Chapter 3

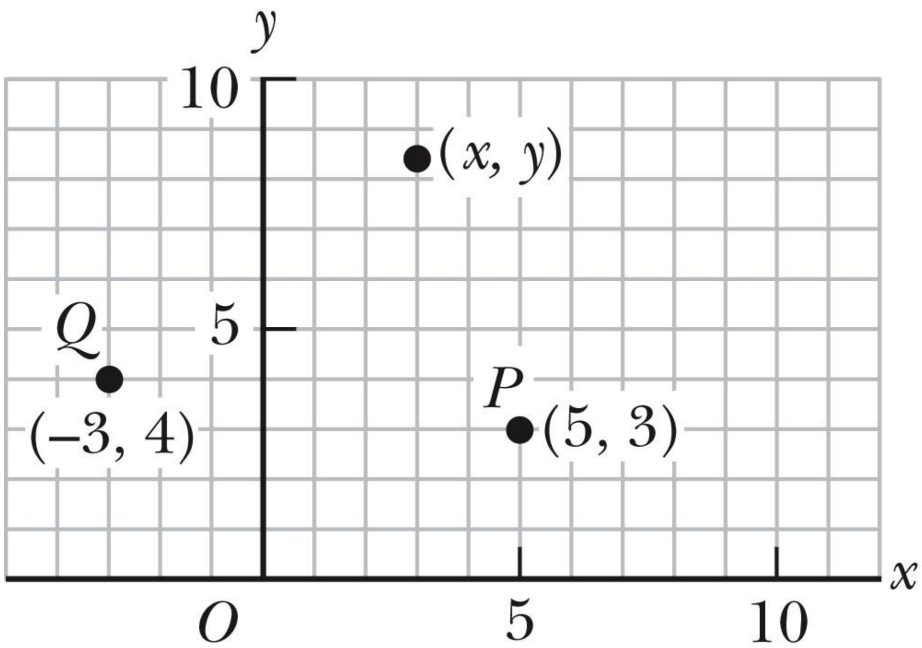
Vectors

Coordinate Systems

- Used to describe the position of a point in space
- Common coordinate systems are:
  - Cartesian
  - Polar

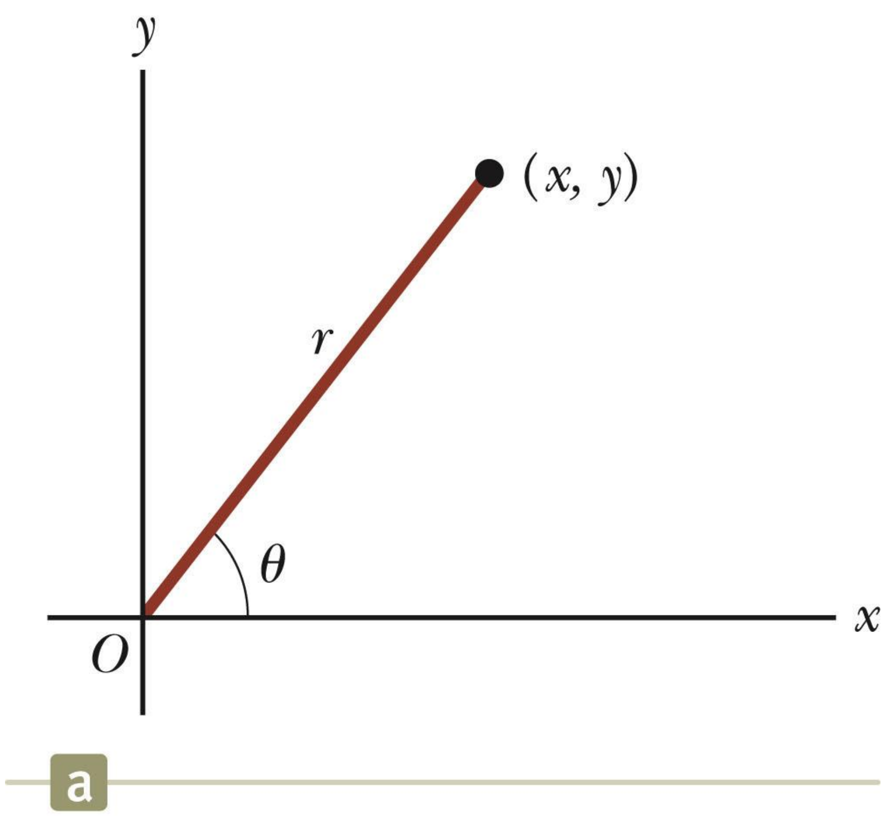
Cartesian Coordinate System

- Also called rectangular coordinate system
- *x*- and *y*- axes intersect at the origin
- Points are labeled (x,y)



Polar Coordinate System

- Origin and reference line are noted
- Point is distance *r* from the origin in the direction of angle θ, ccw from reference line
- Points are labeled **(r,θ)**



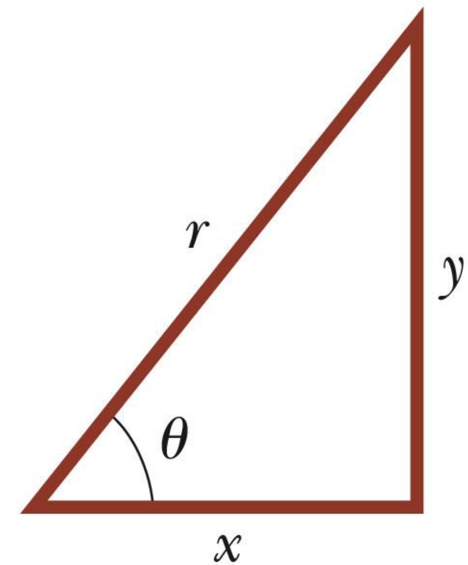
Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- $x = r \cos \theta$
- $y = r \sin \theta$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



b

Cartesian to Polar Coordinates

- *r* is the hypotenuse and θ is an angle

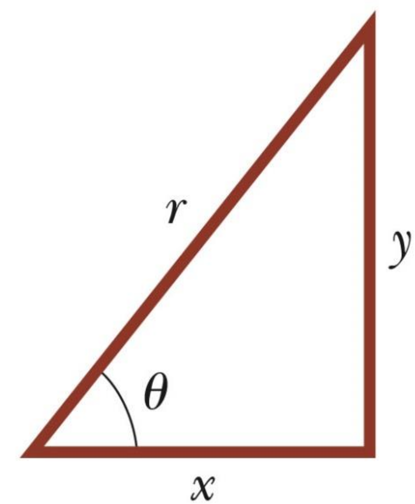
$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

