Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_s s \qquad U_{spring}=\frac{1}{2}k_s s^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots}{m_1+m_2+\ldots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\ldots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

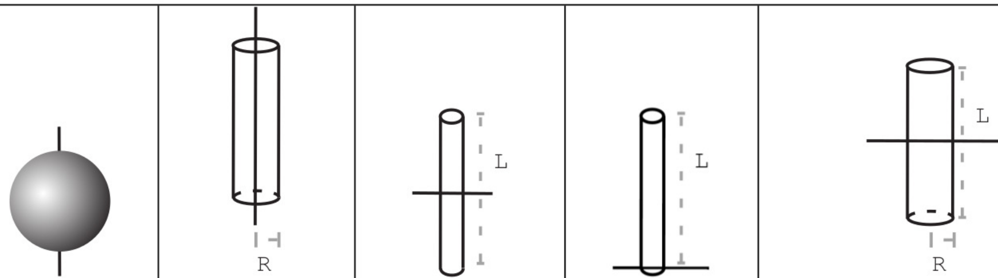
$$\text{prob}(E)\propto\Omega\left(E\right)e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\ldots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\ldots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5} MR^2$	$I = \frac{1}{2} MR^2$	$I = \frac{1}{12} ML^2$	$I = \frac{1}{3} ML^2$	$I = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 Test 4 - Fall 2018

Please circle your lab section and then clearly print your name & GTID

Sections (M) Parker, (N) Yunker		
Day	12-3pm	3-6pm
Monday	M01 N01	M02 N02
Tuesday	M03 N03	M04 N04
Wednesday	M05 N05	M06 N06
Thursday	M07	N07

Name: Key

GTID: ______________________

Instructions

- Please write with a pen or dark pencil to aid in electronic scanning.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- Your solution should be worked out algebraically. Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

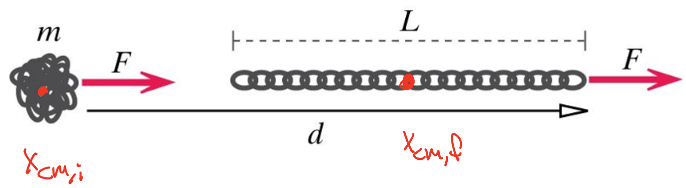
"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."

____

Sign your name on the line above

Problem 1 [25 pts]

A chain of mass M is coiled up into a tight ball on a frictionless table. It is initially at rest. You pull on a link at the end of the chain with a constant force F. Eventually the chain straightens out to its full length L and you keep pulling until you have pulled your end of the chain a total distance d.



A. [3 pts] How much work is done if the system is treated as a point particle at the center of mass?

Constant force, so $W = \int_{x_i}^{x_f} \vec{F}\cdot d\vec{r} = \vec{F}\cdot\Delta\vec{x}$

Point Particle system, so we track center of mass.

$$W = \vec{F}\cdot\Delta\vec{x}_{cm}$$
$$= F\left[\left(d-\frac{L}{2}\right)-0\right]$$

$$\boxed{W = F\left(d-\frac{L}{2}\right)}$$

GRADING

All or Nothing

B. [3 pts] How much work is done on the real system?

Point of contact (POC): right-most chain

$$W = \vec{F}\cdot\Delta\vec{x}_{poc}$$
$$= F\left[d-0\right]$$

$$\boxed{W = Fd}$$

GRADING

All or Nothing

C. [6 pts] Due to friction between the links, the chain increases in temperature as it unwinds. Assume this is a closed system, what is the change in thermal energy?

Using an extended system, by the conservation of energy,

$$W_{hand} = \Delta K_{cm} + \Delta E_{int}$$

S.

$$Fd = F\left(d-\frac{L}{2}\right) + \Delta E_{int}$$

POE from part (B)

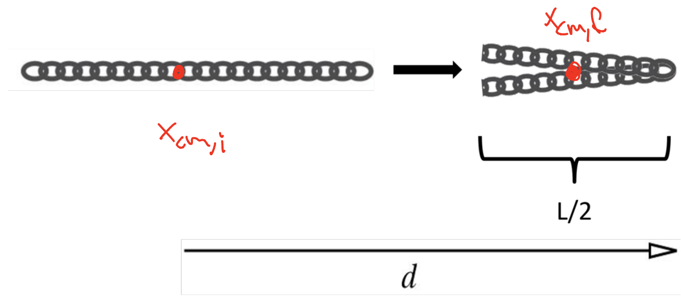
$$\boxed{\Delta E_{int} = F\left(\frac{L}{2}\right)}$$

POE from part (A)

GRADING

-1 Clerical
-2 Minor
-3 Major
-5 BTN

The chain is now fully extended and at rest. You now pull the link in the exact middle of the chain to the right a distance d with the same constant force F as before. In the end, the chain is folded in half.



