Exercise 2.4

Yunhua Zhao

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(a) because

$$x_n(t) = V^\epsilon(t_n + t)$$

and

$$V^\epsilon(t) = \theta_{m(t)}$$

so

$$x_n(t) = \theta_{m(t+t_n)}$$

$$x_n(t+s) = \theta_{m(t+s+t_n)}$$

then

$$x_n(t+s) - x_n(t) = \theta_{m(t+s+tn)} - \theta_{m(t+tn)}$$

which

$$= \sum_{i=m(t_n+t)}^{m(t_n+t+s)-1} \epsilon_i G(\theta_i)$$

Because $X\epsilon(.)$ is piecewise point, $G(X\epsilon(.))$ is also piecewise constant and its jump times are given by $t_n = \sum_{k=1}^{n} \epsilon_k$. Thus the definite integral on $[t_n + t, t_n + t + s] of$G(X$\epsilon$(.)) is a sum that can be approximation expressed as

$$\int_{t_n+t}^{t_n+t+s} G[x_\epsilon(u)]du$$

together

$$\int_{t_n+t}^{t_n+t+s} G[x_\epsilon(u)]du \approx \sum_{i=m(t_n+t)}^{m(t_n+t+s)-1} \epsilon_i G(\theta_i)$$

(b) formula

$$x_n(t+s) - x_n(t) = \theta_{m(t+s+tn)} - \theta_{m(t+tn)} = \sum_{i=m(t_n+t)}^{m(t_n+t+s)-1} \epsilon_i G(\theta_i)$$

contains $m(q)-m(r)-1$ terms. For ϵ sufficiently small, set the ϵ_b is the biggest ϵ and the ϵ_s is the smallest ϵ in interval (r,q) so that the number of terms is bounded by $(\frac{q-r}{\epsilon_b}, \frac{q-r}{\epsilon_s})$. This yields, for small ϵ,

$$|(x_\epsilon(q) - x_\epsilon(r))|_\infty = \sum_{i=m(r)}^{m(q)-1} \epsilon_i G(\theta_i)$$

Because G is bounded, let use L to represent G's bounder, so

$$|x_\epsilon(q) - x_\epsilon(r)|_\infty = L \sum_{i=m(r)}^{m(q)-1} \epsilon_i =< \epsilon_b L(q-r)/\epsilon_s$$

To summarize, for ϵ sufficiently small, we have shown that for any $\eta > 0$, we may let $\delta_\eta = \frac{\eta}{L(\epsilon_b/\epsilon_s)}/$ so that it follows that $|x_\epsilon(q) - x_\epsilon(r)|_\infty =< \eta$ wherever $|q-r| =< \delta_\eta(\epsilon_b/\epsilon_s)$. This establishes equicontinuity in the extended sense.

(c)