Safe Reinforcement Learning via Curriculum Induction

Matteo Turchetta*

Department of Computer Science ETH Zurich matteotu@inf.ethz.ch

Andrey Kolobov

Microsoft Research Redmond, WA-98052 akolobov@microsoft.com

Shital Shah

Microsoft Research Redmond, WA-98052 shitals@microsoft.com

Andreas Krause

Department of Computer Science ETH Zurich krausea@ethz.ch

Alekh Agarwal

Microsoft Research Redmond, WA-98052 alekha@microsoft.com

Abstract

In safety-critical applications autonomous agents may need to learn in an environment where mistakes can be very costly. In such settings, the agent needs to behave safely not only after but also while learning. To achieve this, existing safe reinforcement learning methods make an agent rely on priors that let it avoid dangerous situations during exploration with high probability, but both the probabilistic guarantees and the smoothness assumptions inherent in the priors are not viable in many scenarios of interest such as autonomous driving. This paper presents an alternative approach inspired by human teaching, where an agent learns under the supervision of an automatic instructor that saves the agent from violating constraints during learning. In this model, we introduce the monitor that neither needs to know how to do well at the task the agent is learning nor needs to know how the environment works. Instead, it has a library of reset controllers that it activates when the agent starts behaving dangerously, preventing it from doing damage. Crucially, the choices of which reset controller to apply in which situation affect the speed of agent learning. Based on observing agents' progress, the teacher itself learns a policy for choosing the reset controllers, a curriculum, to optimize the agent's final policy reward. Our experiments use this framework in two environments to induce curricula for safe and efficient learning.

1 Introduction

Safety is a major concern that prevents application of reinforcement learning (RL) [42] to many practical problems [14]. Among the RL safety notions studied in the literature [21], ensuring that the agent does not violate constraints is perhaps the most important. Consider, for exmple, training a policy for a self-driving car's autopilot. Although simulations are helpful, much of the training needs to be done via a process akin to RL on a physical car [24]. At that stage, it is critical to avoid damage to property, people, and the car itself. Safe RL techniques aim to achieve this primarily by imparting the agent with priors about the environment and equipping it with sound ways of updating this information with observations [15, 7, 10, 11]. Some do this heuristically [2], while others provide safety guarantees at the cost of smoothness assumptions [7, 26, 44, 45]. These assumptions hold, e.g., in certain drone control scenarios [6] but are violated in settings such as autonomous driving, where a small delta in control inputs can make a difference between safe passage and collision.

^{*}The author did part of this work while at Microsoft Research, Redmond.

In this paper, we propose Curriculum Induction for Safe Reinforcement learning (CISR, "Caesar"), a safe RL approach that lifts several prohibitive assumptions of existing ones. CISR is motivated by the fact that, as humans, we successfully overcome challenges similar to those in the autopilot training scenario when we help our children learn safely. Children possess inaccurate notions of danger, have difficulty imitating us at tasks requiring coordination, and often ignore or misunderstand requests to be careful. Instead, e.g., when they learn how to ride a bike, we help them do it safely by putting training wheels on their bike, then simply following them while prepared to catch them if they start falling, finally letting them ride freely but with elbow and knee guards for some time, and only then allowing them to ride like grown-ups. Importantly, each "graduation" to the next stage happens based on their observed performance under the previous safeguard mechanism.

Key ideas. In CISR, an artificial teacher helps an agent (student) learn potentially dangerous skills by inducing a sequence of safety-ensuring training stages called *curriculum*. A student is an RL agent trying to learn a policy for a constrained MDP (CMDP) [4]. A teacher has a decision rule – a curriculum policy - for constructing a curriculum for a student given observations of the student's behavior. Each curriculum stage lasts for some number of RL steps of the student and is characterized by an intervention (e.g., the use of training wheels) that the teacher commits to use throughout that stage. Whenever the student runs the risk of violating a constraint (falling off the bike), that stage's intervention automatically puts the agent into a safe state (e.g., the way training wheels keep the bike upright), in effect by temporarily overriding the dynamics of the student's CMDP. The teacher's curriculum policy chooses interventions from a pre-specified set such as {use of training wheels, catching the child if they fall, wearing elbow and knee guards} with the crucial property that any single intervention from this set keeps the agent safe as described above. A curriculum policy that commits to any one of these interventions for the entire learning process is sufficient for safety, but note that in the biking scenario we don't keep the training wheels on the bike forever: at some point they start hampering the child's progress. Thus, the teacher's natural goal is to optimize the curriculum policy with respect to the student's policy performance at the end of the learning process, assuming the process is long enough that the student's rate of attempted constraint violations becomes very small. In CISR, the teacher does this via a round-based process, by playing a curriculum policy in every round, observing a student learn under the induced curriculum, evaluating its performance, and trying an improved curriculum policy on a new student in the next round.

Related Work. CISR is a form of *curriculum learning (CL)* [36]. CL and *learning from demonstra*tion (LfD) [13] are two established classes of approaches that rely on a teacher as an aid in training a decision-making agent, but CISR differs from both. In LfD, a teacher provides demonstrations of a good policy for the task at hand, and the student uses them to learn its own policy by behavior cloning [34], online imitation [32], or apprenticeship learning [1]. In contrast, CISR does not assume that the teacher has a policy for the student's task at all: e.g., a teacher doesn't need to know how to ride a bike in order to help a child learn to do it. CL generally relies on a teacher to structure the learning process. A range of works [31, 20, 19, 38, 48, 35, 47] explore ways of building a curriculum by modifying the learning environment. CISR is closer to Graves et al. [22], which uses a fixed set of environments for the student and also uses a bandit algorithm for the teacher. CISR's major differences from existing CL work is that (1) it is the first approach, to our knowledge, that uses CL for ensuring safety and (2) uses multiple students for training the teacher, which allows it to induce curricula in a more data-driven, as opposed to heuristic, way. With regards to safe RL, in addition to the literature mentioned at the beginning, one work that considers the same training and test safety constraints as ours is Le et al. [28], which proposes a solver for the student's CMDP. In that work, the student avoids potentially unsafe environment interaction altogether by learning solely from batch data, which places strong assumptions on MDP dynamics and data collection policy neither verifiable nor easily satisfied in practice [39, 9, 3]. We use the same solver, but in an online setting.

The ideas introduced in this work may be applicable in several kinds of safety-sensitive settings where CISR can be viewed as a *meta-learning framework* [46], with curriculum policy as a "hyperparameter" being optimized. Our experiments indicate that the number of iterations CISR needs to learn a good curriculum policy is small. This allows its use in robotics, where a curriculum policy is trained on agents with one set of sensors and applied to training agents with different sensor arrays of similar capabilities, e.g., as in Pan et al. [33] for autonomous rovers. Further promising scenarios are training a curriculum policy in simulation and applying it to physical agents and using CISR in *intelligent tutoring systems* [12]. In the latter, the safety notion consists in ensuring that the material is presented to a (human) student in an appropriate manner to avoid them dropping the course.

Contributions. Our main contributions are: (1) We introduce CISR, a novel framework for exploiting prior knowledge to guarantee safe training and deployment in RL that forgoes many unrealistic assumptions made in the existing safe RL literature. (2) We present a principled way of optimizing curriculum policies across generations of students while guaranteeing safe student training. (3) We show empirically in two environments that students trained under CISR-optimized curricula attain reward performance comparable or superior to those trained without a curriculum and remain safe throughout training, while those trained without a curriculum don't.

2 Background: Constrained Markov Decision Processes

In this work, we view a learning agent, which we will call a *student*, as performing constrained RL. This framework has been strongly advocated as a promising path to RL safety [37], and expresses safety requirements in terms of an *a priori unknown* set of feasible safe policies that the student should optimize over. In practice, this feasible policy set is often described by a *constrained Markov decision process* (*CMDP*) [4]. We consider CMDPs of the form $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, r, \mathcal{D} \rangle$, where \mathcal{S} and \mathcal{A} are a state and action space, respectively, $\mathcal{P}(s'|s,a)$ is a transition kernel, $r: \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ is a reward function, and \mathcal{D} is a set of unsafe terminal states. We focus on settings where safety corresponds to avoiding visits to the set \mathcal{D} . The student's objective, then, is to find a policy $\pi: \mathcal{S} \to \Delta_{\mathcal{A}}$, i.e., a mapping from states to action distributions, that solves the following constrained optimization problem, where ρ^{π} is a distribution of trajectories induced by π and \mathcal{P} given some fixed initial state distribution:

$$\pi^* = \underset{\pi}{\arg\max} \ \mathbb{E}_{\rho^{\pi}} \sum_{t=0}^{T} r(s_t, a_t, s_{t+1}), \quad \text{s.t.} \quad \mathbb{E}_{\rho^{\pi}} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}) \le \kappa,$$
(1)

where \mathbb{I} is the indicator function. To ensure complete safety, we restrict our attention to problems where the value of κ makes the constraints feasible. While we have presented the finite-horizon undiscounted version of the problem, both the objective and the constraint can be expressed as the average or the discounted sum over an infinite horizon. For a generic CMDP \mathcal{M} we denote the set of its feasible policies as $\Pi_{\mathcal{M}}$, and the value of any $\pi \in \Pi_{\mathcal{M}}$ as $V_{\mathcal{M}}(\pi)$.

