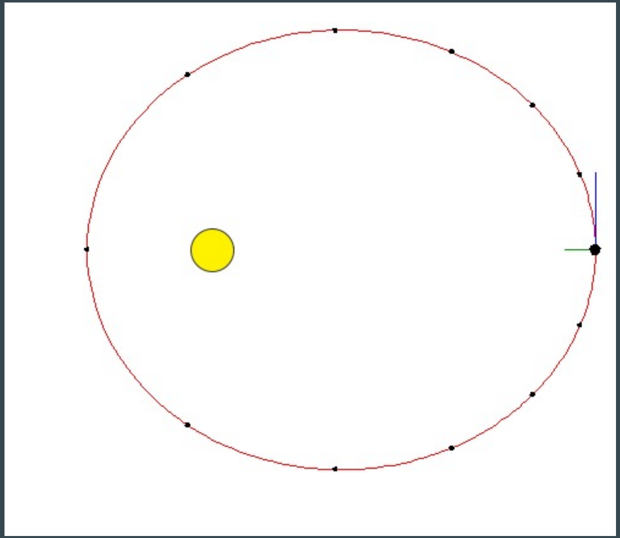
~~6~~ clicker questions today

4



PHYS 2211 K

Week 6, Lecture 1

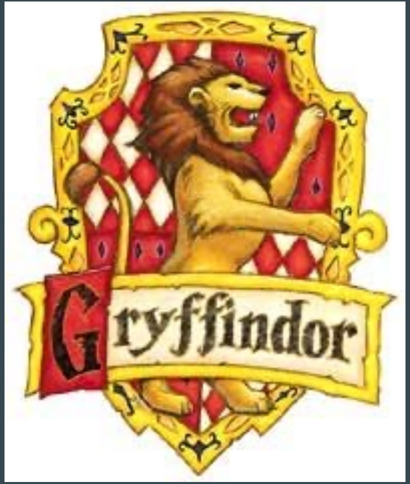
2022/02/15

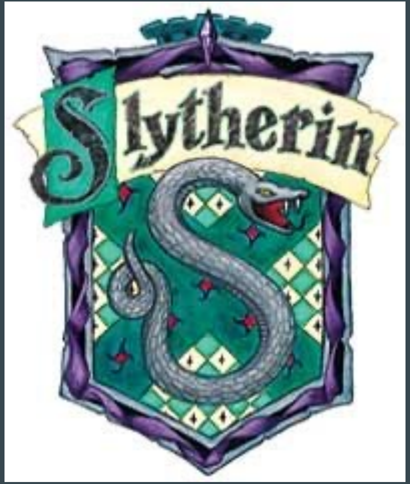
Dr Alicea (ealicea@gatech.edu)

On today’s class…

1. Wrapping up equilibrium
2. Non-equilibrium: curving motion
3. Parallel and perpendicular coordinates

CLICKER 1: Better be…





A. Gryffindor

B. Hufflepuff

C. Ravenclaw

D. Slytherin

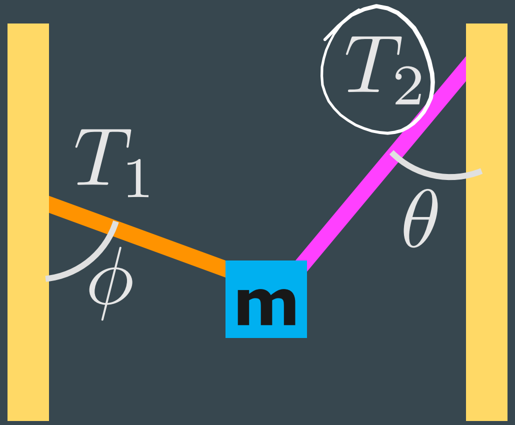
Equilibrium problems

- When $\vec{F}_{\text{net}} = 0$ this means all the forces acting on the system are balanced

- All the x components of all the forces add up to zero
- All the y components of all the forces add up to zero
- All the z components of all the forces add up to zero

$$\vec{F}_{\text{net,x}} = 0 \qquad \vec{F}_{\text{net,y}} = 0 \qquad \vec{F}_{\text{net,z}} = 0$$

Example: A block of mass m hangs motionless tied to two ropes which make different angles on two parallel walls. What is the magnitude of Tension 2?



$F_{net\,x} = 0$

$-T_1 \sin\phi + T_2 \sin\theta = 0$

$T_1 \sin\phi = T_2 \sin\theta$

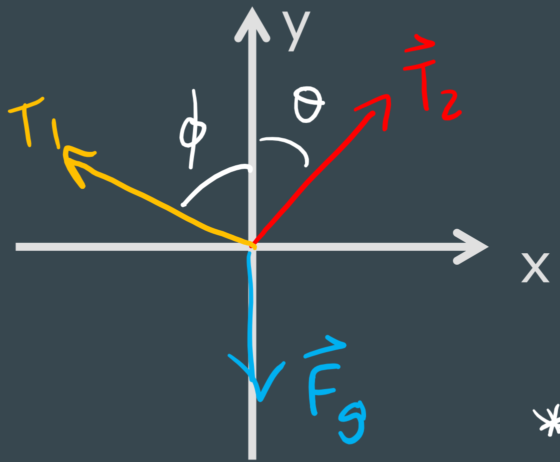
$T_1 = T_2 \dfrac{\sin\theta}{\sin\phi}$

$F_{net\,y} = 0$

$T_1 \cos\phi + T_2 \cos\theta - mg = 0$

$T_1 \cos\phi + T_2 \cos\theta = mg$

$\left(T_2 \dfrac{\sin\theta}{\sin\phi} \cos\phi\right) + T_2 \cos\theta = mg$



$T_2 \left(\dfrac{\sin\theta}{\sin\phi} \cos\phi + \cos\theta\right) = mg$

$T_2 \left(\dfrac{\sin\theta \cos\phi + \cos\theta \sin\phi}{\sin\phi}\right) = mg$

$$* \boxed{T_2 = \frac{mg \sin\phi}{\sin\theta \cos\phi + \cos\theta \sin\phi}} *$$

