Geometric and Physical Constraints Synergistically Enhance Neural PDE Surrogates

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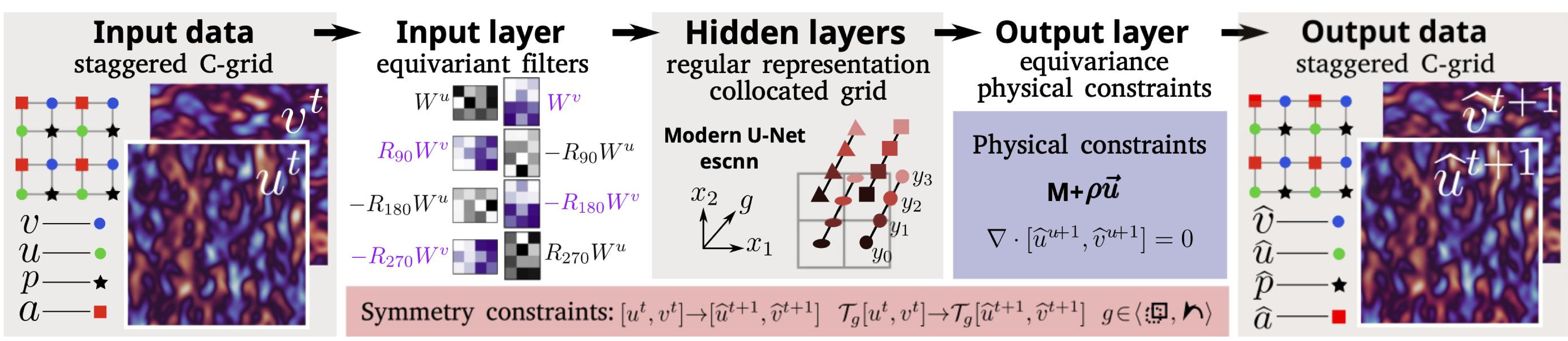
Project website

Introduction & Motivation

- **Challenges of ML for solving PDEs:** Accurate long-term rollout, stability, and generalization.
- **Previous ML Models:** are benified by both **physical and geometric constraints.**
- **Staggered C-grids:** are often used in weather, climate, and fluid dynamics.
- **Our critical scientific question:** how can these constraints be imposed on C-grids, and would their combination be useful or redundant?

Symmetry– and physics–constrained neural surrogate

Network Architecture



- The input and output are on C-grids.
- The network is constrained by both symmetry and physical laws.

Experiments

Shallow Water Equations (SWEs)

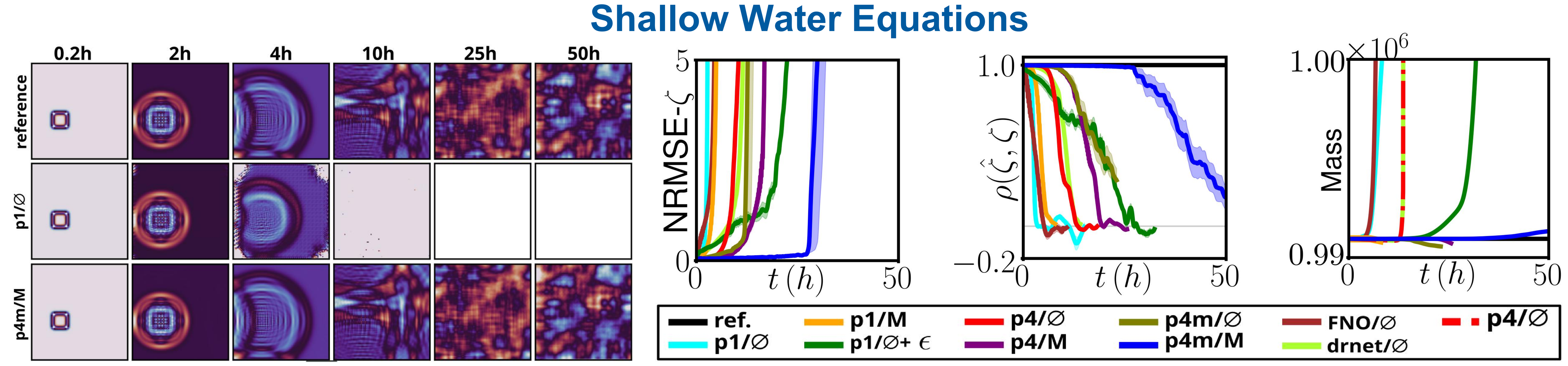
Conservation laws	Symmetries		
None ∅	p1/∅	p4/∅	p4m/∅
Mass M	p1/M	p4/M	p4m/M

