Projet 4 : Platooning - Consensus

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This document presents a cooperative control paradigm based on "consensus" among systems in a SOS. This project is divided into 3 parts, the first part aims to propose a consensus-based control that ensures the control objective and then the second part is to propose simplified model with less connections. And the last part is reserved to establish a model in which robots can travel at constant speed on x-axis having formed the platoon. Besides, analyses of speed of convergence and also of trajectory of all the 5 robots are needed to check the stability of the model proposed.

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1 Theories of collaborative control

Collaborative control is applied widely in the systems of control. It consists generally multiple rovers, between which communications are shared. And all units must agree on goal even though sub goals may be different for each unit. Most importantly, there is no single control unit.

1.1 Non-holonomic constraints

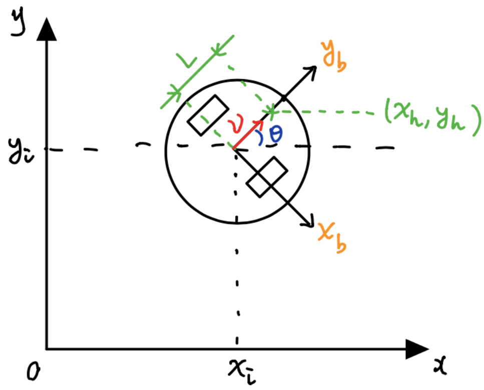


Figure 1: Application example of collaborative control

Taking the example that we analysed during the class : Suppose robot i's kinematic equations are :

$$\dot{x_i} = v_i cos(\theta_i)$$
$$\dot{y_i} = v_i sin(\theta_i)$$
$$\dot{\theta_i} = w_i$$
$$i = 1, 2, ..., N$$

where (x_i, y_i) is the position of the centre of gravity of robot i, θ_i is the orientation and v_i and w_i are the linear and angular velocities.
Non-holonomic constraints :

$$\dot{x_i} sin(\theta_i) - \dot{y_i} cos(\theta_i) = 0$$

that means mobile robots cannot move sideways, which can be expressed as follows :

$$\left(\begin{array}{l} \theta_i = 0 \Rightarrow \dot{y_i} = 0 \\ \theta_i = \frac{\pi}{2} \Rightarrow \dot{x_i} = 0 \end{array} \right.$$

It is difficult to design v_i and w_i such that :

$$x_i \to 0, \quad y_i \to 0, \quad \text{and} \quad \theta_i \to 0$$

1.1.1 Freeback linearization :

Define :

$$x_{h_i} = x_i + L_i \cos(\theta_i)$$
$$y_{h_i} = y_i + L_i \sin(\theta_i)$$
$$\text{where} \quad L_i > 0$$

in which $(x_h i, y_h i)$ is the position of point that is not the centre of gravity of robot. Therefore, we deduce that:

$$\dot{x_{h_i}} = \dot{x_i} + L_i(-\sin(\theta_i))\dot{\theta_i} = v_i \cos(\theta_i) + L_i(-\sin(\theta_i))w_i$$

$$\dot{y_{h_i}} = \dot{y_i} + L_i \cos(\theta_i))\dot{\theta_i} = v_i \sin(\theta_i) + L_i \cos(\theta_i))w_i$$

$$\Rightarrow \begin{pmatrix} \dot{x_{h_i}} \\ \dot{y_{h_i}} \end{pmatrix} = \begin{pmatrix} cos(\theta_i) & -L_i sin(\theta_i) \\ sin(\theta_i) & L_i cos(\theta_i) \end{pmatrix} \begin{pmatrix} v_i \\ w_i \end{pmatrix}$$

$$\mid R_i \mid = L_i cos^2(\theta_i) + L_i sin^2(\theta_i) = L_i > 0$$

Then, let

$$\begin{pmatrix} v_i \\ w_i \end{pmatrix} = R_i^{-1} \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix},$$

$$\Rightarrow \begin{pmatrix} \dot{x_{h_i}} \\ \dot{y_{h_i}} \end{pmatrix} = R_i * R_i^{-1} \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix} \quad = \quad \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix}$$

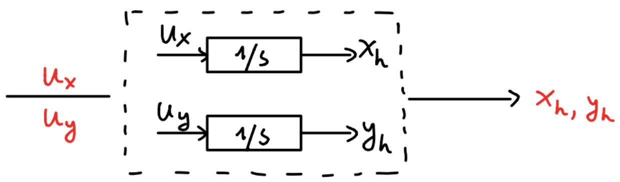


Figure 2: Decoupled systems

1.2 Stabilization problem

In this case, we suppose that robot converges to a fixed point (0,0). Therefore, we design our input as follows :

$$\begin{cases} u_{x_i} = -x_{h_i} \\ u_{y_i} = -y_{h_i} \end{cases}$$

That is,

$$\begin{cases} \dot{x_{h_i}} = -x_{h_i} \\ \dot{y_{h_i}} = -y_{h_i} \end{cases}$$

$$\Rightarrow \begin{cases} x_{h_i} \to 0 \\ y_{h_i} \to 0 \end{cases} \quad \forall x_{h_i}(0), y_{h_i}(0).$$

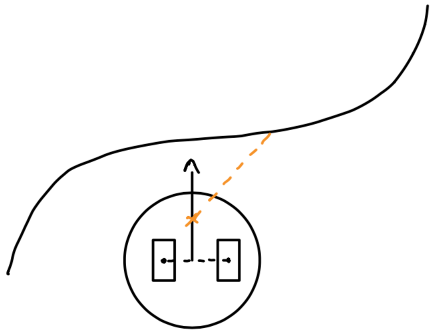


Figure 3: Dynamic problem

1.3 Dynamic trajectory

In this case, we suppose that robot follows a dynamic trajectory of $x_i(t)$ and $y_i(t)$. And we design the input as follows :

$$\begin{cases} u_{x_i} = \dot{x}_i - (x_{h_i} - x_i) \\ u_{y_i} = \dot{y}_i - (y_{h_i} - y_i) \end{cases}$$

$$\text{Then, } \begin{cases} \dot{x_{h_i}} = u_{x_i} = \dot{x}_i - (x_{h_i} - x - i) \\ \dot{y_{h_i}} = u_{y_i} = \dot{y}_i - (y_{h_i} - y - i) \end{cases}$$

$$\Rightarrow \begin{cases} (\dot{x_{h_i}} - \dot{x}_i) = -(x_{h_i} - x_i) \\ (\dot{y_{h_i}} - \dot{y}_i) = -(y_{h_i} - y_i) \end{cases}$$

$$\Rightarrow \begin{cases} x_{h_i} - x_i \to 0 \Rightarrow x_{h_i} \to x_i \\ y_{h_i} - y_i \to 0 \Rightarrow y_{h_i} \to y_i \end{cases}$$

Remark : if the destination coordinates are fixed, then we deduce that $\dot{x}_i = \dot{y}_i = 0$.

