PHYS 2211 Exam 1 - Spring 2018

Please circle your lab section and fill in your contact info below.

Section (K Curtis) and (M Fenton)		
Day	12-3pm	3-6pm
Monday	K01 M01	K02 M02
Tuesday	K03 M03	K04 M04
Wednesday	K05 M05	K06 M06
Thursday	K07 M07	K08 M08

"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

Sign your name on the line above

Instructions

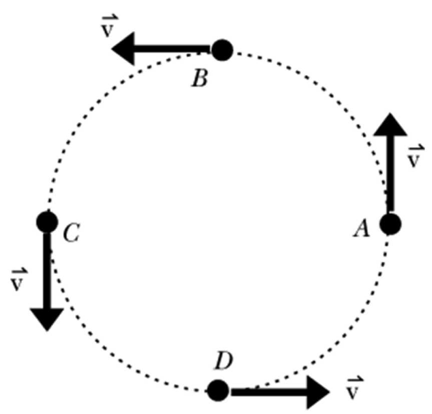
- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

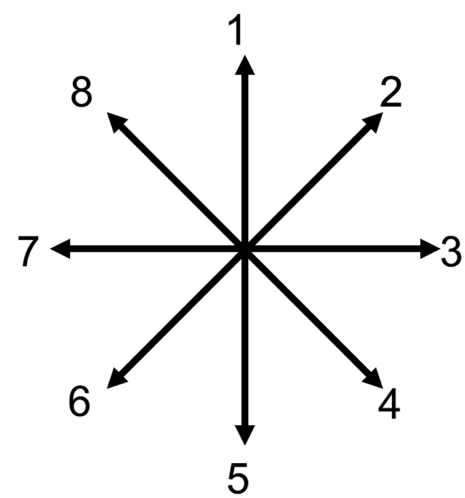
Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Problem 1 [25 pts] Grader & Score: Key

An object follows a circular trajectory in the x-y plane at a constant speed $|\vec{v}|$. At time t = 0 seconds, the object is located at A, and its velocity is $\vec{v} =< 0, 10, 0 >$ m/s. It takes 8 seconds for the object to do one rotation and get back to point A.



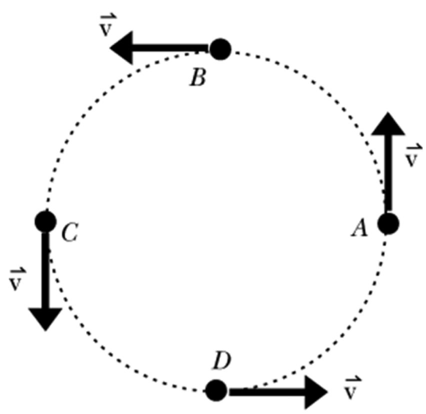


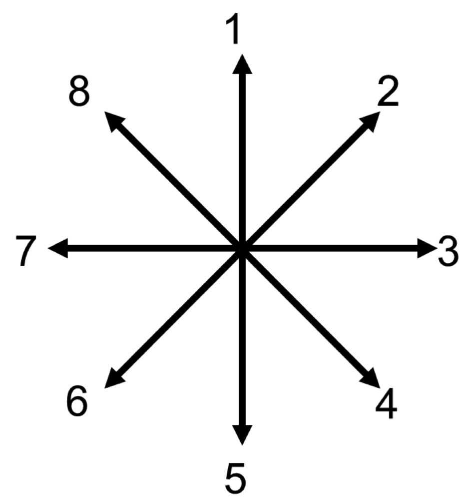
9 zero magnitude
10 undetermined

A. [10 pts] Using the numbered direction arrows shown, indicate (by number) which direction arrow best represents the direction of the quantities listed below. If the quantity cannot be determined, indicate this by using the number 10.