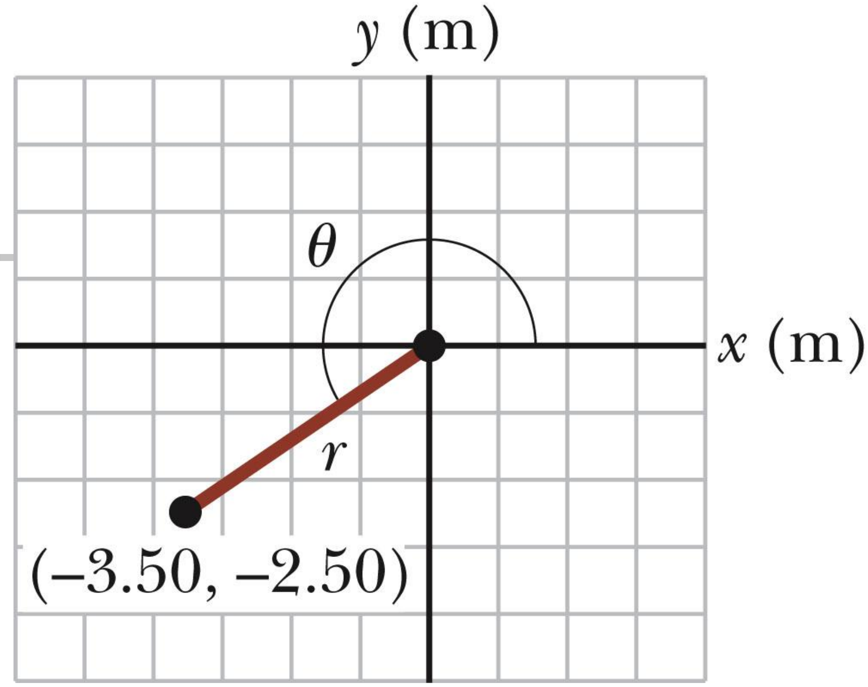
$$\tan\theta = \frac{y}{x}$$



b

- θ must be ccw from positive *x* axis for these equations to be valid

Example 3.1



- The Cartesian coordinates of a point in the *xy* plane are (x,y) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.

- **Solution:** From Equation 3.4,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

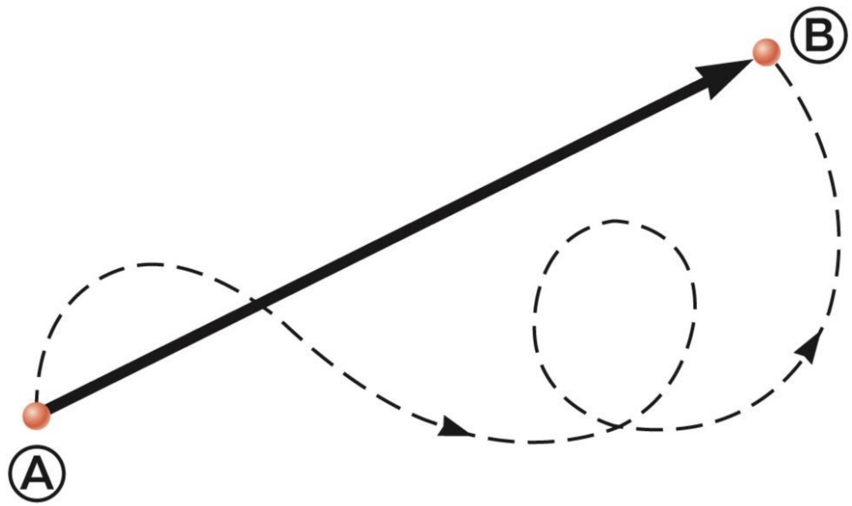
$$\theta = 216°$$

Vectors and Scalars

- A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.
- A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

Vector Example

- A particle travels from A to B along the path shown by the broken line
  - This is the ***distance*** traveled and is a scalar
- The ***displacement*** is the solid line from A to B
  - The displacement is independent of the path taken between the two points
  - Displacement is a vector