1.4 Multi-robot rendez-vous

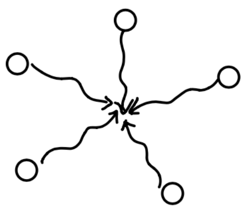


Figure 4: Multi-robot rendez-vous

$$\begin{cases} \dot{x_{h_i}} = u_{h_i} \\ \dot{y_{h_i}} = u_{y_i} \end{cases}$$

Let, $\begin{cases} u_{x_i} = -\sum_{j=1}^{N} g_{ij} k_{ij} (x_{h_i} - x_{h_j}) \\ u_{y_i} = -\sum_{j=1}^{N} g_{ij} k_{ij} (y_{h_i} - y_{h_j}) \end{cases}$
where $g_{ii} = 0$. $g_{ij} = 1$, if robot i receives infomation from robot j, and $g_{ij} = 0$ oterwise, $\forall i \neq j$. k_{ij} is the gain of control term between i and j robot. So in state-space form in closed loop, we can deduce the equation as follow :

$$\dot{X} = AX$$

2 Application of the project 4 : Platooning - Consensus

2.1 Context of the project 4

Suppose the case of five robots, where we con control directly their speeds. The agents should also maintain a linear formation.
We proposed initial positions of the robots : (0,1.5), (0,-1.5), (-3,3), (-3,0), (-3,3) and the objective of this project is to make the robots form the platoon. To achieve this, we give 3 missions as follows :

1. Propose a consensus-based control law that ensures the control objective.
2. Propose another simplified consensus-based control law with less connections.
3. Propose a controller that makes robots continue travel in platooning, on the 'x-axis' at constant speed after having formed the platoon.

Besides, we give two sets of destination positions to simulate our controller, the values of which are as follows :
Destination 1 : (34,0), (31,0), (28,0), (25,0), (22,0)
Destination 2 : (22,0), (28,0), (31,0), (34,0), (25,0)

2.2 Propose a consensus-based control law

The principle of a consensus-based control law, is to assign a leader firstly, then make the rest of the robots share communications between them. Therefore, in this case, we have established 5 connections respectively :
For the destination 1 :

1. We assign robot 1 as leader of the whole system and we give it the destination position of (34,0). Besides, robot 1 receives information from robot 5 and converges to the position $(x_{h5} + (34 - 22), y_{h5} + (0 - 0))$, which means x1 will converge to keep a distance of 12 before robot 5 on x-axis and respectively of 0 on y-axis.
2. Robot 2 receives information from robot 1 and converges to the position $(x_{h1} - (34 - 31))$, which means x2 will converge to keep a distance of 3 behind robot 1 on x-axis and respectively of 0 on y-axis.
3. Robot 3 receives information from robot 2 and converges to the position $(x_{h2} - (31 - 28))$, which means x3 will converge to keep a distance of 3 behind robot 2 on x-axis and respectively of 0 on y-axis.

4. Robot 4 receives information from robot 3 and converges to the position $(x_{h3} - (28 - 25))$, which means x4 will converge to keep a distance of 3 behind robot 3 on x-axis and respectively of 0 on y-axis.

5. Robot 5 receives information from robot 4 and converges to the position $(x_{h4} - (25 - 22))$, which means x5 will converge to keep a distance of 3 behind robot 4 on x-axis and respectively of 0 on y-axis.

2.2.1 Plot the graph of the system

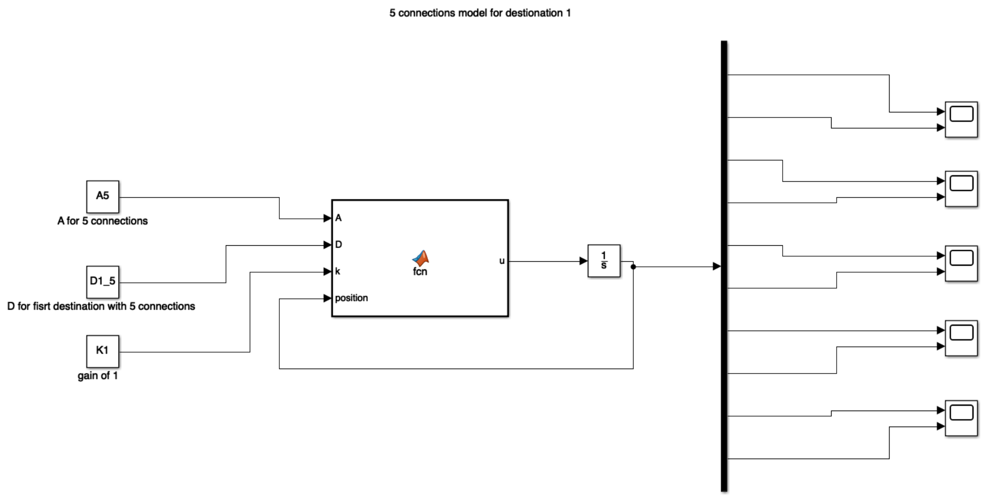


Figure 5: simuink modelization for 5 robots connections

This is a simulink model of 5 connections for the first destination. In this model, we have a controller which has 3 inputs respectively of matrix A, matrix D and gain and also a feedback of position. The role of matrix A is to represent the communication shared between the robots and also the destination position that we gave for the leader robot. Matrix D represents the positions that we want to converge to. And constant K is the gain of speed of the convergence. The reason why we add a feedback of position is that we need to update each time the new position and let it converge finally to the destination position. Besides, this controller has 1 output of u which equals to the derivative of (x_h, y_h). To realize this equation, we add an intergrator between the controller and the final outputs. And eventually, we use 5 scopes to visualize the results of variation of coordinates of the 5 robots.

2.2.2 Consensus-based control law

In this part, we take the example of 5 connections model for the first destination.

