Parameter estimation of age-structured model for SARS-CoV-2 in Seoul and Gyeonggi

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Data

1. Daily confirmed cases in Seoul and Gyeonggi
2. Vaccine
   - Daily number of vaccination for 1st dose (by age)
   - Daily number of vaccination for 2nd dose (by age)
   - Vaccine efficacy
3. Proportion of δ variant

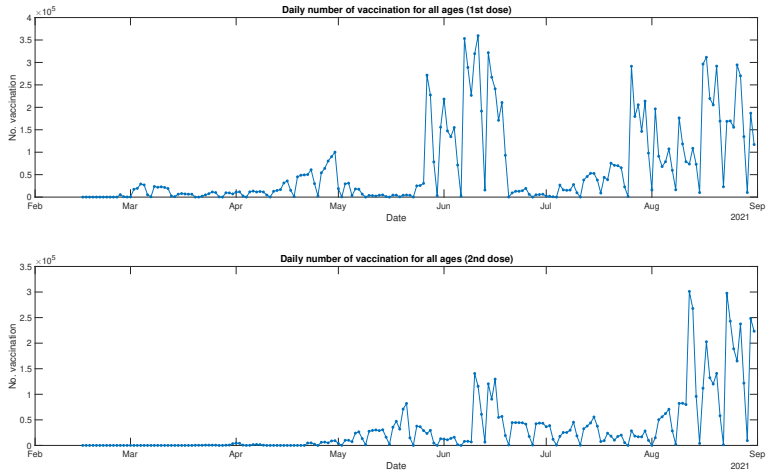**1. Daily number of vaccination for 1st dose (all ages)**

Figure 1: The daily number vaccination for 1st dose and 2nd dose from 2021/02/15 to 2021/09/01

Data processing

1. Daily number of vaccination for 1st dose (by age)

- ▶ The daily number of vaccination by age is generated by the ratio between ages of vaccinated people.
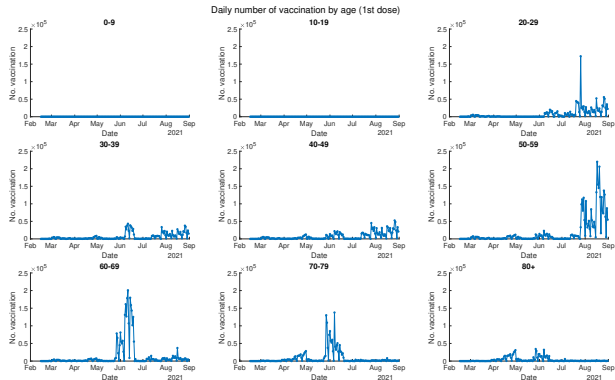- ▶ The ratio is based on KDCA reports.

Figure 2: The daily number vaccination for 1st dose by age from 2021/02/15 to 2021/09/01

2. Daily number of vaccination for 2nd dose (by age)

- ▶ The daily number of vaccination by age is generated by the ratio between ages of vaccinated people.
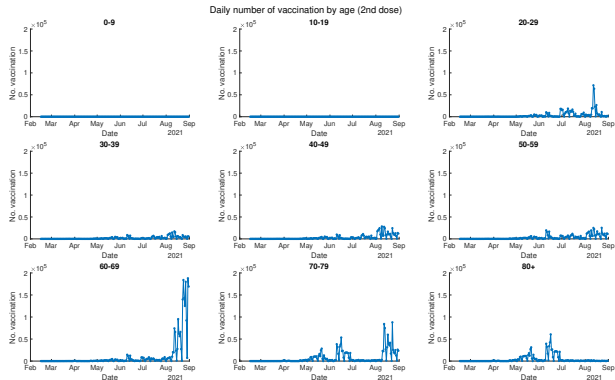- ▶ The ratio is based on KDCA reports.

Figure 3: The daily number vaccination for 2nd dose by age from 2021/02/15 to 2021/09/01

3. Vaccine efficacy

▶ The vaccine efficacies for α variant and δ variant are different.[1]

▶ We use weighted sum of vaccine efficacies where weights are based on proportion of δ variant

		Astrazeneca	Pfizer
α variant	1st dose	48.7%	47.5%
	2nd dose	74.5%	93.7%
δ variant	1st dose	30.0%	35.6%
	2nd dose	67%	88%

