國立中正大學
108學年度碩士班招生考試
試題

[第1節]

系所組別	資訊工程學系－甲組 乙組
科目名稱	數學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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→ 100, 010, 001

1. Let S be the standard basis for R^3, and let $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be another basis for R^3 in which $\mathbf{u}_1 = (1, 2, 1)^T$, $\mathbf{u}_2 = (2, 5, 0)^T$, and $\mathbf{u}_3 = (3, 3, 8)^T$.
   a) (5%) Find the transition matrix from B to S.
   b) (5%) Find the transition matrix from S to B.
   c) (5%) Let $(5, -3, 1)^T$ be the coordinate vector of $\mathbf{w}$ relative to S. Find the coordinate vector of $\mathbf{w}$ relative to B.

2. Let $\mathbf{u} = (1, 1, 1)$ and $\mathbf{a} = (0, 2, -1)$.
   a) (5%) Find the vector component of $\mathbf{u}$ along $\mathbf{a}$.
   b) (5%) Find the vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$.

3. (8%) Find the least square line $y = ax + b$ that fits the three data point $(-2, -11)$, $(0, -2)$, $(4, 2)$.

   thus $y = 2x - 5$

4. (7%) Find a 3×3 symmetric matrix whose eigenvalues are $\lambda_1 = 4$, $\lambda_2 = 2$, $\lambda_3 = 0$ and for which the corresponding eigenvectors are $\mathbf{v}_1 = (1, 1, 0)^T$, $\mathbf{v}_2 = (0, 0, 1)^T$, $\mathbf{v}_3 = (-1, 1, 0)^T$.

   GPT⇒ $f(\lambda) = \lambda^3 - tr(A)\lambda^2 + det(A)\lambda - det(A^T)$

5. Let $W \subset R^3$ be the subspace spanned by $\mathbf{w} = (1, 1, 1)^T$. Let $W^\perp$ be the orthogonal complement of W. Let $\mathbf{v} = (1, 0, 1)^T$.
   a) (6%) Find an orthonormal basis of $W^\perp$.
   b) (4%) Find the projection of $\mathbf{v}$ to $W^\perp$.

6. (7%) Determine whether these statements are true or false.
   a) $\emptyset \in \{\emptyset\}$、 b) $\emptyset \subseteq \{\{\emptyset\}\}$、 c) $\{\emptyset\} \in \{\emptyset, \{\{\emptyset\}\}\}$、 d) $\{\emptyset\} \subseteq \{\{\emptyset\}\}$、
   e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$、 f) $\{\{\emptyset\},\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$、 g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

7. (8%) A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says "At least one of you has a muddy forehead," and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?" The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously.

8. (10%) Suppose that two people play a game taking turns removing, 1, 2, 3 or 4 stones at a time from a pile that begins with 22 stones. The person who removes the last stone wins the game. Show that the first player can win the game no matter what the second player does.

9. (10%) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23} to guarantee that at least three pairs of these numbers add up to 24?

10. (10%) Find the solution to the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n > 1$ with initial conditions $a_0 = 4$, $a_1 = 1$.

11. (5%) The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. If G is a simple graph with 27 edges and $\overline{G}$ has 28 edges, how many vertices does G have?