PHYS 2211 - Test 3 - Spring 2023

Scan and Upload to Gradescope after finishing test

- This quiz/test/exam is closed internet, books, and notes with the following exceptions:
  - You are allowed the formula sheet found on Canvas, blank paper, and a calculator.
  - You should not have any other electronic devices open until time is called.
  - You are not allowed to access the internet until time is called.
  - You must work individually and receive no assistance from any other person or resource.
- Work through all the problems first, and then scan/upload your solutions **at your seat** after time is called.
  - Preferred format is PNG, JPG, or PDF.
  - if your image is unable to be read you will receive a zero.
  - You can upload a single file containing work for multiple problems as long as you upload the file for each problem individually
  - clearly label your work for each sub-part and box final answers.
- To earn partial credit, your work must be legible and the organization must be clear.
  - Your solutions should be worked out algebraically.
  - Numerical solutions should only be evaluated at the last step.
  - Incorrect solutions that are not solved algebraically will receive an 80 percent deduction.
  - You must show all steps in your work, including correct vector notation.
  - **Correct answers without adequate explanation will be counted wrong.**
  - Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want graded
  - Include diagrams and show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^{4}$
  - Give standard SI units with your numeric results. Your symbolic answers should not have units.

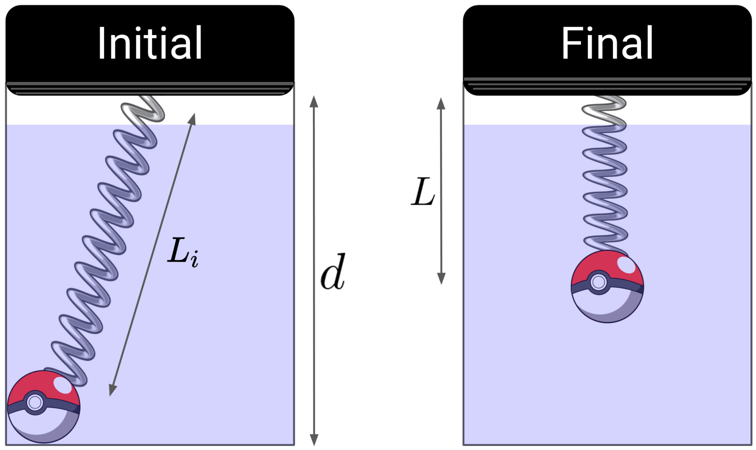
Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it. If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

"In accordance with the Georgia Tech Honor Code,
I have completed this test while adhering to these instructions."

KEY

PRINT your name and GTID on the line above

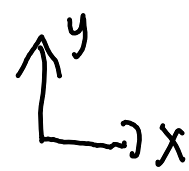
Problem 1 - 30 Points



--- $U_{grav} = 0$

A container of height d is filled with water of mass M and specific heat C. Inside the container a spring is attached to the lid and connected to a ball of mass $M/10$ and specific heat $C/10$. The ball is free to move around inside of the container. The ball is initially held fixed and motionless at the bottom of the container so that the spring has length L_i. The ball is then released from rest. As the ball moves, there is a drag force acting on the ball due to the water that will slow it down and eventually bring it a final state of rest. There is a small buoyant force that can be neglected in all of your calculations.

1. [10pts] Take the stiffness of the spring as k_s and the rest length as L_0. Determine the unknown length of the spring L once the ball comes to rest (i.e. the final state) and the liquid is motionless.



static equilibrium

$$\vec{F}_{net,ball} = \vec{F}_{spring} + \vec{F}_{grav} = \vec{0}$$

$$\Rightarrow \quad -k_s(L-L_0)(-\hat{y}) + \frac{M}{10}g(-\hat{y}) = \vec{0}$$

$$\boxed{L = \frac{Mg}{10k_s} + L_0}$$

-1 clerical

-20% minor

-40% major

-80% min prog

2. [20pts] Calculate the temperature change for the system from the initial to the final state. You can assume that the container is perfectly insulating, the system is in thermal equilibrium in the initial and final state, and that the mass of the spring is negligible.

