ARS1 - Project4: Platooning - Consensus

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Introduction

Presentation of the project

- five robots controllable via:

$$\text{linear velocity} \quad v_i, \text{angular velocity} \quad \omega_i$$

- Robots can have connections with each other.
- The agents should maintain a linear formation.
- Initials positions set by user.
- Destination platooning position set by user.
- From Initials positions to the give platooning position and the platooning formation should be kept in later movement.

Consensus System presentation

Constraint

- Take a point h on robot to satisfy the nonholonomic.

$$\dot{x}_i sin(\theta_i) - \dot{y}_i cos(\theta_i) = 0$$

we have:

$$x_{h_i} = x_i + L_i \cos(\theta_i)$$
$$y_{h_i} = y_i + L_i \sin(\theta_i)$$
$$\text{where} \quad L_i > 0$$

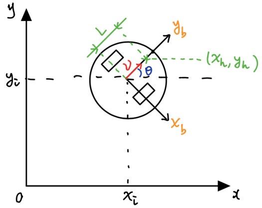


Figure: Robot Schema

Consensus System presentation

Input u_{x_i} and u_{y_i} which can be transfers to v_i, ω_i and also represent the variation of $\dot{x_{h_i}}$, $\dot{y_{h_i}}$.

$$\dot{x_{h_i}} = \dot{x_i} + L_i(-\sin(\theta_i))\dot{\theta_i} = v_i \cos(\theta_i) + L_i(-\sin(\theta_i))w_i$$

$$\dot{y_{h_i}} = \dot{y_i} + L_i \cos(\theta_i))\dot{\theta_i} = v_i \sin(\theta_i) + L_i \cos(\theta_i))w_i$$

$$\begin{pmatrix} \dot{x_{h_i}} \\ \dot{y_{h_i}} \end{pmatrix} = \underbrace{\begin{pmatrix} cos(\theta_i) & -L_i sin(\theta_i) \\ sin(\theta_i) & L_i cos(\theta_i) \end{pmatrix}}_{R_i} \begin{pmatrix} v_i \\ w_i \end{pmatrix}$$

$$\begin{pmatrix} v_i \\ w_i \end{pmatrix} = R_i^{-1} \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix}$$

$$\begin{pmatrix} \dot{x_{h_i}} \\ \dot{y_{h_i}} \end{pmatrix} = R_i * R_i^{-1} \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix} = \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix}$$

Consensus System presentation

Drive $x_{h_i} \rightarrow x_i(t)$, $y_{h_i} \rightarrow y_i(t)$ We design the system:

$$\begin{cases} u_{x_i} = \dot{x}_i - (x_{h_i} - x_i) \\ u_{y_i} = \dot{y}_i - (y_{h_i} - y_i) \end{cases}$$

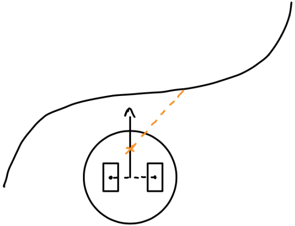


Figure: illustration of robot follows $(x_i(t), y_{h_i})$

Consensus System presentation

Multi-robot rendez-vous We design the system:

$$\begin{cases} \dot{x_{h_i}} = u_{h_i} \\ \dot{y_{h_i}} = u_{y_i} \end{cases}$$

$$\begin{cases} u_{x_i} = -\sum_{j=1}^{N} g_{ij} k_{ij} (x_{h_i} - x_{h_j}) \\ u_{y_i} = -\sum_{j=1}^{N} g_{ij} k_{ij} (y_{h_i} - y_{h_j}) \end{cases}$$

Where k_{ij} is the gain and g_{ij} is the connection between i,j. $g_{ij} = 0$ if there are no connection between robot i,j and is 1 otherwise.