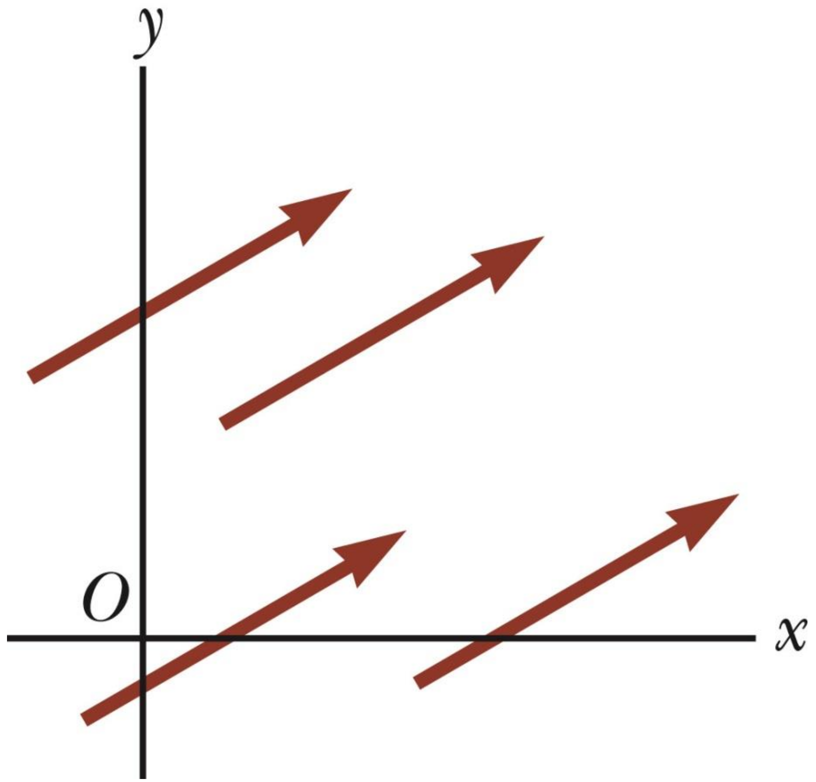


Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print: $\mathbf{A}$ or $\vec{\mathbf{A}}$
- When dealing with just <u>the magnitude of a vector</u> in print, an italic letter will be used: *A* or $|\vec{\mathbf{A}}|$
  - The magnitude of the vector has physical units
  - The magnitude of a vector is always a positive number

Equality of Two Vectors

- Two vectors are ***equal*** if they have the same magnitude and the same direction
- $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if $A = B$ and they point along parallel lines
- All of the vectors shown are equal

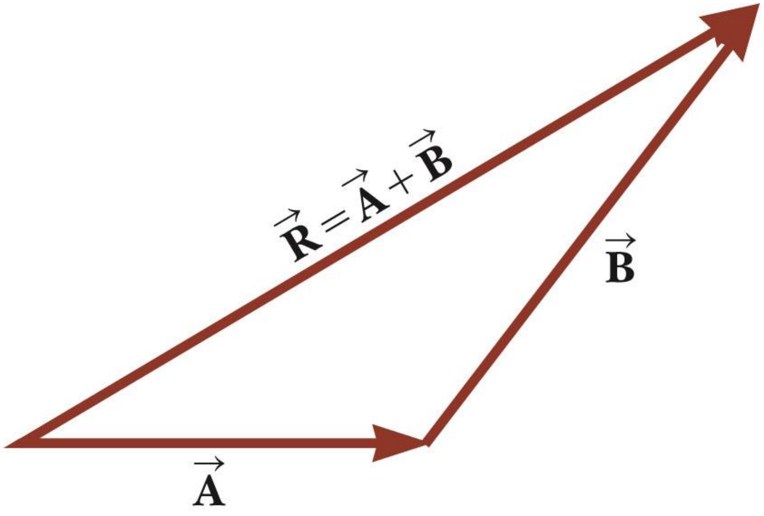


Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same
- Graphical Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient

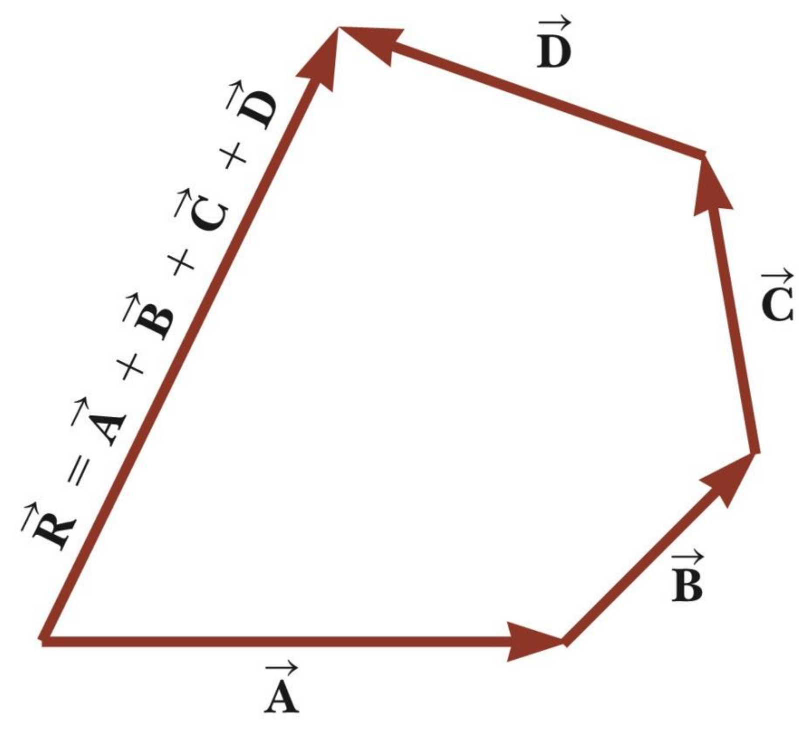
Adding Vectors Graphically

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle
  - Use the scale factor to convert length to actual magnitude