Determination of matrix A

First of all, we need to calculate the matrix A to show the communication shared and also the destination given to the leader robot 1. By converting this image below to the mathematical language, we obtained the matrix A as follow :

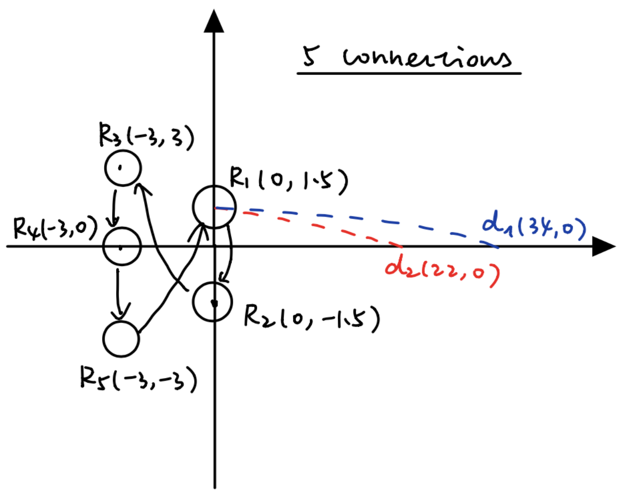


Figure 6: 5 connections model

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

To specify the calculation :
For the robot 1, it is given the destination position and also in the meantime, it receives information shared by robot 5, which tells robot 1 to keep a distance of 12 on x-axis and of 0 on y-axis before it. Therefore, we know $g_{15} = 1$ and the rest equals to 0. And then, to realize this control,

$$\begin{cases} u_{x_1} = -K(x_{h_1} - (x_{h_5} + 12)) - K(x_{h_1} - 34) = K(-2x_{h_1} + x_{h_5} + 34 + 12) \\ u_{y_1} = -K(y_{h_1} - (y_{h_5} + 0)) - K(y_{h_1} - 0) = K(-2y_{h_1} + y_{h_5} + 0 + 0) \end{cases}$$

which shows that the first two lines of matrix A that correspond to robot 1.
For the robot 2, it only receives information from robot 1, which tells robot 2 to keep a distance of 3 on x-axis and of 0 on y-axis behind it. Therefore, we know $g_{21} = 1$ and the rest equals to 0. And then, to realize this control,

$$\begin{cases} u_{x_2} = -K(x_{h_2} - (x_{h_1} - 3)) = K(-x_{h_2} + x_{h_1} - 3) \\ u_{y_2} = -K(y_{h_2} - (x_{h_1} - 0)) = K(-y_{h_2} + y_{h_1} - 0) \end{cases}$$

which shows that the third and the forth lines of matrix A that correspond to robot 2.
For the robot 3, it only receives information from robot 2, which tells robot 3 to keep a distance of 3 on x-axis and of 0 on y-axis behind it.For the robot 4, it only receives information from robot 3, which tells robot 4 to keep a distance of 3 on x-axis and of 0 on y-axis behind it.For the robot 5, it only receives information from robot 4, which tells robot 5 to keep a distance

of 3 on x-axis and of 0 on y-axis behind it. Then we obtained the equations as follows in the same way as in the case of robot 2 :

$$\begin{cases} u_{x_3} = -K(x_{h_3} - (x_{h_2} - 3)) = K(-x_{h_3} + x_{h_2} - 3) \\ u_{y_3} = -K(y_{h_3} - (y_{h_2} - 0)) = K(-y_{h_3} + y_{h_2} - 0) \end{cases}$$

$$\begin{cases} u_{x_4} = -K(x_{h_4} - (x_{h_3} - 3)) = K(-x_{h_4} + x_{h_3} - 3) \\ u_{y_4} = -K(y_{h_4} - (y_{h_3} - 0)) = K(-y_{h_4} + y_{h_3} - 0) \end{cases}$$

$$\begin{cases} u_{x_5} = -K(x_{h_5} - (x_{h_4} - 3)) = K(-x_{h_5} + x_{h_4} - 3) \\ u_{y_5} = -K(y_{h_5} - (y_{h_4} - 0)) = K(-y_{h_5} + y_{h_4} - 0) \end{cases}$$

which shows the last 6 lines of matrix A that correspond to robot 3, 4 and 5.

$\boxed{\text{Determination of matrix D}}$

To facilitate our calculation, we defined the first destination position and the second destination position as follow :

$$d1 = \begin{bmatrix} 34 & 0 \\ 31 & 0 \\ 28 & 0 \\ 25 & 0 \\ 22 & 0 \end{bmatrix} d2 = \begin{bmatrix} 22 & 0 \\ 28 & 0 \\ 31 & 0 \\ 34 & 0 \\ 25 & 0 \end{bmatrix}$$

Therefore, by using the results of the calculation of u presented above, we can obtain the matrix D as follow :

$$D1 = \begin{bmatrix} (d1(1,1) - d1(5,1)) + d1(1,1) \\ (d1(1,2) - d1(5,2)) + d1(1,2) \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix} D2 = \begin{bmatrix} (d2(1,1) - d2(5,1)) + d2(1,1) \\ (d2(1,2) - d2(5,2)) + d2(1,2) \\ d2(2,1) - d2(1,1) \\ 0 \\ d2(3,1) - d2(2,1) \\ 0 \\ d2(4,1) - d2(3,1) \\ 0 \\ d2(5,1) - d2(4,1) \\ 0 \end{bmatrix}$$

And finally, by applying the function as follow, we can obtain the states of each time.