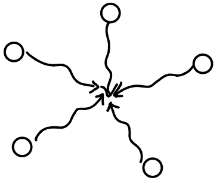


Figure: illustration of robot follows $(x_i(t), y_{h_i})$

Consensus System presentation

An implementation on Simulink

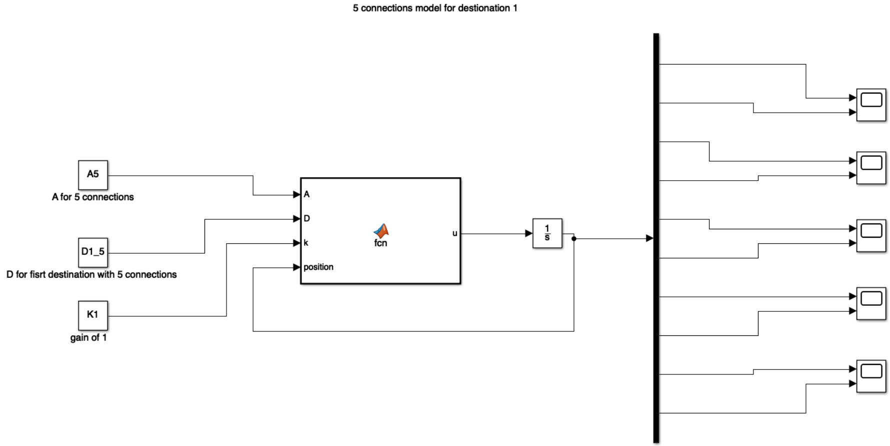


Figure: An example on Simulink

1. Five connections Consensus-based control

Proposed model

- We let the information pass among these five robots
  - $g_{15} = g_{21} = g_{32} = g_{43} = g_{54} = 1$
- Graph of 5 connections model
- We give one robot the position of it's destination and other robots keep the defined distances between each other to form the platooning.

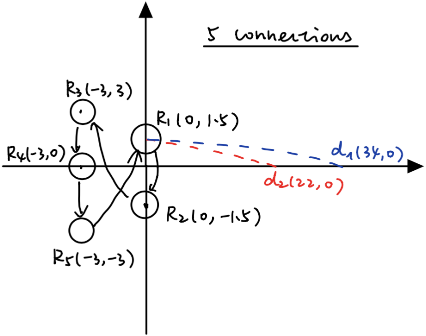


Figure: graph of connections

1. Five connections Consensus-based control

Proposed model

- platooning between robot2 and robot1

$$u_{x_2} = -K(x_{h_2} - (x_{h_1} - 3)) = K(-x_{h_2} + x_{h_1} - 3)$$

$$u = k(A * \text{position} + D)$$

$$A =$$
$$\begin{bmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

```
>> eig(A5)

ans =

  -2.0784 + 0.4969i
  -2.0784 - 0.4969i
  -0.8499 + 0.8975i
  -0.8499 - 0.8975i
  -0.1433 + 0.0000i
  -2.0784 + 0.4969i
  -2.0784 - 0.4969i
  -0.8499 + 0.8975i
  -0.8499 - 0.8975i
  -0.1433 + 0.0000i
```

Figure: eigenvalues

1. Five connections Consensus-based control

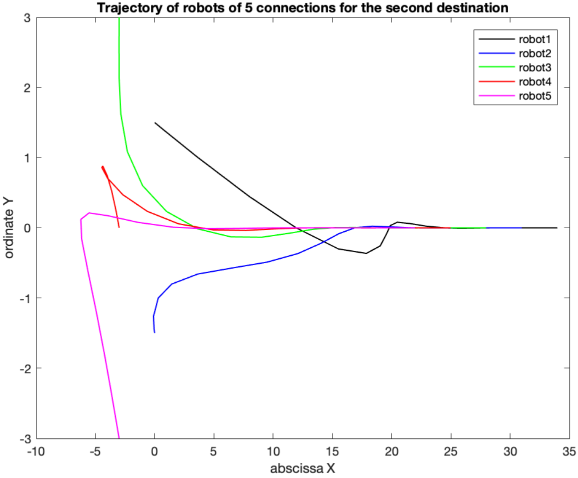
Proposed model

$$D1 = \begin{bmatrix} (d1(1,1) - d1(5,1)) + d(1,1) \\ (d1(1,2) - d2(5,2)) + d(1,2) \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix} \qquad d1 = \begin{bmatrix} 34 & 0 \\ 31 & 0 \\ 28 & 0 \\ 25 & 0 \\ 22 & 0 \end{bmatrix}$$

