PHYS 2211, Summer 2021

Weeks 1 – 3

In this video:

- ✓ vectors & scalars, units
- ✓ position, displacement, velocity, momentum, acceleration, force
- ✓ Newton's 2nd Law (also known as The Momentum Principle)
- ✓ iterative prediction of motion (velocity update, position update, with constant vs non-constant $\vec{F}_{net}$)
- ✓ gravity near the surface of Earth (weight, falling things, projectiles)
- ✓ spring force
- ✓ universal gravitation
- ✓ electric force; reciprocity (Newton's 3rd)

Vectors, Scalars, Units

scalar ⇒ just a number

mass, temperature, speed

magnitude of velocity

vector ⇒ magnitude & direction

position, velocity, force

units: S.I.

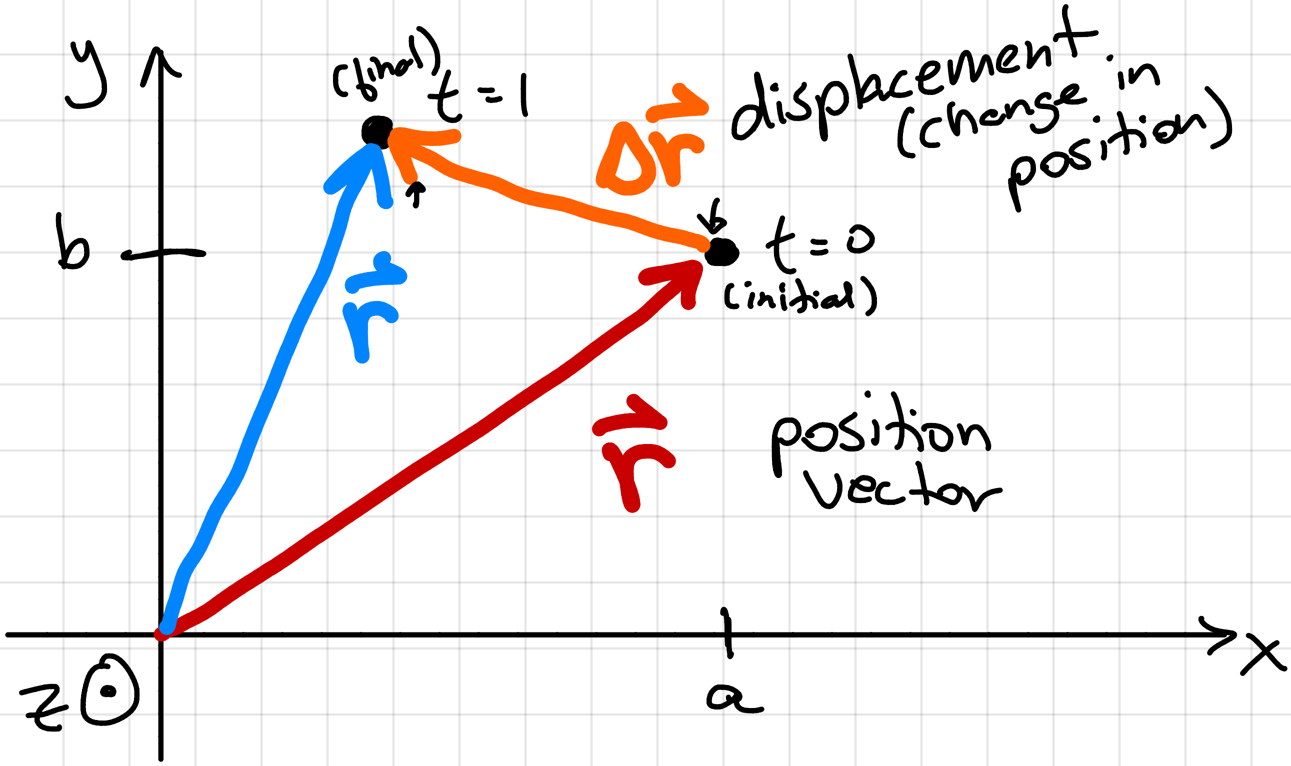
lengths	m
masses	kg
forces	$N = kg\, m/s^2$
time	sec
energy	$\text{Joules} = N m = kg\, m^2/s^2$

When solving problems,

✓ number ⇒ need units!