There are a number of works on solving CMDPs that find a nearly feasible and optimal policy with sufficiently many trajectories, but violate constraints during training [2, 11]. In contrast, we aim to enable the student to learn a policy for CMDP \mathcal{M} without violating any constraints in the process.

3 Curriculum Induction for Safe RL

We now describe CISR, our framework for enabling a student to learn without violating safety constraints. To address the seemingly impossible problem of students learning safely in an unknown environment, CISR includes a *teacher*, which is a learning agent itself. The teacher serves two purposes: (1) mediating between the student and its CMDP in order to keep the student safe, and (2) learning a mediation strategy – a curriculum policy – that helps the student learn faster. First we formally describe the teacher's mediation tools called *interventions* (Section 3.1). Then, in Section 3.2 we show how, from the student's perspective, each intervention corresponds to a special CMDP where training is safe and every feasible policy is also feasible in original CMDP, ensuring that objective (1) is always met. Finally, in Section 3.3 we consider the teacher's perspective and show how, in order to optimize objective (2), it iteratively improves its curriculum policy by trying it out on different students.

3.1 Interventions

In CISR, the teacher has a set of *interventions* $\mathcal{I} = \{\langle \mathcal{D}_i, \mathcal{T}_i \rangle\}_{i=1}^K$. Hereby, each intervention is defined by a set $\mathcal{D}_i \subset \mathcal{S}$ of *trigger states* where this intervention applies and $\mathcal{T}_i : \mathcal{S} \to \Delta_{\mathcal{S} \setminus \mathcal{D}_i}$, a state-conditional reset distribution. The semantics of an intervention is as follows. From the teacher's perspective, \mathcal{D}_i is a set of undesirable states, either because \mathcal{D}_i intersects with the student CMDP's unsafe state set \mathcal{D} or because the student's current policy may easily lead from \mathcal{D}_i 's states to \mathcal{D} 's. Whenever the student enters a state $s \in \mathcal{D}_i$, the teacher can *intervene* by resetting the student to another, safe state according to distribution $\mathcal{T}_i(\cdot|s)$. We assume the following about the interventions:

Assumption 1 (*Intervention set*). a) The intervention set \mathcal{I} is given to the teacher as input and is fixed throughout learning. b) The interventions in \mathcal{I} cannot be applied after student learning.

Assumption a) is realistic in many settings, where the student is kept safe by heuristics in the form of simple controllers such as those that prevent drones from stalling. At least one prior work, Eysenbach

et al. [16], focuses on how safety controllers can be learned, although their safety notion (policy reversibility) is much more specialized than in CISR. Assumption b) is realistic in that a safety controller practical enough to be used beyond training is likely to be part of the agent, removing the need for safety precautions during training. An example of a safety mechanism that cannot be used beyond training is motion capture lab equipment to aid localization and help prevent collisions.

3.2 The student's problem

We now describe the student's learning process under single and multiple interventions of the teacher. As we explain here, training in the presence of an interventions-based teacher can be viewed as learning in a *sequence* of CMDPs that guarantee student safety under simple conditions. Later, in Sec. 3.3, we formalize these CMDP sequences as *curricula*, and show how the teacher can induce them using a *curriculum policy* in order to accelerate students' learning progress.

Intervention-induced CMDPs. Fix an intervention $\langle \mathcal{D}_i, \mathcal{T}_i \rangle$ and suppose the teacher commits to using it throughout student learning. As long as the student avoids states in \mathcal{D}_i , deemed by the teacher too dangerous for the student's ability, the student's environment works like the original CMDP, \mathcal{M} . But whenever the student enters an $s \in \mathcal{D}_i$, the teacher leads it to a safe state $s' \sim \mathcal{T}_i(\cdot|s), s' \notin \mathcal{D}_i$.

Thus, each of teacher's interventions $i \in \mathcal{I}$ induces a student CMDP $\mathcal{M}_i = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}_i, r_i, \mathcal{D}, \mathcal{D}_i \rangle$, where \mathcal{S} and \mathcal{A} are as in the original CMDP \mathcal{M} , but the dynamics are different: for all $a \in \mathcal{A}$, $\mathcal{P}_i(s'|s,a) = \mathcal{P}(s'|s,a)$ for all $s \in \mathcal{S} \setminus \mathcal{D}_i$ and $\mathcal{P}_i(s'|s,a) = \mathcal{T}_i(s'|s)$ for $s \in \mathcal{D}_i$. The reward function is modified to assign $r_i(s,a,s') = 0$ for $s \in \mathcal{D}_i$, $s' \notin \mathcal{D}_i$: all student's actions in these cases get overridden by the teacher's intervention, having no direct cost for the student. However, the teacher cannot supervise the student forever; the student must learn a safe and high-return policy that does not rely on its help. We thus introduce a constraint on the number of times the student can use the teacher's help. This yields the following problem formulation for the student, where ρ_i^{π} is a distribution of trajectories induced by π given some fixed initial state distribution and the modified transition function \mathcal{P}_i :

$$\pi^* = \arg \max \mathbb{E}_{\rho_i^{\pi}} \sum_{t=0}^{T} r_i(s_t, a_t, s'_{t+1}), \text{ s.t. } \mathbb{E}_{\rho_i^{\pi}} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}) \le \kappa_i, \ \mathbb{E}_{\rho_i^{\pi}} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}_i) \le \tau_i, \ (2)$$

where $\kappa_i \geq 0$ and $\tau_i \geq 0$ are intervention-specific tolerances set by the teacher. Thus, although the student doesn't incur any cost for the teacher's interventions, they are associated with violations of teacher-imposed constraints. Our CMDP solver [28], discussed in Section 4, penalizes the student for them during learning, making sure that the student doesn't exploit them in its final policy.

By construction, each intervention-induced CMDP \mathcal{M}_i has two important properties, which we state below and prove in Appendix C. First, if the teacher has lower tolerance for constraint violations than the original CMDP, an optimal learner operating in \mathcal{M}_i will eventually come up with a policy that is safe in its original environment \mathcal{M} :

Proposition 1 (*Eventual safety*). Let $\Pi_{\mathcal{M}}$ and $\Pi_{\mathcal{M}_i}$ be the sets of feasible policies for the problems in Equations (1) and (2), respectively. Then, if $\tau_i + \kappa_i \leq \kappa$, $\Pi_{\mathcal{M}_i} \subseteq \Pi_{\mathcal{M}}$.

Once we remove the teacher, the student can fail either by passing through states where it used to be rescued or by reaching states where the teacher was not able to save it in the first place. Intuitively, $\tau_i + \kappa_i \leq \kappa$ ensures that the probability of either of these cases is sufficiently low that a feasible policy in \mathcal{M}_i is also feasible in \mathcal{M} . While this guarantees the student's *eventual* safety, it doesn't say anything about *safety during learning*. The second proposition states conditions for learning safely:

Proposition 2 (*Learning safety*). Let \mathcal{D} be the set of unsafe states of CMDPs \mathcal{M} and \mathcal{M}_i , and let \mathcal{D}_i be the set of trigger states of intervention i. If $\mathcal{D} \subseteq \mathcal{D}_i$ and $\mathcal{P}(s'|a,s) = 0$ for every $s' \in \mathcal{D}$, $s \in \mathcal{S} \setminus \mathcal{D}_i$, and $a \in \mathcal{A}$, then an optimal student learning in CMDP \mathcal{M}_i will not violate any of \mathcal{M} 's constraints throughout learning.

Informally, Proposition 2 says that if the set of trigger states \mathcal{D}_i of the teacher's intervention "blankets" the set of unsafe states \mathcal{D} , the student has no way of reaching states in \mathcal{D} without triggering the intervention and being rescued first, and hence is safe even when it violates \mathcal{M}_i 's constraints.

Assumption 2 (*Intervention safety*). In the rest of the paper, we assume all teacher interventions to meet the conditions of Proposition 2.

Sequences of intervention-induced CMDPs and knowledge transfer. As suggested by the biking example, the student's learning is likely to be faster under a *sequence* of teacher interventions,

resulting in a sequence of CMDPs $\mathcal{M}_{i_1}, \mathcal{M}_{i_2}, \ldots$ This requires a mechanism for the student to carry over previously acquired skills from one CMDP to the next. We believe that most knowledge transfer approaches for unconstrained MDPs, such as transferring samples [27], policies [18], models [17] and values [43], can be applied to CMDPs as well, with the caveat that the transfer mechanism should be tailored to the environment, teacher's intervention set, and the learning algorithm the student uses. In Section 4, we present the knowledge transfer mechanism used in our implementation.

