MAST20004 Probability

Assignment 4

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HD Education

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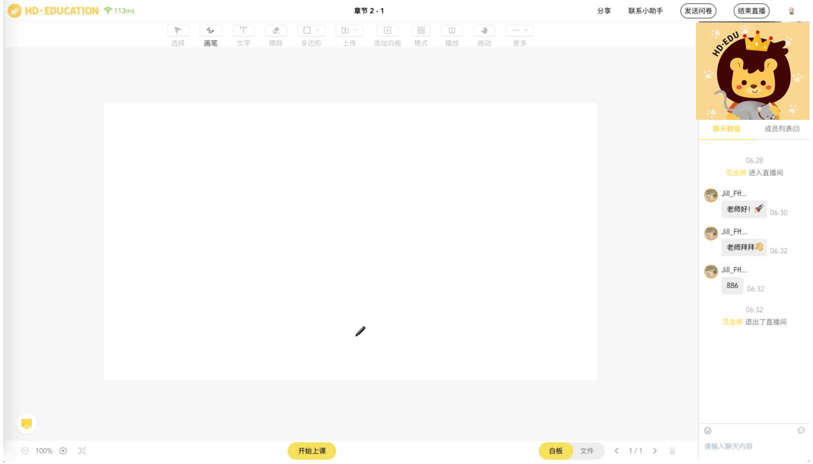

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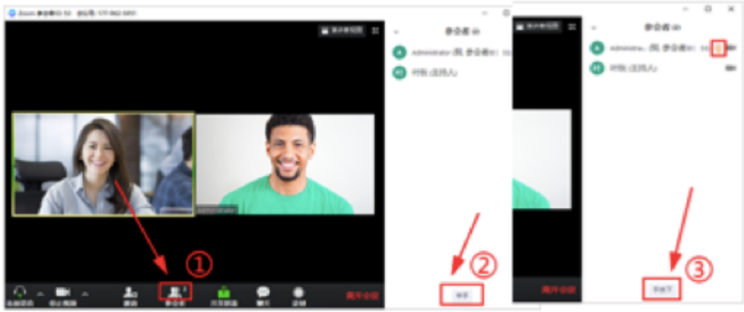

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Revision:Probability Generating Function (PGF)

PGF Expression

$$P_X(z) = E(z^X) = \sum_{n=1}^{\infty} p_X(x) z^x$$

$$P_X(z) \leq 1$$

Inverting pdf to pmf

$$p_X(k) = \mathcal{P}(X = k) = \frac{P_X{}^{(k)}(0)}{k!}$$

Revision:Probability Generating Function (PGF)

Properties of PGF

1) $P_X(1) = 1$
2) $P_X{}^{'}(1) = E(X)$
3) $P_X{}^{''}(1) = E(X(X-1))$
4) $V(X) = P_X{}^{''}(1) + P_X{}^{'}(1) - P_X{}^{'}(1)^2$
5) $P_X(z) = E(z^X) = E(E(z^X|Y))$
6) X and Y are independent, and W = X+Y,
$P_W(z) = P_X(z)P_Y(z) = E(z^{X+Y}) = E(z^X)E(z^Y)$
7) P(X is even) $= \frac{1}{2}(P_X(1) + P_X(-1))$,
P(X is odd) $= 1 - \frac{1}{2}(P_X(1) + P_X(-1))$

Assignment 4 Q1(i)

Assignment 4 Q1(ii)

Assignment 4 Q1(iii)

Assignment 4 Q1(iv)

Revision: Chebyshev's Inequality

Chebyshev's Inequality

$$\mathcal{P}(\frac{|X-\mu|}{\sigma} \geq k) \leq \frac{1}{k^2}$$

Rearrange Inequality

$$\mathcal{P}(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

$$\mathcal{P}(|X-\mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}$$

Revision: Central Limit Theorem

Central Limit Theorem

Assumption: $\mathrm{X}_1, X_2 ... \ are \ i.i.d$, $\mathrm{S}_n = X_1 + X_2 + ... X_n$

$$Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}} \sim \mathcal{N}(0,1)$$

$$S_n \sim \mathcal{N}(n\mu, n\sigma^2)$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, \frac{\sigma^2}{n})$$

Assignment 4 Q2(i) Taylor Approximation

Assignment 4 Q2(ii)