Result Analyse 1

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory
  - Verification of trajectory



Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

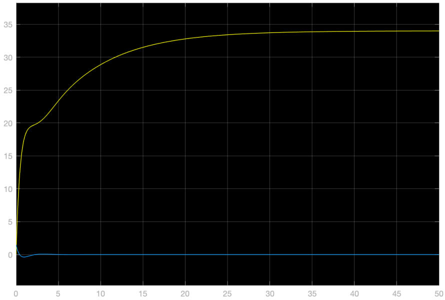


Figure: robot 1

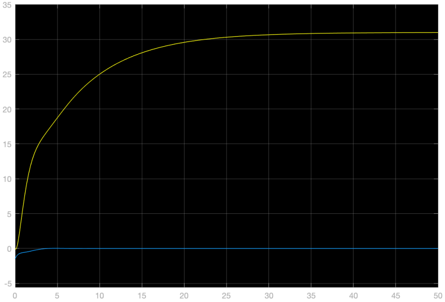


Figure: robot 2

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

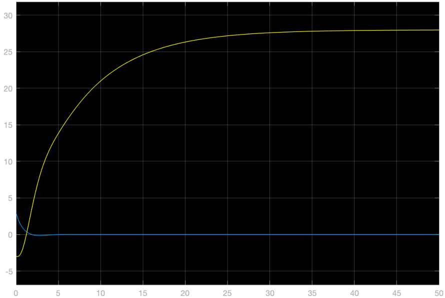


Figure: robot 3

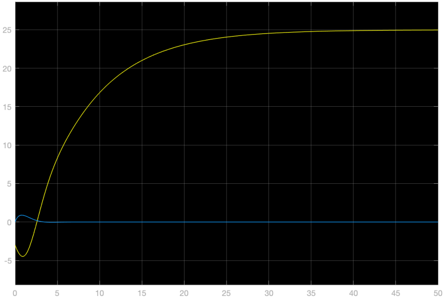


Figure: robot 4

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

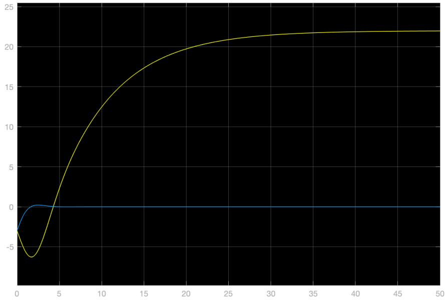


Figure: robot 5

Result Analyse 1

Analysis of trajectory of robots and of influence of gain value

- Influence of gain value
- Robot 1 with first destination

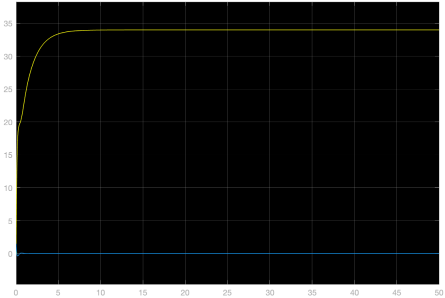


Figure: k=5

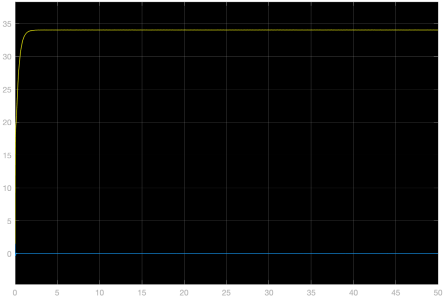
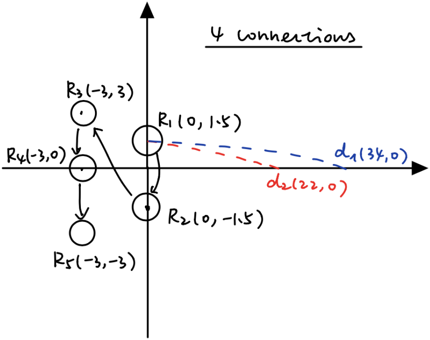