3.3 The teacher's problem

Given that, under simple conditions, any sequence of teacher's interventions will keep the student safe, the teacher's task is to sequence interventions/CMDPs for the student so that the student learns the highest-expected-reward policy. In CISR, the teacher does this by iteratively trying a curriculum policy on different students and improving it after each attempt. This resembles human societies, where curriculum policies are implicitly learned through educating generations of students. At the same time, it is different from prior ap-

Algorithm 1 CISR

```
1: Input: Interventions \mathcal{I}, Initial teacher \pi_0^T
2: for j=0,1,\ldots,N_t do
3: \pi_{0,j} \leftarrow \texttt{get\_student}()
4: for n=0,1,\ldots,N_s do
5: \mathcal{M}_{i_n} \leftarrow \pi_j^T(o_0^T,\ldots,o_n^T)
6: if n>0 then \pi_{n,j} \leftarrow \texttt{transfer}(\pi_{n-1,j})
7: \pi_{n,j} \leftarrow \texttt{student.train}(\mathcal{M}_{i_n})
8: o_n^T \leftarrow \phi(\pi_{n,j})
9: \pi_{j+1}^T \leftarrow \texttt{teacher.train}(\{(\pi_k^T, \hat{V}(\pi_{N_s,k}))\}_{k=1}^j)
```

proaches such as Graves et al. [22], Matiisen et al. [29], which try to learn and apply a curriculum on the same student. These approaches, of necessity, have to rely on pre-conceived heuristics to measure student progress, whereas ours relies on a more data-driven approach to doing this.

What does *not* need to be assumed of the teacher. Note several properties that the teacher does *not* need to satisfy under CISR. In particular, the teacher doesn't need to 1) know the student CMDP's reward or transition function; 2) have a policy for performing the task that the student is trying to learn; 3) be able to communicate the set \mathcal{D}_i of an intervention's trigger states for any i to the student. It only needs to be able to *execute* any intervention in \mathcal{I} and must do so without violating laws governing the CMPD \mathcal{M} 's dynamics. However, the teacher is not assumed to use only the student's action and observation set \mathcal{A} and \mathcal{S} to execute the interventions — the teacher may be able to do things the student itself can't, such as setting the the student upright if it starts falling from the bike.

In CISR the teacher learns online. Abstractly, we view the teacher as an online learner presented in Algorithm 1. In particular, for rounds $j = 1, ..., N_t$:

- 1. The teacher plays a *decision rule* π_j^T that makes a *new* student j learn under an adaptively constructed sequence $C_j = (\mathcal{M}_{i_1}, \dots, \mathcal{M}_{i_{N_s}})$ of intervention-induced CMDPs (lines 4-8).
- 2. Each student j learns via a total of N_s interaction units (e.g., steps, episodes, etc) with an environment. During each unit, it acts in a CMDP in C_j . It updates its policy by transferring knowledge across interaction units (Line 6). The teacher computes features $\phi(\pi_{n,j})$ of student j's performance (lines 8) by evaluating j's policies throughout j's learning process. Based on them, the teacher's decision rule proposes the next intervention MDP in C_j .
- 3. The teacher adjusts its decision rule's parameters (line 9) that govern how a CMDP sequence C_{j+1} will be produced in the next round.

Assumption 3 (Length of student learning). For all potential students, their CMDP solvers student. train are sound and complete.² N_s is much larger than the number of interactions it takes the solver to find a feasible policy $\pi \in \Pi_{\mathcal{M}_i}$ for at least one intervention CMDP \mathcal{M}_i , $i \in \mathcal{I}$.

This assumption ensures that students can, in principle, learn a safe policy in the allocated amount of training N_s under some intervention sequence. It also allows the teacher to learn to induce such sequences, given enough rounds N_t , even though not every student trained in the process will necessarily have a feasible policy for \mathcal{M} at the end of its learning.

This framework's concrete instantiations depend on the specifics of (i) the decision rule that produces a sequence C_j in each round, (ii) the teacher's evaluation of the student to estimate $\hat{V}(\pi_{N_s,k})$ in each

²That is, it finds a feasible policy after enough interactions with the CMDP, assuming one exists.

round. Next, we consider each of these aspects.

Curricula and curriculum policies (i). Before discussing teacher's decision rules that induce intervention sequences in each round, we formalize the notion of these sequences themselves:

Definition 1 (Curriculum). Suppose a student learns via N_s interaction units (e.g., steps or episodes) with an environment, and let \mathcal{I} be a set of teacher's interventions. A curriculum C is a sequence $\mathcal{M}_{i_1},\ldots,\mathcal{M}_{i_{N_s}}$ of length N_s of CMDPs s.t. the student interacts with CMDP \mathcal{M}_{i_n} during unit n, where \mathcal{M}_{i_n} is induced by an intervention $i_n \in \mathcal{I}$.

The difference between a curriculum and a teacher's decision rule that produces it is crucial. While a decision rule for round j can be a mapping $C_j:[N_s]\to\mathcal{I}$ exactly like a curriculum, in general it is useful to make it depend on the student's policy $\pi_{n,j}$ at the start of each interaction unit n. In practice, the teacher doesn't have access to $\pi_{n,j}$ directly, but can gather some statistics $\phi(\pi_{n,j})$ about it by conducting an evaluation procedure that we discuss shortly. Examples of useful statistics include the number of times the student's policy triggers teacher's interventions, features of states where this happens, and, importantly, an estimate of the policy value $\hat{V}(\pi_{n,j})$.

Thus, an adaptive teacher is an agent operating in a partially observable MDP $\langle \mathcal{S}^T, \mathcal{A}^T, \mathcal{P}^T, \mathcal{R}^T, \mathcal{O}^T, \mathcal{Z}^T \rangle$, where $\mathcal{S}^T = \overline{\Pi}_{\mathcal{M}}$ is the space of all student policies for the original CMDP \mathcal{M} (not only feasible ones), $\mathcal{A}^T = \mathcal{I}$ is the set of all teacher interventions, $\mathcal{P}^T: \overline{\Pi}_{\mathcal{M}} \times \mathcal{I} \times \overline{\Pi}_{\mathcal{M}} \to [0,1]$ is governed by the student's learning algorithm, $\mathcal{O}^T = \Phi$ is the space of evaluation statistics the teacher gathers, and $\mathcal{Z}^T = \phi$ is the mapping from the student's policies to statistics about them, governed by the teacher's evaluation setup. The reward function \mathcal{R}^T can be defined as $\mathcal{R}^T(n) = \hat{V}(\pi_{n,j}) - \hat{V}(\pi_{n-1,j})$, with $\mathcal{R}^T(0) = \hat{V}(\pi_{0,j})$ the "progress" in student's policy quality from one curriculum stage to the next. Note, however, that what really matters to the teacher is the student's perceived policy quality at the end of round, $\hat{V}(\pi_{N_s,j}) = \sum_{n=1}^{N_s} \mathcal{R}^T(n)$. Thus, in general, a teacher's decsion rule is a solution to this POMDP:

Definition 2 (Curriculum policy). Let \mathcal{H} be the space of teacher's observation histories. A curriculum policy is a mapping $\pi^T: \mathcal{H} \to \mathcal{I}$ that, for any $n \in [N_s]$, specifies an intervention given the teacher's observation history $\phi(\pi_1), \ldots, \phi(\pi_{n-1})$ at the start of the student's n-th interaction unit.

In the context of curriculum learning, modeling the teacher as a POMDP agent similar to ours was proposed in Matiisen et al. [29] – though not for RL safety. However, from the computational standpoint, CISR's view of a teacher as an online learning agent captures a wider range of possibilities for the teacher's practical implementation. For instance, it suggests that it is equally natural to view the teacher as a bandit algorithm that plays suitably parameterized curriculum policies in each round, which is computationally much more tractable than using a full-fledged POMDP solver. As described in Section 4, this is the approach we take in this work.

Safely evaluating students' policies (ii). Executing and optimizing the curriculum policy requires the evaluation of students' policies to create features and rewards for the teacher. Since a student's intermediate policy is not necessarily safe w.r.t. \mathcal{M} , evaluating it in \mathcal{M} could lead to constraint violations. Instead, we assume that the teacher's intervention set \mathcal{I} includes a special intervention i_0 satisfying Proposition 2. This intervention induces a safe CMDP \mathcal{M}_0 that the teacher uses for evaluation, although it can also be used for student learning. Therefore, for any policy π over the state space \mathcal{S} and action space \mathcal{A} , we define $\hat{V}(\pi) \triangleq V_{\mathcal{M}_0}(\pi)$. Since \mathcal{M}_0 has more constraints than \mathcal{M} , evaluation in \mathcal{M}_0 underestimates the quality of the teacher's curriculum policy. Let $\hat{\pi}^*$ be the optimal policy for \mathcal{M}_0 and π^* the optimal policy for \mathcal{M} . The value of the teacher's curriculum policy is, then, $\hat{V}(\hat{\pi}^*) \leq V_{\mathcal{M}}(\pi^*)$. If the student policy violates a constraint in \mathcal{M}_0 during execution, the teacher gets a reward $-2TR_{\max}$ where R_{\max} is the largest environment reward.

4 Implementation Details

CISR allows for many implementation choices. Here, we describe the those used in our experiments.

Student's training and knowledge transfer. Our students are CMDP solvers based on [28], but train online rather than offline as in [28] since safety is guaranteed by the teacher. This is a primal-dual solver, where the primal consists of an unconstrained RL problem including the original rewards and a Lagrange multiplier penalty for constraint violation. The dual updates the multipliers to increase

the penalty for violated constraints. We use the Stable Baselines [23] implementation of PPO [40] to optimize the Lagrangian of a CMDP for a fixed value of the multipliers, and Exponentiated Gradient [25], a no-regret online optimization algorithm, to adapt the multipliers. Our students transfer both value functions and policies across interventions, but reset the state of the optimizer.