Assignment 4 Q2(iii)

Assignment 4 Q4(i)

Assignment 4 Q4(ii)

Assignment 4 Q4(iii)

Revision: Moment Generating Function (MGF)

MGF

$$M_X(t) = E(e^{tX}) = \int e^{tx} f_X(x)dx$$

$$M_x(t) = \sum_{k=0}^{\infty} E(X^k)\frac{t^k}{k!}$$

Mgf can determine the moments of X if exists, the converse is not true

Revision: Moment Generating Function (MGF)

Properties of the mgf

1) $M_X(0) = 1$
2) $M_X{}'(0) = E(X)$
3) $M_X{}''(0) = E(X^2)$
4) $V(X) = M_X{}''(0) - M_X{}'(0)^2$
5) $Y = aX + b, M_Y(t) = e^{bt}M_X(at) = e^{bt}E(e^{Xat})$
6) $M_X(t) = E(e^{tX}) = E(E(e^{tX}|Y))$
7) If X and Y are independent,
$Z = X + Y, M_Z(t) = M_X(t)M_Y(t) = E(e^{tX})E(e^{tY})$ 8) If X is a discrete random variable: $M_X(t) = P_X(e^t), P_x(z) = M_X(logz)$ (Relationship between mgf and pgf

Revision: Moment Generating Function (MGF)

Commonly used MGF (CHECK THIS IN CHEAT SHEET)

1) X $\sim Ber(p)$:$\mathrm{M}_X(t) = 1 - p - pe^t$
2) X $\sim Bi(n, p)$:$\mathrm{M}_X(t) = (1 - p - pe^t)^n$
3) X $\sim G(p)$:$\mathrm{M}_X(t) = \frac{1}{1-(1-p)e^t}$
4) X $\sim NB(r, p)$:$\mathrm{M}_X(t) = [\frac{pe^t}{1-(1-p)e^t}]^r$
5) X $\sim Pn(\lambda)$:$\mathrm{M}_X(t) = e^{\lambda(e^t-1)}$
6) X $\sim Unif(a, b)$:$\mathrm{M}_X(t) = \begin{cases} 1, \ t = 0 \\ \frac{e^{bt}-e^{at}}{(b-a)t}, \ t \neq 0 \end{cases}$
7) X $\sim exp(\alpha)$:$\mathrm{M}_X(t) = \frac{\alpha}{\alpha-t}$
8) X $\sim \gamma(\gamma, \alpha)$:$\mathrm{M}_X(t) = (\frac{\alpha}{\alpha-t})^\gamma$
9) X $\sim \mathcal{N}(\mu, \sigma^2)$:$\mathrm{M}_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Assignment 4 Q3(i)

Assignment 4 Q3(ii)

Challenge Problem: Assignment 4 Q5(1-i)

Challenge Problem: Assignment 4 Q5(1-ii)

Challenge Problem: Assignment 4 Q5(1-iii)

Challenge Problem: Assignment 4 Q5(2)

Challenge Problem: Assignment 4 Q5(3-i)

Challenge Problem: Assignment 4 Q5(3-ii)

Practical Question

Assignment 4 Q4 Sem 1 2020

A box contains an unknown number of white and black balls. We wish to estimate the proportion p of white balls in the box. Let Z_n be the proportion of white balls obtained after n drawings.
(a)Show that $E(Z_n) = p$ and $V(Z_n) = \frac{p(1-p)}{p}$
(b)Use Chebyshev's Inequality to show that, for all > 0,
$P(|Z_n - p| \geq \epsilon) \leq \frac{1}{4n\epsilon^2}$ (c)Using the result in part (b), find the smallest value of n such that, with probability greater than or equal to 0.9, the proportion Z_n in the sample will estimate p to within an accuracy of 0.05.

Practical Question
(a)Show that $E(Z_n) = p$ and $V(Z_n) = \frac{p(1-p)}{p}$

Practical Question

(b)Use Chebyshev's Inequality to show that, for all > 0,

$$P(|Z_n - p| \geq \epsilon) \leq \frac{1}{4n\epsilon^2}$$

Practical Question

(c)Using the result in part (b), find the smallest value of n such that, with probability greater than or equal to 0.9, the proportion Z_n in the sample will estimate p to within an accuracy of 0.05.

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