Random signals and white noises

Ph. Bonnifait

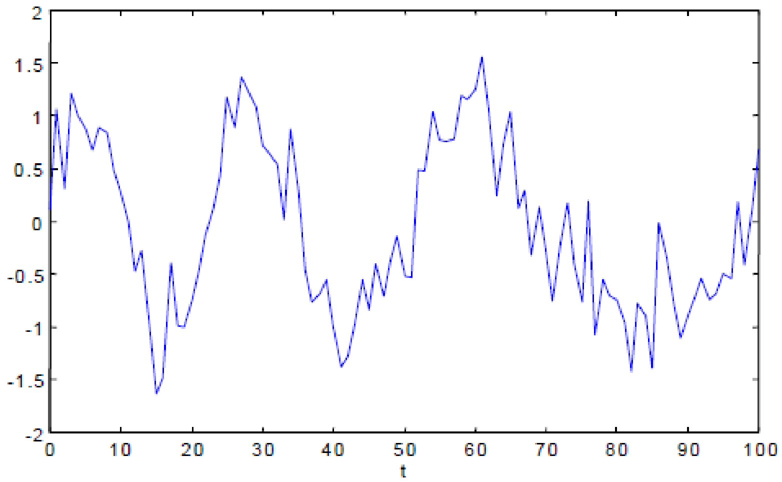
ARS04

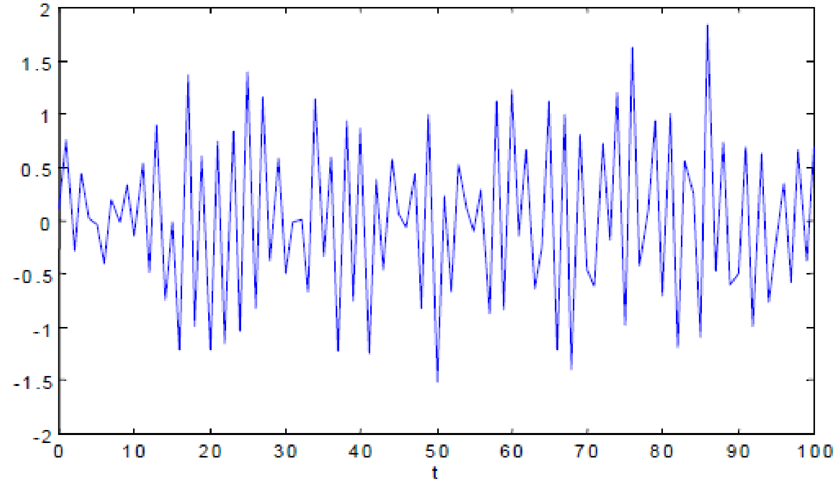
Université de Technologie Compiègne

Second order modeling of a random signal

- A random signal is a function of time and randomness
- It is characterized by:
  - Statistical properties which are also called "spatial distribution"
  - Temporal or frequential properties.

Two statistically different random signals but with the same variance





Université de Technologie Compiègne

Auto-correlation function

$$\Psi_{xx}(t,\tau) = \mathrm{E}\left\langle [\mathrm{x}(\mathrm{t}) - \mathrm{E}(\mathrm{x}(\mathrm{t}))] \cdot [\mathrm{x}(\mathrm{t}+\tau) - \mathrm{E}(\mathrm{x}(\mathrm{t}+\tau))] \right\rangle$$

$$\Psi_{xx}(t,0) = \mathrm{Var}(x(t))$$

- A random signal is characterized at the second order by:
  - Its statistical mean,
  - Its auto-correlation function.

Université de Technologie Compiègne

Stationarity

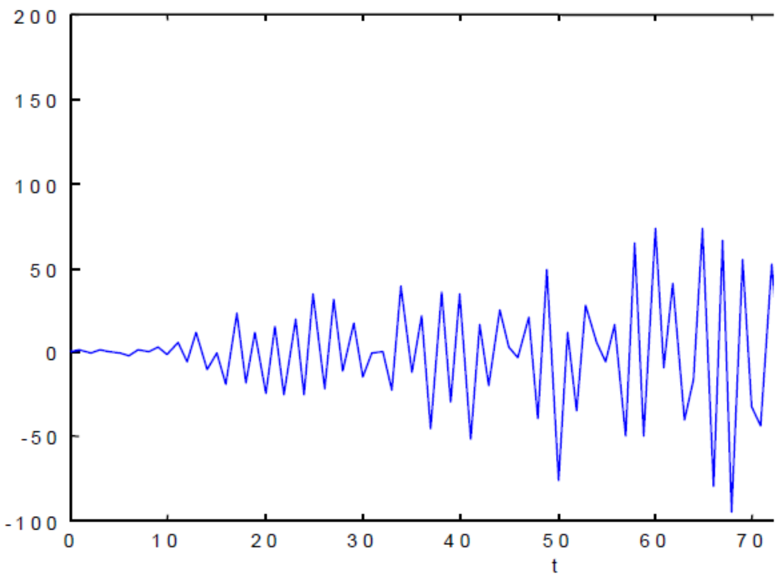
- $x(t)$ is stationary if its spatial and temporal characteristics do not depend on time:

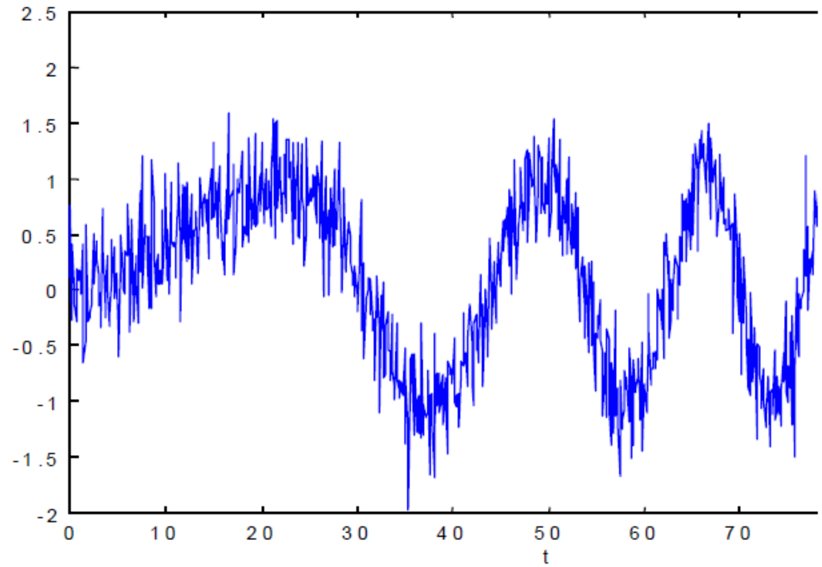
$$\mathrm{E}\left[x(\mathrm{t})\right] = \mathrm{cste}$$

$$\Psi_{xx}\left(t, \tau\right) = \Psi_{xx}\left(\tau\right)$$

Université de Technologie Compiègne

Two non-stationary random signals





Ergodicity

- An ergodic signal has its temporal moments that are identical to its statistical moments
- A realization of an ergodic signal of infinite duration contains the same information as an infinite realization.

Time estimates

$$E[x_k] = \lim_{n \to \infty} \left\{ \frac{1}{n} \cdot \sum_{k=0}^{n} x_k \right\}$$

$$Var[x_k] = \lim_{n \to \infty} \left\{ \frac{1}{n} \cdot \sum_{k=0}^{n} (x_k - m_x)^2 \right\}$$

$$\Psi_{xx}(h) = \lim_{n \to \infty} \left\{ \frac{1}{n} \cdot \sum_{k=0}^{n} (x_k - m_x) \cdot (x_{k+h} - m_x) \right\}$$

$$\hat{m}_x = \frac{1}{n} \cdot \sum_{k=0}^{n} x_k$$

$$\hat{\Psi}_{xx}(h) = \frac{1}{n-h} \cdot \sum_{k=0}^{n-h} (x_k - \hat{m}_x) \cdot (x_{k+h} - \hat{m}_x) \text{ pour } 0 < \mathrm{h} < \mathrm{n} - 1$$

utc Université de Technologie Compiègne

Continuous white noise

Its auto-correlation function $\Psi(t)$ is a Dirac delta function

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

knowledge of $B(t)$ provides no information on $B(t+\tau)$

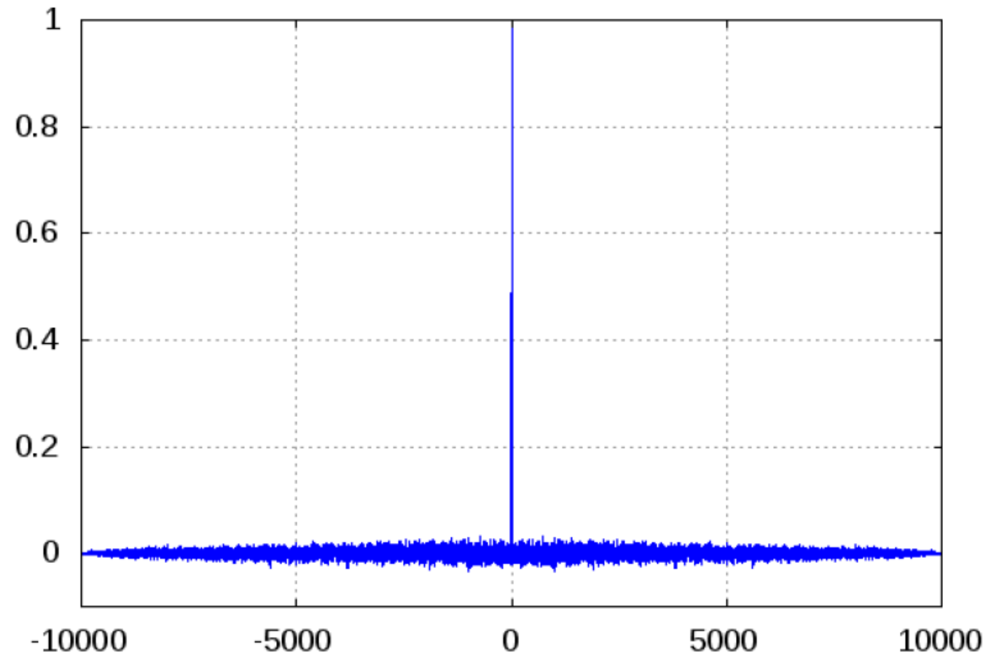
$$Var[B(t)] = \Psi(0) = \infty$$

Physically, such a signal does not exist!

Discrete white noise

$$E[w_k.w_l] = 0 \text{ if } k \neq l$$
$$= q \text{ if } k = l$$

$$E[w_k^2] = var(w_k) = q$$



Auto-correlation of a discrete white noise

The auto-correlation function has a pseudo-Dirac delta at the origin.