Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle	The Energy Principle	The Angular Momentum Principle
Definitions of: velocity, momentum, particle energy, kinetic energy, work, angular velocity, angular momentum, torque		

Other useful formulas

$$\gamma \equiv \frac{1}{\sqrt{1-(|\vec{v}|^2/c^2)}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{\text{grav}} = < 0, -mg, 0 >$$

$$\Delta U_{\text{grav}} = mg\Delta y$$

$$\vec{F}_{\text{grav}} = G\frac{m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$

$$U_{\text{grav}} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\vec{F}_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\vec{F}_{\text{spring}} = -k_s(|\vec{L}| - L_0)\hat{L}$$

$$U_{\text{spring}} = \frac{1}{2}k_s s^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \frac{1}{2}\frac{\vec{F}_{\text{net}}}{m}(\Delta t)^2$$

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}$$

$$K_{\text{rel}} = K_{\text{rot}} + K_{\text{vib}}$$

$$K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{\text{trans,A}} + \vec{L}_{\text{rot}}$$

$$\vec{L}_{\text{rot}} = I\vec{\omega}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

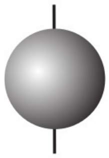
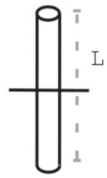
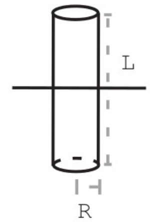
$$\omega = \sqrt{\frac{k_s}{m}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\dots$$

The cross product

$$\vec{A}\times\vec{B} = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x\rangle$$

Moment of inertia for rotation about indicated axis

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Grav accel near Earth's surface	g	9.8 m/s^2
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Planck's constant	h	6.6×10^{-34} J $\cdot$ s
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} J $\cdot$ s
specific heat capacity of water	C	4.2 J/(g $\cdot$ °C)

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 (A/B/C/D/E/F/HP) - Fall 2024 - Test 3

Name: ______________________________ GTID: ______________________________

Instructions

- This quiz/test/exam is closed internet, books, and notes.
  - You are allowed to use the Formula Sheet that is included with the exam.
  - You are allowed to use a calculator as long as it cannot connect to the internet.
  - You cannot have any other electronic devices on or access the internet until time is called.
  - You must work individually and receive no assistance from any person or resource.
- You are not allowed to share or post information, screenshots, files, or any other details of the test anywhere online, not even after the test is over, except for uploading your work to Gradescope for grading.
- Work through all the problems first, then **scan and upload your solutions to Gradescope** (at your seat!) after time is called.
  - You should upload **one single PDF file** to the test assignment on Gradescope.
  - You **must** indicate which page corresponds to each problem or sub-part when you upload your work.
  - Make sure your file is readable. Unreadable files will not be graded and will earn a score of zero.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80% deduction.
  - You must show all your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade.
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams and show what goes into a calculation, not just the final number. For example: $\frac{a \cdot b}{c \cdot d} = \frac{(8 \times 10^{-3})(5 \times 10^{6})}{(2 \times 10^{-5})(4 \times 10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numerical results. Symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the Formula Sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do a portion of a problem, invent a symbol for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

____

Sign your name on the line above

Problem 1 - Cross Products, Torque, Angular Momentum [20 points]

1.1 [2 pt] The cross product of two nonzero vectors that are perpendicular to each other is zero.

(a) True
(b) False

1.2 [2 pt] The cross product of two nonzero vectors that are parallel to each other is zero.

(a) True
(b) False

1.3 [2 pt] If $\vec{A} = \langle 0, 0, 5\rangle$ and $\vec{B} = \langle 5, 0, 0\rangle$, what is the direction of $\vec{A} \times \vec{B}$?

(a) +x
(b) −x
(c) +y
(d) −y
(e) +z
(f) −z
(g) zero magnitude

1.4 [2 pt] If $\vec{A} = \langle 0, 5, 0\rangle$ and $\vec{B} = \langle 0, 0, -5\rangle$, what is the direction of $\vec{A} \times \vec{B}$?