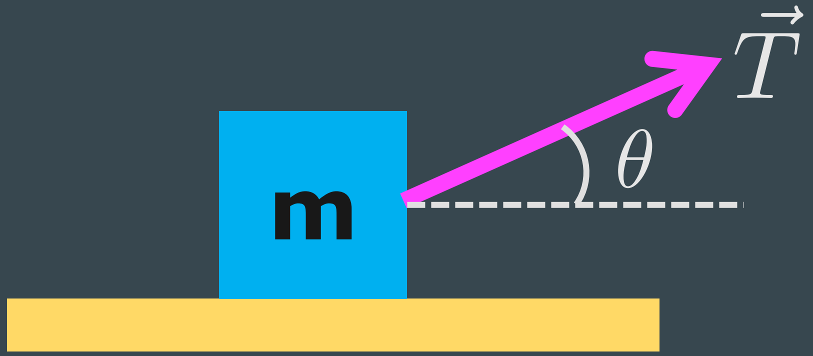
CLICKER 2: A block of mass m is dragged along a table by a string so it moves to the right at constant speed v. The string makes an angle θ with the table, and the coefficient of kinetic friction between table and block is μ. What is the magnitude of the tension force?

A. $T = \dfrac{mg}{\mu\cos\theta + \sin\theta}$

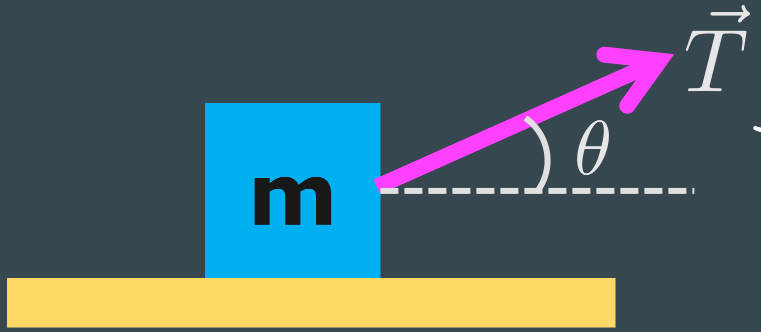
B. $T = \dfrac{\mu N}{\cos\theta}$

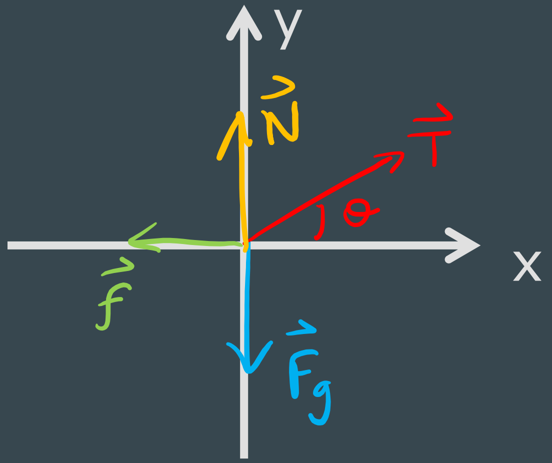
C. $T = N - mg\sin\theta$

D. $T = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$



Solution: A block of mass m is dragged along a table by a string so it moves to the right at constant speed v. The string makes an angle θ with the table, and the coefficient of kinetic friction between table and block is μ. What is the magnitude of the tension force?





$F_{net\,x} = 0$

$-f + T\cos\theta = 0$

$f = T\cos\theta$

$F_{net\,y} = 0$

$N + T\sin\theta - mg = 0$

$N = mg - T\sin\theta$

$$* \ |\vec{f}| = \mu|\vec{N}| \ *$$

$$T\cos\theta = \mu(mg - T\sin\theta)$$

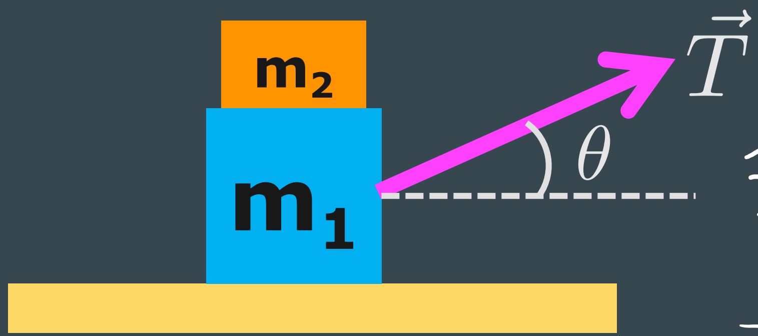
$$T\cos\theta = \mu mg - \mu T\sin\theta$$

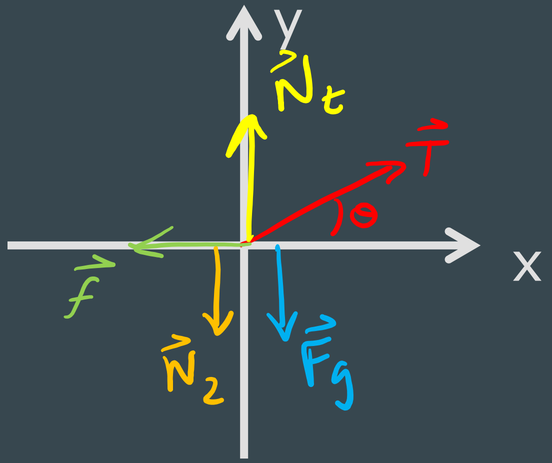
$$T\cos\theta + \mu T\sin\theta = \mu mg$$

$$T(\cos\theta + \mu\sin\theta) = \mu mg$$

$$* \ \boxed{T = \frac{\mu mg}{\cos\theta + \mu\sin\theta}} \ *$$

Question! How would the solution change if there was another block on top of the block?