Teacher's observation. Before every switch to a new intervention i_{n+1} , our teacher evaluates the student's policy in CMDP \mathcal{M}_{i_n} induced by the previous intervention. The features estimated in this evaluation, which constitute the teacher's observation o_n^T , are $V_{\mathcal{M}_{i_n}}(\pi)$, the student's policy value in CMDP \mathcal{M}_{i_n} , and the rate of its constraint violation there, $\mathbb{E}_{\rho_{i_n}^\pi}[\sum_{t=0}^T \mathbb{I}(s_t \in \mathcal{D}_{i_n}) - \tau_{i_n}]$.

Reward. As mentioned in Sec. 3.3, from a computational point of view it is convenient to approach the curriculum policy optimization problem as an online optimization problem for a given parametrization of the teacher's curriculum policy. This is the view we adopt in our implementation, where in round j, the teacher's objective is the value of the student's final policy $\hat{V}(\pi_{N_s,j})$. Moreover, since after N_s curriculum steps the student's training is over, we compute the student's return directly in \mathcal{M} rather than using a separate evaluation intervention \mathcal{M}_0 .

Policy class. To learn a good teaching policy efficiently, we restrict the teacher's search space to a computationally tractable class of parameterized policies. We consider reactive policies that depend only on the teacher's current observation, o_n^T , so $\pi^T(o_0^T,o_1^T,\ldots,o_n^T)=\pi^T(o_n^T)$. Moreover, we restrict the number of times the teacher can switch their interventions to at most $K \leq N_s$. A policy from this class is determined by a sequence of K interventions and by a set of rules that determines when to switch to the next intervention in the sequence. Here, we consider simple rules that require the average return and constraint violation during training to be greater/smaller than a threshold. Formally, we denote the threshold array as $\omega \in \mathbb{R}^2$. The teaching policy we consider switches from the current intervention to the next when $\phi(\pi_{n,j})[0] \geq \omega[0] \wedge \phi(\pi_{n,j})[1] \leq \omega[1]$. Thus, teacher's policies are fully determined by 3K+1 parameters.

Teacher's training with GP-UCB. Given the teacher's policy is low dimensional and sample efficiency is crucial, we use Bayesian optimization (BO) [30] to optimize it. That is, we view the 3K + 1 dimensional parameter vector of the teacher as input to the BO algorithm, and search for parameters that approximately optimize the teacher's reward. Concretely, we use GP-UCB [41], a simple Bayesian optimization algorithm that enjoys strong theoretical properties.

5 Experiments

We present experiments where CISR efficiently and safely trains deep RL agents in two environments: the *Frozen Lake* and the *Lunar Lander* environments from Open AI Gym [8]. While *Frozen Lake* has simple dynamics, it demonstrates how safety exacerbates the difficult problem of exploration in goal-oriented environments. *Lunar Lander* has more complex dynamics and a continuous state space. We compare students trained with a curriculum optimized by CISR to students trained with trivial or no curricula in terms of safety and sample efficiency. *In addition, we show that curriculum policies can transfer well to students of different architectures and sensing capabilities (Table 1).* For a detailed overview of the hyperparameters and the environments, see Appendices A and B.

Frozen Lake. In this grid-world environment (Fig. 1a), the student must reach a goal in a 2D map while avoiding dangers. It can move in 4 directions. With probability 80% it moves in the desired direction and with 10% probability it moves in either of the orthogonal ones. The student only sees the map in Fig. 1a and is not aware of the teacher interventions' trigger states (Figs. 1b and 1c).

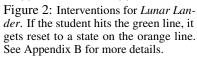
We use three interventions, whose trigger states are shown in Figs. 1b and 1c: soft reset 1 (SR1), soft reset 2 (SR2), and hard reset (HR). SR1 and SR2 have tolerance $\tau=0.1$ and reset the student to the state where it was the time step before being rescued by the teacher. HR has zero tolerance, $\tau=0$ and resets the student to the initial state.

We compare five different teaching policies: (i) No-intervention, where students learn in the original environment; (ii-iii-iv) single-intervention, where students learn under each of the interventions fixed for the entire learning duration; (v) Optimized, where we use a curriculum policy optimized with CISR over 30 students. We let each of these curriculum policies train 10 students. For analysis purposes, we periodically freeze the students' policies and evaluate them in the original environment.

Results. Fig. 3 shows the success rate and the return of the students' policies deployed in the original environment as training progresses. Without the teacher's supervision (*No-interv.*), the students



Figure 1: Interventions for *Frozen Lake*. Maps 1b and 1c show trigger state sets \mathcal{D}_{SR1} and \mathcal{D}_{SR2} for interventions SR1 and SR2, which get triggered at distance = 1 and 2 from lakes (dangers), respectively. Intervention HR has $\mathcal{D}_{HR} = \mathcal{D}_{SR1}$.



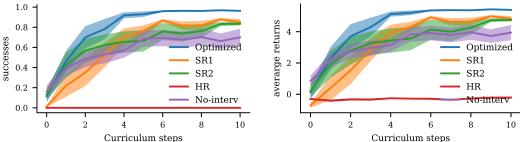


Figure 3: Student success rate (**Left**) and average returns (**Right**) in *Frozen Lake* as student training under different curriculum policies progresses. Under *No-intervention*, students learn reasonable policies, but slowly and at the cost of thousands of constraint violations during training (**see Appendix, Table 6**). *HR* resets the student to the start state upon violations, like *No-intervention*, but has more trigger states (see Fig. 1b vs. Fig. 1a). As a result, students successfully avoid triggering the teacher's intervention (and violating safety constraints), but don't explore enough to reach the goal. *SR1* and *SR2* allow the students to learn about the goal, but their policy performance plateaus, as the soft reset of the training environment lets the students recover from mistakes in a way the deployment environment doesn't. *Optimized* attains the best of both worlds: students initially learn about the goal thanks to *SR* but then learn to be more careful around dangers when the teacher switches to *HR*.

learn sensible policies. However, the training is slow and results in thousands of failures (Table 6 in Appendix B). The *HR* intervention resets the students to the initial state distribution whenever the teacher constraint is violated. However, since it has more trigger states than *No-interv*. (Fig. 1b vs Fig. 1a), the students training exclusively under it do not explore enough to reach the goal. *SR1* and *SR2* allow the student to learn about the goal without incurring failures thanks to their reset distribution, which is more forgiving that *HR*'s one. However, they result in performance plateaus as the consequence of a mistake in the original environment (failure and consequent end of the episode) are quite different from those encountered during training. The *Optimized* curriculum retains the best of both worlds by initially proposing a soft reset intervention that allows the agent to reach the goal and subsequently switching to the hard reset such that the training environment is more similar to the original one. Table 6 in Appendix B shows the confidence intervals of mean performance across 10 students and teachers trained on 3 seeds, indicating CISR's robustness.

Lunar Lander. In this environment, the goal is to safely land a spaceship on the Moon. Crucially, the Moon surface is rugged and differs across episodes. The only constant is a flat landing pad that stretches across \mathcal{X}_{land} along the x dimension (at y=0). Landing safely is particularly challenging since agents do not observe their distance from the ground, only their absolute x and y coordinates.

Since the only way to ensure safe training is to land on the landing pad each time, we use the two interventions in Fig. 2. Each gets triggered based on the student's tilt angle and y-velocity if the student is over the landing pad ($x \in \mathcal{X}_{land}$), and on a funnel-shaped function of its x, y coordinates otherwise ($x \notin \mathcal{X}_{land}$). The former case prevents crashing onto the landing pad, the latter, landing (and possibly crashing) on the rugged surface the student cannot sense. We call the interventions Narrow and Nide (see Fig. 2); both set the student's velocity to 0 after rescuing it to a safe state. The interventions, despite ensuring safety, make exploration harder as they make experiencing a natural ending of the episode difficult.

Similarly to the *Frozen Lake* evaluation, we compare four curriculum policies: (i) *No-intervention*, (ii-iii) single-intervention and (iv) *Optimized*. We let each policy train 10 students and we compare their final performance in the original *Lunar Lander*. Moreover, we use the *Optimized* curriculum to train different students than those it was optimized for, thus showing the transferability of curricula.

	Eval. on	noiseles	s, 2-lay	er student	Eval.	on nois	sy stude	ent	Eval. o	n 1-lay	er stud	lent
	$V_{\mathcal{M}}(\pi_{N_s})$	Succ.	Test	Train	$ V_{\mathcal{M}}(\pi_{N_s}) $	Succ.	Test	Train	$ V_{\mathcal{M}}(\pi_{N_s}) $	Succ.	Test	Train
			fail	fail			fail	fail			fail	fail
Optimized	233.8	88.4%	10.5%	1.6	221.1	86.7%	10.5%	2.95	254.5	92.2%	6.6%	2.2
Narrow	183.0	72.4%	30.0%	1.0	149.1	65.1%	32.4%	1.2	220.4	83.3%	15.9%	1.3
Wide	210.6	81.4%	17.6%	3.4	153.9	75.1%	14.0%	4.05	119.9	67.4%	17.6%	4.0
No-interv.	236.7.3	90.1%	7.9%	1228.8	210.3	85.5%	11.8%	1651.9	248.7	92.4%	6.1%	1368.1

Table 1: *Lunar Lander* final performance summary. **Noiseless, 2-layer student (Left):** The *Narrow* intervention helps exploration but results in policy performance plateau, the *Wide* one slows down student learning due to making exploration more challenging, and the *Optimized* teacher provides the best of both by switching between *Narrow* and *Wide*. Students that learn under the *Optimized* curriculum policy achieve a comparable performance to those training under *No-intervention*, but suffer three orders of magnitude fewer training failures. **Noisy student (Center), One layered-student (Right):** The results are similar when we use the curriculum optimized for students with noiseless observations and a 2-layer MLP policy for students with noisy sensors (center) or a 1-layer architecture (right), thus showing teaching policies can be transferred across classes of students.

