Lab Project

February 22, 2018

Consider a type of nuclear material used in power plant reactors. There are 10 attributes of this material that vary with reactor type and the amount of time that the material spent in the rector. This gives us 30 curves, $g_i(k,t)$, $i = 1, ..., 10$ (attribute), $k = 1, 2, 3$ (type of reactor), $t \in [0, B]$ for some unknown B (time spent in the reactor). See the pictures of the board for visualization.

Suppose that some material is stolen and made into a bomb. This bomb is then detonated and the 10 attributes are measured at the blast site. This is our data, $Y \in \mathbb{R}^{10}$. Our goal is to infer the value of k (the type of reactor from which the material what stolen) and t (the amount of time the material spent in the reactor). Let k^* and t^* be the true values of k and t respectively. Our model is as follows.

$$Y = \sum_{k=1}^{3} \alpha_k g(k,t) + \varepsilon$$

Where $g(k,t) = (g_1(k,t), ..., g_{10}(k,t))^T$, α_k is 1 if $k = k^*$, and $\varepsilon \sim N(0, \sigma^2 I)$. That is, the ith component of Y is equal to $g_i(k^*, t^*)$ plus some normal error. For now we can treat σ^2 as known. It might be interesting to allow for independent but heteroscedastic errors, but that would add unnecessary complication for now.

We want to use a Bayesian approach. It makes sense to use $K \sim Unif(\{1,2,3\})$ and $T \sim Unif[0, B]$ as priors. We should probably treat B as known for now and think about a hyperprior for it later.

We now have a likelihood and priors for our parameters of interest. We can hopefully do some sort of MCMC to get posteriors, $\pi(k|Y)$ and $\pi(t|Y)$. From there we can get credible intervals for k and t (marginal for k, conditional for t), which can surely be used to do something useful.