Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

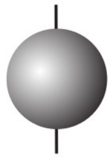
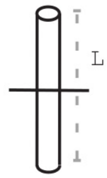
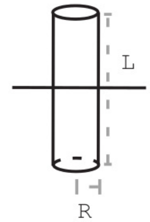
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/F/HP) - Fall 2024 - Test 1

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\frac{a \cdot b}{c \cdot d} = \frac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

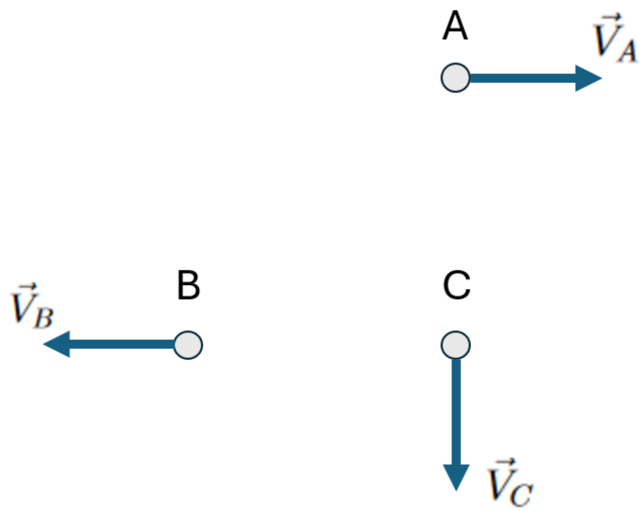
"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

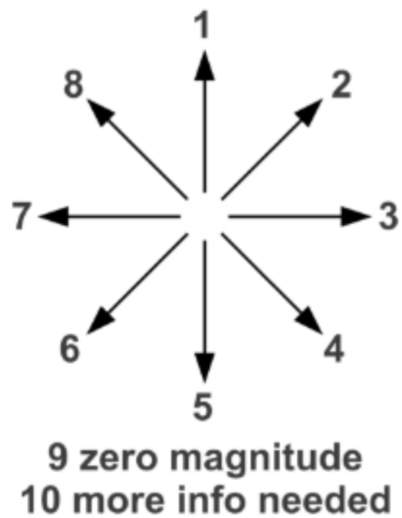
____

Sign your name on the line above

Problem 1 - Vectors [25 points]

An object is initially at location A, later at location B and then, still later, at location C. The object's velocity at each location is represented by arrows as shown in the figure.





Using the numbered direction arrows shown, indicate (by number) which direction arrow best represents the direction of the quantities listed below. If the quantity has zero magnitude or cannot be determined, indicate this with a 9 or 10, respectively.

1.1 [2 pts] ___10___ The position vector for location A.

1.2 [4 pts] ___6___ The net force between location A and location C.

1.3 [3 pts] ___7___ The change in momentum between location A and location B.

1.4 [3 pts] ___5___ The change in position (the displacement) between location A and location C.

1.5 [3 pts] ___4___ The change in momentum between location B and location C.

1.6 [2 pts] ___10___ The position vector for location B.

1.7 [3 pts] ___3___ The change in position (the displacement) between location B and location C.

1.8 [3 pts] ___6___ The average velocity between location A and location B.

1.9 [2 pts] ___10___ The position vector for location C.

Problem 2 - Spaceship [25 points]

A spaceship of mass $m = 1000$ kg is moving at constant velocity $\vec{v}_i = \langle 0, 50, 0\rangle$ km/s, far away from any other objects. At $t = 0$ it passes through location $\langle 0, 0, 0\rangle$ m and the main engine turns on, exerting a constant force $\vec{F}_1 = \langle 2 \times 10^6, 0, 0\rangle$ N on the spaceship. At $t = 10$ s, the main engine turns off and, simultaneously, the side thrusters turn on, exerting a constant force $\vec{F}_2 = \langle 0, 2 \times 10^4, 2 \times 10^4\rangle$ N on the spaceship. What is the position of the spaceship at $t = 30$ s?

Note: the next page is blank to allow for extra space for your work in this problem.

