MAST20004 Probability

Assignment 3

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HD Education

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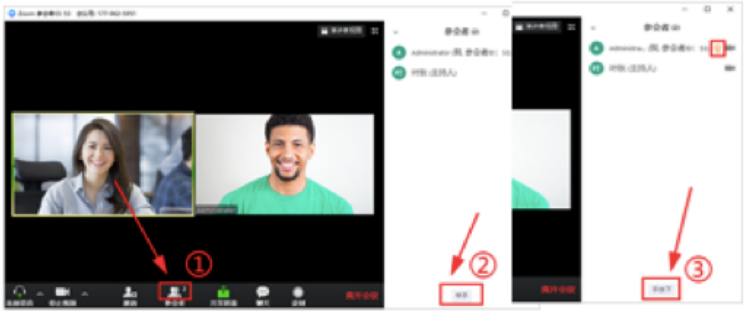

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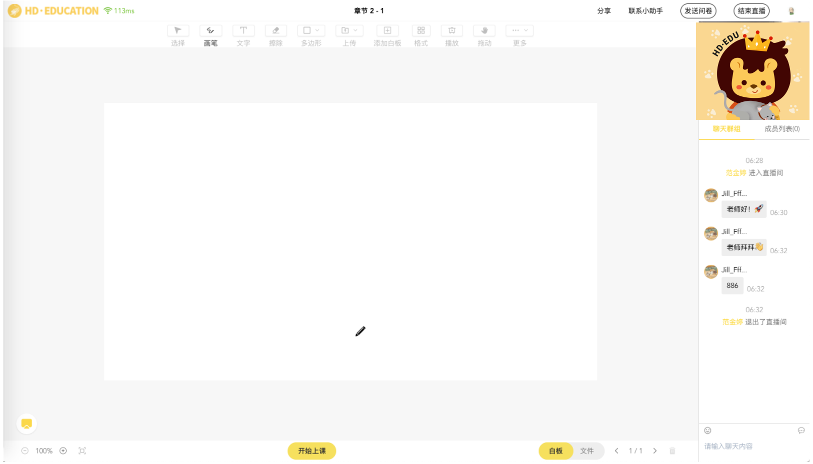

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Review: Bivariate Normal Distribution

Recall: pdf

$$f(x,y)=\frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2-2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)+\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}$$

Recall: Bivariate Standard Normal Distribution pdf

$$f(x,y)=\frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[x^2-2\rho xy+y^2\right]\right\}}{2\pi\sqrt{1-\rho^2}}$$

NOTE: $X\sim\mathcal{N}(0,1)$ and $Y\sim\mathcal{N}(0,1)$

Review: Uncorrelated and Independent

Theroem: General Case

$Cov(X,Y) = 0 \Rightarrow Uncorrelated \nRightarrow Independent$
$Independent \Rightarrow Uncorrelated \Rightarrow\ Cov(X,Y) = 0$

This case: ONLY $X \sim \mathcal{N}(\mu, \sigma^2)$ and $Y \sim \mathcal{N}(\mu, \sigma^2)$ combine as a bivariate normal distribution, $Cov(X,Y) = 0 \Rightarrow Uncorrelated \Rightarrow Independent$
See this in Assignment Q3 (i)

Reiwew: Conditional Brivariate Normal Distribution

We only consider how to find conditional probability

Method 1

$(X|Y=y) \sim \mathcal{N}(\rho y, 1-\rho^2)$
Standardise twice

Method 2

$(X|Y=y) \sim \mathcal{N}(\mu_x + \rho\sigma_x \frac{y-\mu_y}{\sigma_y}, \sigma_x^2(1-\rho^2))$
1) Input all expectation, variance and correlation.
2) Standardise once

Review: Covariance and Correlation

Recall:

If independent:

$E(X+Y) = E(X) + E(Y)$

$V(X+Y) = V(X) + V(Y)$

General case:

$V(X+Y) = V(X) + V(Y) + Cov(X,Y)$

Correlation ρ

$\rho = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

Assignment Q3 (i)

Assignment Q3 (ii)

Assignment Q3 (iii)

Hint: Normal +/- Normal = Normal

Assignment Q3 (iv)