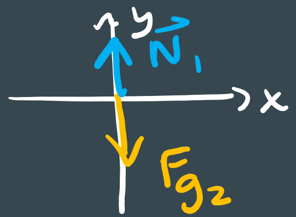




$F_{net_x} = 0$

$f = T\cos\theta$

For m_2



$N_1 - m_2 g = 0$

$N_1 = m_2 g$ *

normal from m_1 on m_2

From Newton's 3rd

$|N_1| = |N_2|$

$F_{net_y} = 0$

$N_t + T\sin\theta - m_1 g - N_2 = 0$

$N_t + T\sin\theta - m_1 g - m_2 g = 0$

$N_t = m_1 g + m_2 g - T\sin\theta$

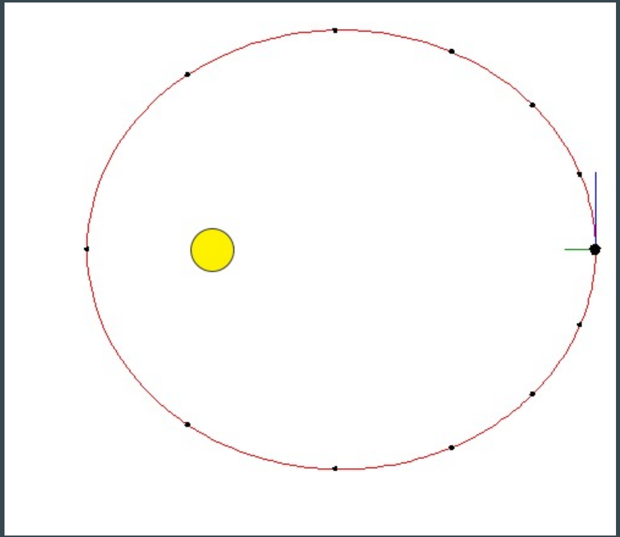
$N_t = \underline{(m_1 + m_2)}g - T\sin\theta$

Then solve as previous problem.

New topic! When $\vec{F}_{\text{net}} \neq 0$

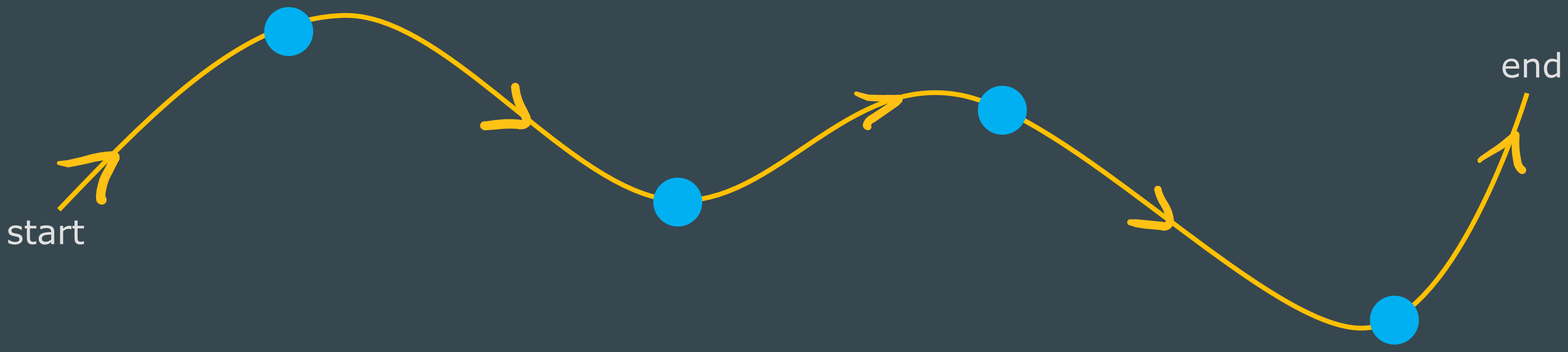
- Fnet = 0 means the system is in equilibrium
  - Static equilibrium if v = 0
  - Dynamic equilibrium if v = constant (nonzero)

- Fnet ≠ 0 means the system is NOT in equilibrium
  - Moving with non-constant speed (speeding up or slowing down)
  - Changing direction of motion (turning)



Parallel and perpendicular coordinates

- Parallel – in the direction of motion (we call this p-hat) $\hat{p}$
- Perpendicular – orthogonal to direction of motion (positive towards where the object’s trajectory is turning; we call this n-hat) $\hat{n}$
- The coordinates move and change with the object’s motion!



