

國立中正大學

110 學年度碩士班招生考試

試題

[第 2 節]

科目名稱	數學
系所組別	資訊工程學系 甲組 - 乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學 110 學年度碩士班招生考試試題

科目名稱：數學

本科目共 2 頁 第 1 頁

系所組別：資訊工程學系-甲組、乙組

1. (12 points) For the matrix A and its reduced row echelon form are given below:

$$A = \begin{bmatrix} 5 & 15 & 5 & 0 & 4 \\ 4 & 12 & 4 & 5 & -3 \\ -2 & -6 & -2 & 0 & -2 \\ -2 & -6 & -2 & 1 & -5 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(1) $\begin{bmatrix} -3 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \end{bmatrix}$

Answer the following questions:

- (a) (3 points) Find a basis for the null space of A . (1) $\begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (1) $\begin{bmatrix} 5 & 4 & -2 & -2 \\ 0 & 5 & 0 & 1 \\ 4 & -3 & -2 & -5 \end{bmatrix}$
- (b) (3 points) Find a basis for the row space of A .
- (c) (3 points) Find a basis for the column space of A .
- (d) (3 points) Find the rank and the nullity of A . (2) rank: 3, nullity: 1

2. (8 points) The following vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span a subspace V of R^3 , but not a basis for V . Answer the following questions.

- (a) (4 points) Choose a subset of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ which forms a basis for V . $\{V_1, V_4\}$
- (b) (4 points) Extend this basis to a basis for R^3 .

(1) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \right\}$

3. (10 points) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$ and let P be the plane through the origin spanned by \vec{v}_1 and \vec{v}_2 .

- (a) (5 points) Find an orthonormal basis of P . (1) $\begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 1/\sqrt{14} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$ (1) $\frac{1}{3}(2, 1, -1)$
- (b) (5 points) Find the point on P which is closest to the point $(1, 0, 0)$.

4. (10 points) Let \vec{v}_1 and \vec{v}_2 denote the following vectors in R^3 .

(1) $\begin{bmatrix} 1 & 1 & 1 \\ 2/3 & -1/3 & -2/3 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \Rightarrow \begin{bmatrix} \sqrt{2} & 1 & -4\sqrt{2} \\ 6 & -6 & 6 \end{bmatrix} \Rightarrow \frac{\sqrt{2}}{6} \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$

$$\vec{v}_1 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix}$$

- (a) (3 points) Find a vector \vec{v}_3 so that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ form an orthonormal basis B of R^3 . How many choices are there for the answer?
- (b) (3 points) Let $T: R^3 \rightarrow R^3$ denote the linear transformation that interchanges \vec{v}_1 and \vec{v}_3 and has \vec{v}_2 as an eigenvector with eigenvalue -5. Write down $[T]_B$, the matrix of T with respect to the basis B .
- (c) (4 points) Write down a product of matrices that equals the standard matrix of T .

5. (10 points) Briefly explain each of the following matrix factorization methods. You also need to specify the existing constraints for each matrix factorization.

(a) (5 points) QR decomposition https://en.wikipedia.org/wiki/QR_decomposition

(b) (5 points) Singular value decomposition https://en.wikipedia.org/wiki/Singular_value_decomposition

國立中正大學 110 學年度碩士班招生考試試題

科目名稱：數學

本科目共 2 頁 第 2 頁

系所組別：資訊工程學系-甲組、乙組

6. (10 points) Determine the truth value of each of these statements if the universe of discourse of each variable consists of all real numbers.

- (a) (2 points) $\forall x \exists y (x^2 = y)$
- (b) (2 points) $\forall x \exists y (x = y^2)$
- (c) (2 points) $\forall x (x^2 \neq x)$
- (d) (2 points) $\forall x (|x| > 0)$
- (e) (2 points) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

7. (10 points) If a and b are integers and m is a positive integer, then a is congruent to b modulo m if m divides $a - b$. We use the notation $a \equiv b \pmod{m}$ to indicate that a is congruent to b modulo m .

- (a) (5 points) Find an inverse of 72 modulo 233.
- (b) (5 points) Solve the congruence $72x \equiv 6 \pmod{233}$

8. (10 points) How many numbers must be selected from the first 10 positive integers to guarantee that at least three pairs of these numbers add up to 11?

9. (10 points) A string that contains only 0s and 1s is called a binary string.

- (a) (5 points) Find a recurrence relation for the number of binary strings of length n that do not contain two consecutive 0s.
- (b) (2 points) What are the initial conditions?
- (c) (3 points) How many binary strings of length 7 do not contain two consecutive 0s?

10. (10 points) The complementary graph \overline{G} of a simple graph G has the same vertices as G . Two vertices are adjacent in \overline{G} if and only if they are not adjacent in G .

- (a) (5 points) If G is a simple graph with 20 edges and \overline{G} has 16 edges, how many vertices does G have?
- (b) (5 points) If the simple graph G has x vertices and y edges, how many edges does \overline{G} have?