system: water, ball, spring, Earth

surr: container

$$\Delta E_{sys} = \cancel{\Delta K_{ball}}^{\,0} + \Delta U_{grav} + \Delta U_{spring} + \Delta E_{thermal} = 0$$

$$\Rightarrow \Delta E_{thermal} = -\Delta U_{grav} - \Delta U_{spring}$$

$L - L_0$ from part a

$$= -\left[\frac{M}{10} g(-L) - \frac{M}{10} g(-d)\right] - \left[\frac{1}{2}k_s\left(\frac{Mg}{10k_s}\right)^2 - \frac{1}{2}k_s(L_i - L_0)^2\right]$$

$$= \frac{M}{10} g(L-d) + \frac{1}{2}k_s\left[(L_i - L_0)^2 - \left(\frac{Mg}{10k_s}\right)^2\right]$$

$$= \frac{M}{10} g\left(\cancel{\frac{\frac{M}{10}g}{k_s}} + L_0 - d\right) + \frac{1}{2}k_s\left[(L_i - L_0)^2 - \cancel{\left(\frac{\frac{M}{10}g}{k_s}\right)^2}\right]$$

$$= \frac{M}{10} g(L_0 - d) + \frac{1}{2}k_s(L_i - L_0)^2$$

By assumption of thermal equilibrium in the initial and final states, $\Delta T_{water} = \Delta T_{ball} = \Delta T_{sys}$.

$$\Delta E_{thermal} = MC\,\Delta T_{water} + \frac{M}{10}\frac{C}{10}\Delta T_{ball}$$

$$= \frac{101}{100} MC\,\Delta T_{sys}$$

$$\Rightarrow \Delta T_{sys} = \frac{100}{101}\frac{1}{MC}\Delta E_{thermal}$$

$$\boxed{\begin{gathered} \Delta T_{sys} = \frac{100}{101}\frac{1}{MC}\left[\frac{M}{10} g (L_0 - d) + \frac{1}{2}k_s (L_i - L_0)^2\right] \\ \text{OR} \\ \Delta T_{sys} = \frac{100}{101}\frac{1}{MC}\left[\frac{M}{10} g (L - d) + \frac{1}{2}k_s \left[(L_i - L_0)^2 - \left(\frac{Mg}{10k_s}\right)^2\right]\right] \end{gathered}}$$

Note: Leaving $\Delta E_{thermal}$ and ΔT_{sys} in terms of L ok

-1 clerical

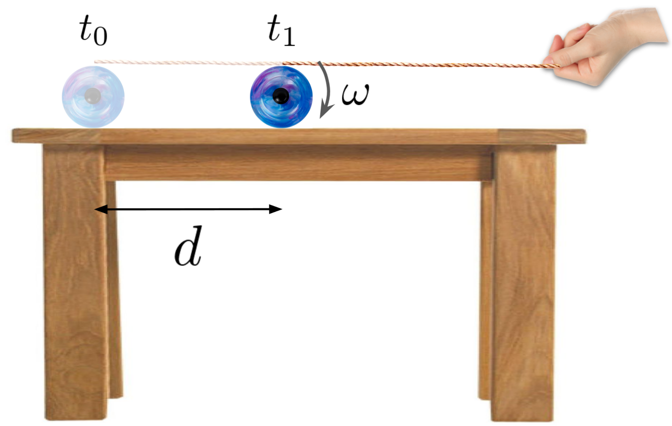
-20% minor

-40% major

-80% min prog

Problem 2 - 30 Points

A yo-yo stands upright on a table so that it can roll across the table parallel to the surface of the Earth. Initially, at $t = t_0$ the yo-yo is at rest. A mysterious hand pulls the string of the yo-yo with a constant force F so that the yo-yo rolls without slipping (i.e. $v = \omega R$) to the right as string unwinds from the yo-yo. At time $t = t_1$ the center of mass for the yo-yo has translated a distance d and the hand has moved a distance L in the same direction. The yo-yo has radius R, mass M, and a moment of inertia $I = \frac{1}{2}MR^2$. Determine the angular speed ω of the yo-yo at time t_1.



system: yo-yo

surr: hand, table

-1 clerical

-20% minor

-40% major

-80% min prog

Method 1 (real system)

$$\Delta E_{sys} = \Delta K_{trans} + \Delta K_{rot} = W_{hand}$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = FL$$