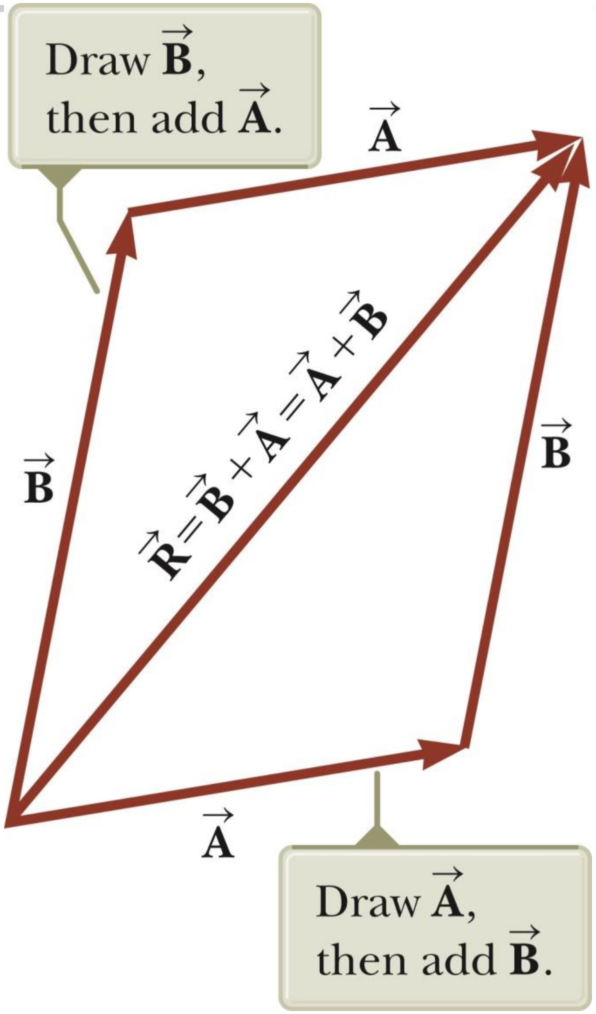
Adding Vectors Graphically, final

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



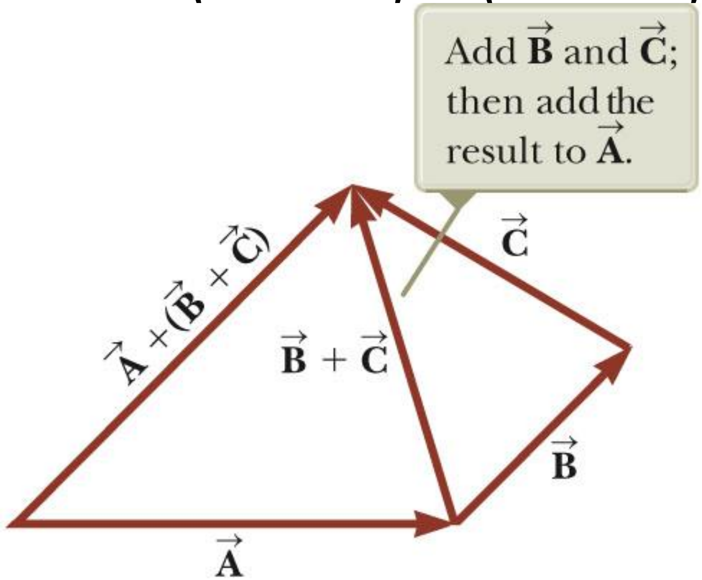
Adding Vectors, Rules

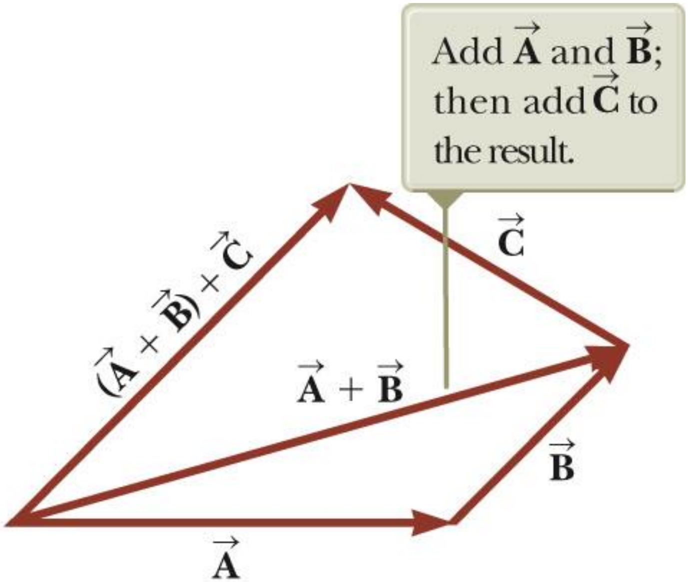
- When two vectors are added, the sum is independent of the order of the addition.
  - This is the ***commutative law of addition***
  - $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$



Adding Vectors, Rules cont.

- When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped
  - This is called the ***Associative Property of Addition***
  - $\vec{\mathbf{A}}+\left(\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)=\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)+\vec{\mathbf{C}}$





Adding Vectors, Rules final

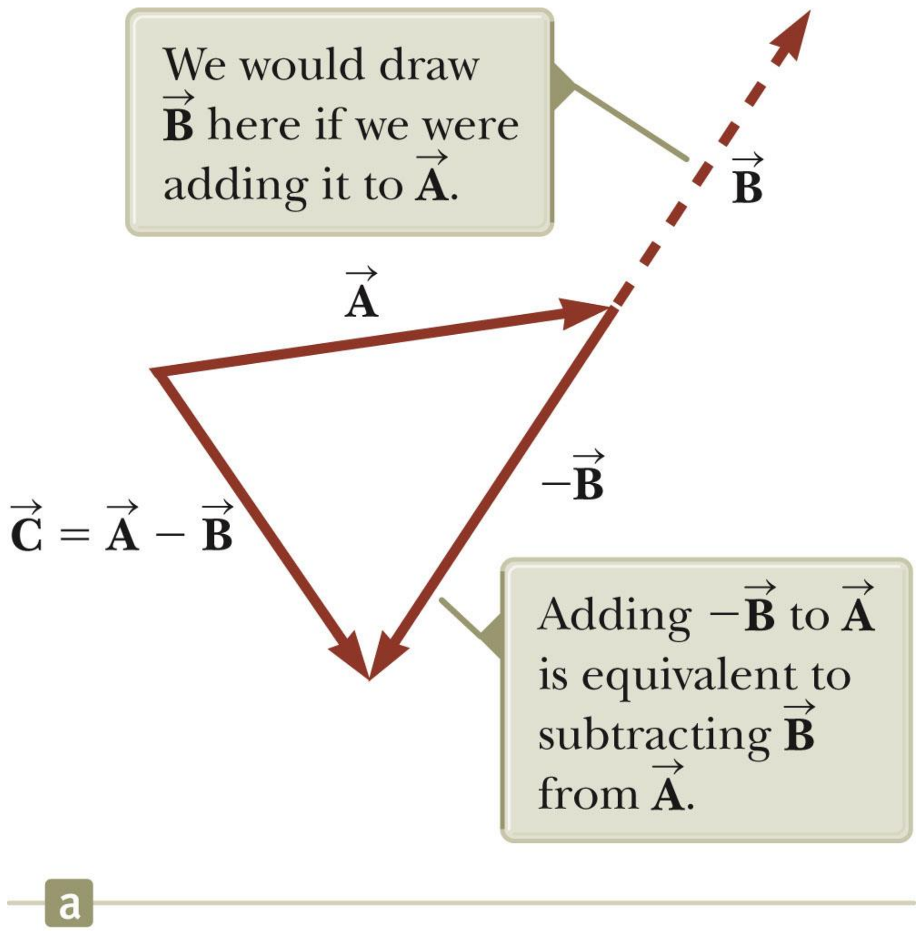
- When adding vectors, all of the vectors must have the same units
- All of the vectors must be of the same type of quantity
  - For example, you cannot add a displacement to a velocity