Results. Table 1 (left) shows for each curriculum policy the mean of the students' final return, success rate and failure rate in the original environment and the average number of failures during training. The *Narrow* intervention makes exploration less challenging but prevents the students from experiencing big portions of the state space. Thus, it results in fast training that plateaus at low success rates. On the contrary, the *Wide* intervention makes exploration more complex but it is more similar to the original environment. Therefore, it results in slow learning that cannot achieve high success rates withing N_s interaction units. *Optimized* retains the best of both worlds by initially using the *Narrow* intervention to speed up learning and subsequently switching to the *Wide* one. In *No-interv*, exploration is easier due to the absence of the teacher that can make it hard for the students to experience a natural ending of the episode. Therefore, *No-interv*, attains a comparable performance to *Optimized*. However, the absence of the teacher results in three orders of magnitude more training failures.

Table 1 shows the results of using the teaching policy optimized for students with 2-layer MLP policies and perfect state information to train students with noisy sensors (center) and different architectures (right). The results for these new classes of students are qualitatively similar to those previously observed: the *Optimized* curriculum attains a comparable performance to the *No interv.* training while greatly improving safety. This shows that teaching policies can be effectively transferred, which is of great interest in many applications. Table 7 in Appendix B shows the confidence intervals for the same experiments over 3 seeds.

6 Conclusion

In this work, we introduce CISR, a novel framework for safe RL that avoids many of the impractical assumptions common in the safe RL literature. In particular, we introduce curricula inspired by human learning for safe training and deployment of RL agents and a principled way to optimize them. Finally, we show how training under such optimized curricula results in performance comparable or superior to training without them, while greatly improving safety.

Acknowledgments and Disclosure of Funding

We would like to thank Cathy Wu (MIT), Sanmit Narvekar (University of Texas at Austin), Luca Corinzia (ETH Zurich) and Patrick MacAlpine (Microsoft Research) for their comments and suggestions regarding this work. This work was supported by the Max Planck ETH Center for Learning Systems.

References

- [1] Abbeel, P. and Ng, A. (2004). Apprenticeship learning via inverse reinforcement learning. In *ICML*
- [2] Achiam, J., Held, D., Tamar, A., and Abbeel, P. (2017). Constrained policy optimization. In *Proceedings of the 34th International Conference on Machine Learning-Volume 70*, pages 22–31. JMLR. org.
- [3] Agarwal, A., Kakade, S. M., Lee, J. D., and Mahajan, G. (2019). Optimality and approximation with policy gradient methods in markov decision processes. *arXiv* preprint arXiv:1908.00261.

- [4] Altman, E. (1999). Constrained Markov decision processes, volume 7. CRC Press.
- [5] Authors, T. G. (2016). GPyOpt: A bayesian optimization framework in python. http://github.com/SheffieldML/GPyOpt.
- [6] Berkenkamp, F., Schoellig, A. P., and Krause, A. (2016). Safe controller optimization for quadrotors with Gaussian processes. In *Proc. of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 493–496.
- [7] Berkenkamp, F., Turchetta, M., Schoellig, A. P., and Krause, A. (2017). Safe model-based reinforcement learning with stability guarantees. In *NiPS*.
- [8] Brockman, G., Cheung, V., Pettersson, L., Schneider, J., Schulman, J., Tang, J., and Zaremba, W. (2016). Openai gym. *arXiv preprint arXiv:1606.01540*.
- [9] Chen, J. and Jiang, N. (2019). Information-theoretic considerations in batch reinforcement learning. *ICML*.
- [10] Chow, Y., Nachum, O., Duenez-Guzman, E., and Ghavamzadeh, M. (2018). A lyapunov-based approach to safe reinforcement learning. In *Advances in neural information processing systems*, pages 8092–8101.
- [11] Chow, Y., Nachum, O., Faust, A., Duenez-Guzman, E., and Ghavamzadeh, M. (2019). Lyapunov-based safe policy optimization for continuous control. *arXiv preprint arXiv:1901.10031*.
- [12] Clement, B., Roy, D., Oudeyer, P.-Y., and Lopes, M. (2015). Multi-armed bandits for intelligent tutoring systems. *Journal of Educational Data Mining*, 7.
- [13] D.Argall, B., Chernova, S., Veloso, M., and Browning, B. (2009). A survey of robot learning from demonstration. *Robotics and Autonomous Systems*, 57.
- [14] Dulac-Arnold, G., Mankowitz, D., and Hester, T. (2019). Challenges of real-world reinforcement learning.
- [15] El Chamie, M., Yu, Y., and Açıkmeşe, B. (2016). Convex synthesis of randomized policies for controlled markov chains with density safety upper bound constraints. In *2016 American Control Conference (ACC)*, pages 6290–6295. IEEE.
- [16] Eysenbach, B., Gu, S., Ibarz, J., and Levine, S. (2018). Leave no trace: Learning to reset for safe and autonomous reinforcement learning. In *ICLR*.
- [17] Fachantidis, A., Partalas, I., Tsoumakas, G., and Vlahavas, I. (2013). Transferring task models in reinforcement learning agents. *Neurocomputing*, 107:23–32.
- [18] Fernández, F., García, J., and Veloso, M. (2010). Probabilistic policy reuse for inter-task transfer learning. *Robotics and Autonomous Systems*, 58(7):866–871.
- [19] Florensa, C., Held, D., Geng, X., and Abbeel, P. (2018). Automatic goal generation for reinforcement learning agents. In *ICML*.
- [20] Florensa, C., Held, D., Wulfmeier, M., Zhang, M., and Abbeel, P. (2017). Reverse curriculum generation for reinforcement learning. In *CoRL*.
- [21] Garcia, J. and Fernández, F. (2015). A comprehensive survey on safe reinforcement learning. *Journal of Machine Learning Research*, 16(1):1437–1480.
- [22] Graves, A., Bellemare, M. G., Menick, J., Munos, R., and Kavukcuoglu, K. (2017). Automated curriculum learning for neural networks. In *ICML*.
- [23] Hill, A., Raffin, A., Ernestus, M., Gleave, A., Kanervisto, A., Traore, R., Dhariwal, P., Hesse, C., Klimov, O., Nichol, A., Plappert, M., Radford, A., Schulman, J., Sidor, S., and Wu, Y. (2018). Stable baselines. https://github.com/hill-a/stable-baselines.
- [24] Kendall, A., Hawke, J., Janz, D., Mazur, P., Reda, D., Allen, J.-M., Lam, V.-D., Bewley, A., and Shah, A. (2019). Learning to drive in a day. In *ICRA*.

- [25] Kivinen, J. and Warmuth, M. K. (1997). Exponentiated gradient versus gradient descent for linear predictors. *information and computation*, 132(1):1–63.
- [26] Koller, T., Berkenkamp, F., Turchetta, M., and Krause, A. (2018). Learning-based model predictive control for safe exploration. In 2018 IEEE Conference on Decision and Control (CDC), pages 6059–6066. IEEE.
- [27] Lazaric, A. and Restelli, M. (2011). Transfer from multiple mdps. In *Advances in Neural Information Processing Systems*, pages 1746–1754.
- [28] Le, H. M., Voloshin, C., and Yue, Y. (2019). Batch policy learning under constraints. *arXiv* preprint arXiv:1903.08738.
- [29] Matiisen, T., Oliver, A., Cohen, T., and Schulman, J. (2019). Teacher-student curriculum learning. In *IEEE Transactions on Neural Networks and Learning Systems*.
- [30] Mockus, J., Tiesis, V., and Zilinskas, A. (1978). The application of bayesian methods for seeking the extremum. *Towards global optimization*, 2(117-129):2.
- [31] Narvekar, S., Sinapov, J., Leonetti, M., and Stone, P. (2016). Source task creation for curriculum learning. In *AAMAS*.
- [32] Osa, T., Pajarinen, J., Neumann, G., Bagnell, J. A., Abbeel, P., and Peters, J. (2018). An algorithmic perspective on imitation learning. *Foundations and Trends in Robotics*, 7(1-2):1–179.
- [33] Pan, Y., Cheng, C.-A., Saigol, K., Lee, K., Yan, X., Theodorou, E., and Boots, B. (2018). Agile autonomous driving using end-to-end deep imitation learning. In *RSS*.
- [34] Pomerleau, D. (1989). ALVINN: An autonomous land vehicle in a neural network. In *Advances in Neural Information Processing Systems*.
- [35] Portelas, R., Colas, C., Hofmann, K., and Oudeyer, P.-Y. (2019). Teacher algorithms for curriculum learning of deep RL in continuously parameterized environments. In *CoRL*.
- [36] Portelas, R., Colas, C., Weng, L., Hofmann, K., and Oudeyer, P.-Y. (2020). Automatic curriculum learning for deep rl: A short survey.
- [37] Ray, A., Achiam, J., and Amodei, D. (2019). Benchmarking safe exploration in deep reinforcement learning.
- [38] Riedmiller, M., Hafner, R., Lampe, T., Neunert, M., Degrave, J., Van de Wiele, T., Mnih, V., Heess, N., and Springenberg, J. T. (2018). Learning by playing solving sparse reward tasks from scratch. In *ICML*.
- [39] Scherrer, B. (2014). Approximate policy iteration schemes: a comparison. In *International Conference on Machine Learning*, pages 1314–1322.
- [40] Schulman, J., Wolski, F., Dhariwal, P., Radford, A., and Klimov, O. (2017). Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*.
- [41] Srinivas, N., Krause, A., Kakade, S. M., and Seeger, M. (2010). Gaussian process optimization in the bandit setting: No regret and experimental design. In *ICML*.
- [42] Sutton, R. S. and Barto, A. G. (2018). Reinforcement learning: An introduction. MIT press.
- [43] Taylor, M. E. and Stone, P. (2005). Behavior transfer for value-function-based reinforcement learning. In *Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems*, pages 53–59.
- [44] Turchetta, M., Berkenkamp, F., and Krause, A. (2016). Safe exploration in finite markov decision processes with gaussian processes. In *Advances in Neural Information Processing Systems*, pages 4312–4320.
- [45] Turchetta, M., Berkenkamp, F., and Krause, A. (2019). Safe exploration for interactive machine learning. In *Advances in Neural Information Processing Systems*, pages 2887–2897.