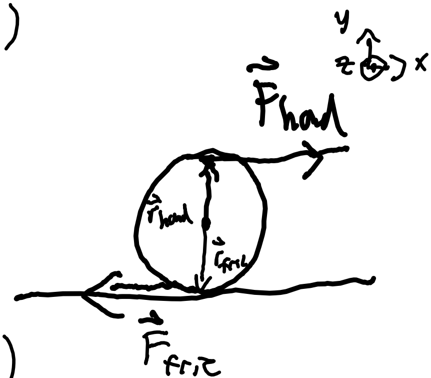
rolling w/out slipping: $v = \omega R$

$$\frac{1}{2}M(\omega R)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = FL$$

$$\frac{3}{4}M\omega^2R^2 = FL$$

$$\boxed{\omega = \sqrt{\frac{FL}{\frac{3}{4}MR^2}}}$$

Method 2 (point-particle + real)



$$\Delta K_{trans} = W_{hand} + W_{fric}$$

$$\vec{F}_{net} = \vec{F}_{hand} + \vec{F}_{fric} = M\vec{a}$$

$$= F(\hat{x}) + F_{fric}(-\hat{x}) = Ma(\hat{x})$$

$$\Rightarrow a = \frac{F - F_{fric}}{M} \quad (*)$$

$$\vec{\tau}_{net} = \vec{r}_{hand} \times \vec{F}_{hand} + \vec{r}_{fric} \times \vec{F}_{fric} = I\vec{\alpha}$$

$$= RF(-\hat{z}) + RF_{fric}(-\hat{z}) = I\alpha(-\hat{z})$$

$$R(F + F_{fric}) = I\alpha \quad \text{no-slip } a = \alpha R$$

$$= I\frac{a}{R} \quad (*)$$

$$R^2(F + \vec{F}_{fric}) = I\left[\frac{F - \vec{F}_{fric}}{M}\right]$$

$$F_{fric} = \frac{\left(\frac{I}{M} - R^2\right)F}{\left(\frac{I}{M} + R^2\right)} = \frac{-\frac{1}{2}}{\frac{3}{2}}F = -\frac{1}{3}F$$

$I = \frac{1}{2}MR^2$

$\vec{F}_{fric}$ points to the right

$$\text{So } \vec{F}_{fric} = F_{fric}(-\hat{x})$$

$$= \frac{1}{3}F(\hat{x})$$

Returning to the energy eq'n,

$$\Delta K_{trans} = W_{hand} + W_{fric}$$

$$\frac{1}{2} M v^2 = Fd + \frac{1}{3} Fd = \frac{4}{3} Fd$$

$$v = \sqrt{\frac{8Fd}{3M}}$$

$$\Rightarrow \boxed{\omega = \frac{v}{R} = \sqrt{\frac{8Fd}{3MR^2}}}$$

(so $\frac{4}{3} L = \frac{8}{3} d$ ⟵ not needed for sol'n
$\Rightarrow L = 2d$)

<u>Note:</u> answer in terms of L or d acceptable

Problem 3 - 40 Points

and catches the ball

At a soccer game, a goalkeeper (mass $M = 60$ kg) jumps and to catch the ball (mass $m = 0.45$ kg). Just when she reaches the highest point of her jump, her center of mass is at height $h = 1.8$ m, her velocity is $\vec{v_0} =< 3.5, 0, 0 >$ m/s, and the ball's velocity is $\vec{v} =< -25, 0, -15 >$ m/s (see diagram). In the three questions below, you should give an algebraic result first, and then evaluate that expression for a numerical value.

$U_{grav} = 0$ ---

1. [15pts] What is the velocity $\vec{v}_{goal}$ of the goalkeeper right after she catches the ball? The result should be a vector.

perfectly inelastic collision - conservation of momentum

$$\vec{P}_i = \vec{P}_f$$

-1 clerical

-20% minor

-40% major

-80% min prog

$$M\vec{v}_0 + m\vec{v} = (M+m)\vec{v}_{goal}$$

$$\vec{v}_{goal} = \frac{M\vec{v}_0 + m\vec{v}}{M+m}$$

$$= \frac{(60\,kg)<3.5, 0, 0> m/s + 0.45\,kg <-25, 0, -15> m/s}{60\,kg + 0.45\,kg}$$

$$\boxed{\vec{v}_{goal} = <3.30, 0, -0.11> m/s}$$

with the ground

2. [15pts] After she catches the ball, she falls on the ground and comes to a rest. Calculate the magnitude of acceleration $|\Delta\vec{v}/\Delta t|$ of the keeper during the duration of the collision $\Delta t = 0.1s$ and determine if she might suffer a concussion (concussions are probable for acceleration $|\Delta\vec{v}/\Delta t| > 7g$, with g the gravitational acceleration near Earth's surface).