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[1] Jamie Lopez Bernal et al. (2021). "Effectiveness of Covid-19 vaccines against the B. 1.617. 2 (Delta) variant". In: *New England Journal of Medicine*
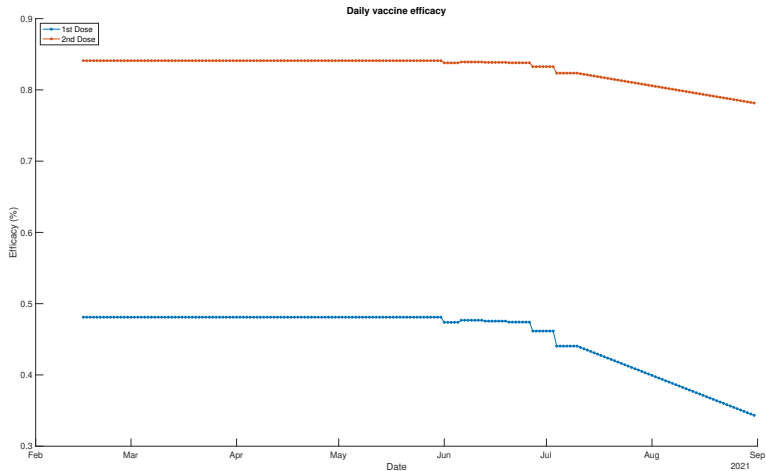
3. Vaccine efficacy

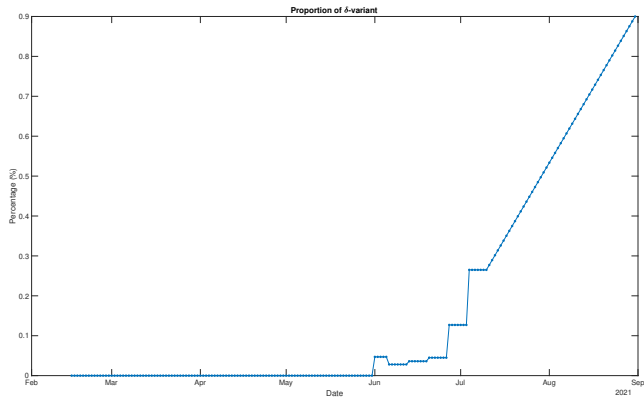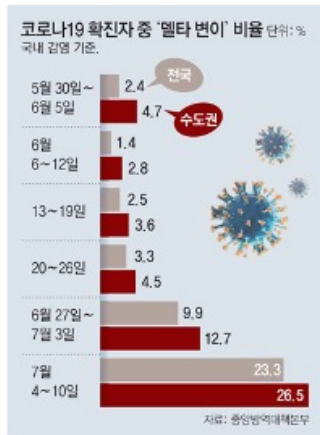Figure 4: The estimated daily vaccine efficacy for 1st dose and 2nd dose.

4. Proportion of δ variant

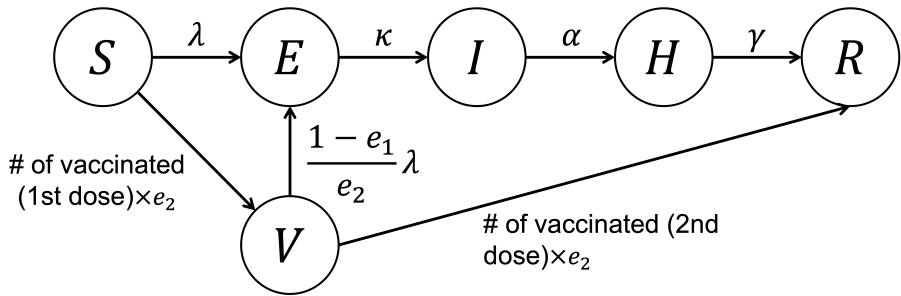

Figure 5: Estimates of proportion of δ variant.

Model

Figure 6: Diagram of age-structured model for SARS-CoV-2.

Model

Notation	Interpretation
S	Susceptibles
E	Exposed
I	Infectious
H	Hospitalized
R	Removed (or recovered)
V	Vaccinated (between 1st dose and 2nd dose)
λ	Force of infection
κ	Latent period
α	Infectious period
γ	Hospitalization period
e_1	Vaccine efficacy for 1st dose
e_2	Vaccine efficacy for 2nd dose

Table 1: Definition of states and parameters.

