Modeling the enriched Max-P region problem as a Mixed-integer programming problem

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1 MIP formulation

Parameters:

$$i, I = \text{Index and set of areas, } I = \{1, ..., n\};$$
$$k = \text{index of potential regions, } k = \{1, ..., n\};$$
$$c = \text{index of contiguity order, } c = \{0, ..., q\} \text{ with } q = (n-1);$$
$$w_{ij} = \begin{cases} 1, \text{ if areas } i \text{ and } j \text{ share a border, with } i, j \in I \text{ and } i \neq j; \\ 0, \text{ otherwise} \end{cases}$$
$$N_i = \{j | w_{ij} = 1\}, \text{the set of areas that are adjacent to area } i;$$
$$d_{ij} = \text{dissimilarity relationships between areas } i \text{ and } j, \text{ with } i, j \in I \text{ and } i < j;$$
$$h = 1 + \left\lfloor log(\sum_i \sum_{j|j>i} d_{ij}) \right\rfloor$$
$$l_i^{con} = \text{the spatially extensive attribute of } con = MIN, MAX, SUM, AVG \text{ for area } i; lower_{con} \quad = \text{the}$$
$$upper_{con} = \text{the upper bound of the range for constraint } con = \{MIN, MAX, AVG, SUM, COUNT\}$$

Decision variables:

$$t_{ij} = \begin{cases} 1, \text{ if areas i and j belong to the same region } k, \text{ with } i < j \\ 0, \text{ otherwise;} \end{cases}$$
$$x_i^{kc} = \begin{cases} 1, \text{ if areas i is assigned to region } k \text{ in order } c \\ 0, \text{ otherwise;} \end{cases}$$

Minimize:

$$Z = \left(-\sum_{k=1}^{n} \sum_{i=1}^{n} x_i^{k0} \right) * 1 -^h + \sum_i \sum_{j|j>i} d_{ij} t_{ij} \tag{1}$$

Subject to:

$$\sum_{i=1}^{n} x_i^{k0} \leq 1 \qquad \forall k = 1, ..., n; \tag{2}$$

$$\sum_{k=1}^{n} \sum_{c=0}^{1} x_i^{kc} = 1 \qquad \forall i = 1, ..., n; \tag{3}$$

$$x_i^{kc} \leq \sum_{j \in N_i} x_j^{k(c-1)} \qquad \forall i = 1, ..., n; \forall k = 1, ..., n; \forall c = 1, ..., q; \tag{4}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \sum_{c=0}^{q} x_j^{kc} l_i^{MIN} \leq Upper_{MIN} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \exists j = 1, ..., n; \forall k = 1, ..., n; \tag{5}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \sum_{c=0}^{q} x_j^{kc} l_i^{MIN} \geq Low_{MIN} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall j = 1, ..., n; \forall k = 1, ..., n; \tag{6}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \sum_{c=0}^{q} x_j^{kc} l_i^{MAX} \leq Upper_{MAX} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall j = 1, ..., n; \forall k = 1, ..., n; \tag{7}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \sum_{c=0}^{q} x_j^{kc} l_i^{MAX} \geq Low_{MAX} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \exists j = 1, ..., n; \forall k = 1, ..., n; \tag{8}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} l_i^{SUM} \leq Upper_{SUM} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{9}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} l_i^{SUM} \geq Lower_{SUM} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{10}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} l_i^{AVG} \leq Upper_{AVG} \sum_{i=1}^{n} x_i^{k0} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{11}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} l_i^{AVG} \geq Lower_{AVG} \sum_{i=1}^{n} x_i^{k0} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{12}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \leq Upper_{COUNT} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{13}$$

$$\sum_{i=1}^{n} \sum_{c=0} x_i^{kc} \geq Lower_{COUNT} \sum_{i=1}^{n} \sum_{c=0}^{q} x_i^{kc} \qquad \forall k = 1, ..., n; \tag{14}$$

$$x_i^{kc} \in \{0,1\} \qquad \forall i = 1,...,n; \forall k = 1,...,n; \forall c = 0,...,q; \tag{15}$$

$$t_{ij} \in \{0,1\} \qquad \forall i,j = 1,...,n | i < j. \tag{16}$$