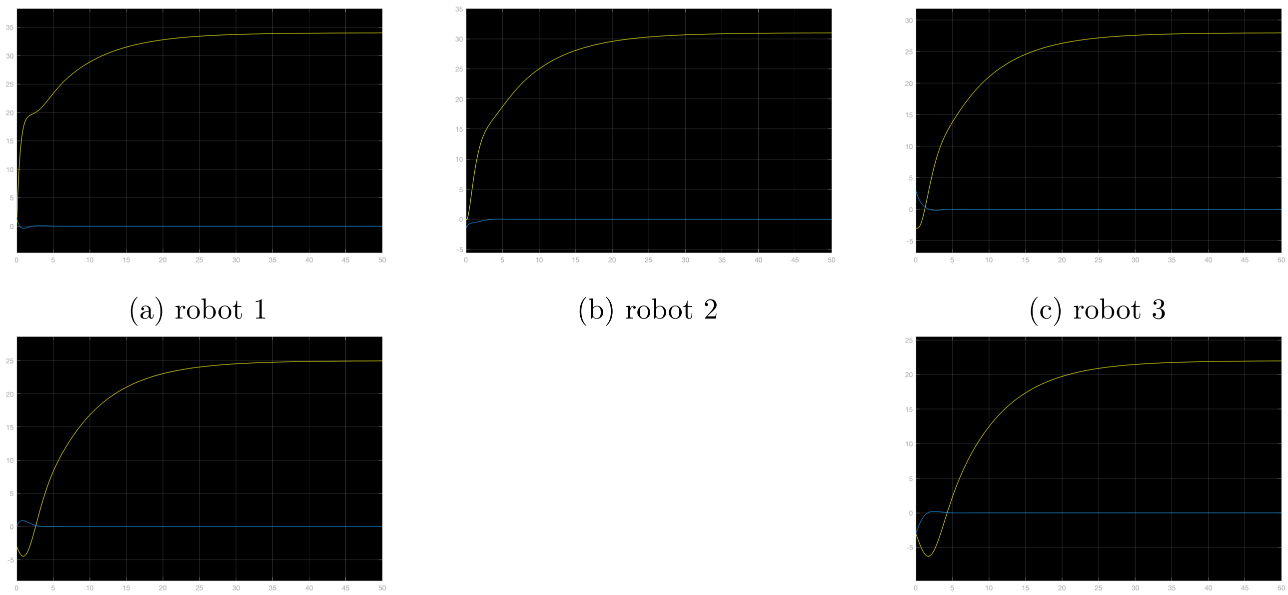
$$u = K(A * \text{position} + D)$$

Now we finally finish our consensus-based control law.

2.2.3 Results of 5 connections model

$\boxed{\text{Verification of destination positions}}$

Verify the case of destination 1 from the results of 5 scopes. For example, we take K=1. From the results below, we see that the all of the 5 robots converge to the correspondent destination positions. That means robot 1 converges to (34,0), robot 2 converges to (31,0), robot 3 converges to (28,0), robot 4 converges to (25,0) and robot 5 converges to (22,0).



(a) robot 1 (b) robot 2 (c) robot 3

(d) robot 4 (e) robot 5

Figure 7: Results of 5 connections model for destination 1

Of course, for different robots, the speeds of convergence are not the same because their real-time coordinates and destination positions are different. But within a sufficiently long time, all the robots can converge to their correspondent destination positions. In addition to that, we also drew the trajectory in the plan (x,y) of the 5 connections model for destination 1 as follow (Figure 8). From the image below, we can see clearly that the 5 robots which are from different initial positions can converge to the destination positions and then form the platooing after a certain period of time. By using the same method, we will also show you the results of the 5 connections model for destination 2 with the same condition of K=1 (Figure 9). And we will also show you the trajectory of 5 connections model for destination 2 (Figure 10).

Discussion of role of gain

For the analysis of gain, we chose 3 different values for gain K, $K = 1$, $k = 5$, and $K = 20$ respectively. We will use results of robot 1 of 5 connections model for destination 1 to show how gain influences the speed of convergence to the destination position. From the below results (Figure 11), we can say that the value of gain K has an influence on the speed of convergence. Because the slope represents the speed of convergence, with increase of the gain value, the slope is greater. So the larger the gain value, the faster the convergence speed.

2.3 Simplified consensus-based control law

2.3.1 Plot the graph of new system

In this case, we propose a consensus-based control law with 4 connnections because we remove 1 connection between 1 and 5. Therefore, the graph of system is shown as follow (Figure 12).

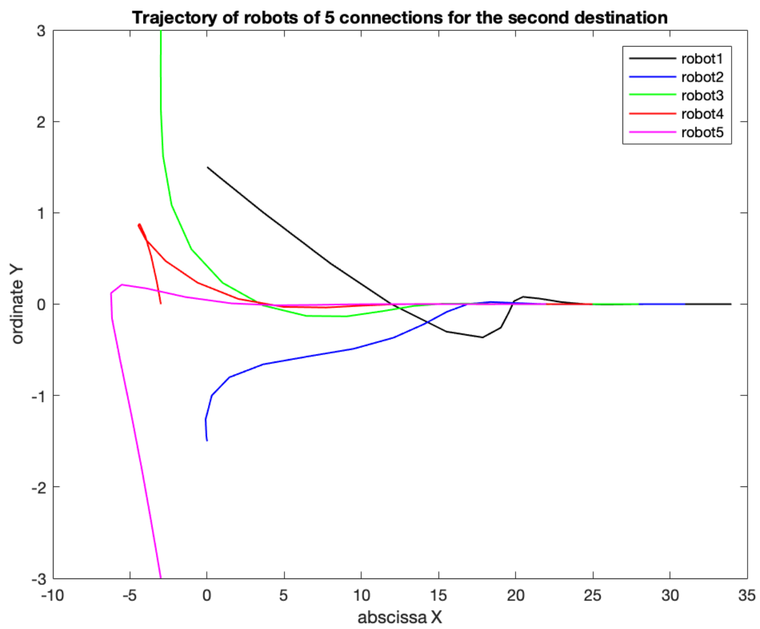


Figure 8: trajectory for 5 connections model for destination 1

2.3.2 Simplified consensus-based control law

Update of new matrix A

By updating the information included in matrix A, the new matrix is shown below :

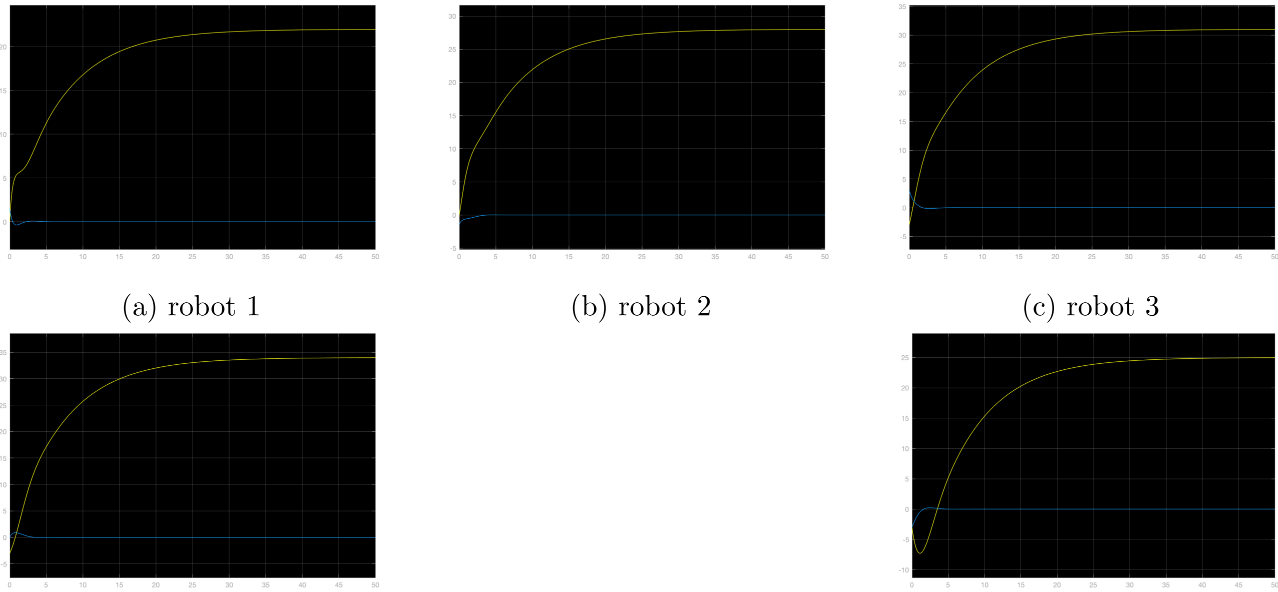
$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