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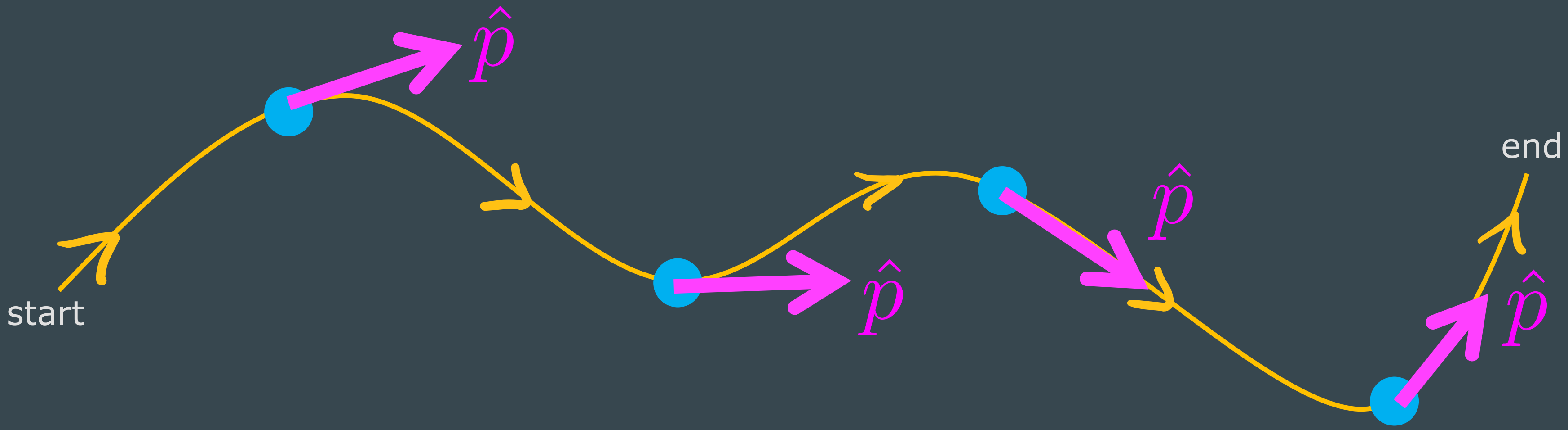

direction of motion/velocity/momentum (tangent to trajectory) is p-hat

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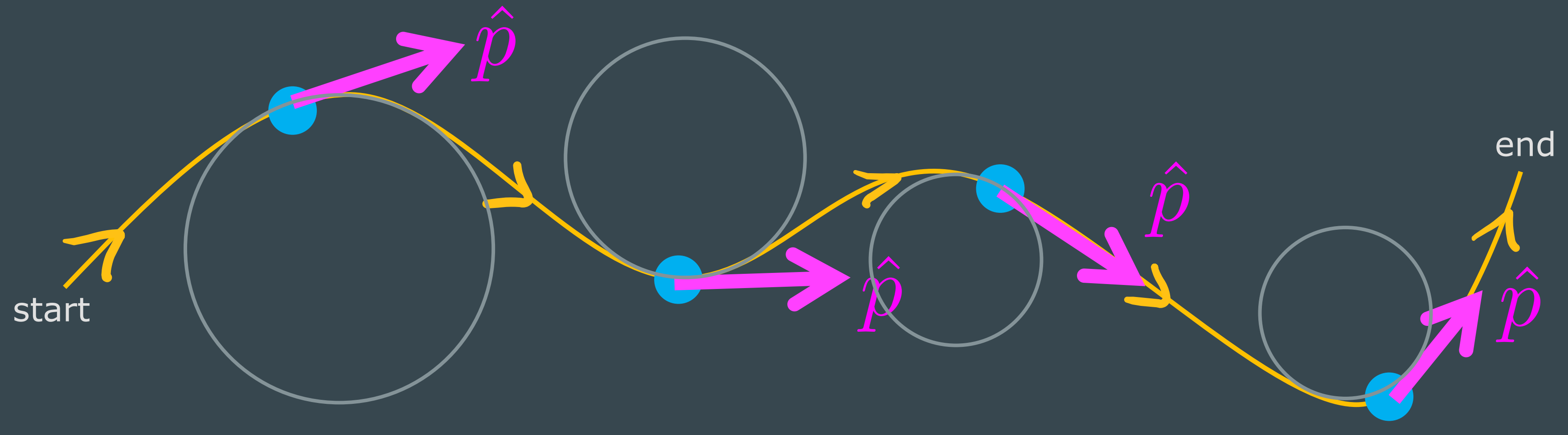

"osculating circle" or "kissing circle" or "turning circle"

