PDE Midterm 2 Topics and Practice problems

Topic 1. The heat kernel function

$$S(x,t) = \frac{1}{\sqrt{4\pi kt}} e^{-\frac{x^2}{4kt}}$$ and approximation of the delta function.

Practice problems. ① Write down the definition of an approximation of the delta function.

② 2.4.8, HW6 #2, #3.

③ Show that the heat kernel $\{\eta_t(x) = S(x,t),\ t>0\}$ is an approximation of the delta function.

Topic 2. Solving the heat equation on the whole line.

Practice problems: 2.4.14, 2.4.16, 2.4.17, 2.4.18 (HW6)

Topic 3. Solving the heat equation and the wave equation on the half line with the homogeneous Dirichlet B.C. or Neumann B.C. or Robin B.C. (6 problems)

Practice problems: ① 3.1.1, HW7 #1.

② HW7 #2.

③ ~~3.2.4~~ 3.1.4 HW7 #4 (which is basically the same as ~~3.2.5~~ 3.1.5)

④ HW8 #2, part (1)

⑤ 3.2.1, 3.2.6.

Topic 4 Solving the heat equation and the wave equation on a finite interval $[a,b]$ with homogeneous Dirichlet B.C. or Neumann B.C. or Robin B.C. on a and on b.

$(2 \times 3 \times 3 = 18$ problems)

Method 1: By the extension method.

Practice problems: HW7 #3, 3.2.8, 3.2.9.
HW7 #6, HW8 #3.

Method 2: By separation of variables. The eigenvalue problem.

Practice problems: All of HW9
HW10 #1, #2.

Topic 5. Orthogonal system of functions and

and Fourier series.

Practice problems: HW10 #3.

Topic 6 Duhamel's principles for the heat equation and the wave equation.

Practice problems: HW7 #7, HW8 #2.

Topic 7 The maximum principle for the heat equation.

Practice problems: ① State both the weak and the strong maximum principle for the heat equation.

② 2.3.3, 2.3.4, 2.3.6.