We can see that the parts of matrix A related to robot 2, robot 3, robot 4 and robot 5 don not change, because the communication shared in these robots stay same. And the only change is related to robot 1, because it can not receive information from robot 5 any more, and it is only given the destination positions now.
So for the robot 1, the new equations for (u_{x_1}, u_{y_1}) for destination 1 is as follows :

$$\begin{cases} u_{x_1} = -K(x_{h_1} - 34) \\ u_{y_2} = -K(y_{h_2} - 0) \end{cases}$$

In the same way, we can also update the new equations for (u_{x_1}, u_{y_1}) for destination 2.

$$\begin{cases} u_{x_1} = -K(x_{h_1} - 22) \\ u_{y_2} = -K(y_{h_2} - 0) \end{cases}$$



(a) robot 1 (b) robot 2 (c) robot 3

(d) robot 4 (e) robot 5

Figure 9: Results of 5 connections model for destination 2

$\boxed{\text{Update of new matrix D}}$

Therefore, we can update our new matrix D for destination 1 and destination 2 as follows :

$$D1 = \begin{bmatrix} d1(1,1) \\ d1(1,2) \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix} D2 = \begin{bmatrix} d2(1,1) \\ d2(1,2) \\ d2(2,1) - d2(1,1) \\ 0 \\ d2(3,1) - d2(2,1) \\ 0 \\ d2(4,1) - d2(3,1) \\ 0 \\ d2(5,1) - d2(4,1) \\ 0 \end{bmatrix}$$

Finally, same law for the equation of u :

$$u = K(A * \text{position} + D)$$

2.3.3 Results of 4 connections model

$\boxed{\text{Verification of destination positions}}$

In the same way, from the results of 5 scopes, we can see if the robots converge to the destination positions. For example K=1, we can see robot 1 converges to (34,0), robot 2 converges to (31,0), robot 3 converges to (28,0), robot 4 converges to (25,0) and robot 5 converges to (22,0). The results are presented below (Figure 13). The trajectory of the 5 robots in 4 connections model for destination 1 is as follow (Figure 14). We will also present the results (K=1) of 4 connections model for destination 2 to you, which show that robot 1 converges to (22,0), robot 2 converges to (28,0), robot 3 converges to (31,0), robot 4 converges to (34,0) and robot 5 converges to (25,0). The trajectory of the 5 robots in 4 connections model for destination 2 is as follow

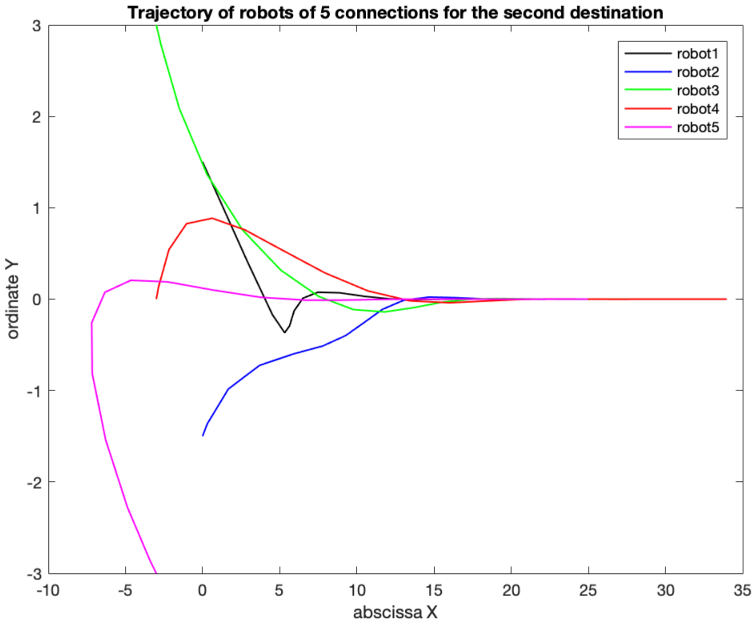
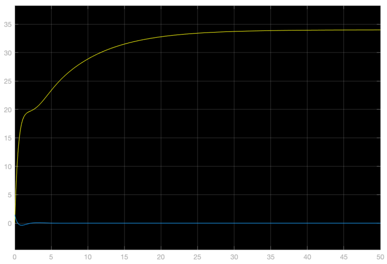
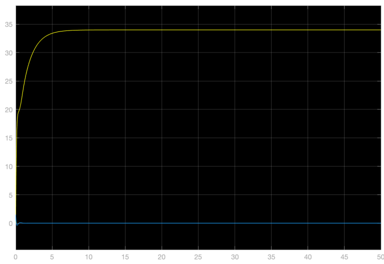


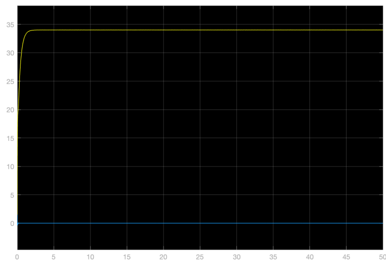
Figure 10: trajectory for 5 connections model for destination 2



(a) $K = 1$



(b) $K = 5$



(c) $K = 20$

Figure 11: Influence of gain in 5 connections model for destination 1 (robot 1)

(Figure 16).

Discussion of role of gain

There are two ways to compare the speed of convergence to the destination positions, which consist of comparing trajectory graphs and also scopes graphs.

First method : compare trajectory graphs

We can see that the 4 connections model have a more faster convergence to the destination positions compared to the 5 connections model (Figure 17). Because the 4 connections model is a simplified model which means it needs less time to communicate, so it can converge more quickly compared to 5 connections model.

Second method : compare scopes graphs

From the images below (Figure 18), we can obtain the same conclusion as in the 5 connections model, which is that gain can have an influence on the speed of convergence. The larger the gain value, the faster the convergence speed.

And we can also see that with the same value of gain value, the speed of convergence in 4 connections model is faster than in 5 connections model.