D. [3 pts] How much work is done if the system is treated as a point particle at the center of mass?

Point of contact (POC): center of mass

$$W = F\Delta x_{poc}$$

$$= F\left[\left(d - \frac{L}{4}\right) - 0\right]$$

$$\boxed{W = F\left(d - \frac{L}{4}\right)}$$

GRADING

All or Nothing

E. [10 pts] Due to friction between the links, and assuming this is a closed system, what is the change in thermal energy?

Just as in part (C), if we use an extended system, the conservation of energy tells us that

$$W_{hand} = \Delta K_{cm} + \Delta E_{int}$$

Our point of contact is the center chain, so

$$W_{hand} = \bar{F} \cdot \Delta \bar{x}_{poc}$$

$$= F(d - 0)$$

$$= Fd$$

Thus,

$$Fd = F\left(d - \frac{L}{4}\right) + \Delta E_{int}$$

POC from part (D)

$$\boxed{\Delta E_{int} = F\left(\frac{L}{4}\right)}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

Problem 2 [25 pts]

Deep frying of frozen turkeys carries a risk of explosion. To be cautious, you decide to warm your turkey in the microwave before frying. Assume the turkey has a mass of 5 kg, and a specific heat of 4 J/gK.

A. [10 pts] The initial turkey temperature is 2 °C. Assume all of the energy from the microwave flows into the turkey as thermal energy. If the oven power is 900 W, how long does it take before the turkey reaches 20 °C?

Convert units: $2°C \rightarrow 275\ °K$, $20°C \rightarrow 293\ °K$, $4\ J/gK \rightarrow 4000\ J/kg\cdot K$

$$\text{energy needed} = Q = m_t C_t \Delta T = (5\,kg)\left(4000\,\frac{J}{kg\cdot K}\right)(293\,°K - 275\,°K) = 360{,}000\,J$$

$$\text{time needed} = \frac{Q}{P} = \frac{360{,}000\,J}{900\,W} = 400\,s$$

$$\boxed{\Delta t = 400\,s}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

B. [5 pts] Suppose 10 liters of frying oil is used, and needs to be heated from an initial temperature of 20 °C to 180 °C. How much thermal energy is needed to do this (the density of the oil is 0.915 kg/liter and the specific heat is 2 J/gK)?

Convert units: $180°C \rightarrow 453°K$, $2\ J/g\cdot K \rightarrow 2000\ \frac{J}{kg\cdot K}$

$$m_{oil} = V_{oil}\,\rho_{oil} = (10L)(0.915\ kg/L) = 9.15\ kg$$

$$Q = m_{oil} C_{oil} \Delta T = (9.15\ kg)(2000\ J/kg\cdot K)(453°K - 293°K)$$

$$\boxed{Q = 2.928 \times 10^6\ J}$$

GRADING

-1 Clerical
-2 Minor
-3 Major

C. [10 pts] After warming the turkey to 20 °C and heating the oil, you carefully put the turkey into the oil. If no additional thermal energy enters or leaves the system, what temperature will the turkey reach?

Closed system → Conservation of energy

$$\Delta Q_t + \Delta Q_{oil} = 0$$

$$M_t C_t \Delta T_t + M_{oil} C_{oil} \Delta T_{oil} = 0$$

$$M_t C_t (T_f - T_{t,i}) + M_{oil} C_{oil} (T_f - T_{oil,i}) = 0$$

So,

$$T_f = \frac{M_t C_t T_{t,i} + M_{oil} C_{oil} T_{oil,i}}{M_t C_t + M_{oil} C_{oil}} = \frac{(5kg)\left(4000\,\frac{J}{kg\cdot K}\right)(293°K) + (9.15\,kg)\left(2000\,\frac{J}{kg\cdot K}\right)(453°K)}{(5kg)\left(4000\,\frac{J}{kg\cdot K}\right) + (9.15\,kg)\left(2000\,\frac{J}{kg\cdot K}\right)}$$

