Further Mathematics: FS1 Topic Questions

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Contents

1. Preface

This is a set of topic questions collected from numerous past papers and exercise sheet solutions designed for further mathematics specification in Edexcel's UK A Levels from 2017.

Most questions were selected from official past papers, but not limited to the Edexcel exam board. In fact I would like to avoid use of official UK Edexcel past paper questions from 2018 onwards as they can be used for mocks and would be a valuable source of pre-exam practices. Most questions here are coming from past papers from other exam boards (AQA, OCR, Edexcel IAL, CAIE, AQA International, WJEC, Cambridge Pre-U, etc.) on the same topics as the Edexcel syllabus, or pre-2017 Edexcel questions, as well as other sources online. References to the original source of questions will be included.

For AS students using this material: some of the questions even on AS topics would include A2-only level questions due to the difference between examination specification and the textbook contents. I will try to mark the "A2-only" questions explicitly but there may be questions I'm mismarking or missing. In such cases please let me (or your further maths teacher) know and we can mark the relevant questions. A more convenient way is to use your Github to access this repository and **create a pull request**.

For the situation above and if you find any other mistakes, or you just want to add the questions, or complain about something, you can also **create a pull request** on Github directly from this project's source repository at https://github.com/YunkaiZhang233/a-level-further-maths-topic-questions-david-game.

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2. Discrete Probability Distributions

Examinable Contents

- Calculation of the mean and variance of discrete probability distributions.
- Extension of expected value function to include $E(g(X))$

2.1. Edexcel IAL June 2018, S1, Q4

A discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2 - x) x = 0, 1 \\ k(3 - x) x = 2, 3 \\ k(x + 1) x = 4 \\ 0 \text{ otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$

(2 marks)

Find the exact value of

(b) $P(1 \leq X < 4)$

(1 mark)

(c) $E(X)$

(2 marks)

(d) $E(X^2)$

(2 marks)

(e) $\mathrm{Var}(3X + 1)$

(3 marks)

2.2. Edexcel IAL S1, Specimen 2018, Q3

The discrete random variable X has probability distribution

$$P(X = x) = \frac{1}{5} \qquad\qquad x = 1, 2, 3, 4, 5$$

(a) Write down the name given to this distribution

(1 mark)

Find

(b) $P(X = 4)$

(1 mark)

(c) $P(X \leq 3)$

(1 mark)

(d) $P(3X - 3 > X + 4)$

(2 marks)

(e) $E(X)$

(1 mark)

(f) $E(X^2)$

(2 marks)

(g) Hence find $\mathrm{Var}(X)$

(2 marks)

Given that $E(aX - 3) = 11.4$

(h) Find $\mathrm{Var}(aX - 3)$

(4 marks)

2.3. OCR (A) AS Further Stat Y532/01, Nov 2021, Q1

The discrete random variable A has the following probability distribution.

a	1	2	5	10	20
$P(A = a)$	0.3	0.1	0.1	0.2	0.3

(a) Find the value of $E(A)$.

(2 marks)

(b) Determine the value of $\mathrm{Var}(A)$.

(3 marks)

(c) The variable A represents the value in pence of a coin chosen at random from a pile. Mia picks one coin at random from the pile. She then adds, from a different source, another coin of the same value as the one that she has chosen, and one 50p coin.

- (i) Find the mean value of the three coins

(2 marks)

- (ii) Find the variance of the value of the three coins.

(1 mark)

2.4. OCR (MEI) AS Further Stat a Y412, Specimen, Q4

The discrete random variable X has probability distribution defined by

$$P(X = r) = k(2r - 1) \text{ for } r = 1, 2, 3, 4, 5, 6, \text{where } k \text{ is a constant}$$

(i) Complete the table below giving the probabilities in terms of k.

r	1	2	3	4	5	6
$P(X = r)$						

(1 mark)

(ii) Show that the value of k is $\frac{1}{36}$.

(2 marks)

(iii) Draw a diagram to illustrate the distribution.

(2 marks)

(iv) **In this question you must show detailed reasoning.** Find:
- $E(X)$
- $\mathrm{Var}(X)$

(5 marks)

A game consists of a player throwing two fair dice. The score is the maximum of the two values showing on the dice.

(v) Show that the probability of a score of 3 is $\frac{5}{36}$.

(2 marks)

(vi) Show that the probability distribution for the score in the game is the same as the probability distribution of the random variable X.

(3 marks)

(vii) The game is played three times. Then find:
- the mean of the total of the three scores.
- the variance of the total of the three scores.

(3 marks)

3. Poisson & Binomial Distributions

Examinable Contents
- The Poisson distribution and its additive properties
- The mean and variance of the Binomial Distribution and the Poisson distribution
- The use of the Poisson distribution as an approximation to the binomial distribution.

4. Geometric and Negative Binomial Distributions

Examinable Contents *All topics A2 only.*
- Geometric and negative binomial distributions.
- Mean and variance of a geometric distribution with parameter p
- Mean and variance of negative binomial distribution with $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

5. Hypothesis Testing

Examinable Contents

- Extend ideas of hypothesis tests to test for the mean of a Poisson distribution
- (A2 only) Extend hypothesis testing to test for the parameter p of a geometric distribution.

5.1. Edexcel IAL S2, June 2021, Q2

Luis makes and sells rugs. He knows that faults occur randomly in his rugs at a rate of 3 every 4mm^2.

(a) Find the probability of there being exactly 5 faults in one of his rugs that is 4m^2 in size.

(2 marks)

(b) Find the probability that there are more than 5 faults in one of his rugs that is 6m^2 in size.

(2 marks)

Luis makes a rug that is 4m^2 in size and finds it has exactly 5 faults in it.

