Test 1 Review

1. Solve the system represented by the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 4 & -2 & 8 & 12 \\ 0 & 1 & -7 & 2 & -4 \\ 0 & 0 & 5 & -1 & 7 \\ 0 & 0 & 1 & 3 & -5 \end{array}\right]$$

2. The augmented matrix of a linear system has been transformed by row operations into the form below. Determine if the system is consistent.

$$\left[\begin{array}{ccc|c} 1 & 5 & 2 & -6 \\ 0 & 4 & -7 & 2 \\ 0 & 0 & 5 & 0 \end{array}\right]$$

3. For what values of h and k is the following system consistent?

$$\begin{aligned} 2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k \end{aligned}$$

4. Row reduce the following matrice to reduced row echelon form, circle the pivot position, and list the pivot columns:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right]$$

5. Find the general solution of the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right]$$

6. Determine if $\vec{b}$ is a linear combination of the vectors formed from the columns of the matrix $\vec{A}$.

$$\vec{A} = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

7. Let $\vec{a_1} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$, $\vec{a_2} = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is $\vec{b}$ in the plane spanned by a_1 and a_2.

8. Given $\vec{A}$ and $\vec{b}$ below, write the augmented matrix that corresponds to $\vec{A}\vec{x} = \vec{b}$. THen solve the system and write the solution as a vector.

$$\vec{A} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$

9. Describe all solutions of $A\vec{x} = \vec{0}$ where

$$A = \begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

10. Suppose the solution set of a certain system of linear equations can be described as $x_1 = 5 - 4x_3, x_2 = -2 - 7x_3$, with x_3 free. Use vectors to describe this set as a line in

$$\mathbb{R}^3$$

11. For what values of h is v_3 in the Span of the other two? For what values of h are the three vectors linearly dependent?

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$$

12. If $\{v_1, v_2, v_3\}$ are a set of linearly dependent vectors, prove that if T is a linear transformation, then $\{T(v_1), T(v_2), T(v_3)$ are linearly dependent.

13. With T defined by $T(\vec{x}) = \vec{A}\vec{x}$, find a vector $\vec{x}$ whose image under T is $\vec{b}$, and determine whether $\vec{x}$ is unique.

$$\vec{A} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$$

14. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$. Find $\vec{x}$ such that $T(\vec{x}) = (3, 8)$.

15. Compute AB, BA, and A-2B where

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

16. Find the transpose of the matrix

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & -8 \\ -.5 & 8 & -1 \end{bmatrix}$$

17. Find the inverses, if they exist, of the matrices below:

$$A = \begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

18. Determine if the following matrix is invertible:

$$\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$$

19. Compute the determinant of the following matrix:

$$\begin{bmatrix} 2 & 3 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{bmatrix}$$

20. Find the determinant of the following matrix by row reduction to echelon form:

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix}$$