Test 3 Review

Things to remember from Test 1 & Test 2

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{net\perp} = \frac{mv^2}{R}\hat{n}$$

$$\vec{F}_{spring} = -k(L - L_0)\hat{L}$$

$$\left.\begin{aligned} \vec{F}_{grav} &= \langle 0, -mg, 0 \rangle \\ \Delta U_{grav} &= mg\Delta h \end{aligned}\right\} \text{near Earth's surface}$$

$$\left.\begin{aligned} \vec{F}_{grav} &= \frac{GMm}{r^2}(-\hat{r}) \\ \Delta U_{grav} &= -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \end{aligned}\right\} \text{in general}$$

$$\vec{F}_{1 on 2} = -\vec{F}_{2 on 1}$$

$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}||\Delta\vec{r}|\cos\theta$$

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

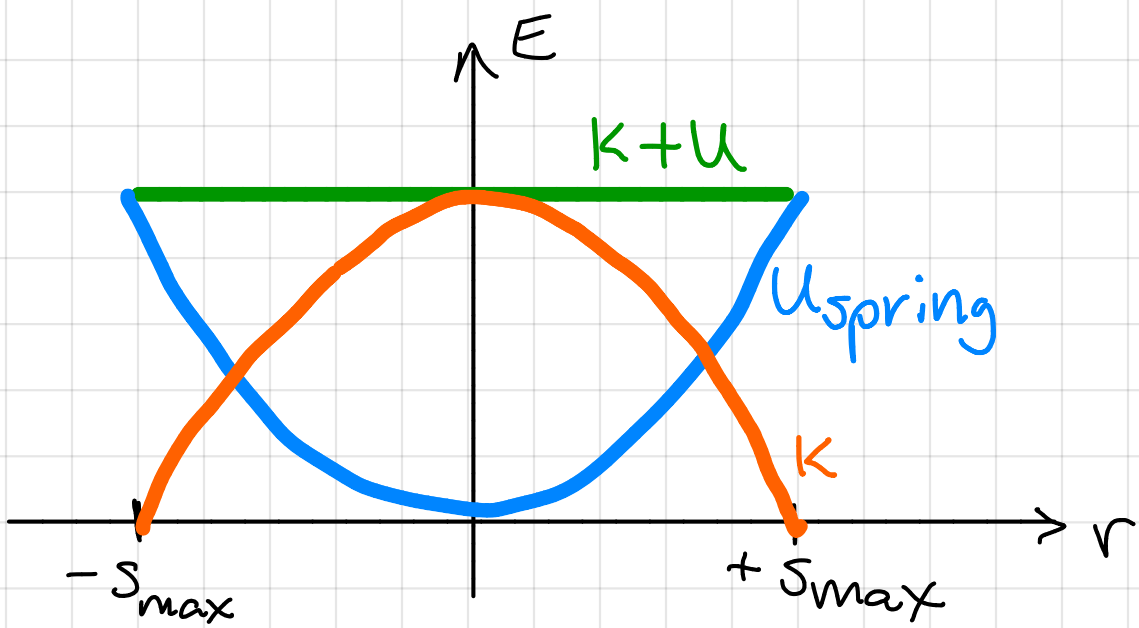
↑ capital K, kinetic energy

Spring Potential Energy

$$\Delta U_s = \frac{1}{2} k (s_f^2 - s_i^2)$$

✓ k = lowercase; spring stiffness

✓ s = stretch or compression of spring; $s = L - L_0$



✓ s_{max} = maximum stretch/compression; $U = max$; $K = 0$

✓ $s = 0$ = relaxed length/equilibrium position; $K = max$, $U = 0$

Thermal Energy and Heat

$$\Delta E_{th} = mC\Delta T$$

✓ m = mass in grams

✓ C = specific heat; units:

$$\frac{J}{g\,^\circ C} \quad \text{or} \quad \frac{J}{g\,K} \leftarrow \text{kelvin}$$

✓ ΔT = change in temperature; in °C or kelvin only

$$\Delta E_{sys} = W_{surr} + Q$$

✓ Q = heat; energy exchange between system and surroundings due to a difference in temperature

$Q > 0 \Rightarrow$ energy goes into system

$Q < 0 \Rightarrow$ energy goes out of system

Power

$$P = \frac{\Delta E}{\Delta t}$$

$$= \frac{Q}{\Delta t} \quad \text{if } W = 0$$

$$= \frac{W}{\Delta t} \quad \text{if } Q = 0$$

$$= \frac{\vec{F} \cdot \Delta\vec{r}}{\Delta t} = \vec{F} \cdot \vec{v}$$

units: $\underset{\text{Watts}}{W} = \underset{\text{Joules}}{J} / \underset{\text{second}}{s}$

