Exercise 3.2

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a) Given initial value $\theta 0$, recursively define the feedback process Y_n through

$$\theta_{n+1} = \theta_n + \epsilon_n Y_n$$

with either fixed step size ϵ or decreasing step size, where we typically assume that

$$\sum_{n=1}^{\infty} \epsilon_n = +\infty$$

$$\sum_{n=1}^{\infty} \epsilon_n^2 < \infty$$

and Y_n given via the feedback function

$$Y_n = \phi(\xi(\theta_n), \theta_n)$$

We assume that all random variables, that is, $\theta 0$ and $(\xi_n(\theta) : n >= 0, \theta \in \Theta)$, are defined on a probability space. Running the stochastic approximation algorithm, we observe the underlying sequence

$$\xi_0(\theta_0), \xi_1(\theta_1), ...$$

Here in the problem,

$$\xi_1(\theta_1) = (1 - \theta_0, \theta_0)$$

$$\xi_2(\theta_2) = (1 - \theta_1, \theta_1)$$

$$\xi_3(\theta_3) = (1 - \theta_2, \theta_2)$$

$$\xi_2(\theta_4) = (1 - \theta_3, \theta_3)$$

and so on, ...

b) set $Y_n = (\xi_{n,1}(\theta_n), \xi_{n,2}(\theta_n))^{\intercal}$ are the independent sequences of unbiased estimators of the target vector field:

$$\xi_n(\theta_n) = (1 - \theta_n, -\theta_n)$$

c) **Under** strict monotonicity, if choose A win, $Y_n = \xi_n(\theta_n) = 1 - \theta_n$ the chosen direction the gradient is bigger than 0, which is always the grow direction;
And the probability that B win, $Y_n = \xi_n(\theta_n) = -\theta_n$ is always a descent direction, which is always the decent direction.
So this means that the field is coercive for the well-posed optimization problem.