Amplitude Death in Directly Coupled Nonlinear Oscillators

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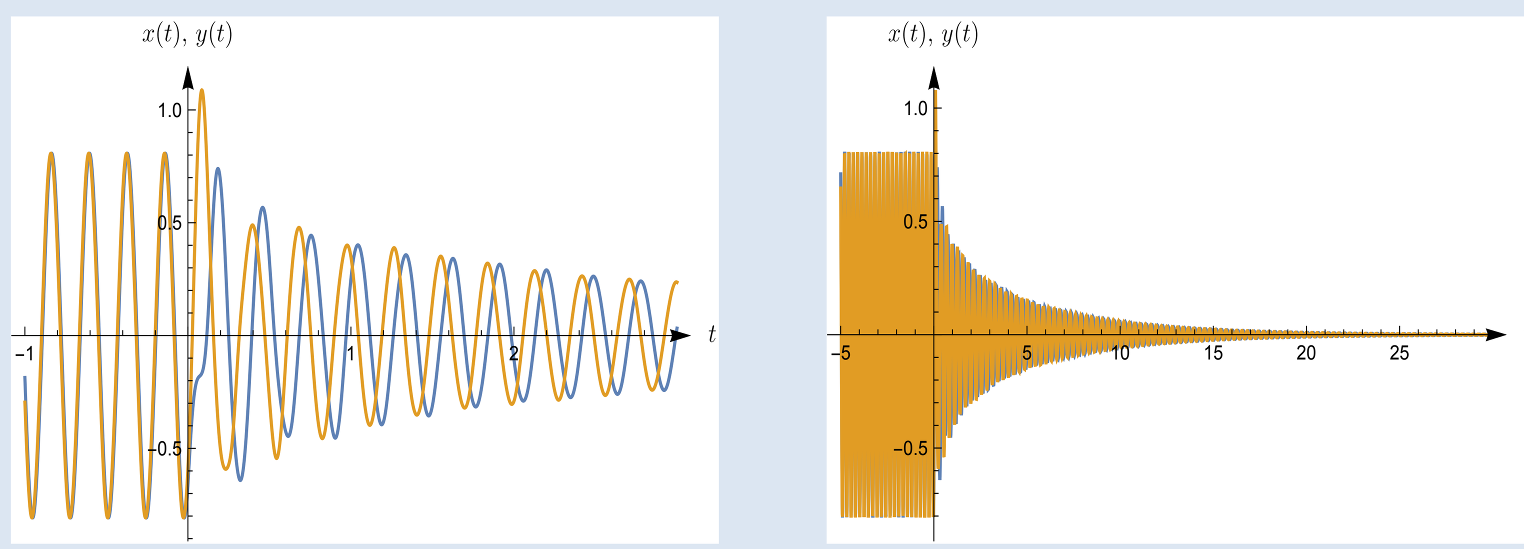
Abstract

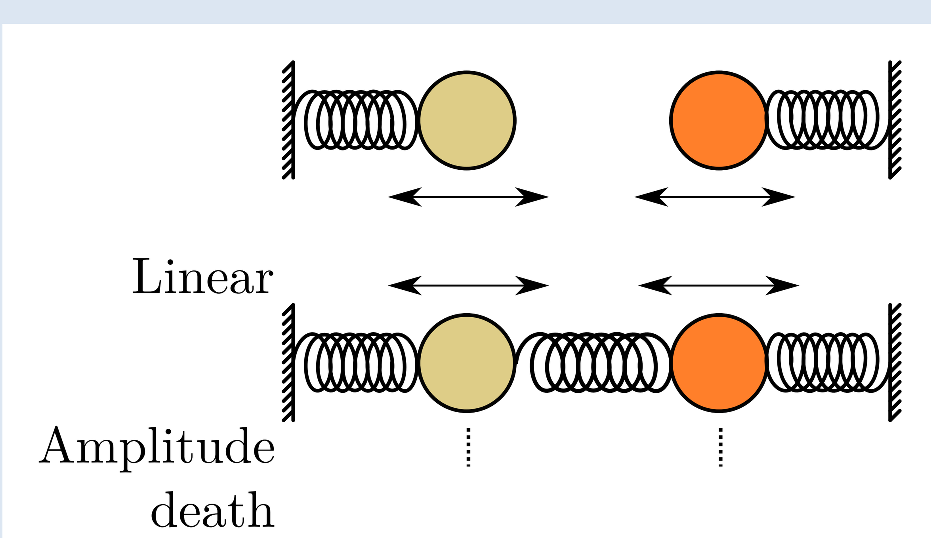
Amplitude death is a fascinating oscillation quenching phenomenon. Continuing a previous study on directly-coupled nonlinear oscillators [1], we demonstrate, both numerically and experimentally, that amplitude death can appear under strong direct coupling given the right choice of system parameters. We also noticed that small mismatches between two oscillators do not inhibit amplitude death.

