MAST20004 Probability

Assignment 3

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HD Education

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Review: Bivariate Normal Distribution

Recall: pdf

$$f(x,y) = \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}$$

Recall: Bivariate Standard Normal Distribution pdf

$$f(x,y) = \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[x^2 - 2\rho xy + y^2\right]\right\}}{2\pi\sqrt{1-\rho^2}}$$

NOTE: $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,1)$

Review: Uncorrelated and Independent

Theroem: General Case

$Cov(X, Y) = 0 \Rightarrow$ *Uncorrelated* $\nRightarrow$ *Independent*
Independent $\Rightarrow$ *Uncorrelated* $\Rightarrow$ $Cov(X, Y) = 0$

This case: ONLY $X \sim \mathcal{N}(\mu, \sigma^2)$ and $Y \sim \mathcal{N}(\mu, \sigma^2)$ combine as a bivariate normal distribution, $Cov(X, Y) = 0 \Rightarrow$ *Uncorrelated* $\Rightarrow$ *Independent*
See this in Assignment Q3 (i)

Reivew: Conditional Brivariate Normal Distribution

We only consider how to find conditional probability

Method 1

$(X|Y=y) \sim \mathcal{N}(\rho y, 1-\rho^2)$

Standardise twice

Method 2

$(X|Y=y) \sim \mathcal{N}(\mu_x + \rho\sigma_x \frac{y-\mu_y}{\sigma_y}, \sigma_x^2(1-\rho^2))$

1) Input all expectation, variance and correlation.

2) Standardise once

Review: Joint pdf, Joint Cdf And Marginal Pdf

From joint pdf to joint cdf

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = F(x,y)$$

From joint cdf to joint pdf

$$\frac{\partial^2}{\partial x \partial y}F(x,y) = f(x,y)$$

From joint pdf to marginal pdf

$$\int_{-\infty}^{\infty} f(x,y)dx = f(y)$$

$$\int_{-\infty}^{\infty} f(x,y)dy = f(x)$$

Review: Conditional joint pdf

from marginal pdf to conditional pdf

$$f(x|y) = \frac{f(x,y)}{f(y)}$$

$$f(y|x) = \frac{f(x,y)}{f(x)}$$

Independence

If X, Y independent:

$$\Rightarrow f(x,y) = f(x)f(y)$$

$$\Rightarrow f(x) = f(x|y)$$

$$\Rightarrow f(y) = f(y|x)$$

Example Question: Double Integral

Assignment 3 Q3 Sem 1 2020:

$$\text{f(x,y)} = \begin{cases} c(x+y), & 0 < x < 1, -x < y < x \\ 0, & \textit{ohterwise} \end{cases}$$

(a) Find the constant c
(b) Find marginal pdf of X
(c) Find the conditional pdf of Y given $X = x$
(d) Are X and Y independent?
(e) Calculate $\mathcal{P}(X \geq \frac{1}{2} | Y \leq 0)$
(f) Calculate $\mathcal{P}(X^2 < Y)$