$$\boxed{T_f = 369.5\ °K}$$

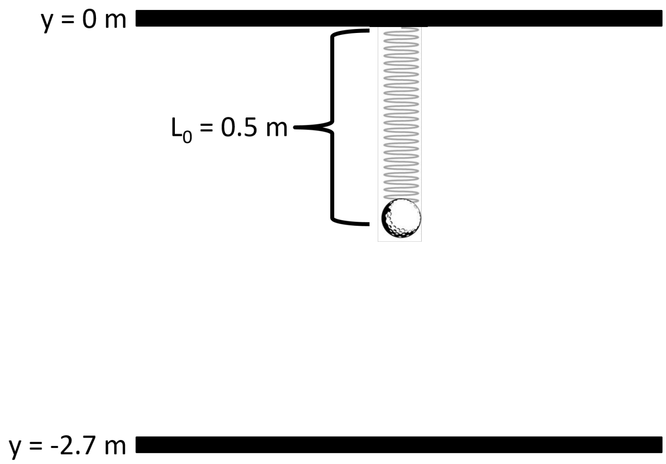
GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

- (The answer you get if you don't convert to kelvin is 96.5 C)

Problem 3 [25 pts]

A spring, equilibrium length 0.5 m and spring constant 100 N/m, hangs from a ceiling, which is a height 2.7 m above the floor. A 2 kg mass is attached to the spring. Your hand is initially holding the mass at rest. Take the system to be the mass, the spring, and the earth.



A. [3 pts] You grab the ball and move it down; the ball starts at $y = -0.5$ m and ends at $y = -0.9$ m. What is the change in spring potential energy in the system?

$$U_s = \frac{1}{2} k s^2 = \frac{1}{2} k (|y| - L_o)^2$$

So,

$$\Delta U_s = U_f - U_i = \frac{1}{2} k \left[(|y_f| - L_o)^2 - (|y_i| - L_o)^2 \right]$$

$$= \frac{1}{2} (100 \tfrac{N}{m}) \left[(.4m)^2 - (0m)^2 \right]$$

$$\boxed{\Delta U_s = 8 \text{ J}}$$

GRADING

-1 Clerical
-2 Minor

B. [3 pts] What is the change in gravitational potential energy in the system?

$$U_g = mgy$$

So,

$$\Delta U_g = mgy_f - mgy_i$$

$$= (2kg)(9.81 \, m/s^2)(-0.9m + 0.5m)$$

$$\boxed{\Delta U_g = -7.85 \text{ J}}$$

GRADING

-1 Clerical
-2 Minor

C. [3 pts] How much work did your hand do to the system?

Conservation of Energy: $E_i + W_{hand} = E_f$

So, $W_{hand} = \Delta E$

$= \Delta U_g + \Delta U_s$ ← watch out for POE

$= -7.85 \text{ J} + 8 \text{ J}$

$$\boxed{W_{hand} = 0.15 \text{ J}}$$

GRADING

All or Nothing

D. [10 pts] You now release the ball. What is the speed of the ball when the spring is at its relaxed length?

At the time of release, $y_i = -0.9\,m$, and $V = 0$, thus

$$E_i = mgy_i + \frac{1}{2}k(|y_i| - L_o)^2$$
$$= (2kg)(9.81\,\tfrac{m}{s^2})(-0.9) + \frac{1}{2}(100\,N/m)(0.9m - 0.5m)^2$$
$$= -9.65\,J$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

By the conservation of energy, $E_i + \cancel{W_{ext}}^{0} = E_f$, so,

$$E_i = E_f = \frac{1}{2}mv^2 + mg(-L_o) + \frac{1}{2}k(|-L_o| - L_o)^2$$

So

$$V = \frac{2}{m}(E_i + mgL_o)^{1/2}$$
$$= \frac{2}{2kg}\left(-9.65\,J + (2kg)(9.81\,\tfrac{m}{s^2})(0.5m)\right)^{1/2}$$

$$\boxed{V = 0.39\,\tfrac{m}{s}}$$

E. [6 pts] After oscillating for a long time, the mass comes to rest at $y = -0.696$ m. If the system is open and does not change temperature, how much energy did the system lose?