What is Amplitude Death?



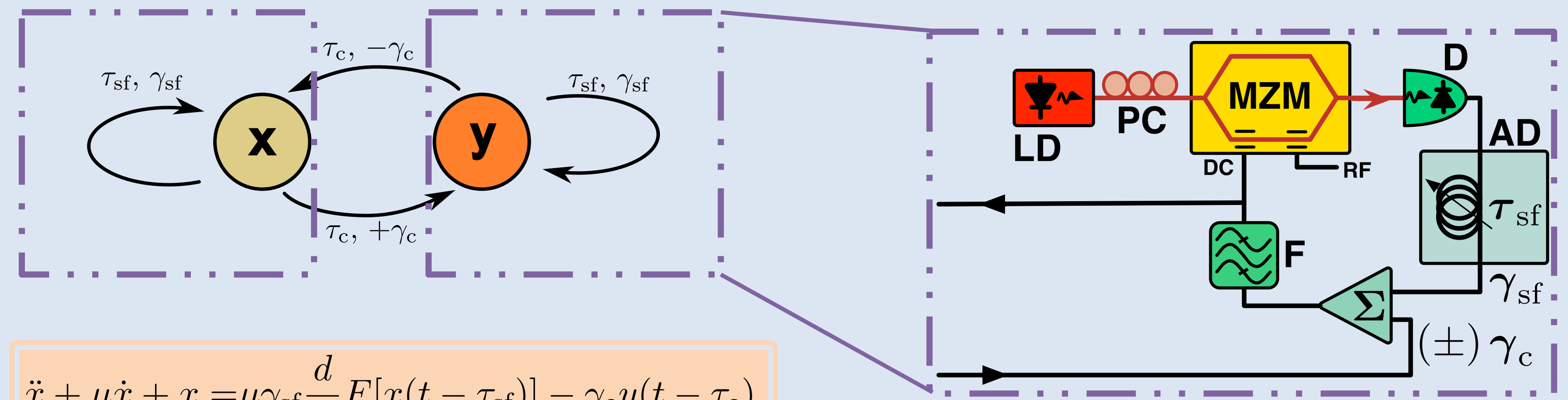
An example of an amplitude death in a two-oscillator network, coupling applied at t = 0.

Amplitude death is a type of **oscillation quenching**. It appears when the coupling pulls each oscillator from its original **limit cycle** (stable periodic oscillation) and forms a **stable steady state** for all oscillators [2]. As an example, shown in the figure above, two oscillators, originally undergoing self-oscillation individually, suppress each other's oscillation after coupling and reach a common signal-level.



- Undamped / driven damped harmonic oscillators: always oscillate
- Nonlinear oscillators: may have amplitude death

Setup of the System



$$\ddot{x} + \mu\dot{x} + x = \mu\gamma_{\text{sf}}\frac{d}{dt}F[x(t-\tau_{\text{sf}})] - \gamma_c y(t-\tau_c),$$
$$\ddot{y} + \mu\dot{y} + y = \mu\gamma_{\text{sf}}\frac{d}{dt}F[y(t-\tau_{\text{sf}})] + \gamma_c x(t-\tau_c),$$
$$F[x] = \cos^2(x+\phi)$$