Negative of a Vector

- The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero
  - Represented as $-\vec{\mathbf{A}}$
  - $\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$
- The negative of the vector will have the <u>same magnitude, but point in the opposite direction</u>

Subtracting Vectors

- Special case of vector addition
- If $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, then use $\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$
- Continue with standard vector addition procedure



Multiplying or Dividing a Vector by a Scalar

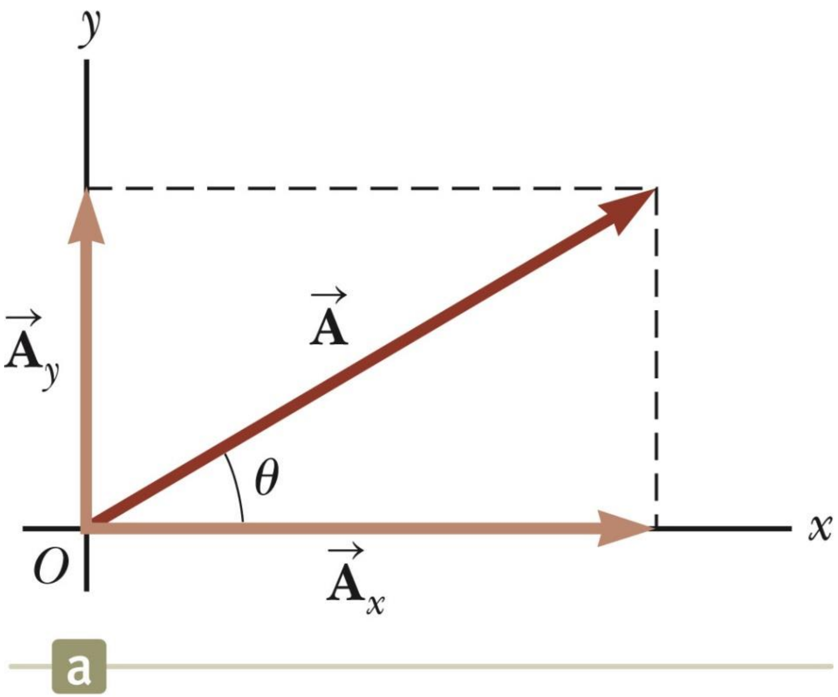
- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Component Method of Adding Vectors

- Graphical addition is not recommended when
  - High accuracy is required
  - If you have a three-dimensional problem
- Component method is an alternative method
  - It uses projections of vectors along coordinate axes

Components of a Vector

- A **component** is a projection of a vector along an axis.
  - Any vector can be completely described by its components
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes



Vector Component Terminology

- $\vec{\mathbf{A}}_x$ *and* $\vec{\mathbf{A}}_y$ are the ***component vectors*** of $\vec{\mathbf{A}}$
  - They are vectors and follow all the rules for vectors
- A_x and A_y are scalars, and will be referred to as the ***components*** of $\vec{\mathbf{A}}$

Components of a Vector, 2

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

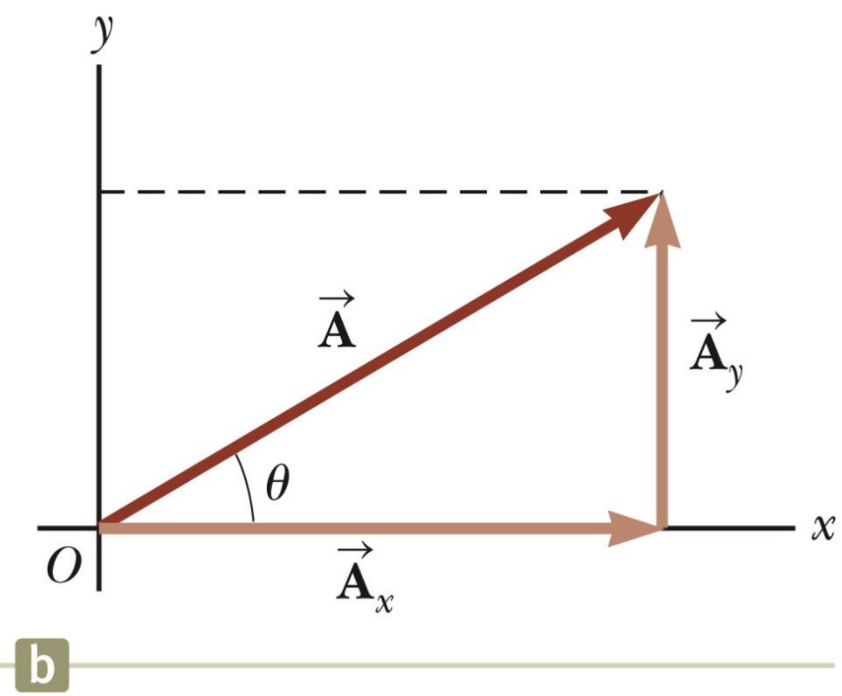
- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