Parallel and perpendicular coordinates

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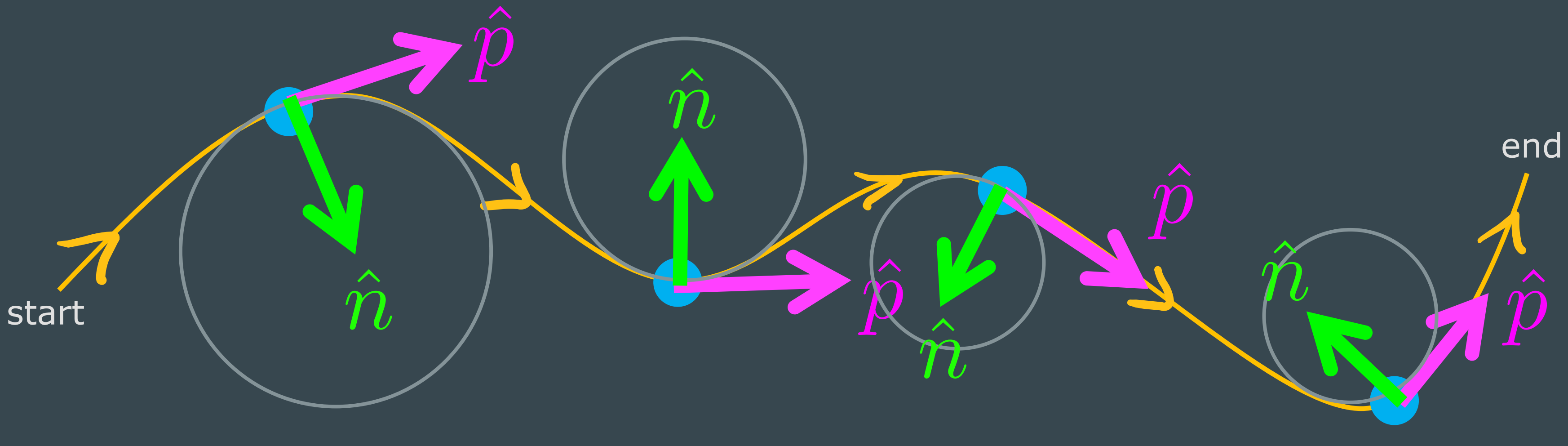
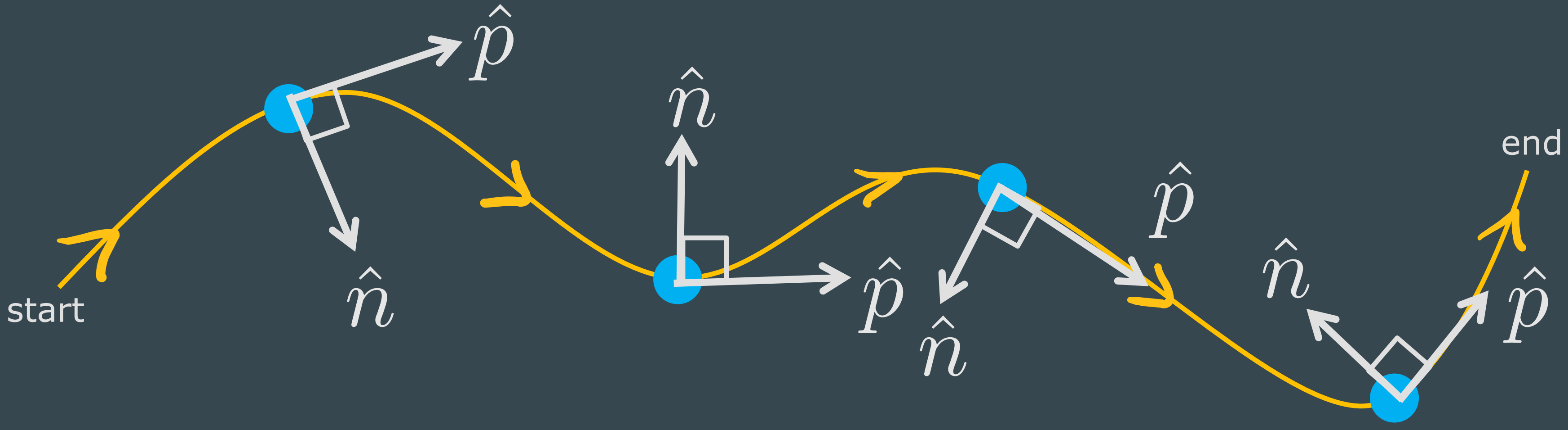


towards the center of the circle (perpendicular to trajectory) is positive n-hat

Parallel and perpendicular coordinates

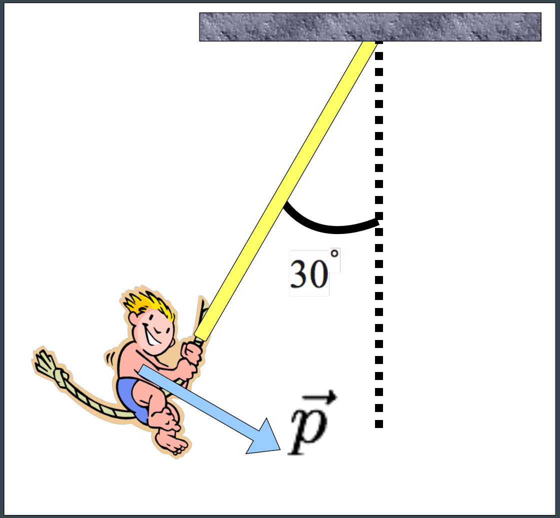
- Parallel – in the direction of motion (we call this p-hat) $\hat{p}$
- Perpendicular – orthogonal to direction of motion (positive towards where the object's trajectory is turning; we call this n-hat) $\hat{n}$
- The coordinates move and change with the object's motion!

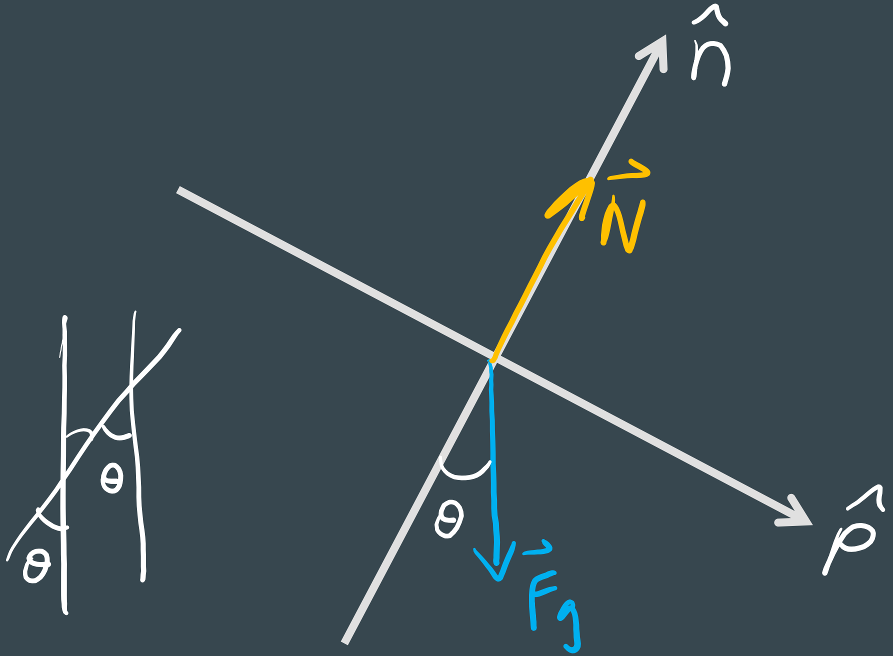


p-hat and n-hat are perpendicular to each other and move with the object

Parallel and perpendicular components

- We can find components of forces on parallel/perpendicular axes in the same way that we would for x-y axes, but we call the axes p-hat and n-hat instead