(a) +x
(b) −x
(c) +y
(d) −y
(e) +z
(f) −z
(g) zero magnitude

1.5 [2 pt] If $\vec{A} = \langle 0, 5, 0\rangle$ and $\vec{B} = \langle 0, 0, -5\rangle$, what is the direction of $\vec{B} \times \vec{A}$?

(a) +x
(b) −x
(c) +y
(d) −y
(e) +z
(f) −z
(g) zero magnitude

1.6 [2 pt] The torque exerted by a force $\vec{F}$ is maximum when the force is antiparallel to the vector $\vec{r}$ that points from the pivot point to the point where the force is applied.

(a) True

(b) False

1.7 [2 pt] Two objects with the same mass m move at the same speed v. If the two objects are at the same distance r from a fixed reference point A, then the angular momentum vectors $\vec{L}$ of both objects with respect to A must be the same.

(a) True

(b) False

1.8 [2 pt] An object of mass m moves with speed v in a clockwise circular path of radius R on the xy plane, centered at the origin. What is the direction of the object's angular momentum?

(a) +x

(b) −x

(c) +y

(d) −y

(e) +z

(f) −z

(g) zero magnitude

1.9 [2 pt] The Moon moves in uniform circular motion around the Earth, in a counterclockwise orbit on the xy plane. What is the direction of the torque exerted by the Earth on the Moon?

(a) +x

(b) −x

(c) +y

(d) −y

(e) +z

(f) −z

(g) zero magnitude

1.10 [2 pt] An object of mass $m = 5$ kg moves with speed $v = 5$ m/s in the −z direction. When the object passes through position $\langle -5, 0, 0 \rangle$ m, what is the direction of its angular momentum with respect to the origin?

(a) +x

(b) −x

(c) +y

(d) −y

(e) +z

(f) −z

(g) zero magnitude

Problem 2 - Thermal Energy [25 points]

In both parts of this problem you can assume the relevant system is **insulated**. You may need to use the following specific heat capacities:

- Water: $C_w = 4.190$ J/(g °C)
- Glass: $C_g = 0.837$ J/(g °C).
- Aluminum: $C_a = 0.900$ J/(g °C)

2.1 [10 pts] Jack did an experiment in which he poured 0.350 kg of water at 45°C into a 0.375 kg glass beaker. The glass beaker was at 25°C initially. What is the thermal equilibrium temperature of the system?

If the glass and water are the system, then

$$\Delta E_{th} = 0.$$

$$\Delta E_g + \Delta E_w = 0$$

$$\Rightarrow \quad m_g C_g (T_f - T_g) + m_w C_w (T_f - T_w) = 0$$

$$\Rightarrow \quad m_g C_g T_f + m_w C_w T_f = m_g C_g T_g + m_w C_w T_w$$

$$\Rightarrow \quad T_f = \frac{m_g C_g T_g + m_w C_w T_w}{m_g C_g + m_w C_w}$$

$$= \frac{(375\,g)(0.837\,\frac{J}{g°C})(25°C) + (350\,g)(4.190\,\frac{J}{g°C})(45°C)}{(375\,g)(0.837\,\frac{J}{g°C}) + (350\,g)(4.190\,\frac{J}{g°C})}$$

$$= \boxed{41.47\ °C}$$

In both parts of this problem you can assume the relevant system is **insulated**. You may need to use the following specific heat capacities:

- Water: $C_w = 4.190$ J/(g °C)
- Glass: $C_g = 0.837$ J/(g °C).
- Aluminum: $C_a = 0.900$ J/(g °C)

2.2 [15 pts] Jill saw Jack's experiment and decided to do an experiment of her own. She poured the same amount of water at the same initial temperature (0.350 kg at 45°C) into another identical glass beaker (0.375 kg at 25°C). She then added a 0.250 kg block of aluminum into the glass beaker with the water. After some time, she measured a final thermal equilibrium temperature of 40°C. What was the initial temperature of the block of aluminum?