Constant Force $\Rightarrow$ Position update: $\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{\vec{F}_{net}}{2m} \Delta t^2$

Velocity update: $\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m} \Delta t$

Main engine stage ($\Delta t_1 = 10$ s):

$$v_{f,x} = \cancelto{0}{v_{i,x}} + \frac{F_{1,x}\Delta t_1}{m} = 2 \times 10^4 \ \frac{m}{s}$$

$$v_{f,y} = v_{i,y} = 50{,}000 \ \frac{m}{s}$$

$$v_{i,z} = v_{f,z} = 0$$

$$r_{f,x} = \cancelto{0}{r_{i,x}} + \cancelto{0}{v_{i,x}}\Delta t_1 + \frac{F_{1,x}}{2m}\Delta t_1^2$$

$$= \frac{F_{1,x}\Delta t_1^2}{2m} = 10^5 \ m$$

$$r_{f,y} = \cancelto{0}{r_{i,y}} + v_{i,y}\Delta t_1 + \frac{\cancelto{0}{F_{1,y}}}{2m}\Delta t_1^2$$

$$= 5 \times 10^5 \ m$$

$$r_{i,z} = r_{f,z} = 0$$

$$\Rightarrow \vec{r}_f = \langle 100 \times 10^3, 500 \times 10^3, 0\rangle \ m$$

$$\vec{v}_f = \langle 20 \times 10^3, 50 \times 10^3, 0\rangle \ m/s$$

$\boxed{\text{Next page}}$

(you can continue your work for Problem 1 on this blank page)

Side thruster stage ($\Delta t_2 = 20$ s):

$$r_{f,z} = \cancel{r_{i,z}}^{\,0} + \cancel{v_{i,z}}^{\,0} \Delta t_2 + \frac{F_{2,z}}{2m} \Delta t_2^2$$

$$= \frac{F_{2,z} \Delta t_2^2}{2m} = 4000 \text{ m}$$

$$r_{f,y} = r_{i,y} + v_{i,y} \Delta t_2 + \frac{F_{2,y}}{2m} \Delta t_2^2$$

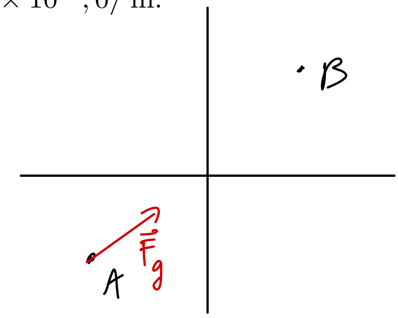
$$= 1.504 \times 10^6 \text{ m}$$

$$r_{f,x} = r_{i,x} + v_{i,x} \Delta t_2 = 0.5 \times 10^6 \text{ m}$$

$$\vec{r}_f = \langle 500 \times 10^3, 1504 \times 10^3, 4 \times 10^3 \rangle \text{ m}$$

Problem 3 - Binary Star [25 points]

A hypothetical binary star system consists of two stars, A and B, each with the same mass (6.0×10^{29} kg). Star A is located at $\langle -2.0 \times 10^{10}, -2.0 \times 10^{10}, 0 \rangle$ m, while Star B is located at $\langle 2.0 \times 10^{10}, 2.0 \times 10^{10}, 0 \rangle$ m.



3.1 [10 pts] Determine the force acting on Star A due to Star B.

$$\vec{r} = \vec{r}_{system} - \vec{r}_{source}$$

$$= \langle -4 \times 10^{10}, -4 \times 10^{10}, 0 \rangle \text{ m}$$

$$\text{Direction} = -\hat{r} = -\frac{\vec{r}}{|\vec{r}|} = -\frac{\langle -4 \times 10^{10}, -4 \times 10^{10}, 0 \rangle}{\sqrt{(-4 \times 10^{10})^2 + (-4 \times 10^{10})^2}}$$

$$\frac{\langle 4 \times 10^{10}, 4 \times 10^{10}, 0 \rangle}{\sqrt{2(4 \times 10^{10})^2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