Social distancing

Social distance level

▶ 0.5단계 감소: transmission rate 전단계 대비 83.22% 증가

▶ 0.5단계 증가: transmission rate 전단계 대비 30% 감소

▶ 1단계 증가: transmission rate 전단계 대비 65% 감소

Date	Social distancing level	Change of transmission rate
2021/02/15-2021/06/30	2	
2021/07/01-2021/07/11	1.5	$\times 1.8322$
2021/07/12-2021/09/01	4 (assumed as 3 or 2.5 or 2)	$\times 0.699 \times 0.35$, $\times 0.35$, $\times 0.699$

Table 2: The change of transmission rate according to the social distancing level from 2021/02/15 to 2021/09/01.

Definition of λ

Motivation

▶ In general, $\lambda(t)$ is defined by $W \times I(t)$ where W is the WAIFW matrix, and $I(t)$ is the number of infectious at time t.

▶ To reflect the non-pharmaceutical intervention, we consider time-dependent $W(t)$.

Let $p(t)$ and $SD(t)$ be the proportion of δ variant and proportionate of the corresponding social distancing level at time t.

1. $W(t) = ((1 - p(t) + p(t)\delta) \times \beta \times SD(t) \times C$

Experiment

Social distancing (2021/07/01-2021/07/11)

- 1.8322
- $1.4161 (:= 1 + 0.8322/2)$

Social distancing (2021/07/12-)

- 0.699×0.35
- 0.699
- 0.35
- $0.5245 (:= (0.699 + 0.35)/2)$

Experiment 1: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.699 \times 0.35$

Parameter	Initial	Estimate
δ	1.0000e+00	2.4596e+00
Cost	7.4796e+04	2.0524e+04
Time	0.0000e+00	3.0104e+01

Table 3: Parameter estimates obtained by maximum likelihood estimation.

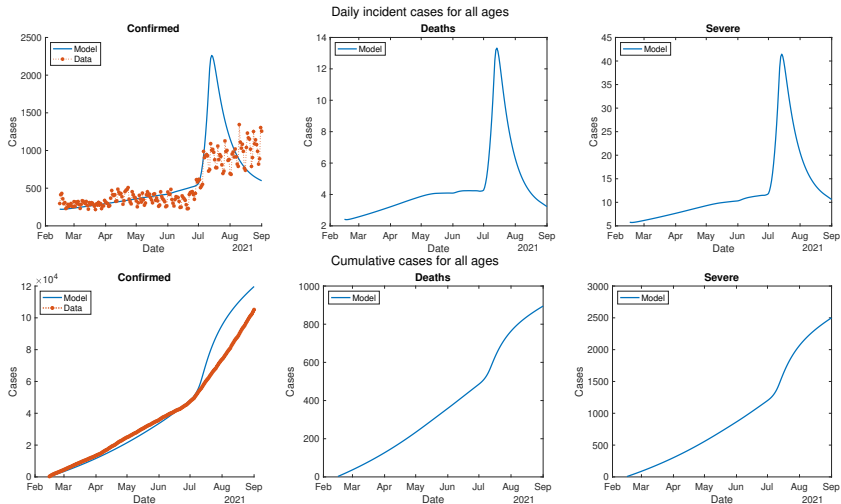Experiment 1: 1st stage = $\beta \times 1.8322$, 2st stage = $\beta \times 1.8322 \times 0.699 \times 0.35$

Figure 7: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 1: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.699 \times 0.35$
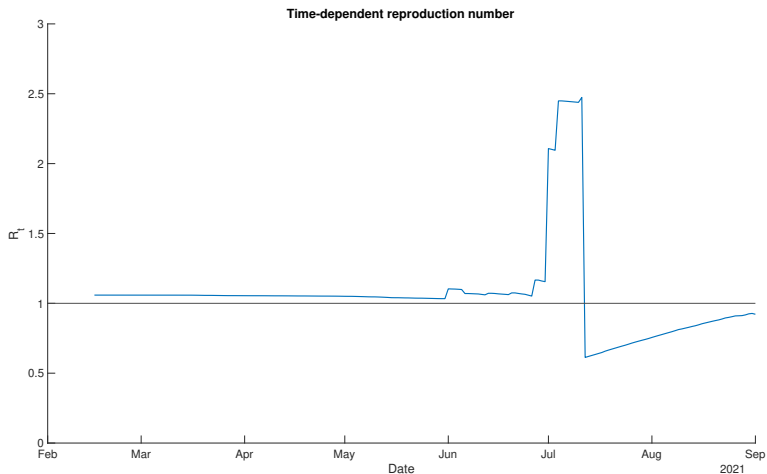

Figure 8: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 2: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.699$

Parameter	Initial	Estimate
δ	1.0000e+00	1.0000e+00
Cost	3.5653e+04	3.5653e+04
Time	0.0000e+00	7.8846e+00

Table 4: Parameter estimates obtained by maximum likelihood estimation.