(c) Write down the probability that the next rug that Luis makes, which is 4m^2 in size, will have exactly 5 faults. Give a reason for your answer.

(2 marks)

A small rug has dimensions 80 cm by 150 cm. Faults still occur randomly at a rate of 3 every 4m^2.

Luis makes a profit of £80 on each small rug he sells that contains no faults but a profit of £60 on any small rug he sells that contains faults.

Luis sells n small rugs and expects to make a profit of at least £4000.

(d) Calculate the minimum value of n

(4 marks)

Luis wishes to increase the productivity of his business and employs Rhiannon. Faults also occur randomly in Rhiannon's rugs and independently to faults made by Luis. Luis randomly selects 10 small rugs made by Rhiannon and finds 13 faults.

(e) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the rate at which faults occur is higher for Rhiannon than for Luis. State your hypotheses clearly.

(5 marks)

6. Central Limit Theorem

Examinable Contents *All topics A2 only.*

- Applications of the central Limit Theorem to other distributions.

7. Chi-Squared Tests

Examinable Contents

- Goodness of fit tests and Contingency Tables
- The null and alternative hypotheses.
- The use $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic.
- Degrees of freedom

7.1. Edexcel IAL Statistics 3, June 2022, Q7

The following table shows observed frequencies, where x is an integer, from an experiment to test whether or not a six-sided die is biased.

Number on die	1	2	3	4	5	6
Observed frequency	$x + 6$	$x - 8$	$x + 8$	$x - 5$	$x + 4$	$x - 5$

A goodness of fit test is conducted to determine if there is evidence that the die is biased.

(a) Write down suitable null and alternative hypotheses for this test.

(1 mark)

It is found that the null hypothesis is not rejected at the 5% significance level.

(b) Hence

- (i) find the minimum value of x

(8 marks)

- (ii) determine the minimum number of times the die was rolled.

(2 marks)

7.2. Edexcel IAL S3, Jan 2021, Q3

The students in a group of schools can choose a club to join. There are 4 clubs available: Music, Art, Sports and Computers. The director collected information about the number of students in each club, using a random sample of 88 students from across the schools. The results are given in Table 1 below.

	Music	Arts	Sports	Computers
Number of students	14	28	27	19

The director uses a chi-squared test to determine whether or not the students are uniformly distributed across the 4 clubs.

(a)

(i) Find the expected frequencies he should use.

Given that the test statistic he calculated was 6.09 (to 3 significant figures)

(ii) use a 5% level of significance to complete the test. You should state the degrees of freedom and the critical value used.

(4 marks)

The director wishes to examine the situation in more detail and takes a second random sample of 88 students. The director assumes that within each school, students select their clubs independently. The students come from 3 schools and the distribution of the students from each school amongst the clubs is given in Table 2 below.

School	Music	Arts	Sports	Computers
School A	3	10	9	8
School B	1	11	13	5
School C	11	6	7	4

The director wishes to test for an association between a student's school and the club they choose.

(b) State hypotheses suitable for such a test.

(1 mark)

(c) Calculate the expected frequency for School C and the Computers club.

(1 mark)

The director calculates the test statistic to be 7.29 (to 3 significant figures) with 4 degrees of freedom.

(d) Explain clearly why his test has 4 degrees of freedom.

(2 marks)

(e) Complete the test using a 5% level of significance and stating clearly your critical value.

(2 marks)

8. Probability Generating Functions

Examinable Contents *All topics A2 only.*

- Definitions, derivations and applications.
- Use of the probability generating function for the negative binomial, geometric, binomial and Poisson distributions.
- Use to find the mean and variance.
- Probability generating function of the sum of independent random variables.

8.1. Cambridge Pre-U 9795/02, June 2022, Q2

(a) The random variable U has the distribution of $\text{Po}(\lambda)$. Show that the probability generating function of U is $e^{\lambda(t-1)}$.

(4 marks)

(b) The random variable V has probability generating function $G(t)$. It is given that $E(V) = 3$ and $\text{Var}(V) = 6.75$.

- (i) Find the values of $G(1)$, $G'(1)$, and $G''(1)$.

(4 marks)

- (ii) Find the mean and variance of the random variable with probability generating function $[G(t)]^4$.

(2 marks)

8.2. CAIE 9321/42 FM Paper 4, June 2022 (v2), Q2

The probability generating function, $G_Y(t)$, of the random variable Y is given by

$$G_Y(t) = 0.04 + 0.2t + 0.37t^2 + 0.3t^3 + 0.09t^4$$

(a) Find $\text{Var}(Y)$.

(4 marks)

The random variable Y is the sum of two independent observations of the random variable X.

(b) Find the probability generating function of X, giving your answer as a polynomial in t.

8.3. Cambridge Pre-U 9795/02, Specimen Paper 2, Q1

The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = at\left(t + \frac{1}{t}\right)^3$$

where a is a constant.

(a) Find, in either order, the value of a and the set of values that X can take.

(4 marks)

(b) Find the value of $E(X)$.

(2 marks)

8.4. CAIE 9321/43 FM Paper 4, Nov 2020 (v3), Q5

Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X.

(1 mark)

Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is 13. The corresponding probability for the third coin is 14. The number of heads obtained by Hassan when he tosses these three coins is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y.

(3 marks)

The random variable Z is the total number of heads obtained by Keira and Hassan.

(c) Find the probability generating function of Z, expressing your answer as a polynomial.

(3 marks)

(d) Use the probability generating function of Z to find $E(Z)$.

(2 marks)

(e) Use the probability generating function of Z to find the most probable value of Z.

(1 mark)

9. Quality of Tests

Examinable Contents *All topics A2 only.*
- Type I and Type II errors.
- Size and Power of Test.
- The power function.