If the glass, aluminum, and water are the system, then

$\Delta E_{th} = 0$.

$$\Delta E_g + \Delta E_w + \Delta E_a = 0$$

$$m_g C_g (T_f - T_g) + m_w C_w (T_f - T_w) + m_a C_a (T_f - T_a) = 0$$

$$\Rightarrow -m_a C_a (T_f - T_a) = m_g C_g (T_f - T_g) + m_w C_w (T_f - T_w)$$

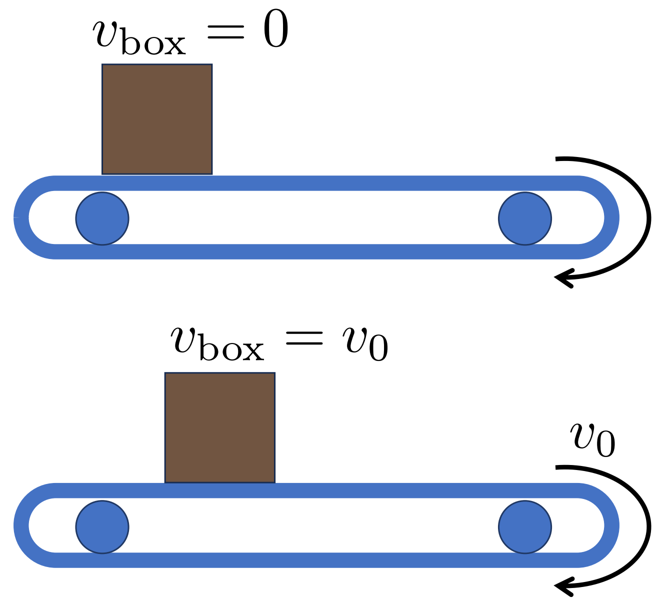
$$\Rightarrow T_a = \frac{m_g C_g (T_f - T_g) + m_w C_w (T_f - T_w)}{m_a C_a} + T_f$$

$$= \frac{(375\,g)\left(0.837 \frac{J}{g°C}\right)(40°C - 25°C) + (350\,g)\left(4.190 \frac{J}{g°C}\right)(40°C - 45°C)}{(250\,g)\left(0.900 \frac{J}{g°C}\right)} + 40°C$$

$$= \boxed{28.34\ °C}$$

Problem 3 - Point Particle/Real System [25 points]

A box of mass m and specific heat C, initially at rest, is set on a conveyor belt that moves with constant speed v_0. Initially, the belt slips as it rubs along the bottom of the box, exerting a constant friction force on the box that causes it to accelerate until the box is moving at the same speed (v_0) as the belt. Note that the coefficient of kinetic friction between the box and the belt is μ.



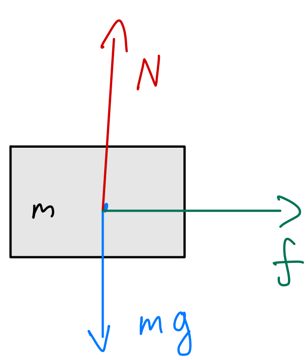
Between the initial state (box at rest set on the belt) and the final state (box first reaches v_0, the same speed as the belt), the temperature of the box has risen by an amount ΔT. At the same time, the temperature of the conveyor belt has also risen. **NOTE: For all parts of this problem, assume Q, the thermal transfer of energy, is zero in the energy principle.**

3.1 [3 pts] Treating the box as a **point particle** system, find the (point particle) work done on the box by the belt's friction force between the initial and final states.

$$\Delta E = \Delta K_{trans} = W_{CM} \Rightarrow \frac{1}{2} m v_0^2 - \frac{1}{2} \cancelto{0}{m v_i^2} = W_{CM}$$

$$\Rightarrow W_{CM} = \boxed{\frac{1}{2} m v_0^2}$$

3.2 [3 pts] Find the magnitude of belt's friction force on the box. Hint: use Newton's 2nd Law applied to the system containing the box, using μ to relate the magnitudes of the friction and normal components of the belt's contact force on the box.



$$\Rightarrow |\vec{f}| = \mu |\vec{N}| = \boxed{\mu m g}$$

3.3 [3 pts] Determine Δt, the time interval between the box's initial and final states. Hint: use Newton's 2nd Law for predicting the motion of the box.