✓ symbolic ⇒ No units needed!

$\vec{r}$, $\Delta\vec{r}$, $\vec{v}$, $\vec{F}$, $\vec{a}$, etc



$$\vec{r} = \langle a, b, c \rangle$$

$$|\vec{r}| = \sqrt{a^2 + b^2 + c^2} \leftarrow \text{distance, magnitude of position}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle a, b, c \rangle}{\sqrt{a^2+b^2+c^2}} \leftarrow \text{direction, unit vector}$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$\vec{r}_f$: arrow head

$\vec{r}_i$: arrow tail

Adding, subtracting vectors

$\vec{r}_1 = \langle a, b, c \rangle$

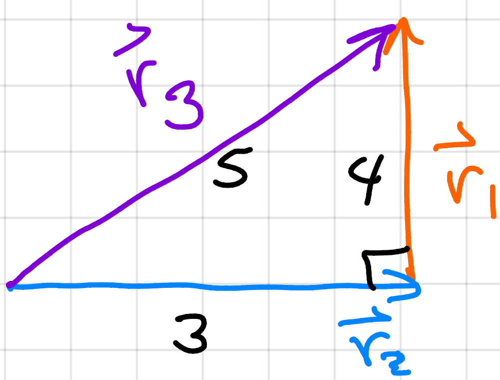
$\vec{r}_2 = \langle d, e, f \rangle$

$$\vec{r}_1 + \vec{r}_2 = \langle a, b, c \rangle + \langle d, e, f \rangle =$$

$$= \langle a+d, \ b+e, \ c+f \rangle$$

Sum of magnitudes $\neq$ magnitude of the sum

☆ $|\vec{r}_1| + |\vec{r}_2| \neq |\vec{r}_1 + \vec{r}_2|$ ☆



$\vec{r}_3 = \vec{r}_1 + \vec{r}_2$

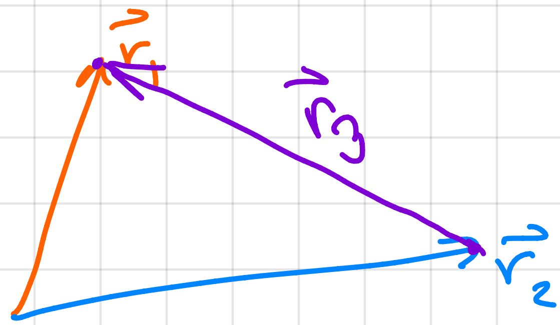
$|\vec{r}_2| + |\vec{r}_1| \neq |\vec{r}_3|$

$$\vec{r}_1 = \langle a, b, c \rangle$$

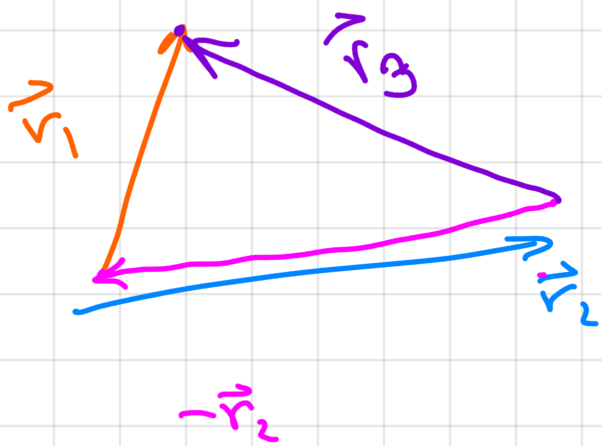
$$\vec{r}_2 = \langle d, e, f \rangle$$

$$\vec{r}_1 - \vec{r}_2 = \langle a, b, c \rangle - \langle d, e, f \rangle =$$

$$= \langle a-d, \quad b-e, \quad c-f \rangle$$



$$\underset{\substack{\text{final} \\ \text{(head)}}}{\vec{r}_3 = \vec{r}_1} - \underset{\substack{\text{initial} \\ \text{(tail)}}}{\vec{r}_2}$$



$$\vec{r}_3 = \vec{r}_1 - \vec{r}_2$$

$$= \vec{r}_1 + (-\vec{r}_2)$$

$\vec{r} \neq \Delta\vec{r}$

Position $\vec{r}$

Displacement $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

Average velocity $\vec{v}_{Avg} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$

