Please remove this sheet before starting your exam.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}}$$

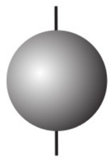
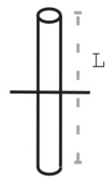
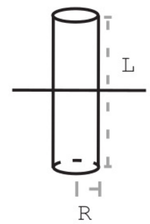
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of inertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

				
$I = \frac{2}{5} M R^2$	$I = \frac{1}{2} M R^2$	$I = \frac{1}{12} M L^2$	$I = \frac{1}{3} M L^2$	$I = \frac{1}{12} M L^2 + \frac{1}{4} M R^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	k	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

PHYS 2211 KMR - Test 3 - Spring 2022

Please clearly print your name & GTID in the lines below

Name: ______________________ GTID: ______________________

Instructions

- This exam is closed internet/books/notes, except for the Formula Sheet which is included with the exam.
- You must work individually and receive no assistance from any person or resource.
- You are not allowed to post screenshots, files, or any other details of the test anywhere online, not even after the test is over.
- Work through all the problems first, then scan/upload your solutions after time is called.
  - Your uploaded files **must** be in either PNG, JPG, or PDF format.
  - Your uploaded files must be readable in order to be graded. Unreadable files will earn a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually.
  - Clearly label your work for each sub-part and box the final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solution should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step. Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
  - Make explanations correct but brief. You do not need to write a lot of prose.
  - Include diagrams!
  - **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

**"In accordance with the Georgia Tech Honor Code,
I have not given or received unauthorized aid on this test."**

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Sign your name on the line above

Stew – Q2 in Gradescope [30 pts]

You get home from school and decide to heat up some vegetable stew for dinner. The stew, which has a heat capacity of $C_s = 3.8$ J/(g°C) and a density of $\rho_s = 1500$ g/L, has been sitting in a large pot in the fridge at a temperature of $T_{si} = 5$°C since you cooked it last night. One serving of stew is $m_s = 650$ g.

1. [15 pts] Your microwave has a power rating of $P = 700$ W. How much time does it take to heat one serving of stew to a temperature of $T_{sf} = 70$°C?

$$\Delta E_{thermal,\,stew} = m_s C_s \Delta T_{stew,\,heating}$$

$$= m_s C_s (T_{sf} - T_{si})$$

$$= (650\,g)(3.8\ J/g°C)(65°C)$$

$$= 1.6 \times 10^5\ J$$

$$t_{heat} = \frac{\Delta E_{th,stew}}{P_{microwave}} = \frac{1.6\times10^5\ J}{700\ J/s} = \boxed{229\,s}$$

2. [10 pts] The volume of a spoonful of stew is $V_s = 0.015$ L. When you bring it to your mouth, you realize the stew is too hot, so you start blowing on it. One blow puts $m_a = 258$ g of air at $T_a = 37°C$ in contact with the contents of the spoon. How many times do you need to blow on the spoon to lower the stew's temperature by 5°C? Assume the system consisting of the spoonful of stew and the air blown on it are a closed system that reaches thermal equilibrium. The heat capacity for the air you blow is $C_a = 0.8$ J/(g°C).

$$\Delta E_{\substack{\text{air-stew}\\\text{mixture}}} \overset{\text{closed}}{=} 0 = \Delta E_{th,stew} + \Delta E_{th,air}$$

$$0 = (\rho_s V_s) \cdot C_s \cdot (\Delta T_{stew,cooling}) + m_a C_a \Delta T_{air}$$

$$0 = (22.5g)(3.8 \text{ J/g°C})(-5°C) + (m_{air,tot})(0.8 \text{ J/g°C})(65°C - 37°C)$$

$$\Rightarrow m_{air,tot} = 19.08 \text{ g} \quad \text{air required}$$

$$\# \text{ blows} = \frac{m_{air,tot}}{\underbrace{m_a}_{\text{mass/blow}}} = \boxed{0.07 \text{ blows}}$$

3. [5 pts] Is your answer to the previous part realistic? (yes/no). Very briefly explain your reasoning.

No, it is only a small fraction of a blow.
The main unjustified assumption we made in the previous part is that the system is closed.

