Week 7

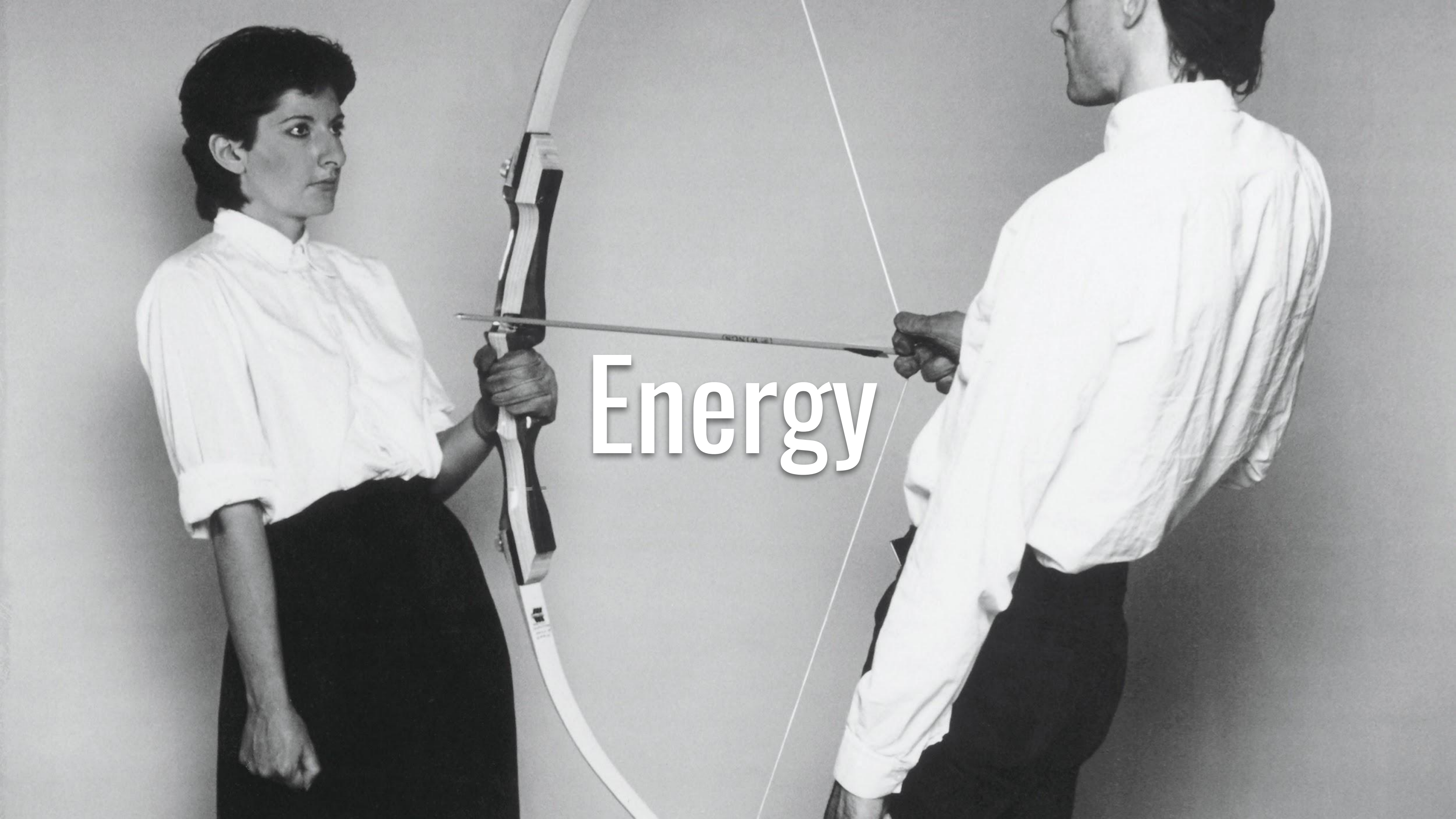
The Energy Principle

Topics for this week

1. Energy conservation
2. Particle Energy
3. Work

By the end of the week

1. Be able to calculate a few types of energy
2. Multiple Vectors


Energy

The origins of energy

- The word energy derives from the Greek word “energeia”.
  - First appearing in the works of Aristotle (400 BC)
- Leibniz (1700's) claimed that it was the mass times the velocity squared that was conserved not the momentum
  - Could Newton be wrong?
- Einstein (1900's) derives a relativistic correction to our model of energy

