Q) For given 2-D p.d.f of a bivariate distribution

$$f(x_1, x_2) = 2 \cdot e^{-x_1 x_2 - x_1 - x_2} \quad (x_1 > 0,\ x_2 > 0),$$

generate random vector $\{(X_{1i}, X_{2i})\}_{i=1}^{n}$ with size $n = 10^3$ using gibbs Sampling considering burn-in period.

① marginal p.d.f of x_1, x_2

$$f_{X_1}(x_1) = \int f(x_1, x_2)\, dx_2 = \int_0^{\infty} 2 \cdot e^{-x_1 x_2 - x_1 - x_2}\, dx_2$$

$$= 2 \cdot e^{-x_1} \int_0^{\infty} e^{-x_2(x_1+1)}\, dx_2$$

$$= 2 \cdot e^{-x_1} \frac{1}{x_1+1} \left[-e^{-x_2(x_1+1)} \right]_0^{\infty}$$

$$= \frac{2 \cdot e^{-x_1}}{x_1+1}$$

$$f_{X_2}(x_2) = \int f(x_1, x_2)\, dx_1 = \frac{2 \cdot e^{-x_2}}{x_2+1}$$

② Conditional p.d.f of x_1, x_2

$$f_{X_1|X_2}(x_1 | x_2) = \frac{f(x_1, x_2)}{f_{X_2}(x_2)} = \frac{2 \cdot e^{-x_1 x_2 - x_1 - x_2}}{\frac{2 \cdot e^{-x_2}}{x_2+1}}$$

$$= (x_2+1) \cdot e^{-x_1 x_2 - x_1}$$

$$= (x_2+1) \cdot e^{-x_1(x_2+1)}$$

$$f_{X_2|X_1}(x_2|x_1) = \frac{f(x_1, x_2)}{f_{X_1}(x_1)} = \frac{2 \cdot e^{-x_1x_2 - x_1 - x_2}}{\frac{2 \cdot e^{-x}}{x_1+1}}$$

$$= (x_1+1) \cdot e^{-x_2(x_1+1)}$$