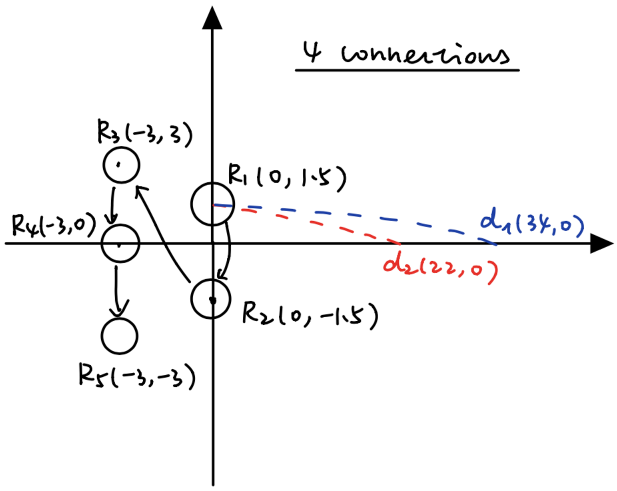


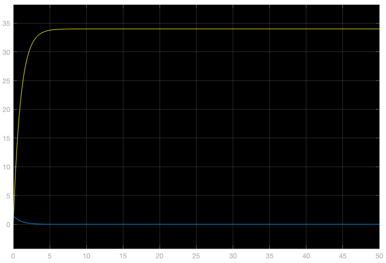
Figure 12: 4 connections model

2.4 Propose a controller that makes robot continue travel in platooning, on the 'x-axis' at constant speed after having formed the platoon

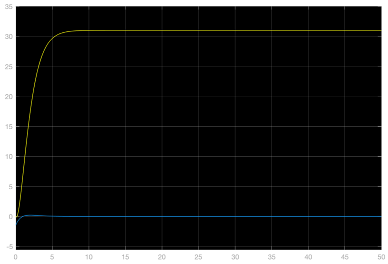
2.4.1 Determination of matrix A

To solve this problem, we propose that we still consider robot 1 as a leader and we give it a constant speed v. Then we will use the 4 connections model to make all of the 5 robots form a platoon and each of them converge to keep a distance of 3 on x-axis and of 0 on y-axis between them.
Therefore, what we have to do is to determine the new matrix of A and new matrix of D.

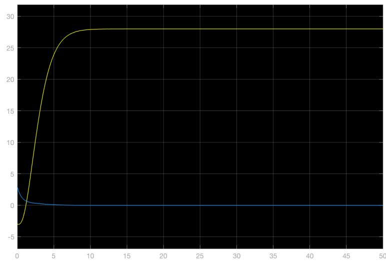
$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$



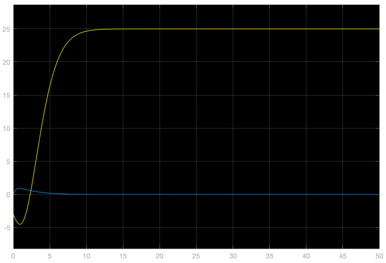
(a) robot 1



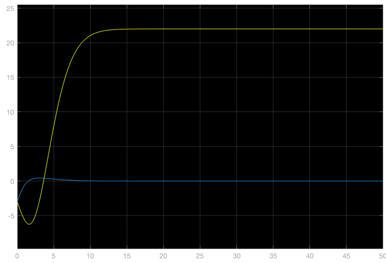
(b) robot 2



(c) robot 3



(d) robot 4



(e) robot 5

Figure 13: Results of 4 connections model for destination 1

2.4.2 Determination of matrix D

$$D = \begin{bmatrix} V \\ 0 \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix}$$

2.4.3 Equation of u

$$u = K(A * \text{position} + D)$$

2.4.4 Results of the simulation

The trajectory for robots at constant speed is presented as follow (Figure 19).

Verificaton of trajectory

We can see that after a certain period of time, the 5 robots form a platoon on x-axis. Then, we will check all the scope figures to verify that robots travel at a constant speed on x-axis after having formed the platoon.

Discussion of role of gain

As usual, we consider the case where K=1. From the images below (Figure 20), we see that after a certain period of time, all of the 5 robots gain the same speed as robot 1 which equals to v. Because the speed of robot 1 is always equal to v, so that we analysed the speed of robot 2. From the images below, a conclusion is always verified which is that gain has an influence

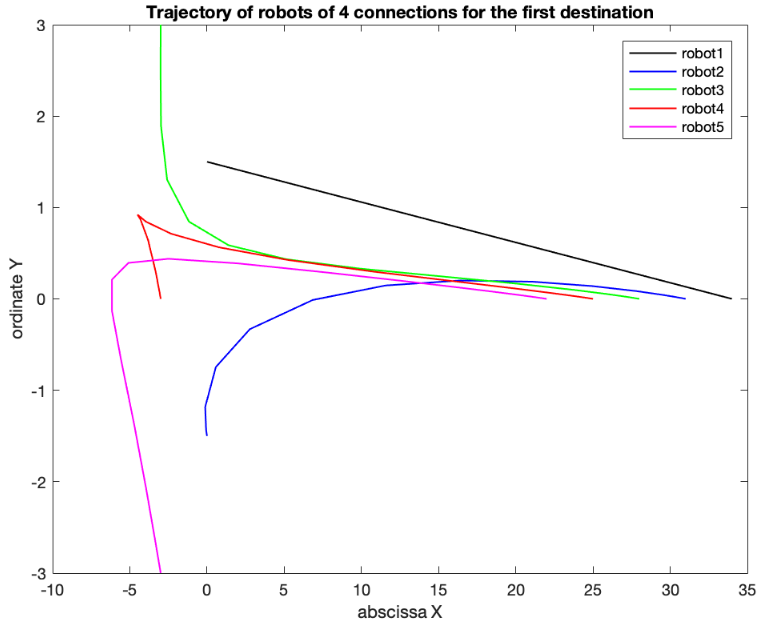


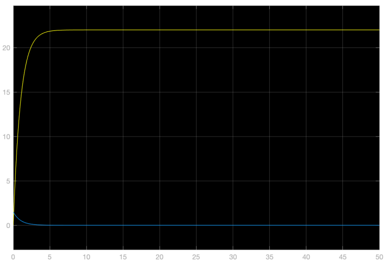
Figure 14: trajectory for 4 connections model for destination 1

on the speed of convergence to the destination positions. However, when we look at the parts where robots travel at a constant speed, we can also find that gain has an influence on speed of robots. With the increase of gain value, the speeds of robots also increase and in a proportional way.

