HD·EDUCATION

MAST20004 Probability

Week 7 Summary 1

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H D · E D U C A T I O N

Summary of Assignment 2

Binomial Distribution

$X \overset{d}{=} Bi(n, p)$ $\begin{cases} n: \text{ \# of success} \\ p: \text{ prob. of success} \end{cases}$

- pmf : $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$
- Expectation : $E(X) = np$
- Variance : $V(X) = np(1-p)$
- Mgf : $M_X(t) = (1-p+pe^t)^n$
- Pgf : $P_X(t) = (1-p+pz)^n$
- Recursive Formula

  - $r(x) = \dfrac{p_X(x)}{p_X(x-1)} = \dfrac{\frac{n+1}{x}-1}{\frac{1}{p}-1} = \begin{cases} x > p(n+1) & \text{pmf} \uparrow \\ x < p(n+1) & \text{pmf} \downarrow \end{cases}$

- Approximation

  - condition : $p \to \frac{1}{2}$

    $\begin{cases} \mu \approx np \\ \sigma^2 \approx np(1-p) \\ Bi \overset{d}{\approx} N(\mu, \sigma^2) \end{cases}$

  - condition $p \to 0$ $(p < 0.05)$

    $\begin{cases} \lambda \approx np \\ Bi \overset{d}{\approx} Pn(\lambda) \end{cases}$

Summary of Assignment 2

Poisson Distribution

$X \stackrel{d}{=} Pn(\lambda)$ discrete

- pmf $p_X(x) = e^{-\lambda}\frac{\lambda^x}{x!}$ (Note: $\sum_{x=0}^{\infty}\frac{\lambda^x}{x!} = e^{\lambda}$)
- Expectation: $E(X) = \lambda$
- Variance : $V(X) = \lambda$
- Mgf: $M_X(t) = e^{\lambda(e^t-1)}$
- Additivity
  - $X = \sum_{i=1}^{n} X_i \sim Bi(n, p)$, $X_i \sim Ber(p_i)$, $p \to 0$, $p_i = p$

  $\Rightarrow X \stackrel{d}{\approx} Pn(\sum_{i=1}^{n} p_i)$

Summary of Assignment 2

Normal Distribution

$X \sim N(\mu, \sigma^2)$ $\quad Z \sim N(0,1)$

- pdf : $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $\quad f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

- Standardisation : $Z = \frac{X-\mu}{\sigma}$

- Mgf : $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ $\quad M_Z(t) = e^{\frac{1}{2}t^2}$

- Higher Moments of Z: $E(Z^n) = (n-1)E(Z^{n-1})$

  $E(Z^{2k+1}) = 0$ $\quad E(Z^{2k}) = \frac{(2k)!}{2^k \cdot k!}$

- Note:
  - 2^{nd} Moment : $E(X^2) = \mu^2 + \sigma^2$
  - $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(x-\mu)^2} dx = 1$