Figure: k=20

2. Four Connections Consensus-based control

Proposed model

- Choose robot 1 as leader and assign a destination position to it
- Communication shared between robot 1, 2, 3, 4 and 5
  - $g_{21} = g_{32} = g_{43} = g_{54} = 1$
- Graph of 4 connections model



Proposed model

- platooning between robot2 and robot1

$$u_{x_2} = -K(x_{h_2} - (x_{h_1} - 3)) = K(-x_{h_2} + x_{h_1} - 3)$$

$$u = k(A * \text{position} + D)$$

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

```
>> eig(A4)

ans =

    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
    -1
```

Figure: eigenvalues

Proposed model

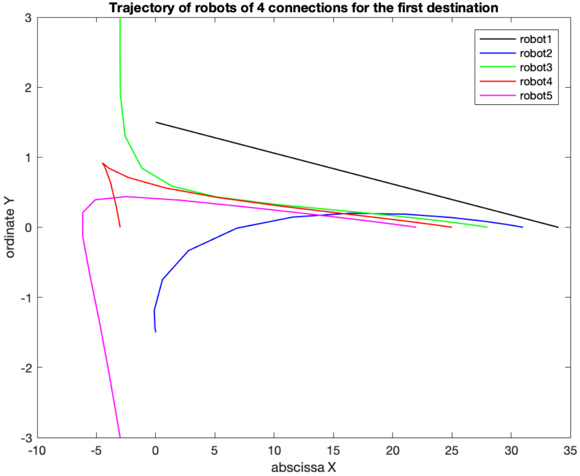
- Determination of matrix D

$$D1 = \begin{bmatrix} d1(1,1) \\ d1(1,2) \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix} \qquad d1 = \begin{bmatrix} 34 & 0 \\ 31 & 0 \\ 28 & 0 \\ 25 & 0 \\ 22 & 0 \end{bmatrix}$$