- The initial position vector 10
- The position vector after 2 seconds 10
- The vector displacement between point A and point C 7
- The change in momentum between point A and point C 5
- The average net force between location A and location C 5
- The vector displacement after 4 seconds 7
- The change in velocity between point B and point C 4
- The change in momentum between point B and point C 4
- The average net force between point B and point C 4
- The vector displacement between point B and point D 5

+ 1 point each





9 zero magnitude
10 undetermined

B. [5 pts] Using the numbered direction arrows shown, indicate (by number) which direction arrow best represents the direction of the quantities listed below. If the quantity cannot be determined, indicate using the number 10.

- The change in momentum between point B and point D __3__
- The average net force between point B and point D __3__
- The position vector at location D __10__
- The average net force between location A and location D __4__
- The change in momentum between point A and point D __4__

+ 1 point each

C. [10 pts] Write "T" next to each true statement below, and write "F" for every false statement.

- If the net force on an object is constant, then the rate of change of its momentum is constant __T__
- An object's acceleration is always in the same direction as its momentum __F__
- The change in an object's momentum is always in the same direction as the net force on the object __T__
- The change in the magnitude of an object's momentum is identical to the magnitude of an object's change in momentum because they are both scalars __F__
- The displacement vector for an object can be in a different direction than its average velocity (during the same time interval) __F__

+ 2 points each

`Problem 2 [25 pts]` Grader & Score: Key

In the following scenarios, please use the momentum principle to **select any answer choice(s) that could be true**. In all cases, the positive x-direction point to the right, positive y-direction points up, and the positive z-direction points out of the page.

A. [4 pts] A truck delivering a birthday cake is moving in the positive x-direction when it approaches a stop sign and slows down.

- The net force is in the positive x-direction with decreasing magnitude.
- The net force is in the positive x-direction with increasing magnitude.
- The net force is zero.
- The net force is in the negative x-direction with decreasing magnitude.
- The net force is in the negative x-direction with increasing magnitude.

+2 for correct choice
-2 for incorrect choice
Minimum score of 0

B. [4 pts] The truck is at a stand still (motionless) at the stop sign.

- The net force is in the positive x-direction with decreasing magnitude.
- The net force is in the positive x-direction with increasing magnitude.
- The net force is zero.
- The net force is in the negative x-direction with decreasing magnitude.
- The net force is in the negative x-direction with increasing magnitude.

All or Nothing

C. [4 pts] The truck accelerates away from the stop sign in the positive x-direction.

- The net force is in the positive x-direction with decreasing magnitude.
- The net force is in the positive x-direction with increasing magnitude.
- The net force is zero.
- The net force is in the negative x-direction with decreasing magnitude.
- The net force is in the negative x-direction with increasing magnitude.

+2 for correct choice
-2 for incorrect choice
Minimum score of 0

D. [4 pts] The truck is traveling at a constant speed in the positive x-direction.

- The net force is in the positive x-direction with decreasing magnitude.
- The net force is in the positive x-direction with increasing magnitude.
- The net force is zero.
- The net force is in the negative x-direction with decreasing magnitude.
- The net force is in the negative x-direction with increasing magnitude.

All or Nothing

E. [3 pts] The truck is traveling at a **constant speed** in the positive x-direction. There are three forces acting on the truck: A drag force from the air in the negative x-direction, a contact force with the road, and the force of gravity in the negative y-direction.

- The x-component of the force from the road is slightly greater than the force from the air.
- The x-component of the force from the road is slightly less than the force from the air.
- The x-component of the force from the road is equal to the force from the air.
- The y-component of the force from the road is slightly greater than the gravitational force on the truck.
- The y-component of the force from the road is slightly less than the gravitational force on the truck.
- The y-component of the force from the road is equal to the gravitational force on the truck.

F. [3 pts] The truck is **speeding up** in the positive x-direction. There are three forces acting on the truck: A drag force from the air in the negative x-direction, a contact force with the road, and the force of gravity in the negative y-direction.

- The x-component of the force from the road is slightly greater than the force from the air.
- The x-component of the force from the road is slightly less than the force from the air.
- The x-component of the force from the road is equal to the force from the air.
- The y-component of the force from the road is slightly greater than the gravitational force on the truck.
- The y-component of the force from the road is slightly less than the gravitational force on the truck.
- The y-component of the force from the road is equal to the gravitational force on the truck.

