國立中興大學

108 學年度

碩士班考試入學招生

試　　題

學系：資訊科學與工程學系

乙組

科目名稱：基礎數學 B

國立中興大學108學年度碩士班招生考試試題

科目：基礎數學B　　系所：資訊科學與工程學系乙組

本科目不得使用計算機　　本科目試題共2頁

Discrete Mathematics

1. (10%) Find all solutions to the system of congruences $x \equiv 1(\text{mod}2)$, $x \equiv 2(\text{mod}3)$, $x \equiv 3(\text{mod}5)$, and $x \equiv 4(\text{mod}11)$.

2. (10%) How many bit strings of length 10 contain at least four 1s?

3. (10%) Solve the following recurrence relation together with the initial conditions given.

$a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

4. (10%) Which of these relations on the set $\{1, 2, 3, 4\}$ are antisymmetric and transitive?

(a) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$
(b) $\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$
(c) $\{(2,4), (4,2)\}$
(d) $\{(1,2), (2,3), (3,4)\}$
(e) $\{(1,1), (2,2), (3,3), (4,4)\}$

5. (10%) Use a K-map to simplify the following sum-of-products expansion.

$wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}\bar{z}$

Part II Linear Algebra

1. Let A be a 4×3 matrix of rank 3.
   (a) Give a simple example of such a matrix A. Justify your answer. (5 %)
   (b) What is the dimension of the null space of A? (3 %)
   (c) What is the dimension of the column space of A? (3 %)
2. Find the point Q on the line $y = x/3$ that is closest to the point $(1, 4)$. (6 %)
3. Let $\{u_1, u_2, u_3\}$ be an orthonormal basis for an inner product space. If $x = c_1u_1 + c_2u_2 + c_3u_3$ is a vector with properties $||x|| = 5$, $\langle x, u_1\rangle = 4$ and $x \perp u_2$, then what are the possible values of c_1, c_2, c_3? (9 %)
4. Suppose A is a 3×3 matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$. Suppose also that $A - I$ has rank one.
   (a) Which eigenvalue of A is repeated? (3 %) Justify your answer. (5 %)
   (b) Give a simple matrix B which is symmetric and similar to A. Justify your answer. (6 %)
5. Consider the vector space of polynomials of the form $p(x) = ax^3 + bx^2 + cx + d$. Are the following subspaces? Explain briefly.
   (a) Those $p(x)$ for which $p(1) = 0$. (3 %)
   (b) Those $p(x)$ for which $p(0) = 1$. (3 %)
   (c) Those $p(x)$ for which $a + b = c + d$. (4 %)