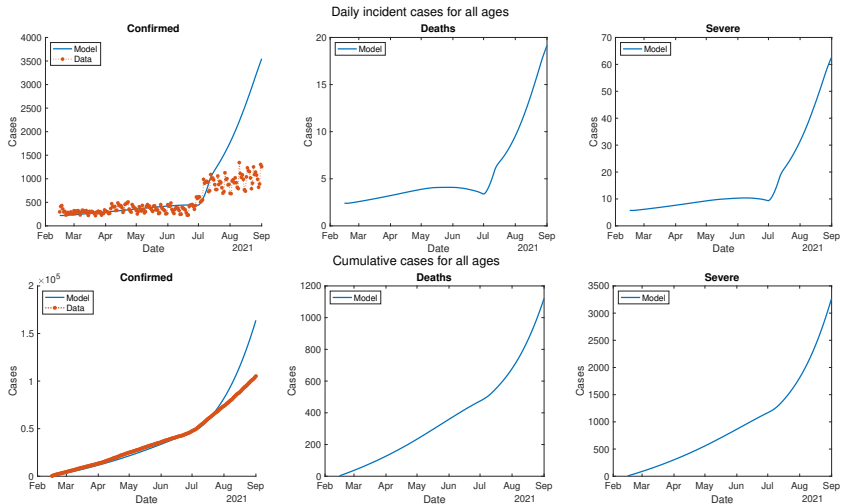Experiment 2: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.699$

Figure 9: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 2: 1st stage = $\beta \times 1.8322$, 2st stage = $\beta \times 1.8322 \times 0.699$
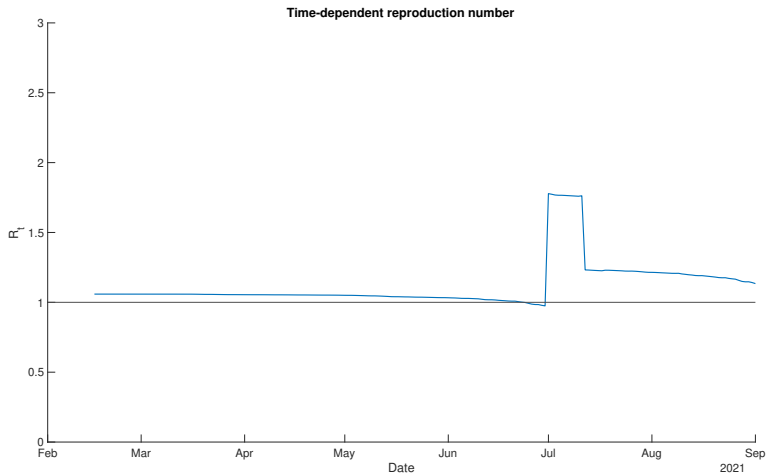

Figure 10: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 3: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.35$

Parameter	Initial	Estimate
δ	1.0000e+00	1.7928e+00
Cost	4.5014e+04	1.5073e+04
Time	0.0000e+00	2.7888e+01

Table 5: Parameter estimates obtained by maximum likelihood estimation.

Experiment 3: 1st stage = $\beta \times 1.8322$, 2st stage = $\beta \times 1.8322 \times 0.35$
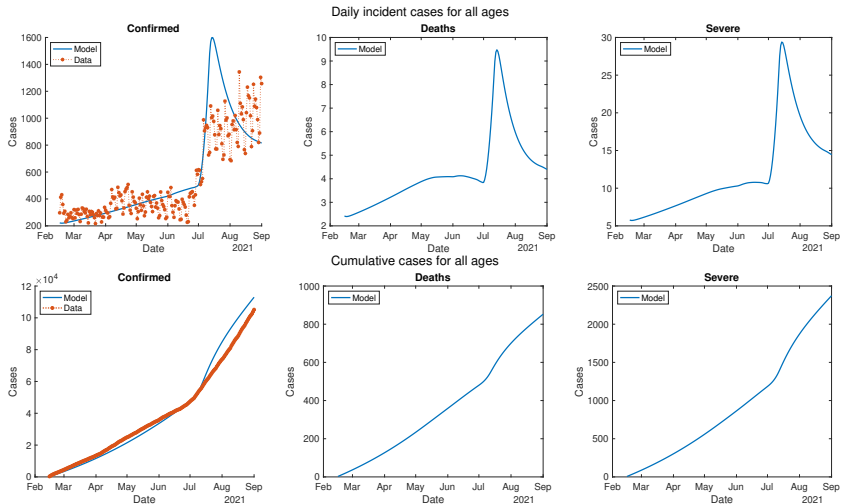