G. [3 pts] The truck is **stopped (motionless)**. There are three forces acting on the truck: A drag force from the air in the negative x-direction, a contact force with the road, and the force of gravity in the negative y-direction.

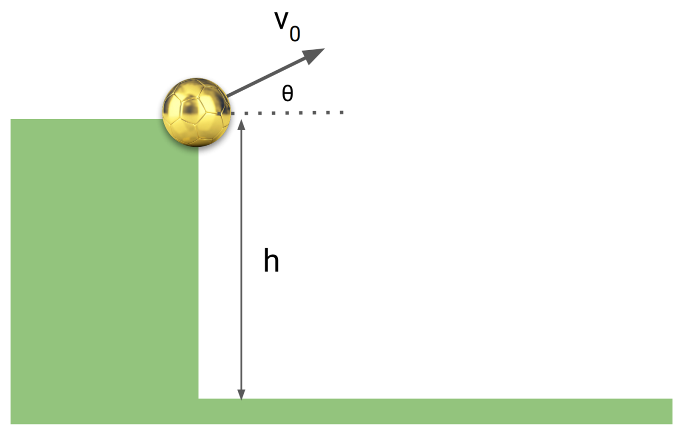
- The x-component of the force from the road is slightly greater than the force from the air.
- The x-component of the force from the road is slightly less than the force from the air.
- The x-component of the force from the road is equal to the force from the air.
- The y-component of the force from the road is slightly greater than the gravitational force on the truck.
- The y-component of the force from the road is slightly less than the gravitational force on the truck.
- The y-component of the force from the road is equal to the gravitational force on the truck.

-2 for circling wrong answer
+2 for circling one correct answer
+3 for circling both correct answers
Minimum score of 0

Problem 3 [25 pts]

Grader & Score: Key

After successfully taking Physics 2211 at GT, a game developer comes to you and ask for help. He has created a game where a ball is kicked off the edge of a cliff and lands on the ground below. The developer needs help determining how to scale an image of the balls trajectory so that it will properly fit onto the screen. The given initial height of the cliff is h and the initial speed of the ball when it leaves the cliff is v_0. The initial velocity of the ball makes an angle θ with the horizontal.



A. [10 pts] Calculate the maximum height of the ball relative to the ground below.

Consider y-direction only

$V_{iy} = V_0 \sin\theta$

$V_{fy} = 0$

$\Delta y = ?$

$\Delta t = ?$

-1 Clerical/included units

-2 Minor/Math error

-4 Major physics error

-8 BTN

Constant force

$$V_{avg\text{-}y} = \frac{V_{iy}+V_{fy}}{2} = \frac{V_0 \sin\theta}{2}$$

But by definition

$$V_{avg\text{-}y} = \frac{\Delta y}{\Delta t} = \frac{V_0 \sin\theta}{2} \quad (*)$$

Common errors:

-2 incorrect trig

No points lost for not writing down second law

Momentum Principle

$$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\Delta t$$

If we consider only y-direction

$$P_{yf} = P_{yi} + F_y \Delta t$$

$$m V_{fy} = m V_{iy} - mg\Delta t$$

$$V_{fy} = V_{iy} - g\Delta t$$

$$0 = V_0 \sin\theta - g\Delta t$$

Solve for $\Delta t = \dfrac{V_0 \sin\theta}{g}$

Plug $\Delta t = \dfrac{V_0 \sin\theta}{g}$ into $(*)$

$$\Delta y = \frac{V_0 \sin\theta}{2}\Delta t$$

$$= \frac{V_0 \sin\theta}{2}\frac{V_0 \sin\theta}{g}$$

$$= \frac{(V_0 \sin\theta)^2}{2g}$$

$$y_f = y_i + \Delta y = \boxed{h + \frac{(V_0 \sin\theta)^2}{2g}}$$

B. [15 pts] Determine how far the ball will travel in the x-direction before reaching the ground.

First consider y-direction to find Δt

$v_{iy} = v_0 \sin\theta$

$v_{fy} = ?$

$y_i = h$

$y_f = 0$

$v_{avg\text{-}y} = \frac{\Delta y}{\Delta t} = \frac{y_f - y_i}{\Delta t} = -\frac{h}{\Delta t} \quad (*)$