“Energy is a numerical quantity, which does not change when something happens....” - Feynman

Energy is everyone

- How many types of energy can you identify?
  - Mass energy
  - Kinetic energy
  - Potential energy
  - Chemical energy
  - Thermal energy
  - Rotational energy
  - Sound energy
- It is often difficult for us to identify every type of energy present in a system
  - We must also account for transfers of energy in and out of the system

The energy principle

- The change in the energy of a system is equal to the net transfer of energy into or out of the system
- We distinguish between macroscopic and microscopic energy transfers
  - Macro: The work (W) done by forces from the surroundings
  - Micro: The flow of energy (Q) between system and surroundings due to a temperature difference.

$$\Delta E = W + Q$$

- A fundamental principle
  - We have never found a violation
  - A relationship between scalar quantities

Experiment is the judge of physics

- Émilie du Châtelet 1706 – 1749
  - French mathematician, physicist, and author.
  - Her crowning achievement was her translation and commentary on Isaac Newton's work Principia Mathematica.
  - Repeated and publicized an experiment originally devised by Gravesande in which balls were dropped from different heights into a sheet of soft clay.
    - The depth of penetration depended on the mass times the square of velocity

$$E \sim mv^2$$

Mass-Energy equivalence

- "Does the mass of an object depends on its energy content?" - Einstein 1905
- The energy of a single particle is given by:

$$E_{particle} = \gamma mc^2$$

  - The SI units of energy is the joule "J"
  - When the velocity is zero, gamma equals one
- For non-zero velocity the difference between the energy of the total particle and the rest energy is called the kinetic energy

$$E_{particle} = mc^2 + K$$

The energy principle

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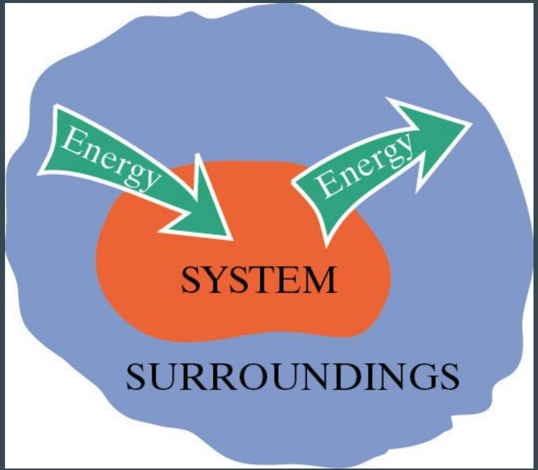


- A fundamental principle
  - The first law of Thermodynamics
    - We have never found a violation
  - A relationship between scalar quantities

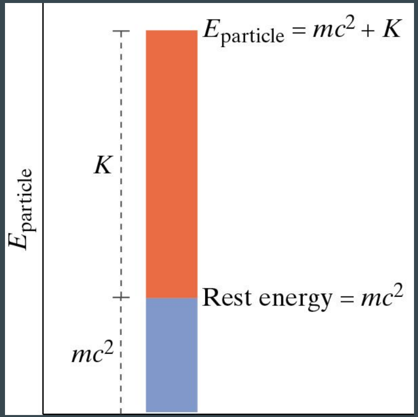
The kinetic energy of a particle

- The kinetic energy can be written in a way to facilitate taking the limit v << c