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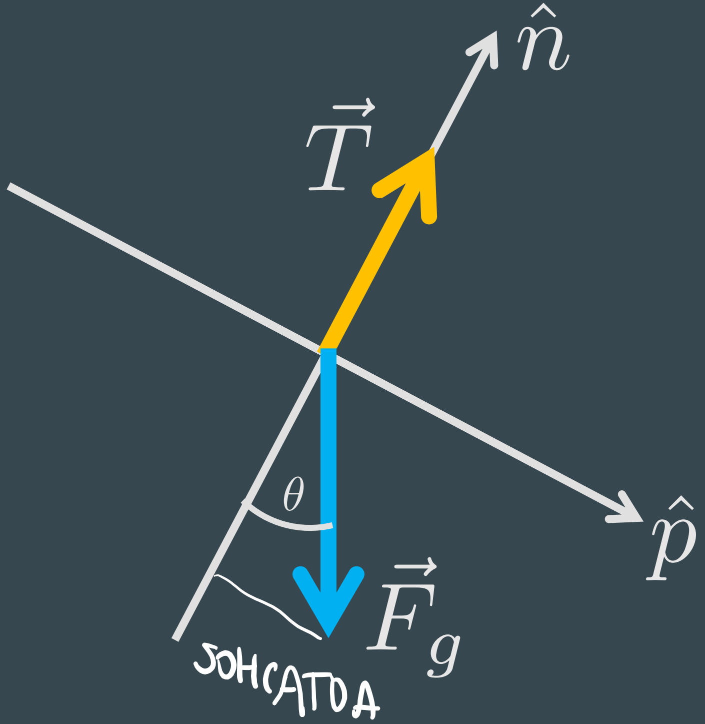


$$\vec{F}_{\text{net}\parallel} = mg\sin\theta\;(\hat{p})$$

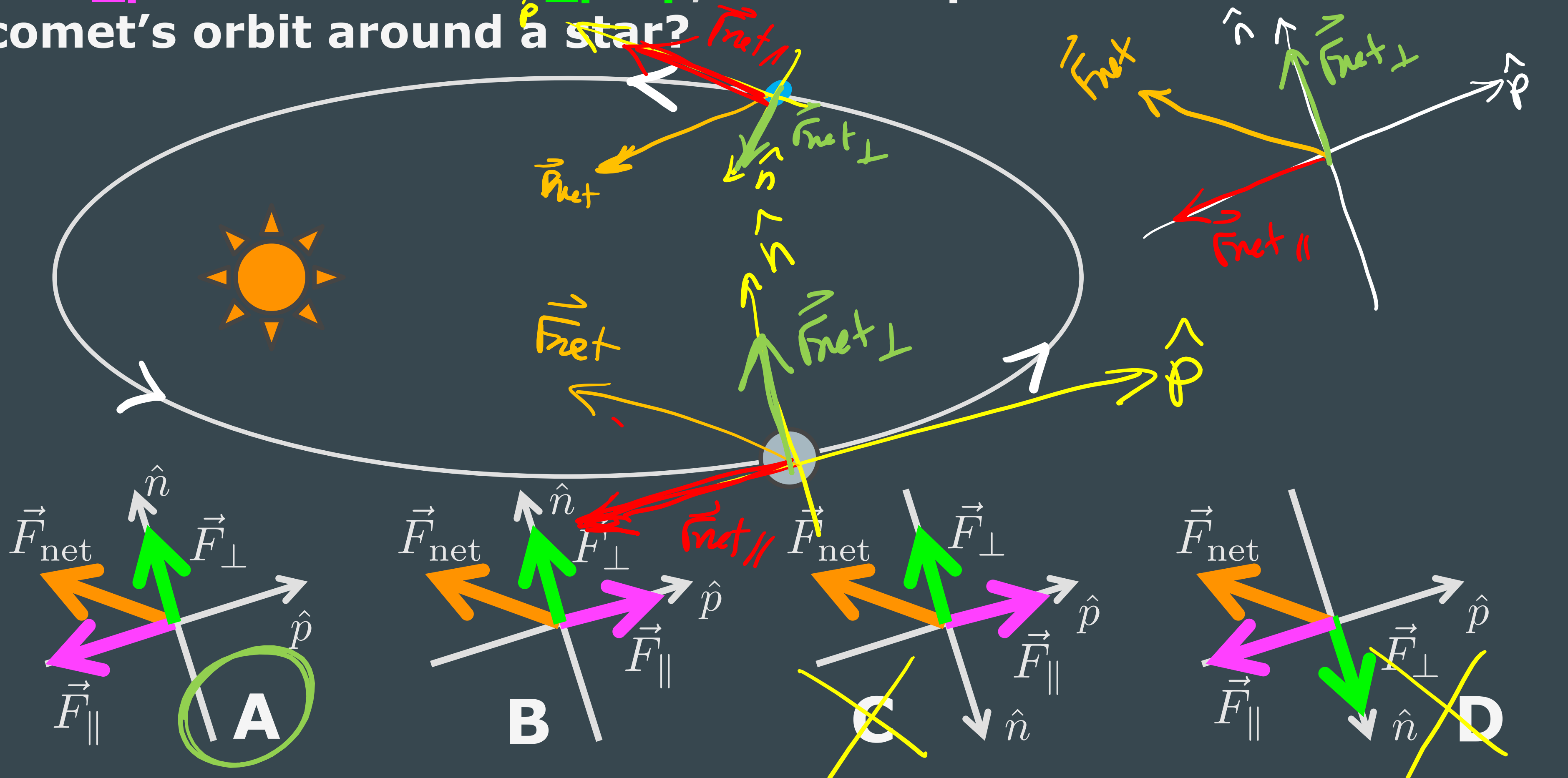
(parallel)

$$\vec{F}_{\text{net}\perp} = T(\hat{n}) + mg\cos\theta\,(-\hat{n})$$

(perpendicular)

$$= (T - mg\cos\theta)\,\hat{n}$$

CLICKER 3: What is the correct FBD, indicating Fnet, Fnet_parallel and Fnet_perp, for this specific location in a comet's orbit around a star?