$\Delta t \to 0$ $\vec{v} = \frac{d\vec{r}}{dt}$ instantaneous velocity

velocity is the time derivative of position

Momentum $\vec{p} \equiv m\vec{v}$

Change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

Change in momentum $\Delta\vec{p} = m\,\Delta\vec{v}$

$\Delta\vec{v} \neq \vec{v}$

Newton's 2nd Law
the Momentum Principle

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \qquad \Delta t \to 0$$

net force acting on an object

change in time of object's momentum

acceleration

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = m\vec{a}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \Rightarrow \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{F}_{net}\Delta t = \Delta\vec{p}$$

(impulse formula)

$$\vec{p}_f - \vec{p}_i = \vec{F}_{net}\Delta t$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

momentum update

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

velocity update formula

Velocity update

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m} \Delta t$$

Position update

$$\vec{r}_f = \vec{r}_i + \vec{v}_{Avg} \Delta t$$

Iterative Prediction of Motion

$$\left.\begin{array}{l} ① \ \vec{F}_{net} \\ ② \ \vec{v}_f = \vec{v}_i + (\vec{F}_{net}/m) \Delta t \\ ③ \ \vec{r}_f = \vec{r}_i + \underline{\vec{v}_{Avg}} \Delta t \end{array}\right\} \text{over one time step}$$

Canvas
→ modules
→ getting started
→ useful coding stuff

Inside while loop

Fnet = ~~

ball.vel = ball.vel + (Fnet/ball.m) * deltat

ball.pos = ball.pos + ball.vel * deltat

What's the deal with $\vec{v}_{AVG}$?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{AVG}\,\Delta t \quad \leftarrow$$

If $\vec{F}_{net} = \text{Constant}$

$$\vec{v}_{Avg} = \frac{\vec{v}_i + \vec{v}_f}{2} \quad \checkmark$$

If $\vec{F}_{net} \neq \text{constant}$ (depends on position, velocity, time)

$$\vec{v}_{AVG} \approx \vec{v}_f \quad \checkmark$$

Constant forces

Weight $= \langle 0, -mg, 0\rangle$

$\langle F, -F, 0\rangle$

$\langle 0, 0, 3\rangle$ N

Non-constant forces

$\vec{F}_{spring}$

$\vec{F}_{air}$

$\vec{F}_{grav}$

$\vec{F}_{electric}$

Gravity Near Surface of Earth
(weight)

$$\vec{F}_g = \langle 0, -mg, 0 \rangle$$

<u>Constant force</u>

magnitude: mg

direction: down $(-\hat{y})$

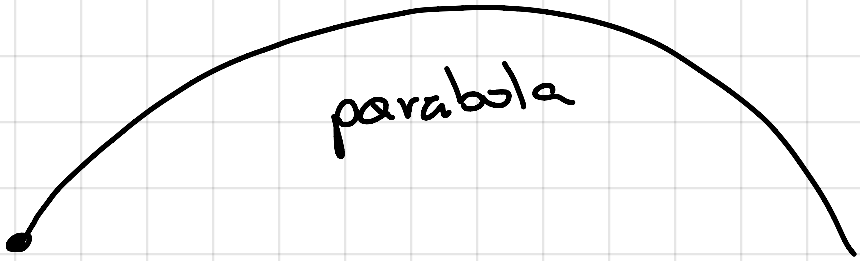
$$g = 9.8 \, m/s^2$$

magnitude of acceleration due to gravity @ surface of Earth

$$\vec{g} = g(-\hat{y})$$

$$= \langle 0, -g, 0 \rangle$$

Projectile Motion



in X direction: $F_{net_x} = 0$

in y direction: $F_{net_y} = mg\,(-\hat{y})$

Kinematic formula $y_f = y_i + v_{iy}\Delta t - \frac{1}{2} g \Delta t^2$

$$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

$$2\vec{v}_{Avg} - \vec{v}_i = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