Figure 11: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 3: 1st stage = $\beta \times 1.8322$, 2st stage = $\beta \times 1.8322 \times 0.35$
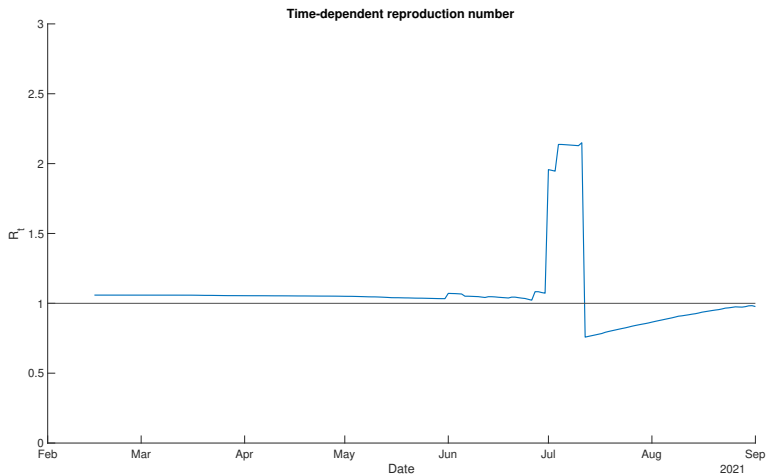

Figure 12: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 4: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.5245$

Parameter	Initial	Estimate
δ	1.0000e+00	1.1112e+00
Cost	1.4410e+04	1.3009e+04
Time	0.0000e+00	2.3580e+01

Table 6: Parameter estimates obtained by maximum likelihood estimation.

Experiment 4: 1st stage = $\beta \times 1.8322$, 2st stage = $\beta \times 1.8322 \times 0.5245$
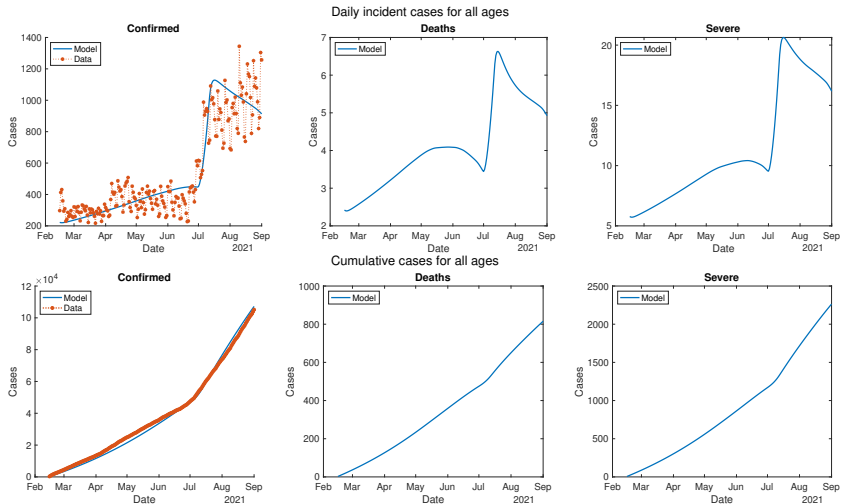

Figure 13: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

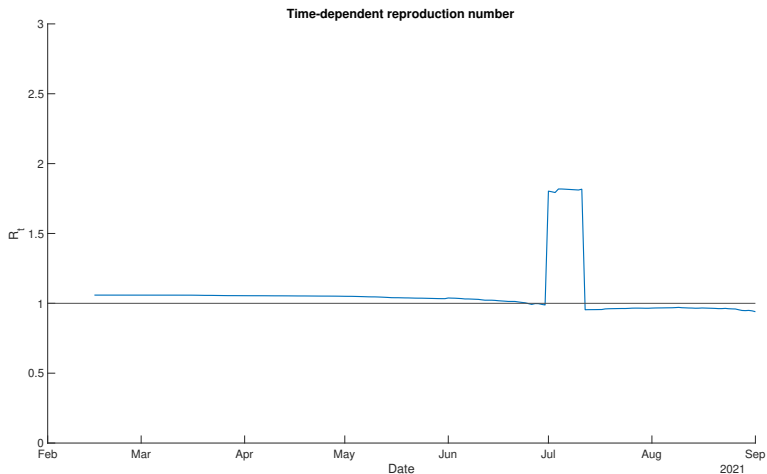Experiment 4: 1st stage $= \beta \times 1.8322$, 2st stage $= \beta \times 1.8322 \times 0.5245$

