國立中正大學
110學年度碩士班招生考試
試題
[第2節]

科目名稱	數學
系所組別	資訊工程學系 甲組 － 乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

科目名稱：數學　　　　　　　　　　　　　　　　　　本科目共 2 頁　第 1 頁

系所組別：資訊工程學系-甲組、乙組

1. (12 points) For the matrix A and its reduced row echelon form are given below:

$$A = \begin{bmatrix} 5 & 15 & 5 & 0 & 4 \\ 4 & 12 & 4 & 5 & -3 \\ -2 & -6 & -2 & 0 & -2 \\ -2 & -6 & -2 & 1 & -5 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer the following questions:

(a) (3 points) Find a basis for the null space of A. (a) t[0,-1,1,0,0]

(b) (3 points) Find a basis for the row space of A. (b) [l,3,1,0,0] [0,0,0,1,0] [0,0,0,0,1]

(c) (3 points) Find a basis for the column space of A. (c) [1 0 0 0] [0 1 0 0] [0 0 0 1]

(d) (3 points) Find the rank and the nullity of A. (d) rank: 3, nullity: 1

2. (8 points) The following vectors

$$\overrightarrow{v_1} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \overrightarrow{v_2} = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \overrightarrow{v_3} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \overrightarrow{v_4} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span a subspace V *of* R^3, but not a basis for V. Answer the following questions.

(a) (4 points) Choose a subset of $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}, \overrightarrow{v_4}\}$ which forms a basis for V. $\{V_1, V_4\}$

(b) (4 points) Extend this basis to a basis for R^3.

(b) $\left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right\}$

3. (10 points)Let $\overrightarrow{v_1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\overrightarrow{v_2} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$ and let P be the plane through the origin spanned by $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$.

(a) (5 points) Find an orthonormal basis of P.

(b) (5 points) Find the point on P which is closest to the point (1,0,0).

(a) $\overline{\sqrt{85}}$ [-4,5,6] (b)

4. (10 points) Let $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ denote the following vectors in R^3.

(a) $\begin{bmatrix} i & j & k \\ 2/3 & -1/3 & -2/3 \\ -\sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix} \Rightarrow \left(\frac{\sqrt{2}}{6} i - \frac{4\sqrt{2}}{6} j + \frac{\sqrt{2}}{6}\right) \Rightarrow \frac{\sqrt{2}}{6}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ X

$$\overrightarrow{v_1} = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix} \qquad \overrightarrow{v_2} = \begin{bmatrix} -\sqrt{2}/2 \\ 0 \\ -\sqrt{2}/2 \end{bmatrix}$$

$[T]_B = \begin{bmatrix} 0, 1, 0 \\ 1, 0, 0 \\ 0, 0, -5 \end{bmatrix}$

(a) (3 points) Find a vector $\overrightarrow{v_3}$ so that $\overrightarrow{v_1}$, $\overrightarrow{v_2}$, $\overrightarrow{v_3}$ form an orthonormal basis B of R^3. How many choices are there for the answer?

(b) (3 points) Let T: $R^3 \to R^3$ denote the linear transformation that interchanges $\overrightarrow{v_1}$ and $\overrightarrow{v_3}$ and has $\overrightarrow{v_2}$ as an eigenvector with eigenvalue -5. Write down $[T]_B$, the matrix of T with respect to the basis B.

(c) (4 points) Write down a product of matrices that equals the standard matrix of T. (c) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

5.(10 points) Briefly explain each of the following matrix factorization methods. You also need to specify the existing constraints for each matrix factorization.

(a) (5 points) QR decomposition https://en.wikipedia.org/wiki/QR_decomposition

(b) (5 points) Singular value decomposition https://en.wikipedia.org/wiki/Singular_value_decomposition

6. (10 points) Determine the truth value of each of these statements if the universe of discourse of each variable consists of all real numbers.

(a) (2 points) $\forall x \exists y (x^2 = y)$

(b) (2 points) $\forall x \exists y (x = y^2)$

(c) (2 points) $\forall x (x^2 \neq x)$

(d) (2 points) $\forall x (|x| > 0)$

(e) (2 points) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

7. (10 points) If a and b are integers and m is a positive integer, then a is congruent to b modulo m if m divides $a - b$. We use the notation $a \equiv b \pmod{m}$ to indicate that a is congruent to b modulo m.

(a) (5 points) Find an inverse of 72 modulo 233.

(b) (5 points) Solve the congruence $72\,x \equiv 6 \pmod{233}$

8. (10 points) How many numbers must be selected from the first 10 positive integers to guarantee that at least three pairs of these numbers add up to 11?

9. (10 points) A string that contains only 0s and 1s is called a binary string.

(a) (5 points) Find a recurrence relation for the number of binary strings of length n that do not contain two consecutive 0s.

(b) (2 points) What are the initial conditions?

(c) (3 points) How many binary strings of length 7 do not contain two consecutive 0s?

10. (10 points) The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

(a) (5 points) If G is a simple graph with 20 edges and $\overline{G}$ has 16 edges, how many vertices does G have?

(b) (5 points) If the simple graph G has x vertices and y edges, how many edges does $\overline{G}$ have?