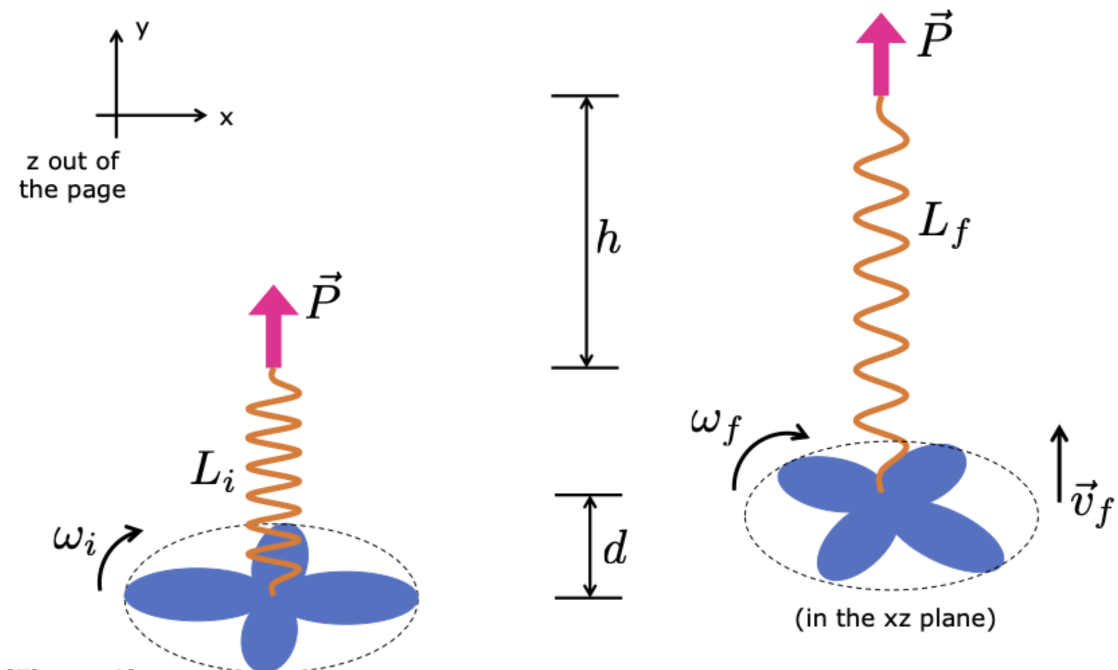
Note on grading: Full credit for "system is not closed".
Partial credit for other sensible answers.

Propeller – Q3 in Gradescope [50 pts]

A propeller toy consists of four circular blades, each of mass m and radius r, connected to each other and attached to a massless spring of stiffness k and relaxed length L_0, as seen in the diagram. You pull upwards $(+\hat{y})$ on the spring with a force of constant magnitude P, causing the spring to be stretched to a length L_i. When this happens, the blades are rotating in the xz plane (parallel to the ground) with angular speed ω_i. This is the initial state (Figure 1).

You continue pulling and some time later your hand has moved up a distance h, causing the spring to be stretched to a new length L_f. The center of mass of the system has moved up a distance d and now moves upwards with an unknown speed v_f. Each blade feels a force from the air of magnitude $F = \alpha v^2$ pointing **opposite to the velocity** of the object (α is a positive constant) that causes their angular speed to change to ω_f. This is the final state (Figure 2).

Consider the system to consist of the four circular blades AND the spring.



(Figure 1) **(Figure 2)**

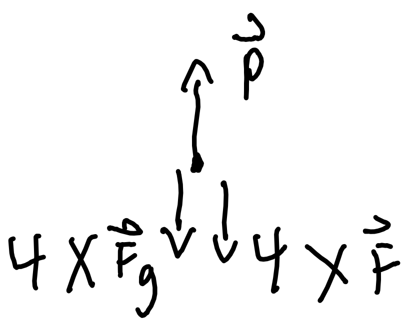
1. [5 pts] What is the magnitude of the net force acting on the **point particle system**?

   (a) $F_{\text{net}} = P - mg - \alpha v^2$

   (b) $F_{\text{net}} = P - 4mg + \alpha v^2$

   (c) $F_{\text{net}} = P - mg + 4\alpha v^2$

   (d) $F_{\text{net}} = P - 4mg - 4\alpha v^2$



2. [5 pts] What is the work done on the **point particle system**?

(a) $W_{\text{cm}} = Pd - 4mgd - 4\alpha \int_0^d v^2 dy$

(b) $W_{\text{cm}} = Ph - 4mgh + \alpha \int_0^h v^2 dy$

(c) $W_{\text{cm}} = P(L_f - L_i) - 4mg(L_f - L_i) + 2\alpha \int_0^{L_f - L_i} v^2 dy$

(d) $W_{\text{cm}} = P(h-d) - mg(h-d) - \alpha \int_0^{h-d} v^2 dy$

$W_{cm} = \vec{F}_{net} \cdot \vec{r}_{cm}$, $\vec{r}_{cm}$ = disp. of center of mass

3. [10 pts] If the system **started at rest**, what is the final speed v_f? Do not evaluate any integrals in your result.

$$\Delta E_{pt\text{-}particle} = \Delta K_{trans} = W_{cm}$$

$$\Delta K_{trans} = K_f - \cancel{K_i}^{\,0}$$

$$\Rightarrow K_f = \frac{1}{2}(4m)v_f^2 = Pd - 4mgd - 4\alpha \int_0^d v^2 dy$$

$$\Rightarrow \boxed{v_f = \sqrt{\frac{1}{2m}\left[Pd - 4mgd - 4\alpha \int_0^d v^2 dy\right]}}$$

4. [5 pts] What is the change in **spring potential energy** (ΔU_s) of the system?

