f = 14.2 m/s

C. v_f = 14.9 m/s

D. v_f = 221 m/s



Solution: A box of mass m = 3 kg is pushed off the roof of a building from an initial height of h = 10 m above the ground. The initial velocity of the box is $\vec{v}_i$ = <5, 0, 0> m/s. Some time later the box hits the ground, d = 7.14 m away from the building. How fast was the box moving when it hit the ground?

A box of mass m = 3 kg is pushed off the roof of a building from an initial height of h = 10 m above the ground. The initial velocity of the box is $\vec{v}_i = <5, 0, 0>$ m/s. Some time later the box hits the ground, d = 7.14 m away from the building. How fast was the box moving when it hit the ground?

How would we solve this same problem using Newton’s 2nd Law?

Comparison: Using the Energy Principle vs Using Newton's 2nd Law

A box of mass m = 3 kg is pushed off the roof of a building from an initial height of h = 10 m above the ground. The initial velocity of the box is $\vec{v}_i$ = <5, 0, 0> m/s. Some time later the box hits the ground, d = 7.14 m away from the building. How fast was the box moving when it hit the ground?

Using energy

Displacement $\Delta\vec{r} = \vec{r}_f = \vec{r}_i$

Work $W = \vec{F}_g \cdot \Delta\vec{r}$

Apply energy principle, solve

$$\Delta K = W$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = W$$

$$v_f = \sqrt{\frac{2W}{m} + v_i^2}$$

Using forces

Kinematics to find Δt

$$y_f = y_i + v_{iy}\Delta t - \frac{1}{2}g(\Delta t)^2$$

$$0 = h - \frac{1}{2}g(\Delta t)^2$$

$$h = \frac{1}{2}g(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

Newton's 2nd to find v_{fy}

$$v_{fy} = v_{iy} + \frac{F_{\text{net,y}}}{m}\Delta t$$

$$v_{fy} = -g\Delta t$$

$$v_{fy} = -g\sqrt{\frac{2h}{g}}$$

Find magnitude of v_f vector

$$|\vec{v}_f| = \sqrt{\left(-g\frac{2h}{g}\right)^2 + (v_{ix})^2}$$

In general, use the Energy Principle when there's no information about time and directions don't matter, and use Newton's 2nd Law when you have time and directions matter

Work done by non-constant forces

- When the force is not constant, we need to integrate to find work

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

- Example of a non-constant force: springs!

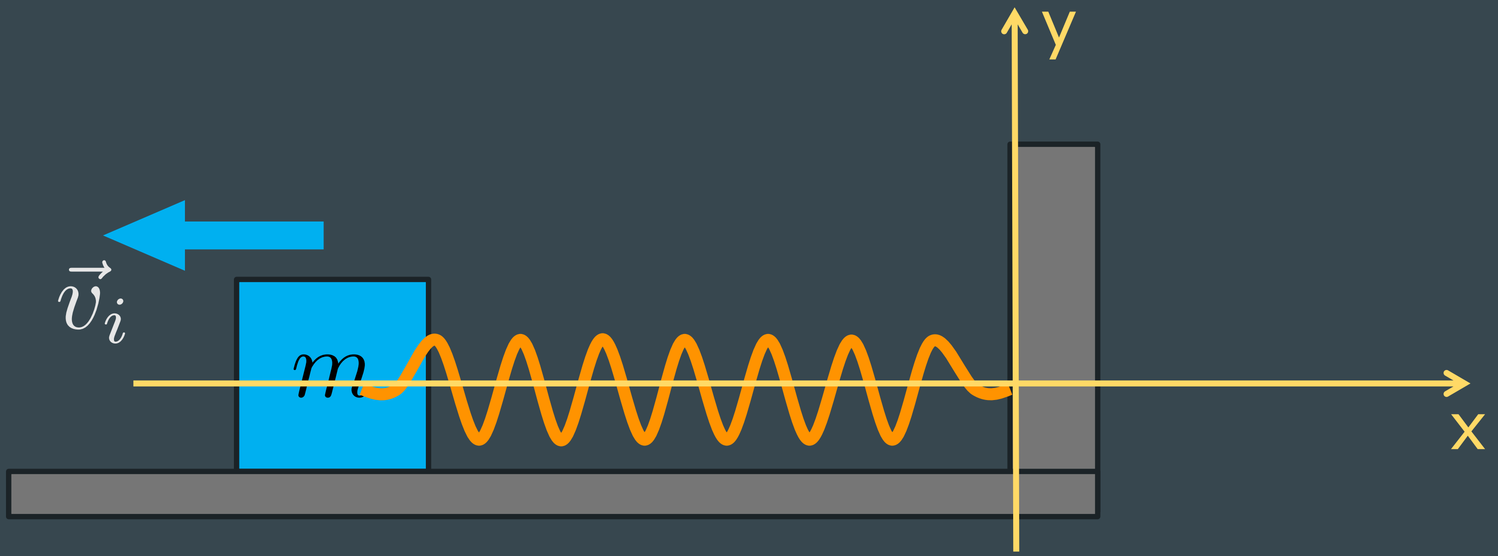
$$\vec{F}_s = -k(L - L_0)\hat{L} = -ks\hat{L}$$

where we have replaced $(L-L_0)$ with s

Work done by a one-dimensional spring

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} =$$

Example: A horizontal spring with stiffness **$k = 15$ N/m** and relaxed length **$L_0 = 4$ m** is fixed to a wall and attached to a block of mass **$m = 7$ kg** on the other end. Right now, the spring is compressed to a length **$L = 2.8$ m** and the block moves to the left with an initial speed of **2 m/s**. How fast will the block move **when the spring is relaxed**?



Solution: A horizontal spring with stiffness $k = 15$ N/m and relaxed length $L_0 = 4$ m is fixed to a wall and attached to a block of mass $m = 7$ kg on the other end. Right now, the spring is compressed to a length $L = 2.8$ m and the block moves to the left with an initial speed of 2 m/s. How fast will the block move when the spring is relaxed?