Review: Joint pdf, Joint Cdf And Marginal Pdf

From joint pdf to joint cdf

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = F(x,y)$$

From joint cdf to joint pdf

$$\frac{\partial^2}{\partial x \partial y} F(x,y) = f(x,y)$$

From joint pdf to marginal pdf

$$\int_{-\infty}^{\infty} f(x,y)dx = f(y)$$

$$\int_{-\infty}^{\infty} f(x,y)dy = f(x)$$

Review: Conditional joint pdf

from marginal pdf to conditional pdf

$$f(x|y) = \frac{f(x,y)}{f(y)}$$

$$f(y|x) = \frac{f(x,y)}{f(x)}$$

Independence

If X, Y independent:

$$\Rightarrow f(x,y) = f(x)f(y)$$

$$\Rightarrow f(x) = f(x|y)$$

$$\Rightarrow f(y) = f(y|x)$$

Assignment Q2 (i)

Assignment Q2 (ii)

Assignment Q2 (iii)

Assignment Q2 (iv)

Assignment Q1 (i)

Assignment Q1 (ii)

Assignment Q1 (iii)

Assignment Q1 (iv)

Review: How to Find Shadow Area

Basic Example: $\begin{cases} 0 < x < 1 \\ |y| < x \end{cases}$

Assignment Q4 (i)

Assignment Q4 (ii)

Assignment Q4 (iii)

Introduction to *min*() and *max*() function

min(X, Y) function

$$Z = \text{min(X,Y)} = \begin{cases} X, & X < Y \\ Y, & X > Y \end{cases}$$

$\mathcal{P}(Z > z) = \mathcal{P}(X > x)\mathcal{P}(Y > y)$ if X, Y independent

max(X, Y) function

$$Z = \text{max(X,Y)} = \begin{cases} X, & X > Y \\ Y, & X < Y \end{cases}$$

$\mathcal{P}(Z \leq z) = \mathcal{P}(X \leq x)\mathcal{P}(Y \leq y)$ if X, Y independent

Let's try question 5...

Assignment Q5 (i)

Assignment Q5 (ii) Method 1

Assignment Q5 (ii) Method 2

Further Knowledge: Mixed Distribution of $min(X, M)$

Assumption: X is a non-negative random variable and M is a positive constant.

$min(X, M)$ expression

$$Y = min(X, M) = \begin{cases} X, & X \leq M \\ M, & X > M \end{cases}$$

$min(X, Y)$ cdf and pdf

$$\mathrm{F}_Y(x) = \begin{cases} F_X(x), & x < M \\ 1, & x \geq M \end{cases}, \quad f_Y(x) = \begin{cases} f_X(x), & x < M \\ 1 - F_X(M), & x = M \end{cases}$$

$min(X, Y)$ higher moments

$$\mathrm{E}(Y^n) = \int_0^M x^n f_X(x)dx + M^n[1 - F_X(M)] = n\int_0^M x^{n-1}[1 - F_X(x)]dx$$

Further Knowledge: Mixed Distribution of $max(0, X - M)$

Assumption: X is a non-negative random variable and M is a positive constant.

$max(0, X - M)$ expression

$$Z = min(X, M) = \begin{cases} 0, & X \leq M \\ X - M, & X > M \end{cases}$$

$max(0, X - M)$ cdf and pdf

$$\mathrm{F}_Z(x) = \begin{cases} F_X(x), & x = 0 \\ F_X(x + M), & x > 0 \end{cases}, \quad f_Z(x) = \begin{cases} F_X(x), & x = 0 \\ f_X(x + M), & x > 0 \end{cases}$$

$min(X, Y)$ higher moments

$$\mathrm{E}(Z^n) = \int_0^M x^n f_X(M + x) dx = \int_M^\infty (x - M)^{n-1}[1 - F_X(x)] dx$$

Example Question: Double Integral

Assignment 3 Q3 Sem 1 2020:

$$f(x,y) = \begin{cases} c(x+y), & 0 < x < 1, -x < y < x \\ 0, & \textit{ohterwise} \end{cases}$$

(a) Find the constant c

(b) Find marginal pdf of X

(c) Find the conditional pdf of Y given $X = x$

(d) Are X and Y independent?

(e) Calculate $\mathcal{P}(X \geq \frac{1}{2} | Y \leq 0)$

(f) Calculate $\mathcal{P}(X^2 < Y)$

Example Answer (a)

Example Answer (b)

Example Answer (c)

Example Answer (d)

Example Answer (e)

Example Answer (f)

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