Velocity update: $\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t$. We focus only on the x-component. $\frac{F_{net}}{m} = \frac{\mu m g}{m} = \mu g$.

$$\Rightarrow \quad v_f = v_0 = \cancel{v_i}^{\,0} + \mu g \Delta t \quad \Rightarrow \quad \boxed{\frac{v_0}{\mu g}} = \Delta t$$

3.4 [3 pts] Find the change in thermal energy of the box between the initial and final states.

$$\Delta E_{th} = \boxed{m C \Delta T}$$

3.5 [4 pts] Treating the box as a **real system**, find the (real) work done on the box by the belt's friction force between the initial and final states.

$$W_{real} = \Delta K_{trans} + \Delta E_{th}$$

$$= \boxed{\frac{1}{2} m v_0^2 + m C \Delta T}$$

3.6 [3 pts] Calculate the displacement of the belt (i.e., how far the belt, moving at constant speed v_0, has traveled) during the time interval Δt.

$$\Delta r = V_0 \Delta t$$

$$= V_0 \left(\frac{V_0}{\mu g} \right) = \boxed{\frac{V_0^2}{\mu g}}$$

3.7 [3 pts] To keep the belt moving at constant speed v_0, the motor driving the conveyor must exert a force on the belt equal in magnitude to the friction force. Find the work done by the motor during the time interval Δt. (Hint: Use the result from part 6 together with the definition of work.)

$$W_{motor} = \vec{F} \cdot \Delta \vec{r} = \mu m g \left(\frac{V_0^2}{\mu g} \right)$$

$$= \boxed{m V_0^2}$$

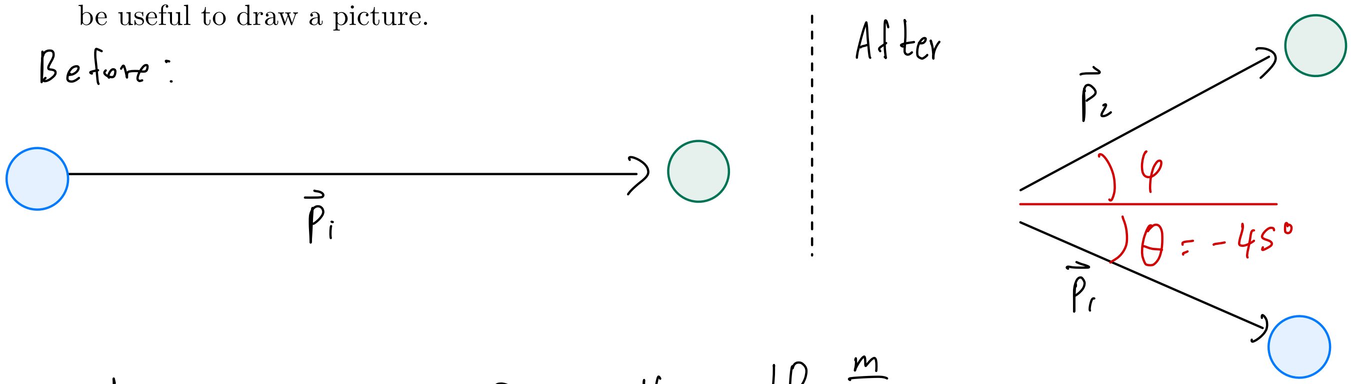
3.8 [3 pts] Do you expect the work done by the motor on the belt (answer to part 7) to be the same as the real work done on the box by the belt's friction force (answer to part 5)? If yes, briefly justify why they are the same; if no, briefly justify why there is a difference.

No, the motor's work is not the same as the box's real work because some of the energy goes into changing the temperature of the belt and box.

Problem 4 - Collisions [30 points]

Consider a puck with mass 0.2 kg initially at rest on a frictionless surface. Another identical puck is traveling towards the first with a kinetic energy of 10 J, with its momentum along the $+\hat{x}$ axis.