$$\gamma mc^2 = mc^2 + K$$

$$K = (\gamma - 1)mc^2$$

$$\gamma = \left(1 - (v/c)^2\right)^{-\frac{1}{2}}$$



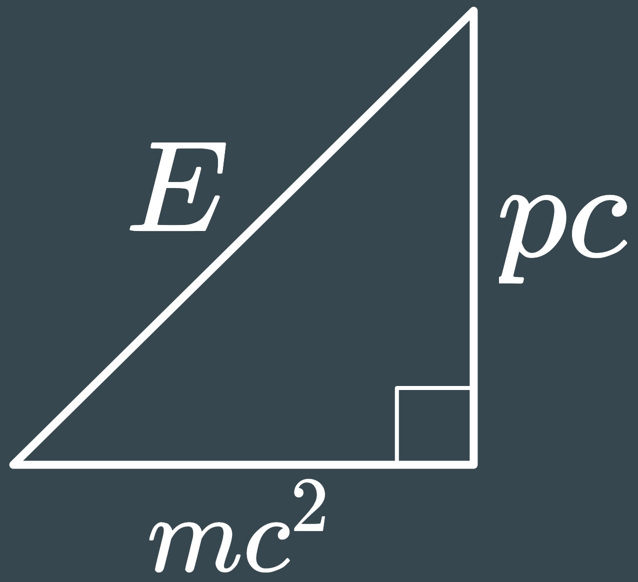
- In the limit where v << c we can approximate gamma

$$\gamma \approx 1 + \frac{1}{2}(v/c)^2 \quad \longrightarrow \quad K \approx \frac{1}{2}mv^2$$

Energy decomposition

- Energy and momentum both contain gamma
  - We can write an expression that relates these two quantities using gamma
  - The rest mass energy is often much larger

$$E^2 = (pc)^2 + (mc^2)^2$$



- From this expression we can see that it is possible to have energy even if you don’t have mass
  - Photos of light have momentum but not mass

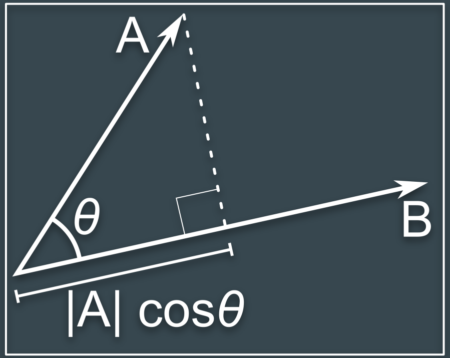
Work

- Work is the transfer of energy into or out of the system due to interactions with the surroundings
  - Only forces parallel to the motion change the magnitude of momentum!

$$W = \int_{r_i}^{r_f} \vec{F}_{net} \cdot d\vec{l}$$

- The scalar (or dot) product finds the proportion of a vector that points in the direction of the other vector

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{3} A_i B_i = |\vec{A}||\vec{B}| \cos\theta$$