Figure 14: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 5: 1st stage $= \beta \times 1.4161$, 2st stage $= \beta \times 1.4161 \times 0.699 \times 0.35$

Parameter	Initial	Estimate
δ	1.0000e+00	3.4985e+00
Cost	1.0711e+05	1.9969e+04
Time	0.0000e+00	3.0754e+01

Table 7: Parameter estimates obtained by maximum likelihood estimation.

Experiment 5: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.699 \times 0.35$
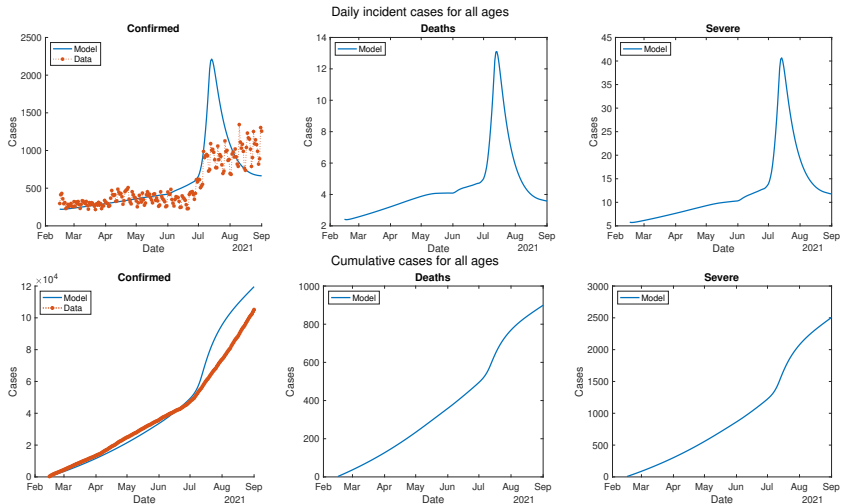

Figure 15: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 5: 1st stage $= \beta \times 1.4161$, 2st stage $= \beta \times 1.4161 \times 0.699 \times 0.35$
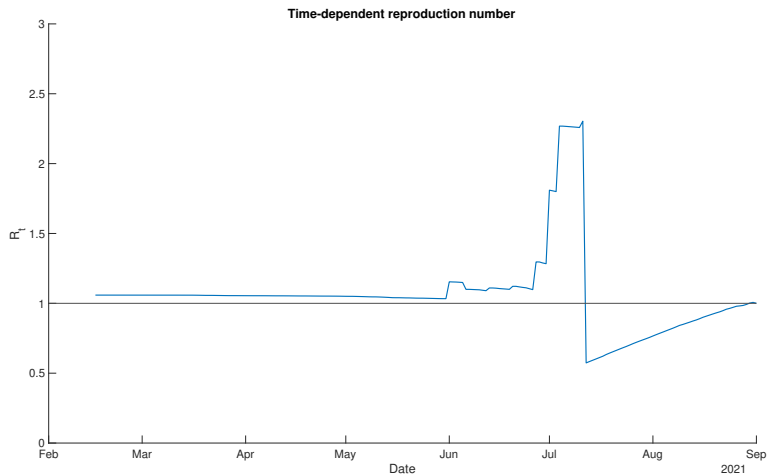

Figure 16: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 6: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.699$

Parameter	Initial	Estimate
δ	1.0000e+00	1.2532e+00
Cost	1.9417e+04	1.2841e+04
Time	0.0000e+00	2.6832e+01

Table 8: Parameter estimates obtained by maximum likelihood estimation.