Result Analyse 2

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory



Result Analyse 2

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

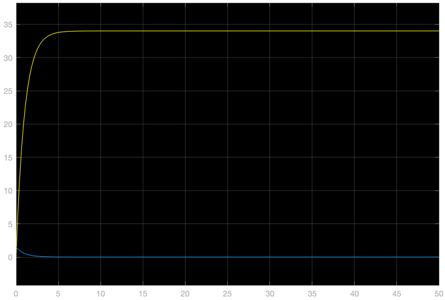


Figure: robot 1

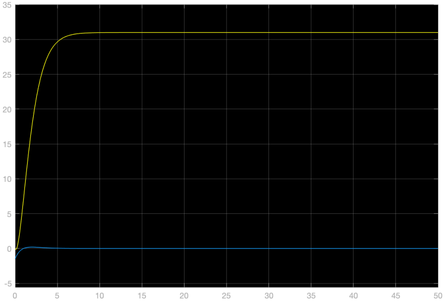


Figure: robot 2

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

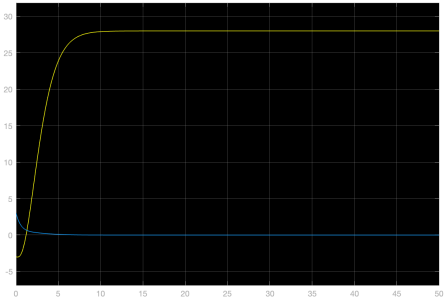


Figure: robot 3

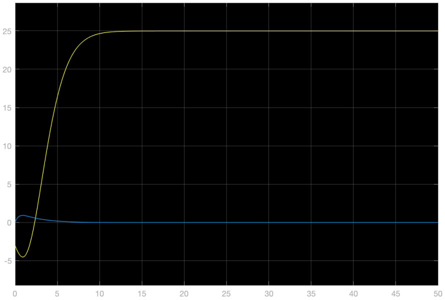


Figure: robot 4

Result Analyse 2

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

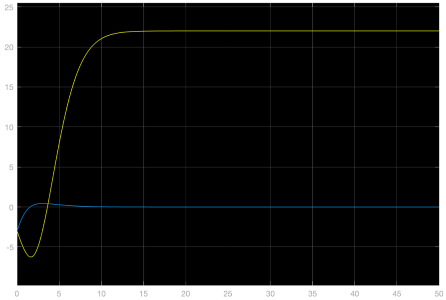


Figure: robot 5

Result Analyse 2

Analysis of trajectory of robots and of influence of gain value

- Influence of gain value
- Robot 1 with first destination

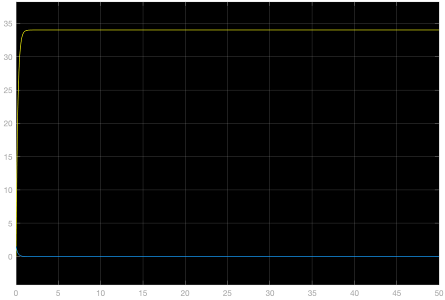


Figure: k=5

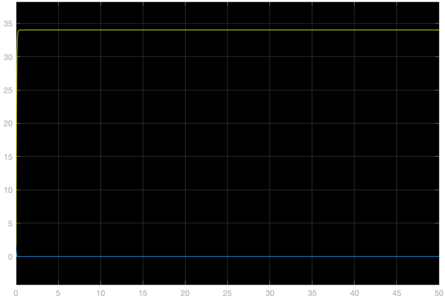


Figure: k=20

Proposed model

- Choose robot 1 as leader and assign a constant speed v to it
- Communication shared between robot 1, 2, 3, 4 and 5
    - $g_{21} = g_{32} = g_{43} = g_{54} = 1$
- Determination of matrix A

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

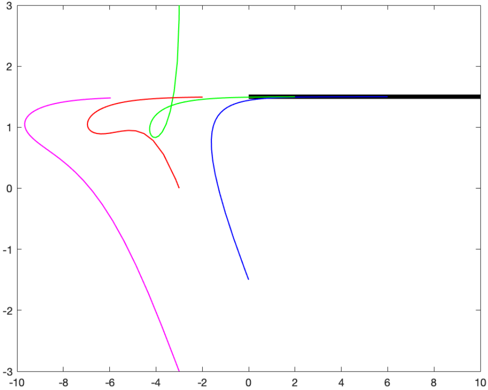
Proposed model

- By applying 4 connection model and keep a distance of 3 on x-axis (d1).
  - $u = k(A * \text{position} + D)$
    - ex. $u_{x_2} = -K(x_{h_2} - (x_{h_1} - 3)) = K(-x_{h_2} + x_{h_1} - 3)$
  - $d1 = \begin{bmatrix} 34 & 0 \\ 31 & 0 \\ 28 & 0 \\ 25 & 0 \\ 22 & 0 \end{bmatrix}$ $D = \begin{bmatrix} V \\ 0 \\ d1(2,1) - d1(1,1) \\ 0 \\ d1(3,1) - d1(2,1) \\ 0 \\ d1(4,1) - d1(3,1) \\ 0 \\ d1(5,1) - d1(4,1) \\ 0 \end{bmatrix}$

