國立中興大學

109 學年度

碩士班考試入學招生

試　　題

學系：資訊科學與工程學系

乙組

科目名稱：基礎數學 B

Part I: Discrete Mathematics

1. (20%) Find the number of ways to give 5 identical red pens and 4 identical blue pens to 3 children under the following conditions. (**Hint**: You can assume 3 children to be A, B, C, then discuss how to allocate red pens and blue pens to them respectively.)
   (a) (10%) Any combination of pens for the three children.
   (b) (10%) If one specific child needs to be assigned at least one pen (either red or blue).

2. (10%) Find the number of ways to divide 10 students into 4 groups [G1, G2, G3, G4] so that two of the four groups contain 3 students and the other two groups contain 2 students.

3. (10%) Consider the congruence equation $7x \equiv 1 (\mathrm{mod}\ 9)$, find the general solution of x

4. (10%) Solve the following Boolean algebra.
   (a) (5%) Express $E(x, y, z) = x(xy + y' + x'y)$ as its complete sum-of-products form (Hint: needs to consider z terms)
   (b) (5%) Find the Boolean expression $E = E(x, y, z)$ corresponding to the truth tablet $T(E) = 01001001$

Part II Linear Algebra

1. Define a sequence of numbers in the following way: $G_0 = 0$, $G_1 = 1/2$. and $\forall k \geq 0$, $G_{k+2} = (G_{k+1} + G_k)/2$.

   (a) (4 %) What are the values of G_4 and G_5?

   (b) (5 %) Give a matrix A such that
   $$A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix} = \begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix}.$$

   (c) (5 %) Find an explicit formula for G_k.

   (d) (6 %) What is the limit of G_k as $k \to \infty$?

2. Suppose A is 3 by 4, and $Ax = 0$ has exactly 2 special solutions:
   $$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \text{ and } x_2 = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

   (a) (6 %) Find the reduced row echelon form of$A$.

   (b) (3 %) What are dimensions of Col(A), Row(A), and Null(A)?
   (6 %) Justify your answers.

3. Let H be a plane and y be a given point in R^3

   (a) Let p be the orthogonal projection of y onto the plane H. Show that p is the point in H that is closest to y. (6 %)

   (b) Let $H = \{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} | x_1 + 2x_2 + x_3 = 4 \}$ and $y = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$. Find p. (4 %)

   (c) (5 %)
   Find the distance between y and p.