- [46] Vanschoren, J. (2018). Meta-learning: A survey. arXiv preprint arXiv:1810.03548.
- [47] Wang, R., Lehman, J., Clune, J., and Stanley, K. O. (2019). Paired open-ended trailblazer (POET): endlessly generating increasingly complex and diverse learning environments and their solutions. *CoRR*, abs/1901.01753.
- [48] Wu, Y. and Tian, Y. (2017). Training agent for first-person shooter game with actor-critic curriculum learning. In *ICLR*.

APPENDIX

A Hyperparameters

In this section, we report the hyperparameters that we use for the students, which are CMDP solvers based on an online version of [28], and for the teachers, which are based on the GP-UCB algorithm for multi-armed bandits [41].

A.1 Students

The students comprise two components: an unconstrained RL solver and a no-regret online optimizer. The first component is used to solve the unconstrained RL problem that results from optimizing the Lagrangian of a given CMDP for a fixed value of the Lagrange multipliers. For this, we use the Stable Baselines [23] implementation of the Proximal Policy Optimization (PPO) algorithm [40]. The second component is used to adapt the Lagrangian multipliers online. As suggested in [28], we use the Exponentiated Gradient algorithm [25] for this. In the following, we use the hyperaparameters naming convention from Stable Baselines [23] for PPO and from [28] for Exponentiated Gradient.

Frozen Lake. In Table 2, we show the hyperparameters used for the students in the *Frozen Lake* experiments, except for those that determine their policy class. In these experiments, the student's policies are parametrized as convolutional neural networks with 2 convolutional layers followed by a fully connected layer. The first convolutional layer has 32 filters of size 3 and stride 1. The second one has 64 filters of size 3 and stride 1. The fully connected layers contains 32 neurons. We use ReLU as activation function.

Lunar Lander. In Table 3, we show the hyperparameters used for the students in the *Lunar Lander* experiments, except for those that determine their policy class. In these experiments, the student's policies are parametrized as MLP networks with 2 hidden layers with 20 neurons each. We use ReLU as activation function.

A.2 Teachers

Our teachers are based on the the GPyOpt [5] implementation of GP-UCB.

The teacher's hyperparameters are those of the Gaussian process (GP) model used by GP-UCB. In all the experiments, we use a GP with radial basis function (RBF) kernel with automatic relevance determination (ARD) and a Gaussian likelihood. Therefore, the teacher has the following hyperparameters: the signal variance, σ_f^2 , an array of 3K+1 lengthscales $l \in \mathbb{R}^{3K+1}$, where 3K+1 is the number of parameters that determines the teacher's policy for a fixed number of intervention switches, K, see Sec. 4, and the noise variance σ_n^2 . Rather than fixing the hyperparameters a priori, we define hyperpriors over them and use their maximum a posteriori (MAP) estimate, which we update after every newly acquired data point.

The data is normalized before being fed to the GP.

Frozen Lake. In the *Frozen Lake* experiments, we allow for up to two switches between interventions; that is, K=2. Therefore, $l \in \mathbb{R}^7$. In Table 4, we show the mean and the variance of the Gamma hyperprior of each hyperparameter.

Lunar Lander. In the *Lunar Lander* experiments, we allow for up to one switch between interventions; that is, K=1. Therefore, $l \in \mathbb{R}^4$. In Table 5, we show the mean and the variance of the Gamma hyperprior of each hyperparameter.

Name	Value		
n_steps	128	Name	Value
ent_coef	0.05	\overline{B}	0.5
$learning_rate$	0.001	η	1.0
nopte pochs	9	(b) Exponer	tiated Gradient
(a) PPO			

Table 2: Student's hyperparameters for the Frozen lake environment.

Name	Value		
n_steps	500	Name	Value
ent_coef	0.001	\overline{B}	120
$learning_rate$	0.005	η	1.0
nopte pochs	32	(b) Exponen	tiated Gradient
(a) PPO			

Table 3: Student's hyperparameters for the *Lunar Lander* environment.

Hyperparameter	σ_f^2	l_1	l_2	l_3	l_4	l_6	l_5	l_7	σ_n^2
μ	1	1	0.05	1	0.05	0.2	0.2	0.2	0.01
σ^2	0.2	1	0.02	1	0.02	0.2	0.2	0.2	0.1

Table 4: Mean and variance of the Gamma hyperpriors for the teacher's hyperparameters for the *Frozen Lake* environment.

B Experiments

In this section, we provide a detailed explanation of our experimental setup and we present the results we obtained repeating the curriculum optimization and evaluation for multiple random seeds.

B.1 Frozen Lake

Environment. In the *Frozen Lake* experiments, we use the 10×10 map in Fig. 1a. The student receives the full map as observation but is not aware of the areas of influence of the teacher, Figs. 1b and 1c. In each location, it can take one of four actions: up, right, left or down. With probability 80% it moves in the desired direction and with 10% probability it moves in either of the orthogonal ones. After each move, it can end up in one of three kind of tiles: goal, which results in a successful termination of the episode, danger, which results in a failure and the consequent termination of the episode and safe. The agent receives a reward of 6 for reaching the goal and -0.01 otherwise (entering dangerous tiles is discouraged via the constraint rather than with low rewards).

An interaction unit between the student and the teacher consists of 10000 time steps. A curriculum lasts for 11 of such interaction units.

Teacher's training. We consider curriculum policies that allow for up to two intervention switches, i.e., K=2. To initialize the GP model, we sample 10 curriculum policies at random, train a student with each of those and feed their final performance to the GP model. To optimize the curriculum, we run GP-UCB for 20 iterations, where each iteration corresponds to training a single student with the curriculum policy proposed by GP-UCB.

Teacher's evaluation. To evaluate the quality of a curriculum policy, we get 10 new students, we let them train with the curricula induced by such policy and we record their failures during training as well as their returns and their successes when they are deployed in the original environment (i.e. without supervision) for 10000 time steps. Fig. 3 reports the mean of these quantities over the 10 students for all the curriculum policies that we consider at the end of each interaction unit. Notice that evaluating after each intervention unit is done solely for analysis purposes as, in practice, one should not deploy a student in the original environment before the curriculum is completed.

For each of the teaching policies considered, we report the mean returns and success rates at the end of the curriculum over the 10 students as well as their mean number of failures during training in Table 6. Here, the confidence intervals are obtained by optimizing 3 curriculum policies independently with different random sees and repeating the evaluation procedure for each one.

B.2 Lunar Lander

The observation space of the $Lunar\ Lander$ environment is 8-dimensional and it includes: x and y position, tilt angle, linear and angular velocities and two Booleans that indicate whether each leg is in contact with the ground. At each time step, the lander can take one of four actions: fire the main, the left or the right engine or do nothing. The agent receives a reward of 100 for a successful landing,

Hyperparameter	σ_f^2	l_1	l_2	l_3	l_4	σ_n^2
${\mu}$	1	20	1	0.2	0.2	0.01
σ^2	0.2	4	0.3	0.2	0.2	0.1

Table 5: Mean and variance of the Gamma hyperpriors for the teacher's hyperparameters for the *Lunar Lander* environment.