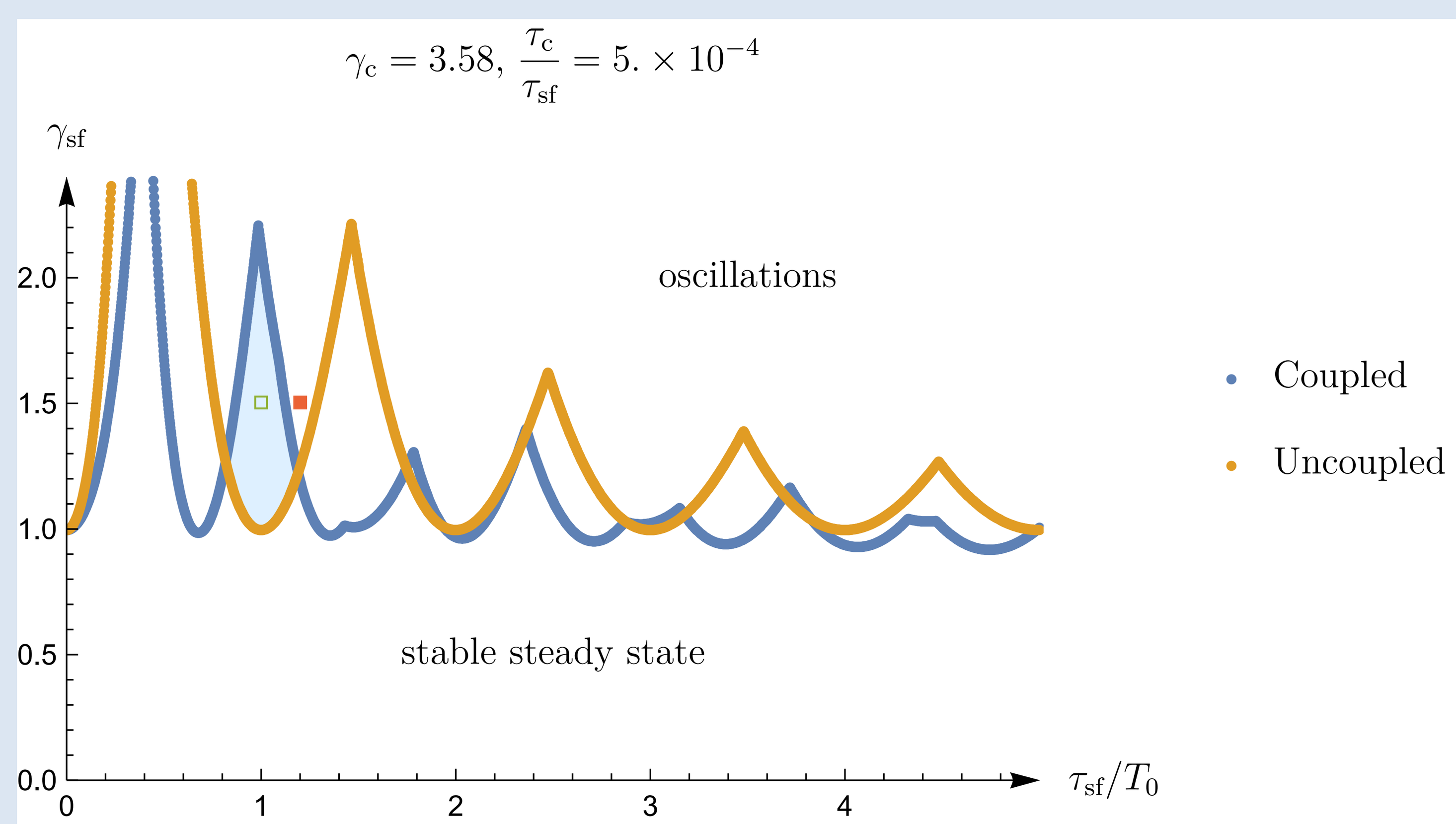
Parameters:

- μ: dimensionless bandwidth of the filter
- γ_{sf}: self-feedback gain ✓
- τ_{sf}: self-feedback delay ✓
- γ_c: coupling gain
- τ_c: coupling delay

Components:

- LD: Laser Diode
- PC: Polarization Control
- MZM: Mach-Zehnder Modulator
- D: Photodetector
- AD: Audio Delay
- Σ: Summing Amplifier
- F: Bandpass Filter