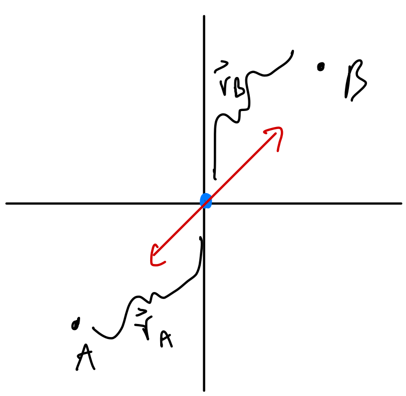
$$\text{Magnitude} = \frac{G m^2}{r^2} = 7.54 \times 10^{27} \text{ N}$$

$$\vec{F}_g = \frac{G m^2}{r^2}(-\hat{r}) = \boxed{\langle 5.33 \times 10^{27}, 5.33 \times 10^{27}, 0 \rangle \text{ N}}$$

3.2 [5 pts] A small space probe with a mass of 10 kg is located at $\langle 0, 0, 0 \rangle$ m. Find the net force acting on the probe.

$$r_A = r_B, \quad m_{probe} m_A = m_{probe} m_B$$

$$\Rightarrow |\vec{F}_{A\text{-on-probe}}| = |\vec{F}_{B\text{-on-probe}}|$$



$$\text{And} \quad \hat{F}_{A\text{-on-probe}} = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$\hat{F}_{B\text{-on-probe}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$\vec{F}_{net} = \vec{F}_{A\text{-on-probe}} + \vec{F}_{B\text{-on-probe}} = 0$$

Alternatively, if student explains that probe is in midpoint and the forces cancel eachother, that's fine.

3.3 [10 pts] Some time later the probe is located at $\langle 0, 0, 2.0 \times 10^9 \rangle$ m. Find the net force acting on the probe.

$$\vec{r}_{A\text{-to-probe}} = \langle 2 \times 10^{10}, \; 2 \times 10^{10}, \; 2 \times 10^{9} \rangle \text{ m}$$

$$\vec{r}_{B\text{-to-probe}} : \langle -2 \times 10^{10}, \; -2 \times 10^{10}, \; 2 \times 10^{9} \rangle \text{ m}$$

$$\left| \vec{F}_{A\text{-on-probe}} \right| = \frac{G \, m_A \, m_{probe}}{\left| \vec{r}_{A\text{-to-probe}} \right|^2} = \frac{G \, m_A \, m_{probe}}{\left| \vec{r}_{B\text{-to-probe}} \right|^2} = \left| \vec{F}_{B\text{-on-probe}} \right|$$

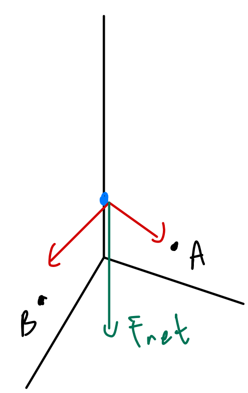
$$\Rightarrow \quad \vec{F}_{A\text{-on-probe}} + \vec{F}_{B\text{-on-probe}}$$

$$= \frac{-G \, m_A \, m_{probe}}{\left| \vec{r}_{A\text{-to-probe}} \right|^2} \frac{\langle 2 \times 10^{10}, \; 2 \times 10^{10}, \; 2 \times 10^{9} \rangle}{\left| \vec{r}_{A\text{-to-probe}} \right|}$$

$$+ \quad \frac{-G \, m_A \, m_{probe}}{\left| r_{A\text{-to-probe}} \right|^2} \frac{\langle -2 \times 10^{10}, \; -2 \times 10^{10}, \; 2 \times 10^{9} \rangle}{\left| \vec{r}_{A\text{-to-probe}} \right|}$$