Result Analyse 3

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory



Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

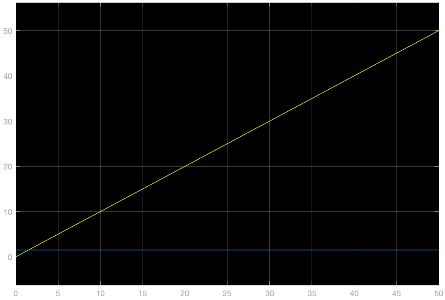


Figure: robot 1

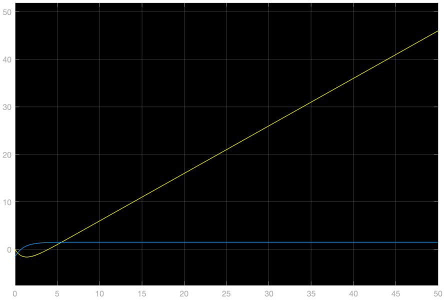


Figure: robot 2

Result Analyse 3

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

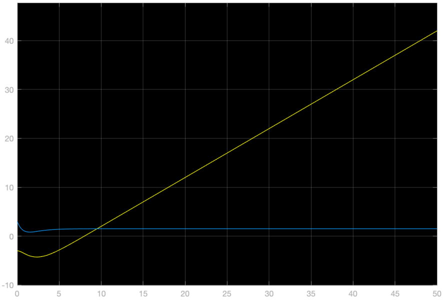


Figure: robot 3

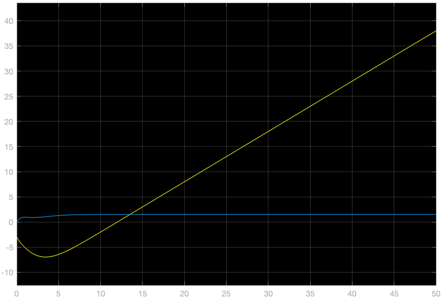


Figure: robot 4

Result Analyse 3

Analysis of trajectory of robots and of influence of gain value

- Verification of trajectory

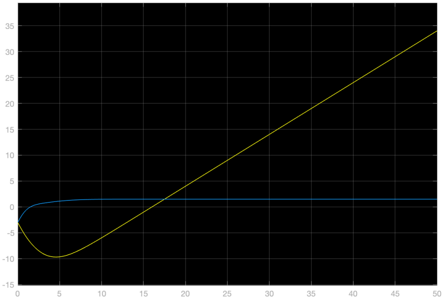


Figure: robot 5

Analysis of trajectory of robots and of influence of gain value

- Influence of gain value

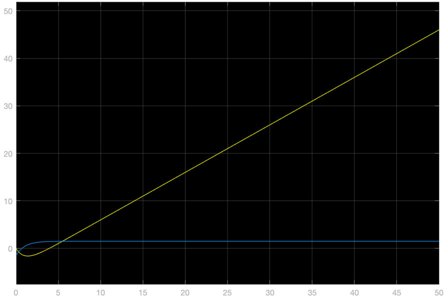


Figure: k=1

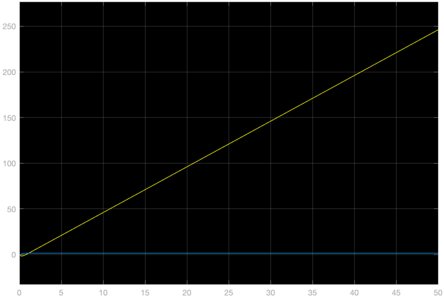


Figure: k=5

Result Analyse 3

Analysis of trajectory of robots and of influence of gain value

- Influence of gain value

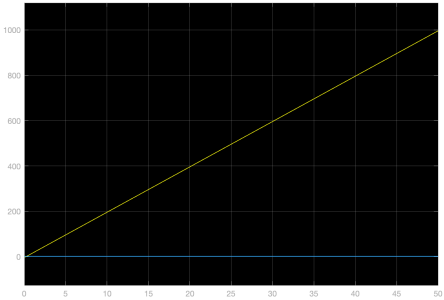


Figure: k=20

Result Analyse 3

Analysis of trajectory of robots and of influence of gain value

- Influence of gain value
  - The greater the gain value, the faster the convergence speed.
  - Speed is proportional to gain value.

Test of stability

- Matrix A of constant speed model

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

```
>> eig(A_C)

ans =

    -1
    -1
    -1
    -1
     0
    -1
    -1
    -1
    -1
     0
```

Figure: eigenvalues

Remark : all of the eigenvalues of matrix A are not positive, which means that the system control is stable.

Conclusion

- This consesus-based SOS get a great convergence.
- The linear control system for each robot is stable.
- Gain value(K) decides the speed of convergence
- 4 connections model converges more quickly than 5 connections model with the same K
- The system solve the bandwidth limit problem. (We pass the position or speed to only one robot and they will form platooning)
- The system achieve the platooning mission