After catching the ball but before collision w/ the ground,

$$\Delta K_{GK+ball} + \Delta U_{grav} = 0$$

-1 clerical

-20% minor

-40% major

-80% min prog

$$\frac{1}{2}(M+m)\left[\vec{v}_{ground}^{\,2} - \vec{v}_{goal}^{\,2}\right] + (M+m)g\left[0-h\right] = 0$$

$$\Rightarrow |\vec{v}_{ground}| = \sqrt{2gh + \vec{v}_{goal}^{\,2}}$$

$$|\vec{a}| \approx \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \left|\frac{\vec{0} - \vec{v}_{ground}}{\Delta t}\right| = \frac{|\vec{v}_{ground}|}{\Delta t}$$

$$\boxed{|\vec{a}| = \frac{\sqrt{2gh + \vec{v}_{goal}^{\,2}}}{\Delta t} = 67.96\ m/s^2} \lesssim \underbrace{68.6\ m/s^2}_{7g}$$

concussion is possible

3. [10pts] Determine the change of internal energy ΔE_{int} of the keeper (still holding the ball) from right before to right after after her collision with the ground, considering the Earth's kinetic and internal energies as negligible (careful with sign of ΔE_{int}).

Assume E_{int} of the ball doesn't change.

-1 clerical

-20% minor

-40% major

-80% min prog

$$\Delta K_{GK+ball} + \Delta E_{int} = 0$$

$$\Delta E_{int} = -\Delta K_{GK+ball}$$

$$= -\left[0 - \frac{1}{2}(M+m)\vec{v}_{ground}^{\,2}\right]$$

$$= \frac{1}{2}(M+m)\vec{v}_{ground}^{\,2}$$

$$= \frac{1}{2}(60.45\,kg)(46.18\ m^2/s^2)$$

$$\boxed{\Delta E_{int} = 1395\ J}$$

Q3.2 Solving for $\vec{v}_{ground}$ w/ energy

Since $\vec{F}_{net} = mg(-\hat{z})$, $v_{ground,x} = v_{goal,x}$ and $v_{ground,y} = v_{goal,y}$

$$\vec{v}_{ground}^{\,2} = \underbrace{v_{ground,x}^{2}}_{v_{goal,x}^{2}} + \underbrace{v_{ground,y}^{2}}_{0} + v_{ground,z}^{2} = 2gh + \vec{v}_{goal}^{\,2}$$

$$\Rightarrow v_{ground,z} = \cancel{+}\!\!-\sqrt{2gh + \vec{v}_{goal}^{\,2} - v_{goal,x}^{2}}$$

$$\vec{v}_{ground} = \langle 3.30, 0, -5.94 \rangle \ m/s$$

(continued on next pg)

For the collision w/ the ground,

$$\Delta\vec{v} = -\vec{v}_{ground}$$

so

$$\frac{\Delta\vec{v}}{\Delta t} = -\frac{\vec{v}_{ground}}{\Delta t}$$

$$= -\frac{\langle 3.30, 0, -5.89 \rangle \ m/s}{0.1 s}$$

$$= \langle -33, 0, -58.9 \rangle \ m/s^2$$

and

$$\boxed{\left|\frac{\Delta\vec{v}}{\Delta t}\right| = 67.51 \ m/s^2} \lesssim 68.6 \ m/s^2$$

Q3.2 Solving for $\vec{v}_{ground}$ w/ kinematics

pos. update (const. acc.)

$$\langle x_f, y_f, \underset{\overset{\shortparallel}{0}}{z_f} \rangle = \langle x_i, y_i, \overset{\overset{h}{\shortparallel}}{z_i} \rangle + \vec{v}_{goal}\,\Delta t + \frac{1}{2}\frac{\vec{F}_{net}}{m_{tm}}\Delta t^2$$

In the z-direction,

$$0 = h + v_{goal,z}\,\Delta t + \frac{1}{2}(-g)\,\Delta t^2$$

$$\Delta t = \frac{-v_{goal,z} \pm \sqrt{v_{goal,z}^2 - 4\left(-\frac{g}{2}\right)(h)}}{\cancel{2}\left(-\frac{g}{\cancel{2}}\right)}$$

$$= \frac{0.11\ m/s \pm \sqrt{(0.11\ m/s)^2 - 4(-4.9\ m/s^2)(1.8\ m)}}{-9.8\ m/s^2}$$

$$= -\cancel{0.62\,s},\ 0.59\,s$$

vel. update

$$\vec{v}_f \equiv \vec{v}_{ground} = \vec{v}_{goal} + \frac{\vec{F}_{net}}{m_{tm}}\Delta t$$

$$= \langle 3.30, 0, -0.11 \rangle\ m/s + \langle 0, 0, -9.8 \rangle\ m/s^2\ (0.59\,s)$$

$$= \langle 3.30, 0, -5.89 \rangle\ m/s$$