$v_{avg\text{-}y} = \frac{v_{iy} + v_{fy}}{2} = \frac{v_0 \sin\theta + v_{fy}}{2} \quad (**)$

$p_{yf} = p_{yi} + F_y \Delta t$

$m v_{fy} = m v_{iy} - mg\Delta t$

$v_{fy} = v_{iy} - g\Delta t = v_0 \sin\theta - g\Delta t$

Plug into $(**)$

$v_{avg\text{-}y} = \frac{v_0 \sin\theta + v_0 \sin\theta - g\Delta t}{2}$

$= \frac{2v_0 \sin\theta - g\Delta t}{2} = v_0 \sin\theta - \frac{g\Delta t}{2}$

-1 Clerical/included units

-3 Minor/Math error

-6 Major physics error

-12 BTN

Common errors:

-6 $\triangle$t in final answer

-6 no gravitational force

-3 incorrect trig

Plug into $(*)$

$v_0 \sin\theta - \frac{g\Delta t}{2} = -\frac{h}{\Delta t}$

$v_0 \sin\theta \Delta t - \frac{g}{2}(\Delta t)^2 = -h$

$-\frac{g}{2}(\Delta t)^2 + v_0 \sin\theta \Delta t + h = 0$

$\Delta t = \frac{-v_0 \sin\theta \pm \sqrt{(v_0 \sin\theta)^2 + 2gh}}{-g}$

$= \frac{v_0 \sin\theta \mp \sqrt{(v_0 \sin\theta)^2 + 2gh}}{g}$

We choose (+) because we want the positive Δt

$\Delta t = \frac{v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2gh}}{g}$

$(***)$

Now consider x-direction

There is no force in x-direction so v_x is constant

$v_{avg\text{-}x} = v_{ix} = v_{fx}$

$= v_0 \cos\theta$

$v_{avg\text{-}x} = \frac{\Delta x}{\Delta t}$

$\Delta x = v_{avg\text{-}x} \Delta t$

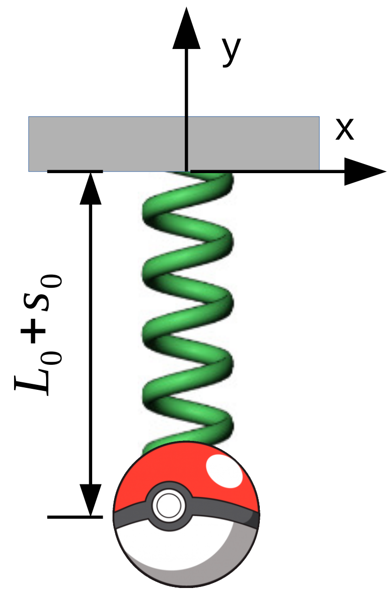
$= v_0 \cos\theta \Delta t$

Plug in Δt from $(***)$

$$\boxed{\Delta x = \frac{v_0 \cos\theta}{g}\left(v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2gh}\right)}$$

Problem 4 [10 pts] Grader & Score: Key

On Earth where gravity points in the $-\hat{y}$ **direction**, a spring with stiffness k_s and relaxed length L_0 is hung from a ceiling. The origin is taken to be the point where the spring meets the ceiling as shown in the figure. A mass M is attached to the bottom of the spring so that it came move up and down. At time $t = 0$, the spring is stretched to an initial length $L_0 + s_0$ and is moving down with velocity $\vec{v} =< 0, -v_0, 0 >$.