$$2\vec{v}_{Avg} = 2\vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$$

$$\vec{v}_{Avg} = \vec{v}_i + \frac{1}{2}\frac{\vec{F}_{net}}{m}\Delta t$$

$$\vec{F}_{net} = \langle 0, -mg, 0 \rangle$$

$$\vec{v}_{Avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$2\vec{v}_{Avg} = \vec{v}_i + \vec{v}_f$$

$$\vec{v}_f = 2\vec{v}_{Avg} - \vec{v}_i$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{Avg}\,\Delta t$$

$$\vec{v}_{Avg} = \vec{v}_i + \frac{1}{2}\frac{\vec{F}_{net}}{m}\Delta t$$

$$= \vec{r}_i + \left(\vec{v}_i + \frac{1}{2}\frac{\vec{F}_{net}}{m}\Delta t\right)\Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\,\Delta t + \frac{1}{2}\frac{\vec{F}_{net}}{m}(\Delta t)^2$$

$\underline{X}$: $F_{net} = 0$

$$\boxed{x_f = x_i + v_{ix}\,\Delta t} \quad *$$

y: $F_{net} = mg(-\hat{y}) = -mg$

$$y_f = y_i + v_{iy}\,\Delta t + \frac{1}{2}\frac{(-\cancel{m}g)}{\cancel{m}}(\Delta t)^2$$

