國立中興大學

110 學年度

碩士班考試入學招生

試　題

學系：資訊科學與工程學系

乙組

科目名稱：基礎數學 B

國立中興大學110學年度碩士班招生考試試題

科目： 基礎數學B　　　　系所： 資訊科學與工程學系 乙組

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1. Which of the following is a subspace of R^2? (3%)

(A) $\left\{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in R^2 : u_1 u_2 = 0\right\}$

(B) $\left\{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in R^2 : 2u_1 - 5u_2 = 0\right\}$.

(C) $\left\{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in R^2 : u_1 > 0\right\}$.

(D) $\left\{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in R^2 : u_1^2 + u_2^2 \leq 1\right\}$.

2. Which of the following statements is <u>False</u>? (3%)

(A) If x is orthogonal to y and y is orthogonal to z, then x is orthogonal to z.

(B) For any matrix A, $(\text{Null}A)^{\perp} = \text{Row } A$.

(C) For any subspace W of R^n, the only vector in both W and $W^{\perp}$ is 0.

(D) If P is a matrix such that $P^T = P^{-1}$, then P is an orthogonal matrix.

3. Which of the following statements is <u>True</u> about linear transformation? (3%)

(A) If $T: R^2 \to R^3$ is linear, then its standard matrix has size 2×3.

(B) If T is a linear transformation, then $T(0) = 0$.

(C) If f is a function and $f(u) = f(v)$, then $u = v$.

(D) A function is onto if its range equals it domain.

4. Which of the following statements about symmetric matrix is <u>False</u>? (3%)

(A) Every real symmetric matrix is diagonalizable.

(B) If A is a symmetric matrix, then $A = A^T$.

(C) If A is symmetric, then distinct eigenvectors are orthogonal to each other.

(D) If A is an $n \times n$ matrix and A is diagonalizable, then A must have n distinct eigenvalues.

5. Which of the following statements about linear equation systems is <u>False</u>? (3%)

(A) The rank of a matrix equals to the number of pivot columns in the matrix.

(B) If the reduced echelon form of $[A|b]$ contains a zero row, then $Ax = b$ must have infinitely many solutions.

(C) If the equation $Ax = b$ is inconsistent, then the rank of $[A|b]$ is greater than the rank of A.

(D) If R is an $n \times n$ matrix in reduced row echelon form that has rank n, then $R = I_n$.

6. Consider the following linear equation systems, express these equations as the matrix form $A\mathbf{x}=\mathbf{b}$. Then find the solution of the vector $\mathbf{x}$. (5%)

$$\begin{cases} x_1 + 2x_2 + \ \ 3x_3 = 1 \\ x_1 + 3x_2 + \ \ 6x_3 = 3 \\ 2x_1 + 6x_2 + 13x_3 = 5 \end{cases} \quad \text{find } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

7. Find the basis of the vector space $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : v_1 - 2v_2 + 3v_3 = 0 \right\}$ (5%)

8. Given the following linear transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ x_2 + x_3 \\ x_1 + 3x_2 + 2x_3 \end{bmatrix}$.

   (a) Find the standard matrix **A** of this linear transformation T. (5%)

   (b) Please find the null space of the column space of **A**. (5%)

9. Let a matrix $\mathbf{A} = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ find $\mathbf{A}^{20}$. (5%). (Hint: Diagonalize **A** first.)

10. Given the following three data points (x_i, y_i) , i=1 to 3, find the least square error approximation line $\hat{y}_i = ax_i + b$ by projection matrix approach that fits them: (1,2), (3,4), (1,5).

   - Hint 1: For data points $(x_i, y_i)'s$, $\hat{\mathbf{y}} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \triangleq Cv.$
   - Hint2: The projection matrix $\boldsymbol{P}$ is defined as $\hat{\mathbf{y}} = \boldsymbol{P}\mathbf{y} = \boldsymbol{P} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

   (a) Find the parameter a, b for the least square error approximation line, where $\hat{y}_i = ax_i + b$ (5%)

   (b) Find the projection matrix $\boldsymbol{P}$ (5%)

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11. Define a sequence $s_0, s_1, s_2, \ldots$ as follows: $s_0 = 0$, $s_1 = 4$, $s_n = 6s_{k-1} - 5s_{k-2}$ for all integers $k \geq 2$.
    (a) What are the third and fourth terms of this sequence? **(4%)**
    (b) Prove if $s_n = 5^n - 1$? **(6%)**

12. Explain how to achieve the Kruskal's algorithm. Given a planar graph G, what is the output of G after performing the Kruskal's algorithm? **(10%)**

13. Prove that $(2n-1) + (2n-3) + \ldots + 3 = n^2-1$. **(5%)**

14. Let G be an undirected graph containing two subgraphs G_1 and G_2. λ is the number of colors for graph coloring. If $G = G_1 \cup G_2$ and $G_1 \cap G_2 = K_n$, where $n \in \mathbf{Z}^+$. Prove the polynomial function $P(G, \lambda)$ as follows: **(5%)**

$$P(G,\lambda) = \frac{P(G_1,\lambda)\cdot P(G_2,\lambda)}{\lambda^n}.$$

15. Simplify the expression $wx + \overline{\overline{x}z} + (y + \overline{z})$, where w, x, y, and z are Boolean variables. **(5%)**

16. Prove every subgroup of a cyclic group is cyclic. **(5%)**

17. Place the following sets {3,6,7,8}, {1, 3, 4, 7}, {2,3,4,7}, {1,3,5,6}, {4,6,7,8}, and {2,3,5,6} in the lexicographic order. **(5%)**

18. Prove both $b_n = 2^n$, and $b_n = n \cdot 2^n$ are the solutions for the second order recurrence relation $b_n = 4b_{n-1} - 4b_{n-2}$ for $n \geq 2$. **(5%)**