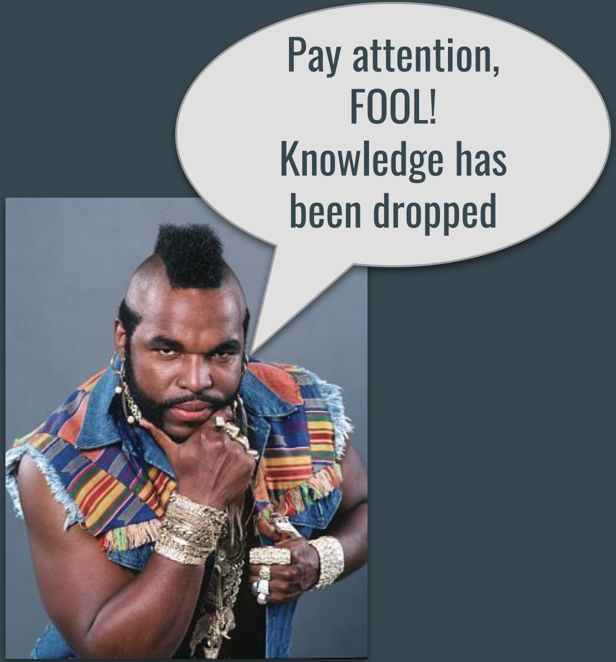


Constant force motion

- For a particle with a constant net force the work calculation simplifies

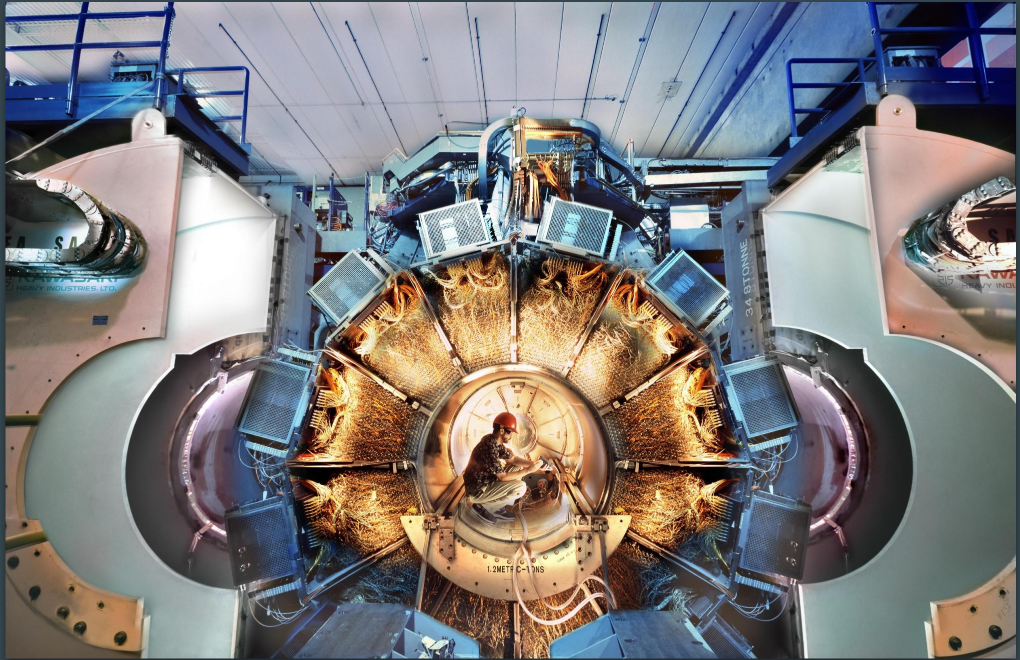
$$\Delta E = \vec{F}_{net} \cdot \Delta \vec{r}$$

- For a constant force, this calculation is path independent.
  - Only the initial and final states are required
  - We ignoring the complex dynamics taking place between the initial and final states
- Places a limit on what is possible
  - Energy analysis gives less detail
    - How fast does the process proceed?
    - Is the object changing direction?



Example: The SLAC revisited

At the Stanford Linear Accelerator Center, electrons are accelerated through a vacuum tube of length L. Initially at rest, the electrons are subject to a constant force F along the entire length of the tube. How fast is an electron traveling when it reaches the end of the tube?



