$R3 - R1 = nR3$
$2-(-3) = 2+3 = 5$

Homework 3 1.5 {2, 9, 13, 17, 19}

$$\begin{aligned} x_1 - 3x_2 + 7x_3 &= 0 \\ -2x_1 + x_2 - 4x_3 &= 0 \\ x_1 + 2x_2 + 9x_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ \boxed{-2} & 1 & -4 & 0 \\ 1 & 2 & 9 & 0 \end{array}\right]$$

$-2+2(1)=0$
$1+2(-3)=-5$
$-4+2(7)=$
$-4+14=10$

$\Downarrow$ $R2 - 2R1 = nR2$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ \boxed{1} & 2 & 9 & 0 \end{array}\right]$$

$\Longleftarrow$ $R3 - R1 = nR3$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & \boxed{5} & 2 & 0 \end{array}\right]$$

$\Downarrow$ $R3 + R2 = nR3$

$0+0=0$
$5+(-5)=0$
$2+10=12$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & 12 & 0 \end{array}\right]$$

The matrix has no row the form $[0\ 0\ 0 \mid 1]$ so the system is consistent.

There are no free variables since all columns have pivots

There are three pivots, one in each column

we have a case of the num of pivots equals the number of variables, which means there are no free variables.

This system only has the trivial solution.

9.) $\begin{bmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{bmatrix}$ Swap R1 and R2 $\begin{bmatrix} -1 & 3 & -2 \\ 3 & -9 & 6 \end{bmatrix}$

$\Downarrow$ $-R1 = \text{new } R1$

$\begin{bmatrix} 1 & -3 & 2 \\ \boxed{3} & -9 & 6 \end{bmatrix}$

$-9-3(-3)$
$-9+9=0$
$6-3(2)=0$

$\Downarrow$ $R2 - 3R1 = \text{new } R2$

$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

- we have a pivot in col 1
- column 2 nor 3 has a pivot
- therefore x_2 and x_3 are free variables.

$$x_1 - 3x_2 + 2x_3 = 0$$

$$x_1 = -2x_3 + 3x_2$$
$$x_1 = 3x_2 - 2x_3$$

coefficients
$x_1 = 1$ $x_3 = -2$ $x_2 = 3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{bmatrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$x_1 = 3x_2 - 2x_3$ ✓

$x_2 = x_2 + 0x_3$ ✓

$x_3 = 0x_2 + x_3$ ✓

$$x = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

where $x_2, x_3 \in \mathbb{R}$ are free variables

13)

$$x = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

expanding

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$x_1 = 3x_2 - 2x_3$ ✓
$x_2 = x_2 + 0x_3$ ✓
$x_3 = 0x_2 + x_3$ ✓

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Ax \Rightarrow \begin{bmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{bmatrix} \begin{bmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

Row1

$$(3)(3x_2 - 2x_3) - 9(x_2) + 6x_3$$
$$9x_2 - 6x_3 - 9x_2 + 6x_3 = 0$$

Row2 $(-1)(3x_2 - 2x_3) + 3(x_2) - 2(x_3)$

$$-3x_2 + 2x_3 + 3x_2 - 2x_3 = 0$$

Since every row in Ax equals 0, we confirm

$$Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore we have shown that the solution is indeed a homogeneous solution.

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 39 \end{array}\right]$$

17) Suppose the solution set of a certain system of a certain system of linear eqns can be described as

$$x_1 = 5 + 4x_3, \quad x_2 = -2 - 7x_3 \text{ with } x_3 \text{ free.}$$

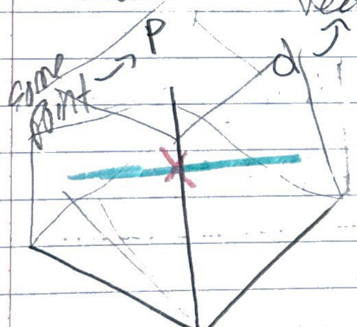
Use vectors to describe this set as a line in $\mathbb{R}^3$

$$\begin{aligned} x_1 &= 5 + 4x_3 \\ x_2 &= -2 - 7x_3 \\ x_3 &= x_3 \end{aligned} \implies x = \begin{bmatrix} 5 + 4x_3 \\ -2 - 7x_3 \\ x_3 \end{bmatrix}$$

$$x = \underbrace{\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}_{\text{solution}} + x_3 \underbrace{\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}}_{\text{direction vector}}$$

This is the equation of a line in $\mathbb{R}^3$.

The free variable x_3 scales d, moving the point along the line.