$$\boxed{y_f = y_i + v_{iy}\,\Delta t - \frac{1}{2}g(\Delta t)^2} \quad *$$

Spring Force

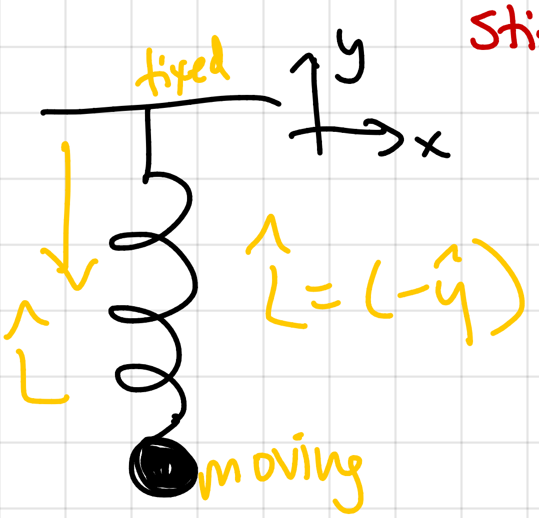
$$\vec{F}_s = -K\,(L - L_0)\,\hat{L}$$

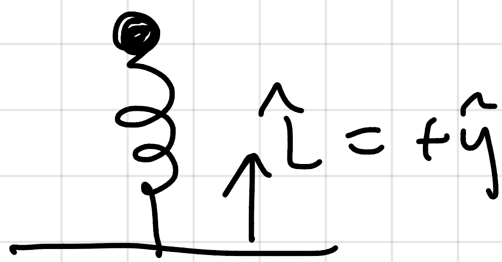
Stiffness

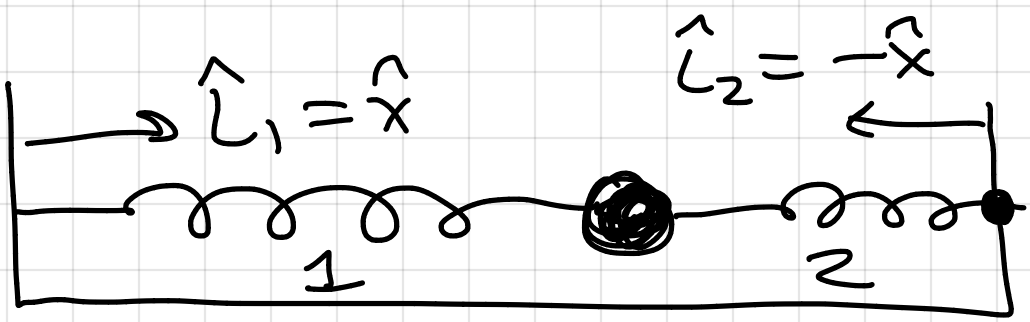
current length

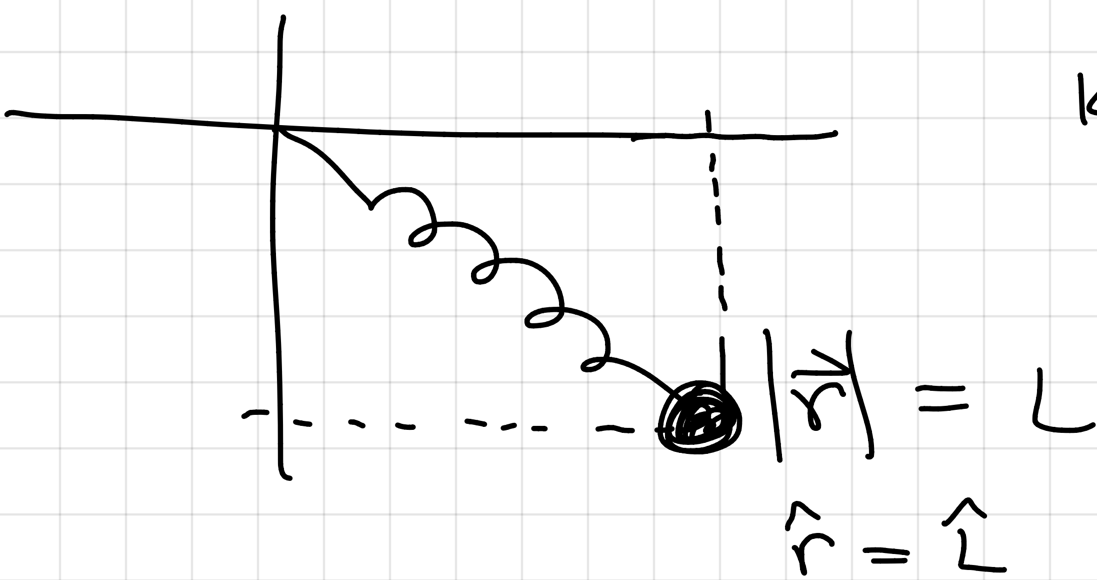
relaxed length

from fixed to moving



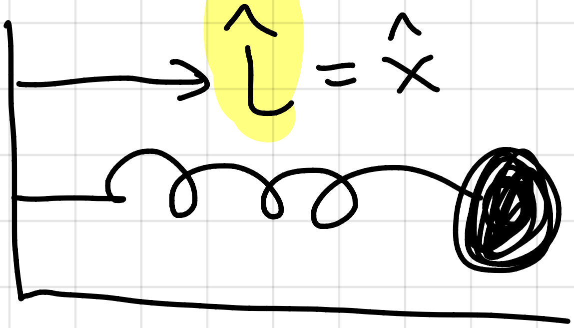






K, L_0 physical characteristics of a spring

$\hat{L} = \hat{x}$



$$\vec{F}_s = -k\underbrace{(L - L_0)}_{s = L - L_0}\hat{L}$$

Stretched