Experiment 6: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.699$
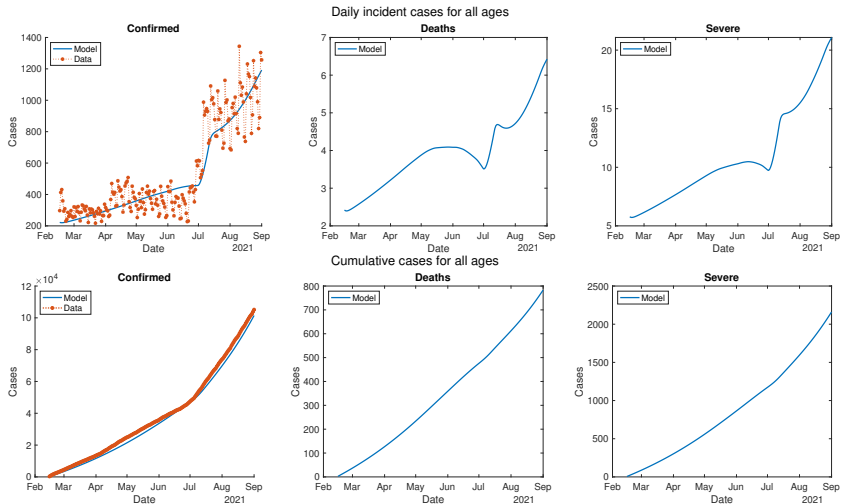

Figure 17: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

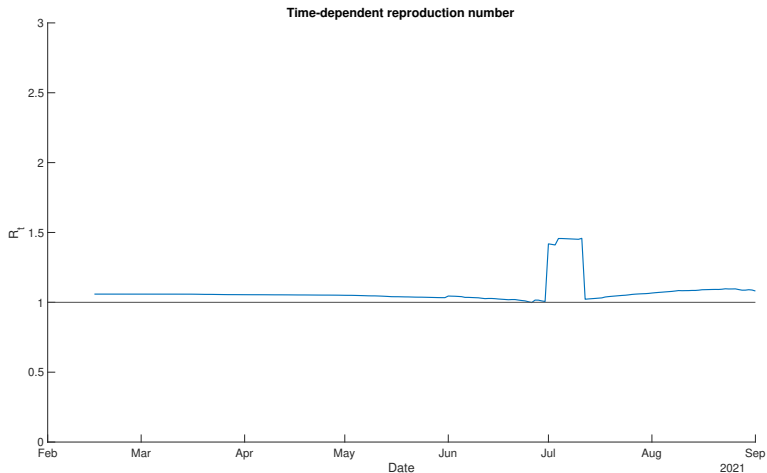Experiment 6: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.699$

Time-dependent reproduction number

Figure 18: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 7: 1st stage $= \beta \times 1.4161$, 2st stage $= \beta \times 1.4161 \times 0.35$

Parameter	Initial	Estimate
δ	1.0000e+00	2.6626e+00
Cost	8.3531e+04	1.4292e+04
Time	0.0000e+00	3.0413e+01

Table 9: Parameter estimates obtained by maximum likelihood estimation.

Experiment 7: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.35$
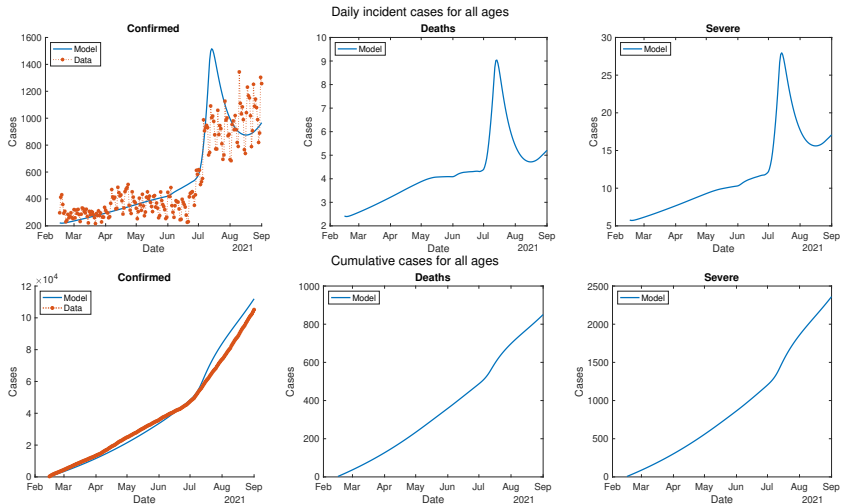

