Physics 2211 GPS Week 8

Problem #1

Ed and Mike are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.5 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Ed exerts a force $\langle -400, 0, 200\rangle$ N and Mike exerts a force $\langle 150, 0, 300\rangle$ N.

(a) How much work does Ed do?

✓ $\Delta\vec{r}_{boat} = \vec{r}_f - \vec{r}_i = \langle 4,0,2\rangle - \langle 2,0,3\rangle = \langle 2,0,-1\rangle$ m

$\Rightarrow W_{Ed} = \vec{F}_{Ed} \cdot \Delta\vec{r}_{boat} = \langle -400, 0, 200\rangle \cdot \langle 2,0,-1\rangle =$

$= (-400)(2) + (200)(-1) = -800 - 200 = \boxed{-1000\text{ J}}$

(b) How much work does Mike do?

✓ $\Delta\vec{r}_{boat}$ = same as above = $\langle 2,0,-1\rangle$ m

$\Rightarrow W_{Mike} = \vec{F}_{Mike} \cdot \Delta\vec{r}_{boat} = \langle 150, 0, 300\rangle \cdot \langle 2,0,-1\rangle =$

$= (150)(2) + (300)(-1) = 300 - 300 = \boxed{0}$

(c) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

$$\Delta E = W_{total}$$

$$\Delta K = W_{Ed} + \cancel{W_{Mike}}^{0}$$

$$K_f - K_i = W_{Ed}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = W_{Ed}$$

$$v_f^2 - v_i^2 = \frac{2}{m} W_{Ed}$$

$$v_f^2 = \frac{2W}{m} + v_i^2$$

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-1000)}{3000} + (1.5)^2} = \boxed{1.26 \text{ m/s}}$$

(d) What effect does Mike have on the boat's motion?

Steering (changing direction of motion)

Problem #2

A lighthouse keeper spots a sailboat of mass M at location A $\langle x_0, 0, 0\rangle$ moving with speed v_0. After dozing off for a quick nap, the lighthouse keeper awakens to find the sailboat at location B $\langle 0, y_0, 0\rangle$. Having no way to measure the passage of time, the keeper decides to use her vast knowledge of the sea to estimate the speed of the sailboat. The keeper estimates that the net force acting on the sailboat is constant and given by $\langle a, b, 0\rangle$ where both a and b are positive constants. What would the lighthouse keeper predict for the speed of the sailboat at location B?

$$\vec{r}_A = \langle x_0, 0, 0 \rangle$$

$$|\vec{v}_A| = v_0$$

$$\vec{r}_B = \langle 0, y_0, 0 \rangle$$

$$\vec{F}_{net} = \langle a, b, 0 \rangle \quad \begin{Bmatrix} a > 0 \\ b > 0 \end{Bmatrix}$$

$$\Delta E = W$$

$$\Delta K = \int \vec{F}_{net} \cdot d\vec{l}$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \vec{F}_{net} \cdot \Delta \vec{L} \qquad \text{where } \Delta\vec{L} = \vec{r}_f - \vec{r}_i$$

$$= \vec{F}_{net} \cdot \langle -x_0, +y_0, 0 \rangle \qquad = \langle -x_0, +y_0, 0 \rangle$$

$$\frac{1}{2} m (v_f^2 - v_0^2) = -a x_0 + b y_0$$

$$\rightarrow \quad v_f = \sqrt{\frac{2}{m}(b y_0 - a x_0) + {v_0}^2}$$

Problem #3 After graduating from GT you are asked to help in the design of a "bungee jump" apparatus for adults. During a typical bungee jump, a person falls from a high platform with two elastic cords tied to their ankles. The jumper falls freely for a while, with the cords slack. Then the jumper falls an additional distance with the cords increasingly tense. Assume that you have cords that are 10 m long, and that the cords stretch in the jump an additional $\underbrace{23\text{ m}}_{\Delta L}$ for a jumper whose mass is $\underbrace{100\text{ kg}}_{m}$, the heaviest adult you will allow to use your bungee jump.

(a) It will help you a great deal in your analysis to make a series of 5 simple diagrams, like a comic strip, showing the platform, the jumper, and the two cords at the following times in the fall and the rebound

