HD·EDUCATION

MAST20004 Probability

Week 7 Summary 2

Jiacheng Min / Mint
minjm@student.unimelb.edu.au

Summary of Assignment 2

Pdf, Expectation and Variance

- convert Pdf to Cdf: $F(x) = \int_{-\infty}^{x} f(x)\,dx$
- Expectation / Higher Moments
  1. $E(x^n) = \int_{-\infty}^{\infty} x^n f(x)\,dx$
  2. $E(x^n) = n\int_{0}^{\infty} x^{n-1}(1-F(x))\,dx$

  NOTE: $\begin{cases} X \geq 0 \\ P(X \geq 0) = 1 \end{cases}$
- Variance

  $V(x) = E(x^2) - E(x)^2$

Summary of Assignment 2

Normal Distribution Stock Model

$X \sim N(2, 1)$ Find $100\,E(e^X)$

Transform to $Z \sim N(0,1)$ $\quad \frac{X-2}{1} = Z \Rightarrow X = Z+2$

$100\,E(e^{Z+2}) = 100\,E(e^Z \cdot e^2) = 100\,e^2\,E(e^Z)$

$$E(e^Z) = \int_{-\infty}^{\infty} e^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2 + z} dz \qquad (z+a)^2$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2-2z+1-1)} dz = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-1)^2 + \frac{1}{2}} dz$$

$$= e^{\frac{1}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-1)^2} \underbrace{dz}_{d(z-1)}$$

$m = z-1$, $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$

$$= e^{\frac{1}{2}} \int_{-\infty}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(m)^2} dm}_{=1}$$

$\int_{-\infty}^{\infty} f(z)\,dz = 1$

$$= e^{\frac{1}{2}}$$

$$\Rightarrow 100\,E(e^X) = 100\,e^2 \cdot e^{\frac{1}{2}} = 100\,e^{\frac{5}{2}}$$

Summary of Assignment 2

Answer