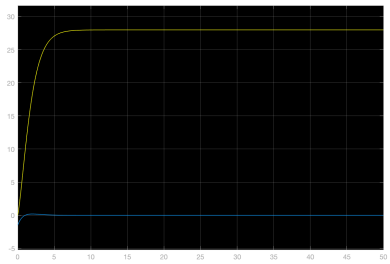
2.5 Test of stability

From the below figure (Figure 22), we know that the part which belongs to negative real number field is stable, and the other part is not stable. So next, we will check the eigenvalue of matrix A to see if the control system is stable. If all eigenvalues of matrix A is negative, which means this control system is stable, otherwise, this control system is not stable.
Matrix A of 5 connections model
Eigenvalues of matrix A is calculated automatically on Matlab and the results are as follows (Figure 22). We noticed that all of the eigenvalues of matrix A belong to negative real number field, which means that this control system is stable.

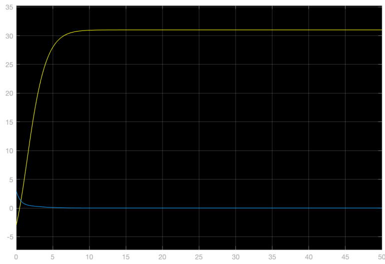
$$A = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$



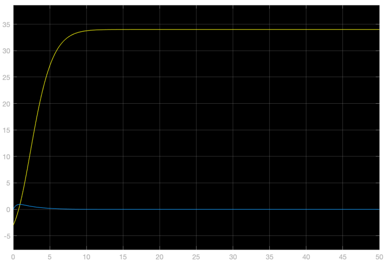
(a) robot 1



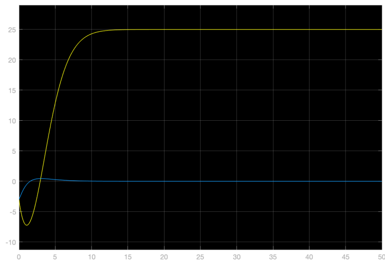
(b) robot 2



(c) robot 3



(d) robot 4



(e) robot 5

Figure 15: Results of 4 connections model for destination 2

Matrix A of 4 connections model

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

All of the eigenvalues of matrix A are negative, which means that this control system is stable.

Matrix A of constant speed model

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

All of the eigenvalues of matrix A are not positive, which also means that this control system is stable.

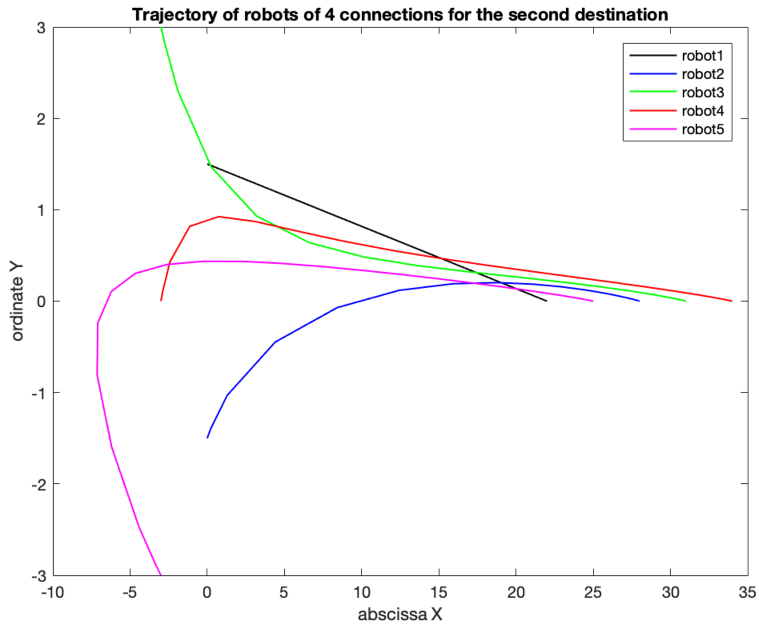
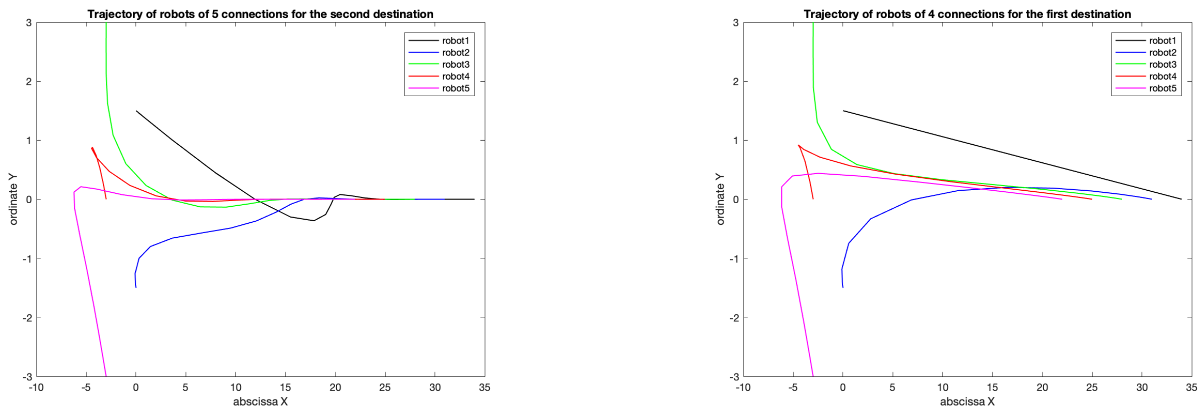


Figure 16: trajectory for 4 connections model for destination 2

3 Conclusion

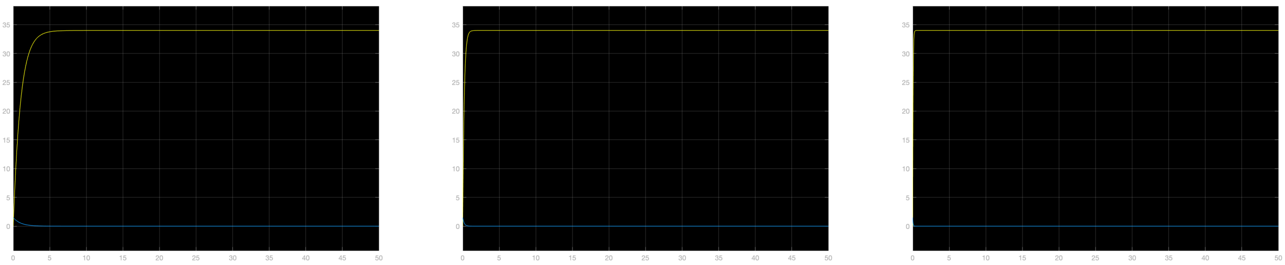
In this project 4 : platooning - consensus, we use collaborative control to let robots communicate with each other and finally form the platooning. To achieve this objective, we proposed different kinds of control system, and after all of the tests, we know that they are all stable. In addition to that, we also analysed the trajectory of 5 robots and the role of gain value. And we know that the larger gain value, the faster the convergence speed.