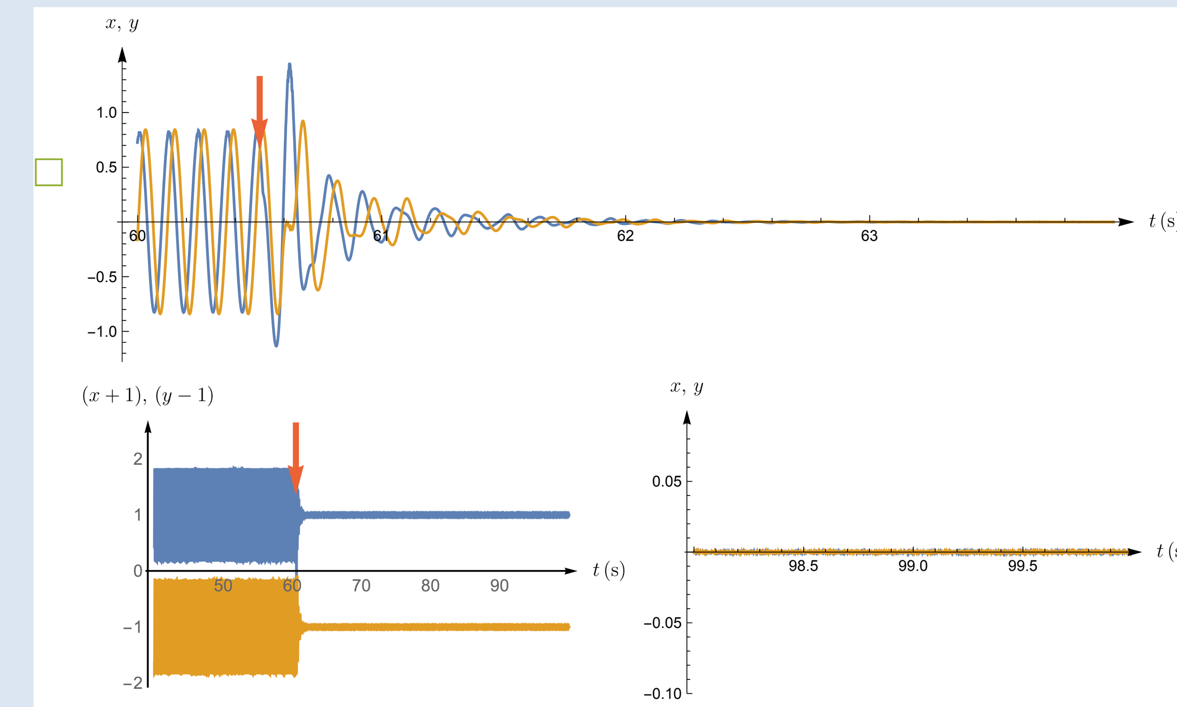
Numerical Bifurcation Analysis



Stability and bifurcation?

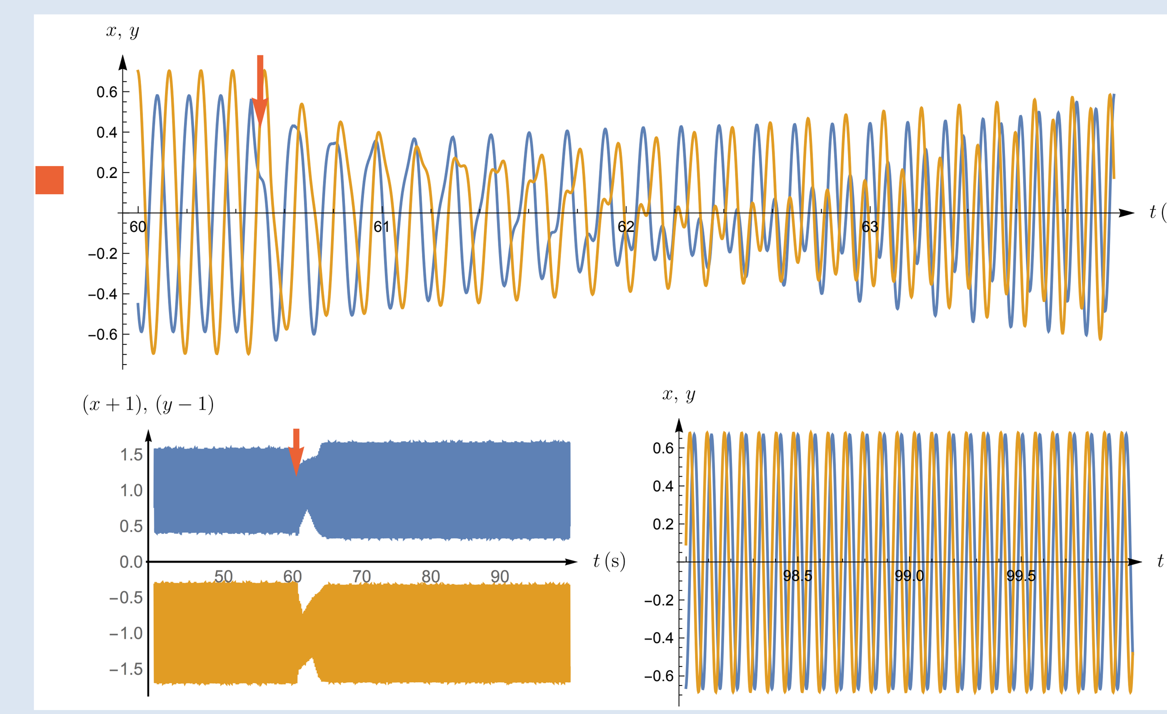
- **Stable steady state**: solution to a differential equation which remains unchanged from its initial condition and "attracts" nearby solutions.
- **Bifurcation**: Change of qualitative behavior of system as a system parameter is varied; here, change of steady state from stable to unstable and appearance of a stable periodic solution.

Experiment Data



Amplitude death:

Arrow indicates initiation of coupling



Non amplitude death:

Arrow indicates initiation of coupling

Conclusion and Acknowledgment

Under strong direct coupling, two nonlinear oscillators can exhibit amplitude death. From numerical simulations, we learn that amplitude death does not necessarily appear for all possible parameters. Experiments show that this oscillation quenching depends on self-feedback gain and self-feedback delay, and amplitude death can persist even under small mismatches in filter characteristics and gain parameters.

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References

[1] Illing, Lucas. "Amplitude death of identical oscillators in networks with direct coupling." *Phys. Rev. E* 94.2 (2016): 022215.

[2] Koseska, Aneta, Evgeny Volkov, and Jürgen Kurths. "Oscillation quenching mechanisms: Amplitude vs. oscillation death." *Physics Reports* 531.4 (2013): 173-199.