19) Follow method from #3 to describe solns of the following system in parametric form.

ex 3) $A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & 8 \end{bmatrix}$ $b = \begin{bmatrix} 7 \\ -1 \\ 4 \end{bmatrix}$

$-1+7=6$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ -3 & -2 & 4 & -1 \\ 6 & 1 & 8 & 4 \end{array}\right] \xrightarrow{R2+R1=nR2} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ 0 & 3 & 0 & 6 \\ 16 & 1 & 8 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ 0 & 3 & 0 & 6 \\ 0 & -9 & 16 & -10 \end{array}\right]$$

$-3+3=0$

$-2+5=3$

$4+(-4)=0$

$4-2(7) = 4-14 = -10$

$R3 - 2(R1) = nR3$

$8 - 2(-4)$
$= 8+8$

$16 + 3(0$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 16 & 11 \end{array}\right] \xrightarrow{\frac{1}{3}R2 = nR2} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 16 & 11 \end{array}\right]$$

$R3 + 4(R1) = nR3$

$16 + 4(-4) = 16 - 16$

$11 + 4(7) = 11 + 28$

$x_1 + 5x_2 - 4/3\, x_3 = 7/3$

$x_3 = \frac{1}{2}$ $\quad x_2 = 2$

$x_1 + \frac{20}{6} - \frac{4}{6} = \frac{14}{6}$

$x_1 + \frac{16}{6} = \frac{14}{6}$

$x_1 + \frac{5}{3}(2) - \frac{4}{3}\left(\frac{1}{2}\right) = \frac{7}{3}$

$x_1 + \frac{10}{3}\left(\frac{2}{2}\right) - \frac{4}{6} = \frac{7}{3}\left(\frac{2}{2}\right)$

$x_1 = \frac{14}{6} - \frac{16}{6}$

$x_1 + \frac{10}{3} - \frac{4}{6} = \frac{7}{3}$

$x_1 = -\frac{1}{3}$

$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & 8 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ -1 \\ 4 \end{bmatrix}$

$x_1 = -\frac{2}{6}$

$Aug = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 7 \\ -3 & -2 & 4 & -1 \\ 6 & 1 & 8 & 4 \end{array}\right] \Longrightarrow R1 \cdot \frac{1}{3} = nR1 \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 7/3 \\ -3 & -2 & 4 & -1 \\ 6 & 1 & 8 & 4 \end{array}\right]$

$R2 + 3R1 = \text{new } R2$

$-2 + 3\left(\frac{5}{3}\right) = -2 + 5 = 3$

$4 + 3\left(-\frac{4}{3}\right) = 4 + (-4) = 4 - 4 = 0$

$-1 + 3\left(\frac{7}{3}\right) = -1 + 7 = 6$

$\left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 3 & 0 & 6 \\ 0 & -9 & 16 & -10 \end{array}\right]$

$R3 - 6R1 = \text{new } R3$

$6 - 6(1) = 6 - 6 = 0$

$1 - 6\left(\frac{5}{3}\right) = 1 - 10 = -9$

$8 - 6\left(-\frac{4}{3}\right) = 8 + 8 = 16$

$4 - 6\left(\frac{7}{3}\right) = 4 - 14 = -10$

$R2/3$ or $R2 \cdot \frac{1}{3} = nR2$

$\left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 1 & 0 & 2 \\ 0 & -9 & 16 & -10 \end{array}\right]$

$0 + 9(0) = 0$

$-9 + 9(1) = -9 + 9 = 0$

$R3 + 9(R2) = nR3$

$16 + 9(0) = 16$

$-10 + 9(2) = -10 + 18 = 8$

$\left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 16 & 8 \end{array}\right]$

$\boxed{x_1 = -\frac{1}{3},\ x_2 = 2,\ x_3 = \frac{1}{2}}$

$R3 \cdot \frac{1}{16} = nR3 \quad \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$

REF

19.) Follow method of ex 3 to describe the solns of the following system in parametric form

$$x_1 + 3x_2 + x_3 = 1$$
$$-4x_1 - 9x_2 + 2x_3 = -1$$
$$-3x_2 - 6x_3 = -3$$

$-9+4(3) =$
$-9+12=3$
$2+4(1)=2+4=$
$0+4(0)=0$

$$\overset{A}{\begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix}} \overset{b}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \xrightarrow{R2+4R1=\text{new }R2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right]$$

$R2 \cdot \frac{1}{3} = \text{new } R2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right]$$

$R3+3R2=\text{new }R3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Row echelon form

$-3+3(1)$
$-3+3$
$-6+3(2)=-6+6=0$

we have a row of 0's x_3 is the free variable since there is no pivot in the 3rd column

$x_1 + 3x_2 + x_3 = 0$
$x_2 + 2x_3 = 0 \rightarrow \boxed{x_2 = -2x_3}$

$x_3 = -3x_2 - x_1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix}$$

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix},\ x_3 \in \mathbb{R}}$$

$x_1 + 3(-2x_3) + x_3 = 0$
$x_1 - 6x_3 + x_3 = 0$
$x_1 - 5x_3 = 0 \rightarrow \boxed{x_1 = 5x_3}$

$(5x_3) + 3(-2x_3) + x_3 = 0$
$5x_3 - 6x_3 + x_3 = 0$
$5x_3 - 5x_3 = 0$ free