(a) 5 connections model for destination 1

(b) 4 connections model for destination 1

Figure 17: Influence of model for speed of convergence (K=1)



(a) K=1 (b) K=5 (c) K=20

Figure 18: Influence of gain in 4 connections model for destination 1 (robot 1)

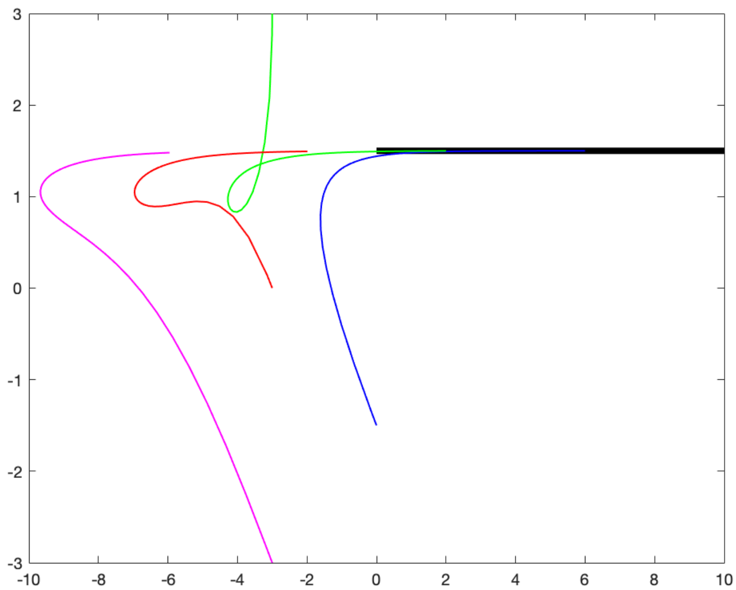
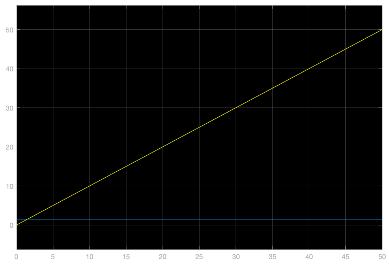
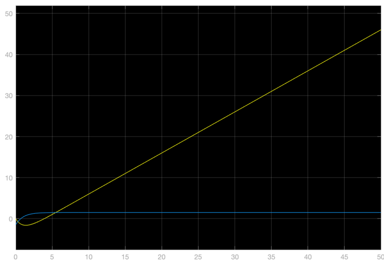


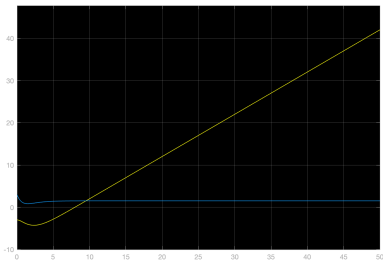
Figure 19: trajectory for robots at constant speed



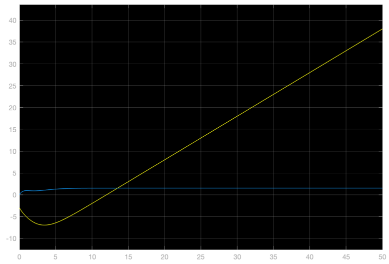
(a) robot 1



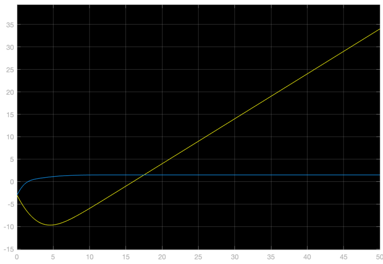
(b) robot 2



(c) robot 3

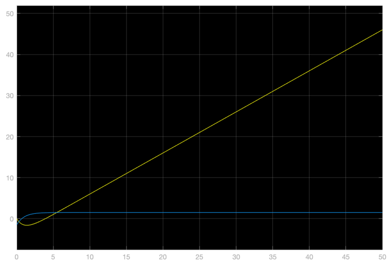


(d) robot 4

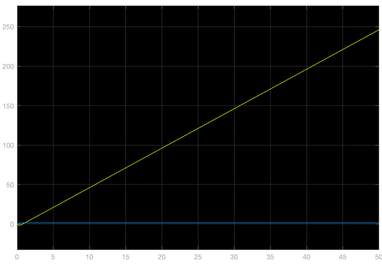


(e) robot 5

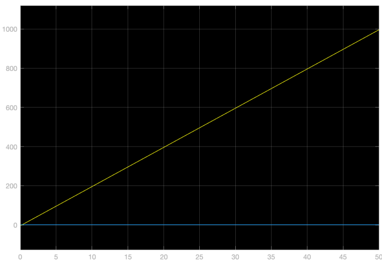
Figure 20: Results of robots at constant speed



(a) K=1

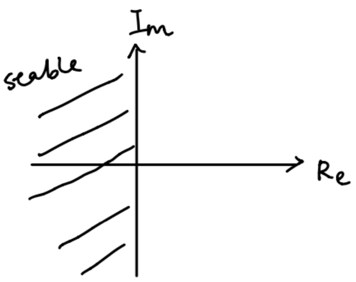


(b) K=5



(c) K=20

Figure 21: Influence of gain on speed of robots (robot 2)



(a) Theory of stability

```
>> eig(A5)

ans =

  -2.0784 + 0.4969i
  -2.0784 - 0.4969i
  -0.8499 + 0.8975i
  -0.8499 - 0.8975i
  -0.1433 + 0.0000i
  -2.0784 + 0.4969i
  -2.0784 - 0.4969i
  -0.8499 + 0.8975i
  -0.8499 - 0.8975i
  -0.1433 + 0.0000i
```

(b) Eigenvalues of matrix A of 5 connections model

```
>> eig(A4)

ans =

    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
```

(c) Eigenvalues of matrix A of 4 connections model

```
>> eig(A_C)

ans =

    -1
    -1
    -1
    -1
     0
    -1
    -1
    -1
    -1
     0
```

(d) Eigenvalues of matrix A of constant speed model

Figure 22: Test of stability