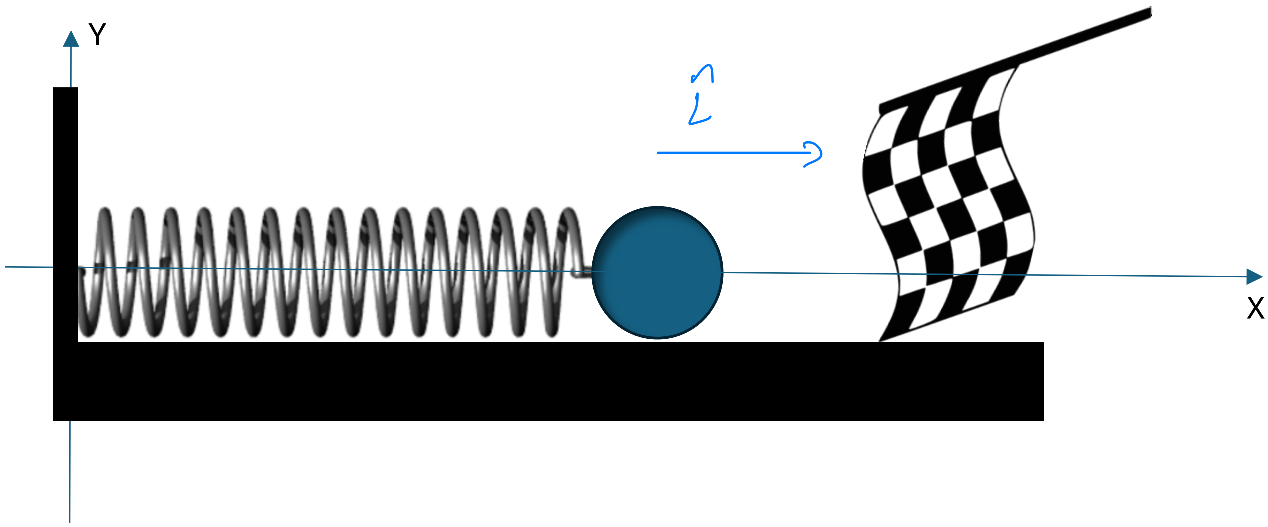
$$= 2 \frac{G \, m_A \, m_{probe}}{\left| r_{A\text{-to-probe}} \right|^3} (2 \times 10^9) \langle 0, 0, -1 \rangle$$

$$= \langle 0, 0, -0.07 \rangle \text{ N}$$



Problem 4 - Spring [25 points]

Little Ryan is playing with a small ball (mass m) that is attached to a spring. The spring sits on a frictionless table, has relaxed length L_0, stiffness k, and one end fixed at the origin, $\langle 0,0,0\rangle$. Ryan compressed and released the spring with the intention of hitting a flag he kept in front of the spring. Ryan's big sister, a college student who has taken PHYS 2211, was helping Ryan with his project. At $t=0$, the spring is compressed to a length L, and the ball's velocity is $\vec{v}_i = \langle v,0,0\rangle$. The flag is located at $\langle D,0,0\rangle$. Note that $D > L_0$, and v is positive.



4.1 [5 pts] Determine the **magnitude** and **direction** of the spring force at $t=0$.

$\Rightarrow \quad \vec{F}_s = -k(L - L_0)\,\hat{x}$

Direction: spring is compressed, so direction is $\hat{x}$ or $\langle 1,0,0\rangle$.

Magnitude $= k|L - L_0| = k(L_0 - L)$

4.2 [20 pts] By making an iterative prediction using one time step, Ryan's sister realized that the ball hits the flag after a short time Δt. Assuming that the ball is a point mass and the flag is hanging vertically, find an expression for D. Your answer must be expressed in terms of the variables given in the problem: L, L_0, k, v, m, and Δt.

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_s}{m} \Delta t$$

$$= \left\langle v + \frac{k(L_0 - L)}{m} \Delta t, 0, 0 \right\rangle$$

$$\vec{r}_f = \vec{r}_i + \underbrace{v_{avg}}_{\approx v_f \text{ for non-constant force.}} \Delta t$$

All motion is in the x-direction, thus:

$$D = L + \left(v + \frac{k(L_0 - L)}{m} \Delta t \right) \Delta t$$

$$= L + v\Delta t + \frac{k(L_0 - L)}{m} (\Delta t)^2$$

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