

國立中正大學

111 學年度碩士班招生考試

試題

[第 1 節]

科目名稱	數學
系所組別	資訊工程學系-甲組、乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1.

<https://www.quora.com/Let-u-2-3-1-and-a-1-1-2-How-would-I-find-the-vector-component-of-u-orthogonal-to-a>

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1. (10%) Let $\mathbf{u} = (2, 0, 1)$ and $\mathbf{a} = (1, 2, 3)$. Find the vector component of \mathbf{u} along \mathbf{a} and the vector component of \mathbf{u} orthogonal to \mathbf{a} .

$$\frac{\mathbf{u} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \mathbf{a} = \frac{5}{14} \mathbf{a} \Rightarrow \mathbf{u} - \mathbf{w}_1 = \frac{23}{14}, \frac{10}{14}, \frac{1}{14}$$

2. (10%) Find a vector that is orthogonal to both $\mathbf{u} = (0, 2, -2)$ and $\mathbf{v} = (1, 3, 0)$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -2 \\ 1 & 3 & 0 \end{vmatrix} = 6\mathbf{i} - 2\mathbf{j} - 2\mathbf{k} \Rightarrow \text{Ans} = \langle 6, -2, -2 \rangle$$

3. (10%) Find the coordinate vector of $\mathbf{x} = (3, 4, 3)$ relative to the basis $S = \{(3, 2, 1), (-2, 1, 0), (5, 0, 0)\}$.

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -2 & -2 \end{bmatrix}$$

4. (10%) Find A^{13} , where

$$A^2 = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 4 & 3 \\ 3 & 0 & 1 \end{bmatrix}, A^4 = \begin{bmatrix} -14 & 0 & -7 \\ 15 & 16 & 15 \\ -3 & 0 & -11 \end{bmatrix}, A^5 = \begin{bmatrix} -20 & 0 & -6 \\ 21 & 22 & 21 \\ 12 & 0 & 27 \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 2^{n-2} & 0 & 2^{n-1}-2 \\ 2^{n-1} & 2^{n-1} & 2^{n-1}-1 \\ -1 & 0 & 2^{n-1}-2 \end{bmatrix}$$

$$\begin{bmatrix} -8190 & 0 & -16382 \\ 8191 & 8192 & 8191 \\ 8191 & 0 & 16383 \end{bmatrix}$$

5. (10%) Let the vector space P_2 have the inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^1 p(x)q(x)dx$$

Apply the Gram-Schmidt process to transform the standard basis $\{1, x, x^2\}$ for P_2 into an orthogonal basis $\{\varphi_1(x), \varphi_2(x), \varphi_3(x)\}$.

$$\left\{ \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}x, \frac{\sqrt{5}}{2\sqrt{2}}x(3x^2-1) \right\}$$

6. (10%) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

a) a proof by contraposition. (5%)

b) a proof by contradiction. (5%)

(10%) Prove whether the given pair of graphs G_1 and G_2 represented by incidence matrices is isomorphic or not.

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, G_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It is isomorphic

8. (10%) Show that the set of positive rational numbers is countable.

$$\begin{array}{cccc} 1/1 & 1/2 & 1/3 & \dots \\ 1/2 & 2/2 & 2/3 & \dots \\ 1/3 & 2/3 & 3/3 & \dots \end{array}$$

 \Rightarrow thus

is countable

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9. (10%) Consider the recurrence relation $a_n = 2a_{n-1} + 3n$.

(a) (2%) Write the associated homogeneous recurrence relation.

$$a_n = 2a_{n-1}$$

(b) (2%) Find the general solution to the associated homogeneous recurrence relation.

$$a_n = C \cdot 2^n$$

(c) (2%) Find a particular solution to the given recurrence relation.

$$A_n = \frac{3n}{2}$$

(d) (2%) Write the general solution to the given recurrence relation.

$$h_n = C \cdot 2^n + \frac{3n}{2}$$

(e) (2%) Find the particular solution to the given recurrence relation when $a_0 = 1$.

$$a_n = \frac{3n}{2} + 1$$

10. (10%)

a) (5%) Draw all non-isomorphic **trees** with 5 vertices.

b) (5%) Draw all non-isomorphic **rooted trees** with 4 vertices.

