國立中興大學

110學年度

碩士班考試入學招生

試　題

學系：資訊科學與工程學系

甲組

科目名稱：基礎數學A

本科目不可以使用計算機　　　　本科目試題共 2 頁

PART I

一. 選擇題 (單選或多重選擇，每題二分)

1. Let A={a,b,c} B=P(A), the power set of A. Which of the following properties is correct?
   (a) $\phi \subseteq B$ (b) $\phi \in B$ (c) $A \in B$ (d) $A \subseteq B$ (note: ϕ is the empty set)
2. Let R be an equivalence relation on a set A. Both a and b are elements of A and are equivalent. [a] stands for the equivalence class of a. Which of the followings is correct?
   (a) aRb (b) bRa (c) [a] = [b] (d) $[a] \cap [b] \neq \phi$
3. Which of the following propositions is equivalent to "$p \to q$"?
   (a) $\neg q \to \neg p$ (b) $\neg p \vee \neg q$ (c) $\neg(q \wedge p)$ (d) $\neg p \vee q$
4. Which of the following numbers is a primitive root in Z_{13}?
   (a) 2 (b) 3 (c) 5 (d) 7
5. Which of the following numbers is prime?
   (a) $2^6 - 1$ (b) $2^7 - 1$ (c) $2^8 - 1$ (d) $2^9 - 1$
6. What is the value of the postfix expression "7 2 3 * - 4 ^ 9 3 / +", where ^ stands for exponentiation?
   (a) 3 (b) 4 (c) 8 (d) 12
7. Let T be a full m-ary tree, which a node has either 0 or m child nodes, with n vertices. If m=3 and n=100, which of the following statements about this tree is correct?
   (a) there are 98 edges (b) there are 33 internal vertices (c) the height of T is 4 (d) there are 66 leaf nodes
8. Which of following sets is countable?
   (a) (0,1) (b) Z^+ (c) {N,Z,Q} (d) P(N), the power set of natural numbers
9. Let P(n) be a propositional function. Which of the following statements is enough to verify P(n) is true for all positive integers n?
   (a) P(k) is true for large k (b) $P(1) \wedge [\forall k>1\ (P(k) \to P(k+1))]$ is true (c) $P(1) \wedge [\forall k>1\ (P(1) \wedge \ldots \wedge P(k) \to P(k+1))]$ is true (d) $P(1) \wedge P(n)$ is true for some n >1
10. Which of the following graphs is planar?
    (a) complete, K_4 (b) 3-cube, Q_3 (c) complete bipartite, $K_{3,3}$ (d) wheel, W_4

二. 是非題 (每題二分，答錯倒扣一分)

T 1. The incidence matrix for representation of any simple graph is a symmetric matrix.
F 2. The cardinality of Q is the same as the cardinality of Z.
F 3. Among 100 people there are at least 9 who were born in the same month.
T 4. $(P(S), \subseteq)$ is a partially ordered set, where P(S) is a power set of S={1,2,4}.
T 5. "$\neg p \to q$" is logically equivalent to "$\neg(q \leftrightarrow p)$", where $\neg$ stands for "not".
T 6. There are 81 ways to put 4 distinguishable balls into 3 different boxes.
T 7. Traveling salesman problem is the problem to find a Euler circuit of least cost.
T 8.? There are 1250 positive integers less than 100000 having the sum of their digits equal to 12.
F 9. The least number of colors needed for coloring a planar graph is no longer than 5.
T 10. A simple weighted graph is connected if and only if it has a minimum spanning tree.

三. 計算題 (每題五分)

1. Over the set of {1,2,3,4,5,6}, what is the next permutation in lexicographic order after 326541?
2. How many one-to-one functions are there from a set with 5 elements to a set with 7 elements?

$\frac{7!}{5!2!}$

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PART II

一、是非題（每題二分，答錯倒扣一分）

F 1. A square matrix A is called skew-symmetric if $A^t = -A$. ~~If~~ B is a square matrix, then $B - B^t$ is skew-symmetric.

F 2. If $a, b, c, \cdots, i \in R$, then $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \neq \det\begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix}$.

$\begin{bmatrix} -1 \\ 0 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$

T 3. $p_1 = 6 - x^2$ and $p_2 = 1 + x + 4x^2$ are linear independent in P_2.

~~T~~ F 4. The set of vectors $(1,6,4)$, $(2,4,-1)$ *and* $(-1,2,5)$ is a basis for a vector space R^3.

T 5. Let $T: R^2 \to R^2$ be the rotation of R^2 through the angle $\pi/4$. T has rank=2 and nullity=1.

~~F~~ T 6. If A and B are similar matrices, then $\det(A) = det(B)$.

T 7. A square matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable.

1 2 -1
0 -8 8
0 1 1
vectorR3

F 8. If A, B and C are three square matrices, then $\mathrm{Trace}(ABC) = Trace(ACB)$.

T 9. If A is an $n \times n$ matrix, then the sum of the multiplicities of the eigenvalues of A equals n.

T 10. The matrix $A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & 4 \\ -3 & 4 & 9 \end{bmatrix}$ is not positive definite.

二、選擇題（單選或多重選擇，每題四分）

1. Which of the following are elementary matrices?

(a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

2. Which of the following are linear combinations of $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -2 \\ 0 & -2 \end{bmatrix}$?

(a) $\begin{bmatrix} 6 & 3 \\ 0 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 7 \\ 5 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -1 \\ -8 & -8 \end{bmatrix}$

3. Which of the following are Hermitian matrices?

(a) $\begin{bmatrix} 1 & 1+i \\ 1-i & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & i \\ i & 2 \end{bmatrix}$ (c) $\begin{bmatrix} i & i \\ -i & i \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. Which of the following are the singular values of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$?

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}$

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\boxed{\sqrt{6}}$

5. Suppose that $T: R^2 \to R^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) = ?$

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 2 \\ 1 & 2 & 5-\lambda \end{vmatrix}$

$(1-\lambda)^2(5-\lambda) \ldots = 0$

$-\lambda^3 + 7\lambda^2 - 11x + 5 \Rightarrow$

$\Rightarrow$ ~~$\lambda \ldots + 5$~~ $\sqrt{1}$ $\sqrt{5}$.

三、計算題（十分）

1. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$, express A in the form $A = LDU$, where L is lower triangular with 1's along the main diagonal, U is upper triangular, and D is a diagonal matrix.

L = [(1, 0, 0), (-1, 1, 0), (1, -1, 1)]
D = [(2, 0, 0), (0, -1, 0), (0, 0, 1)]
U = [(2, -1, 1), (0, -1, 2), (0, 0, 1)]