Example: SLAC Solution

- The energy principle relates the change in energy change to the work from SLAC

$$\Delta E_{rest} + \Delta K = W_{surr}$$

- Calculate work, and use our models for energy change to solve for v

$$K_{final} = FL \quad \Rightarrow \quad (\gamma - 1)mc^2 = FL$$

$$v = c\sqrt{\frac{1}{1 + \frac{mc^2}{FL}}}$$

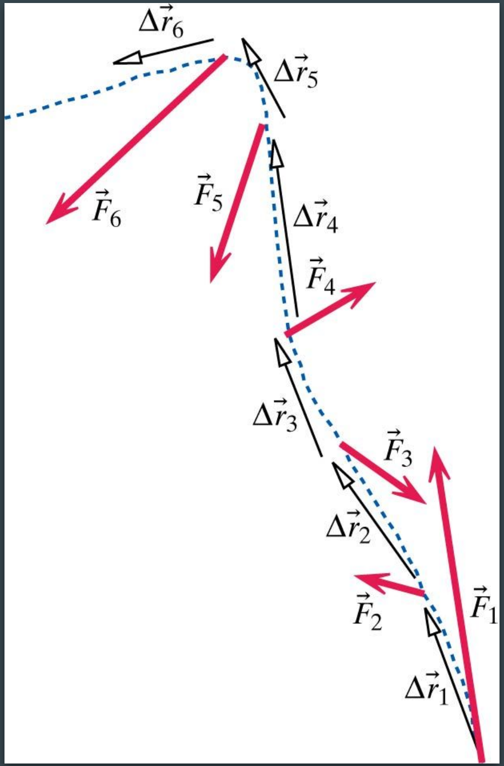
Work by non-constant forces

- When the net force is changing in magnitude or direction, the total work is no longer the product of the net force and the displacement
  - Instead, the path of the system must be split into small pieces such that the net force is approximately constant along each piece
  - The total work is then the sum of the individual work across each segment

$$W_{total} = W_1 + W_2 + \ldots W_6$$

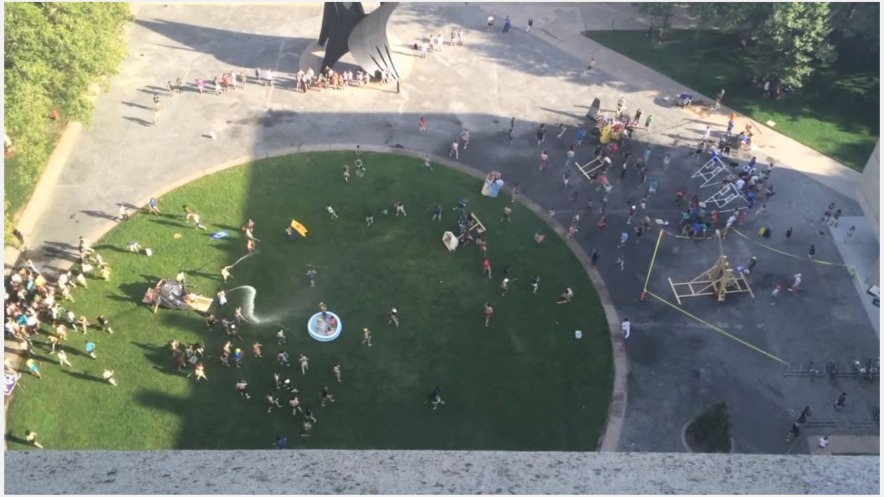
- In the limit where the steps are infinitesimal

$$W_{total} = \sum_{i=1}^{N} \vec{F}_i \cdot \Delta\vec{r}_i \quad \Rightarrow \quad W_{total} = \int_i^f \vec{F} \cdot d\vec{r}$$



Example: Water Balloon Fight

During the spring semester at MIT, residents of the parallel buildings of the East Campus Dorms battle one another with large slingshots made from surgical hose mounted to window frames and other makeshift devices. Water balloons are placed in a pouch attached to the hose, which is then stretched nearly the width of the room. If the hose obeys Hooke's Law, estimate how fast is the balloon traveling when it leaves the dorm room?



Epic Water Balloon Battle at MIT!!!

Example: Water Balloon Fight Solution

- The energy principle relates the work on the water balloon to its final velocity

$$\Delta E_{total} = \Delta K_{balloon} = W_{spring} \quad \Rightarrow \quad v_f^2 = \frac{2W_{spring}}{m_{balloon}}$$

- The work of the balloon is the integral of the spring force

$$W_{spring} = \int_{x_{max}}^{0} (-k_s x\hat{x}) \cdot dx\hat{x} = \frac{1}{2} k_s x_{max}^2$$

$$v_f = x_{max} \sqrt{\frac{k_s}{m}}$$