- Then, $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$

Components of a Vector, 3

- The *y*-component is moved to the end of the *x*-component
- This is due to the fact that any vector can be moved parallel to itself without being affected
  - This completes the triangle



Components of a Vector, 4

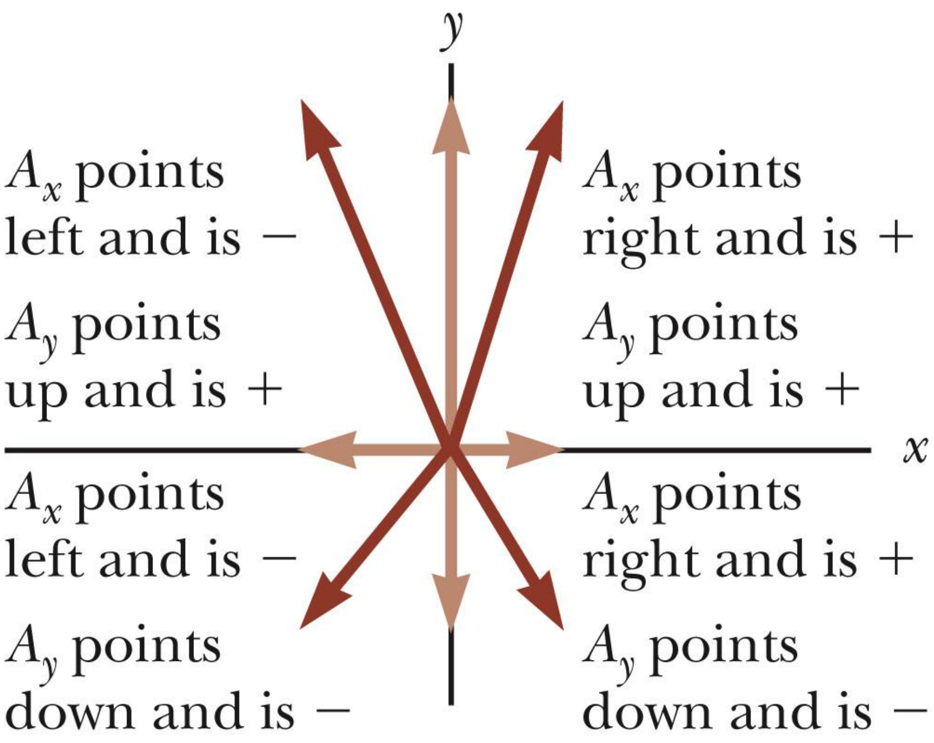
- The previous equations are valid **only if θ is measured with respect to the x-axis**
- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

  - May still have to find θ with respect to the positive *x*-axis

Components of a Vector, final

- The components can be positive or negative and will have the same units as the original vector
- The signs of the components will depend on the angle



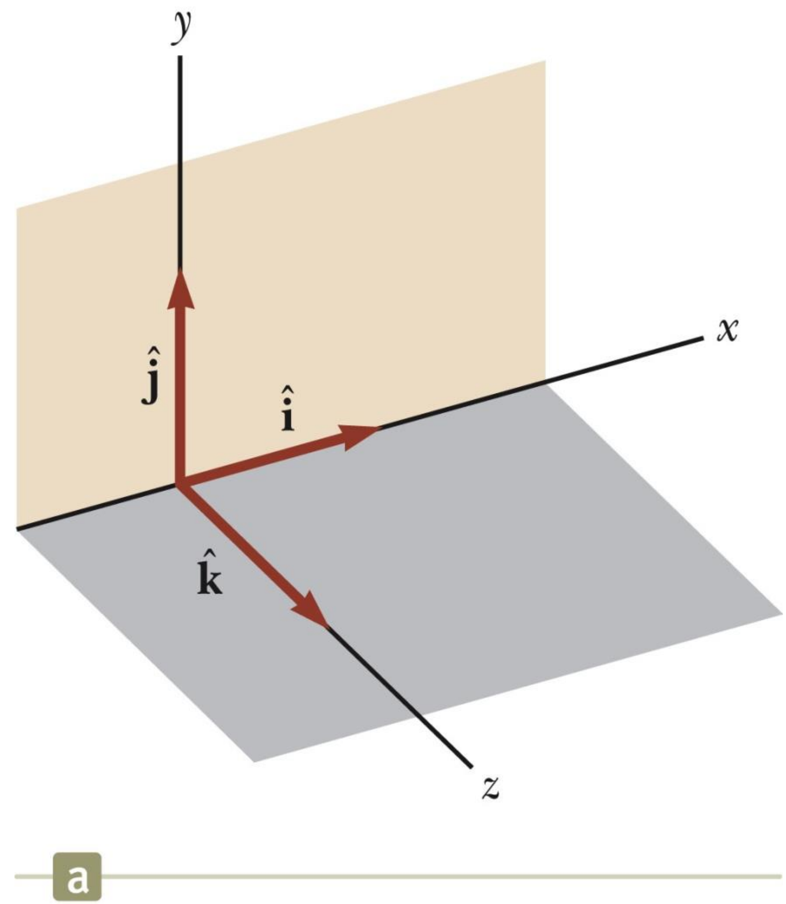
Unit Vectors

- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance

Unit Vectors, cont.

