國立中正大學
111 學年度碩士班招生考試
試 題

[第 1 節]

科目名稱	數學
系所組別	資訊工程學系-甲組、乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1.
https://www.quora.com/Let-u-2-3-1-and-a-1-1-2-How-would-I-find-the-vector-component-of-u-orthogonal-to-a

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1. (10%) Let **u** = (2, 0, 1) and **a** = (1, 2, 3). Find the vector component of **u** along **a** and the vector component of **u** orthogonal to **a**.

2. (10%) Find a vector that is orthogonal to both **u** = (0, 2, -2) and **v** = (1, 3, 0).

3. (10%) Find the coordinate vector of $v = (3, 4, 3)$ relative to the basis $S = \{(3,2,1), (-2,1,0), (5,0,0)\}$.

4. (10%) Find A^{13}, where

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

5. (10%) Let the vector space P_2 have the inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)dx$$

Apply the Gram-Schmidt process to transform the standard basis $\{1, x, x^2\}$ for P_2 into an orthogonal basis $\{\varphi_1(x), \varphi_2(x), \varphi_3(x)\}$.

6. (10%) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
   a) a proof by contraposition. (5%)
   b) a proof by contradiction. (5%)

7. (10%) Prove whether the given pair of graphs G_1 and G_2 represented by incidence matrices is isomorphic or not.

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

8. (10%) Show that the set of positive rational numbers is countable.

9. (10%) Consider the recurrence relation $a_n = 2a_{n-1} + 3n$.

(a) (2%) Write the associated homogeneous recurrence relation. $a_n = 2a_{n-1}$

(b) (2%) Find the general solution to the associated homogeneous recurrence relation. $a_n = C \times 2^n$

(c) (2%) Find a particular solution to the given recurrence relation. $A_n = \frac{3n}{2}$

(d) (2%) Write the general solution to the given recurrence relation. $h_n = C \times 2^n + \frac{3n}{2}$

(e) (2%) Find the particular solution to the given recurrence relation when $a_0 = 1$. $a_n = \frac{3n}{2} + 1$ ※

10. (10%)

a) (5%) Draw all non-isomorphic **trees** with 5 vertices.

b) (5%) Draw all non-isomorphic **rooted trees** with 4 vertices.

(a)

(b)