ECON 8022/4422
Non-Walrasian Markets: Search and Matching Frictions and Unemployment

References: Read Chapter 6 in Ljungqvist and Sargent, and, Chapter 18 in Miao. You are advised to try these problems below seriously as they will appear in the exam.

Problem 1 Consider again the search and matching model discussed in lectures *à la* Mortensen and Pissarides. Suppose now workers have to pay a proportional tax on labour income. That is, if a worker's earnings per period when employed is w, his after-tax earnings will be $(1-\tau)\,w$ where $\tau \in [0,1)$ is the tax rate. Suppose the aggregate matching function is $M(u,v) = Au^{\alpha}v^{1-\alpha}$ as explained in class. Worker's and firms determine wages via Nash bargaining, where the worker's bargaining strength is ϕ. *Focus your analysis only on the steady state*:

1. Derive the Beveridge curve relationship between unemployment and vacancies for our assumption on the matching function. Verify that this relationship is convex to the origin in (u,v) space.

2. Write down the firm's optimal value function in terms of a Bellman equation and derive the relevant job-creation function.

3. Write down the worker's optimal value function in terms of a Bellman equation. Define the firm and worker match surpluses and derive the relevant wage-setting curve under Nash bargaining.

4. Define the steady-state equilibrium in terms of (u,θ,w). Compare the steady-state equilibria when $\tau = 0$ with $\tau \in (0,1)$. Is unemployment higher or lower when $\tau \in (0,1)$? Depict your results using relevant diagrams.

Problem 2 *This not explicitly covered in lectures but is just another application of dynamic programming (DP). This is a good test to see if you have really understood DP ideas in a new application. Note that in this application, the DP problem involves a discrete action set, instead of a continuous one like in our growth, RBC, NK and Mortensen-Pissarides models.*
Mr. Borat Sagdiyev who is currently unemployed draws one wage offer w from a fixed probability distribution function $F(w)$ each period he is unemployed. If he accepts the job offer w he remains employed forever at w. $F(w)$ has the properties: $F(0) = 0$, and there exists $B < +\infty$ such that $F(B) = 1$. If Mr. Sagdiyev rejects the offer, he receives unemployment compensation $b \geq 0$ and waits for the next period to draw a new offer. He seeks to maximize the expected discounted value of his earnings, where per period earnings is w_t. His discount factor is $\delta \in (0,1)$.

1. Write down Mr. Sagdiyev's Bellman equation.

2. Denote Mr. Sagdiyev's optimal reservation wage as $\overline{w}$. Show that his optimal decision rule (fixed point solution of this Bellman equation) is of a reservation wage form, $\overline{w}$. What is the expression for this reservation wage, $\overline{w}$?

3. Consider solving Mr. Sagdiyev's Bellman equation using the method of value function iteration.

   (a) Prove that there exists a unique value function solving the Bellman equation. Show that it is continuous on $[0,B]$ and is non-decreasing.

(b) Show that the Bellman equation fixed-point problem can be characterized in terms of the reservation wage form.

(c) Show that for any initial guess of the reservation wage solution $\overline{w}_0$, the Bellman equation defines a δ-contraction on the set $[0, B]$. That is, show that $\overline{w}_n \to \overline{w}$ at modulus $\delta < 1$.

The following extended exercises are for the motivated student. Try this on your own and come and discuss your work with your tutor when you're ready.

Problem 3 (Optional practice.) Do Exercises 6.2, 6.3 and 6.4 in LS.

Problem 4 (More practice.) Do Exercises 1, 2 and 3 from Miao's Chapter 18.