the probability answers

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15.1 $\log_2 1000$

Use binary search, the height of the binary tree is $\log_2 1000$

15.4

- **(a)** $32 = 1 + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$

  Situations: only contains x_1, it is 1; x_1 and one other element, it is $\binom{5}{1}$; x_1 and two other elements, it is $\binom{5}{2}$; x_1 and three other elements, it is $\binom{5}{3}$; x_1 and four other elements, it is $\binom{5}{4}$; x_1 and five other elements, it is $\binom{5}{5}$;

- **(b)** $8 = 1 + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$

  Situations: only contains x_2 x_3, it is 1; x_2 x_3 and one other element from x_1 x_4 x_5, it is $\binom{3}{1}$; x_2 x_3 and two other elements, it is $\binom{3}{2}$; x_2 x_3 and three other elements, it is $\binom{3}{3}$;

15.5

- **a** (3 letters+3 digits)∪(5 letters)∪(2 characters) $= (\binom{26}{1} * \binom{26}{1} * \binom{26}{1} * \binom{10}{1} * \binom{10}{1} * \binom{10}{1}) \cup (\binom{26}{1} * \binom{26}{1} * \binom{26}{1} * \binom{26}{1} * \binom{26}{1}) \cup (\binom{36}{1} * \binom{36}{1})$

- **b** $29458672 = 1757600 + 11881376 + 1296$

15.10 Suppose that $(n-1)$ elements has 2^{n-1} subsets. If now there is 1 element more, all the 2^{n-1} subsets can choose if include the new element or not, so it is $2 * 2^{n-1}$, which equals to 2^n. So $(n-1)$ elements has 2^{n-1} subsets in turn, which prove the hypothesis is right.

15.13 $\binom{6}{1}^{36}$

15.15 $2877 = \binom{6}{1} * \binom{9}{4} + \binom{6}{2} * \binom{9}{3} + \binom{6}{3} * \binom{9}{2} + \binom{6}{4} * \binom{9}{1} + \binom{6}{5}$

Use w represents woman, m represents man, the solution contains:1w+4m, 2w+3m, 3w+2m, 4w+1m, 5w.

15.21 (a) $\binom{21}{5} * (5!) * (21!)$

Put every vowel appears to the left of a consonant so that no two vowels appear consecutively and the last letter in the ordering is not a vowel, then there are 5 pairs (vowel, consonant) which has $\binom{21}{5} * 5!$ ways. Then there are 16 consonant left with 5 pairs, so there are 21! ways.

15.33

- **(a)** $10240 = \binom{10}{1} * \binom{4}{1} * \binom{4}{1} * \binom{4}{1} * \binom{4}{1} * \binom{4}{1}$

  There are total 10 sequence, every card has 4 suits.

- **(b)** $617760 = \binom{4}{1} * \binom{13}{5} * 5!$

  There are 4 suits totally, so choose one of them, each card number in an exact suit has one. And there are totally 13 cards with different numbers.

- **(c)** $40 = \binom{10}{1} * \binom{4}{1}$

  There are total 10 sequence, and 4 suits.

- **(d)** $10200 = 10240 - \binom{10}{1} * \binom{4}{1}$

  According to (a), there are 10240 sequence, and there are 40 matching suits for these sequences as (c).

- **(e)** $617720 = 617760 - 40$

  According to (b) there are 617760 matching suit, and both a sequence and a matching suit which is a straight flush is 40, according to (c).