A. while cords are slack $\Rightarrow L_0 = 10\text{ m}$

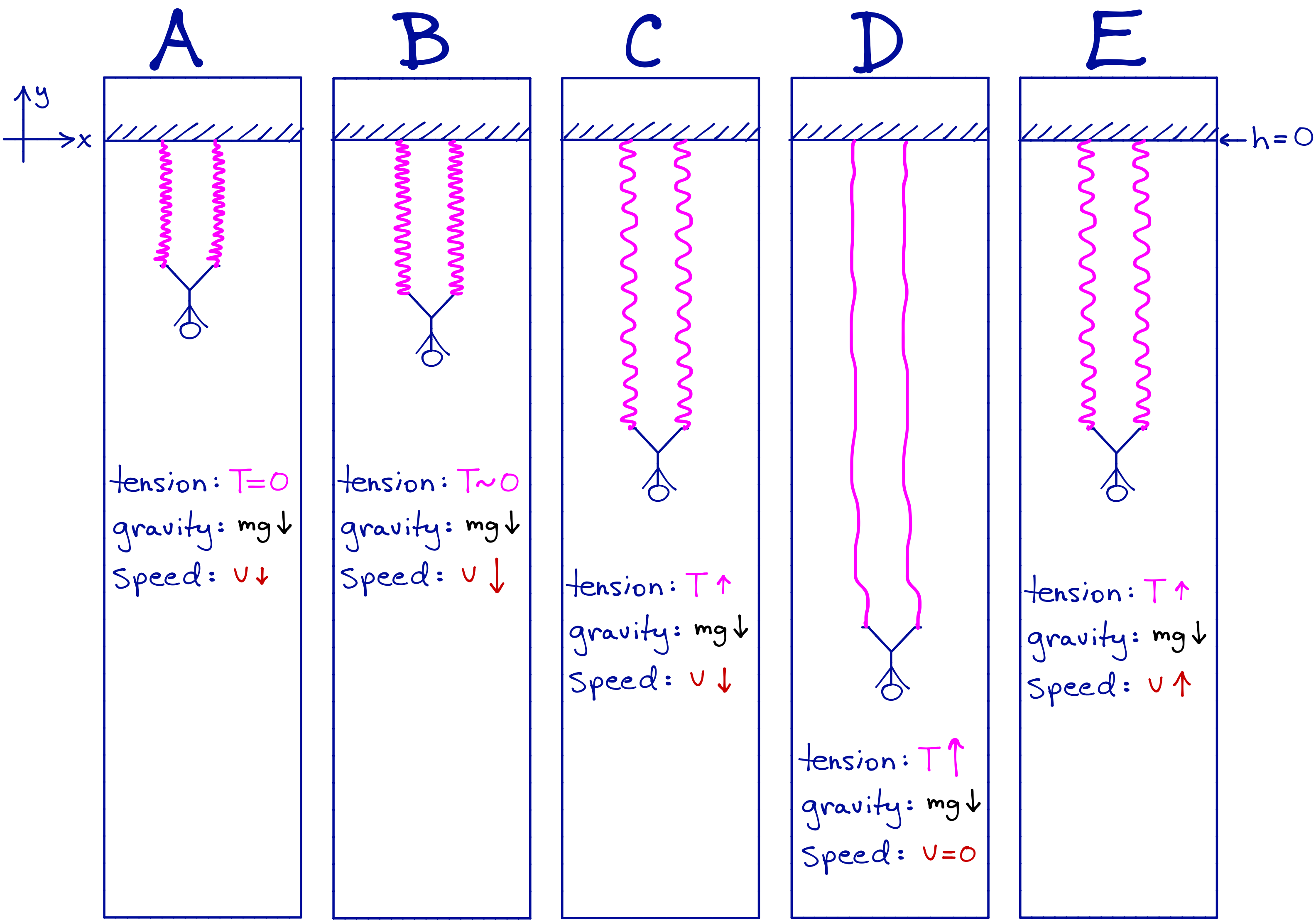
B. when the two cords are just starting to stretch

C. when the two cords are half stretched

D. when the two cords are fully stretched $L = L_0 + \Delta L = 10 + 23 = 33\text{ m}$

E. when the two cords are again half stretched, on the way up

On each diagram, draw and label vectors representing the forces acting on the jumper, and the jumper's velocity. Make the relative lengths of the vectors reflect their relative magnitudes.



(b) At what instant is there the greatest tension in the cords? (How do you know?)

Scene D, at the maximum stretch

(c) What is the jumper's speed at this instant, when the tension is greatest in the cords?

$V = 0$ b/c turning point

(d) Is the jumper's momentum changing at this instant or not? Briefly explain how you know this.

Yes $\frac{d\vec{p}}{dt} \neq 0$ b/c momentum changes from pointing up to pointing down

(e) Focus on this instant of greatest tension and, starting from a fundamental principle, determine the spring stiffness k_s for each of the two cords.

System: jumper + bungees + earth

(initial @ top; final @ bottom)

$$\Delta E = \Delta K + \Delta U_s + \Delta U_g = 0$$

$$\tfrac{1}{2}m(\cancel{v_f^2} - \cancel{v_i^2}) + \tfrac{1}{2}k(s_f^2 - \cancel{s_i^2}) + mg(h_f - \cancel{h_i}) = 0$$

$$\tfrac{1}{2}ks_f^2 + mgh_f = 0$$

$$\tfrac{1}{2}k(\Delta L)^2 + mg(-L) = 0$$

$$\tfrac{1}{2}k(\Delta L)^2 = mgL$$

$$k = \frac{2mgL}{(\Delta L)^2}$$

Numbers:

$$k = \frac{(2)(100)(9.8)(33)}{(23)^2} = 122 \text{ N/m}$$

Note that this is the effective k for the two springs in parallel. Need to find k_i for each individual spring:

$$k_{eff} = k_1 + k_2 = 2k_i$$

$$\Rightarrow k_i = \tfrac{1}{2}k_{eff} = \tfrac{1}{2}(122) = \boxed{61 \text{ N/m}}$$

(f) What is the maximum tension that each one of the two cords must support without breaking? (This tells you what kind of cords you need to buy.)

$$T_{max,i} = |\vec{F}_{spring,max,i}| = k_i s_{max} = k_i \Delta L = (61)(23) = \boxed{1403 \text{ N}}$$

(g) What is the maximum acceleration that the jumper experiences? What is the direction of this maximum acceleration?

$$\vec{F}_{max} = \vec{F}_{s,max} + \vec{F}_g = (k_{eff} s_{max} - mg)\,\hat{y} = m\vec{a}_{max}$$

$$\Rightarrow |\vec{a}_{max}| = \frac{k_{eff}\Delta L - mg}{m} = \frac{(122)(23) - (100)(9.8)}{100} = \boxed{18.26 \text{ m/s}^2}$$