PHYS 2211 Test 3
Fall 2011

Name(print)____

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- Correct answers without adequate explanation will be counted wrong.
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you don't want us to read!
- Make explanations correct but brief. Don't write a lot of prose.
- Include diagrams!
- Show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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Sign your name on the line above

The final exam is scheduled for Thursday December 15th from 8:00 to 10:50 AM. If you have a conflict, please contact Dr. Greco by November 28th.

Problem 1 (25 Points)

Jack and Jill are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.3 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Jack exerts a force $\langle -400, 0, 200\rangle$ N and Jill exerts a force $\langle 150, 0, 300\rangle$ N.

(a 5pts) How much work does Jack do?

$$W = \vec{F} \cdot \Delta\vec{r}, \quad \vec{F} = \langle -400, 0, 200\rangle, \quad \Delta\vec{r} = \langle 4,0,2\rangle - \langle 2,0,3\rangle = \langle 2,0,-1\rangle$$

$$W = \langle -400, 0, 200\rangle \cdot \langle 2, 0, -1\rangle = (-400)(2) + (200)(-1) = \boxed{-1000\ \text{J}}$$

(b 5pts) How much work does Jill do?

$$W = \vec{F} \cdot \Delta\vec{r}, \quad \vec{F} = \langle 150, 0, 300\rangle, \quad \Delta\vec{r} = \langle 2, 0, -1\rangle$$

$$W = \langle 150, 0, 300\rangle \cdot \langle 2, 0, -1\rangle = (150)(2) + (300)(-1) = \boxed{0\ \text{J}}$$

(c 10pt) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

$$\Delta E = W + \cancel{Q}$$

$$\frac{1}{2}m|v_f|^2 - \frac{1}{2}m|v_i|^2 = W_{Jack} + W_{Jill}$$

$$|v_f| = \sqrt{\frac{2}{m}\left(W_{Jack} + W_{Jill} + \frac{1}{2}mv_i^2\right)}$$

$$|v_f| = \sqrt{\frac{2}{(3000\ \text{kg})}\left(-1000\ \text{J} + 0\ \text{J} + \frac{1}{2}(3000\ \text{kg})(1.3\ \text{m/s})^2\right)}$$

$$|v_f| = \boxed{1.01\ \text{m/s}}$$

(d 5pts) What effect does Jill have on the boat's motion?

The force that Jill exerts changes the direction of motion of the boat.

Problem 2 (25 Points)

A deuteron, the nucleus of heavy hydrogen, consists of one proton plus one neutron (so its charge is $+e$, where $e = 1.6 \times 10^{-19}$ C). If two deuterons make contact with each other, they can fuse to form an alpha particle, the nucleus of helium, consisting of two protons and two neutrons (with charge $+2e$). The mass of the deuteron is 2.0136 u, and the mass of the alpha particle is 4.0015 u, where one atomic mass unit u $= 1.66 \times 10^{-27}$ kg.

(a 10pts) Start with two deuterons far from each other. If you shoot them straight at each other with equal kinetic energies, sufficient that they approach and touch each other at a center-to-center distance of 3×10^{-15} m, the nuclear force can act to fuse the two deuterons and form an alpha particle. What is the smallest initial kinetic energy you need to give each deuteron when they are far apart so that they can get close enough to be able to fuse?

$$\Delta K + \Delta U = 0 \quad \Rightarrow \cancel{K_f} - K_i + U_f - \cancel{U_i} = 0$$

$$K_i = U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r|} = \left(9 \times 10^9 \frac{N m^2}{C^2}\right)\frac{(1.6 \times 10^{-19} C)^2}{(3 \times 10^{-15} m)} = 7.68 \times 10^{-14} J$$

Divide by two for the kinetic energy to give each deuteron : $\boxed{3.84 \times 10^{-14} J}$

(b 15pts) Next the two touching deuterons fuse to form an alpha particle. In this fusion process, a high-energy photon is emitted. What is the energy of this photon? You may neglect the small kinetic energy of the newly formed alpha particle.

$$2 E_{d,rest} + U = E_{\alpha,rest} + E_{photon}$$

$$2 m_d c^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r|} = m_\alpha c^2 + E_{photon}$$

where $m_d = (2.0136\ u)(1.66 \times 10^{-27} \frac{kg}{u}) = 3.3426 \times 10^{-27} kg$