- The symbols $\hat{i}, \hat{j}, and\ \hat{k}$ represent unit vectors
- They form a set of mutually perpendicular vectors in a right-handed coordinate system
- Remember,

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

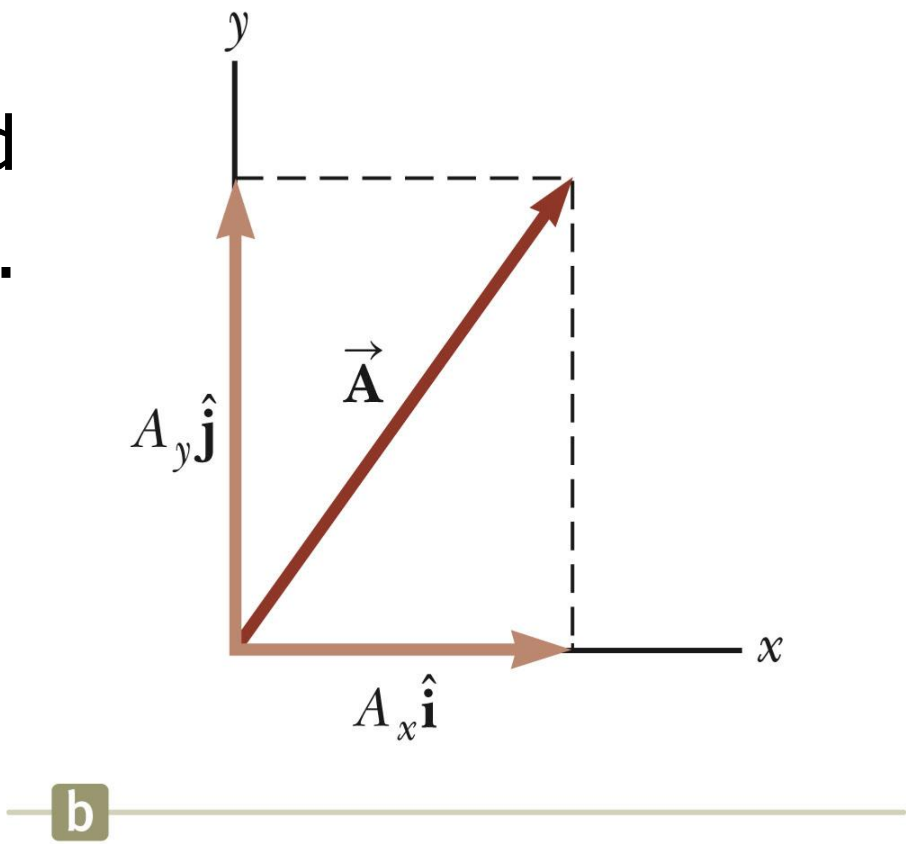


Unit Vectors in Vector Notation

- $\vec{\mathbf{A}}_x$ is the same as $A_x\,\hat{\mathbf{i}}$ and $\vec{\mathbf{A}}_y$ is the same as $A_y\,\hat{\mathbf{j}}$ etc.
- The complete vector can be expressed as

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

$$= A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}$$

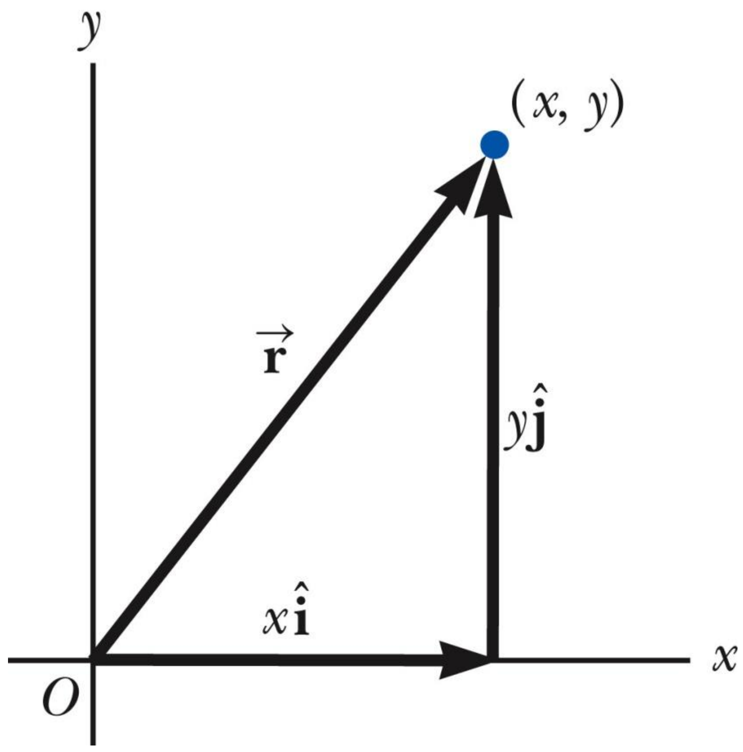


Position Vector, Example

- A point lies in the *xy* plane and has Cartesian coordinates of (x, y).
- The point can be specified by the position vector.

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

- This gives the components of the vector and its coordinates.



Adding Vectors Using Unit Vectors

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then $\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