A. [5 pts] At $t = 0$, calculate the net force on the mass and express symbolically as a vector in component form.

$\vec{F}_{grav} = \langle 0, -Mg, 0 \rangle$

$\vec{F}_{spring} = -k_s s \hat{L}$

$s = (L_0 + s_0) - L_0 = s_0$

$\hat{L} = \langle 0, -1, 0 \rangle$

$\vec{F}_{spring} = \langle 0, k_s s_0, 0 \rangle$

$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring}$

$= \langle 0, k_s s_0 - Mg, 0 \rangle$

+ 2 F_grav
+ 2 F_spring
+1 F_net = F_spring+F_grav

B. [10 pts] Find the velocity of the mass (expressed as a vector in component form) at a time $t = \Delta t$. Where Δt can be treated as a small, single, time step.

Momentum principle

$\vec{P}_f = \vec{P}_i + \vec{F}_{net} \Delta t$

$= m \langle 0, -v_0, 0 \rangle + \langle 0, k_s s_0 - Mg, 0 \rangle \Delta t$

$= \langle 0, -mv_0 + (k_s s_0 - Mg)\Delta t, 0 \rangle$

$\vec{v}_f = \frac{\vec{P}_f}{m} = \langle 0, -v_0 + \frac{(k_s s_0 - Mg)\Delta t}{M}, 0 \rangle$

$= \boxed{\langle 0, -v_0 + \left(\frac{k_s s_0}{M} - g\right)\Delta t, 0 \rangle}$

-1 Clerical/included units
-2 Minor/Math error
-4 Major physics error
-8 BTN

Common errors:
-8 no momentum principle
-4 assumed constant acceleration

Watch for POE

C. [10 pts] Find the new force acting on the mass (expressed as a vector in component form) at a time $t = \Delta t$.

First we must find the new position

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{v}_{avg} \approx \vec{v}_f$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_f \Delta t$$

$$= \langle 0, -(L_0+S_0), 0\rangle + \langle 0, -V_0 + \left(\frac{K_s S_0}{m} - g\right)\Delta t, 0\rangle \Delta t$$

$$= \langle 0, -(L_0+S_0) - V_0\Delta t + \left(\frac{K_s S_0}{m} - g\right)(\Delta t)^2, 0\rangle$$

$$s = |\vec{L}| - L_0 = |\vec{r}_f| - L_0$$

since (L_0+S_0) is positive, V_0 is positive, and Δt is very small and positive

$$|\vec{L}| = L_0 + S_0 + V_0\Delta t - \left(\frac{K_s S_0}{m} - g\right)(\Delta t)^2$$

$$s = S_0 + V_0\Delta t - \left(\frac{K_s S_0}{m} - g\right)(\Delta t)^2$$

$$\vec{F}_{spring} = -K_s s \hat{L}$$

$$\hat{L} = \langle 0, -1, 0\rangle$$

$$\vec{F}_{spring} = \langle 0, K_s\left[S_0 + V_0\Delta t - \left(\frac{K_s S_0}{m} - g\right)(\Delta t)^2\right], 0\rangle$$

$$\vec{F}_{grav} = \langle 0, -Mg, 0\rangle$$

$$\vec{F}_{net} = \boxed{\langle 0, K_s\left[S_0 + V_0\Delta t - \left(\frac{K_s S_0}{m} - g\right)(\Delta t)^2\right] - Mg, 0\rangle}$$

Watch for POE

-1 Clerical/included units
-2 Minor/Math error
-4 Major physics error
-8 BTN

Common errors:
-4 Didn't use v_avg = v_f

OK if stretch is left as an absolute value

This page is for extra work, if needed.

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Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_s s \qquad U_{spring}=\frac{1}{2}k_s s^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots}{m_1+m_2+\ldots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\ldots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y=\frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

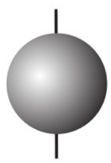
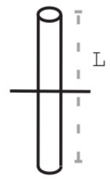
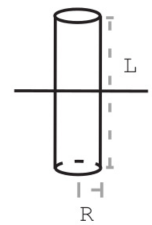
$$\text{prob}(E)\propto\Omega\left(E\right)e^{-\dfrac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2} \text{ where } N=1,2,3\ldots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\ldots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}