Incompressible Navier–Stokes (INS) & Decaying turbulence

Conservation laws	Symmetries		
None ∅	p1/∅	p4/∅	p4m/∅
Momentum $\rho\vec{u}$	p1/$\rho\vec{u}$	p4/$\rho\vec{u}$	p4m/$\rho\vec{u}$
Mass/momentum M+$\rho\vec{u}$	p1/M+$\rho\vec{u}$	p4/M+$\rho\vec{u}$	p4m/M+$\rho\vec{u}$

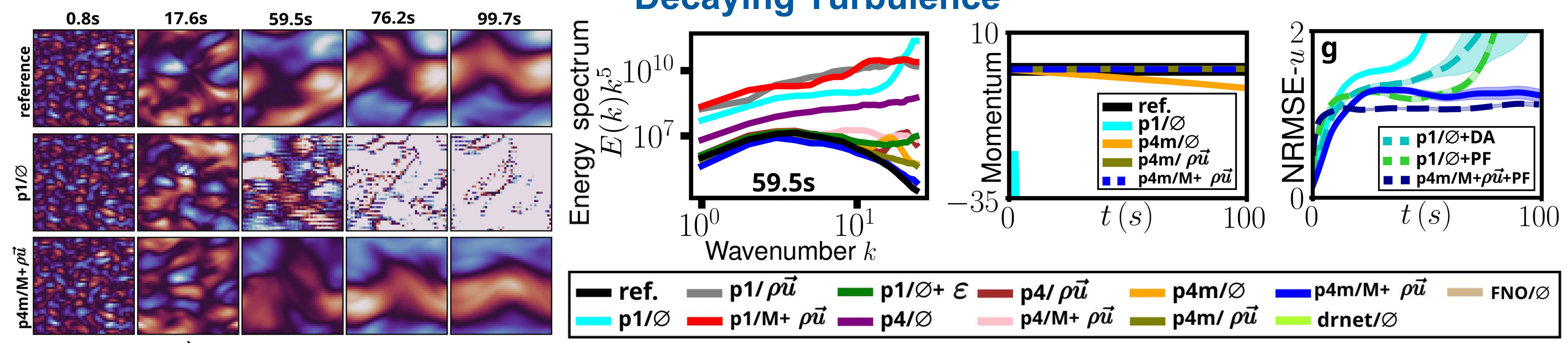
Double–constrained models outperform other networks on SWEs & INS

Shallow Water Equations



- p4m/M (symmetry+physics constraints) outperforms other networks.
- Accuracy over 50h rollouts, with standard error of the meanover 20 ICs.