$m_\alpha = (4.0015\ u)(1.66 \times 10^{-27} \frac{kg}{u}) = 6.6425 \times 10^{-27} kg$

$$E_{photon} = 2 m_d c^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r|} - m_\alpha c^2$$

$$E_{photon} = 2(3.3426 \times 10^{-27} kg)(3 \times 10^8 m/s)^2 + 7.68 \times 10^{-14} J - (6.6425 \times 10^{-27} kg)(3 \times 10^8 m/s)^2$$

$$E_{photon} = \boxed{3.92 \times 10^{-12} J}$$

Problem 3 (25 Points)

You drop a single coffee filter from a tall building, and it takes 90 seconds to reach the ground. Next you drop a stack of 5 of these coffee filters. There is a drag force from the air on the filters given by $\vec{F} = -\frac{1}{2}C\rho Av^2\hat{v}$. About how long will they take to hit the ground? Explain briefly, including any approximations or simplifying assumptions you had to make.

Begin by calculating the terminal velocity for each case. Recall that when terminal velocity is reached, we have $|F_{grav}| = |F_{air}|$.

<u>1 Coffee Filter</u>

$$|F_{grav}| = |F_{air}|$$

$$mg = \frac{1}{2} C\rho A v_1^2$$

$$|v_1| = \sqrt{\frac{2mg}{C\rho A}}$$

<u>5 Coffee Filters</u>

$$|F_{grav}| = |F_{air}|$$

$$5mg = \frac{1}{2} C\rho A v_5^2$$

$$|v_5| = \sqrt{\frac{10mg}{C\rho A}}$$

So, we see that the two terminal velocities are related by $|v_5| = \sqrt{5}|v_1|$.

Now, we make the assumption that terminal velocity is reached quickly, so that $v_{avg} \approx v_{terminal}$. This is valid from what we observed in Lab 9.

Since the coffee filters fall the same distance in both cases, we have:

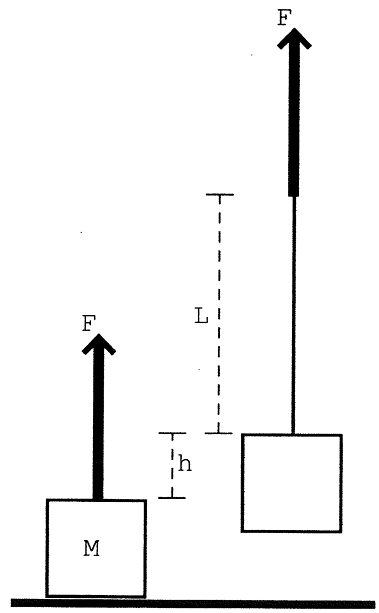
$$\Delta r = v_{avg}\Delta t \Rightarrow |v_1|t_1 \approx |v_5|t_5 \Rightarrow |v_1|(90s) \approx \sqrt{5}|v_5|t_5$$

$$\Rightarrow t_5 \approx \frac{90}{\sqrt{5}} = \boxed{40.2\,s}$$

Assumptions made include:
- coffee filters reach terminal velocity quickly, so $v_{avg} \approx v_{terminal}$
- coffee filters fall straight (i.e. there is no wind)
- $|F| = mg$ is constant

Problem 4 (25 Points)

A box contains machinery that can rotate. The total mass of the box plus machinery is M. A string wound around the machinery comes out through a small hole in the top of the box. Initially the box sits on the ground, and the machinery inside the box is not rotating. Then you pull upwards on the string with a force whose magnitude F is constant. At an instant when you have pulled a length of string L out of the box, the box has risen a height h.



(a 10pts) Consider the point particle system and calculate the speed of the box at this instant. Start from a fundamental principle, and show all your work. Express your answer in terms of the given quantities.

$$\Delta K_{trans} = \vec{F}_{net} \cdot \Delta \vec{r}_{CM}$$

$$\frac{1}{2} M v_f^2 = (F - Mg)h$$

$$\boxed{|v_f| = \sqrt{\frac{2}{M}(F - Mg)h}}$$

(b 15pts) Consider the real system and calculate the rotational kinetic energy of the machinery inside the box at this instant. Start from a fundamental principle, and show all your work. Express your answer in terms of the given quantities.

$$\Delta K_{trans} + \Delta K_{rot} = W$$

$$\frac{1}{2} M v_f^2 + K_{rot,f} = F(L+h) - Mgh$$

$$K_{rot,f} = F(L+h) - Mgh - \frac{1}{2} M v_f^2$$

$$K_{rot,f} = F(L+h) - Mgh - (Fh - Mgh)$$

$$\boxed{K_{rot,f} = FL}$$