(a) $\Delta U_s = \frac{1}{2}kh^2 - \frac{1}{2}kd^2$
(b) $\Delta U_s = \frac{1}{2}kL_f^2 - \frac{1}{2}kL_i^2$
(c) $\Delta U_s = \frac{1}{2}k(L_f - L_i)^2$
(d) $\Delta U_s = \frac{1}{2}k(L_f - L_0)^2 - \frac{1}{2}k(L_i - L_0)^2$

5. [5 pts] What is the total work done on the **extended (multiparticle) system**?

(a) $W_{\mathrm{R}} = Pd - 4mgh + \alpha \int_0^d v^2 dy$
(b) $W_{\mathrm{R}} = Ph - 4mgd - 4\alpha \int_0^d v^2 dy$
(c) $W_{\mathrm{R}} = PL_f - 4mgd + 4\alpha \int_0^{L_f} v^2 dy$
(d) $W_{\mathrm{R}} = P(h - d) - mg(h - d) - \alpha \int_0^{h-d} v^2 d$

Use the displacement of the point where each force is applied.

6. [20 pts] Determine what is the **final angular speed** ω_f **of the system**. The moment of inertia of a single blade about its edge is $I = (2/3)mr^2$. To simplify the algebra, you may use any of the quantities you've so far determined as variables, if needed (F_{net}, W_{cm}, v_f, ΔU_s, and/or W_{R}). Do not evaluate any integrals in your result.

$$\Delta E_{\text{real system}} = \Delta K_{\text{trans}} + \Delta K_{\text{rot}} + \Delta U_s = W_R$$

$$\frac{1}{2}(4m)v_f^2 + 4 \cdot \frac{1}{2} I (w_f^2 - w_i^2) + \Delta U_s = W_R$$

$$\Rightarrow w_f = \sqrt{\frac{1}{2I}\left[W_R - 2mv_f^2 + 2Iw_i^2 - \Delta U_s\right]}$$

$$\boxed{w_f = \sqrt{\frac{3}{4mr^2}\left[W_R - 2mv_f^2 - \Delta U_s\right] + w_i^2}}$$

Collision – Q4 in Gradescope [20 pts]

An point mass $m_1 = m$ moves with velocity $\vec{v}_{1i} =< v, 0, 0 >$ when it collides with another point mass $m_2 = m$ which was stationary, $\vec{v}_{2i} = 0$. Prove mathematically that if this is an **elastic** collision, all the momentum of point mass 1 will be transferred to point mass 2. In other words, show that $\vec{v}_{1f} = 0$ and $\vec{v}_{2f} =< v, 0, 0 >$.

Hint: think about what two quantities are conserved during an elastic collision.

Initial

$m \quad \vec{v}_{1i} = \langle v, 0, 0 \rangle$

$\cdot\ m$

$\vec{v}_{2i} = 0$

Final

?

Because this is a (head-on) collision of two point masses, we only need to consider the x-direction.

Conservation of momentum: $\vec{p}_i = \vec{p}_f$

$$\cancel{m}\vec{v}_{1i} + \cancel{m}\vec{v}_{2i} = \cancel{m}\vec{v}_{1f} + \cancel{m}\vec{v}_{2f}$$

$$x: \quad v + 0 = v_{1f,x} + v_{2f,x}$$

$$\Rightarrow \underbrace{v_{1f,x} = v - v_{2f,x}} \qquad (*)$$

Conservation of energy: $KE_i = KE_f$

$$\cancel{\tfrac{1}{2}m}\vec{v}_{1i}^{\,2} + \cancel{\tfrac{1}{2}m\vec{v}_{2i}^{\,2}}^{\,0} = \cancel{\tfrac{1}{2}m}\vec{v}_{1f}^{\,2} + \cancel{\tfrac{1}{2}m}\vec{v}_{2f}^{\,2}$$

$$v^2 = v_{1f,x}^2 + v_{2f,x}^2 \qquad (**)$$

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Subbing in (*) into (* *), we get

$$v^2 = (v - v_{2f,x})^2 + v_{2f,x}^2$$

$$\cancel{v^2} = \cancel{v^2} - 2v v_{2f,x} + 2v_{2f,x}^2$$

$$\Rightarrow v_{2f,x}(v_{2f,x} - v) = 0$$

$$v_{2f,x} = \cancel{0}, v$$

this solution implies no collision -> reject

Plugging in this solution back into (*), we get

$$v_{1f,x} = v - (v) = 0$$

So we have $\vec{v}_{1f} = 0$, $\vec{v}_{2f} = \langle v, 0, 0 \rangle$. $\square$