$A = LU$

L2.

$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ by row elimination.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

so want to get 1^{st} column/row

$\begin{bmatrix} 2 & 3 \\ 4 & \underline{6} \end{bmatrix}$ and + $\begin{bmatrix} 0 & 0 \\ 0 & \_ \end{bmatrix}$

rank 1 rank 1.

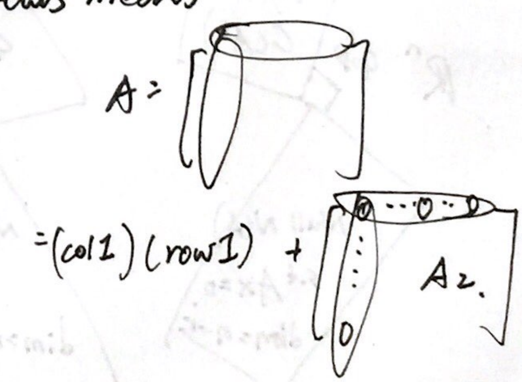
if is rank 1,

than the blank is 6

think of 1^{st} matrix as $l^* u^T$ $= \begin{bmatrix} l_1 \end{bmatrix} \begin{bmatrix} u_1^T \end{bmatrix} + \begin{bmatrix} l_2 \end{bmatrix} \begin{bmatrix} u_2^T \end{bmatrix} = LU$

this is what limination is doing.

this means



4 fundamental subspaces A $m \times n$ rank.

column space $C(A)$ dim $= r$ (where $r \le n$)

row space $C(A^T)$ dim $= r$

null space $N(A)$ dim $= n - r$.

null space $N(A^T)$ dim $= m - r$.

what's null space? A set of solutions. s.t. $Ax = 0$.
↑ vectors.

what's the space of vector?

closed, which means I could add vectors in the space → could do linear algebra.

e.g.

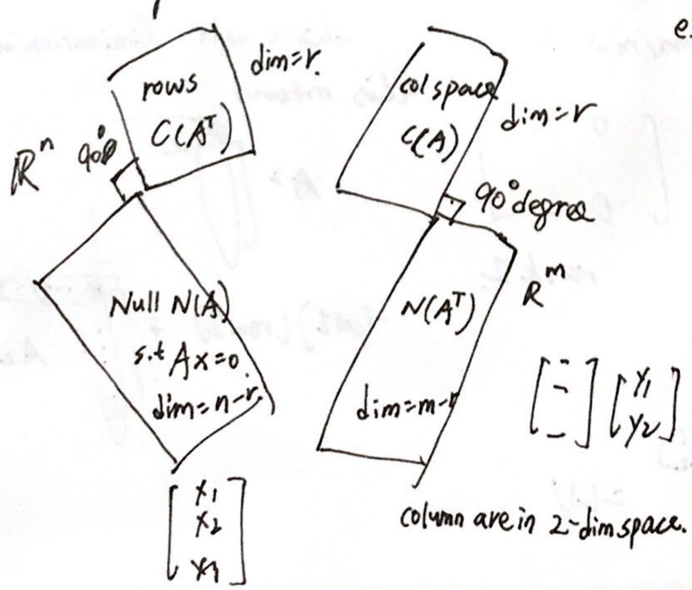
$$\begin{array}{r} Ax = 0 \\ + \\ Ay = 0 \\ \hline A(x+y) = 0. \end{array}$$

$A(cx) = 0$

if $x, y \in$ Null space then sum is in null space

How many independent vectors in the null space?

e.g. 2×3 matrix $m \times n$.



column are in 2-dim space.

example.

dim $\underline{r}$ $\underline{n-r}$ dim $\underline{r}$ $\underline{m-r}$

row null space

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$m = 2$

$n = 3$

$r = 1$
rank.

$n - r = 2$. This means 2 vectors exist in the null space.

$$Ax = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

rows x.

So the point is these 2 spaces are orthogonal.

row space $\perp$ null space.

column space $\perp$ null^T space.