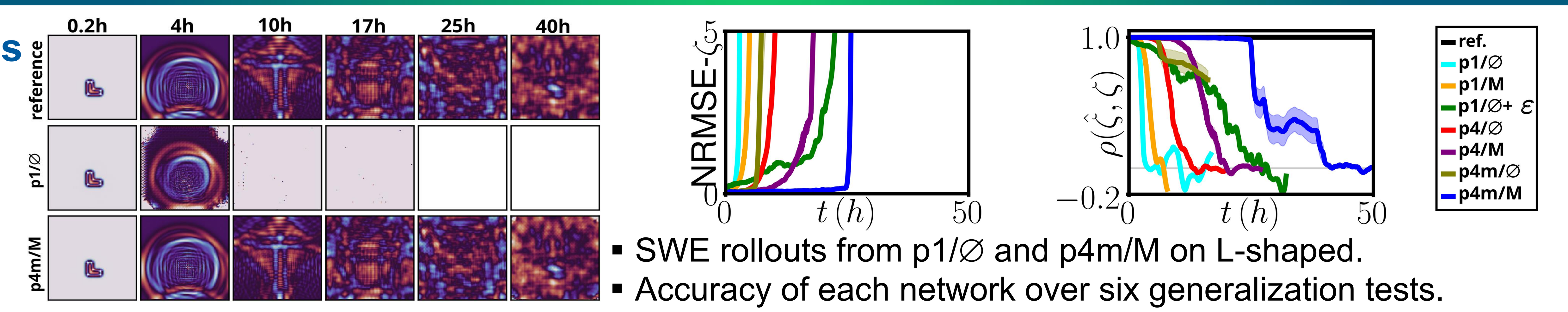
Decaying Turbulence



- p4m/M+ρu⃗ outperforms other networks with similar parameter counts on INS.
- Accuracy of energy spectrum and momentum with standard error of the mean over 30 ICs.

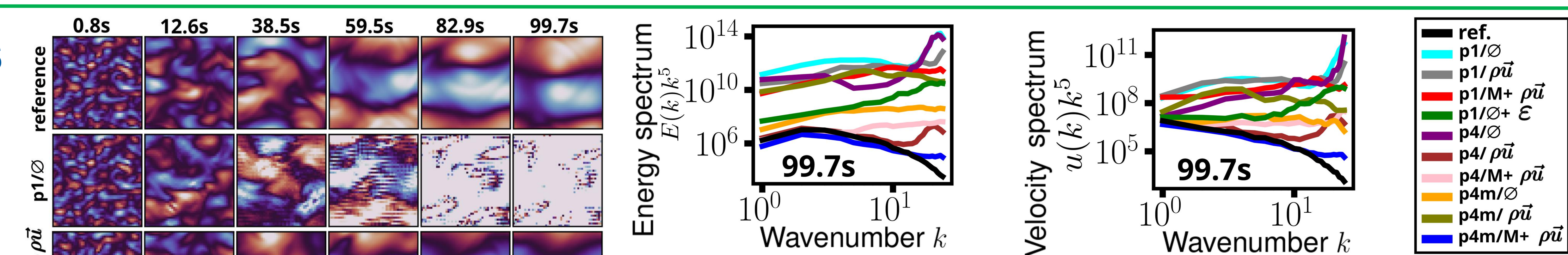
Generalization beyond training data

SWEs



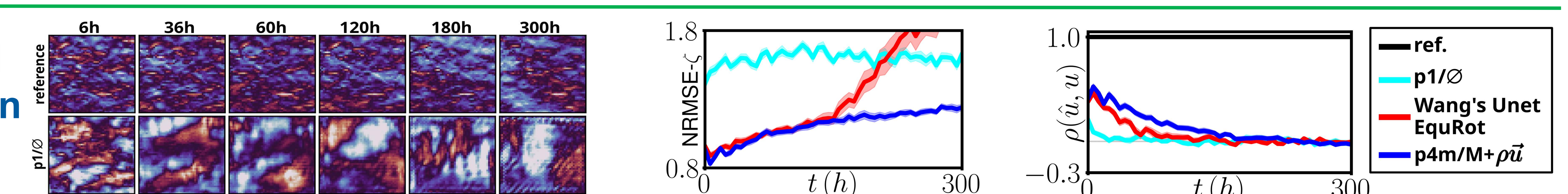
- SWE rollouts from p1/∅ and p4m/M on L-shaped.
- Accuracy of each network over six generalization tests.

INS



- INS rollouts from p4m/M+ρu⃗ on ICs with peak wavenumber 8.
- Energy- and velocity spectra at t = 99.7 s, averaged over 10 ICs.

Real ocean



- The prediction of ocean current observations by p4m/M+ρu⃗
- The accuracy of forecasts over 300h (averaged over 30 ICs).

Conclusion

- **A double-constrained model with input and output layers on C-grids.**
- **Symmetries are more effective than physical constraints, but combining both is best.**
- **Our model improves predictions in terms of generalisation and the real ocean data.**