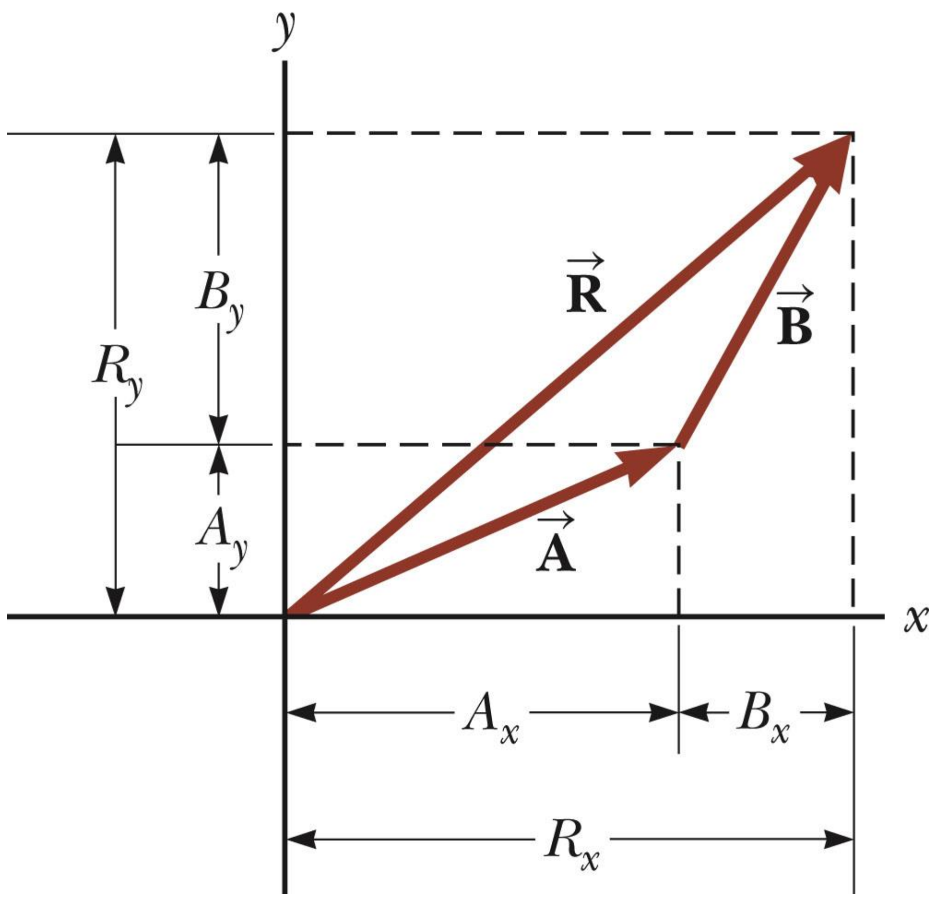
$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

- and so $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Adding Vectors with Unit Vectors

- Note the relationships among the components of the resultant and the components of the original vectors
- $R_x = A_x + B_x$
- $R_y = A_y + B_y$



Three-Dimensional Extension

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

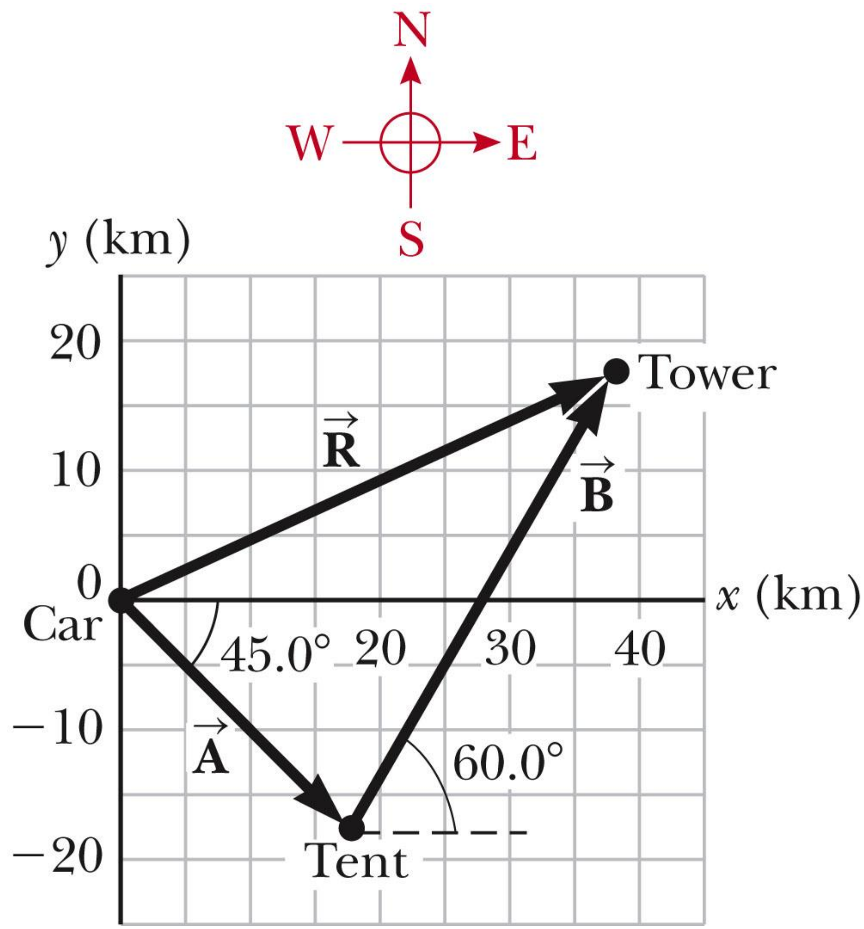
$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}}$$

- and so $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_x + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \cos^{-1}\frac{R_x}{R}, etc.$$

Example 3.5 – Taking a Hike

- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.



(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

SOLUTION

Use Equation 3.15 to find the components of the resultant displacement $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = \boxed{37.7 \text{ km}}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = \boxed{16.9 \text{ km}}$$

Write the total displacement in unit–vector form:

$$\vec{\mathbf{R}} = \boxed{(37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}}) \text{ km}}$$

Finalize Looking at the graphical representation in Figure 3.17, we estimate the position of the tower to be about (38 km, 17 km), which is consistent with the components of $\vec{\mathbf{R}}$ in our result for the final position of the hiker. Also, both components of $\vec{\mathbf{R}}$ are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.17.

What If? After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

Answer The desired vector $\vec{\mathbf{R}}_{\text{car}}$ is the negative of vector $\vec{\mathbf{R}}$:

$$\vec{\mathbf{R}}_{\text{car}} = -\vec{\mathbf{R}} = (-37.7\hat{\mathbf{i}} - 16.9\hat{\mathbf{j}}) \text{ km}$$

The heading is found by calculating the angle that the vector makes with the x axis:

$$\tan\theta = \frac{R_{\text{car},y}}{R_{\text{car},x}} = \frac{-16.9 \text{ km}}{-37.7 \text{ km}} = 0.448$$

which gives an angle of $\theta = 204.1°$, or $24.1°$ south of west.