Once the ball comes to rest, at $y_f = -0.696\,m$,

$$E_f = mgy_f + \frac{1}{2}k(|y_f| - L_o)^2$$
$$= (2kg)(9.81\,m/s^2)(-0.696m) + \frac{1}{2}(100\,\tfrac{N}{m})(0.696m - .5m)^2$$

$$= -11.73\,J$$

The ball started with

GRADING

-1 Clerical
-2 Minor
-3 Major
-5 BTN

$$E_i = -9.65\,J$$

So the ball "gained"

$$\Delta E = (-11.73\,J) - (-9.65\,J)$$

$$\boxed{\Delta E = -2.07\,J}$$

Problem 4 [25 pts]

The code below models the interaction between an alpha particle and a gold nucleus interacting through the electric force. The code is similar to the one you completed in your homework except in this example the gold nucleus remains stationary. **Add in the missing lines of code below to complete the program.**

```
GlowScript 2.7 VPython
b = 1e-14 ## impact parameter
m_alpha = 4*1.67e-27  ## mass of the alpha particle in kg
qa=2*1.6e-19 ## charge of alpha particle
qg=79*1.6e-19 ## charge of gold nucleus
oofpez=9e9 ## one over four pi epsilon_0
alpha = sphere(pos=vector(-2e-13,b,0), radius=2e-15, color=color.cyan)
gold = sphere(pos=vector(0,0,0), radius=8e-15, color=color.yellow)
alpha.p = vector(1.46e-19,0,0) ## The initial momentum of the alpha particle
deltat = 1e-25
t = 0
while t < 2e-20:
    ## A. [8 pts] calculate the force on alpha particle by gold nucleus
    r = alpha.pos - gold.pos
    rhat = norm(r)
    rmag = mag(r)
    F_alpha = oofpez*qa*qg/(rmag**2)*rhat
```

GRADING

-1 Clerical/Syntax (per occurrence)
-2 Minor
-4 Major
-7 BTN

```
    ## B. [2 pts] update the momentum of the alpha particle
    alpha.p = alpha.p + F_alpha*deltat
```

GRADING

-1 Clerical/Syntax (per occurrence)
-1 Minor (like an obvious mental lapse due to rushing.

```
    ## C. [2 pts] update position of the alpha particle
    alpha.pos= alpha.pos + (alpha.p/m_alpha)*deltat
```

GRADING

-1 Clerical/Syntax (per occurrence)
-1 Minor (like an obvious mental lapse due to rushing.

```
    ## B. [3 pts] update the total energy E of the alpha+gold system
(+1)    K_alpha = mag(alpha.p)**2/(2*m_alpha)
(+1)    U = oofpez*qa*qg/rmag
(+1)    E = K_alpha + U
    t=t+deltat
```

GRADING

-1 Clerical/Syntax (per occurrence)
+1 for each correct statement

This problem continues on the next page

E. [10 pts] Using the numerical values given in the code, calculate by hand the shortest distance between the alpha particle and the gold nucleus if the impact parameter is set to zero ($b = 0$).

For $b=0$, the closest point occurs at the turning point of the particle, right when $v=0$. Lets use this turning point as our final state. Then, by the conservation of energy

$$\frac{P_{i,a}^2}{2m_a} + K\frac{q_a q_g}{|r_i|} = \cancel{\frac{P_{f,a}^2}{2m_a}} + K\frac{q_a q_g}{|r_f|}$$

0

So,

$$|r_f| = K q_a q_g \left(\frac{P_{ia}^2}{2m_a} + K\frac{q_a q_g}{|r_i|}\right)^{-1}$$

$$= (9e9)(2\times 1.6\times 10^{-19}\,C)(79\times 1.6\times 10^{-19}\,C)\left(\frac{(1.46\times 10^{-19}\,kg\frac{m}{s})^2}{2(4\times 1.67\times 10^{-27}\,kg)} + (9e9)\frac{(2\times 1.6\times 10^{-19}\,C)(79\times 1.6\times 10^{-19}\,C)}{(2\times 10^{-13}\,m)}\right)^{-1}$$

$$= 2.05\times 10^{-14}\ m$$

Since the particle is to the left of the origin (gold nucleus)

$$\boxed{r_f = -2.05\times 10^{-14}\ m}$$

GRADING

-1 Clerical
-2 Minor
-4 Major
-8 BTN

This page is for extra work, if needed.

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