Parallel and perpendicular components

- Remember Newton's 2nd law:

$$\vec{F}_{\mathrm{net}} = \frac{d\vec{p}}{dt}$$

- If forces can be decomposed into parallel and perpendicular components, then we can do the same to changes in momentum, and this will give us information about the effect that the net force acting on the object has on the object's motion

Parallel and perpendicular components

- Start from this:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

- Remember that any vector can be expressed as the product of its magnitude and its direction

$$\vec{p} = |\vec{p}|\hat{p}$$

- Therefore:

$$\vec{F}_{\text{net}} = \frac{d}{dt}(|\vec{p}|\hat{p})$$

Parallel and perpendicular components

- Product rule! $\dfrac{d}{dt}(AB) = A\dfrac{dB}{dt} + B\dfrac{dA}{dt}$

$$\vec{F}_{\text{net}} = \frac{d}{dt}\left(|\vec{p}|\hat{p}\right) =$$

$$= \underbrace{\hat{p}\,\frac{d|\vec{p}|}{dt}}_{\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \vec{F}_{\text{net}\,\parallel}} + \underbrace{|\vec{p}|\,\frac{d\hat{p}}{dt}}_{\left(\frac{d\vec{p}}{dt}\right)_{\perp} = \vec{F}_{\text{net}\,\perp}}$$

The parallel component

$$\vec{F}_{\mathrm{net}\|} = \left(\frac{d\vec{p}}{dt}\right)_{\|} = \frac{d|\vec{p}|}{dt}\hat{p} = \left(\frac{|\vec{p}_f| - |\vec{p}_i|}{\Delta t}\right)\hat{p}$$

- Forces parallel (or anti-parallel) to the direction of motion

- Changes the magnitude of the momentum (speeding up or slowing down)

The parallel component

- REMEMBER! "the change in the magnitude" is not the same as "the magnitude of the change"

change in the magnitude $$\Delta|\vec{p}| = |\vec{p}_f| - |\vec{p}_i|$$

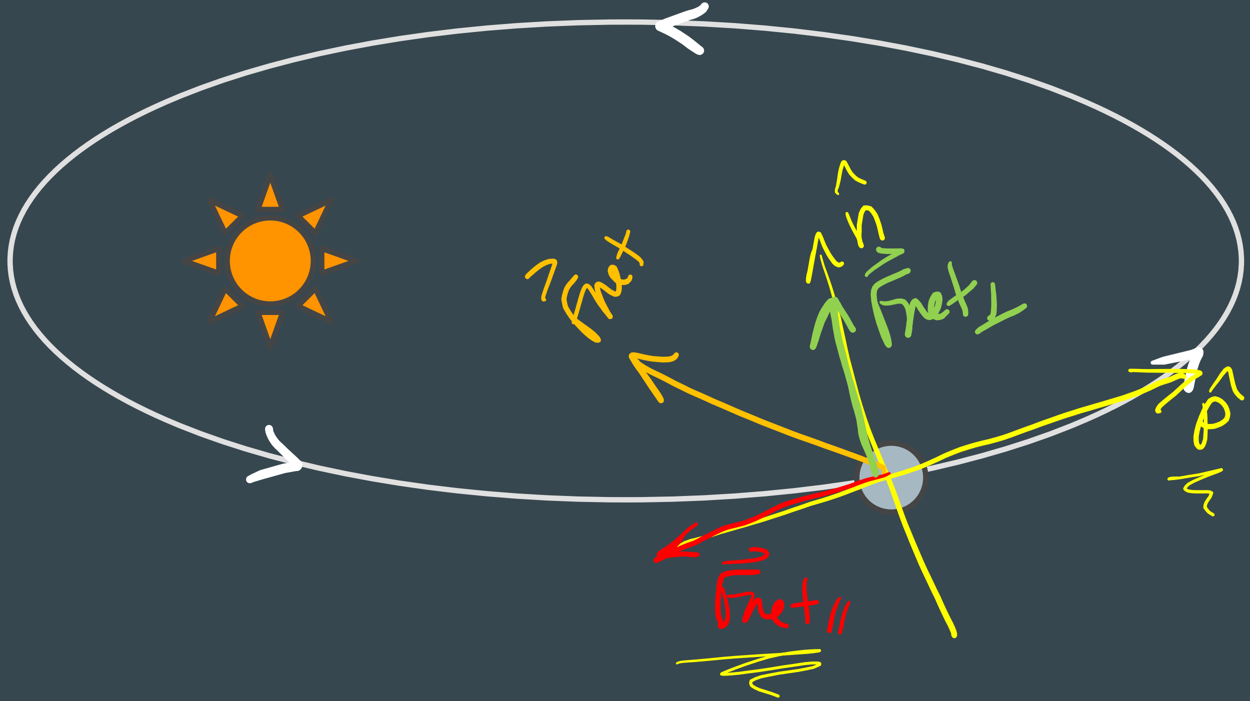
magnitude of the change $$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i|$$

CLICKER 4: Is the comet speeding up, slowing down, or moving at constant speed at this location in its orbit?