Figure 19: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 7: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.35$
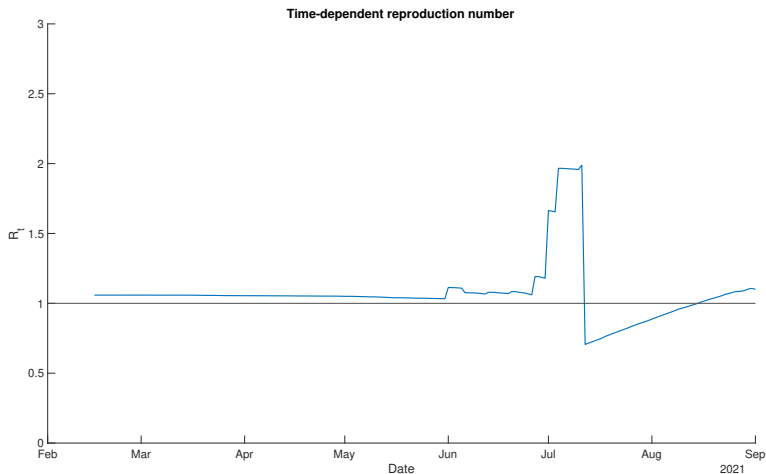

Figure 20: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Experiment 8: 1st stage $= \beta \times 1.4161$, 2st stage $= \beta \times 1.4161 \times 0.5245$

Parameter	Initial	Estimate
δ	1.0000e+00	1.7865e+00
Cost	4.5783e+04	1.2556e+04
Time	0.0000e+00	2.8626e+01

Table 10: Parameter estimates obtained by maximum likelihood estimation.

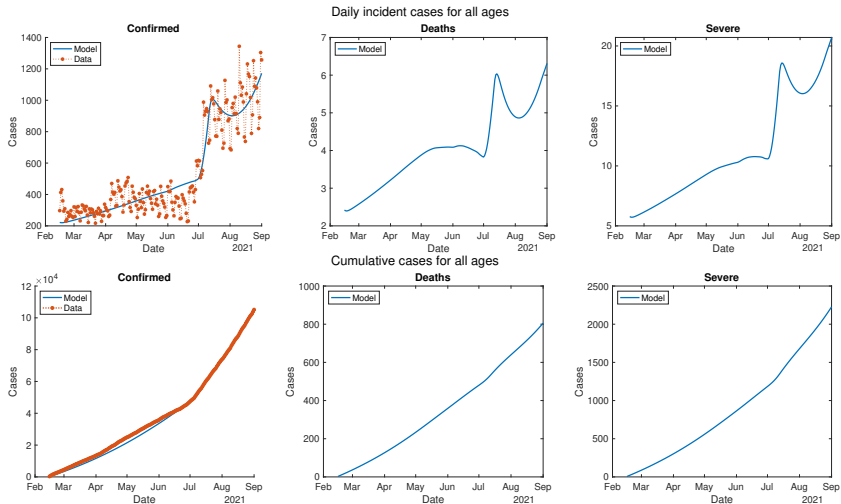Experiment 8: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.5245$

Figure 21: The model prediction and data for daily confirmed cases (top) and cumulative confirmed cases (bottom).

Experiment 8: 1st stage = $\beta \times 1.4161$, 2st stage = $\beta \times 1.4161 \times 0.5245$
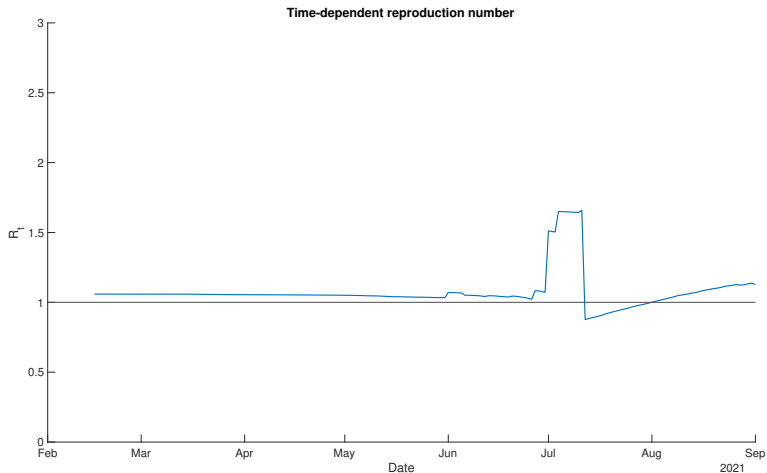

Figure 22: The estimated reproduction number from 2021/02/15 to 2021/09/01.

Conclusion

▶ 1st stage $= \beta \times 1.4161$, 2nd stage $= \beta \times 1.4161 \times 0.5245$가 δ값이나 피팅 면에서 가장 적합.

▶ 큰 δ 값이 나오기 위해서는 social distancing의 효과가 커야 함.

▶ Social distancing의 효과가 크면 산 형태의 dynamics가 나올 수 밖에 없음.

▶ 결론적으로 social distancing 효과와 δ 효과가 서로 compensate하는 관계