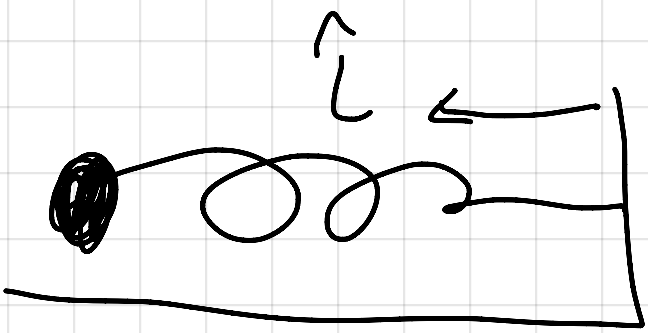
$L > L_0$, $s > 0$

$$\vec{F}_s = (-)(+)(+\hat{x}) \Rightarrow -\hat{x}$$

Compressed

$L < L_0$, $s < 0$

$$\vec{F}_s = (-)(-)(+\hat{x}) \Rightarrow +\hat{x}$$



$\hat{l} = -\hat{x}$

Stretched $\quad L - L_0 > 0$

$$\vec{F}_s = (-)(+)(-\hat{x})$$

$$\Rightarrow +\hat{x}$$

Compressed $\quad L - L_0 < 0$

$$\vec{F}_s = (-)(-)(-\hat{x})$$

$$\Rightarrow -\hat{x}$$

Universal Gravitation

$$\vec{F}_g = \frac{G M_1 m_2}{r^2}\,(-\hat{r})$$

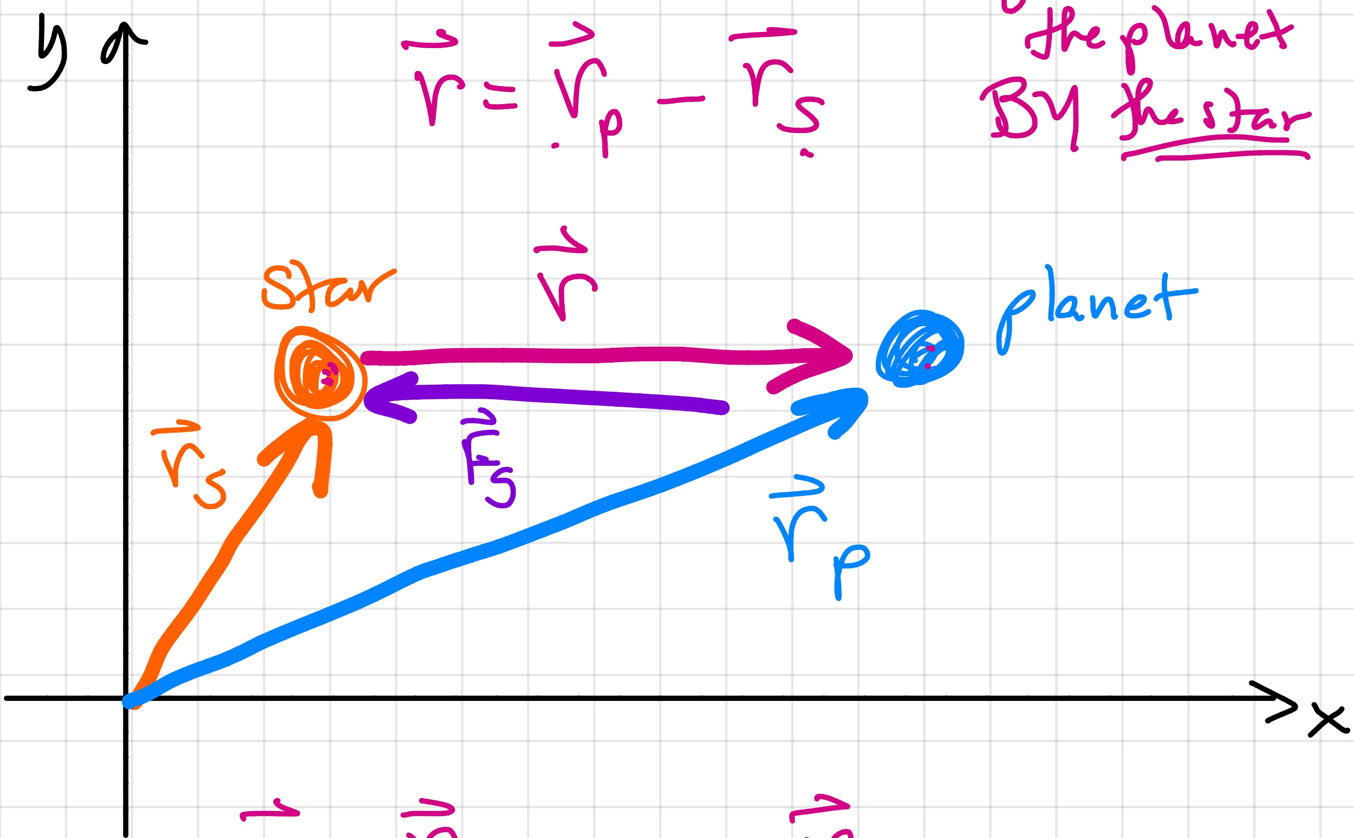
$$G = 6.7 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$$