A. Speeding up

B. Slowing down

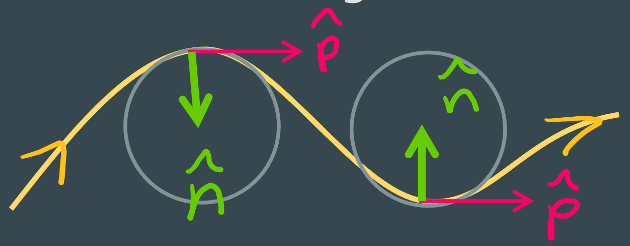
C. Constant speed



The perpendicular component

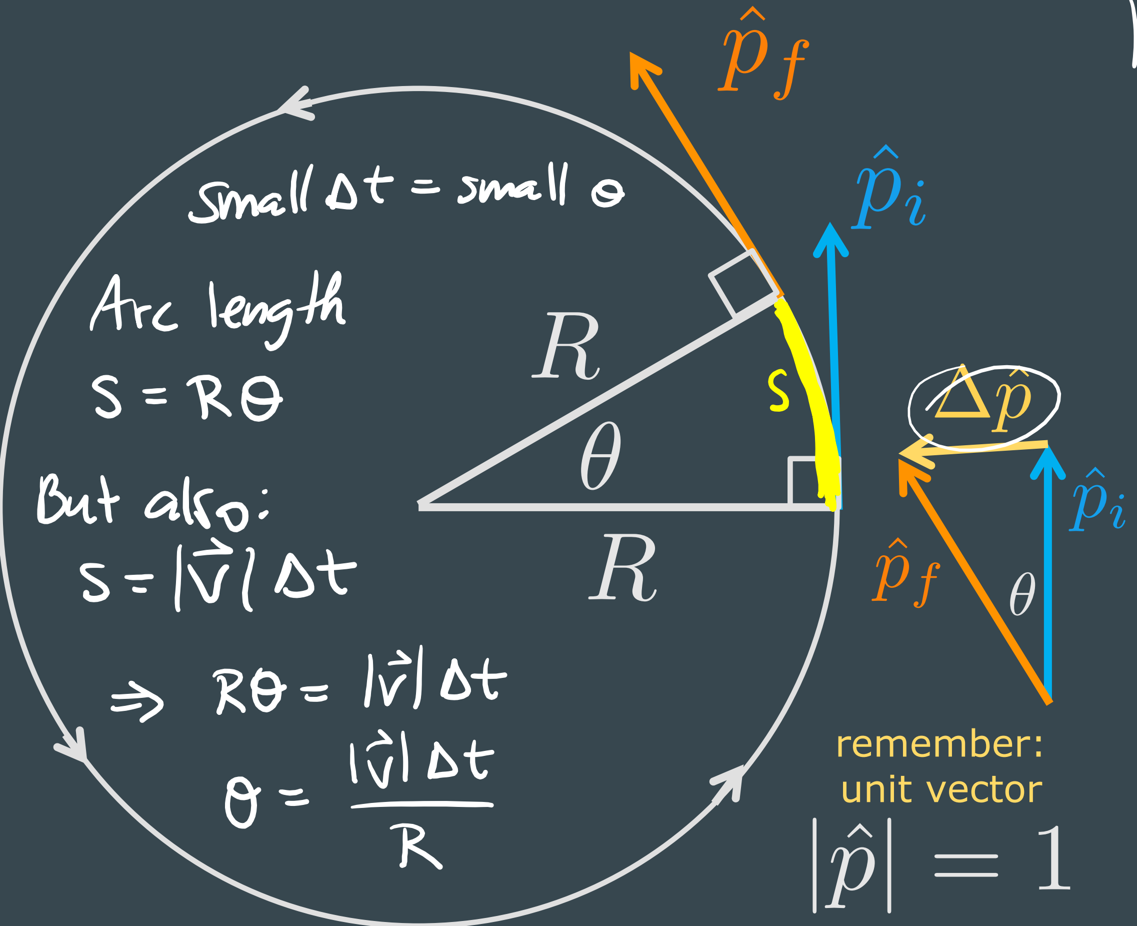
$$\vec{F}_{\text{net}\perp} = \left(\frac{d\vec{p}}{dt}\right)_\perp = |\vec{p}|\frac{d\hat{p}}{dt} = \frac{mv^2}{R}\hat{n}$$

- Forces perpendicular to the direction of motion; positive n-hat points towards the center of the turning circle



- Changes the direction of the momentum (turning)

The perpendicular component



Small Δt = small θ

Arc length

$s = R\theta$

But also:

$s = |\vec{v}|\Delta t$

$\Rightarrow R\theta = |\vec{v}|\Delta t$

$\theta = \dfrac{|\vec{v}|\Delta t}{R}$

remember:
unit vector

$$|\hat{p}| = 1$$

$$|\Delta\hat{p}| = \theta\,|\cancel{\hat{p}_i}|^{\,1}$$

$$\Delta\hat{p} = \theta\,\hat{n}$$

$$\Delta\hat{p} = \frac{|\vec{v}|\Delta t}{R}\,\hat{n}$$

$$\frac{\Delta\hat{p}}{\Delta t} = \frac{|\vec{v}|}{R}\,\hat{n}$$

$$\frac{d\hat{p}}{dt} = \frac{|\vec{v}|}{R}\,\hat{n}$$

F_{net}

$$|\vec{p}|\,\frac{d\hat{p}}{dt} = |\vec{p}|\,\frac{|\vec{v}|}{R}\,\hat{n} = \frac{mv^2}{R}\,\hat{n}$$

The perpendicular component

$$\vec{F}_{\text{net}\perp} = |\vec{p}|\frac{d\hat{p}}{dt} \approx |\vec{p}|\frac{\Delta\hat{p}}{\Delta t} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \boxed{\frac{mv^2}{R}\hat{n}}$$

- n-hat points towards the center of the turning circle
- R is the radius of the turning circle
- v is the speed of the object
- The term v^2/R is called centripetal acceleration
- When |v| is constant, Fnet_parallel = 0, which means Fnet = Fnet_perp, and we call this uniform circular motion