4.1 [15 pts] After colliding, one of the pucks is observed moving at an angle $-45°$, half-way between the $+\hat{x}$ and $-\hat{y}$ axes with a kinetic energy of 5 J. What is the angle of the other puck's velocity to the $+\hat{x}$ axis? It may be useful to draw a picture.



$$\frac{1}{2} m v_i^2 = 10 \text{ J} \Rightarrow v_i = 10 \ \frac{m}{s}$$

$$\frac{1}{2} m v_1^2 = 5 \text{ J} \Rightarrow v_1 = \sqrt{50} \ \frac{m}{s} = 5\sqrt{2} \ \frac{m}{s}$$

$$\vec{p}_i = \vec{p}_f = \vec{p}_1 + \vec{p}_2$$

$$\Rightarrow \langle m v_i, 0, 0 \rangle = \langle m v_1 \cos(\theta), m v_1 \sin(\theta), 0 \rangle + \vec{p}_2$$

$$\vec{p}_2 = \langle m v_i - m v_1 \cos(\theta), - m v_1 \sin(\theta), 0 \rangle$$

$$= \langle (0.2 \text{ kg})(10 \tfrac{m}{s}) - (0.2 \text{ kg})(5\sqrt{2} \tfrac{m}{s}) \cos(-45°), -(0.2 \text{ kg})(5\sqrt{2} \tfrac{m}{s}) \sin(-45°), 0 \rangle$$

$$= \langle 1 \text{ kg} \tfrac{m}{s}, 1 \text{ kg} \tfrac{m}{s}, 0 \rangle$$

$$\tan(\varphi) = \frac{1 \text{ kg} \frac{m}{s}}{1 \text{ kg} \frac{m}{s}} = 1$$

$$\Rightarrow \varphi = \arctan(1) = \boxed{45°}$$

4.2 [5 pts] Is the collision elastic, (partially) inelastic, or maximally inelastic? You must justify your answer in order to receive full credit.

$$\frac{\vec{p}_2}{m} = \frac{\langle 1 \text{ kg}\frac{m}{s}, 1 \text{ kg}\frac{m}{s}, 0\rangle}{0.2 \text{ kg}} = \langle 5, 5, 0\rangle \frac{m}{s}$$

$$\Delta K = K_f - K_i = 10 \text{ J} - 5 \text{ J} - \frac{1}{2}(0.2)(5^2+5^2) \text{ J}$$

$$= 0 \text{ J} \Rightarrow \boxed{\text{elastic}}$$

4.3 [10 pts] During the collision, the pucks momentarily compress. At one particular instant, the velocity of one of the pucks is observed to be $\langle 6.5,\ 1.5,\ 0\rangle$ m/s. What was the increase in the internal energy of the pucks, at this instant, due to the compression? Hint: you do not need to have answered the previous questions in order to determine this.

$$\vec{p}_i = (0.2 \text{ kg})\langle 10, 0, 0\rangle \frac{m}{s} = \langle 2, 0, 0\rangle \text{ kg}\frac{m}{s}$$

$$\vec{p}_1 = (0.2 \text{ kg})\langle 6.5, 1.5, 0\rangle \frac{m}{s} = \langle 1.3, 0.3, 0\rangle \text{ kg}\frac{m}{s}$$

$$\vec{p}_2 = \vec{p}_i - \vec{p}_1 = \langle 0.7, -0.3, 0\rangle \text{ kg}\frac{m}{s}$$

$$\frac{\vec{p}_2}{m} = \langle 3.5, -1.5, 0\rangle \frac{m}{s}$$

$$\Delta E_{int} + \Delta K = 0 \Rightarrow \Delta E_{int} = K_i - K_f$$

$$\Rightarrow \Delta E_{int} = 10 \text{ J} - \frac{1}{2}(0.2)(3.5^2 + 1.5^2) \text{ J} - \frac{1}{2}(0.2)(6.5^2 + 1.5^2) \text{ J}$$

$$= \boxed{4.1 \text{ J}}$$

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