$m_1, m_2 \Rightarrow$ masses

$r \Rightarrow$ separation

$$\vec{r} = \vec{r}_p - \vec{r}_s$$

force ON the planet BY the star



$$\vec{r} = \underline{\vec{r}_{system}} - \underline{\vec{r}_{\text{"source"}}}$$

$\vec{r}_s = \langle x_s, y_s, z_s \rangle$

$\vec{r}_p = \langle x_p, y_p, z_p \rangle$

force ON planet BY star

star ↑ "source"

planet ↑ system

$\vec{r} = \vec{r}_{system} - \vec{r}_{source} =$

$= \vec{r}_p - \vec{r}_s = \langle x_p - x_s,\ y_p - y_s,\ z_p - z_s \rangle$

$|\vec{r}| = \sqrt{(x_p - x_s)^2 + (y_p - y_s)^2 + (z_p - z_s)^2}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{F}_{\substack{\text{on planet} \\ \text{by the star}}} = \frac{G m_p m_s}{r^2}(-\hat{r})$

$\frac{G M_E \cancel{m}}{R_E^2} = \cancel{m} g$

$\boxed{g = \frac{G M_E}{R_E^2}}$

for any planet

$g_{mars} = \frac{G M_{mars}}{R_{mars}^2}$

$g_{moon} = \frac{G M_{moon}}{R_{moon}^2}$

Electric Force

$$\vec{F}_e = \frac{k\, q_1 q_2}{r^2}\,\hat{r}$$

Coulomb = unit of electric charge (C)

$$k = 9\times10^{9}\ Nm^2/C^2$$

electric charge can be + or −

electrons − , protons +

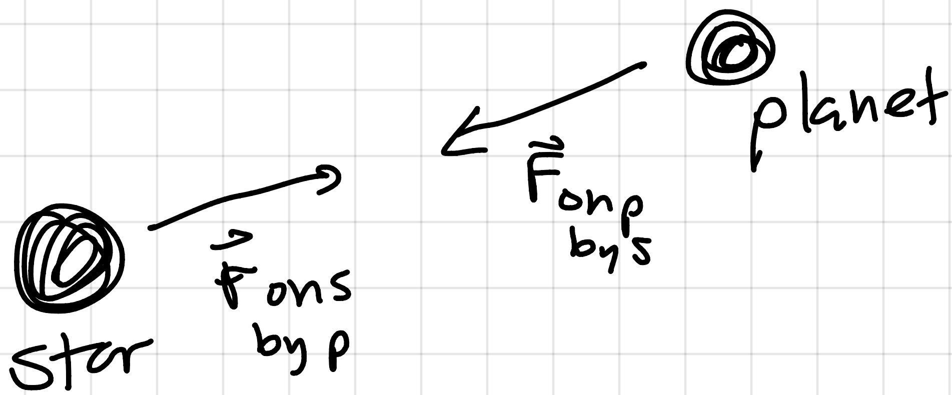
+ − ⇒ opposite charges attract

$$\vec{F}_e = \frac{k(-Q)(+Q)}{r^2}\,\hat{r} = \frac{kQ_1Q_2}{r^2}(-\hat{r})$$

looks exactly like F_{grav}

+ + or − − } like charges repel ⇒ repulsive

Reciprocity (Newton's 3rd)



Force ON the planet BY the star

$$\vec{F}_{\substack{\text{on } p\\ \text{by } s}} = \frac{G m_p m_s}{r^2}\left(-\hat{r}_{\text{from } s \text{ to } p}\right)$$

Force ON the star BY the planet

$$\vec{F}_{\substack{\text{on } s\\ \text{by } p}} = \frac{G m_p m_s}{r^2}\left(-\hat{r}_{\text{from } p \text{ to } s}\right)$$

$$\vec{F}_{\substack{\text{on } p\\ \text{by } s}} = -\vec{F}_{\substack{\text{on } s\\ \text{by } p}}$$

Newton's 3rd law