	Success	Training failures	$V_{\mathcal{M}}(\pi_{N_s})$
Optimized	0.960 ± 0.004	0 ± 0	$\boldsymbol{5.368 \pm 0.025}$
SR2	0.827 ± 0.027	0 ± 0	4.669 ± 0.168
SR1	0.850 ± 0.011	0 ± 0	4.839 ± 0.065
HR	0.000 ± 0.000	0 ± 0	-0.222 ± 0.013
No-interv.	0.768 ± 0.028	3075.6 ± 492.1	4.329 ± 0.160

Table 6: Final deployment performance in *Frozen Lake* with confidence intervals obtained by training and evaluating the teachers with three different random seeds. The students trained with the optimized curriculum outperform both naive curricula and training in the original environment in terms of success rate and return. All the agents supervised by a teacher are safe during training. In contrast, training directly in the original environment results in many failures. These results are consistent across random seeds, thus showing the robustness of CISR.

of -0.3 for firing the main engine and of -0.03 for firing the side engines. Additionally, there is a potential based reward shaping that encourages the contact of the legs with the ground and moving toward the origin, i.e., the center of the landing pad. At the beginning of each episode, a random force is applied to the agent and the surface of the Moon is generated at random, with the only constant being the flat surface of the landing pad in the center of the map. Since the agent does not observe its distance from the ground and since the surface of the Moon is generated at every episode, the only way to guarantee safety is to land on the landing pad. In the original environment, each episode can terminate with either a successful landing or a failure (either a crash or exiting the game window from the sides, which we call an out of map, OOM, outcome). However, since the teacher's interventions make it hard for inexperienced students to come across a natural ending of the episode, we introduce a timeout, which we set to 500 during training and to 2000 during deployment (a well trained agent usually requires between 150 and 250 steps to land). Every time an episode ends because of a timeout, the student receives a reward of -100.

The trigger function of the interventions depends on whether the student's is above the landing pad, i.e., $x \in \mathcal{X}_{land}$, or not. In particular, let us denote with x and y the position of the agent, with \dot{x} and \dot{y} its linear velocities, with α its tilt angle and with $\dot{\alpha}$ its angular velocity. The landing pad stretches between -0.2 and 0.2, while the whole map extends from -1 to 1. For a fixed steepness of the funnel a, the trigger function of the interventions are of the form:

$$trigger(x, y, \dot{x}, \dot{y}, \alpha, \dot{\alpha}) = \begin{cases} \mathbb{I}(\dot{y} \ge 0.3 + 10y) \vee \mathbb{I}(\alpha \ge 0.5 + 10y) & \text{if } x \in [-0.2, 0.2] \\ y \le a(-0.2 - x) & \text{if } x < -0.2 \\ y \le a(x - 0.2) & \text{if } x > -0.2 \end{cases}$$
(3)

The reset distribution that determines the student's state after the teacher intervenes also depends on whether the teacher rescues the student above the landing pad or not. We denote with $(x,y,\dot{x},\dot{y},\alpha,\dot{\alpha})$ the state where the students gets rescued and with $(x',y',\dot{x}',\dot{y}',\alpha',\dot{\alpha}')$ the state where the student gets reset. First of all, the teacher always stabilizes the student and, therefore, we have $\dot{x}'=\dot{y}'=\alpha'=\dot{\alpha}'=0$. Thus, the reset distributions only differ based on the location where the teacher steers the student to make it stay clear from danger. In particular, if $x\in[-0.2,0.2]$, we have x'=x and y'=y-0.1. However, if x>0.2, the reset location of the student is determined by a geometric construction: we reset the student at the intersection between the line of that forms and angle of 135° with the horizontal axis passing through x and y and the line a'(x-0.2), for a given a'>a (the orange line in Fig. 2). A symmetric construction is used in case x<-0.2.

The Narrow intervention corresponds to a=20 and a'=100, while the Wide intervention corresponds to a=0.5 and a'=1.

An interaction unit between the student and the teacher consists of 100000 time steps. A curriculum lasts for 15 of such interaction units.

	Succ.	Crash	OOM	$V_{\mathcal{M}}(\pi_{N_s})$	Training failures
Optimized	$89.2 \pm 0.4\%$			236.1 ± 0.9	1.7 ± 0.13
Wide	$79.2 \pm 2.2\%$	$18.7 \pm 1.5\%$	$0.4 \pm 0.1\%$	199.5 ± 8.3	3.5 ± 0.15
Narrow	$72.7 \pm 3.1\%$	$23.4 \pm 2.1\%$	$0.4 \pm 0.2\%$	187.2 ± 9.0	0.9 ± 0.02
No-interv.	$88.5 \pm 1.6\%$	$7.3 \pm 0.1\%$	$1.9\pm1.4\%$	225.1 ± 7.4	1251.0 ± 33.72

(a) Evaluation on noiseless, 2-layer students.

	Success	Crashes	OOM	$V_{\mathcal{M}}(\pi_{N_s})$	Training
					failures
Optimized	$83.4 \pm 2.3\%$	$13.2 \pm 2.2\%$	$0.3 \pm 0.1\%$	211.5 ± 6.8	2.6 ± 0.23
Wide	$78.8 \pm 2.6\%$	$13.8 \pm 0.8\%$	$0.7 \pm 0.4\%$	184.7 ± 21.8	4.2 ± 0.12
Narrow	$63.2 \pm 1.3\%$	$32.9 \pm 1.0\%$	$0.6 \pm 0.2\%$	139.1 ± 7.1	1.8 ± 0.46
No-interv.	$86.0 \pm 0.4\%$	$10.8 \pm 0.1\%$	$0.8 \pm 0.2\%$	214.7 ± 3.1	1695.8 ± 31.04

(b) Evaluation on 2-layer students with noisy sensors.

	Success	Crashes	OOM	$V_{\mathcal{M}}(\pi_{N_s})$	Training failures
Optimized	$92.1 \pm 1.6\%$	$5.1 \pm 0.6\%$	$0.0 \pm 0.0\%$	253.4 ± 5.0	1.9 ± 0.13
Wide	$72.2 \pm 2.9\%$	$16.6 \pm 0.9\%$	$2.4 \pm 1.1\%$	151.6 ± 18.5	3.5 ± 0.25
Narrow	$81.7 \pm 1.0\%$	$16.3 \pm 0.2\%$	$0.0 \pm 0.0\%$	221.0 ± 2.1	1.3 ± 0.03
No-interv.	$94.5 \pm 1.0\%$	$4.5 \pm 0.9\%$	$0.1 \pm 0.0\%$	256.4 ± 3.7	1175.0 ± 80.56

(c) Evaluation on noiseless, 1-layer students.

Table 7: Lunar Lander final deployment performance summary for three different kinds of students with confidence intervals obtained by training and evaluating the teachers with three different random seeds. Noiseless, 2-layer student (Top): The Narrow intervention helps exploration but results in policy performance plateau, the Wide one slows down student learning due to making exploration more challenging, and the Optimized teacher provides the best of both by switching between Narrow and Wide. Students that learn under the Optimized curriculum policy achieve a comparable performance to those training under No-intervention, but suffer three orders of magnitude fewer training failures. Noisy student (Center), One layered-student (Bottom): The results are similar when we use the curriculum optimized for students with noiseless observations and a 2-layer MLP policy for students with noisy sensors (center) or a 1-layer architecture (bottom), thus showing teaching policies can be transferred across classes of students. These results are consistent across random seeds, thus showing the robustness of CISR.

Teacher's training. We consider curriculum policies that allow for up to one intervention switch, i.e., K=1. Since the student's learning dynamics are quite noisy in this environment, we evaluate each curriculum policy for a class of 10 students in parallel and use the mean final performance of the students as a signal for GP-UCB. To initialize the GP model, we use 4 curriculum policies, one for each possible combination of interventions allowed by the policy class considered. To optimize the curriculum, we run GP-UCB for 10 iterations, where each iteration corresponds to training a class of 10 students in parallel with the curriculum policy proposed by GP-UCB.

Teacher's evaluation. The evaluation of teaching policies is analogous to the *Frozen Lake* case: we let each curriculum policy train 10 newly sampled students and we deploy them in the original environment for 200000 time steps to measure their performance. Since a much longer deployment time compared to *Frozen Lake* is required to obtain accurate estimates of the student's performance, we only record it at the end of the curriculum rather than after each interaction unit.

In these experiments, we also investigate the transferability of teaching policies, which is of great importance for many practically relevant scenarios. To this end, we apply the teaching policy optimized for students with perfect state information to students with noisy sensors. In particular, we consider students that observe $\tilde{x} = x + w_x$ and $\tilde{y} = y + w_y$, where $w_x \sim \mathcal{N}(0, 10^{-4})$ and $w_y \sim \mathcal{N}(0, 10^{-4})$. This level of noise is quite challenging as one standard deviation covers 2.5% of the width of the landing pad. In these experiments, the teacher uses the noiseless state information to rescue the student. This captures a scenario that is common in real-world applications where we have hardware that helps preserving safety during training, such as motion capture systems, that we cannot use during deployment. Since training in these conditions is harder and more prone to constraint violation, we let the training run for 20 interaction units rather than 15 and we allow for

higher penalty for constrain violation through the Lagrange multipliers by considering a higher upper bound on them (we set B=160 rather than B=120).

In a separate experiment, we apply the teaching policies optimized for the student's architecture presented in Appendix A to students that only have one hidden layer with 20 neurons rather than two.

For each of the curriculum policies considered and for each of the experiments described above, we report the mean returns, success rates and failure rates at the end of the curriculum over the 10 students as well as their mean number of failures during training in Table 7. Notice that the fact that the rates do not sum to 100% is due to timeouts. The confidence intervals in Tables 7a–7c are obtained by optimizing 3 curriculum policies independently with different random sees and repeating the evaluation procedure for each one.

C Proof

In this section, we provide proofs for Propositions 1 and 2.