Collisions

$$\Delta \vec{p}_{sys} = 0 \quad \Rightarrow \quad \vec{p}_i = \vec{p}_f$$

✓ total linear momentum ($\vec{p}$) is conserved during a collision

✓ total $\vec{p}$ includes all objects colliding

$$\Delta E_{sys} = 0$$

✓ total energy is conserved during a collision

Types of collision

✓ Elastic $\Rightarrow \Delta K = 0$ (K = kinetic energy)

✓ Inelastic $\Rightarrow \Delta K \neq 0$

✓ Maximally inelastic $\Rightarrow \Delta K \neq 0$ and the colliding objects stick together

Point Particle & Real System

✓ point particle system = center of mass system: the entire system is reduced to one point at the center of mass

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$$\Delta K_{trans} = W_{cm} = \vec{F}_{net} \cdot \Delta\vec{r}_{cm}$$

✓ ΔK_{trans} = translational kinetic energy

$$\Delta K_{trans} = \frac{1}{2} M_{total} \, v_{cm}^2$$

total mass of system

speed of center of mass

✓ W_{cm} = net work done on the center of mass

$$W_{cm} = \vec{F}_{net} \cdot \Delta\vec{r}_{cm}$$

sum of all forces acting on the system

displacement of center of mass

Point Particle & Real System

✓ real system = extended system:
need to take into consideration the geometry of the system and account for internal energies

$$\Delta E_{sys} = W_{real} = W_1 + W_2 + \ldots$$
$$= (\vec{F}_1 \cdot \Delta\vec{r}_1) + (\vec{F}_2 \cdot \Delta\vec{r}_2) + \ldots$$

✓ ΔE_{sys} = all the changes in energy of the system

$$\Delta E_{sys} = \Delta K_{trans} + \Delta K_{rot} + \Delta K_{vib}$$
$$+ \Delta E_{th} + \Delta E_{ch} + \Delta E_{int} + \Delta U \ldots$$

✓ W_{real} = sum of all the works done by all the forces acting on the system

* note that each force has its own displacement

Rotational Kinetic Energy

$$K_{rot} = \frac{1}{2} I \omega^2 ; \quad \Delta K_{rot} = \frac{1}{2} I \left(\omega_f^2 - \omega_i^2\right)$$

✓ I = moment of inertia

* for a point particle:

$$I = m r_{\perp}^2$$

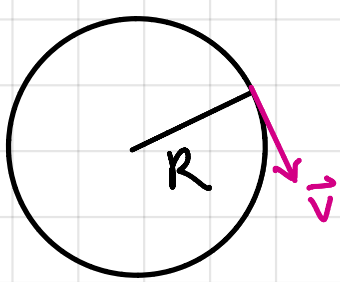
← perpendicular distance to axis of rotation

* for a system of many point particles:

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + m_3 r_{3\perp}^2 + \ldots$$

* for a rigid object (disk, cylinder, etc)
⇒ consult formula sheet

✓ ω = angular speed



$$v = R\omega$$

$$\omega = \frac{2\pi}{T}$$

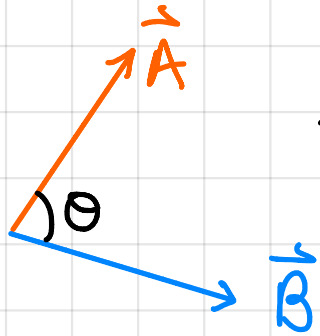
← rotation period

✓ Parallel axis theorem:

$$I_{PA} = I_{cm} + m d^2$$

The Cross Product

$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta$ in the direction of the right hand rule



$\vec{A} \times \vec{B}$ = into the page $\otimes$

$\vec{B} \times \vec{A}$ = out of the page $\odot$

Determinant form:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$= \hat{x}\left(A_y B_z - A_z B_y\right)$$

$$- \hat{y}\left(A_x B_z - A_z B_x\right)$$

$$+ \hat{z}\left(A_x B_y - A_y B_x\right)$$

Translational Angular Momentum & Torque

$$\vec{L}_{trans,A} = \vec{r}_A \times \vec{p}$$

✓ $\vec{L}_{trans,A}$ = angular momentum about the reference point A

✓ $\vec{r}_A$ = position of the object relative to the reference point A

✓ $\vec{p}$ = linear momentum of the object

$$\vec{\tau}_A = \vec{r}_A \times \vec{F}$$

✓ $\vec{\tau}_A$ = torque about reference point A

✓ $\vec{r}_A$ = vector that goes from the reference point A (the pivot) to the point where the external force $\vec{F}$ is applied

✓ $\vec{F}$ = external force applied