(a). Let $x^T = [x_1, \ x_2 \ \cdots \ x_n]$

$a^T = [a_1 \ a_2 \ \cdots \ a_n]$

$$\therefore \ x^T a = \sum_{i=1}^{n} a_i x_i = a^T x$$

$$\nabla x^T a = \begin{bmatrix} \frac{\partial \sum a_i x_i}{\partial x_1} \\ \vdots \\ \frac{\partial \sum a_i x_i}{\partial x_n} \end{bmatrix} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = a$$

$$\therefore \ \nabla_x x^T a = \nabla_x a^T x = a$$

(b) $$x^T A x = [x_1, \ x_2 \ \cdots \ x_n] \begin{bmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \vdots & \vdots \ \ddots \ \vdots \\ a_{n1} & a_{n2} \quad a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \left[\sum_{i=1}^{n} a_{1i} x_i \ \ \sum_{i=1}^{n} a_{2i} x_i \ \cdots \ \sum_{i=1}^{n} a_{ni} x_i \right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \sum_{j=1}^{n} \sum_{i=1}^{n} a_{ij} x_i x_j$$

$$\nabla_x x^T A x = \nabla_x \left(x_1 \sum_{j=1}^{n} a_{1j} x_j + \cdots + x_k \sum_{j=1}^{n} a_{kj} x_j + \cdots + x_n \sum_{j=1}^{n} a_{nj} x_j \right)$$

$$\nabla_x \left(x_1 \sum_{j=1}^{n} a_{1j} x_j \right) = \begin{bmatrix} 2a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \\ a_{12} x_1 \\ \vdots \\ a_{1n} x_1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + \sum a_{1i} x_i \\ a_{12}x_1 \\ \vdots \\ a_{1n}x_1 \end{bmatrix}$$

Similarly

$$\nabla_x x^T A x = \begin{bmatrix} \sum a_{i1} x_i + \sum a_{1i} x_i \\ \vdots \\ \sum a_{in} x_i + \sum a_{ni} x_i \end{bmatrix} = Ax + A^T x$$

$$= (A + A^T)x$$

c) $\because$ A is symetric

$\therefore A = A^T$

$\therefore \nabla_x x^T A x = (A + A^T)x = (A + A)x = 2Ax$

(d) $(Ax+b)^T (Ax+b)$

$= (x^T A^T + b^T)(Ax+b) = x^T A^T A x + x^T A^T b + b^T A x + b^T b$

$\therefore \nabla\left[(Ax+b)^T (Ax+b)\right]$

$= \nabla(x^T A^T A b + x^T A^T b + b^T A x + b^T b)$

$= 2A^T A x + 2A^T b$

$= 2A^T (Ax+b)$