Proposition 1 (*Eventual safety*). Let $\Pi_{\mathcal{M}}$ and $\Pi_{\mathcal{M}_i}$ be the sets of feasible policies for the problems in Equations (1) and (2), respectively. Then, if $\tau_i + \kappa_i \leq \kappa$, $\Pi_{\mathcal{M}_i} \subseteq \Pi_{\mathcal{M}}$.

Proof. The main idea of the proof is to show that the constraints in Equation (2), which are based on expectations with respect to ρ_i^π , are stricter than the constraint in Equation (1), which are based on expectations with respect to ρ^π . To this end, we need to distinguish between trajectories, or segments thereof, that have the same probability under ρ^π and ρ_i^π for any π from those that do not. Let us denote with $\xi = (s_0, s_1, \ldots, s_T)$, a generic trajectory in $\mathcal M$ and with Ξ the set of all possible trajectories in $\mathcal M$ (this is the set that the distributions ρ and ρ_i are defined over). Moreover, for a given set of trigger states $\mathcal D_i$, we indicate the set of trajectories where at least one state belongs to $\mathcal D_i$ with $\Xi_{\mathcal D_i} = \{\xi \in \Xi \mid \xi \cap \mathcal D_i \neq \emptyset\}$ and with $\Xi_{\mathcal D_i}^C = \Xi \setminus \Xi_{\mathcal D_i}$ its complement. With this notation, the constraint $\mathbb E_{\rho_i^\pi} \sum_{t=0}^T \mathbb I(s_t \in \mathcal D_i) \leq \tau_i$ is equivalent to $\sum_{\xi \in \Xi} \rho_i^\pi(\xi) |\xi \cap \mathcal D_i| \leq \tau_i$. Therefore, for a $\pi \in \Pi_{\mathcal M_i}$, we know that:

$$\sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}_i| + \sum_{\xi \in \Xi_{\mathcal{D}_i}^{\mathcal{D}}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}_i| \le \tau_i. \tag{4}$$

Since, by definition, we know that $|\xi \cap \mathcal{D}_i| = 0$ for all $\xi \in \Xi_{\mathcal{D}_i}^C$, (4) simplifies to

$$\sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}_i| \le \tau_i. \tag{5}$$

Every trajectory $\xi \in \Xi_{\mathcal{D}_i}$ can be divided in two segments: $\xi_1 = (s_0, s_1, \dots, s_m)$, which contains all the states up to the first one in the sequence that belongs to \mathcal{D}_i , i.e., $s_0, s_1, \dots, s_{m-1} \notin \mathcal{D}_i$ and $s_m \in D_i$, and $\xi_2 = (s_{m+1}, \dots, s_T)$, which contains the remaining part of the trajectory. Thus, we can say:

$$\tau_i \ge \sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}_i|,\tag{6}$$

$$= \sum_{(\xi_1, \xi_2) \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi_1, \xi_2) | (\xi_1, \xi_2) \cap \mathcal{D}_i |, \tag{7}$$

$$\geq \sum_{(\xi_1, \xi_2) \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi_1, \xi_2) |\xi_1 \cap \mathcal{D}_i|, \tag{8}$$

$$= \sum_{\xi_1 \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi_1) |\xi_1 \cap \mathcal{D}_i| \sum_{\xi_2 \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi_2 | \xi_1), \tag{9}$$

$$= \sum_{\xi_1 \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi_1) |\xi_1 \cap \mathcal{D}_i| \sum_{\xi_2 \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi_2 | \xi_1), \tag{10}$$

$$= \sum_{(\xi_1, \xi_2) \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi_1, \xi_2) |\xi_1 \cap \mathcal{D}_i|, \tag{11}$$

$$\geq \sum_{(\xi_1, \xi_2) \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi_1, \xi_2) | (\xi_1, \xi_2) \cap \mathcal{D} | = \sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi) | \xi \cap \mathcal{D} |.$$
 (12)

In the previous chain of inequalities, (10) holds because ρ and ρ_i are the same for the portion of the trajectory before the teacher intervenes for the first time, i.e., ξ_1 , and because $\sum_{\xi_2 \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi_2|\xi_1) = \sum_{\xi_2 \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi_2|\xi_1) = 1$. Furthermore, (11) holds because $|\xi_1 \cap \mathcal{D}_i| = 1$ by definition of ξ_1 and because $|(\xi_1, \xi_2) \cap \mathcal{D}| \leq 1$ since the states in \mathcal{D} are terminal.

Moreover, for a $\pi \in \Pi_{\mathcal{M}_i}$, we know that:

$$\sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}| + \sum_{\xi \in \Xi_{\mathcal{D}_i}^{\mathcal{D}}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}| \le \kappa_i.$$
(13)

Since the teacher does not modify the original dynamics unless the student enters a trigger state, we know that $\rho_i^{\pi} = \rho$ for every $\xi \in \Xi_{\mathcal{D}_i}^C$. Therefore, (13) becomes:

$$\sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}| + \sum_{\xi \in \Xi_{\mathcal{D}_i}^C} \rho^{\pi}(\xi) |\xi \cap \mathcal{D}| \le \kappa_i.$$
 (14)

By summing (12) and (14), we obtain:

$$\sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho^{\pi}(\xi) |\xi \cap \mathcal{D}| + \sum_{\xi \in \Xi_{\mathcal{D}_i}^{\mathcal{D}}} \rho^{\pi}(\xi) |\xi \cap \mathcal{D}| \le \tau_i + \kappa_i - \sum_{\xi \in \Xi_{\mathcal{D}_i}} \rho_i^{\pi}(\xi) |\xi \cap \mathcal{D}| \le \tau_i + \kappa_i, \quad (15)$$

which means
$$\mathbb{E}_{\rho^{\pi}} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}) \leq \tau_i + \kappa_i \leq \kappa$$
, which implies that $\pi \in \Pi_{\mathcal{M}}$.

Proposition 2 (*Learning safety*). Let \mathcal{D} be the set of unsafe states of CMDPs \mathcal{M} and \mathcal{M}_i , and let \mathcal{D}_i be the set of trigger states of intervention i. If $\mathcal{D} \subseteq \mathcal{D}_i$ and $\mathcal{P}(s'|a,s) = 0$ for every $s' \in \mathcal{D}$, $s \in \mathcal{S} \setminus \mathcal{D}_i$, and $a \in \mathcal{A}$, then an optimal student learning in CMDP \mathcal{M}_i will not violate any of \mathcal{M} 's constraints throughout learning.

Proof. During learning, the student may use any policy π from the set of all possible policies for the original environment \mathcal{M} , $\overline{\Pi}_{\mathcal{M}}$ (which includes unfeasible and, therefore, unsafe policies). However, during training, for any intervention i the student transitions according to the dynamics \mathcal{P}_i rather than \mathcal{P} . Therefore, we aim to show that $\mathbb{E}_{\rho_i^\pi} \sum_{t=0}^T \mathbb{I}(s_t \in D) \leq \kappa$ for all $\pi \in \overline{\Pi}_{\mathcal{M}}$. If $s_t \in \mathcal{S} \setminus \mathcal{D}$, we can either have (i) $s_t \in \mathcal{S} \setminus \mathcal{D}_i$ or (ii) $s_t \in \mathcal{D}_i \setminus \mathcal{D}$. Let us consider these two cases separately. In (i), we know that $\mathcal{P}(s_{t+1}|s_t,a_t)=0$ for any action a_t and any $s_{t+1} \in \mathcal{D}$ by assumption. Moreover, since $\mathcal{P}(s_{t+1}|s_t,a_t)=\mathcal{P}_i(s_{t+1}|s_t,a_t)$ for all $s_t \notin \mathcal{D}_i$, we have $\mathcal{P}_i(s_{t+1}|s_t,a_t)=0$ for all $s_t \in \mathcal{S} \setminus \mathcal{D}_i$ and $s_{t+1} \in \mathcal{D}$. In case (ii), we know that $\mathcal{P}_i(s_{t+1}|s_t,a_t)=0$ for all $s_{t+1} \in \mathcal{D}_i \supseteq \mathcal{D}$ and $s_t \in \mathcal{D}_i$ by definition of reset distribution. Therefore, we have shown that for all $s_t \in \mathcal{S} \setminus \mathcal{D}$, $s_t \in \mathcal{A}$ and $s_{t+1} \in \mathcal{D}$, we have $s_t \in \mathcal{S}_i \in \mathcal{S}_i \in \mathcal{S}_i$ and $s_t \in \mathcal{S}_i \in \mathcal{S}_i$ is if $s_t \in \mathcal{S}_i \in \mathcal{S}_i$, which corresponds to starting an episode in an unsafe state under the dynamics $s_t \in \mathcal{S}_i$ is if $s_t \in \mathcal{S}_i$, which corresponds to starting an episode in an unsafe terminal state, which only depends on the initial state distribution and not on the policy. If the initial state distribution is such that it is not possible for the student to start an episode in an unsafe state, then we have $s_t \in \mathcal{S}_i \in \mathcal{S}_i$ and $s_t \in \mathcal{S}_i$ is if $s_t \in \mathcal{S}_i$. Otherwise, we have assumed $s_t \in \mathcal{S}_i$ then we have $s_t \in \mathcal{S}_i$ is feasible. Therefore, it must be such that all the trajectories starting with $s_t \in \mathcal{S}_i$ can be tolerated.